Gravitational Lensing

Sut-Ieng Tam 譚雪瑩 (NYCU)

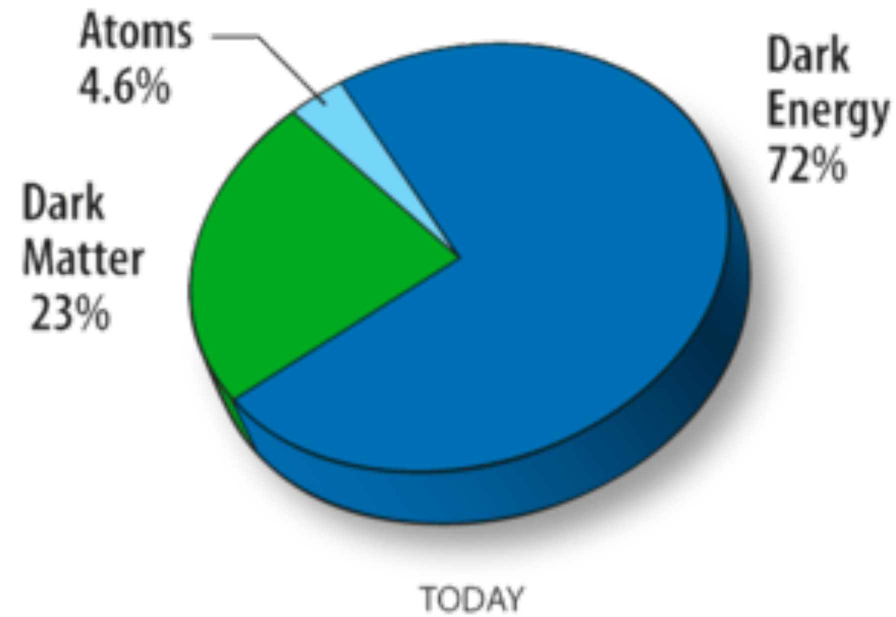
Image Credit: NASA, ESA, A. Koekemoer, M. Jauzac, C. Steinhardt, and the BUFFALO team

Outline

- ΛCDM model
- Introduction to Gravitational Lensing
- Strong Lensing & its application
- Weak Lensing & its application
- Microlensing & its application
- Studying DM collision-less nature with GL?
- Probing FDM with GL?

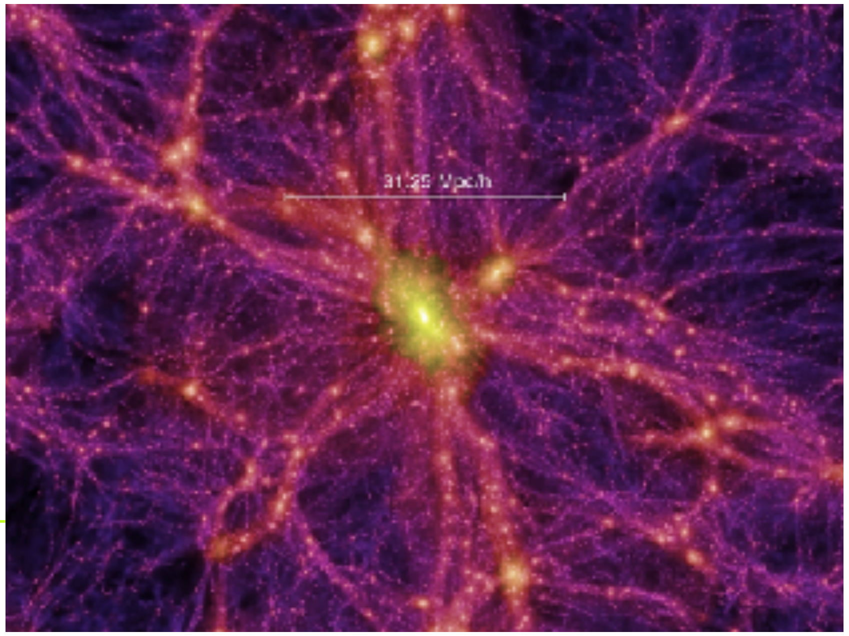
Standard Model-Lambda CDM

- Lambda CDM Λ-冷暗物質模型
  Λ：a cosmological constant associated with dark energy
  CDM：cold dark matter
- Dark Matter: A form of matter that does not emit, absorb, or reflect light, making it invisible to traditional telescopes and other electromagnetic radiation detectors.
- Gravitational Influence
  —> gravitational lensing
  —> use this to detect dark matter



NASA / WMAP Science Team

Millennium Simulation



Einstein's General Relativity

- Describe how the space-time affect by mass and energy
- Gravity corresponds to the curvature of spacetime caused by the presence of mass and energy.
  —> influences the motion of objects and the propagation of light, manifesting as the force of gravity.
- Einstein Field Equation:

energy-momentum tensor

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Space-time curvature

Matter tells spacetime how to curve,
spacetime tells matter how to move.

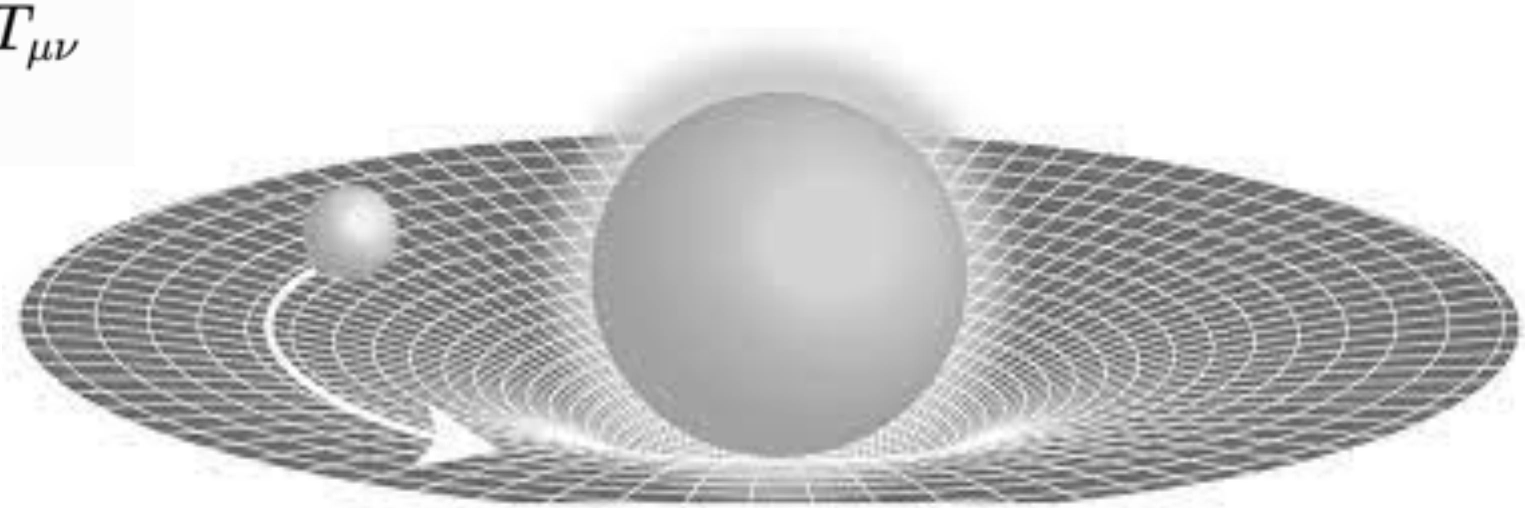
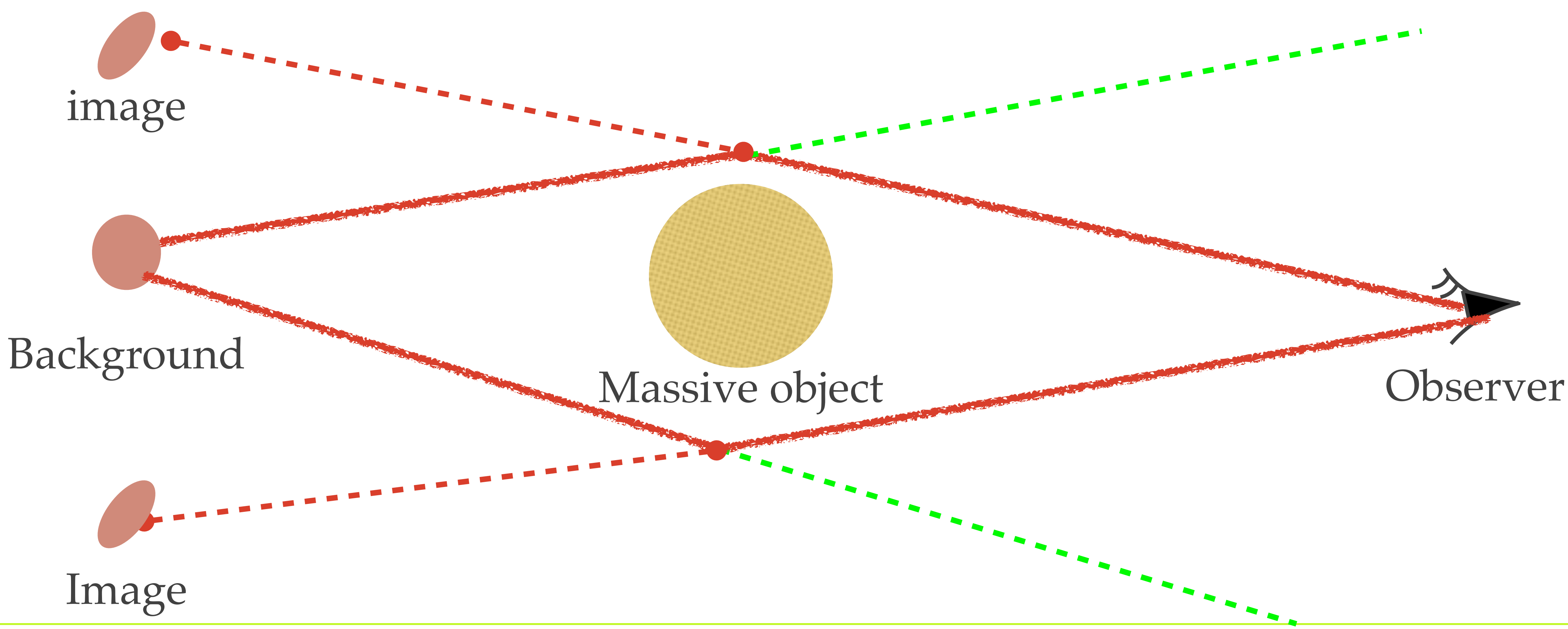


Photo Credit: PanSci 泛科學

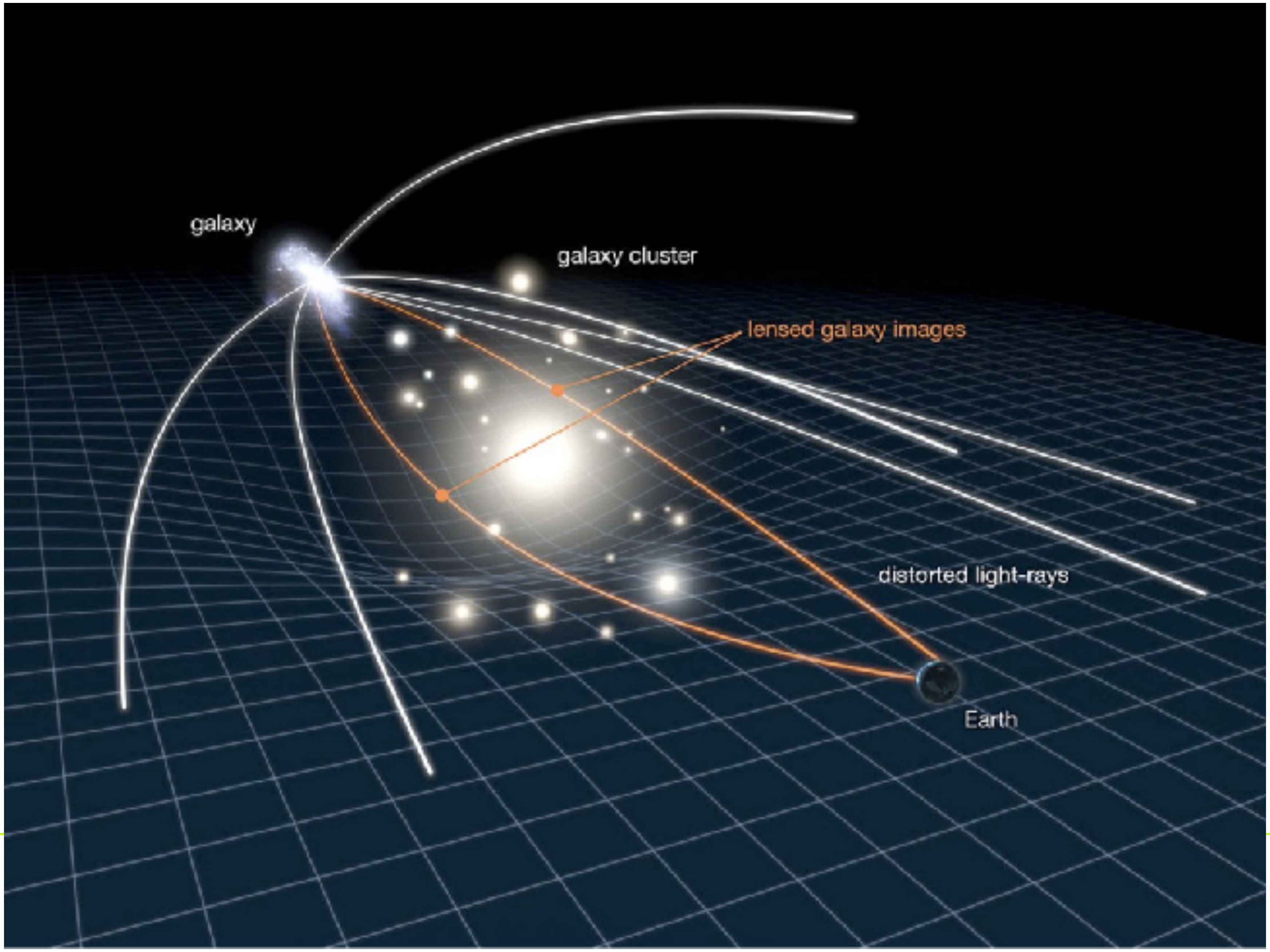
Gravitational Lensing 重力透鏡

- A natural telescope in our Universe
- According to Einstein's General Relativity, the presence of mass and energy in the universe can distort the space-time
- Light travels through the distorted space-time —> the bending of light paths



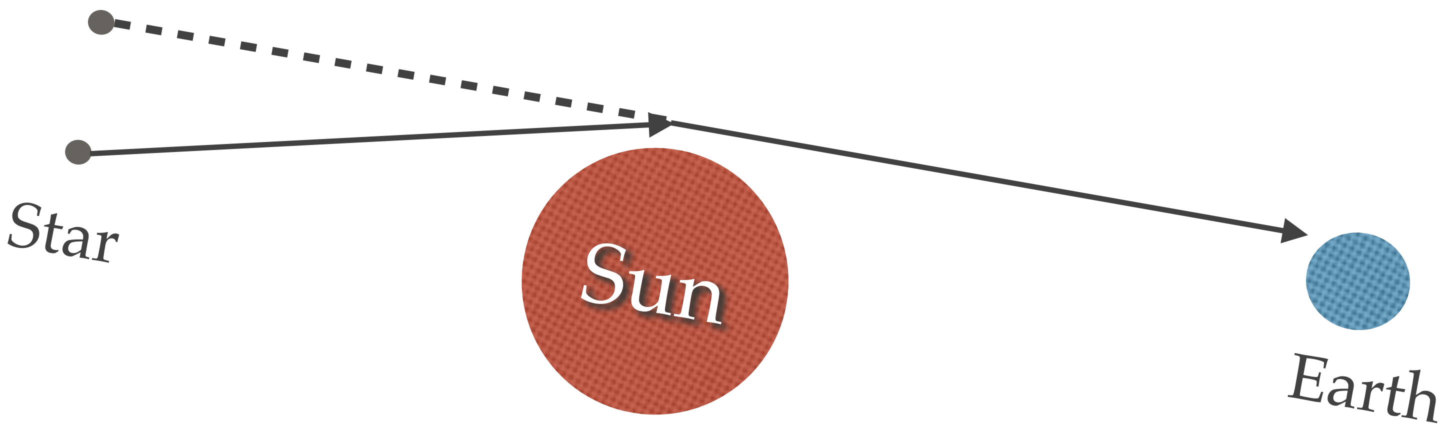
Gravitational Lensing

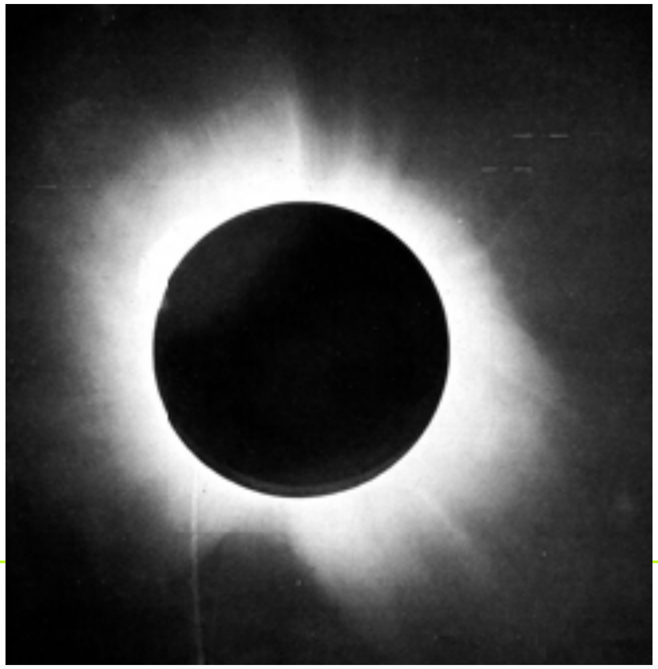
- The light ray of the background source is bended, leading to the distorted shape of its image.



First time to observe gravitational lensing effect

- In 1919, Sir Arthur Eddington led an expedition to measure the apparent shift in the positions of stars near the Sun.
- The Sun's gravity can produce observable bending, but its intense light can obscure nearby starlight. Therefore, a solar eclipse provided the best opportunity to verify this theory at the time.
- This shift was consistent with the predictions of Einstein's General Relativity —> first confirmation of the General Relativity and offering evidence gravitational lensing effect.



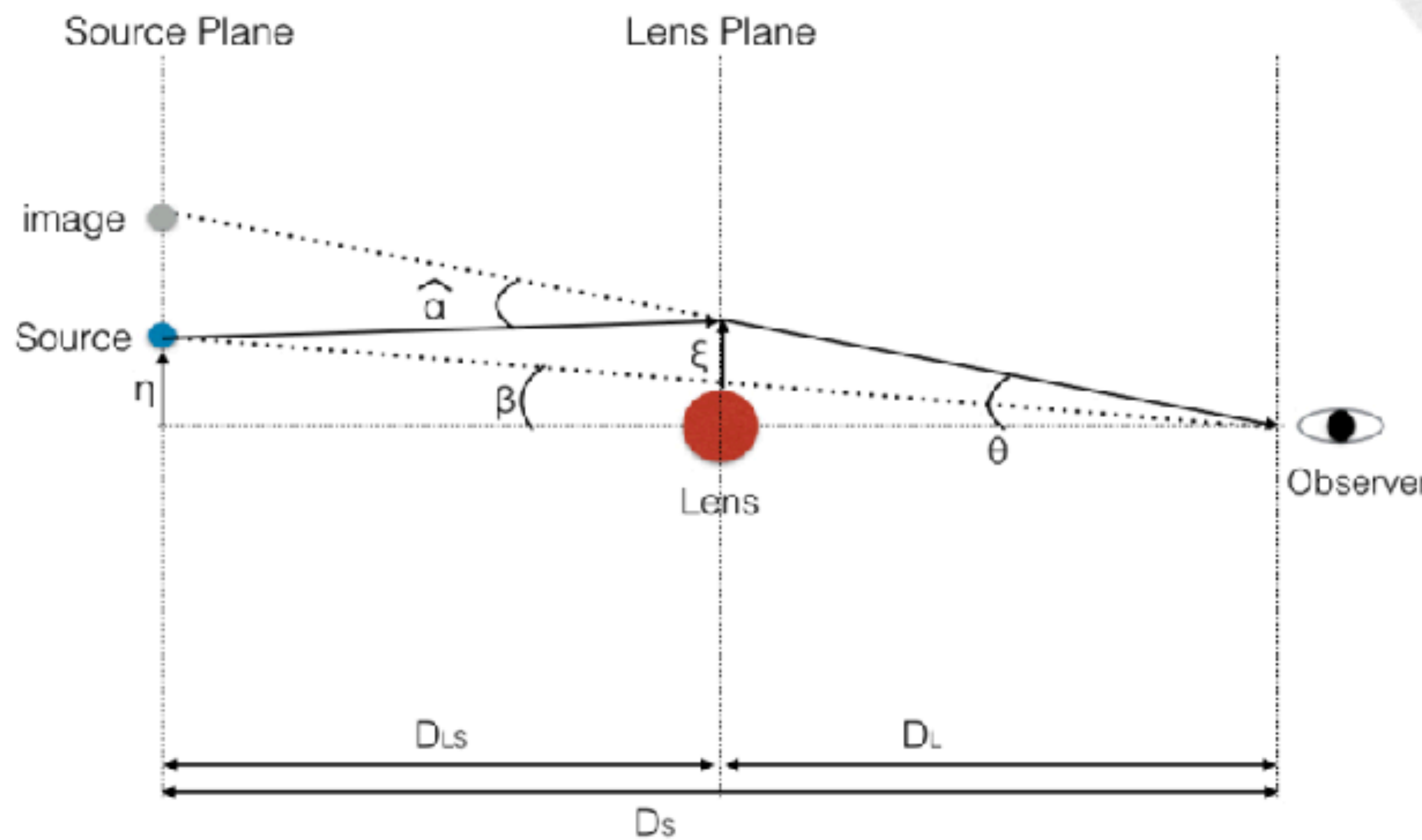


F. W. Dyson, A. S. Eddington, and C. Davidson (1920)

Lens Equation

$$\beta = \theta - \alpha(\theta)$$

$$\alpha(\theta) = \frac{D_{ds}}{D_s}\hat{\alpha}(D_d\theta)$$



- Mapping between source position β and image position θ.
- Deflection angle α is determined by the mass distribution of the lens.
- For point mass, $\hat{\alpha} = \frac{4GM}{c^2 b}$. (twice the Newtonian prediction, confirmed by the 1919 solar eclipse)

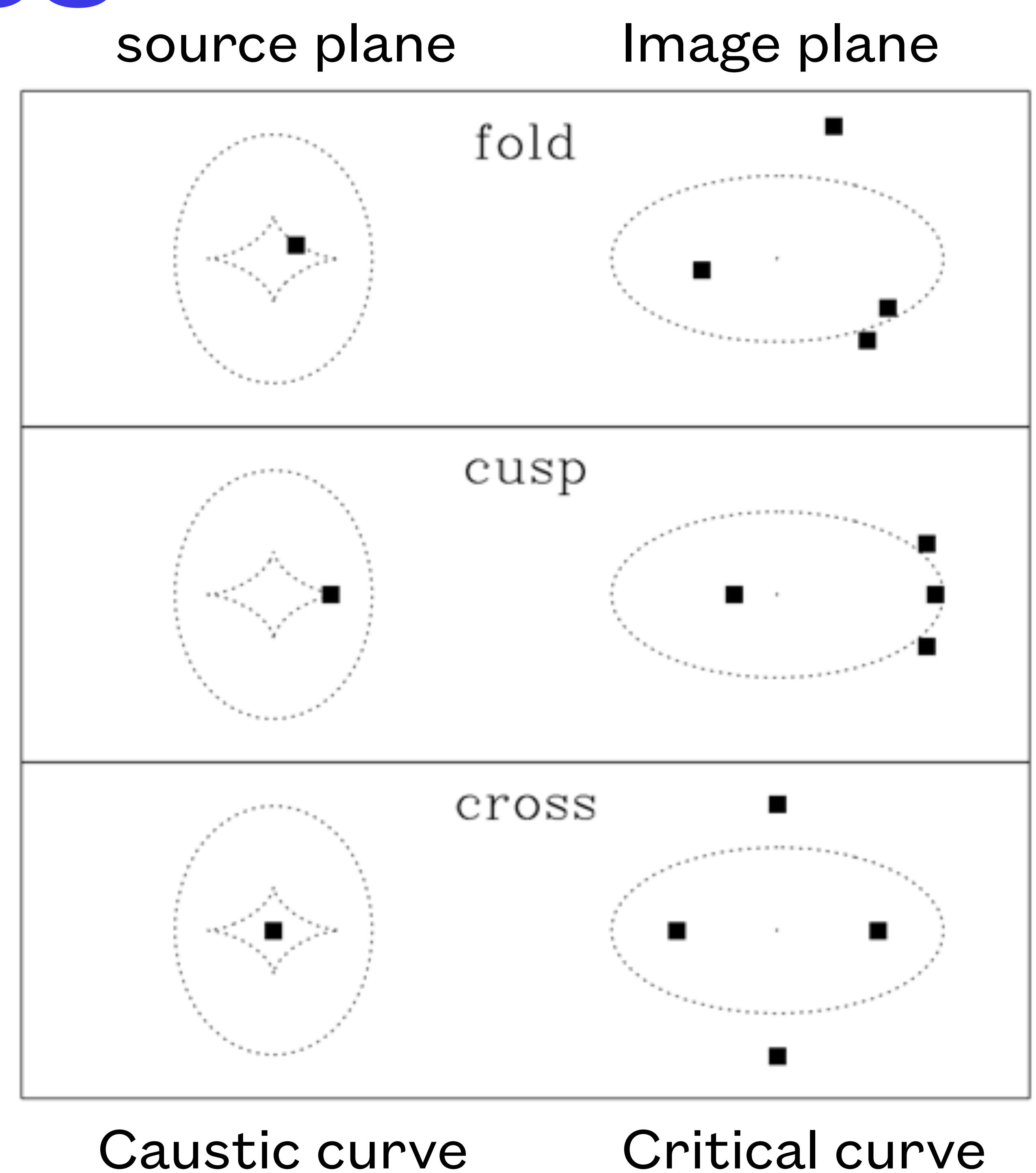
Multiple images

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta})$$

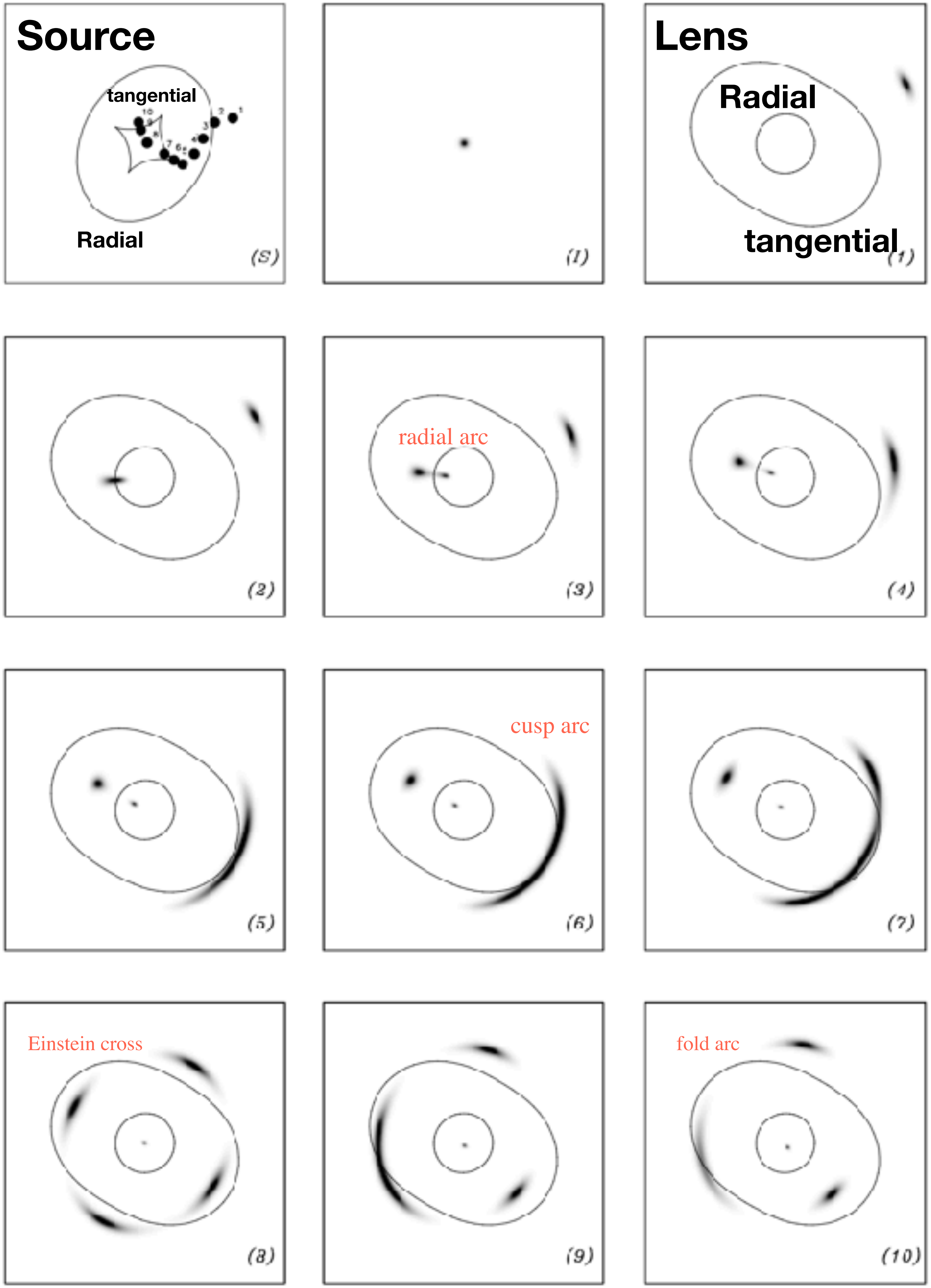
- It's not one-to-one relation

$$\frac{d\beta}{d\theta} = 0$$ Vanishing derivative

- The image position undergo large shifts for a very small shift in the source position
- Caustic curves (source plane)
Critical curves (image plane)
—> infinite magnification $\mu = \frac{1}{|\det A|}$.
- Tangentially distorted images are found near tangential critical lines and radially distorted ones near the radial critical lines



Keeton et al . 2002



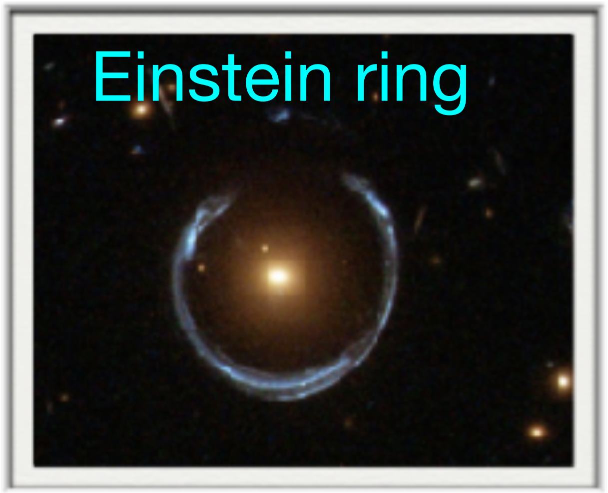
elliptical mass distribution

one crosses a caustic line in the source plane two additional lensed images

Kneib & Natarajan 2012

Strong Gravitational Lensing

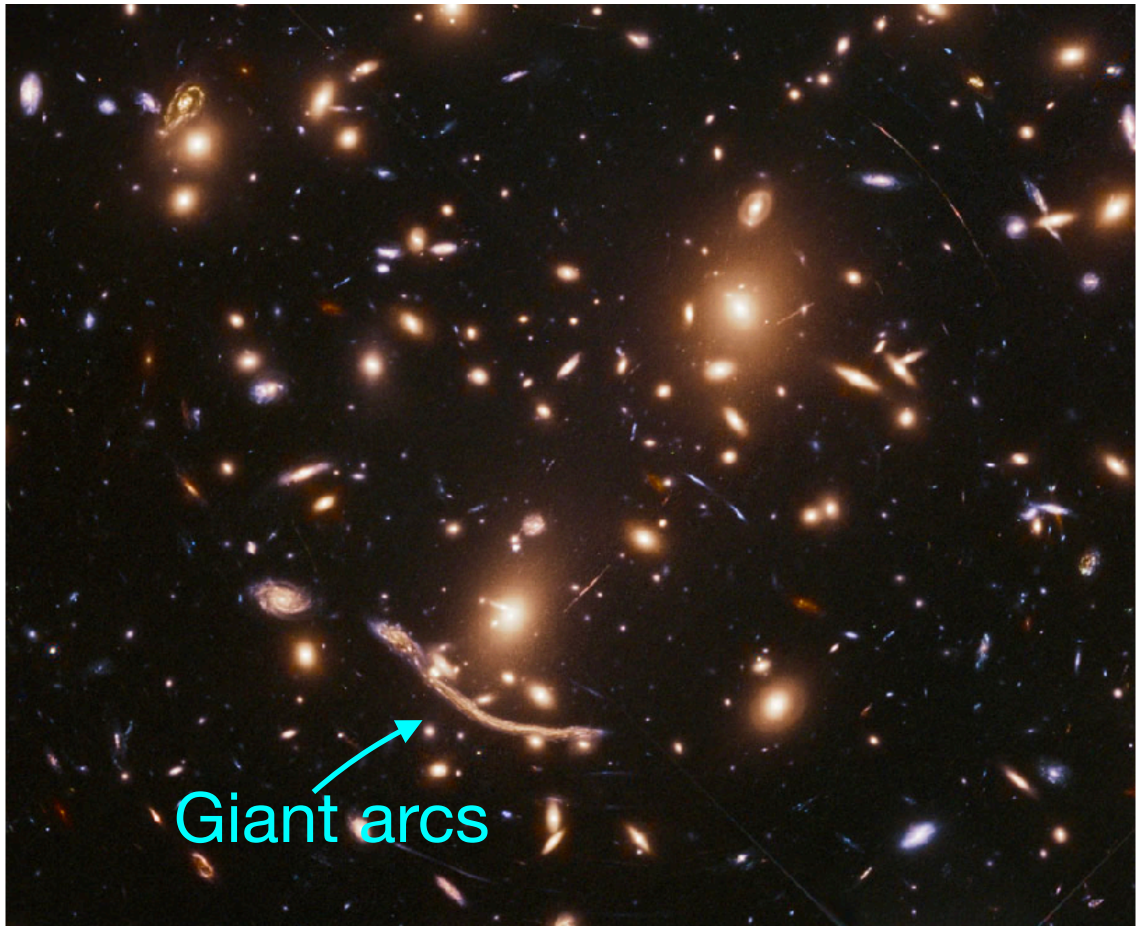
- Multiple images of the background appear, giant arcs, Einstein rings
- Dark Matter Mass in the core region

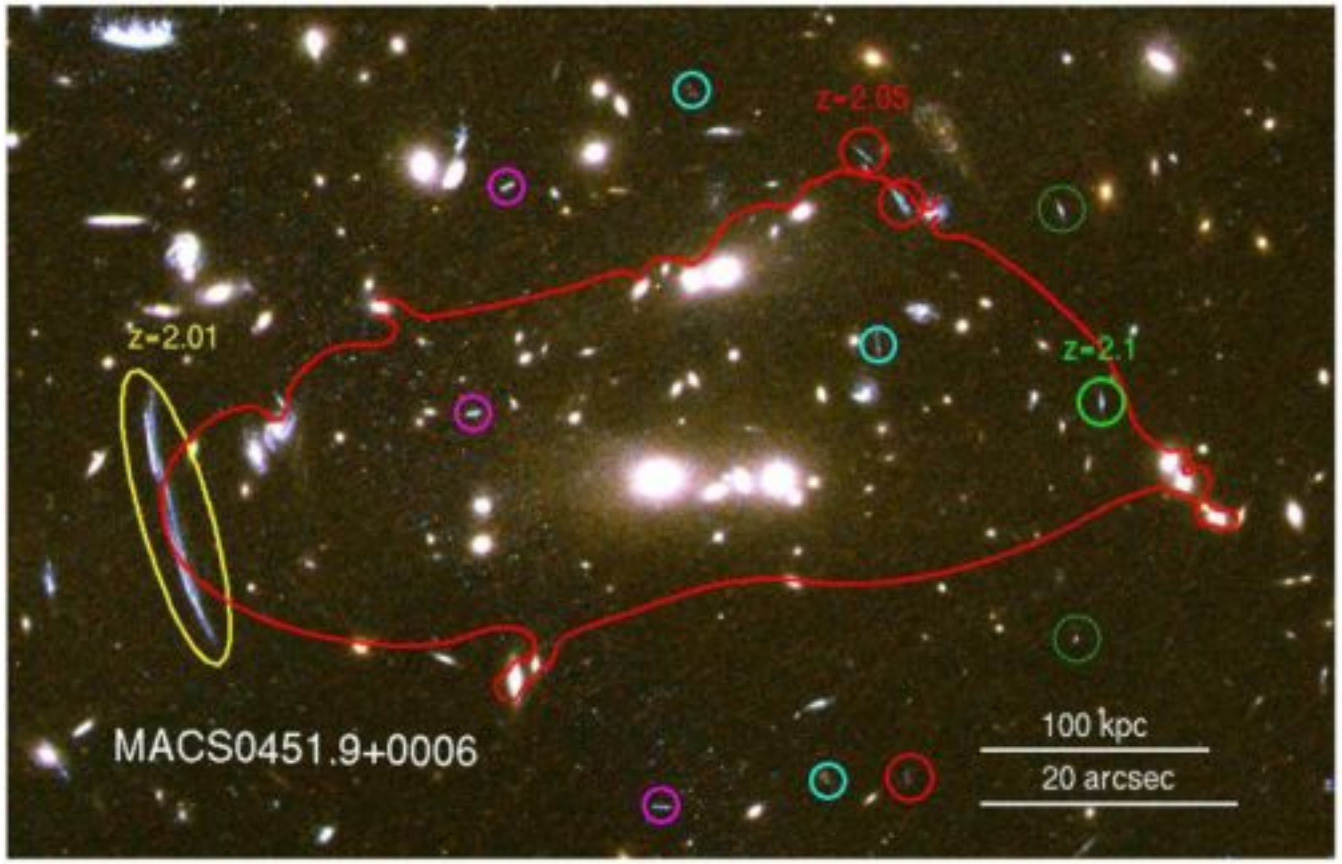


Galaxy-Galaxy Lenses

Galaxy-QSO Lenses

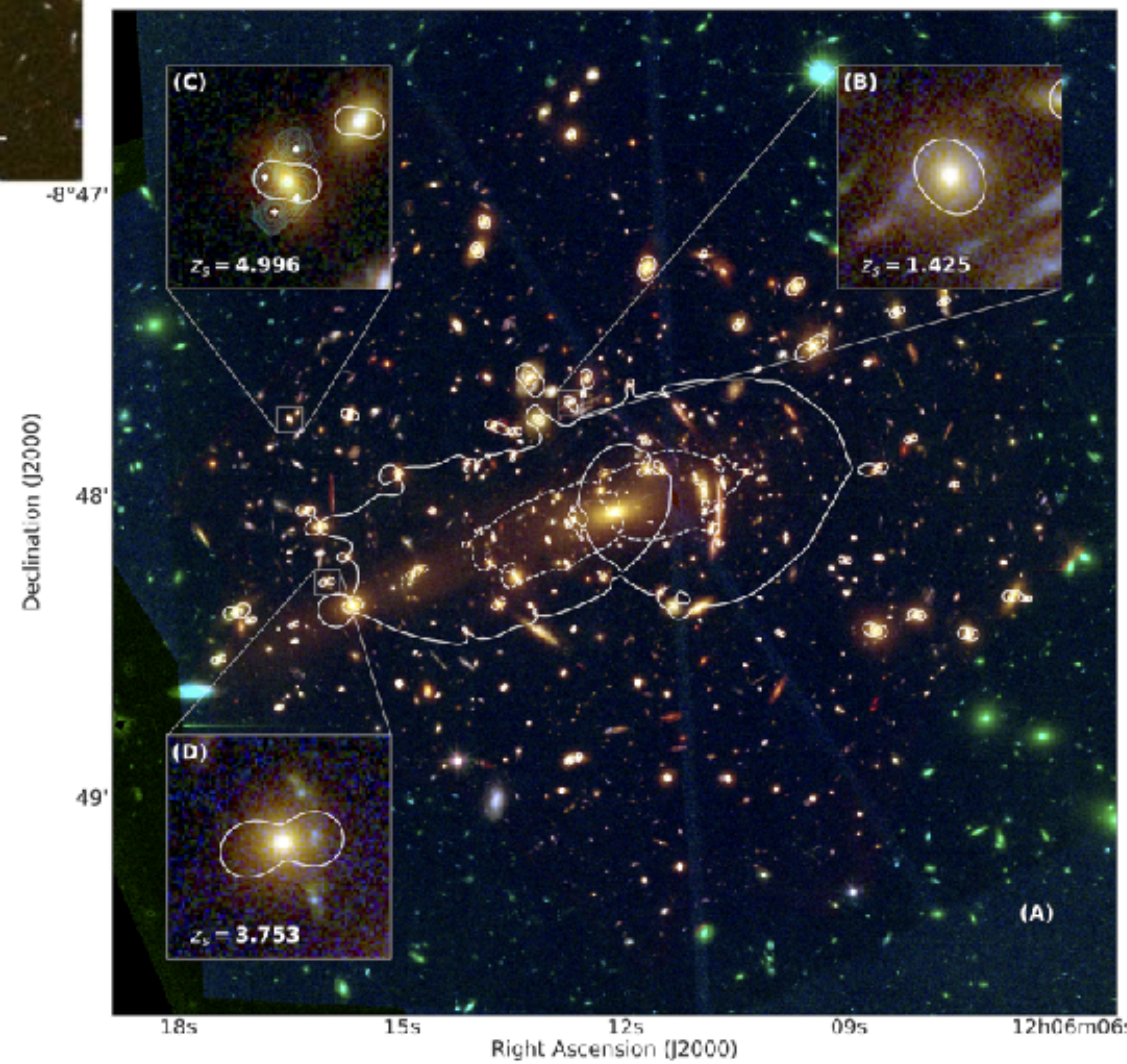
Galaxy-Cluster Lenses





Kneib & Natarajan et al. 2011

Meneghetti et al. 2020



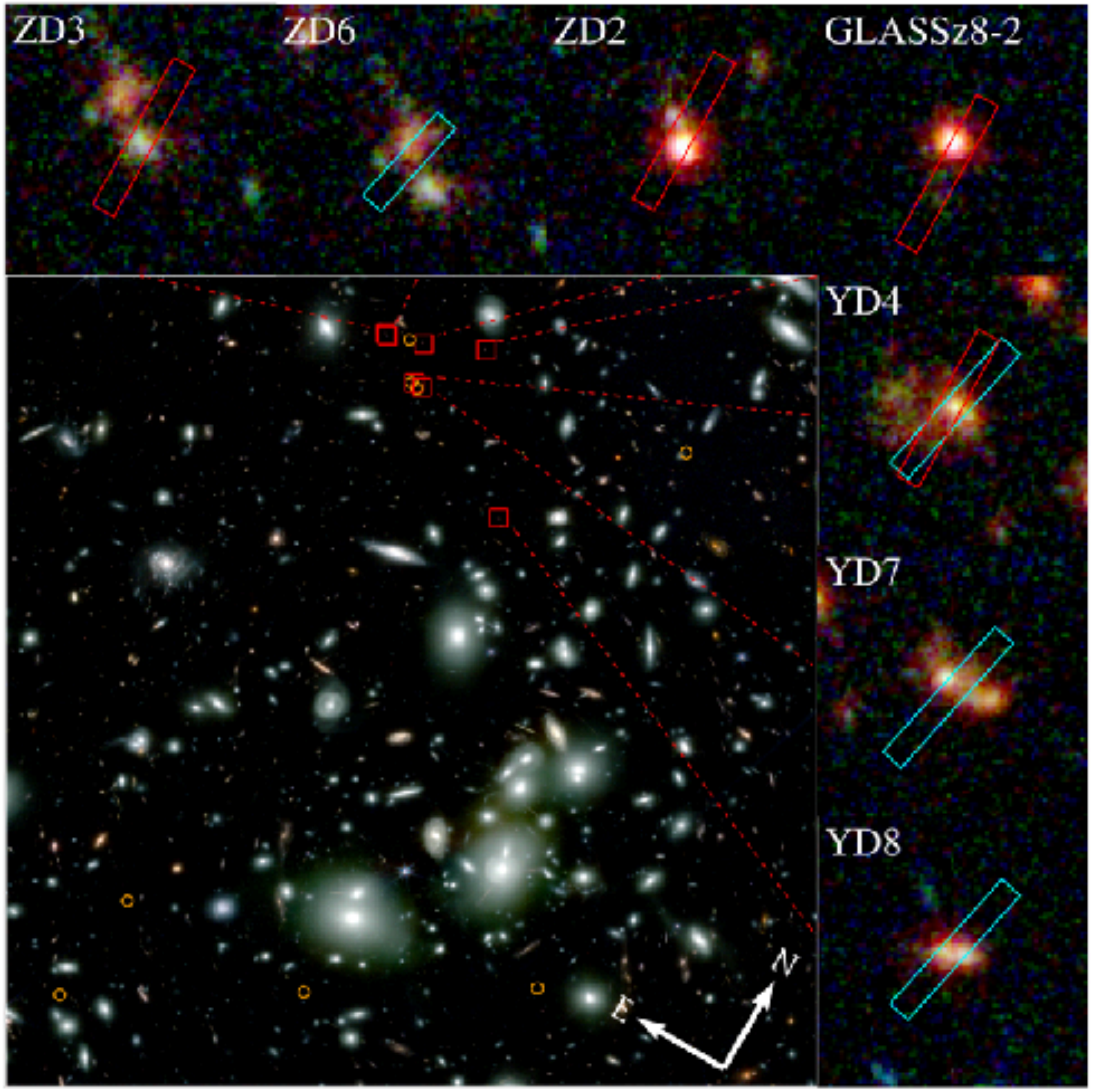
Cosmic telescopes

- Magnification: Gravitational lensing preserves the surface brightness, but the it changes the solid angle of a source.
  Magnification = image area/ source area $$\mu = \frac{d^2\theta}{d^2\beta} = \frac{1}{|\det A|}.$$

- Source located at Einstein ring —> highly magnified
$$\mu = \frac{1}{0} \to \infty.$$

- Studying very high redshift and faint sources

Morishita et al. 2023



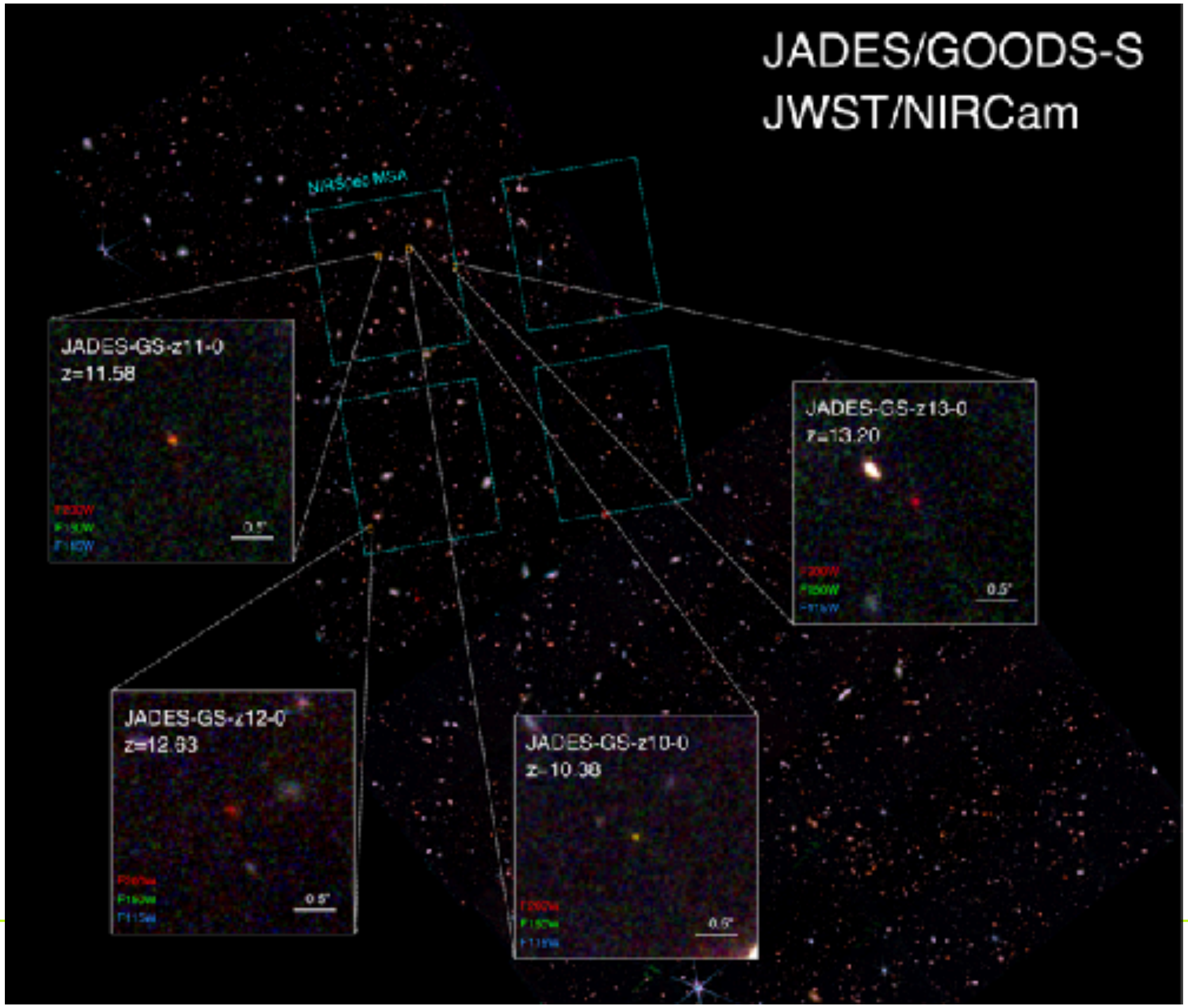
Abell 2744: JWST's NIRCam image

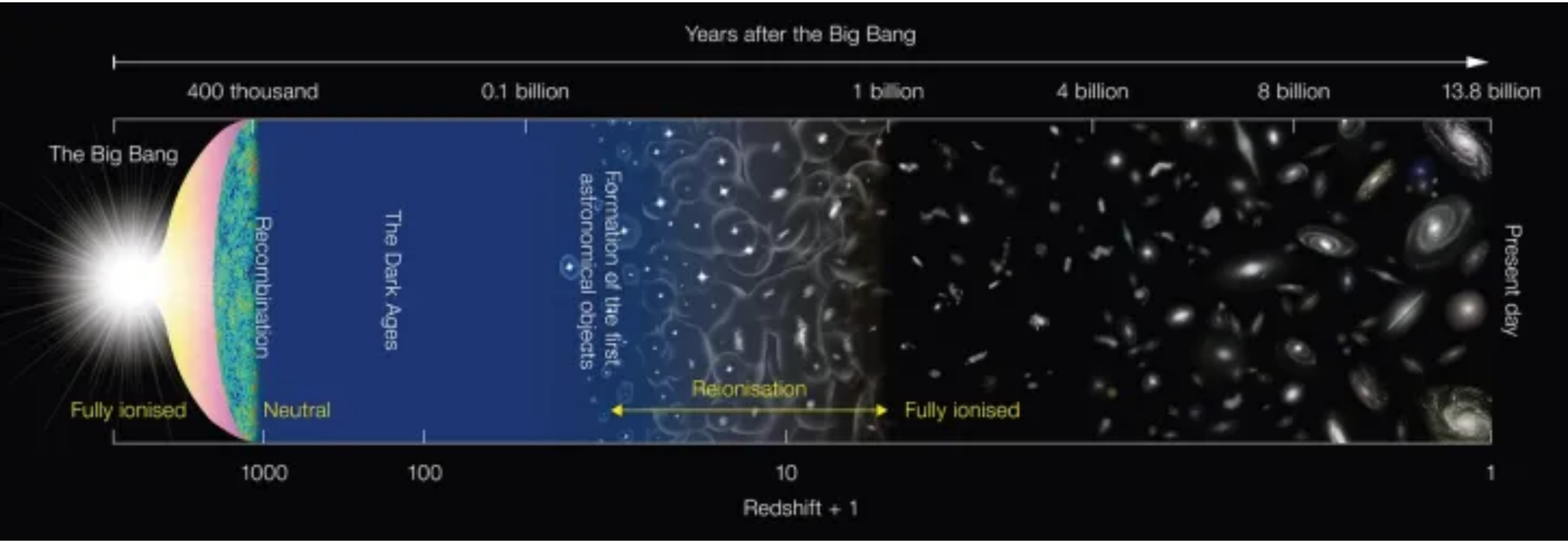
7 background galaxies at z~7.9

James Webb Space Telescope (JWST)

- James Webb Space Telescope (JWST)
  — Infrared wavelength, allows it to see through dust clouds and observe the early universe (i.e., the first galaxies and stars).
  — Galaxies at High Redshift: early stages of cosmic structure formation.

Morishita et al. 2023




Years after the Big Bang
400 thousand
0.1 billion
1 billion
4 billion
8 billion
13.8 billion
The Big Bang
Recombination
The Dark Ages
Formation of the first astronomical objects
Present day
Fully ionised
Neutral
Reionisation
Fully ionised
1000
100
10
1
Redshift + 1

Time Delay-Cosmography

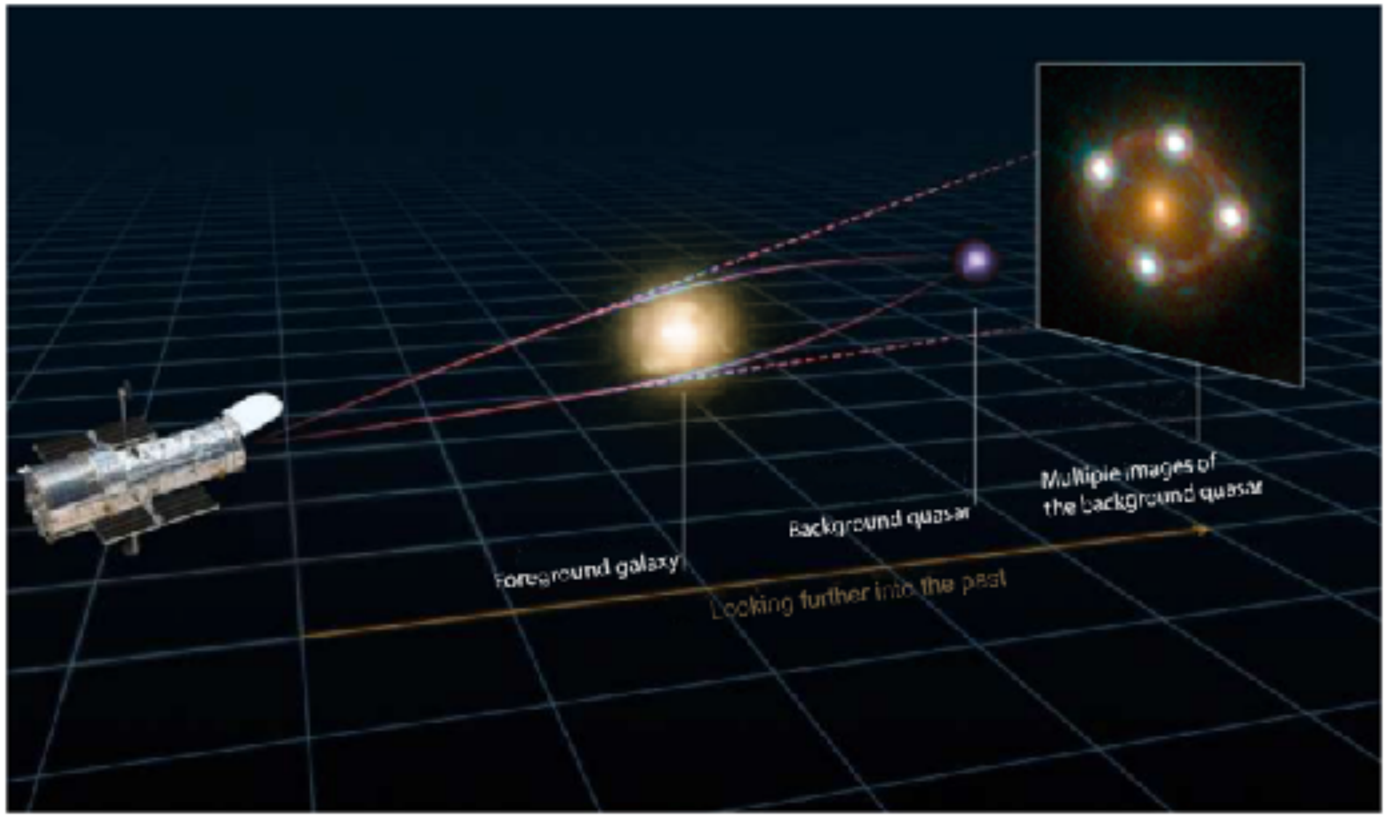
- Strong lensing of quasars— variable sources

$$D_{\Delta t} \equiv (1 + z_{\rm d}) \frac{D_{\rm d} D_{\rm s}}{D_{\rm ds}} \propto H_0^{-1}, \qquad \Delta t = \frac{D_{\Delta t}}{c} \Delta\phi(\theta, \beta)$$

- For cosmography to constrain Hubble constant, need:
  >Time delay measurement — from the light curves
  >Lens mass modeling

- Accurate mass modeling: external structures along line of sight

- H0LiCOW collaboration used six lensed quasar systems inferred $H0 = 73.3^{+1.7}_{-1.8}$ km $s^{-1}Mpc^{-1}$ (2% precision) (Wong et al. 2019)

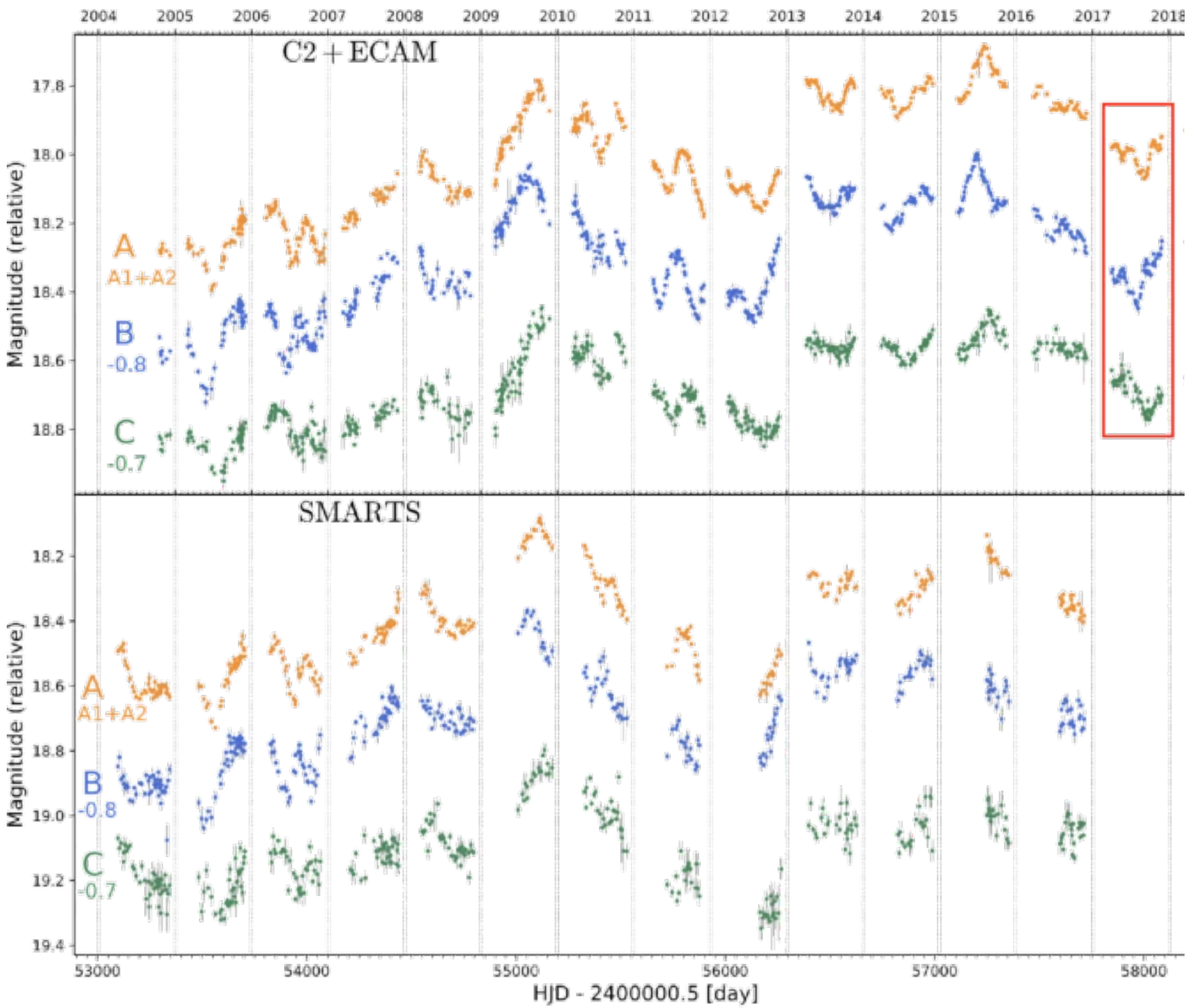


Wang et al. 2017

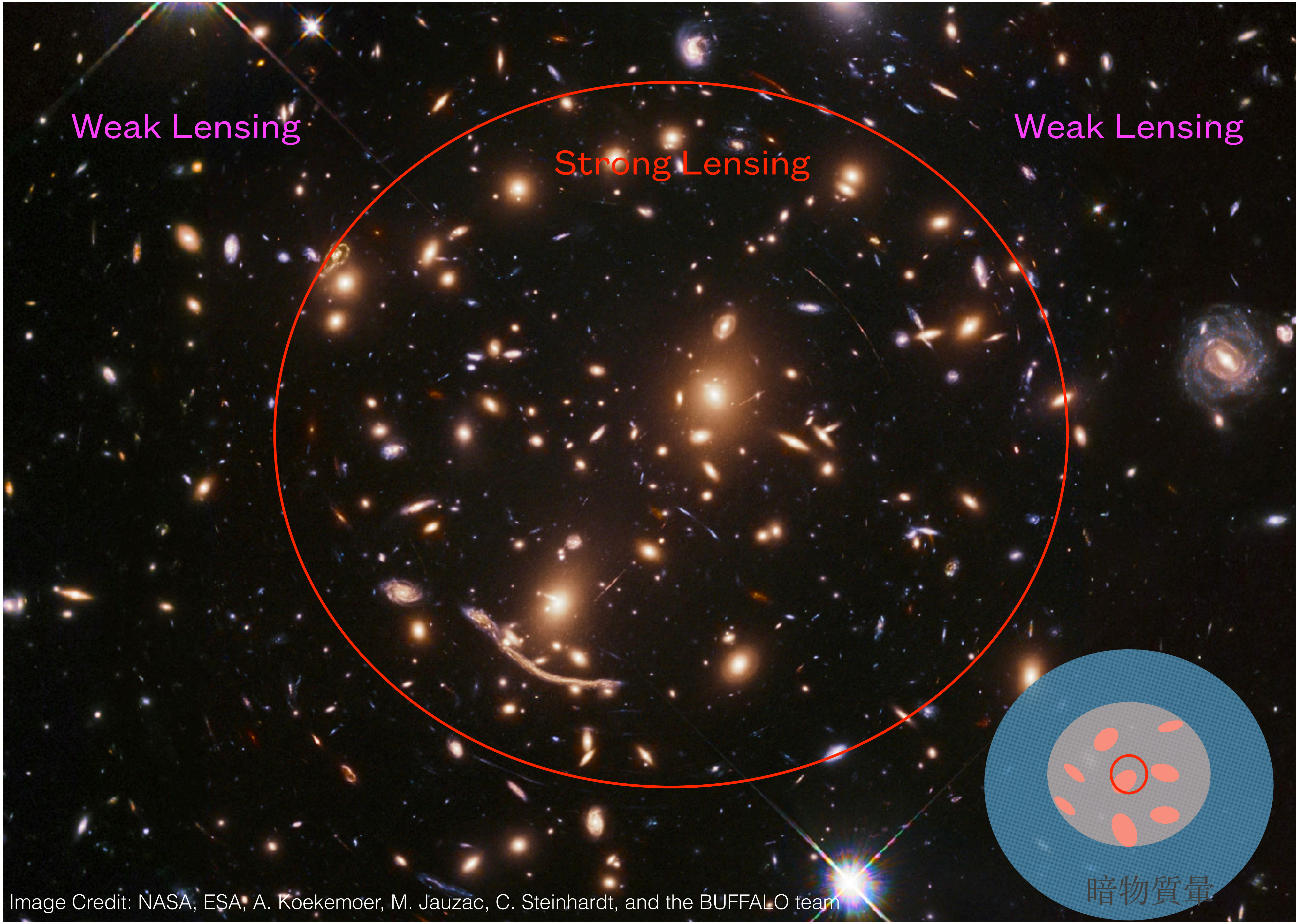
Time Delay-Cosmography

- Strong lensing of quasars— variable sources
- **Monitor the Source**: Observe brightness variations in the lensed images over time to determine precise time delays.
- **Model the Lens**: Use high-resolution imaging (e.g., from the Hubble Space Telescope) and spectroscopy to map the lensing mass distribution.
- **Estimate Time-Delay Distance**: Combine the time delays and lens model to calculate $\Delta t = \frac{D_{\Delta t}}{c} \Delta\phi(\theta, \beta)$
- **Infer H_0^{-1}**: use the relationship between $D_{\Delta t} \equiv (1 + z_d) \frac{D_d D_s}{D_{ds}} \propto H_0^{-1}$

Hubble Tension

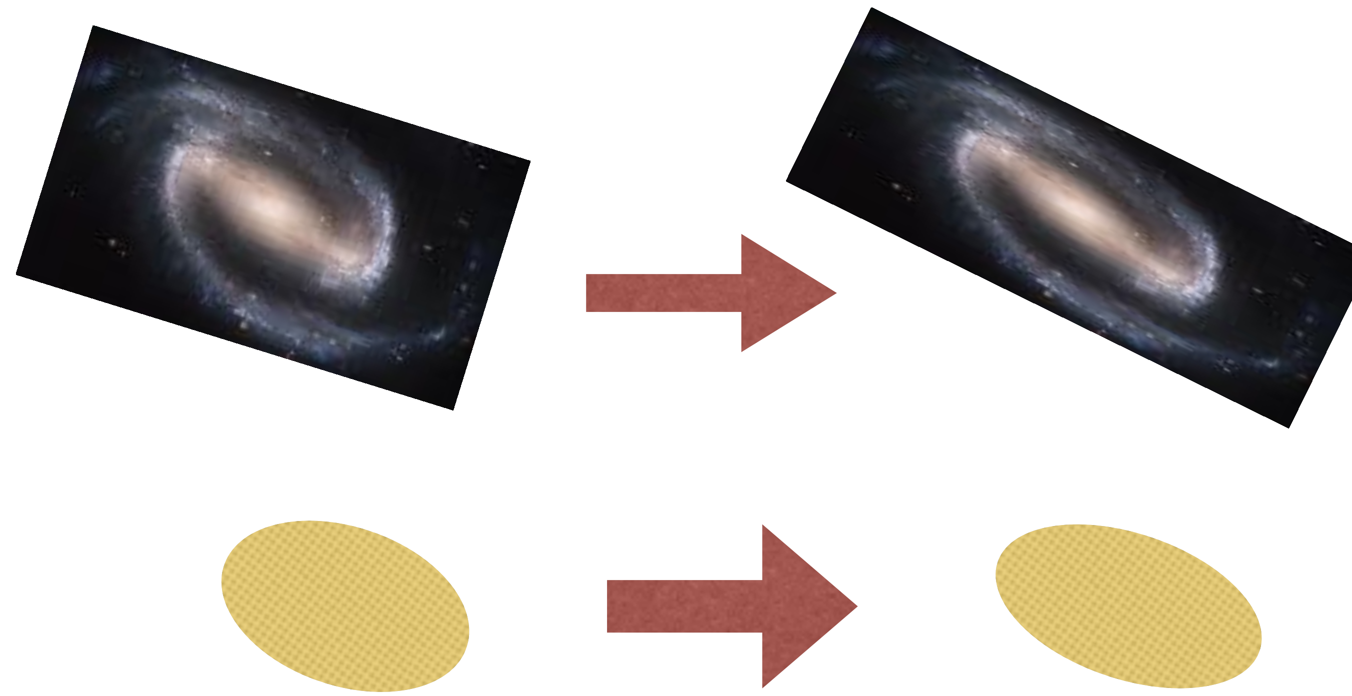


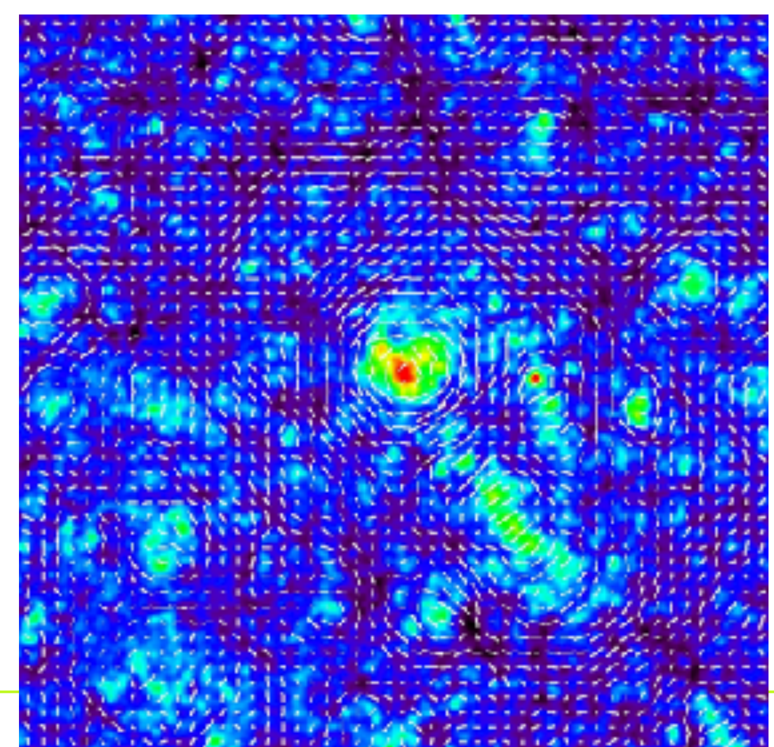
Comparison of two observing cadences. 14-year-long light curves of WFI 2033-4723, observed with a 4-day cadence at the 1.2 m Euler telescope at La Silla and at the SMARTS telescope at Las Campanas. (Bonvin et al. 2019; Millon et al. 2020a)


Weak Lensing
Strong Lensing
Weak Lensing
暗物質量
Image Credit: NASA, ESA, A. Koekemoer, M. Jauzac, C. Steinhardt, and the BUFFALO team

Weak Gravitational Lensing

- Occurs in low-density regions, causing a tiny distortion of background images.
- Important to detect the dark matter distribution in large-scale
- Very difficult to detect for a single source because of noise
- Need to measure statistically; Need a lot of background galaxies





Color : projected mass distribution
White bars : shear

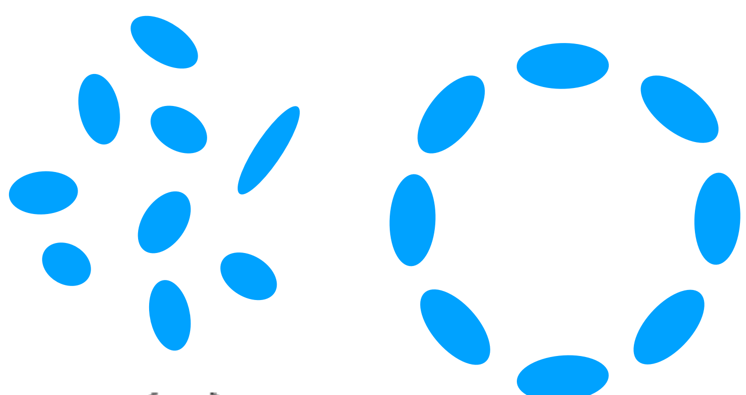
Convergence & Shear

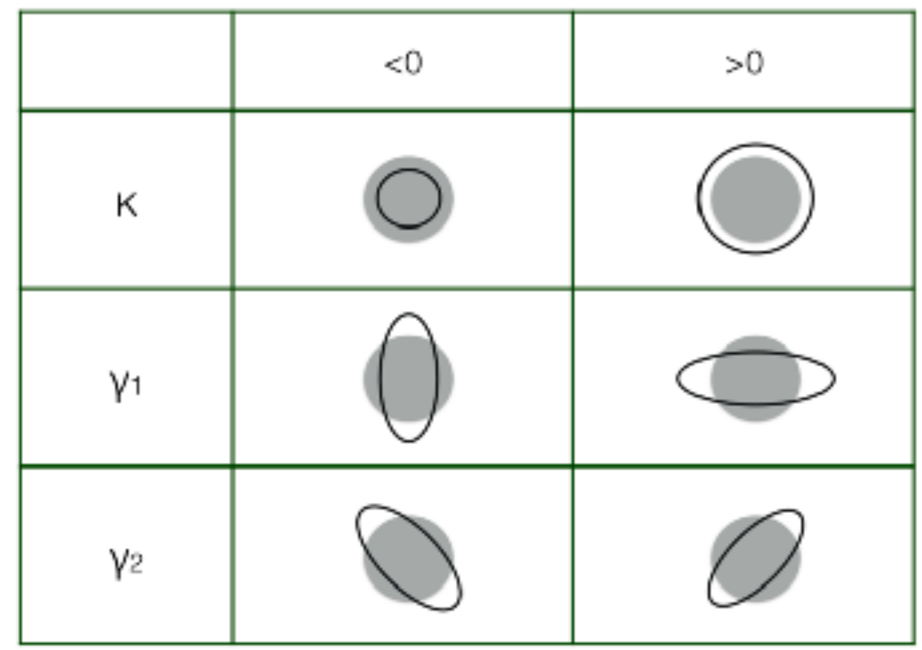
Lens equation: $\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta})$, $\quad \boldsymbol{\alpha} = \nabla_{\boldsymbol{\theta}} \psi$

$$A = \frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{\theta}} = \begin{pmatrix} 1 - \psi_{,11} & -\psi_{,12} \\ -\psi_{,21} & 1 - \psi_{,22} \end{pmatrix} = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix}$$

$$\kappa = \frac{1}{2}(\psi_{,11} + \psi_{,2})$$

$$\gamma_1 = \frac{1}{2}(\psi_{,11} + \psi_{,22}) \qquad \gamma_2 = \frac{1}{2}(\psi_{,12} + \psi_{,21})$$





$$\epsilon_i^{obs} = \epsilon_i^{(s)} + \gamma_i \rightarrow E(\epsilon^{(s)}) = E(\gamma)$$

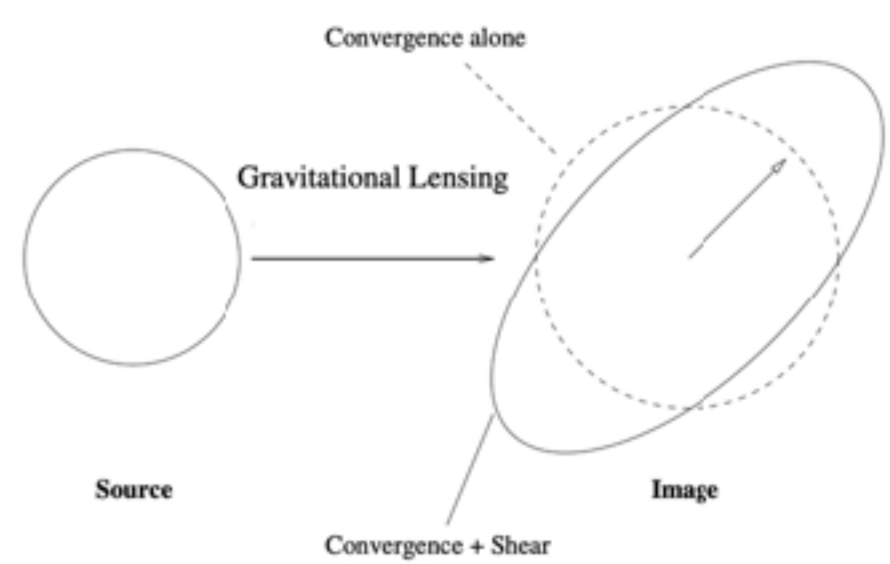
Intrinsic galaxy shape

random orientation

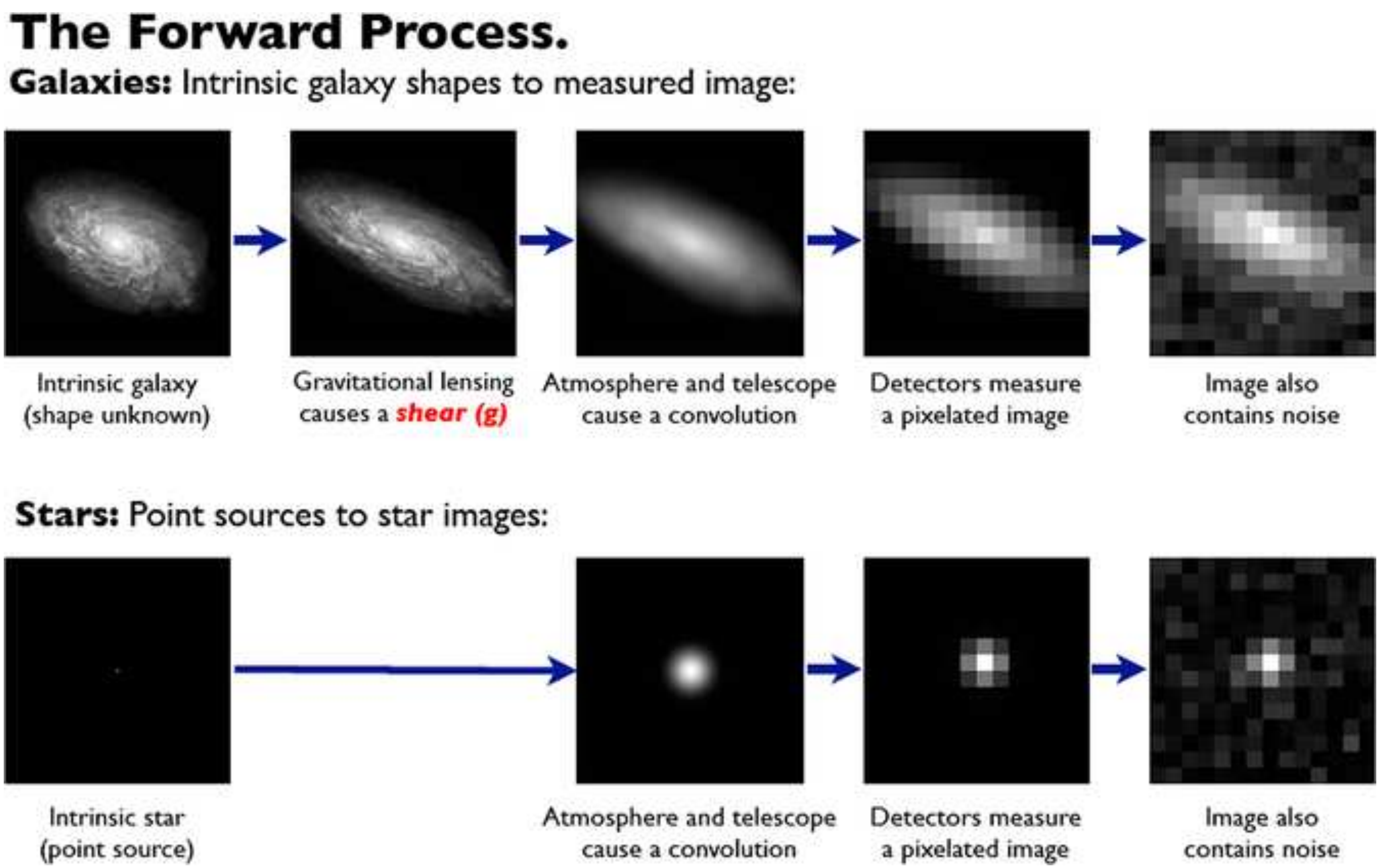
Weak Lensing Shear

Coherent distortion

Ensemble averaging $\rightarrow E(\epsilon^{(s)}) \approx 0$



Weak Gravitational Lensing

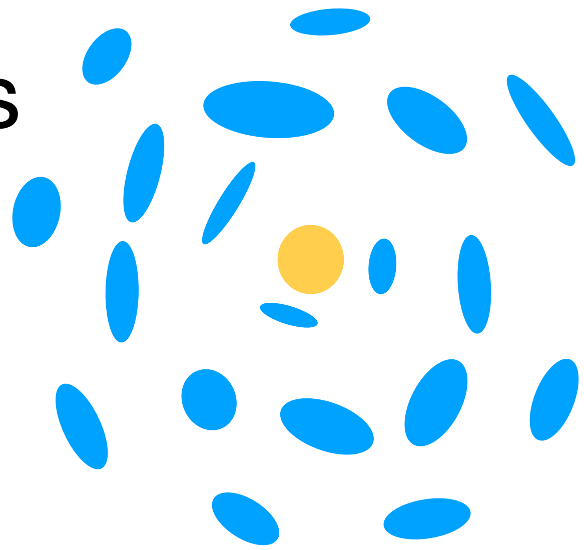


Bridle et al. 2008

1D Weak Lensing Analysis

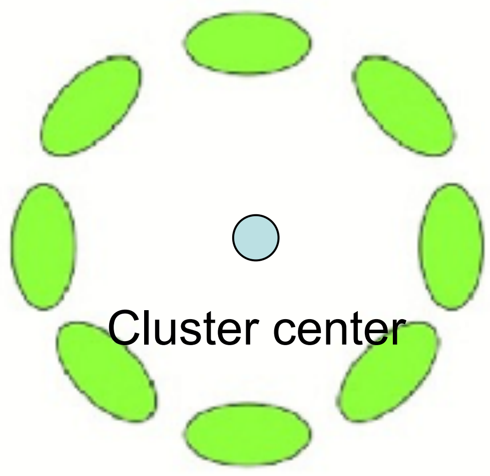
- Tangential shear profile —> extract the mass
- What is tangential profile ?

  Shear (γ) split into two terms:



tangential distortion (γ_+ : E-mode): generated from lensing

45 degree-rotated component ($\gamma_\times$: B-mode): used to check systematic errors



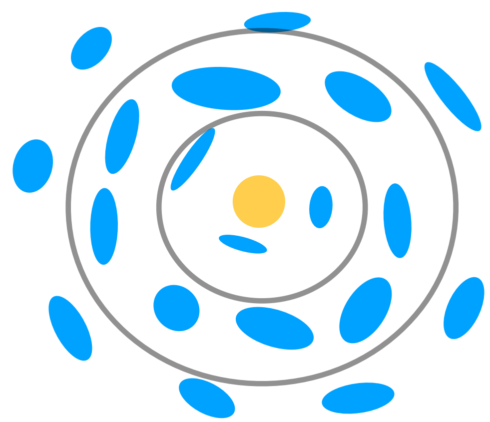
$$\gamma_+(\vartheta) := \oint \frac{d\varphi}{2\pi}\, \gamma_+(\vartheta, \varphi) = \bar{\kappa}(\vartheta) - \kappa(\vartheta) \equiv \frac{\Delta\Sigma(\vartheta)}{\Sigma_{\rm cr}},$$

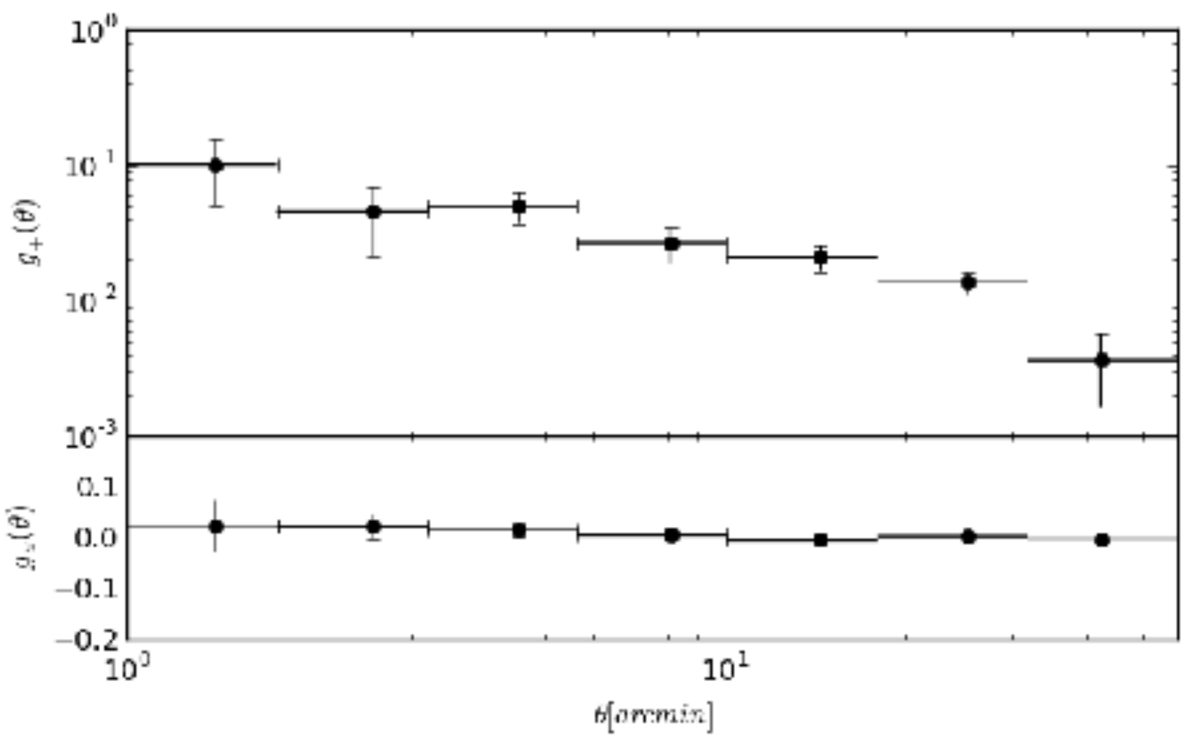
$$\gamma_\times(\vartheta) := \oint \frac{d\varphi}{2\pi}\, \gamma_\times(\vartheta, \varphi) = 0,$$

1D Weak Lensing Analysis

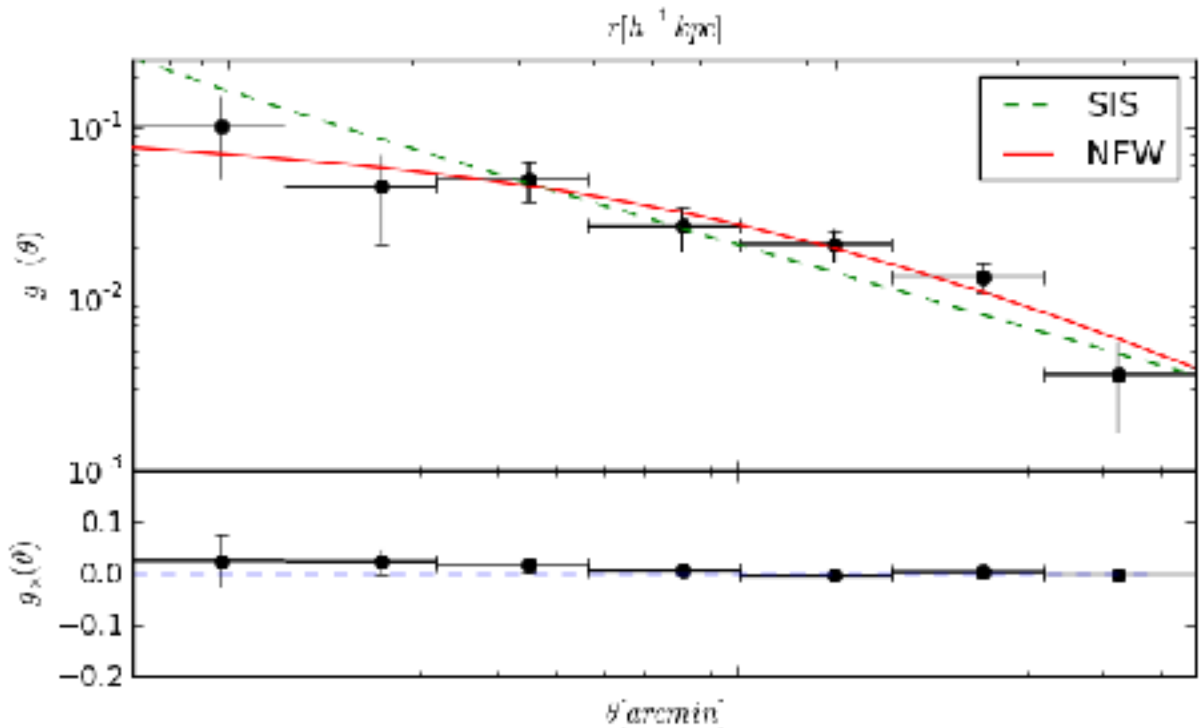
- Averaging tangential shear of all galaxies in the annulus

$$\gamma_+(\vartheta) := \oint \frac{d\varphi}{2\pi}\,\gamma_+(\vartheta,\varphi)$$





Abell 478



Fitted with some analytic halo models (e.g. NFW, Einasto, SIS) to infer the properties, e.g. M200, c200

2D Weak Lensing Mass Reconstruction

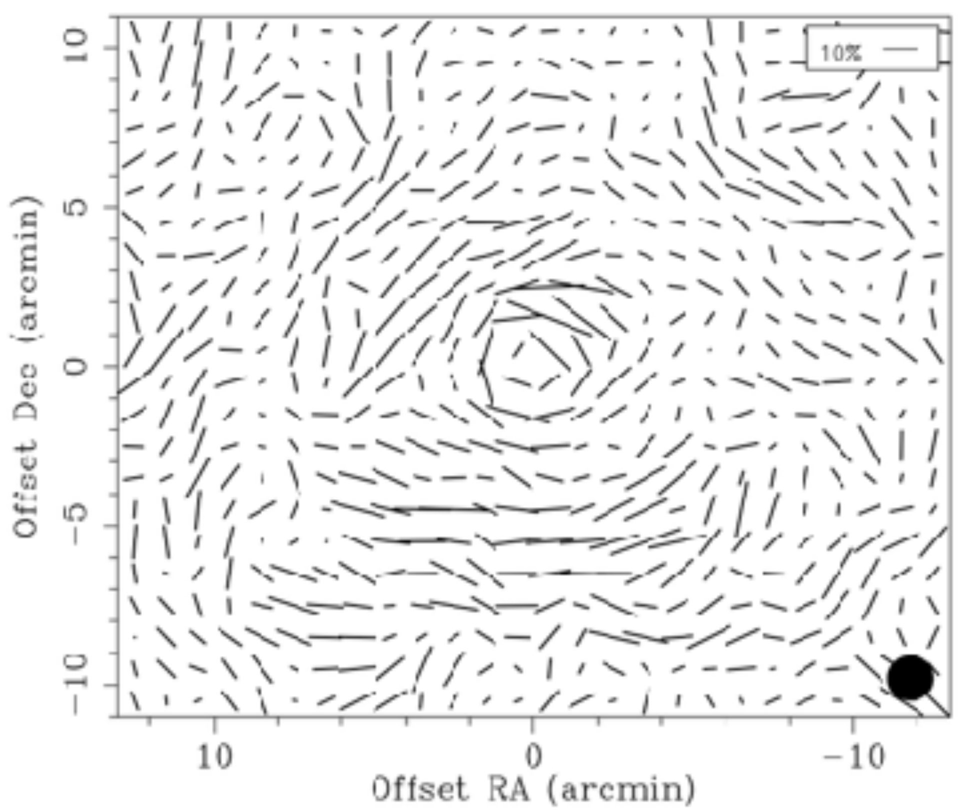
γ : shear

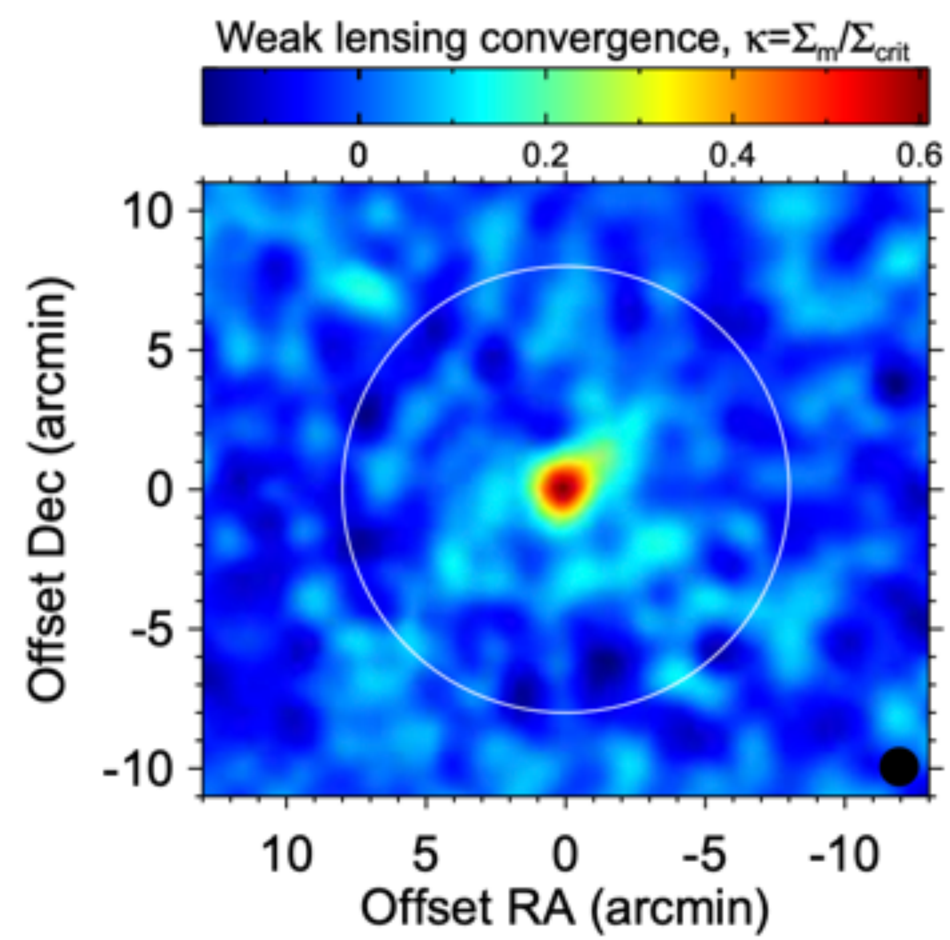
$$\kappa(\theta) = \frac{1}{\pi}\int d^2\theta' D(\theta - \theta')\gamma(\theta')$$

$$D(\theta) = \frac{\theta_1^2 - \theta_2^2 + 2i\theta_1\theta_2}{|\theta|^4}$$

κ: dimensionless surface mass density

Shear map





Umetsu et al. 2010

Clusters as gravitational telescopes

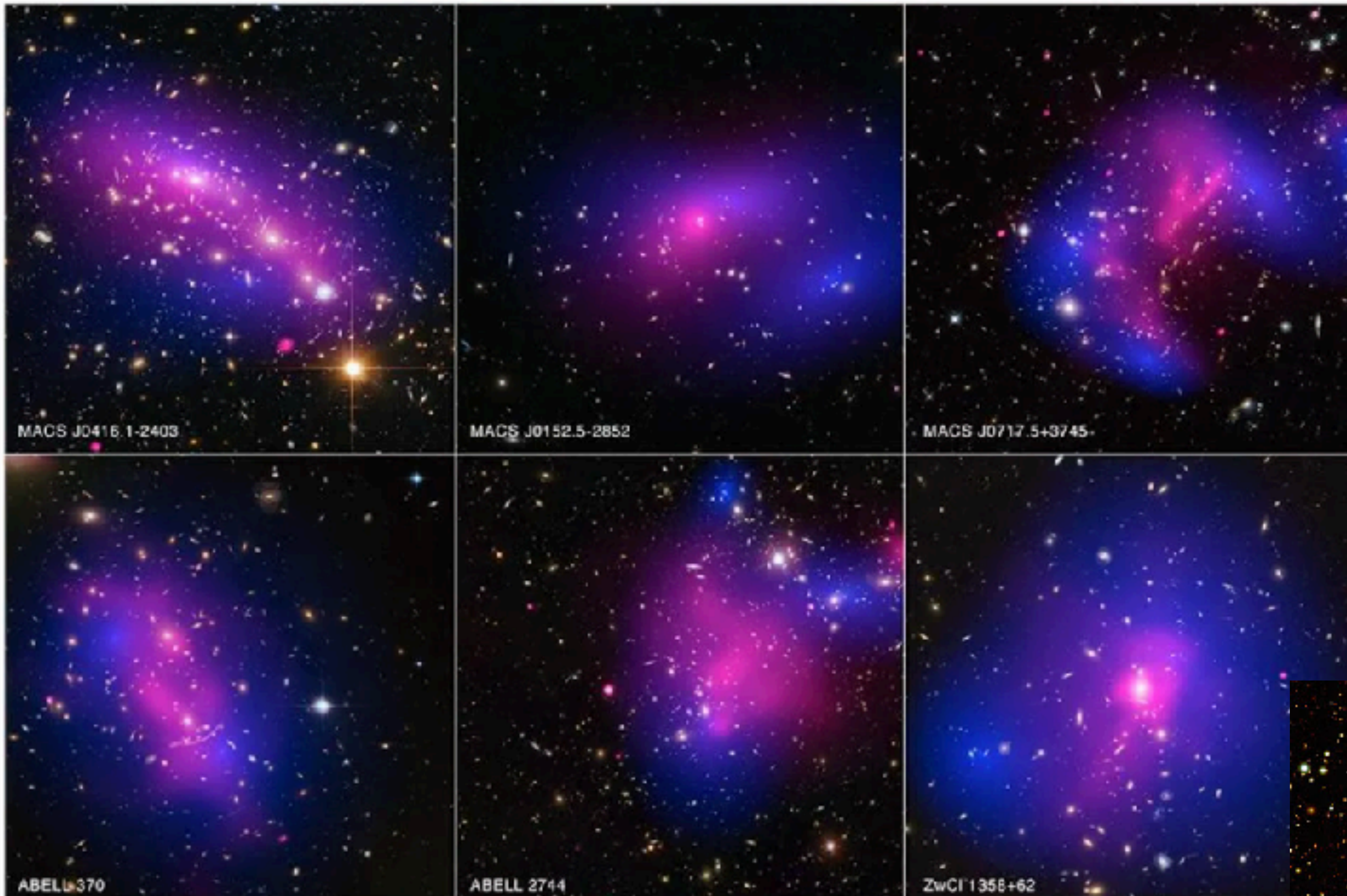
- The most massive objects in the Universe
- —> consists of hundreds to thousands of galaxies and hot gas bound by gravity
  —>consists of 85% dark matter, 10% hot gas and 5% stars
  —>ideal laboratory for studying the dark Universe and testing the cosmological models

Galaxy

Galaxy cluster

Galaxy Cluster

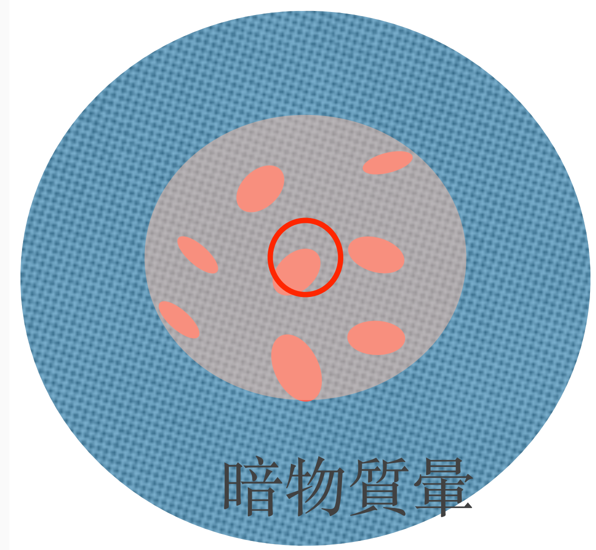
Dark Matter Distribution in Galaxy Cluster

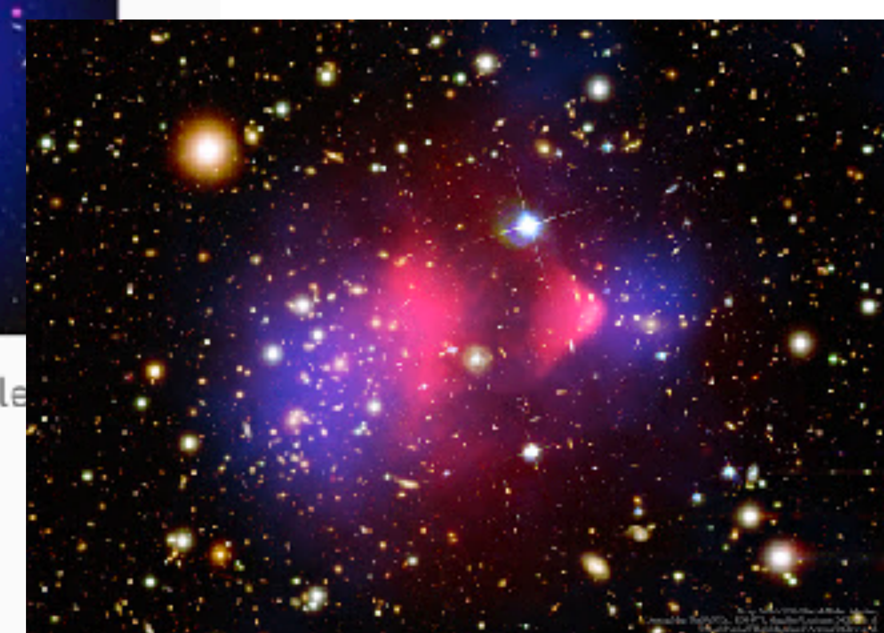


The X-ray (pink) and overall matter (blue) maps of various colliding galaxy clusters show a cle

[+] X-RAY: NASA/CXC/ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE, SWITZERLAND/D.HARVEY NASA/CXC/DURHAM UNIV/R.MASSEY; OPTICAL/LENSING MAP: NASA, ESA, D. HARVEY (ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE, SWITZERLAND) AND R. MASSEY (DURHAM UNIVERSITY, UK)

Pink: X-ray gas
Blue: dark matter

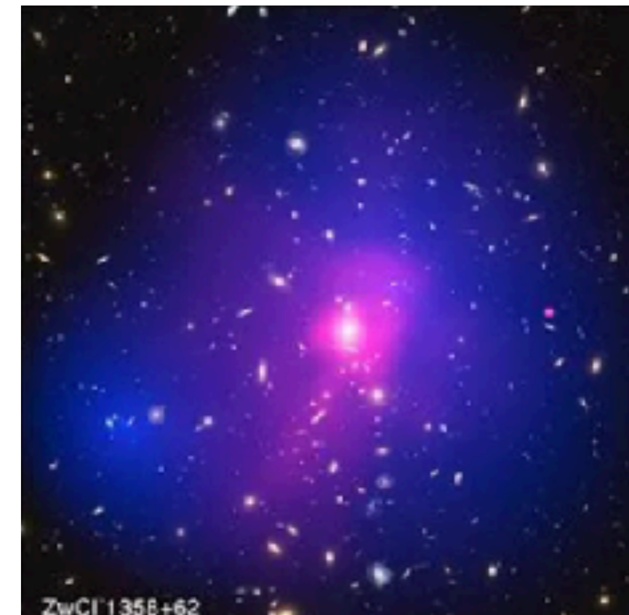


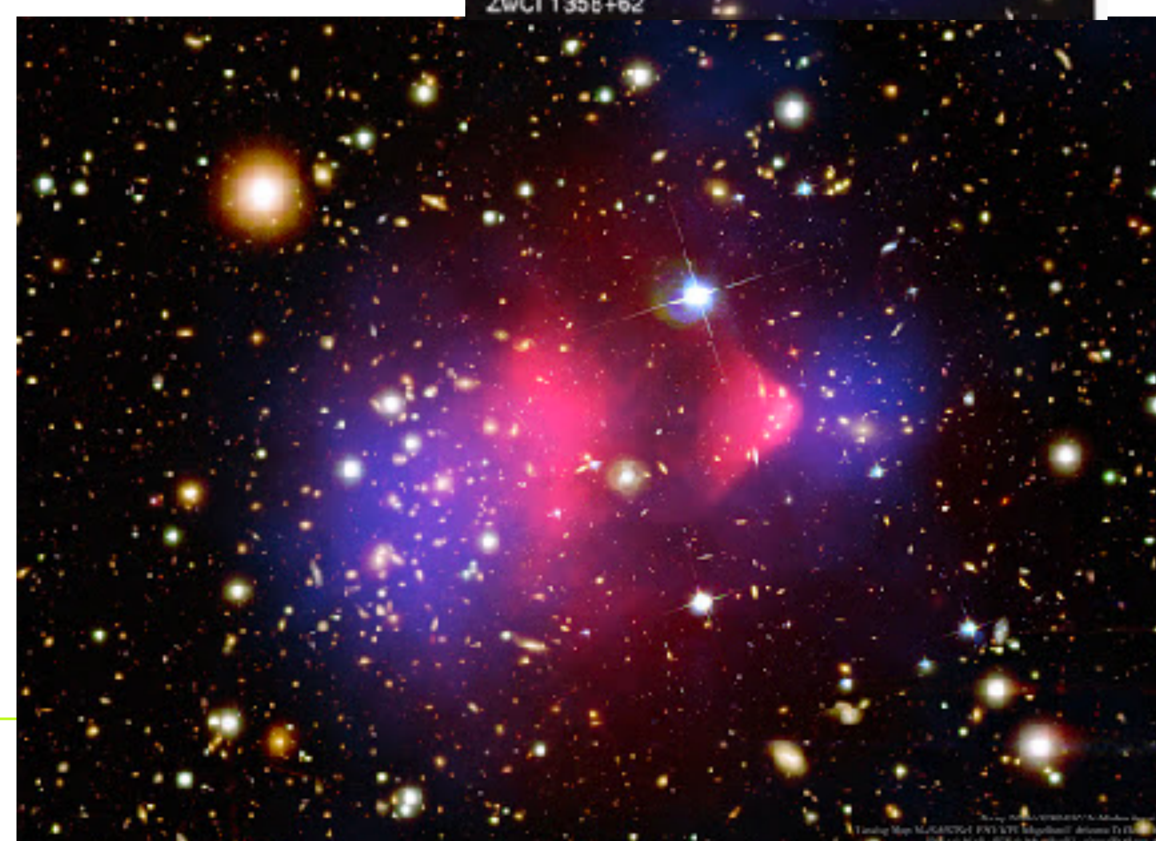


Why does dark matter exhibit a distribution distinct from that of gas?

Pink: X-ray gas (ordinary matter)

Blue: dark matter

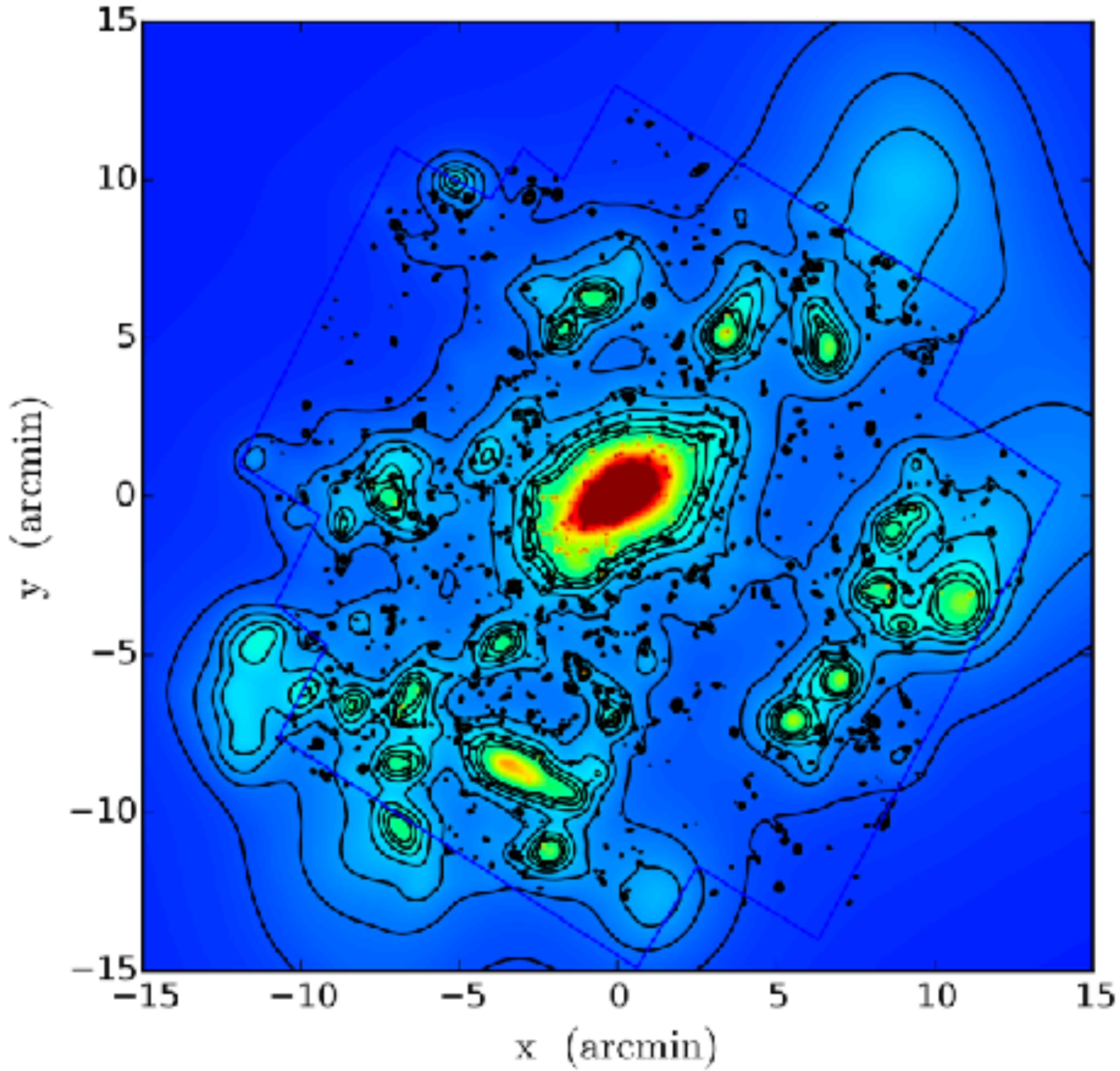


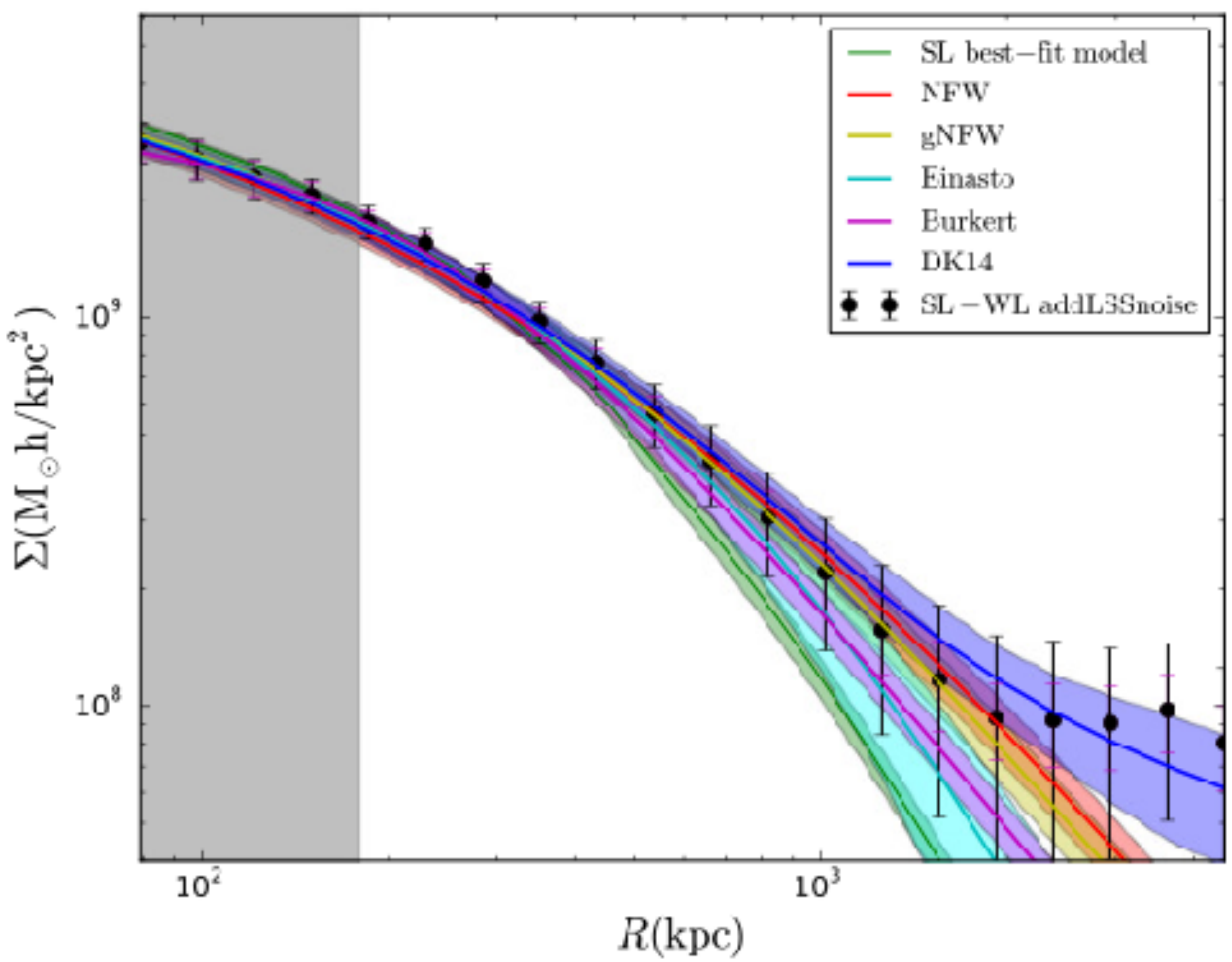


- dark matter: only gravity
  —> exhibits a diffuse, extended distribution
  gas: additional forces such as pressure and radiation, leading to a more concentrated distribution
- In a merging system
  dark matter: pass through each other and do not experience significant frictional forces.
  gas: hydrodynamic drag and shocks, during mergers. They gather central regions of merging system.

Combined Strong+Weak lensing Analysis

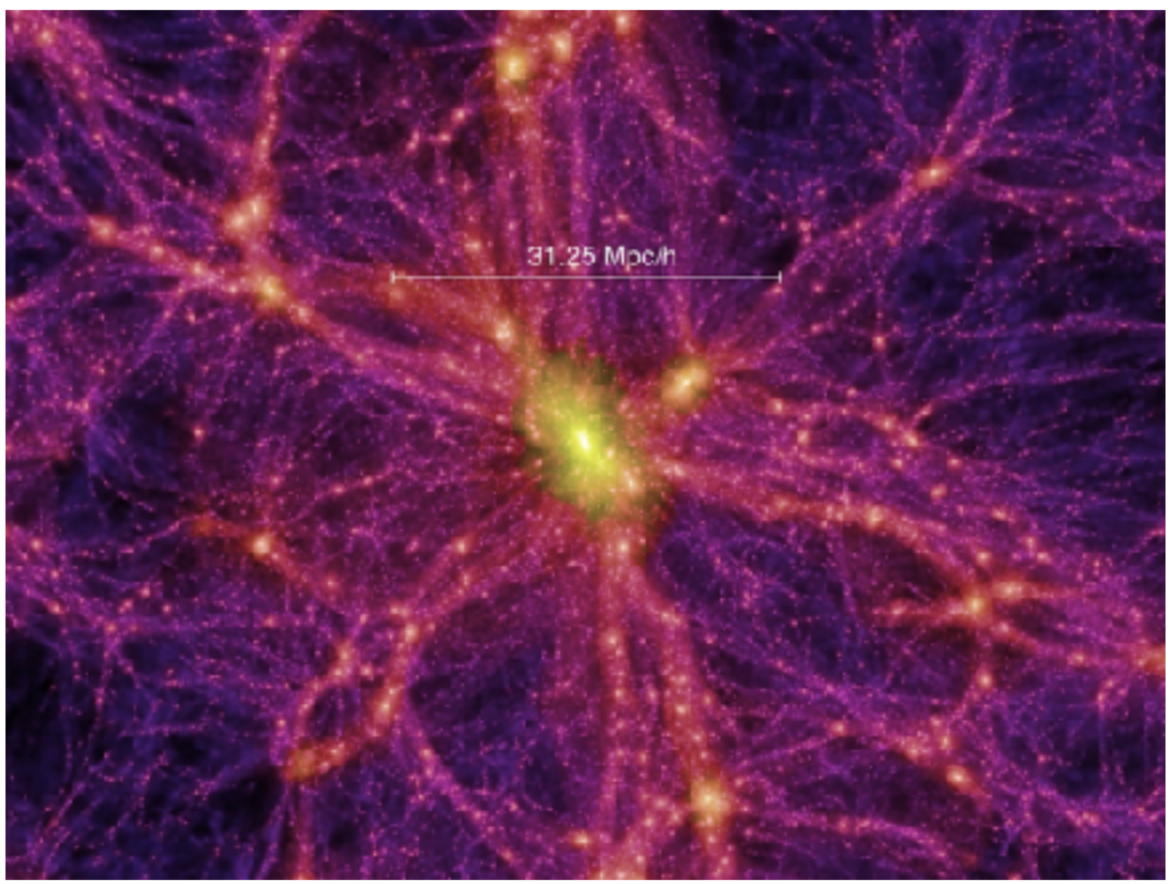
a wide field HST mosaic of 41 images (a total area of ~ 460 arcmin^2) for MS0451
—> use high resolution space-based WL data to confirm large-scale structures

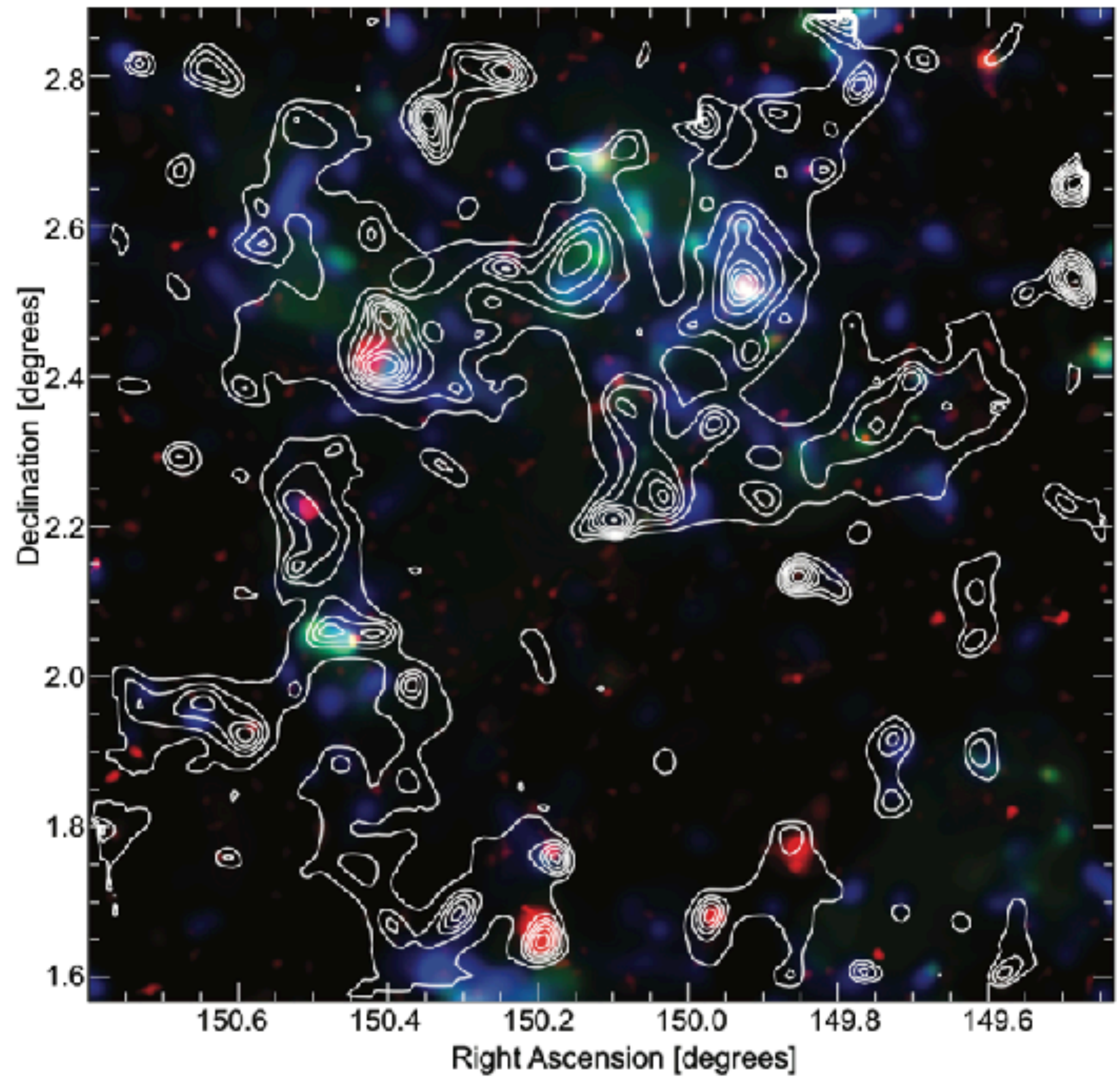




Tam et al. 2020

Projected DM distribution in larger scales

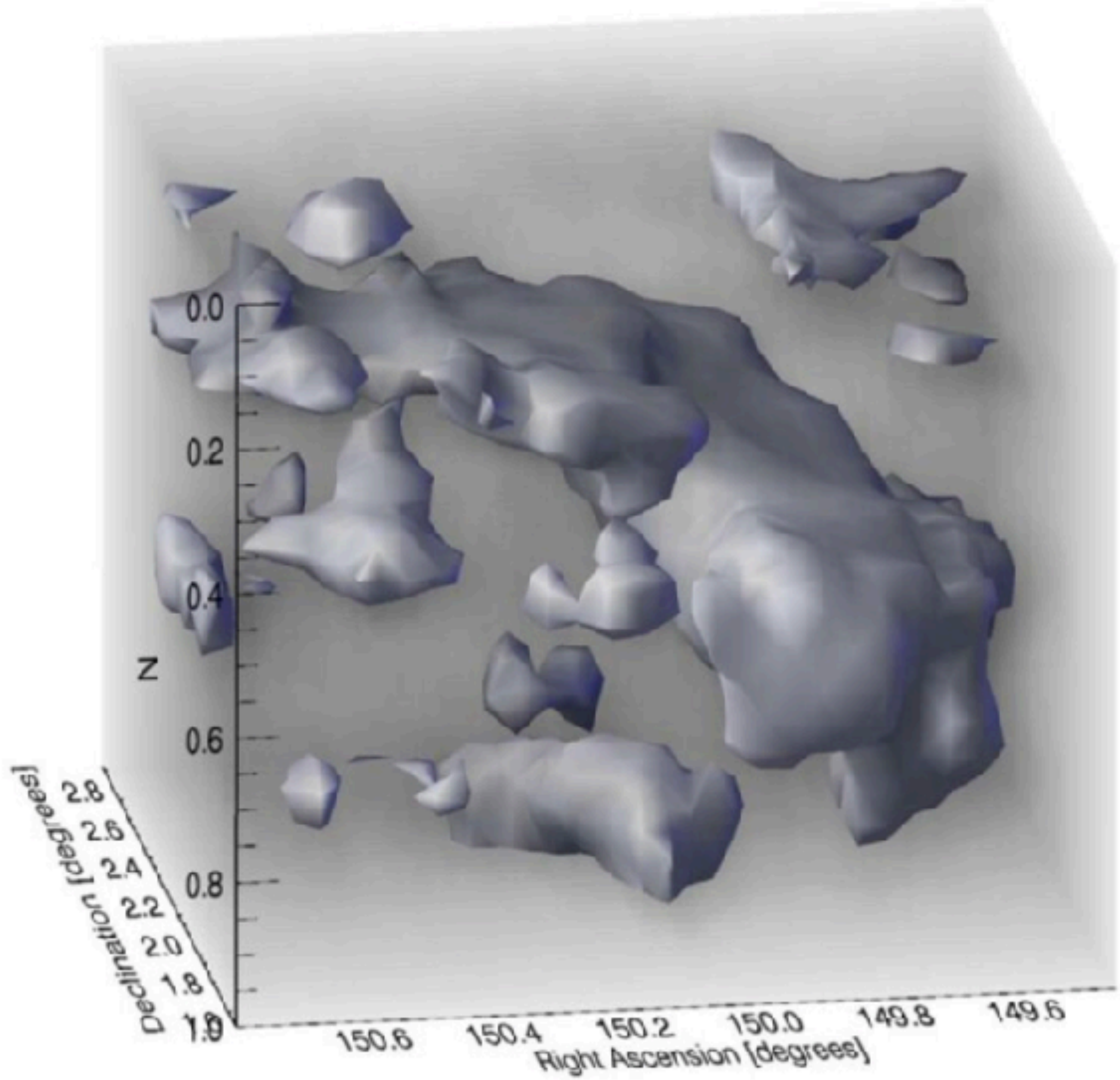




Large-scale structures in COSMOS field
Massey et al. 2007

3D LSS distribution

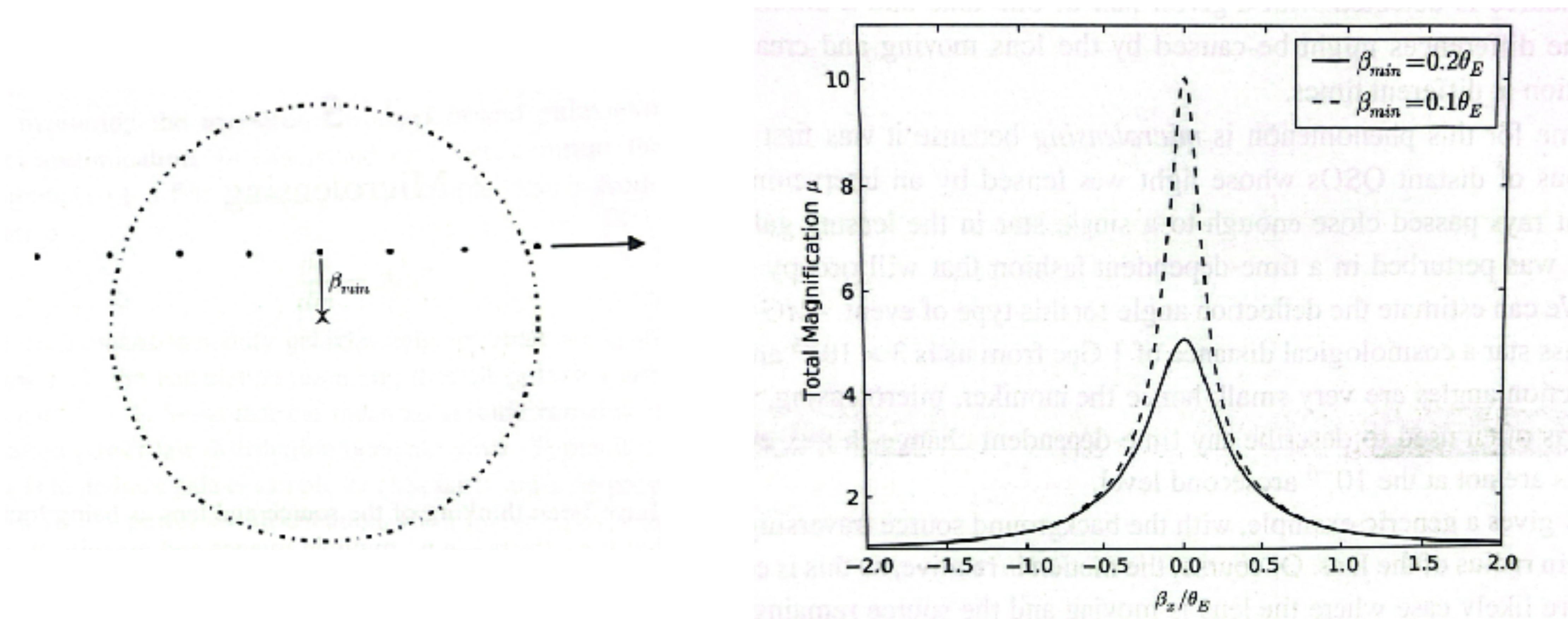
- 3D mapping of DM
- Need source populations at different redshifts



Massey et al. 2007

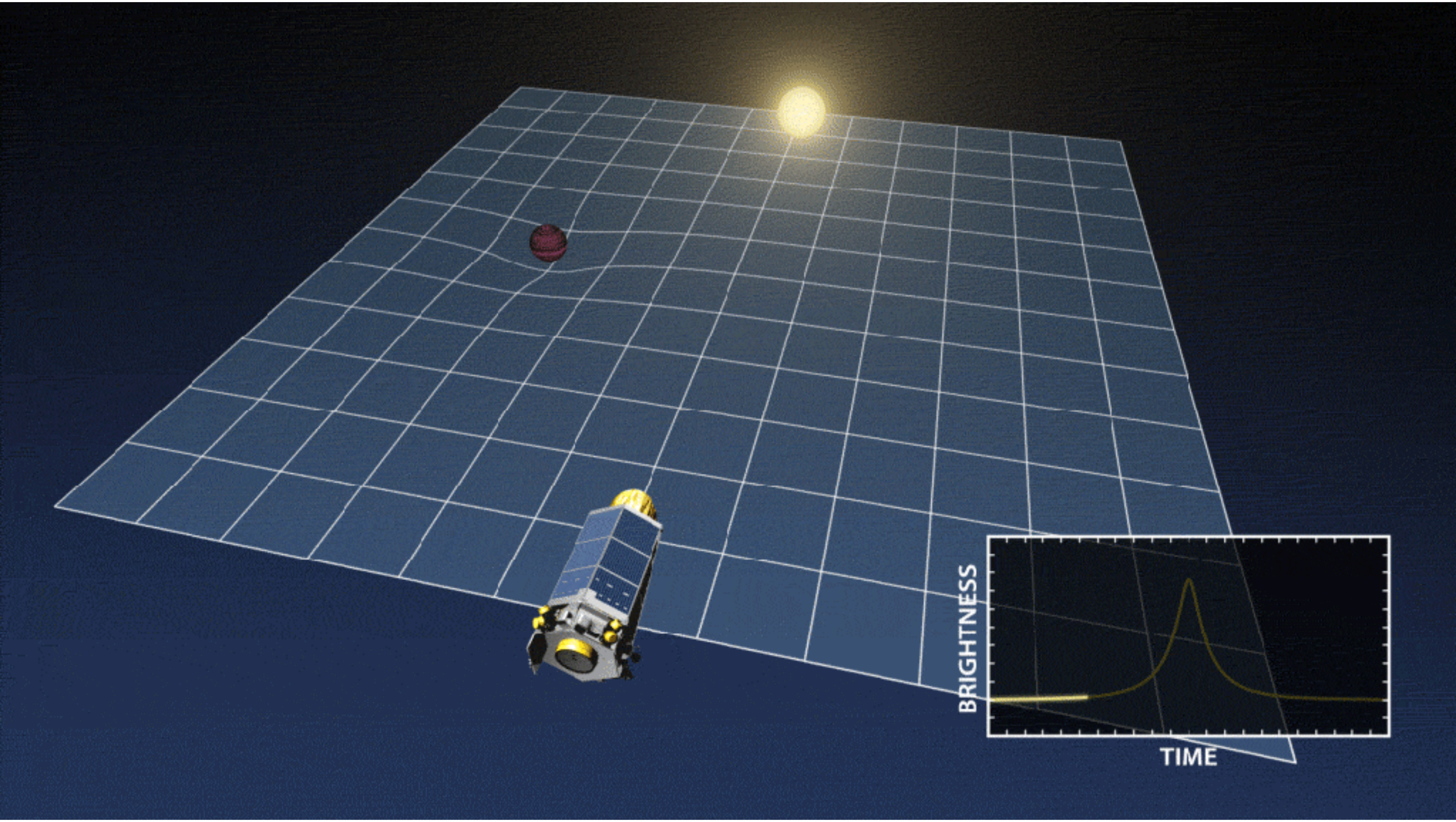
Microlensing

- Relevant position of the lenses and sources change with time —> magnification will change with time
- Brightening and dimming effects



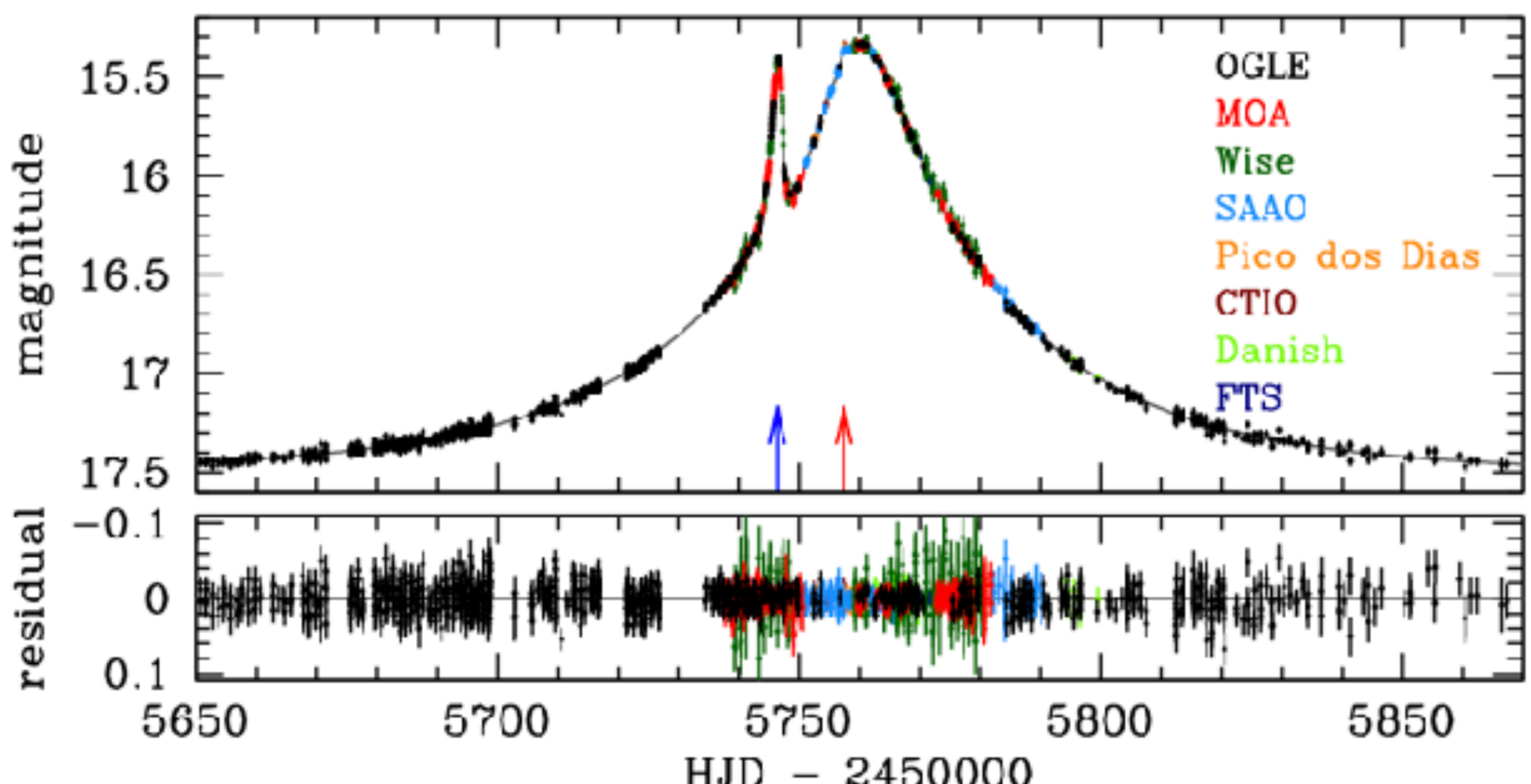
Gravitational Lensing, Scott Dodelson

Microlensing

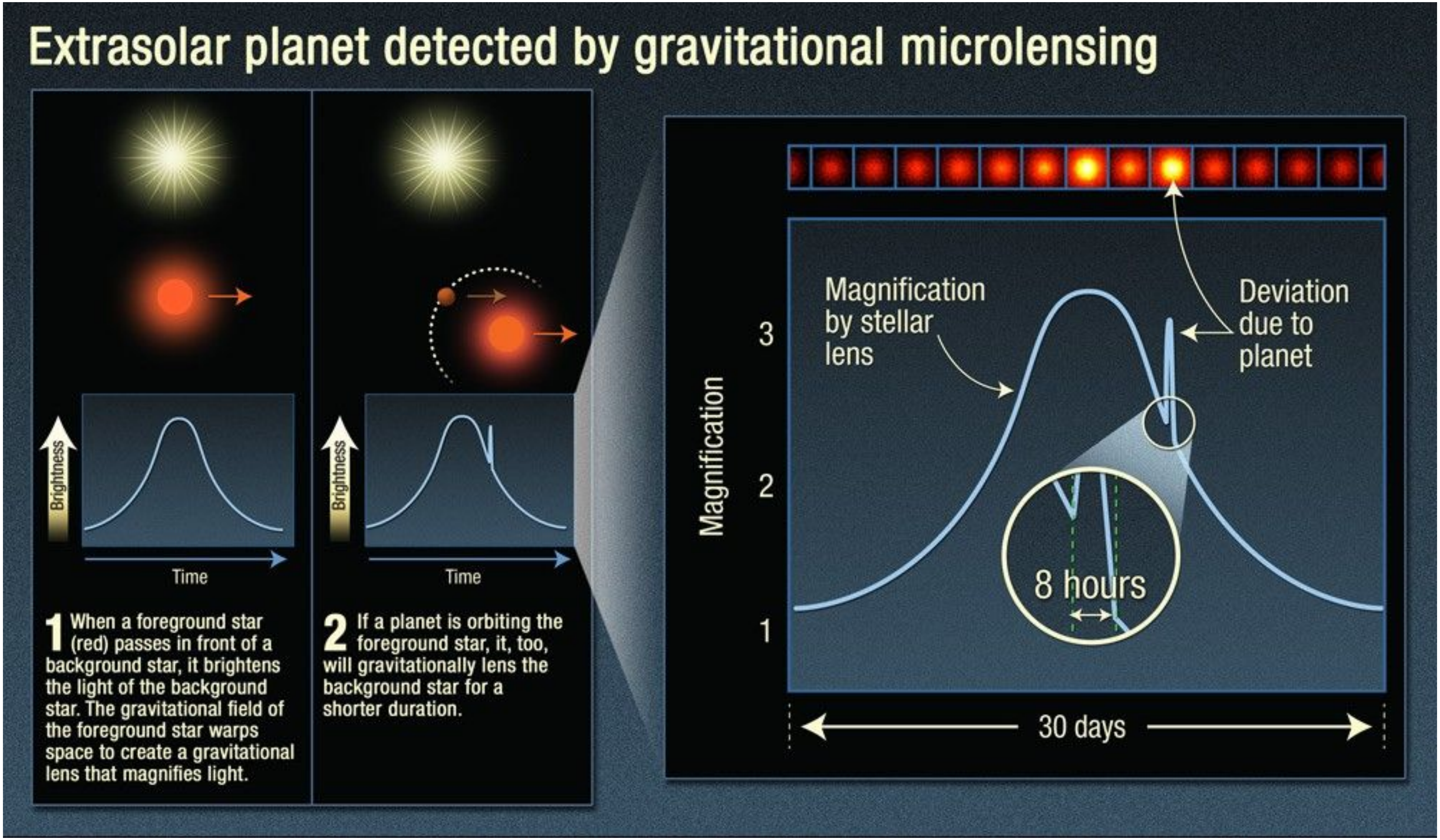


Application: microlensing

- Search for exoplanets: planetary microlensing
  —> cause perturbation in magnification
- A planet orbits the microlensing star, it can cause a secondary, smaller amplification event in the light curve.



Skowron et al. 2015

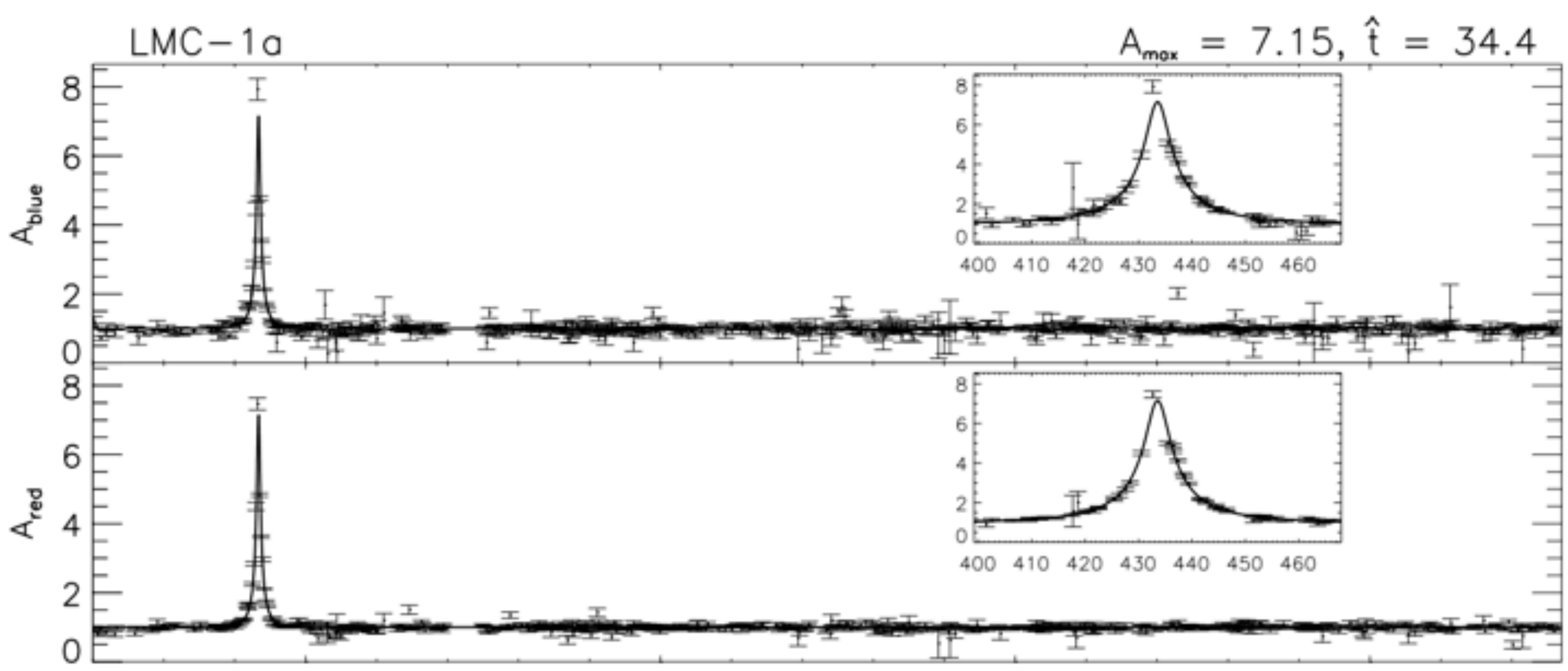


Credit: NASA, ESA, And K. Sahu (STScI)

Application: microlensing

- determine if dark matter consisting of massive compact halo objects (MACHOs): the number of events determine the number density of MACHOs

- Number of events—> how many MACHOs exit in the Galactic halo
  Event Duration & Peak Magnification —> about the masses



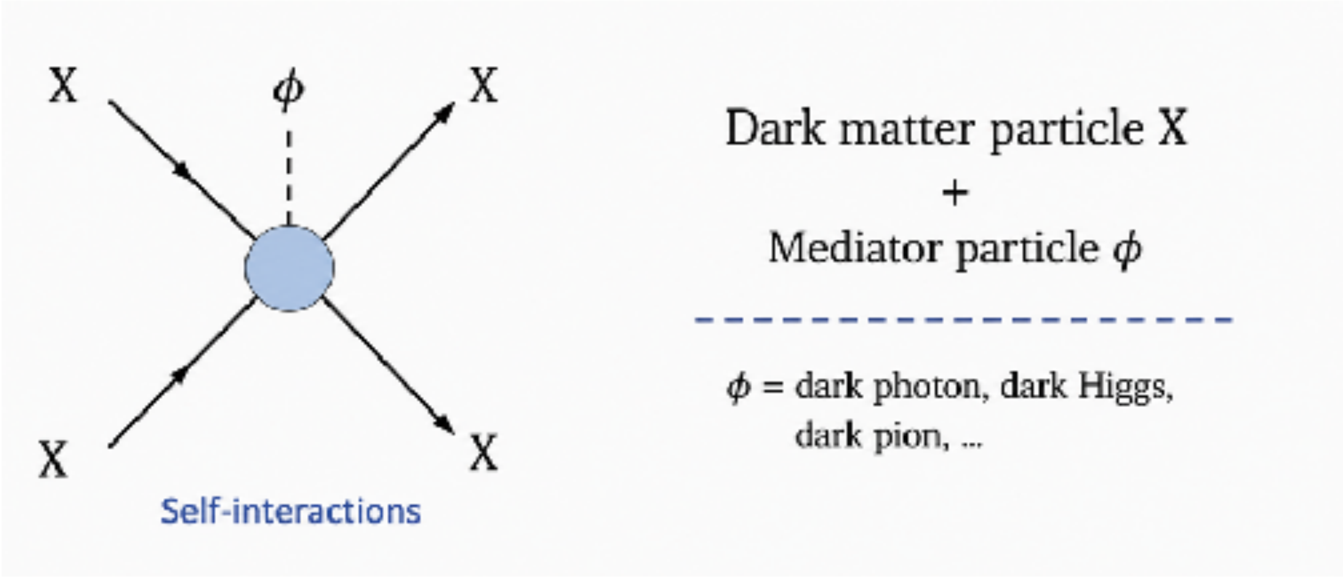
Alcock et al. 2000 (MACHO project)

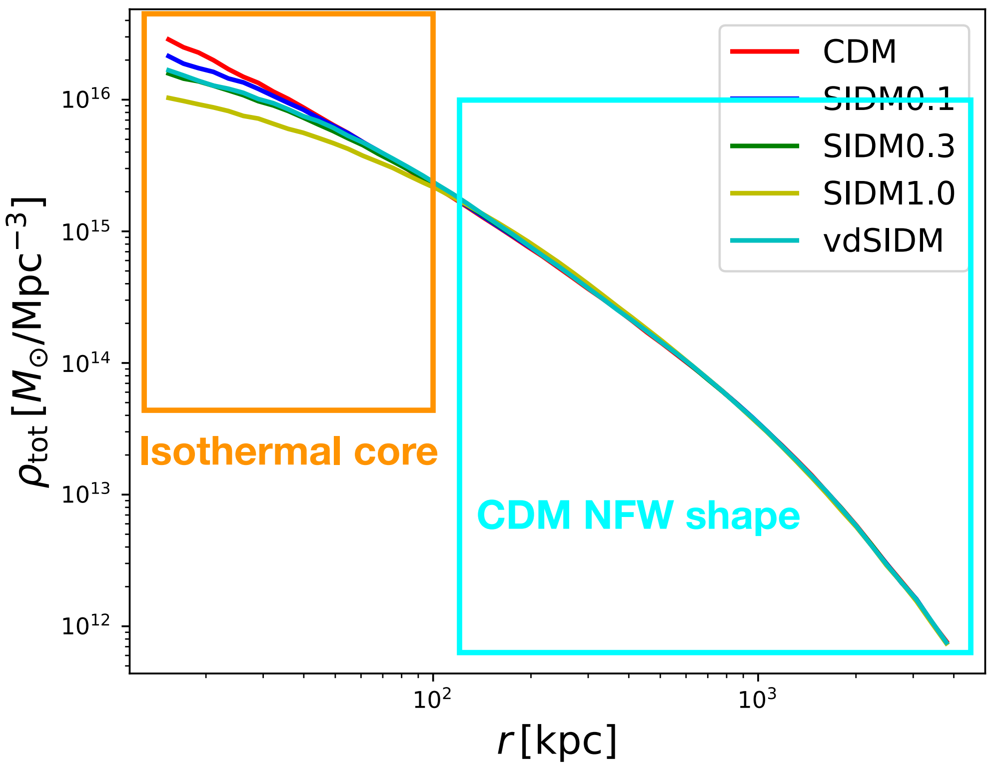
SIDM model

- SIDM was proposed to solve the small-scale problems while preserving the successful predictions on large-scales in the ΛCDM model.
- Dark matter particles in halos elastically scatter with other dark matter particles. (Spergel & Steinhardt 2000)

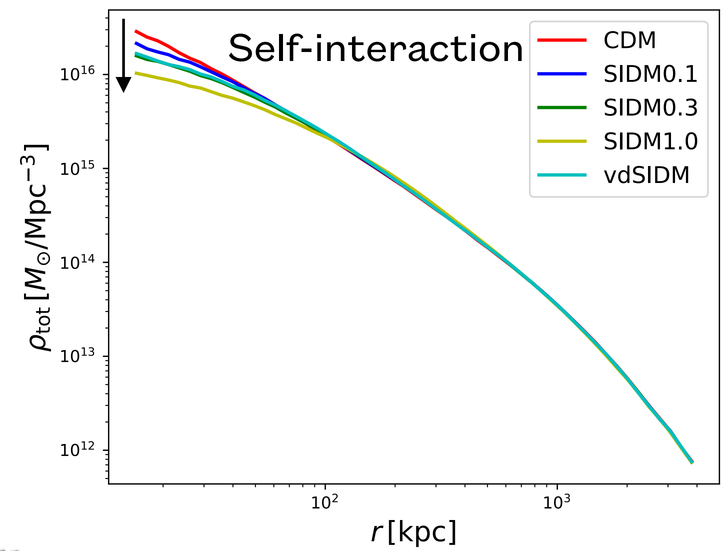
scattering rate = σ (ρ/m) velocity

$\Gamma \, t_{\rm age} \sim 1$



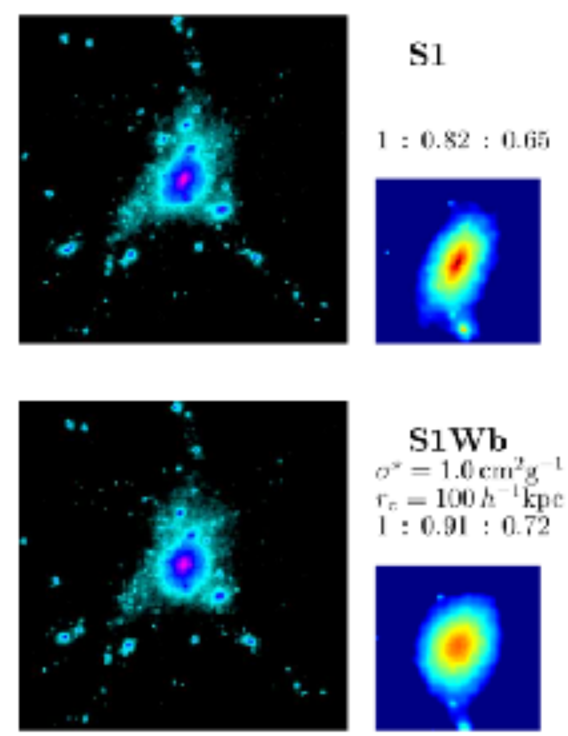


Use Gravitational Lensing to Constrain Self-Interacting Cross-Section



- Inner density profile—central slope of the internal density profile of 12 massive X-COP galaxy clusters —> SIDM with σ/m < 0.19 cm²/g (Eckert et al. 2022)

- Dark matter halo shape σ/m < 1 cm²/g (Peter et al. 2013)

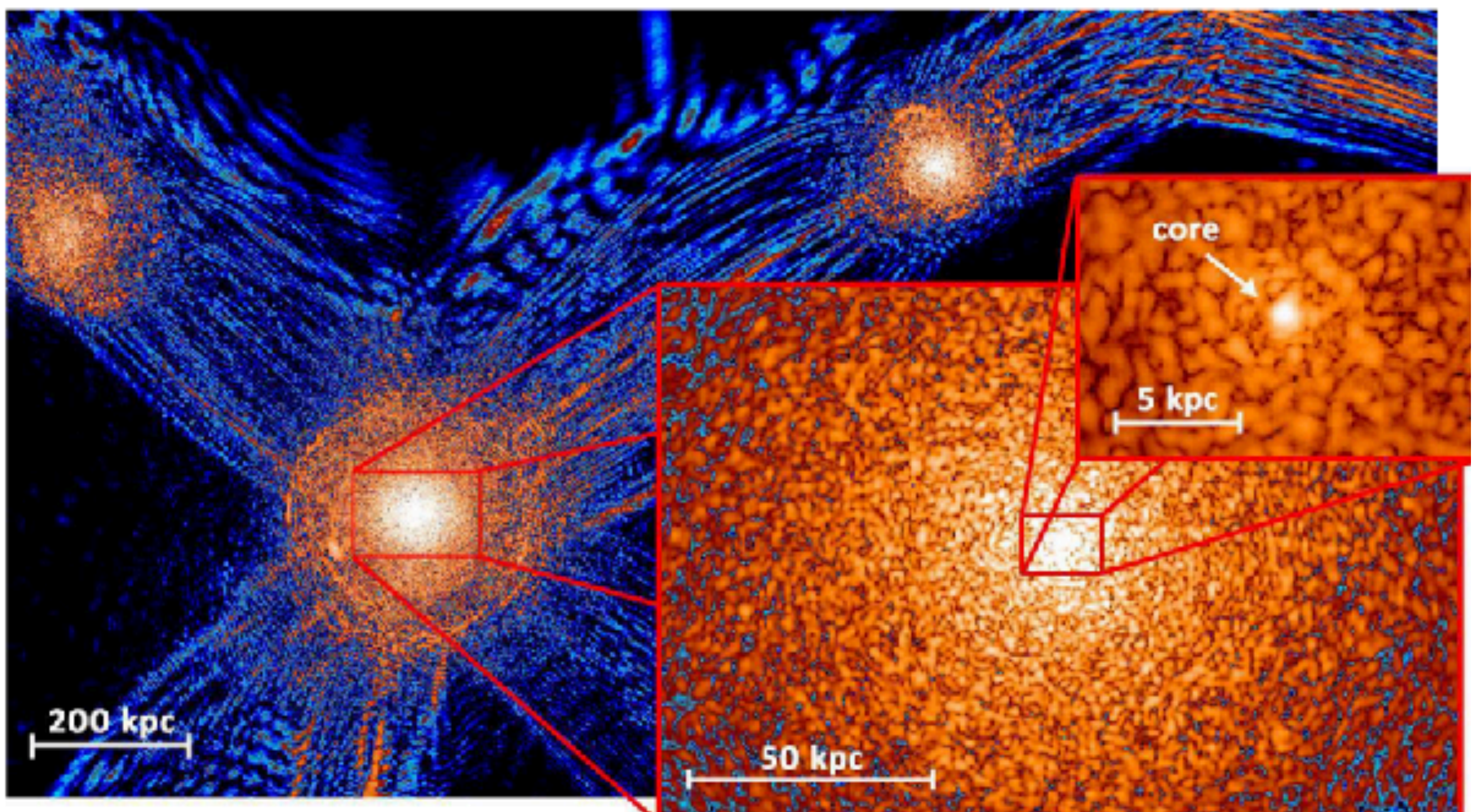


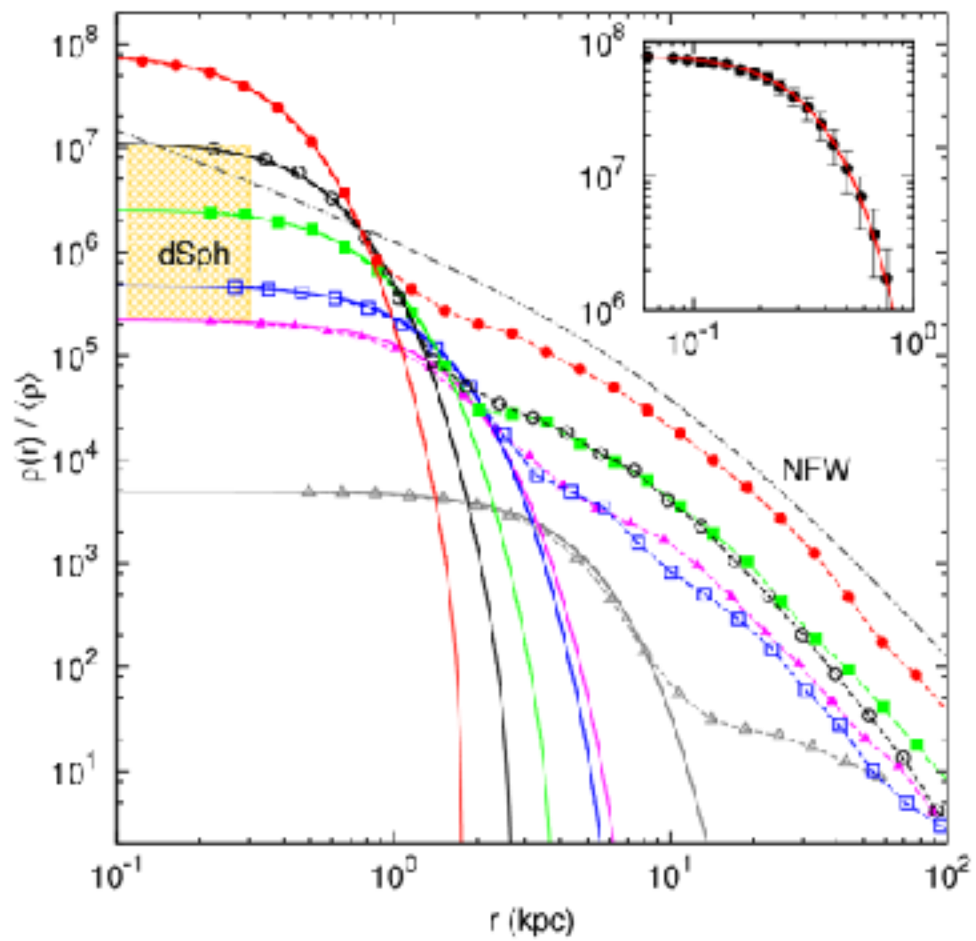
Yoshida et al. 2000

- Offset between dark matter and baryons in merging systems:
  σ/m < 1.25 cm²/g (Randall et al. 2007)
  σ/m < 0.47 cm²/g (Harvey et al. 2015)

Probing FDM with Gravitational Lensing?

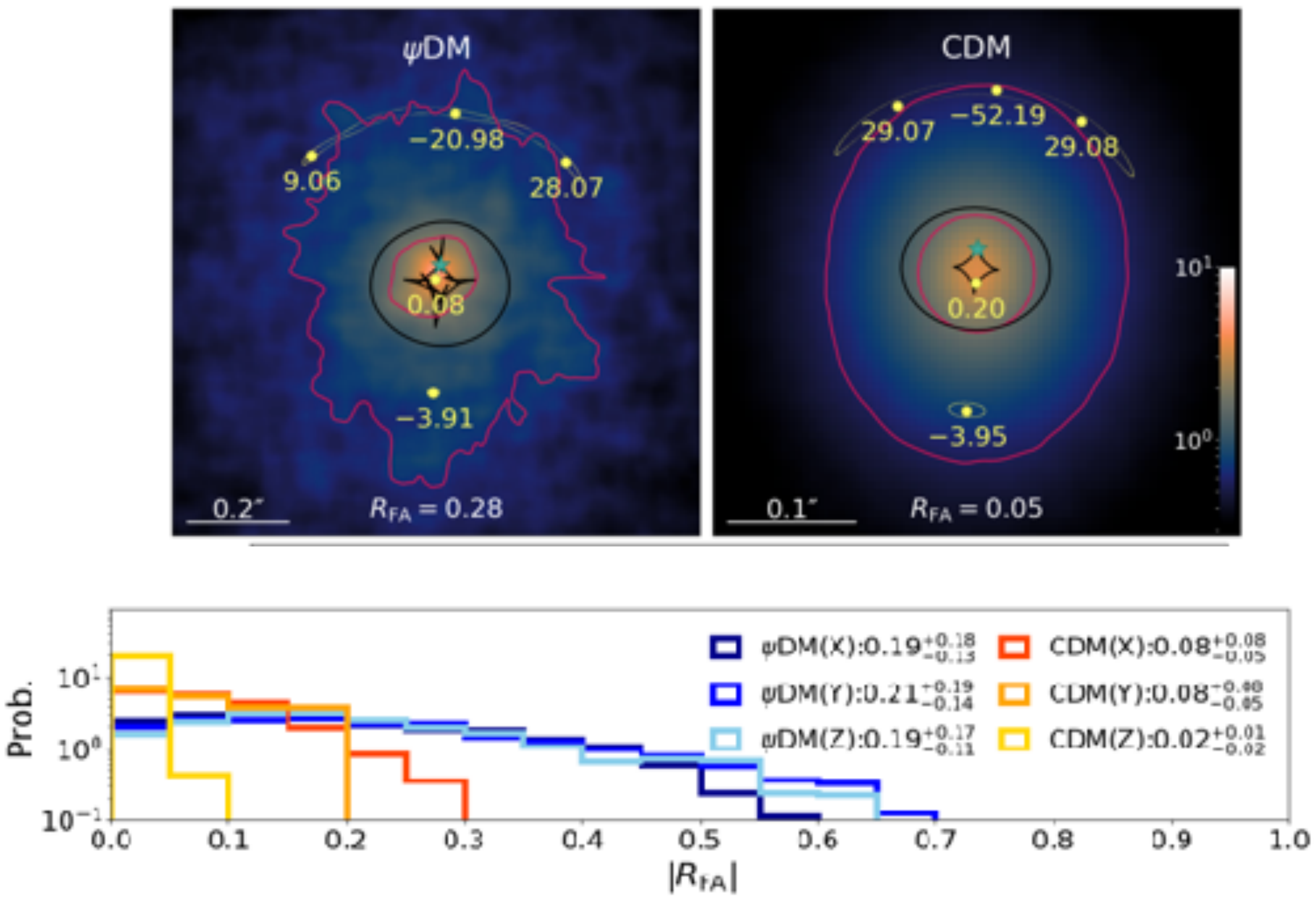
- Fuzzy dark matter
  —ultra-light DM that exhibits a ~kpc-scale de Broglie wavelength
  —> Suppressed halo mass function at low masses; produced Cored density profiles; Granules structures due to wave interference



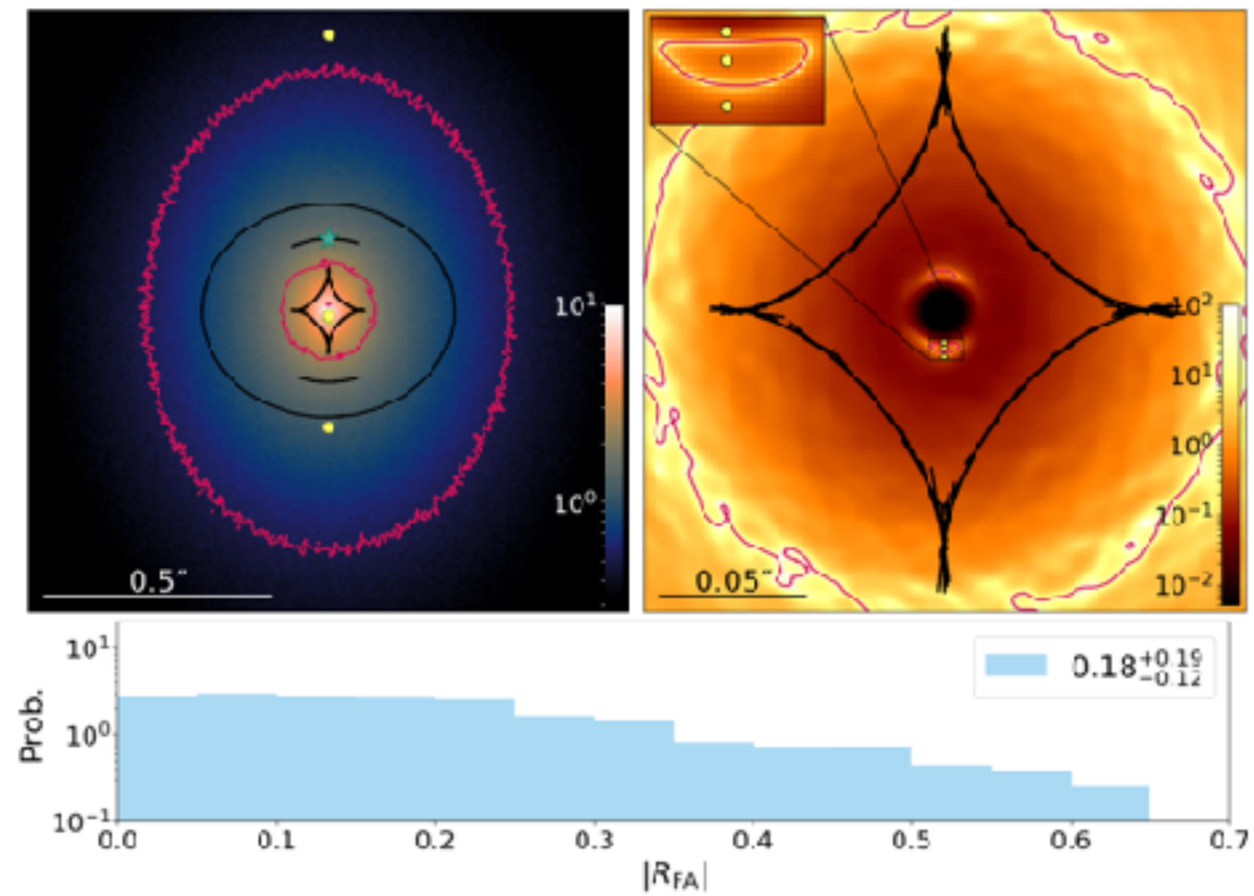


Schive, Chiueh, & Broadhurst 2014, Nature Physics, 10, 496

Fluctuating critical curve in the fuzzy lens

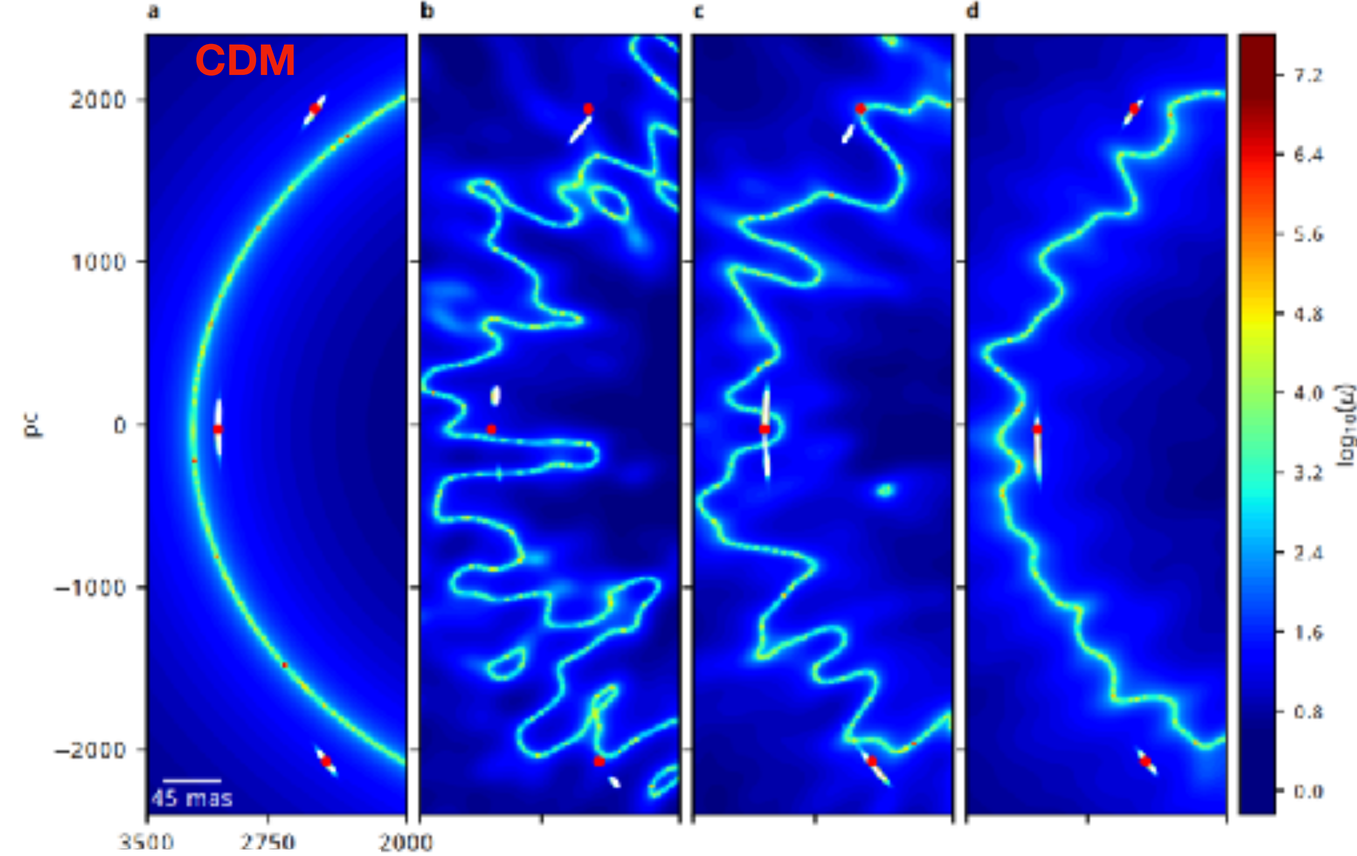


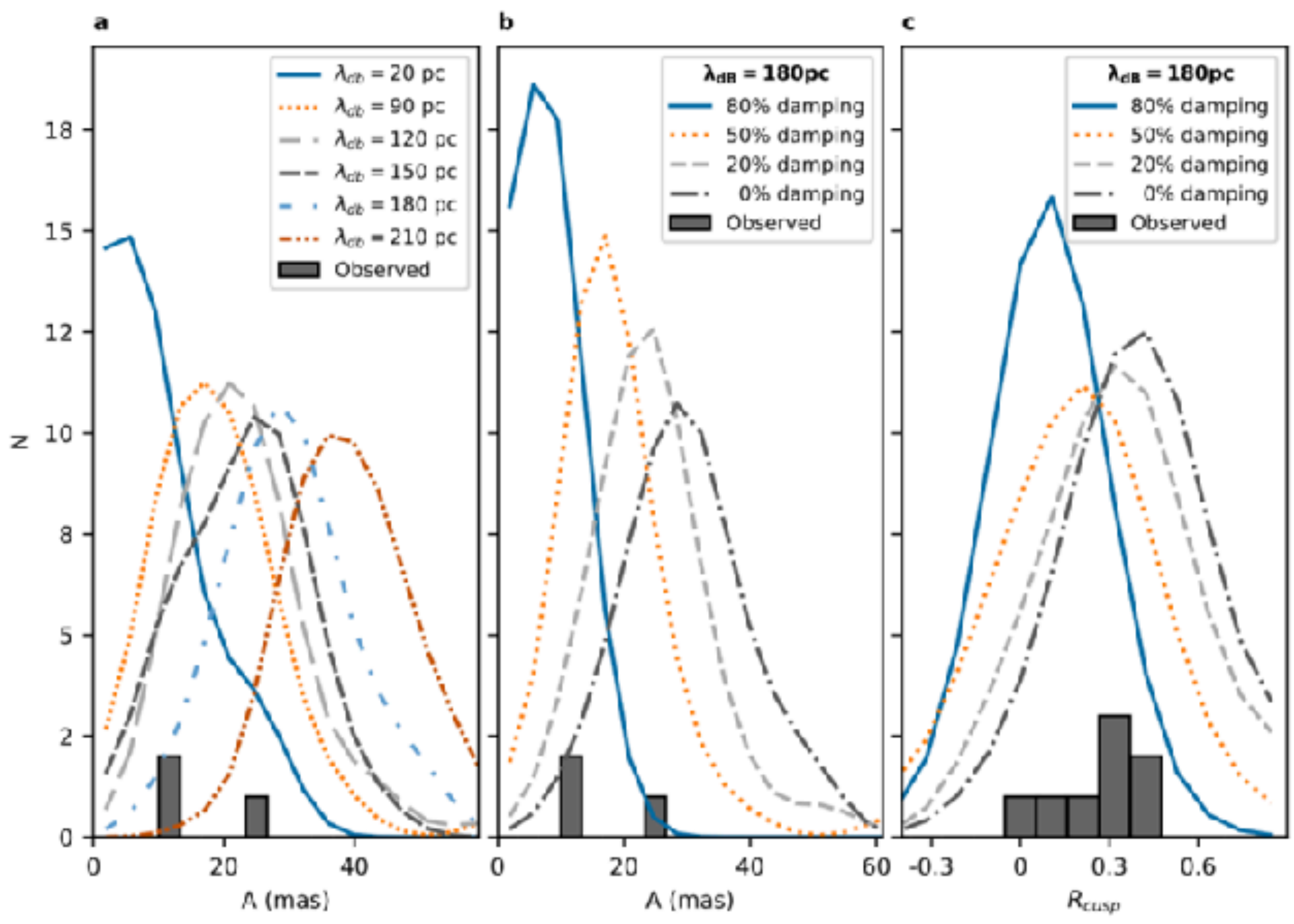
Axion mass $m_a = 8 \times 10^{-22}$ eV



Flux ratio anomalies: $R_{\rm FA} = \sum_j \mu_j / \sum_j |\mu_j|,$

(Chen et al. 2020)





Probability distribution of position, A, and brightness, Rcusp,

Summary

- Gravitational Lensing:
  —> bending of light ray by strong gravity (massive objects)
  —> powerful tool from exoplanets to cosmology

- Strong Lensing
  —> inner region of dark matter halo
  —> High redshift Universe
  —> Cosmography - H0 determination

- Weak Lensing
  —> Dark matter distribution in large-scale
  —> Weak Lensing Mass Calibration for Cosmological studies

- Microlensing
  —> Detection of small massive compact objects, faint stars, MACHOs
  —> Search for exoplanets

Gravitational lensing provides a crucial test of dark matter models.