

General Relativity & Cosmology

廣義相對論及宇宙論

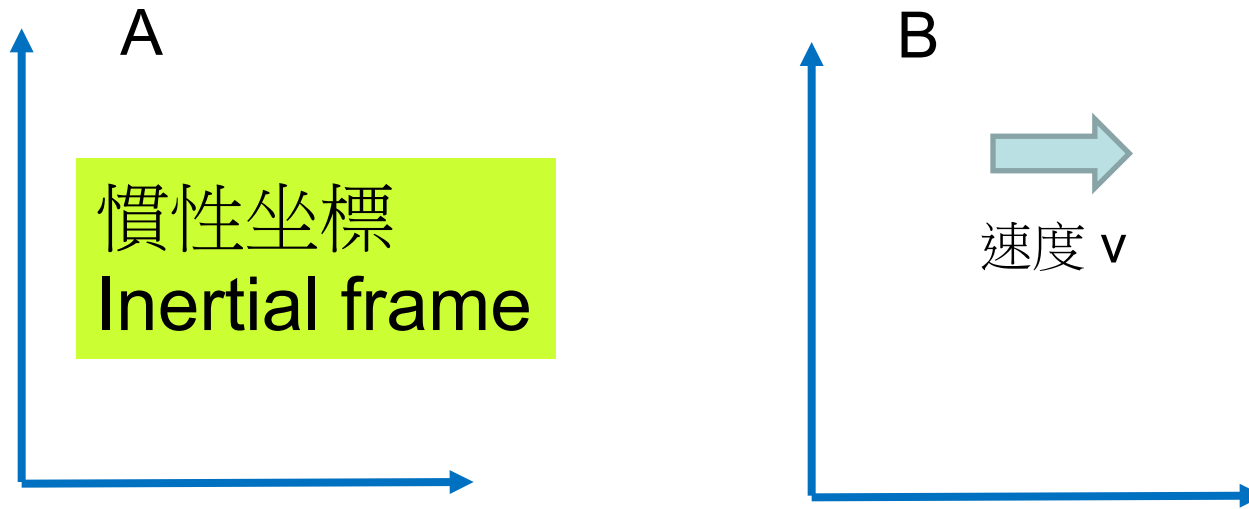
Kin-Wang Ng (吳建宏)



ASIoP (中央研究院物理所) &
ASIAA (中央研究院天文所)

NCTS-HEP workshop 2026
NCTS-NTU
July 1-3, 2026

Special Relativity 狹義相對論



A坐標:
A先生發現並測
試了物理定律

B坐標:
B小姐發現並測
試了物理定律

1905 愛因斯坦: 他們的物理定律完全相同，光速是速度極限的普遍常數。

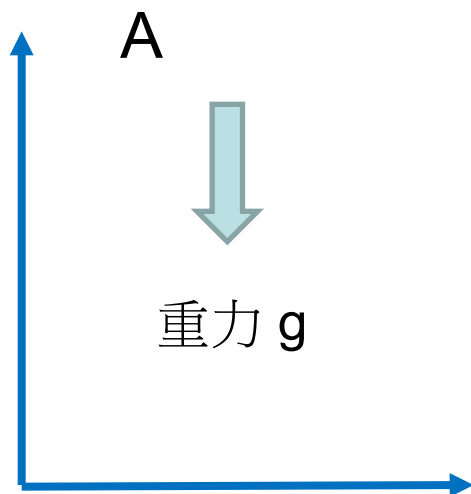
Galileo伽利略變換到Lorentz勞侖茲變換

$$\left\{ \begin{array}{l} x' = x - vt \\ y' = y \\ z' = z \\ t' = t \end{array} \right. \quad \longrightarrow \quad \left\{ \begin{array}{l} x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \\ y' = y \\ z' = z \\ t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}} \end{array} \right.$$

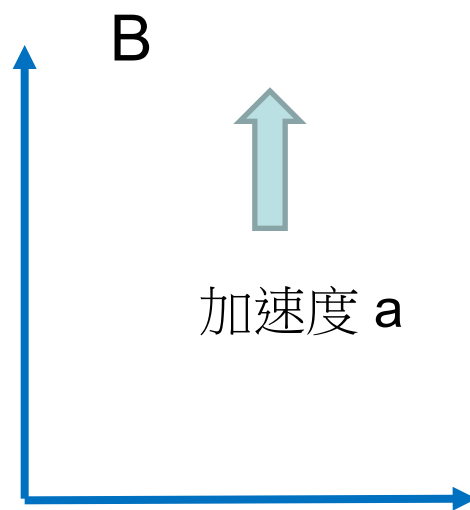
能量質量交換

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad E = m_0 c^2$$

General Relativity 廣義相對論

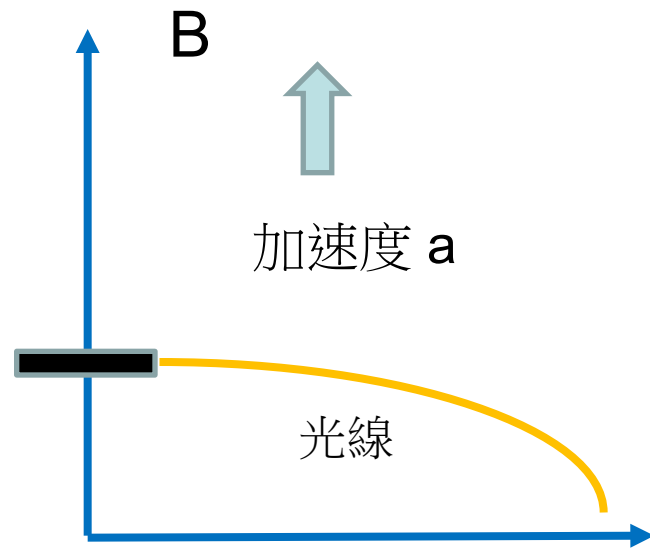
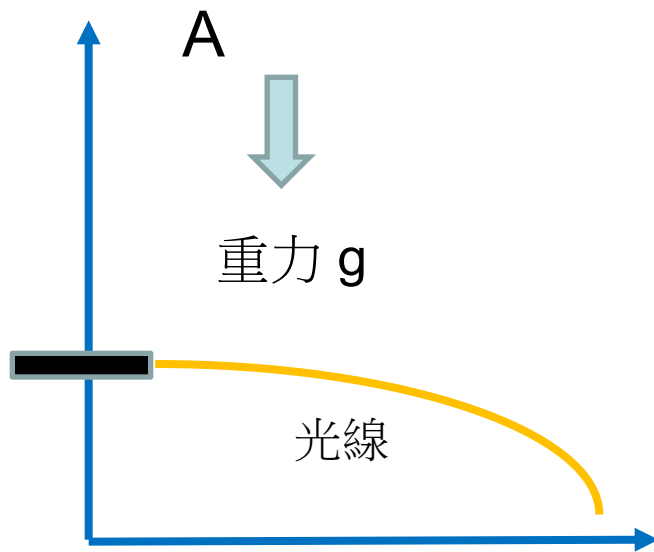


A坐標:
A先生發現並測
試了物理定律

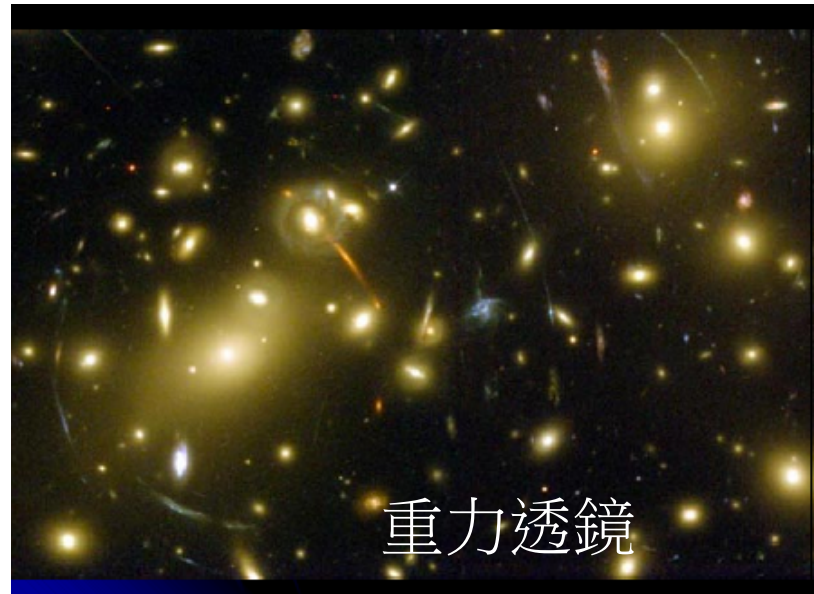
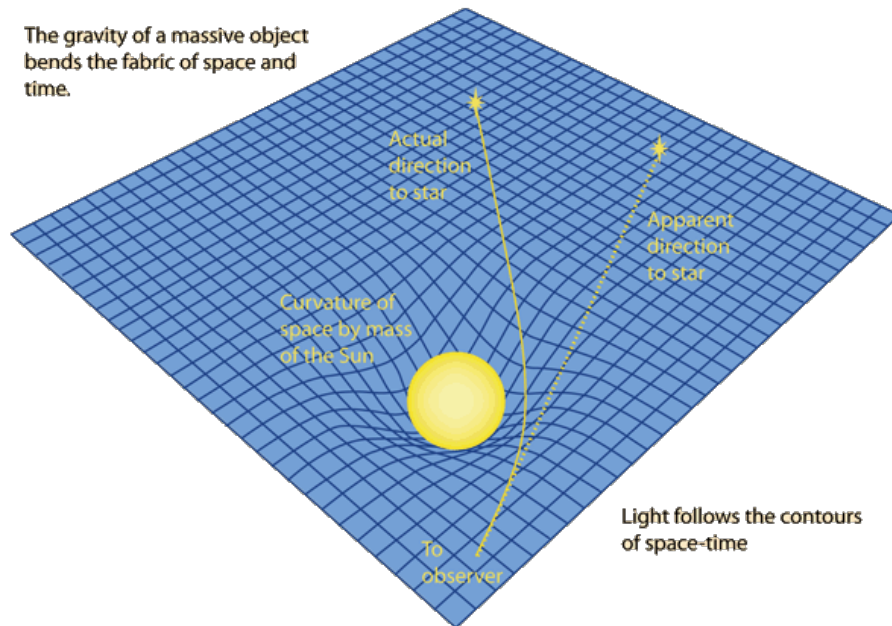


B坐標:
B小姐發現並測
試了物理定律

1915 愛因斯坦: 他們的物理定律完全相同，光速是速度極限的普遍常數。

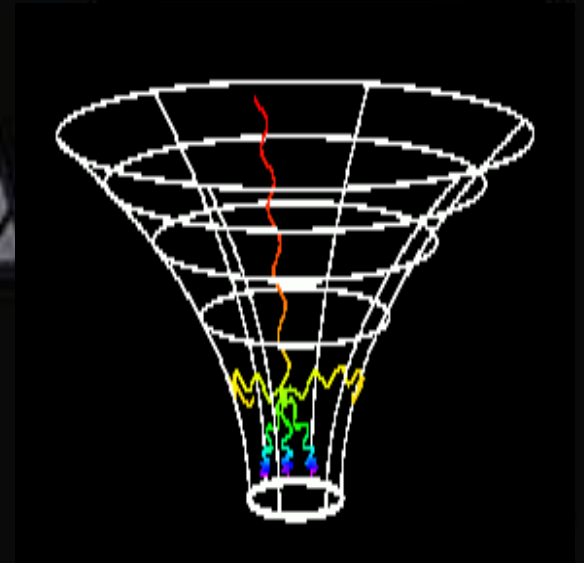
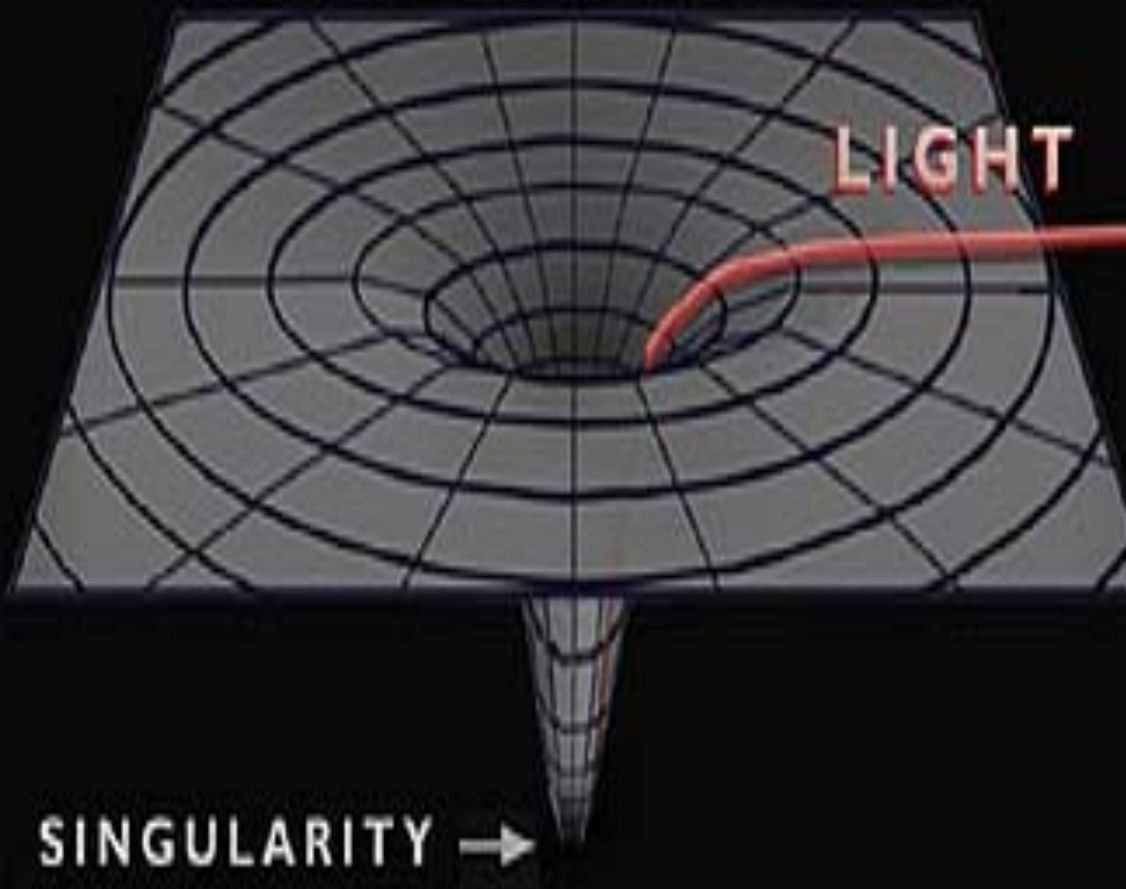


The gravity of a massive object bends the fabric of space and time.



GRAVITATIONAL RED SHIFT

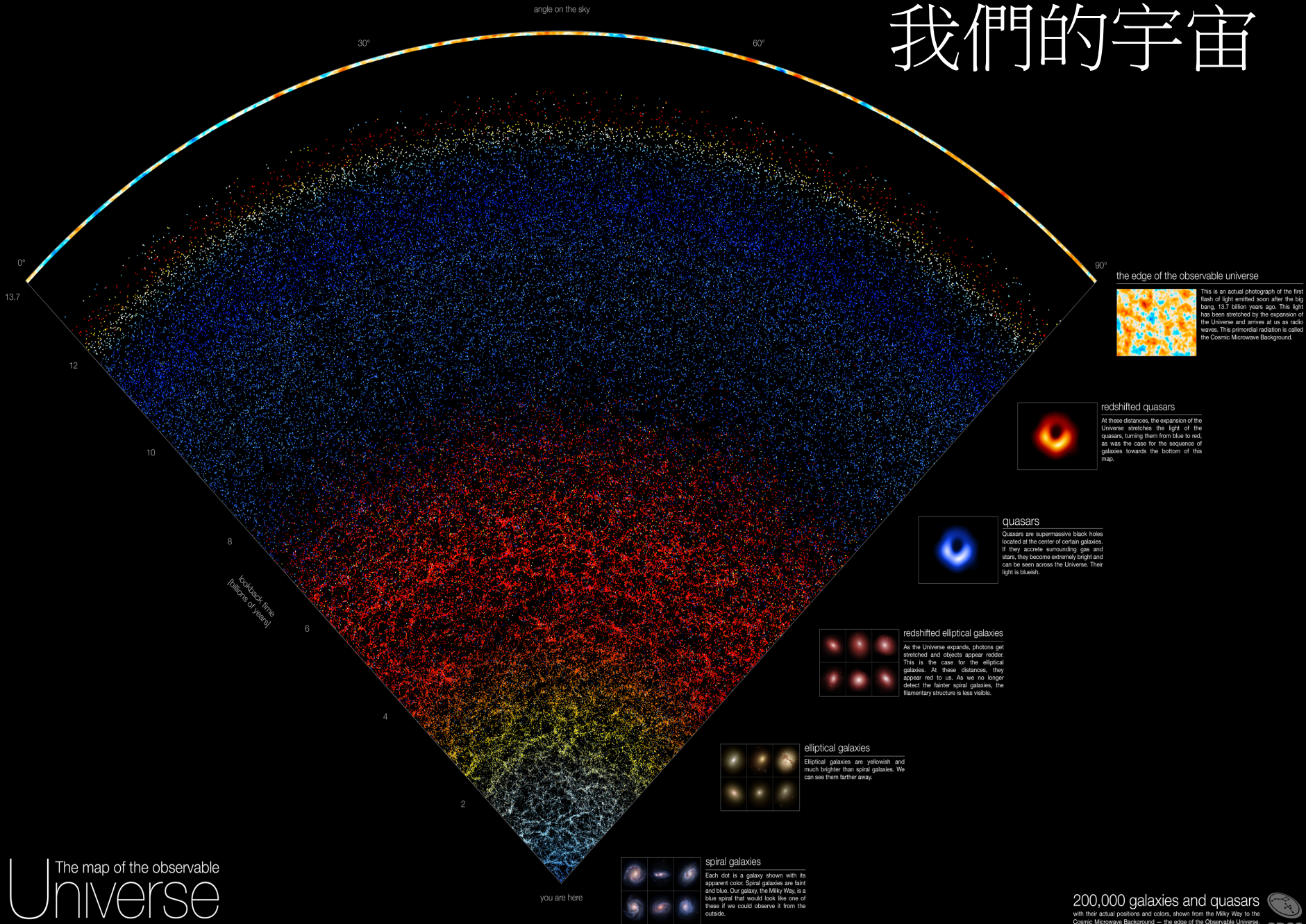
<http://archive.ncsa.uiuc.edu/Cyberia/NumRel/Images/redshift.jpeg>

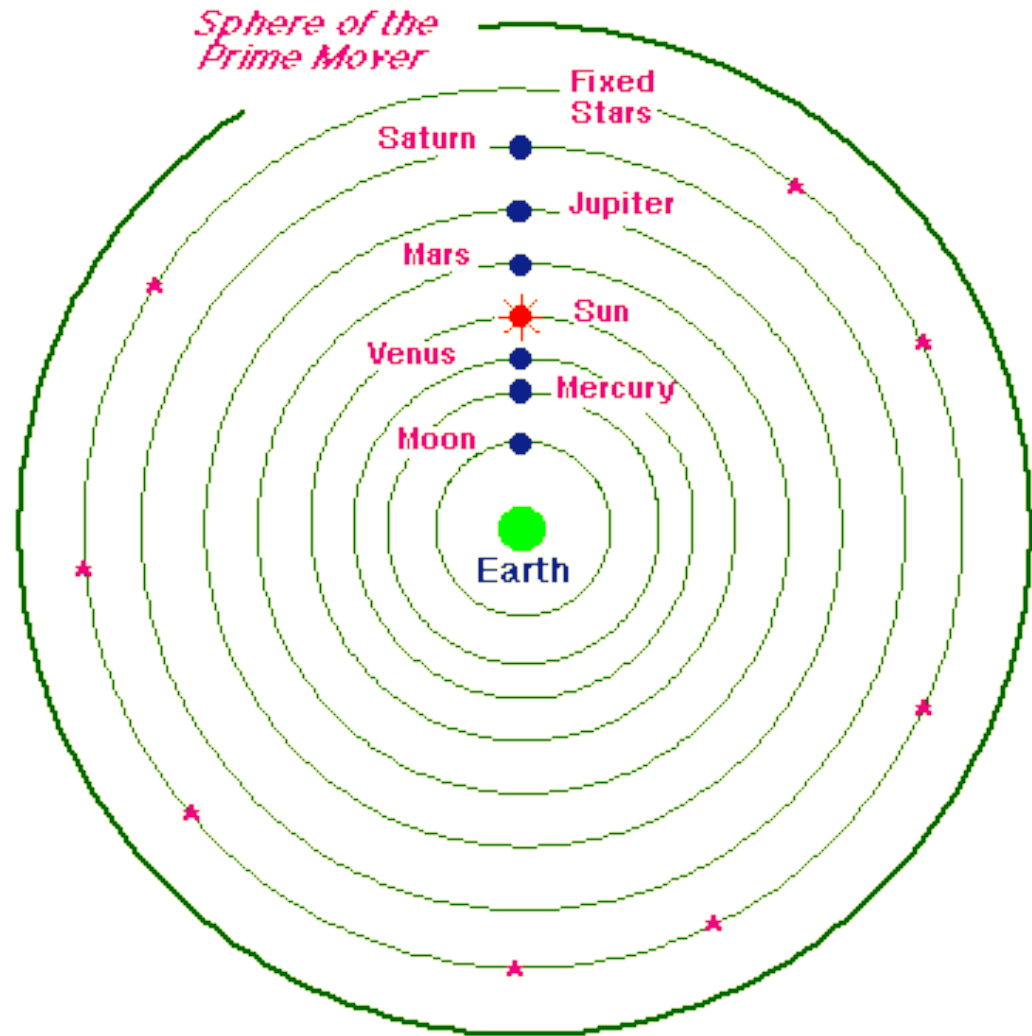


http://casa.colorado.edu/~ajsh/redsbig_gif.html

Tested by : white dwarf 1924, solar 1960s, earthbound 1960, and neutron stars

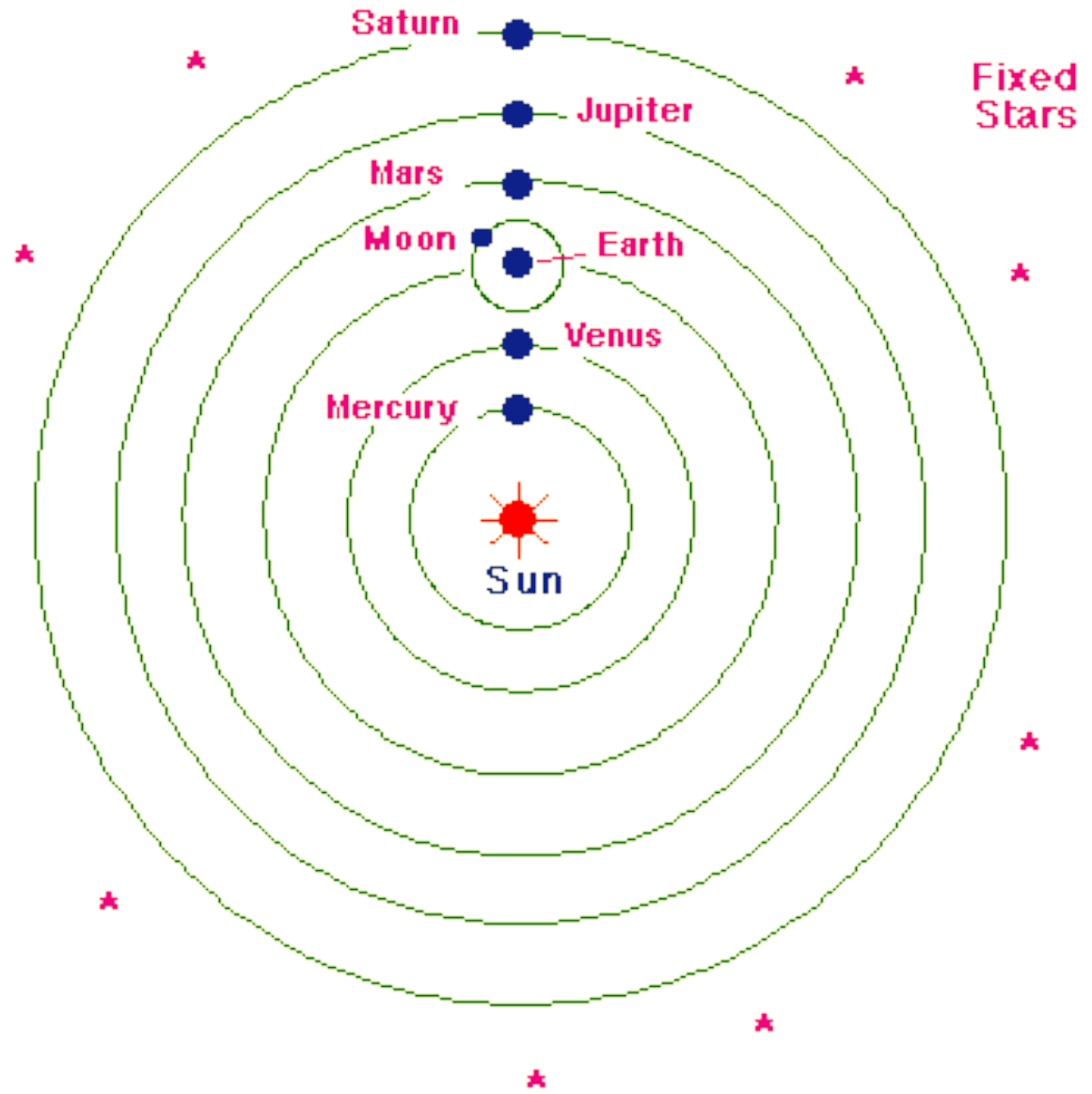
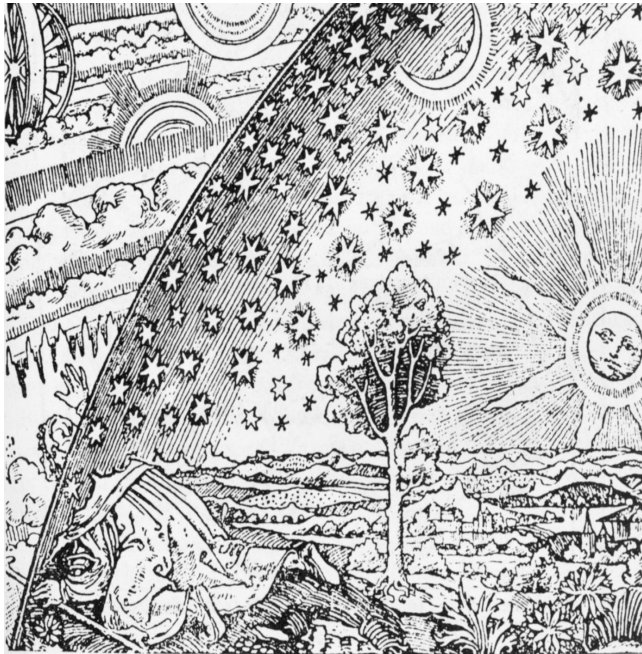
我們的宇宙





Aristotle's Universe

亞里斯多德(400 B.C.)地心體系(Geocentric)



哥白尼(Nicolaus Copernicus, 1473-1543)日心體系(Heliocentric)

Hubble's Law 哈勃定律

- 膨脹的宇宙



(1929)

Hubble's Law:

$$z = \frac{H_0}{c} r$$

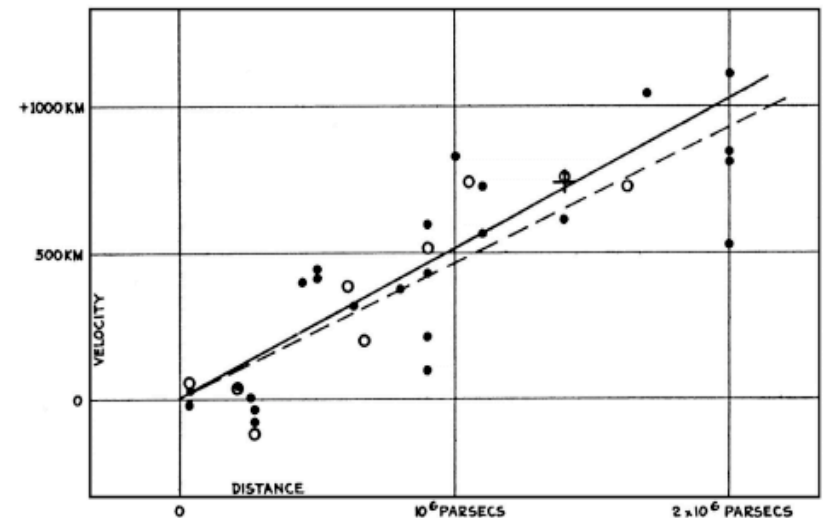
or

$$v_r = H_0 r$$

From the diagram:

$$H_0 = 500 \text{ km s}^{-1} \text{ Mpc}^{-1} \quad \text{宇宙膨脹速度參數}$$

Hubble Diagram



Metric

$$\left[\begin{array}{l} \text{Units:} \quad \hbar = c = k_B = 1 \\ 1 \text{ GeV} = 5.07 \times 10^{13} \text{ cm}^{-1} = 1.52 \times 10^{24} \text{ s}^{-1} = 1.16 \times 10^{13} \text{ K} \end{array} \right]$$

$$d\tau^2 = -g_{\mu\nu} dx^\mu dx^\nu \quad \mu, \nu = 0, 1, 2, 3$$

$$\text{where } g^{\mu\nu} g_{\nu\lambda} = \delta^\mu_\lambda \quad \& \quad T^{\mu\lambda} g_{\lambda\nu} = T^\mu_\nu \text{ for a tensor } T^{\mu\nu}$$

e.g.

$$\text{A flat metric } g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{For light } 0 = d\tau^2 = dt^2 - d\vec{x}^2 \Rightarrow \left| \frac{d\vec{x}}{dt} \right| = 1$$

Expanding Universe

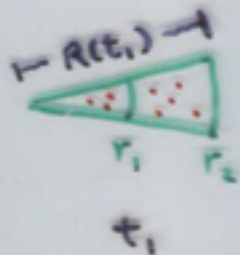
Robertson-Walker metric

$$d\tau^2 = dt^2 - R^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right]$$

$$k = \begin{cases} +1 & \text{closed} \\ -1 & \text{open} \\ 0 & \text{flat} \end{cases}$$

$R(t)$: Scale factor

(r, θ, ϕ) : Comoving coordinates



History of the Universe



宇宙演化的標準模型

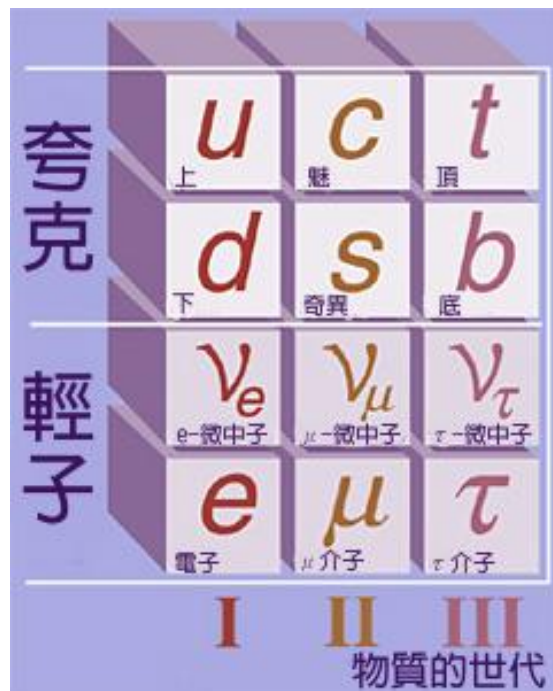
星系的形成

由基本粒子
構成的高溫
等離子體

大霹靂

粒子物理的標準模型

基本粒子



+反物質

+



載力者

希格斯粒子

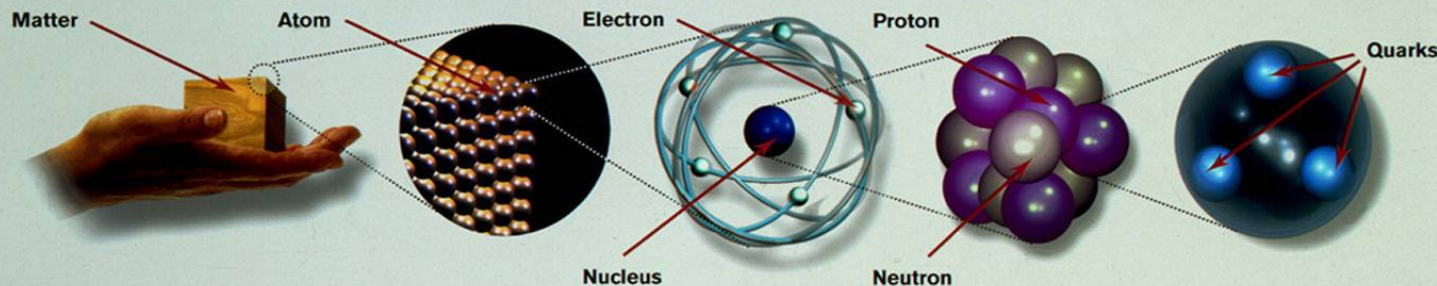
Higgs

(07/04/2012)

「已瞭解」的萬物

電磁力
原分子

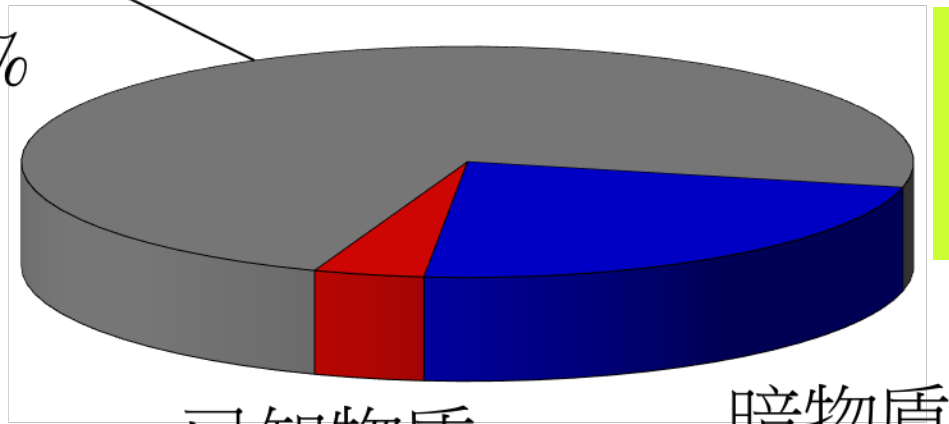
強作用力
原子核



宇宙的物質與能量

暗能量

73%



90% 以上不發光
但可以利用重力
得知它們的存在

已知物質

4%

暗物質

23%

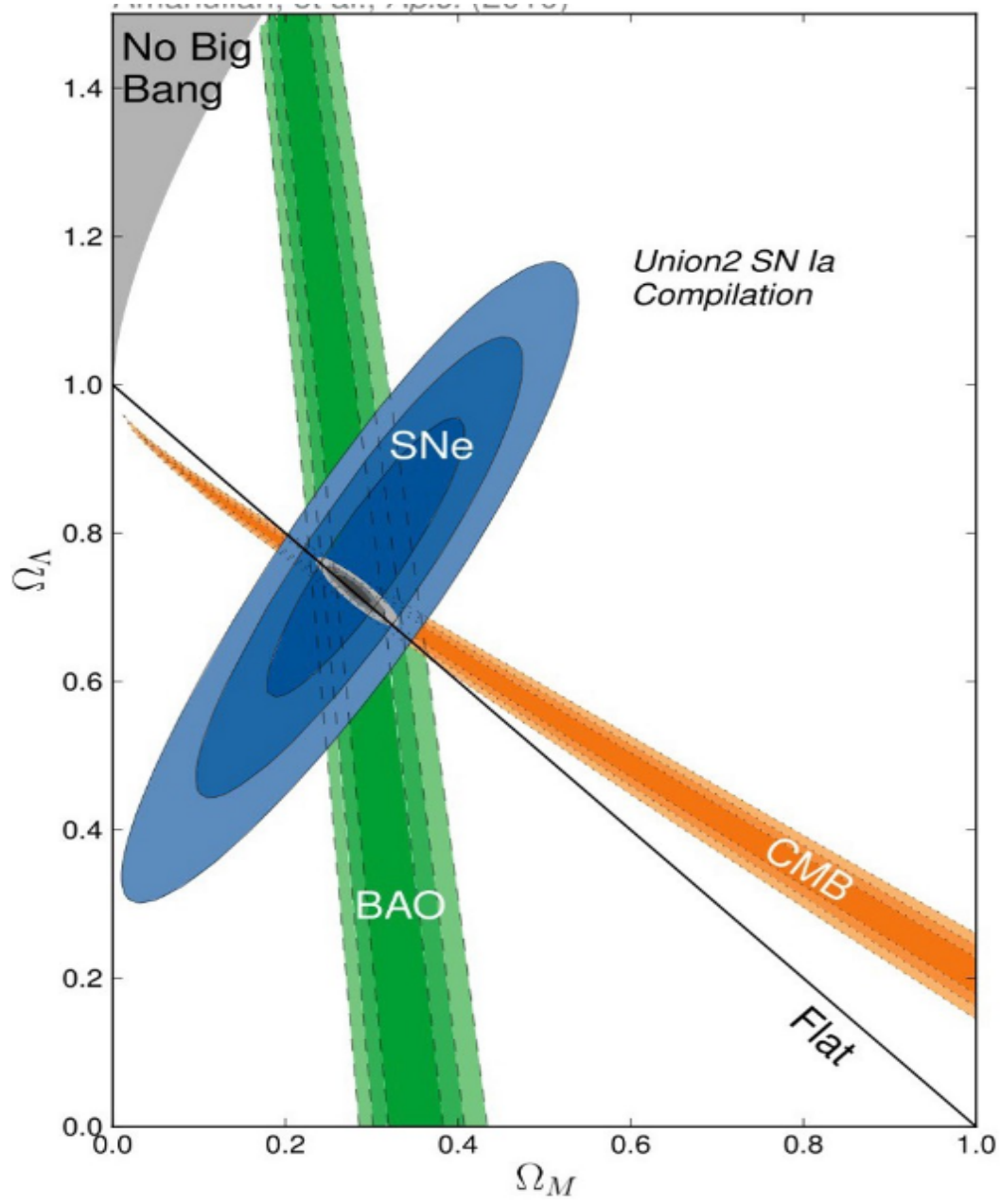
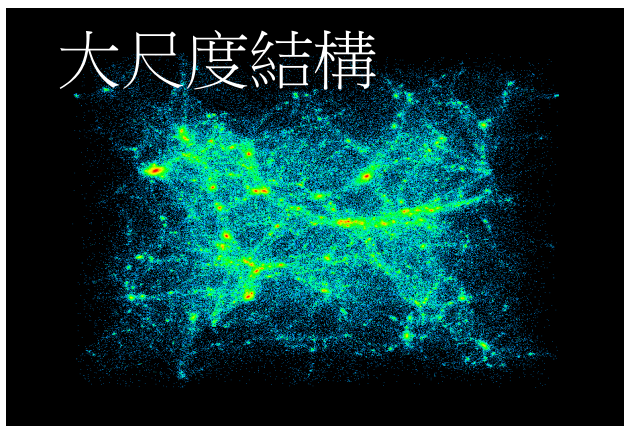
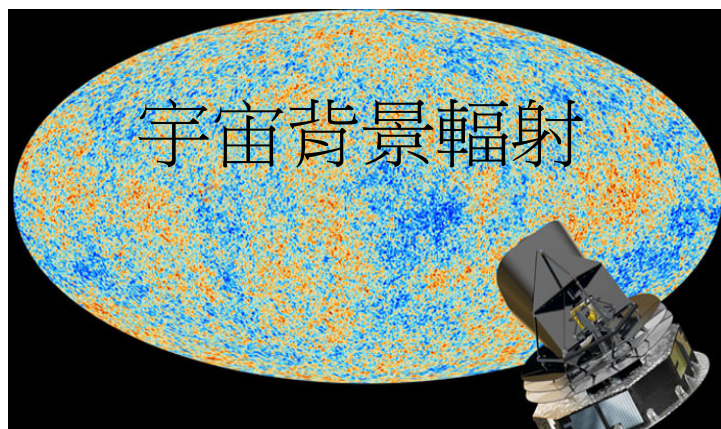
暗物質存在的證據

大尺度結構

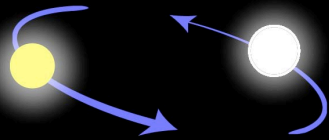
重力透鏡

星系團對撞

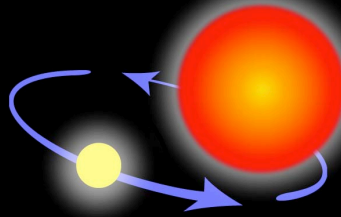
暗能量存在的證據



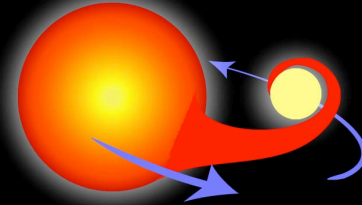
The progenitor of a Type Ia supernova



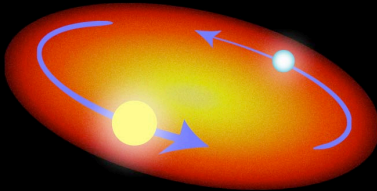
Two normal stars are in a binary pair.



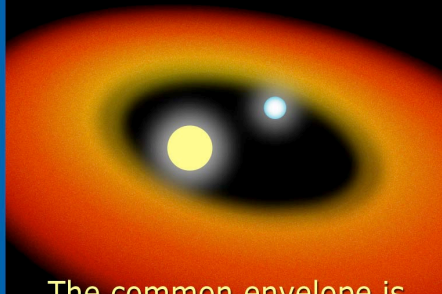
The more massive star becomes a giant...



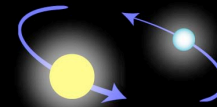
...which spills gas onto the secondary star, causing it to expand and become engulfed.



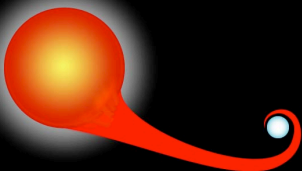
The secondary, lighter star and the core of the giant star spiral inward within a common envelope.



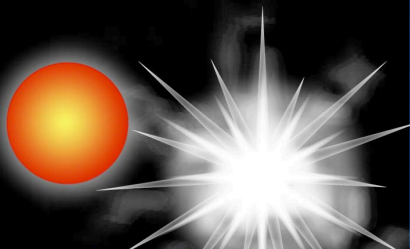
The common envelope is ejected, while the separation between the core and the secondary star decreases.



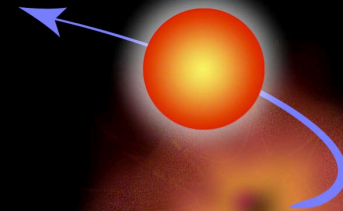
The remaining core of the giant collapses and becomes a white dwarf.



The aging companion star starts swelling, spilling gas onto the white dwarf.

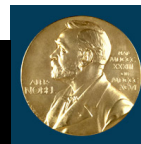


The white dwarf's mass increases until it reaches a critical mass and explodes...

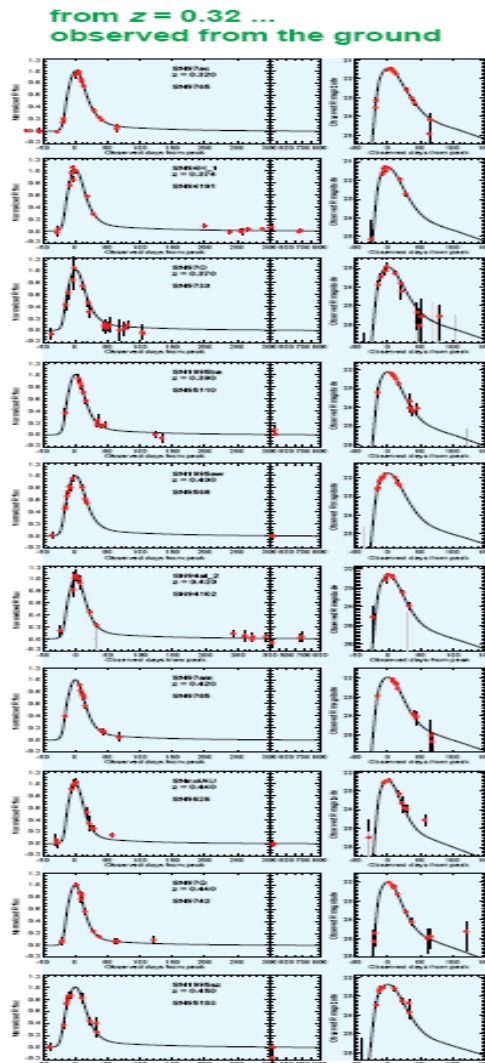


...causing the companion star to be ejected away.

保證 Ia 型超新星可做為 標準燭光 測量遙遠星系的距離



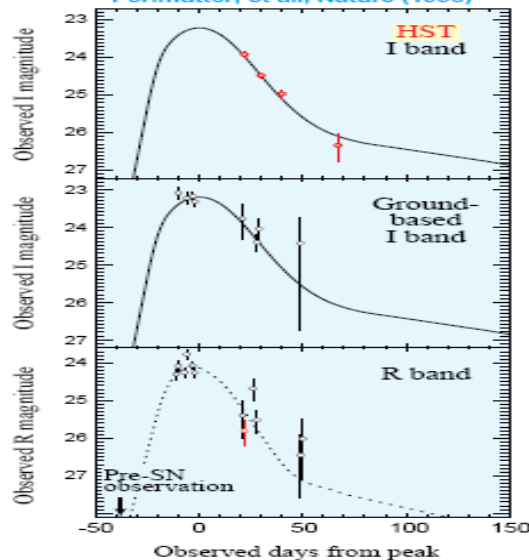
Nobel Prize in Physics 2011
Perlmutter, Brian P. Schmidt, Adam G. Riess



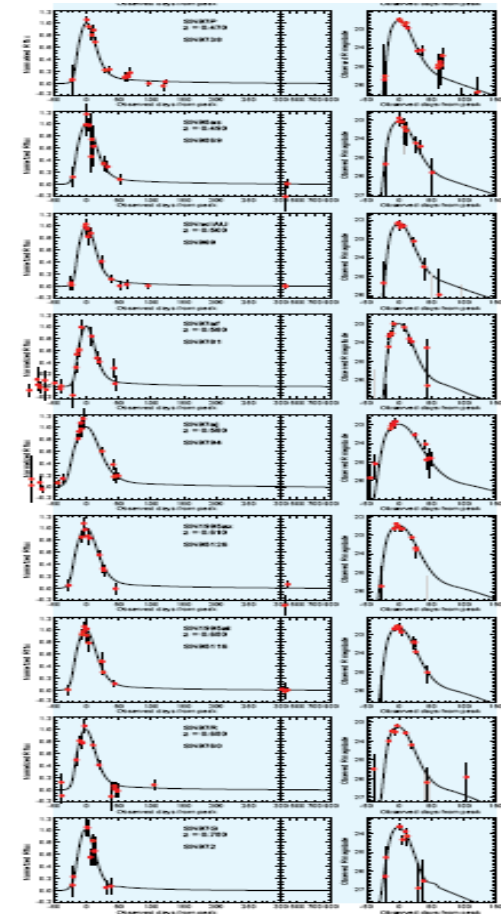
Light Curves

SN 1997ap at $z = 0.83$
observed from the
ground and with the HST

Perlmutter, et al., Nature (1998)

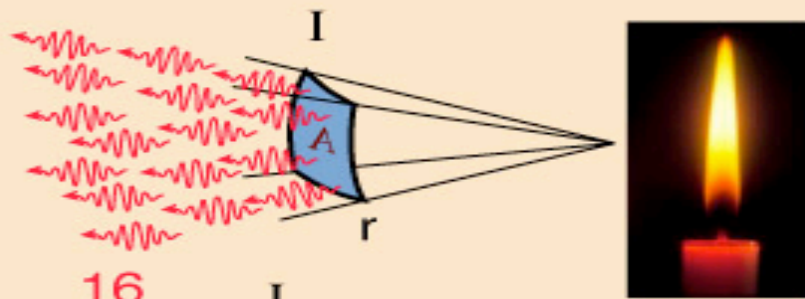


We observe most of the supernovae for approximately two months in both the R and I bands (corresponding approximately to the restframe B and V bands for the median redshift). At high redshifts, a significant fraction of this host galaxy light is within the seeing disk of the supernova, so final observations about one year later are usually necessary to observe (and subtract) the host galaxy light after the supernova has faded. The plots to the left and the right show just the R band light curves for about half of the 40 supernovae that have been completely observed and analyzed so far. The plots above show the highest redshift spectroscopically confirmed supernova, which was observed with the Hubble Space Telescope.



Perlmutter, et al., Nature (1998)

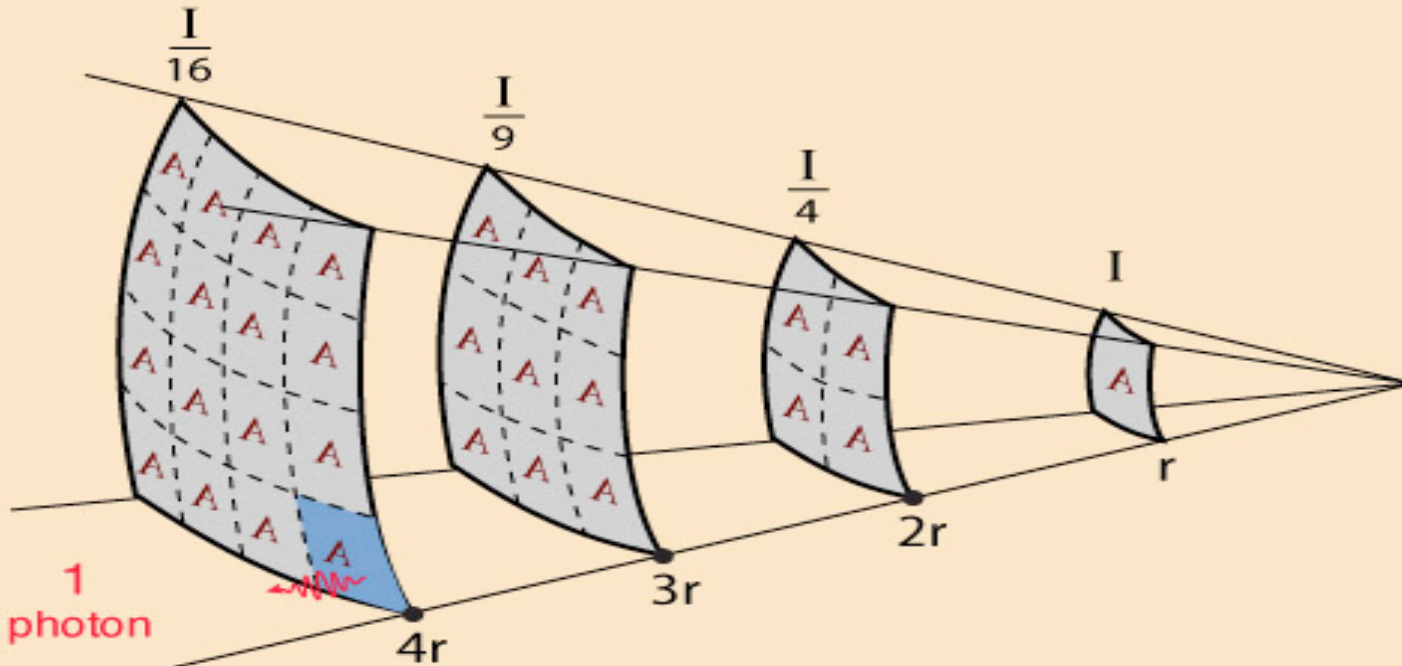
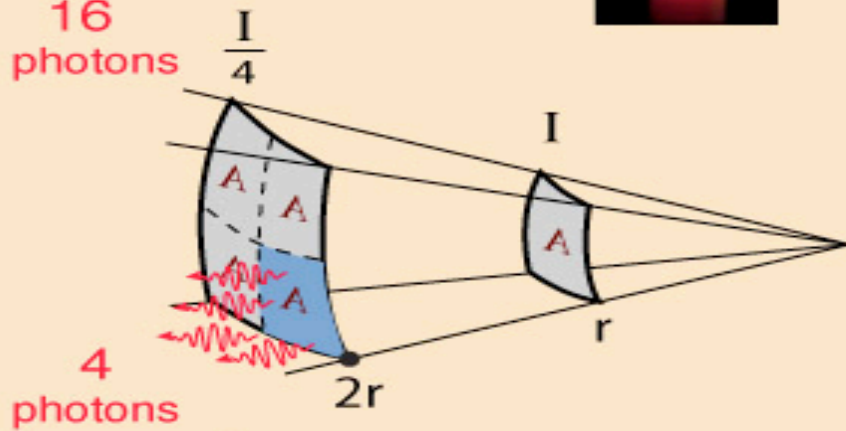
標準燭光



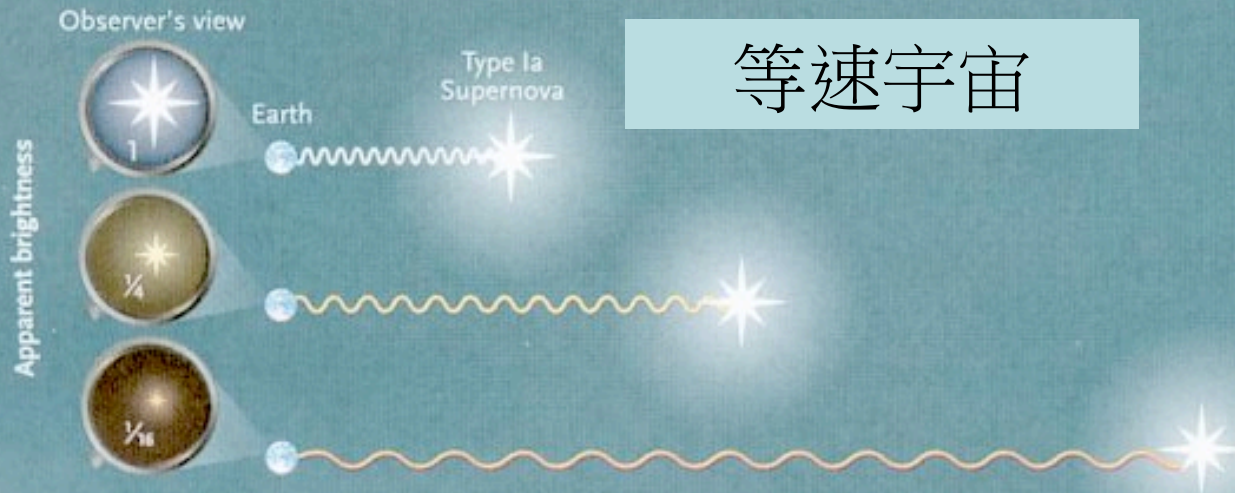
If you know you have the same source strength of light, then counting the number of photons through a standard area detector tells you the distance to the source.



Light from a point source drops off according to the inverse square law, a strictly geometrical relationship.



等速宇宙



加速宇宙

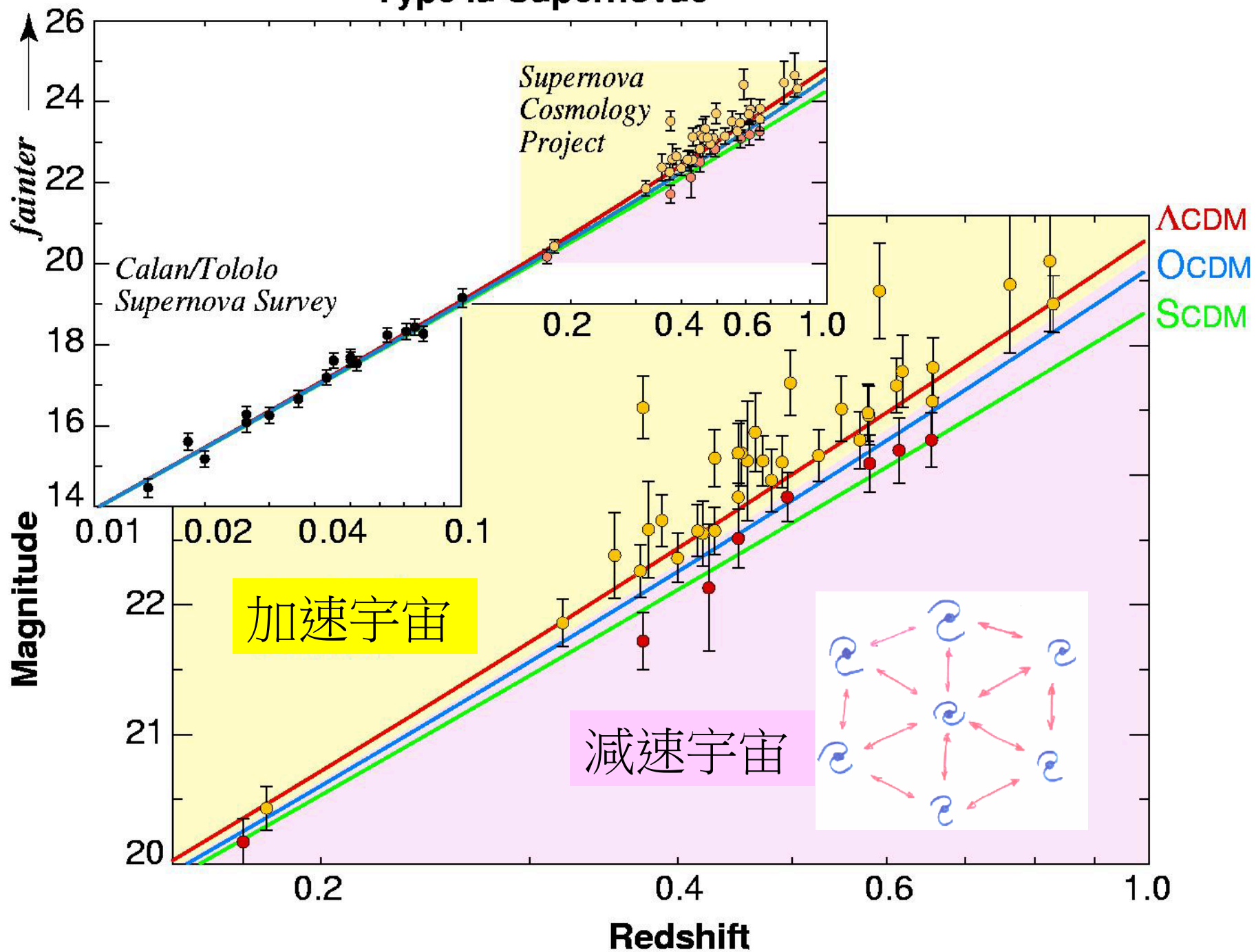


In 1998 two teams discovered that the universe's expansion rate is accelerating by studying Type Ia supernovae. The brightness of a supernova indicates its distance, and the color indicates its redshift due to cosmic expansion.

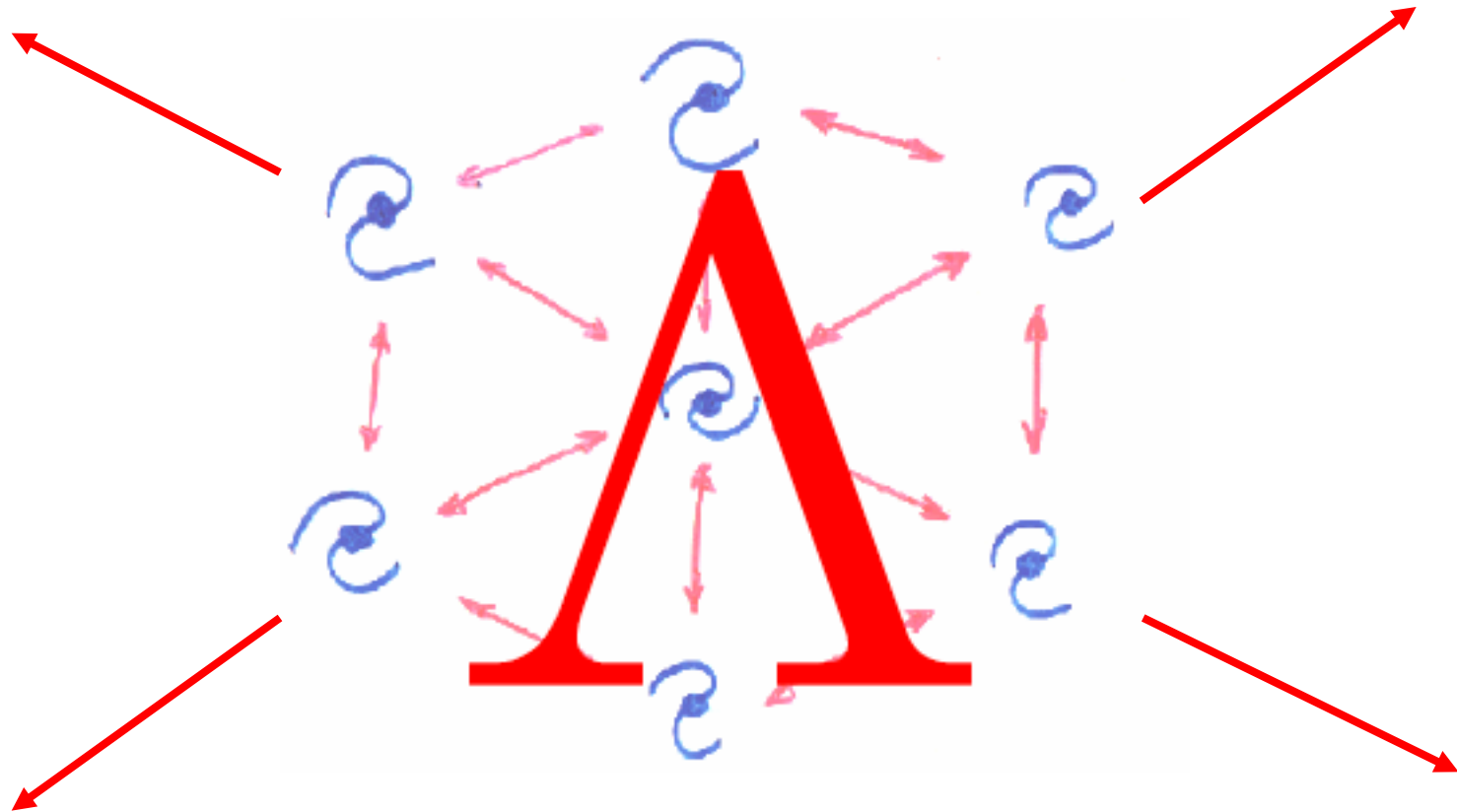
Top: In a uniformly expanding universe, the distance increases (the brightness decreases) as the redshift grows larger, as illustrated.

Bottom: The relation between distance and redshift changes in an accelerating universe. High-redshift supernovae appear more distant (and thus fainter) than is the case for uniform expansion.

Type Ia Supernovae



帶有負壓力的暗能量 驅駛宇宙加速膨脹



暗能量是什麼？

宇宙常數 Λ

(1917)

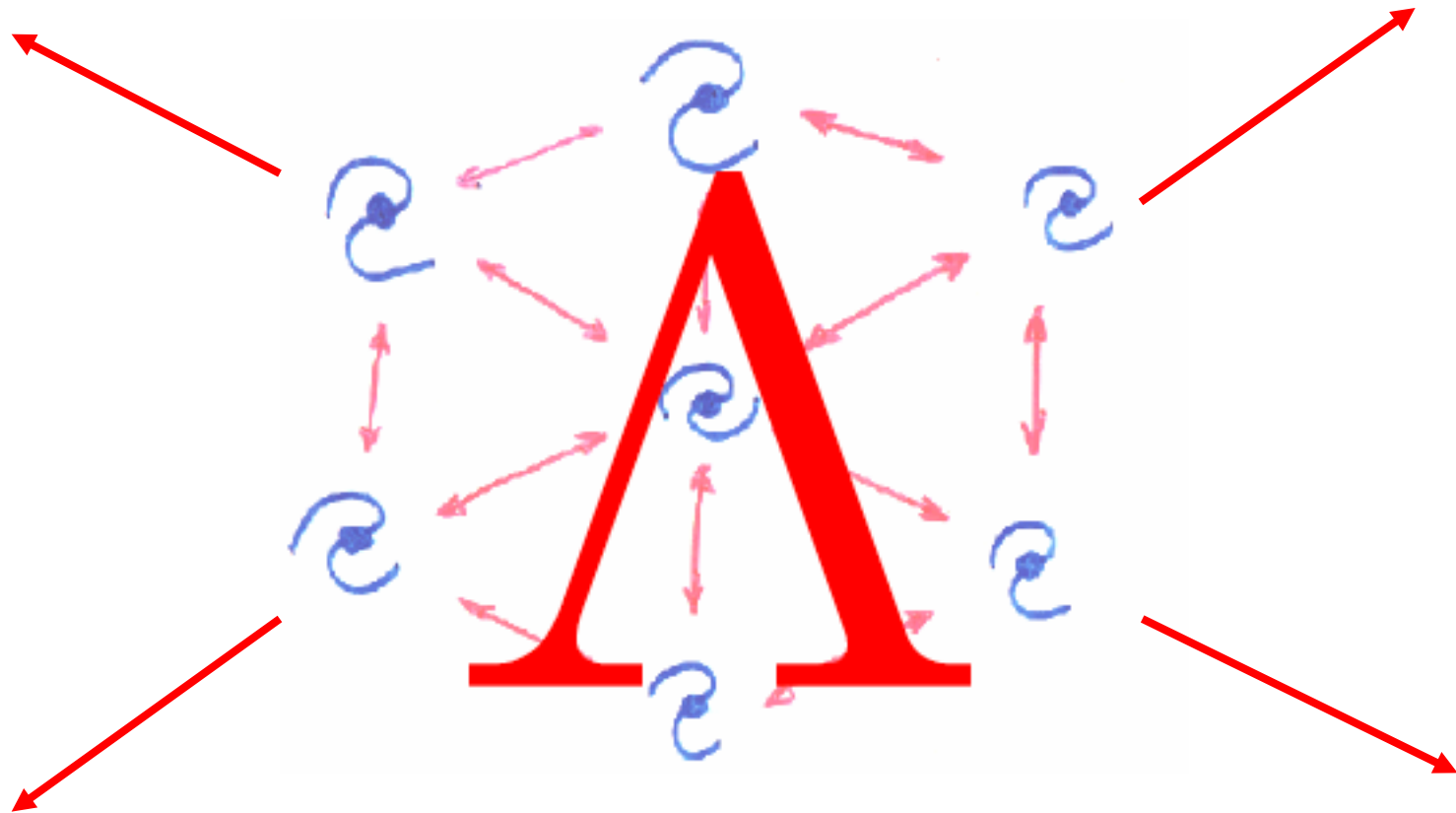
Einstein's Invention

我堅持一個
永恆不變
靜止的宇宙

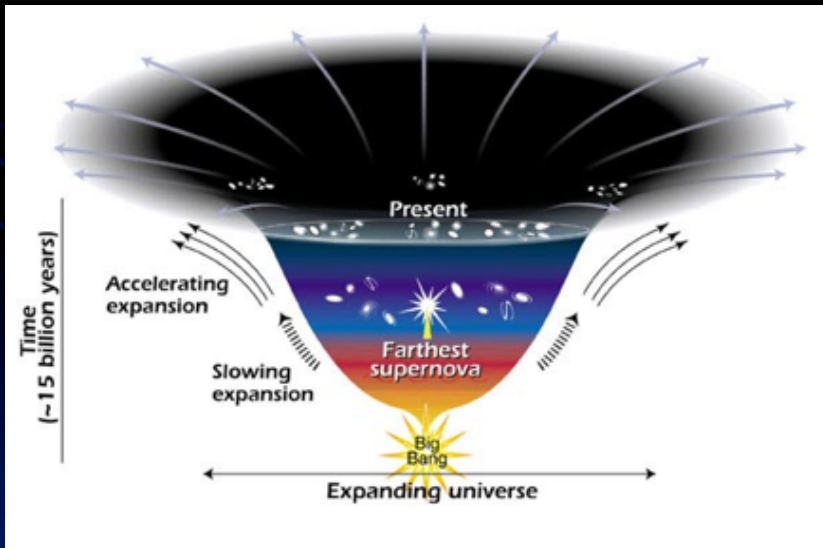
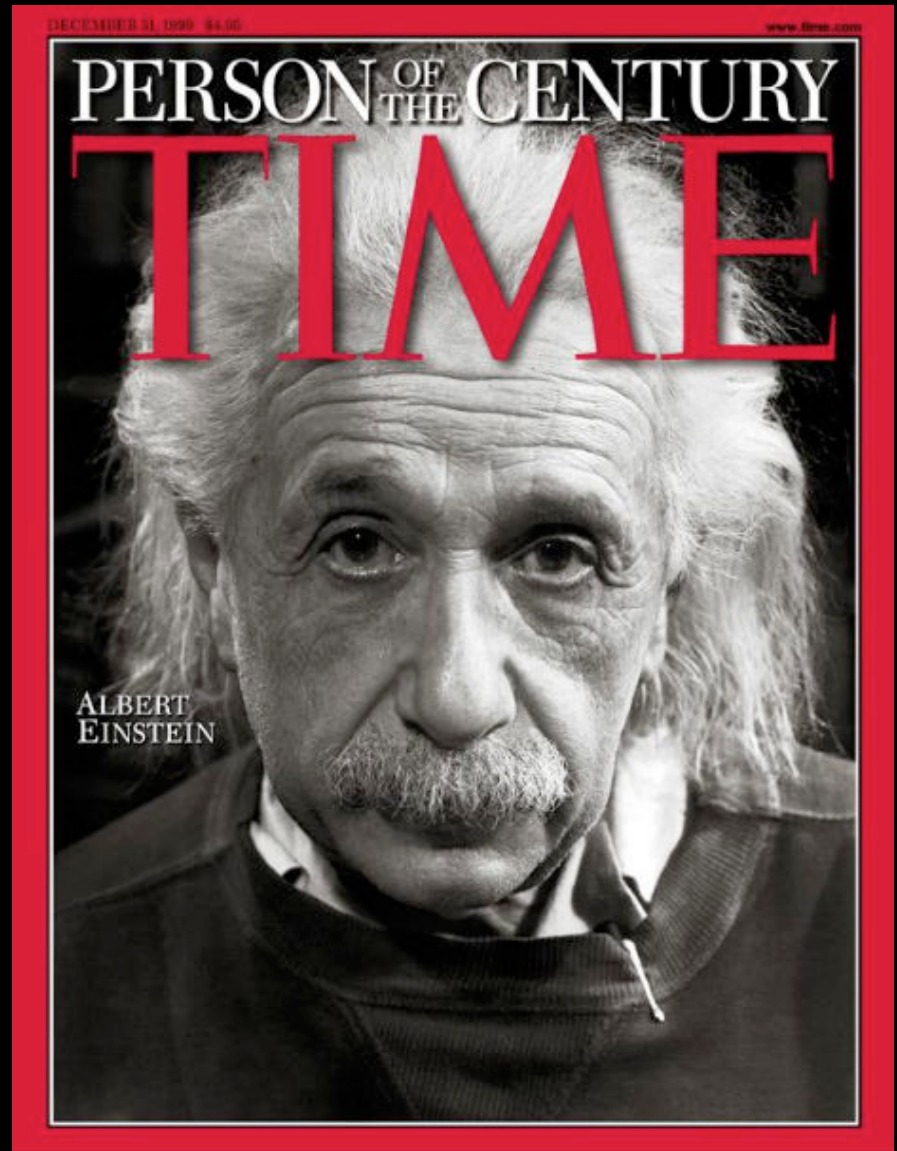
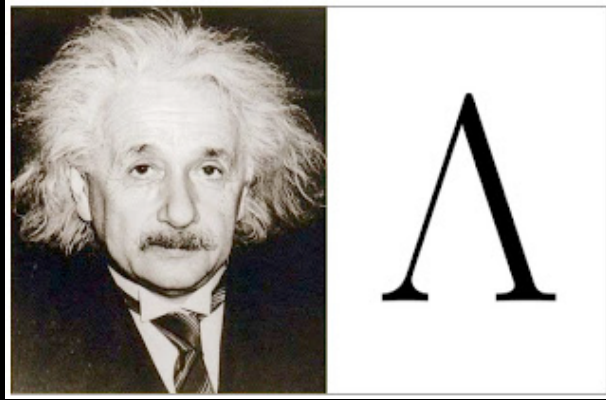
$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

愛因斯坦一生中最大的錯誤！

「宇宙常數」是帶有負壓力的能量

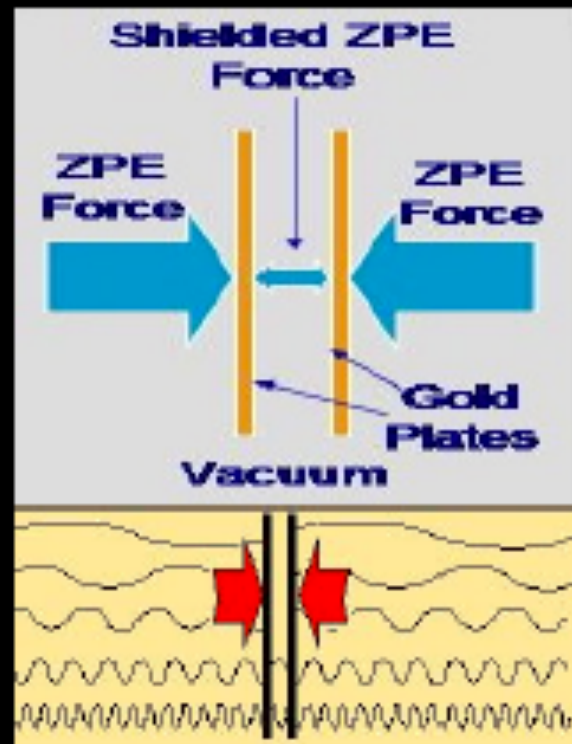
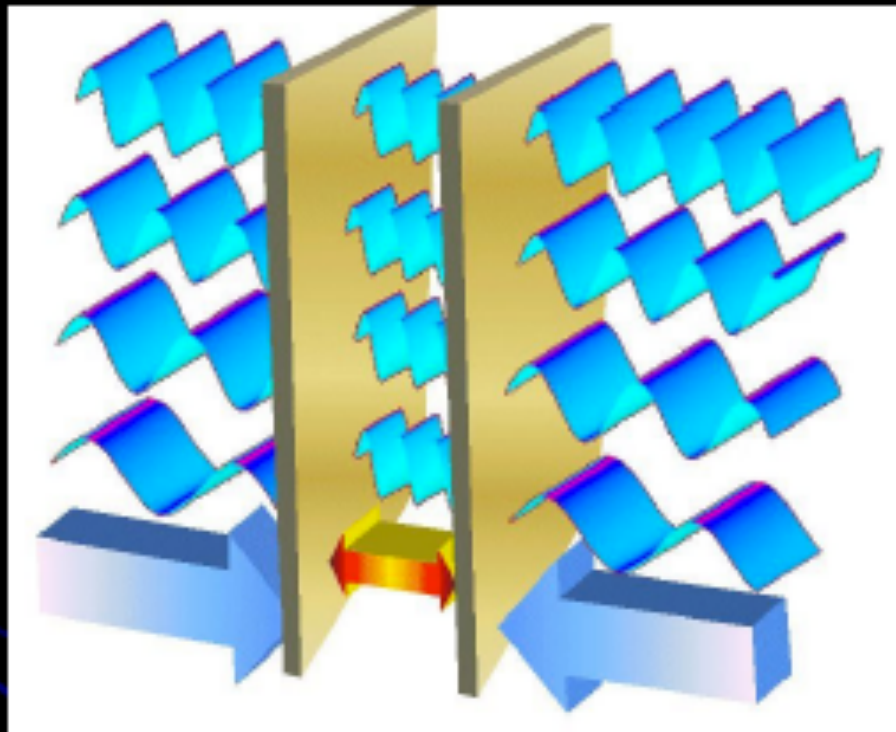


暗能量可能是愛因斯坦的「宇宙常數」



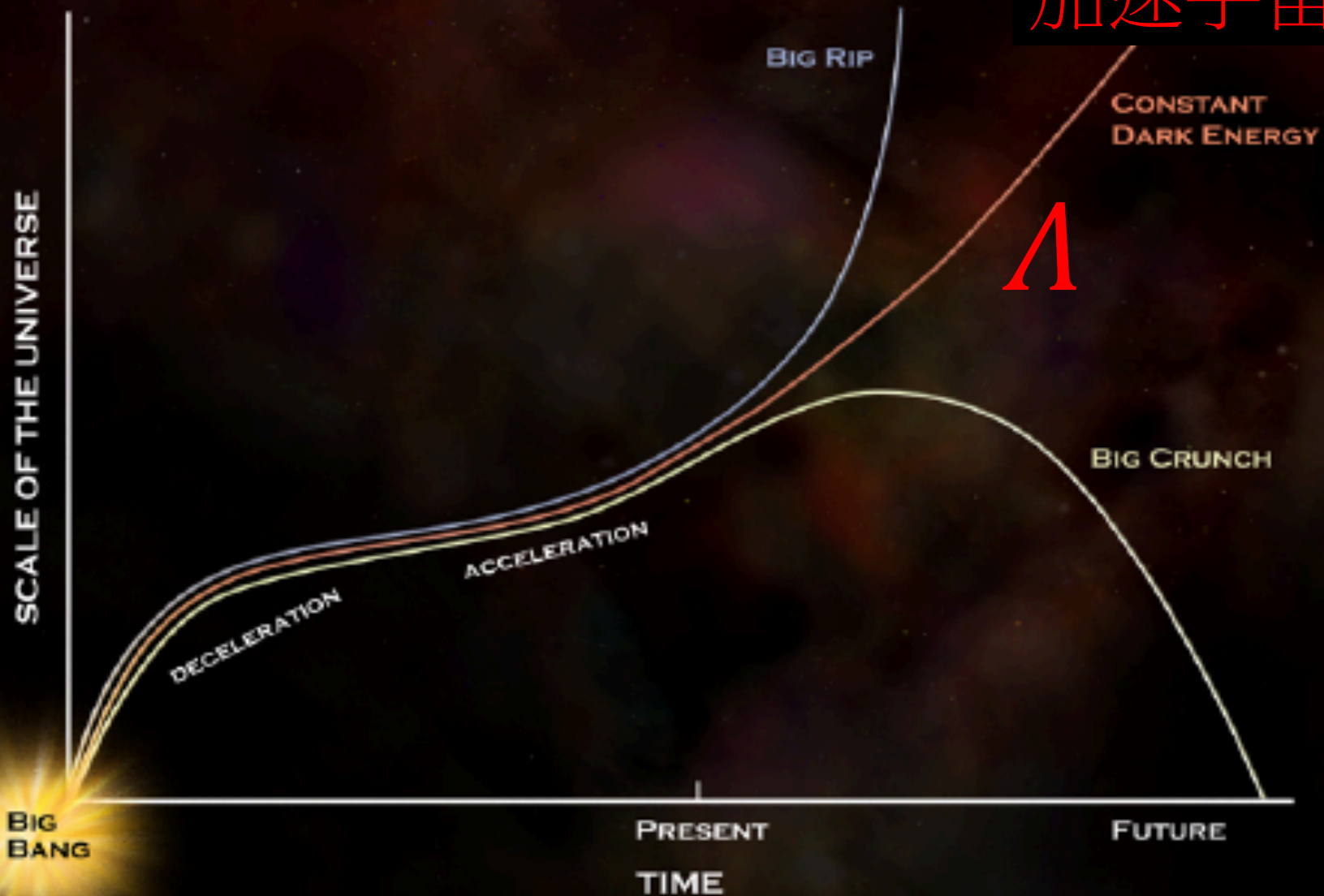
暗能量可能是真空能量

真空能量具有負壓力



宇宙的未來命運

加速宇宙

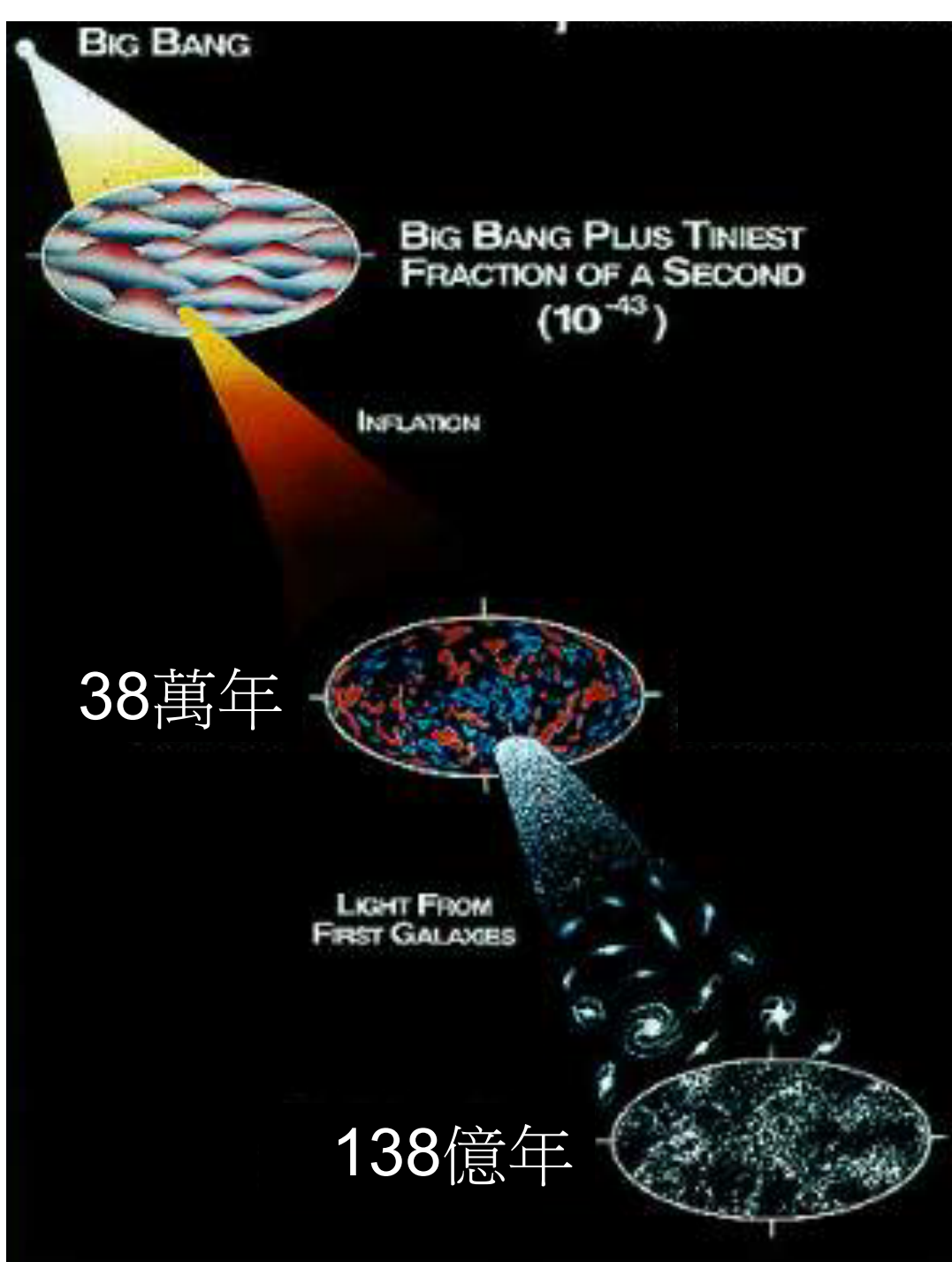


宇宙的溫度

10^{26} K

3000 K

3 K 宇宙微波背景輻射
(-270°C) (CMB)



1965

CMB Milestones

AT&T Bell

Penzias and
Wilson

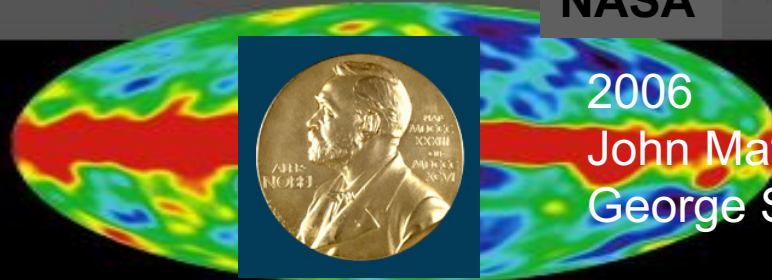


1978
Arno Penzias
Robert Wilson

1992

NASA

COBE

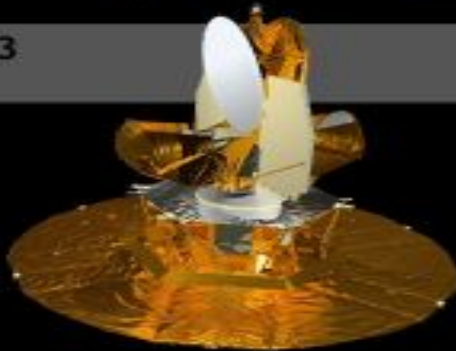


2006
John Mather
George Smoot

2003

NASA

WMAP

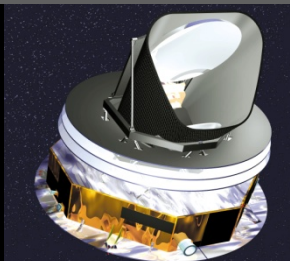


2010, 2018
Charles Bennett
Lyman Page
David Spergel

2013

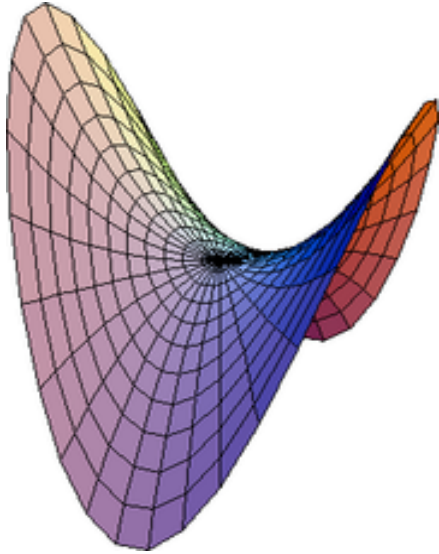
ESA

Planck



Plus many ground-
based experiments

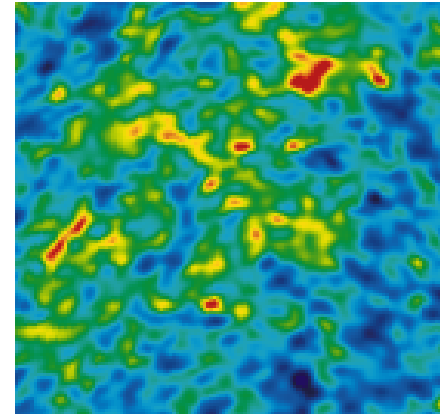
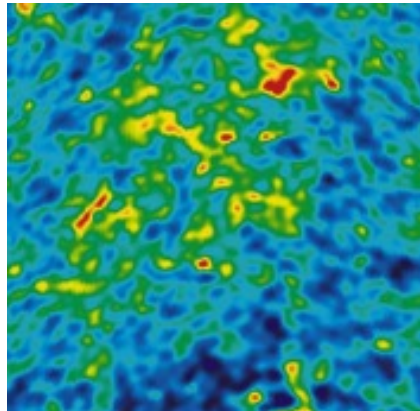
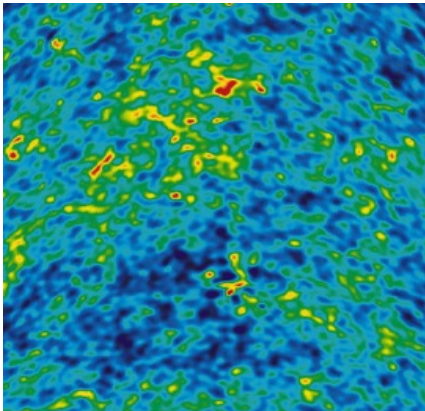
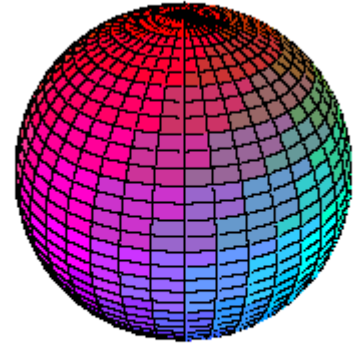
$k=-1$



$k=0$

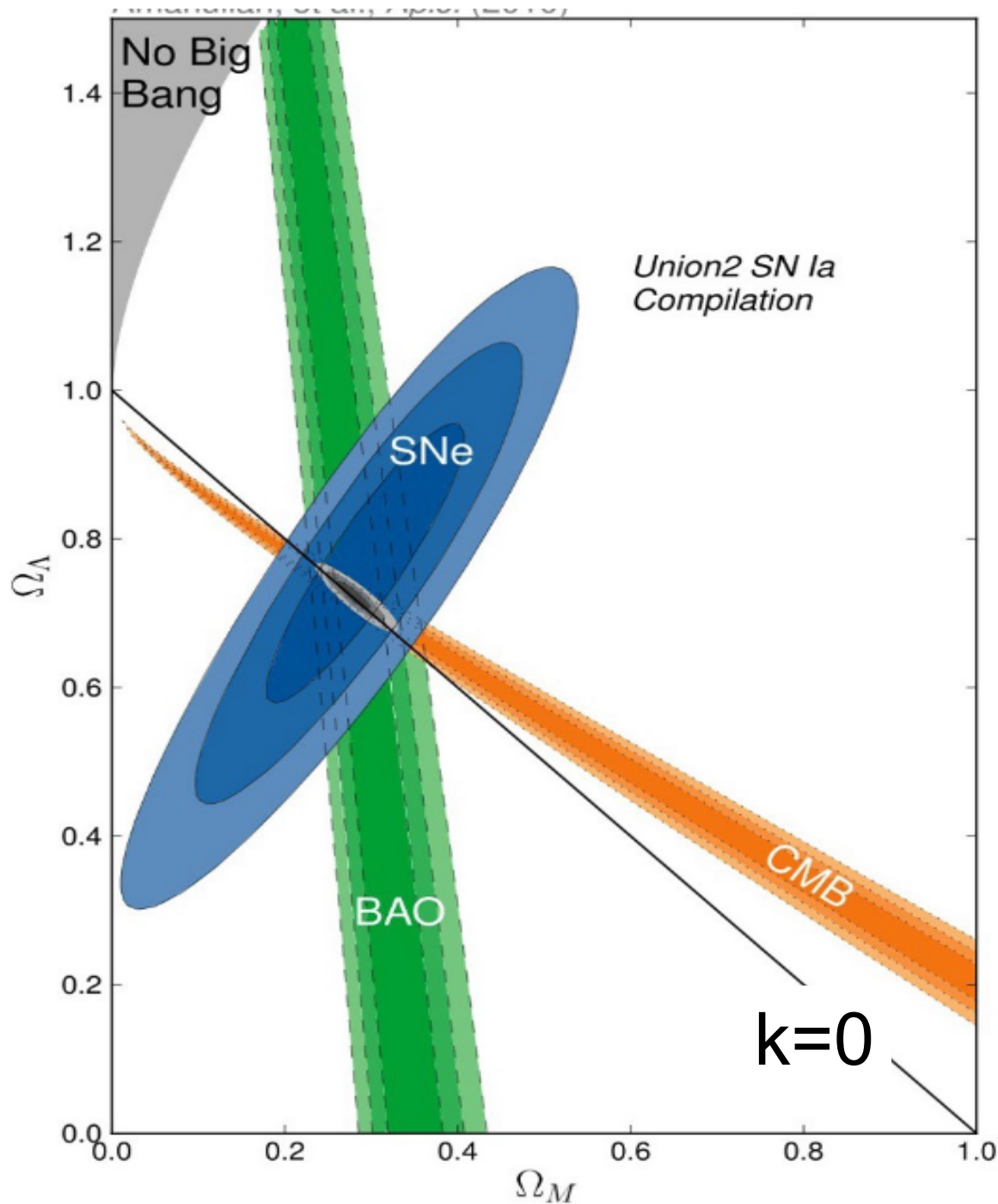


$k=1$



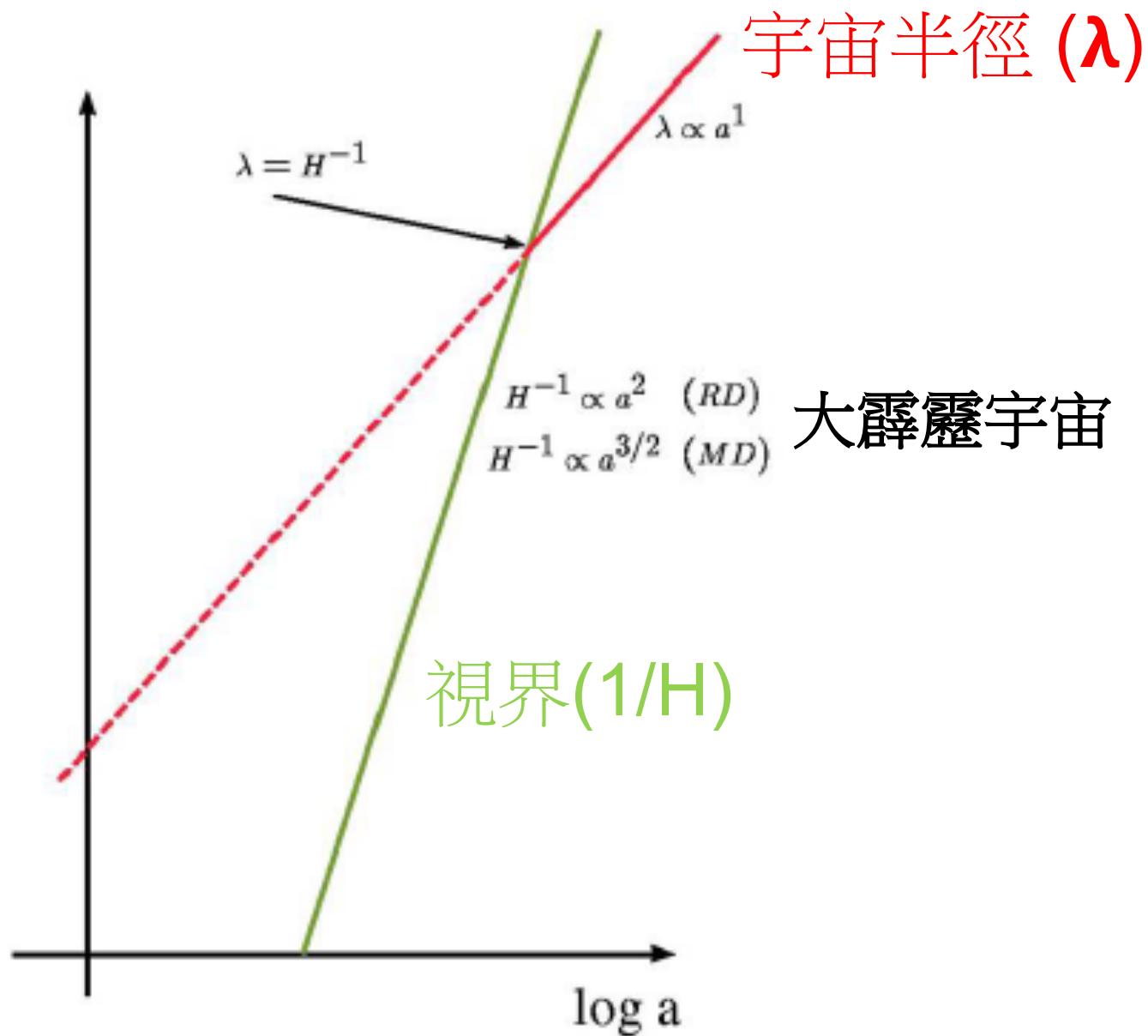
$$1 = \Omega_M + \Omega_\Lambda - k / (R^2 H^2)$$

$$\Omega_M + \Omega_\Lambda = 1.099^{+0.100}_{-0.085}$$

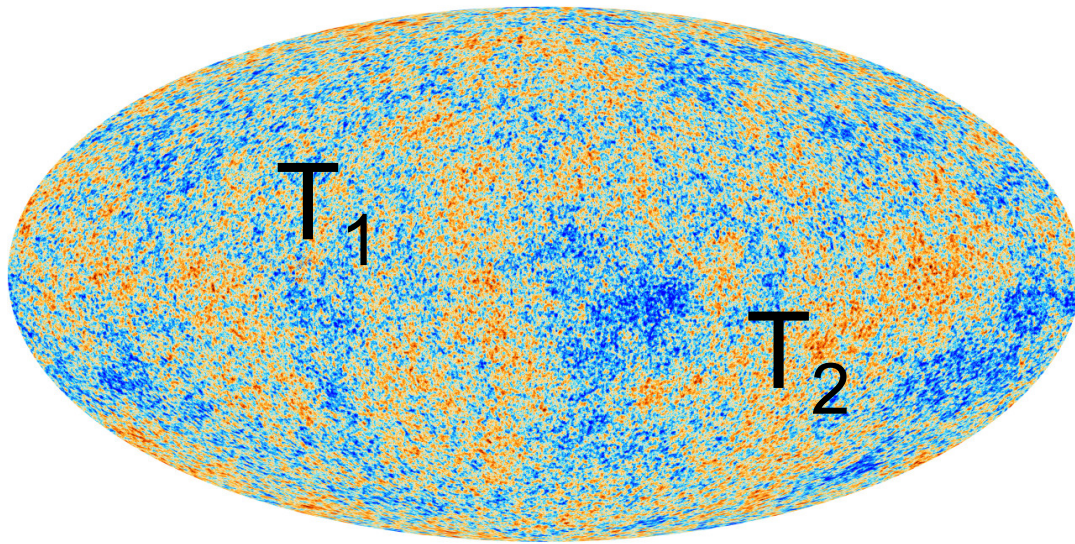


宇宙是
平坦的！
為什麼？

視界問題



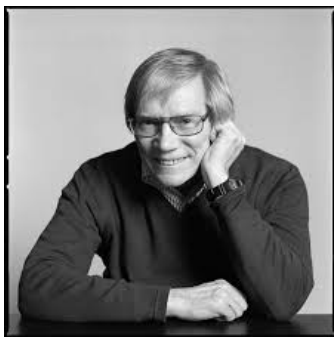
宇宙微波背景輻射的各向異性



$$T_1 \sim T_2 \sim 3 \text{ K}$$

$$T_1 - T_2 \sim 10^{-5} \text{ K}$$

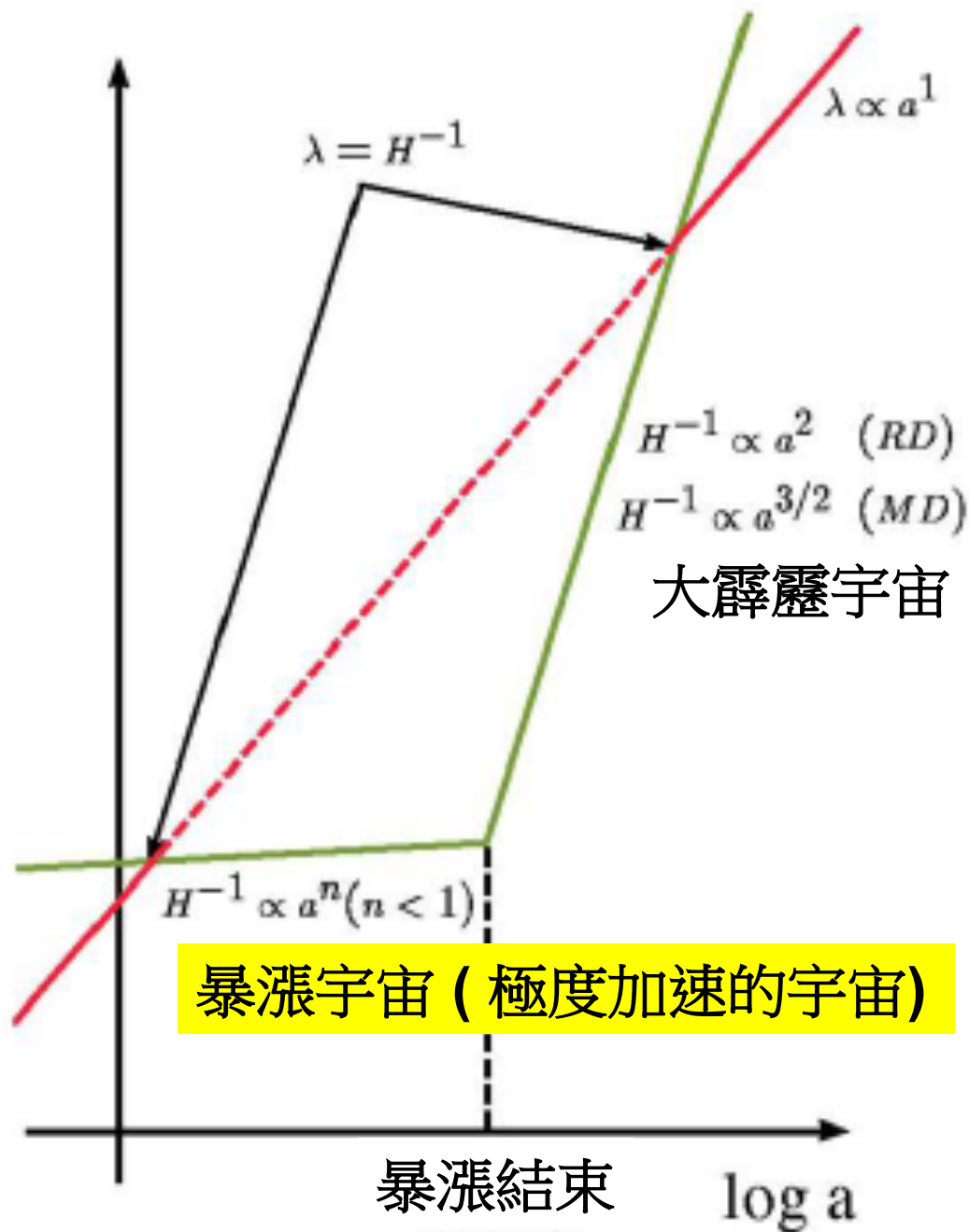
宇宙是非常均勻的！
為什麼？



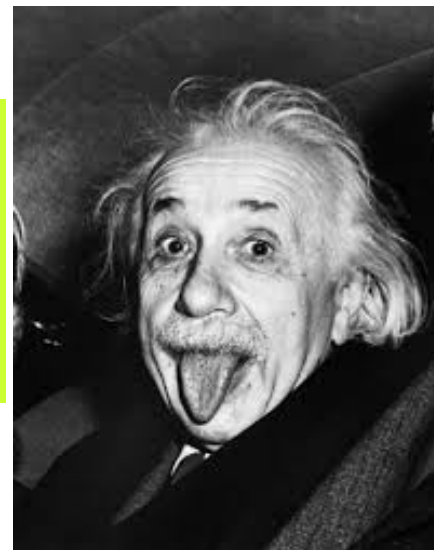
**1982年 MIT
Alan Guth 提出「暴漲宇宙模型」一起解決宇宙難題**



2012
Breakthrough Prize



又見真空能量



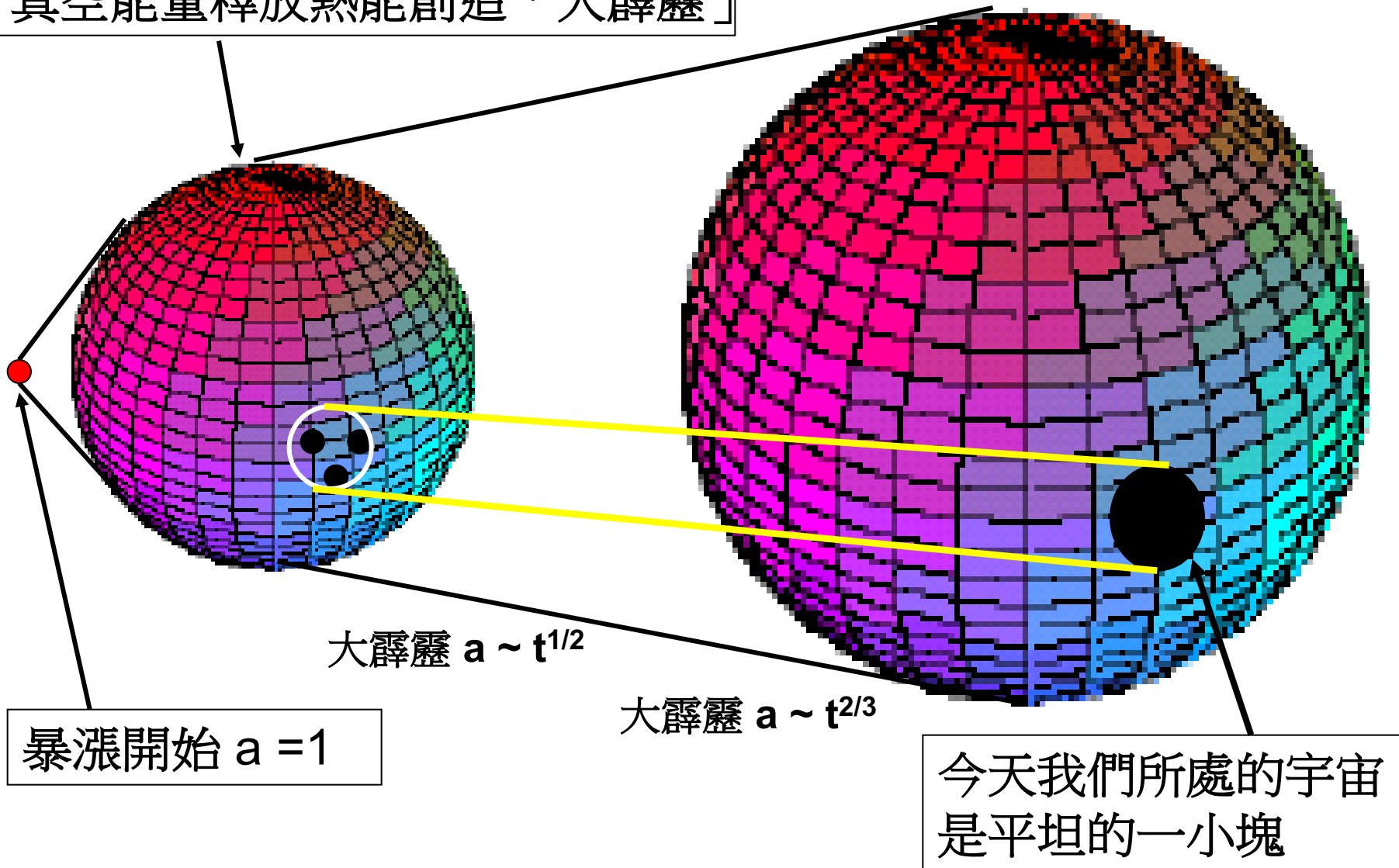
真空
能量

$$\Lambda_{\text{暴漲}} > > \Lambda_{\text{暗能量}}$$

膨脹
速度

$$H_{\text{暴漲}} > > H_{\text{暗能量}}$$

暴漲結束 $a \sim e^{Ht}$ ($Ht=60$)
真空能量釋放熱能創造「大霹靂」

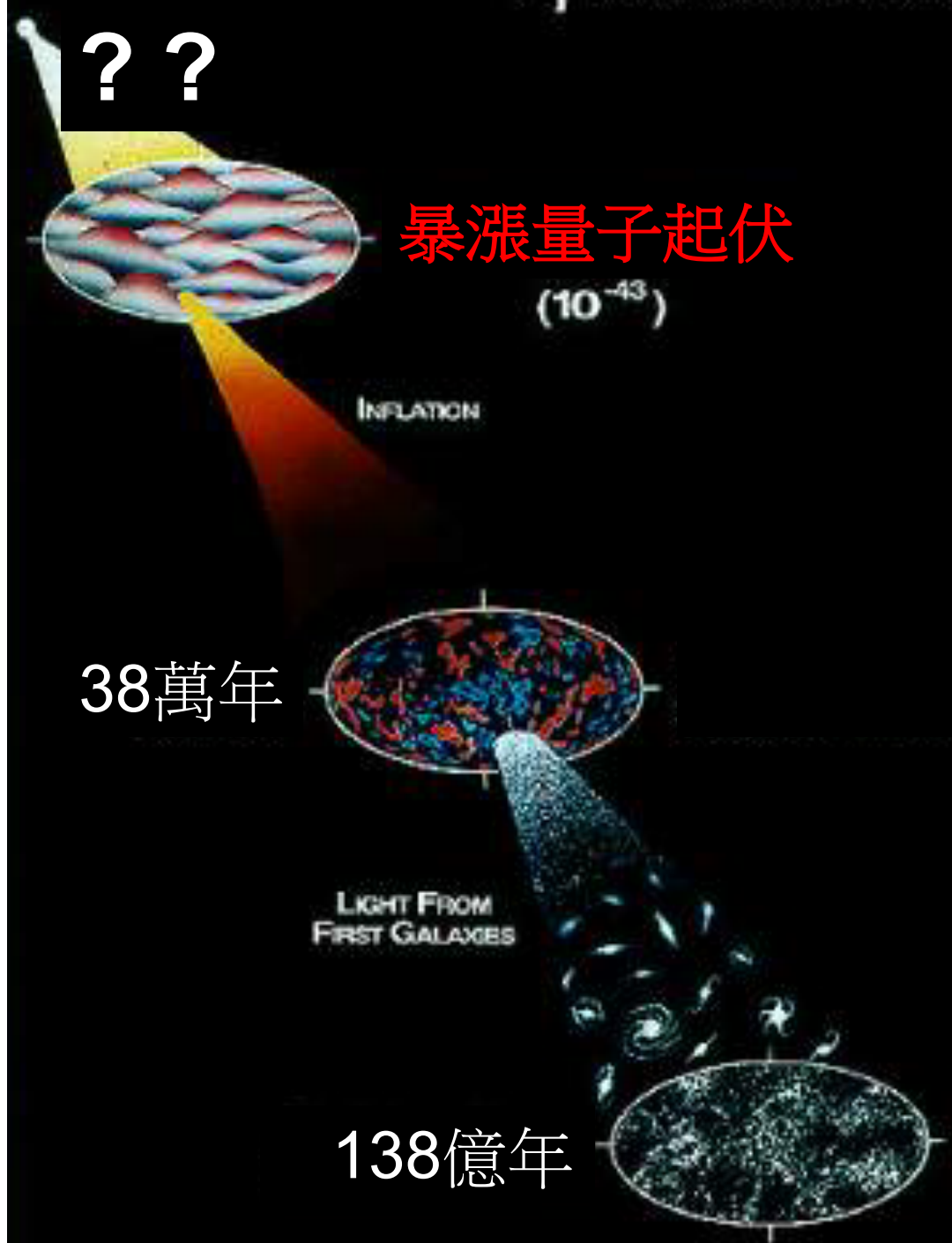


暴漲開始 $a = 1$

今天我們所處的宇宙
是平坦的一小塊

宇宙大尺度結構 之起源

暴漲時真空能
量的量子起伏
散播了宇宙結
構的種子



真空能量是什麼 ??

$\Lambda_{\text{暴漲}}$ >> $\Lambda_{\text{實驗室}}$ >> $\Lambda_{\text{暗能量}}$

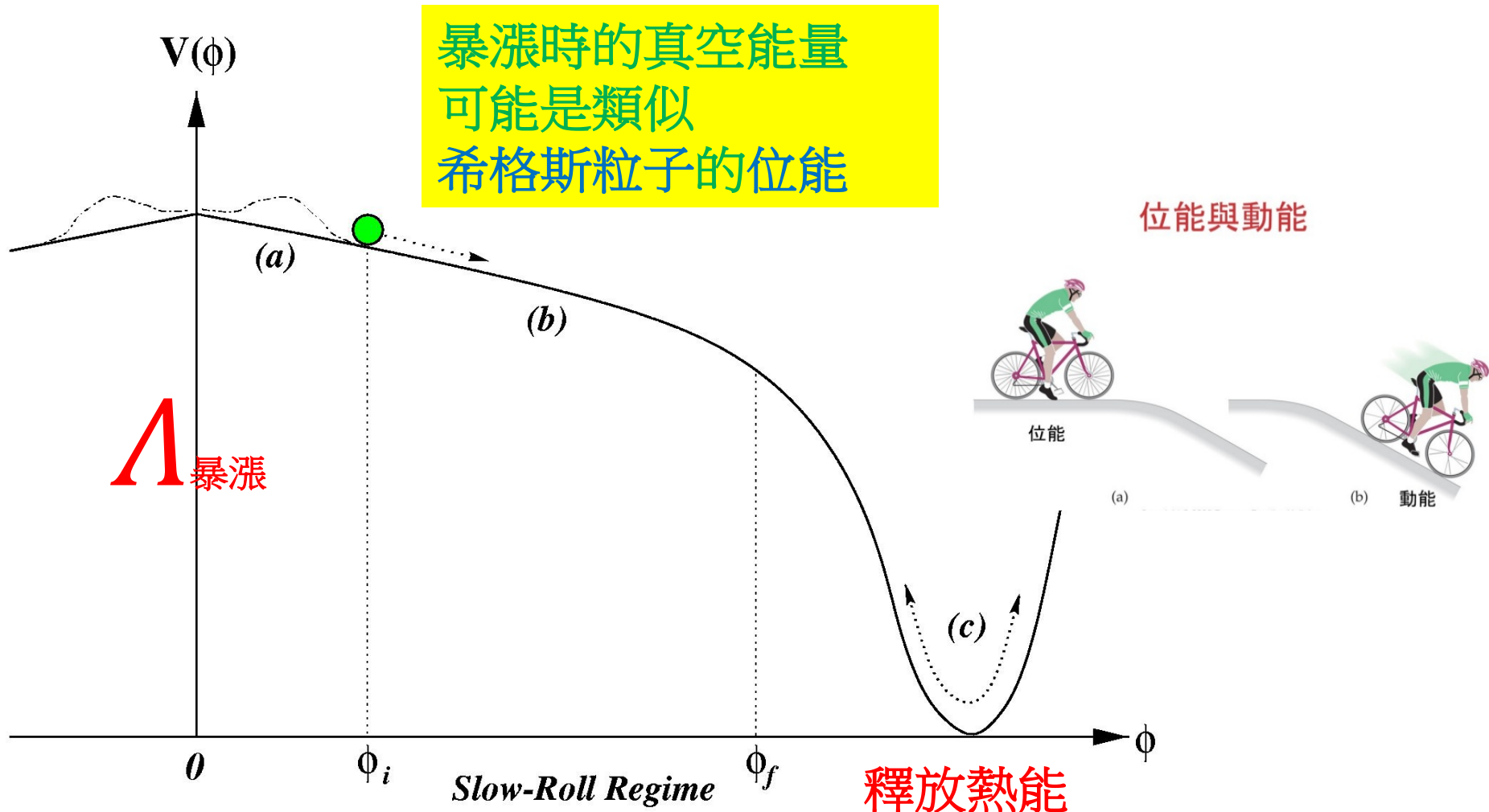
為什麼能量階如此不同 ??

真心話: 搞什麼鬼? 把一個難題換做另一個難題

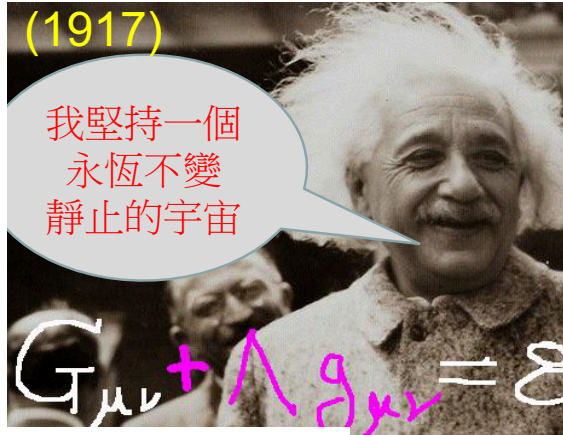
共勉之: 大膽假設 小心求證

真空能量是什麼？

真空能量釋放熱能創造「大霹靂」？



結語



Hubble's Law 哈勃
- 膨脹的宇宙



加速的宇宙

Λ 是

愛因斯坦一生中最大的錯誤？

我們透過暴漲宇宙及暗能量的介
紹, 認識到

科學家的堅持睿智 +
艱難的實驗觀測

把宇宙學從科幻故事發展成
精準的科學