Tensor Renormalization Group Analysis of Critical Transition in Random tensor network model

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A prior study, Ref. [2], investigated the transition point of the Random Tensor Network (RTN) model. In that study, the transition point of the effective permutation spin model was predicted using duality analysis, a standard method for analyzing phase transitions in spin models. Furthermore, the transition point of the original RTN model was estimated via extrapolation based on the results for the effective model. However, since this derivation relies on non-trivial assumptions, verification using alternative methods is required.Meanwhile, high-precision analysis of the transition point in the RTN model using Markov Chain Monte Carlo (MCMC) methods has proven difficult due to critical slowing down [1]. In this study, we report on the verification of the transition point predicted in Ref. [2] by employing the Tensor Renormalization Group (TRG), a numerical approach distinct from MCMC.First, we applied the replica method to the RTN model, which contains intractable randomness, converting it into a spatially uniform effective spin model. Next, we calculated the physical quantities of this effective model across various system sizes using TRG and identified the transition temperature via finite-size scaling of the specific heat peaks. Finally, following the procedure of the replica method, we determined the transition temperature of the original RTN model by taking the replica limit $n \to 0$. The validity of the extrapolation functions was confirmed through preliminary experiments using the Potts model and the Random Bond Ising Model (RBIM). One of the primary motivations for analyzing the RTN model is its relevance to Random Quantum Circuits (RQC). Quantum circuits are computational models that process information via operations and measurements on qubits. Qubits share information through entanglement, and by establishing global correlations, they achieve computational supremacy over classical computers. However, in physical implementations, entanglement is degraded by environmental interactions, gate operations (unitary operators), and noise introduced by measurements. Modern devices, known as Noisy Intermediate-Scale Quantum (NISQ) computers, typically contain around 1,000 qubits and are highly susceptible to noise. If noise levels are too high, the quantum advantage over classical computers may be lost. Therefore, understanding the entanglement properties of the system is crucial. When the entire system is entangled, the entanglement entropy follows a "volume law," proportional to the size of the subsystem. Conversely, when entanglement is local, it follows an "area law," proportional to the boundary length of the subsystem. In the volume-law phase, the complexity of entanglement makes classical simulation of the system's state computationally intractable. In contrast, in the area-law phase, the state can be efficiently simulated on classical computers using techniques such as tensor network methods. It is known that the scaling of entanglement entropy in RQCs undergoes a phase transition from an area law to a volume law (known as the Measurement-Induced Phase Transition, or MIPT) at a certain measurement probability. Determining this transition point with high precision is expected to clarify the threshold for quantum advantage over classical computation in the presence of noise. Using the replica method, this model can be treated within a statistical mechanics framework that incorporates randomness; in other words, the MIPT can be analyzed in a manner analogous to a ferromagnetic-paramagnetic phase transition. However, the effective spin model derived from RQCs involves complex lattice geometries and interactions. Therefore, by analyzing the phase transition of the RTN model, which shares similar properties, we aim to verify the applicability of TRG to the analysis of phase transitions in RQCs.

[1] Ryuki Ito. “Numerical analysis of phase transitions in permutation spin models”. MA thesis. Institute of Science Tokyo, Department of Physics, 2025.

[2] Masayuki Ohzeki. “Duality analysis in a symmetric group and its application to random tensor network models”. In: Progress of Theoretical and Experimental Physics 2024.12 (2024), 121A01.