Solving the Gross-Pitaevskii Equation with Quantic Tensor Trains: Ground States and Nonlinear Dynamics

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We develop a tensor network framework based on the quantic tensor train (QTT) [1] format to efficiently solve the Gross-Pitaevskii equation (GPE) [2, 3], which governs Bose-Einstein condensates under mean-field theory. For sufficiently regular functions, the QTT approach enables exponentially fine grids with computational cost that grows only linearly. By adapting the time-dependent variational principle (TDVP) [4] and gradient descent methods, we accurately handle the GPE's nonlinearities within the QTT structure. Our approach enables high-resolution simulations with drastically reduced computational cost on highly refined grids. We benchmark the ground states and dynamics of BECs, including vortex lattice formation and breathing modes, demonstrating superior performance over regular-grid-based methods and stable long-time evolution due to saturating bond dimensions. This establishes QTT as a powerful tool for nonlinear quantum simulations [5].

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