Quantized 6-vertex model on a torus and tetrahedron equations

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The 6-vertex model is an integrable statistical model of (1+1) dimensional space time, whose symmetry is governed by the famous Yang-Baxter equation (YBE). The quantized 6-vertex model (q-6v) was introduced by Kuniba-Matsuike-Yoneyama [1], by replacing the Boltzmann weight of the 6-vertex model with the elements of the q-Weyl algebra. This gives (2+1)-dimensional lattice models whose symmetry is governed by the tetrahedron equation (TE) introduced by Zamolodchikov in 1980 as a higher dimensional version of the YBE. The YBE and the TE have graphical expressions as shown in Figure 1. We assign elements of some vector space to each edge, and each intersection represents a linear transformation of the tensor product of the vector spaces. The equations mean that the corresponding Boltzmann weights are equal when summed on the interior edges.

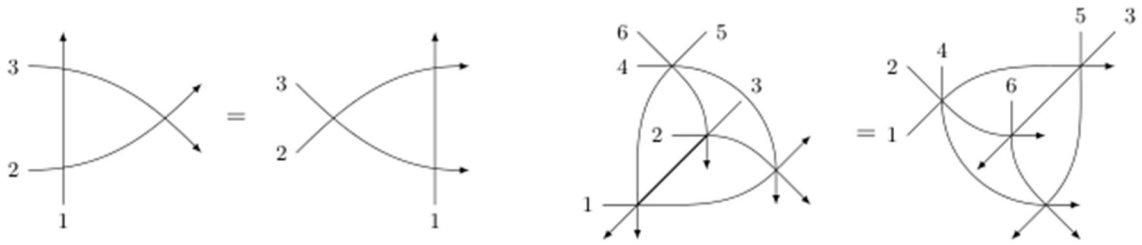

Figure 1: YBE (left) and TE (right)

In this talk, based on a joint work with Atsuo Kuniba, Yuji Terashima and Junya Yagi [2], we show the integrability of the q-6v model on a certain class of directed graphs on a torus (including square lattice) by applying various tetrahedron equations.

[1] A. Kuniba, S. Matsuike, A. Yoneyama, New solutions to the tetrahedron equation associated with quantized six-vertex models, Commun. Math. Phys. Volume 401, 3247-3276 (2023).
[2] R. Inoue, A. Kuniba, Y. Terashima, J. Yagi, Quantized six-vertex model on a torus, arXiv: 2505.08924.