Tensor-Network Study on Infinite Hyperbolic Dodecahedral Lattice

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We propose a tensor-network-based algorithm to study the classical Ising model on an infinitely large hyperbolic lattice with a regular 3D tesselation of identical dodecahedra [1]. We reformulate the corner transfer matrix renormalization group (CTMRG) algorithm from 2D to 3D to reproduce the known results on the cubic lattice. Consequently, we generalize the CTMRG to the hyperbolic dodecahedral lattice, which is an infinite-dimensional lattice. We analyze the spontaneous magnetization, von Neumann entropy, and correlation length to find a continuous non-critical phase transition on the dodecahedral lattice. The phase transition temperature is estimated to be $T_{pt} \approx 4.66$. We find the magnetic critical exponents $\beta = 0.4999$ and $\delta = 3.007$ that confirm the mean-field universality class in accord with predictions of Monte Carlo and high-temperature series expansions. The algorithm can be applied to arbitrary multi-state spin models.

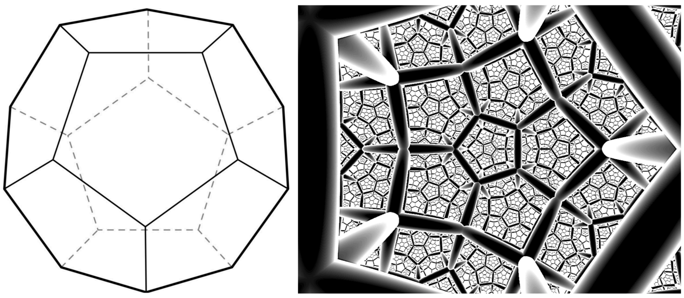

Figure 1: The regular dodecahedron (on the left) serves as a basic cell for constructing the hyperbolic lattice through the uniform 3D tessellation of an infinite number of identical dodecahedra. Around each dodecahedral edge and vertex, there are 4 and 8 dodecahedra, respectively, without leaving free space. Such a generalized 3D tessellation of the infinite lattice can only be embedded in the infinite-dimensional space. The local visualization from the inside of the hyperbolic dodecahedral lattice is shown on the right and is denoted as a (5, 3, 4) order-4 dodecahedral (honeycomb) lattice.

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[1] M. Moško and A. Gendiar, arXiv:2510.20939.