On observers in holographic maps

Chris Akers

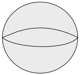
CU Boulder

With Gracemarie Bueller, Oliver DeWolfe, Kenneth Higginbotham,
Johannes Reinking, and Rudolph Rodriguez

Based on [2503.09681]

Goal: Quantum mechanics of closed universes

- We'd like to understand the quantum mechanics of spacetime itself.
- Part of this is understanding the quantum mechanics of closed universes.



- This has turned out to be very subtle: The holographic principle and the gravitational path integral (GPI) seem to tell us

$$|\mathcal{H}_{\text{fun}}| = 1 \ .$$

- This is far too trivial. Either we can rule out that we live in a closed universe, or there's something we haven't included in the arguments.
- Recently there have been proposals to fix this problem by appropriately incorporating an *observer* into the GPI:

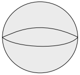
AAIL [Abdalla-Antonini-Iliesiu-Levine '25]

HUZ [Harlow-Usatyuk-Zhao '25]

- I will present a new rule for incorporating an observer into the holographic map.
- Further, I'll argue this matches the rule for the GPI proposed by AAIL and elucidates the difference between AAIL and HUZ.

Arguments for $|\mathcal{H}_{\rm fun}| = 1$

- First, why think $|\mathcal{H}_{\rm fun}| = 1$ for closed universes?
- Many arguments have been given [Penington-Shenker-Stanford-Yang '19, Marolf-Maxfield '20, Usatyuk-Zhao '24, Harlow-Usatyuk-Zhao '25, ...]. I'll give two quick ones.
- The holographic principle seems to suggest this simply because there is no boundary!



- The gravitational path integral also suggests this when you compute inner products with it. A quick way is to compute the variance of the inner product.
- Say we have $\mathcal{H}$ of unknown dimension. All we are given are k random vectors

$$|i\rangle \in \mathcal{H}$$

- It holds that

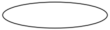
$$\sigma^2 = \overline{|\langle i|j\rangle|^2} - \overline{|\langle i|j\rangle|}^2 \simeq \frac{1}{|\mathcal{H}|}$$

- Hence we can estimate $|\mathcal{H}_{\rm fun}| = 1$ by computing $\overline{|\langle i|j\rangle|^2}$ and $\overline{|\langle i|j\rangle|}^2$ with the GPI and plugging it in.

Variance in inner product with GPI

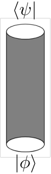
- To draw these calculations with the GPI, we'll represent states as

$$|\psi\rangle =$$



- Compute inner products by summing over connecting geometries:

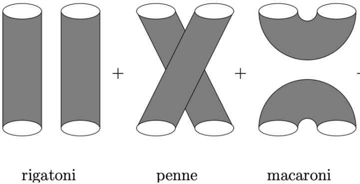
$$\overline{\langle\psi|\phi\rangle} =$$



$$+\mathcal{O}(e^{-2S_0})$$

- Meanwhile,

$$\overline{|\langle\psi|\phi\rangle|^2} =$$



$$+ \ \mathcal{O}(e^{-2S_0})$$

- Large variance, small $|\mathcal{H}_{\text{fun}}|$,

$$\sigma^2 \simeq 1 + |\langle\psi^*|\phi\rangle|^2 + \ldots \implies |\mathcal{H}_{\text{fun}}| \simeq 1$$

Observers as a fix

- The usual rules for computing inner products with holography or the GPI gave an answer that seems too trivial.
- What calculation in a $1d$ Hilbert space could predict the outcome of any given experiment?
- The main point of this talk will be to present new computational rules that ameliorate this issue. The key component is keeping track of the *observer* doing the experiment.
- The proposal I'll present was developed after two others that incorporated observers:

$$\text{AAIL}$$ [Abdalla-Antonini-Iliesiu-Levine '25]

$$\text{HUZ}$$ [Harlow-Usatyuk-Zhao '25]

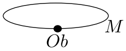
- I'll first review these and comment on their differences.

HUZ proposal

- Consider the Hilbert space of semiclassical gravity

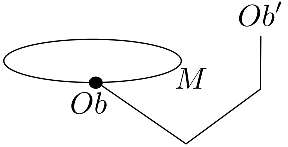
$$\mathcal{H}_{\text{eff}} = \mathcal{H}_{Ob} \otimes \mathcal{H}_M$$

  partitioned into observer and environment.
- Draw states in $\mathcal{H}_{\text{eff}}$ like

$|\psi\rangle =$ 

- New rule: before taking their inner products with GPI, act on each with an isometry cloning the observer into its "pointer basis"

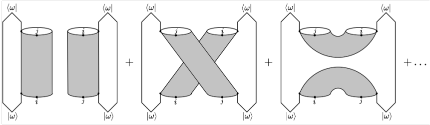
$$\mathcal{H}_{Ob} \to \mathcal{H}_{Ob} \otimes \mathcal{H}_{Ob'}$$



HUZ proposal continued

- Take the inner product treating Ob' as a non-gravitational reference:

$$\overline{|\langle\psi|\phi\rangle|^2} =$$



- This suppresses the cross terms:

$$\overline{|\langle\psi|\phi\rangle|^2} = \overline{|\langle\psi|\phi\rangle|}^2 + O\left(\frac{1}{d_{Ob}}\right) + O\left(e^{-2S_0}\right)$$

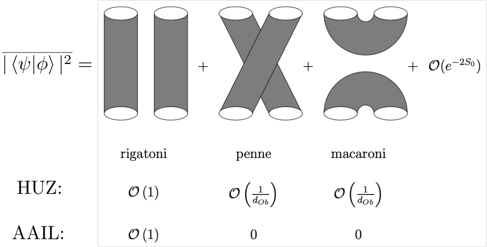
- Small variance implies large $|\mathcal{H}_{\text{fun}}|$

$$|\mathcal{H}_{\text{fun}}| \simeq \frac{1}{\sigma^2} \simeq \min\left(d_{Ob}, e^{2S_0}\right)$$

AAIL proposal

- A different rule:

1. Instead of requiring the observer comes in a particular highly entangled state after cloning, let it be in *any* state.
2. In the GPI, simply discard the cross terms:



- Note this is not simply $d_{Ob} \to \infty$ of HUZ rule, because of step 1.
- Orthogonal observer states stay orthogonal

$$d \simeq \frac{1}{\sigma^2} \simeq d_{Ob} e^{-2S_0}$$

HUZ and AAIL

- Two different proposals for $|\mathcal{H}_{\text{fun}}|$ with observers:

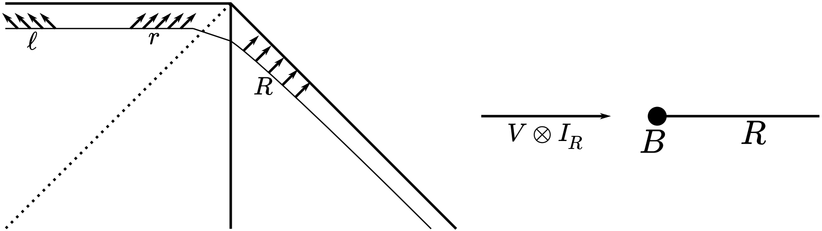
$$\begin{aligned} \text{HUZ}: &\quad \mathcal{H}_{\text{fun}} \simeq \min(d_{Ob}, e^{-2S_0}) \\ \text{AAIL}: &\quad \mathcal{H}_{\text{fun}} \simeq d_{Ob} e^{-2S_0} \end{aligned}$$

- To better compare them, we would like to understand the physics underlying each.
- This is tricky with the rules formulated just as rules for the GPI. The GPI often has a difficult physical interpretation, for example allowing us to derive that black holes have entropy $S = A/4G$ without telling us what states it's counting.
- One path is to understand the *Hilbert space description* of each rule, explicitly in terms of microstates.
- HUZ provided such a description of their rule. AAIL did not.
- The main result of this talk is to present a new rule for including an observer in Hilbert space terms, which I'll argue is equivalent to the AAIL GPI rule.

Hilbert space description of evaporating black holes

- Our starting point is the modern developments understanding evaporating black holes and deriving the Page curve [Penington '19, Almheiri-Englardht-Marolf-Maxfield '19, CA-Engelhardt-Harlow-Penington-Vardhan '22]
- In any quantum system with emergent behavior, we can formulate this emergence mathematically in terms of a linear encoding map

$$V : \mathcal{H}_{\text{eff}} \to \mathcal{H}_{\text{fun}}$$

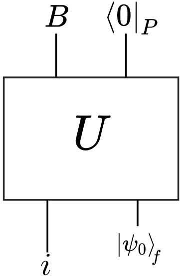


For an evaporating black hole we have an encoding map $V : \mathcal{H}_\ell \otimes \mathcal{H}_r \to \mathcal{H}_B$, where ℓ and r are interior modes and B is the microstate degrees of freedom. We also assume a trivial encoding map for the Hawking radiation R.

Model of an evaporating black hole

- We can make a concrete model of an evaporating black hole with a random unitary matrix U:

$$V = \sqrt{|P|}$$



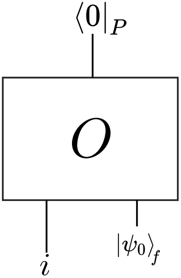
  where we combined ℓ and r into i.

- This simple model already reproduces the famous Page curve calculations of [Penington '19, Almheiri-Engelhardt-Marolf-Maxfield '19]. After the Page time this is a non-isometric embedding $i \to B$, and one can show that generically the non-isometric nature of the code cannot be detected by any observer who cannot perform operations exponential in the black hole entropy [CA-Engelhardt-Harlow-Penington-Vardhan '22].

Model of a closed universe

- We can model a closed universe by taking $|B| \to 1$, recovering the naive $1d$ fundamental Hilbert space:

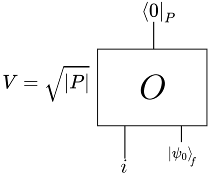
$$V = \sqrt{|P|}$$



where we have used an orthogonal matrix O because in a closed universe $\mathcal{CPT}$ should be gauged, implying a real Hilbert space [Harlow-Numasawa '23, Witten '25].

Inner products in closed universe model

- This encoding map doesn't do a great job preserving the physics of the effective description. We can see this by looking at what it does to inner products on average over the Haar measure on $O(d)$:



$$\int dO \, \langle\phi|V^\dagger V|\psi\rangle = \langle\phi|\psi\rangle$$

$$\int dO |\langle\phi|V^\dagger V|\psi\rangle - \langle\phi|\psi\rangle|^2 = 1 + |\langle\phi|\psi\rangle|^2 + |\langle\phi^*|\psi\rangle|^2 + O(1/d)$$

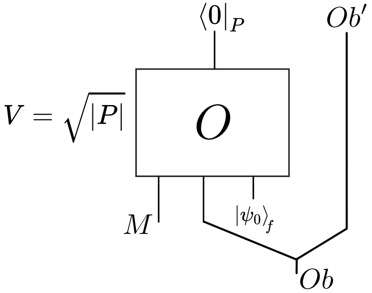
  - On average inner products are preserved, but the fluctuations are huge with the particular choice of O.
  - This issue is analogous to the large fluctuations in the inner product for closed universes from the GPI. Here we will discuss proposals for "adding an observer" that makes the effective description better encoded.

Observers in these models

- HUZ included an observer in this model by first dividing the input into

$$H_i = H_{Ob} \otimes H_M$$

- Then V is concatenated with an observer-cloning map:



- This leads to

$$|\mathcal{H}_{\rm fun}| = d_{Ob}$$

One way to see: variance in the inner products is now $O(1/d_{Ob})$.

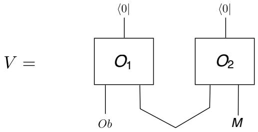
- A quicker way to see: $\mathcal{H}_{Ob'}$ is left over after the action of V_{Ob}.

Incorporating local structure

- This didn't fully model

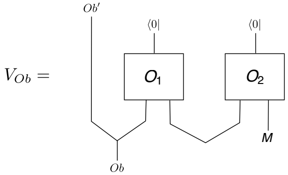
$$|\mathcal{H}_{\text{fun}}| \simeq \min\left(d_{Ob}, e^{2S_0}\right)$$

- We can get this with a more complete model recognizing the local structure of the effective description and map:



Angled lines represent maximally entangled states with dimension e^{2S_0}.

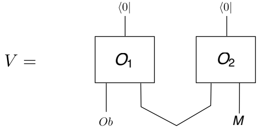
- HUZ version:

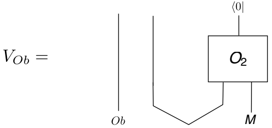


with $|\mathcal{H}_{\text{fun}}| \simeq \min\left(d_{Ob}, e^{2S_0}\right)$ because there can be a bottleneck if S_0 is too small.

New proposal for adding an observer

- I'll now propose a way to incorporate an observer into the encoding map that uses this local structure.
- Simply remove the part of the map acting on Ob:



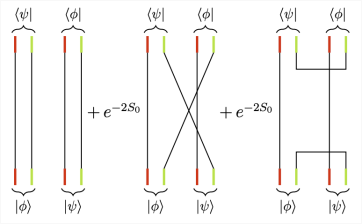


This leads to $|\mathcal{H}_{\text{fun}}| \simeq d_{Ob} e^{2S_0}$. The quickest way to see: that's the dimension of the legs leftover after acting V_{Ob}.

New rule $\simeq$ AAIL rule

- This new rule should be understood as implementing the AAIL GPI rule at the level of the holographic map.
- To see this, consider

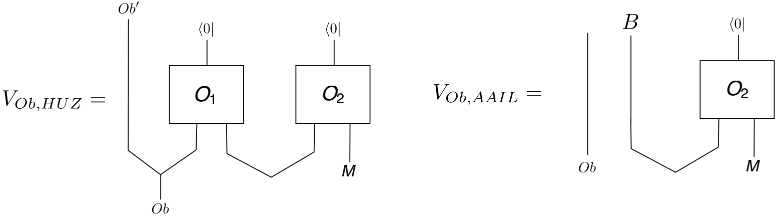
$$\overline{|\langle\psi|\phi\rangle|^2} =$$



The key point is that in every diagram, the observer leg connects the correct ket and bra. There are no cross terms.

Observer rule comparison

- Let's compare the two rules for adding observers into the encoding map:



HUZ tries to encode the effective description into Ob', and requires Ob to start entangled in a certain way. The AAIL map encodes everything into $Ob \otimes B$ with no requirement on the state of Ob.

Discussion

- Standard GPI and holography rules suggest closed universes have $1d$ Hilbert spaces.
- We designed a way to modify the holographic map to include special rules for an observer.
- This matched the rules for the GPI propose by AAIL, providing insight into their rules physical interpretation.
- Furthermore, this elucidated the difference with the rule proposed by HUZ.
- Interesting future work: are there physical reasons to prefer one proposal to the other?