DEPARTMENT OF PHYSICS, NATIONAL TAIWAN UNIVERSITY
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CHAIR PROFESSORSHIP AWARD

Quantum Entanglement and Gravitational spacetime

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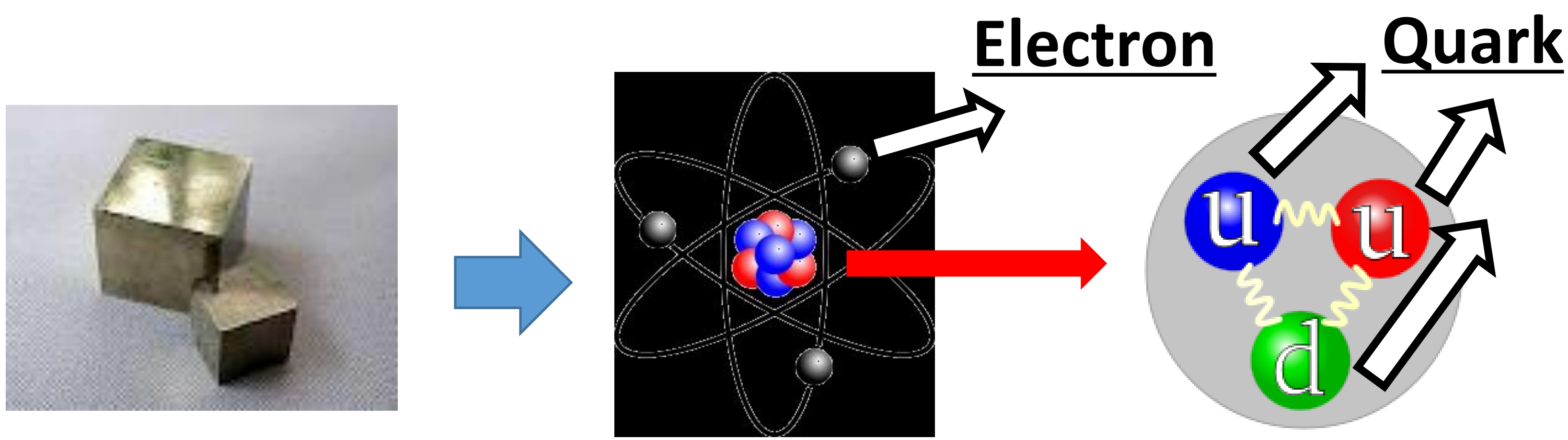
Grant-in-Aid for Transformative Research Areas A
Extreme Universe

①Introduction

What is particle physics ?

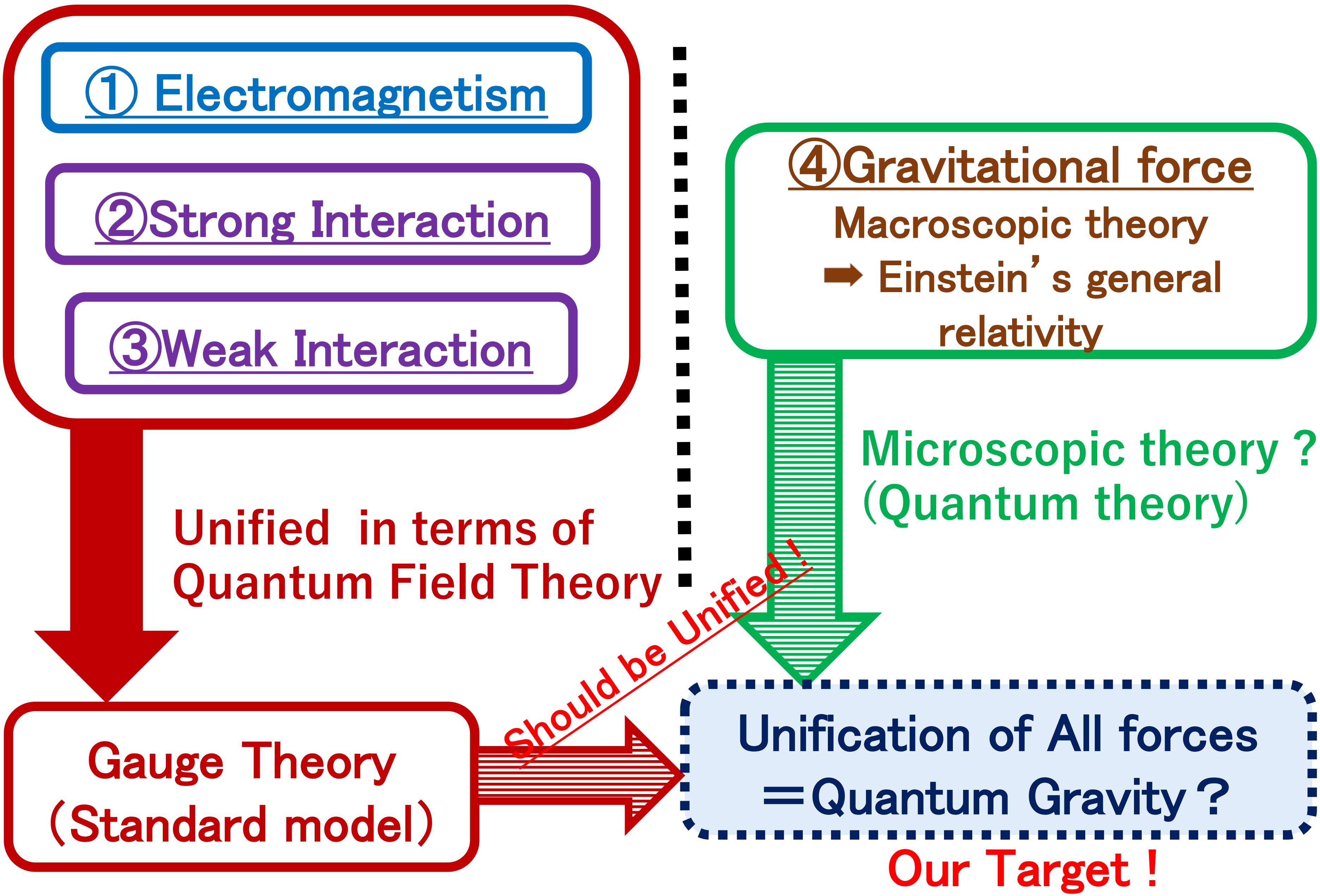
The study of fundamental particles and forces that constitute matter and radiation.

⇒**Exploring the minimal unit of matter**



Matter　Atom＝Electrons＋Nucleus　**Proton**

Four fundamental forces of Nature and Unification



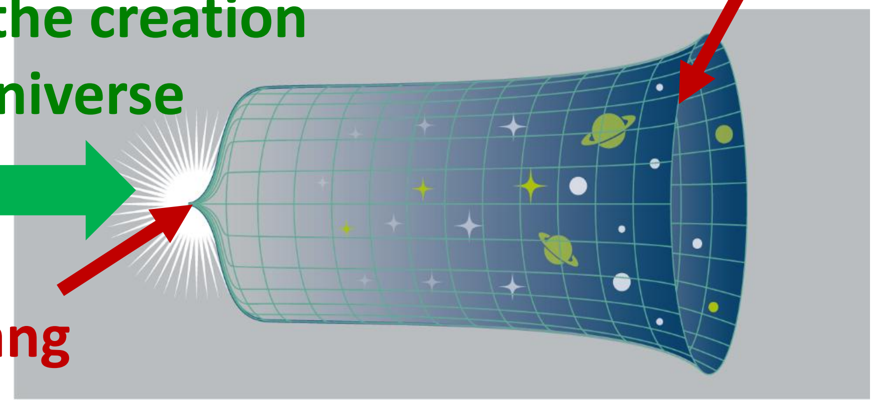
To understand the creation of Universe, we need quantum gravity !

<u>Quantum Gravity</u>

= Microscopic theory of gravity

Explain the creation of the Universe

Present Universe



Big Bang

Time

First of all, we want to magnify the Universe.

We need a “good microscope” !

What is the minimal unit of spacetime ?

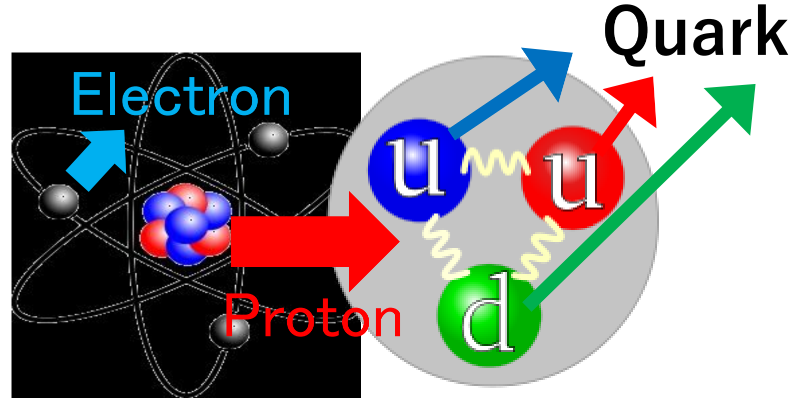
Microscope for Quantum Gravity ➡ Holography

High Energy Physics

Matter

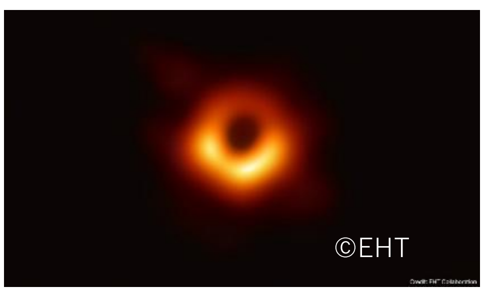
Microscope

Accelerators



Elementary particles

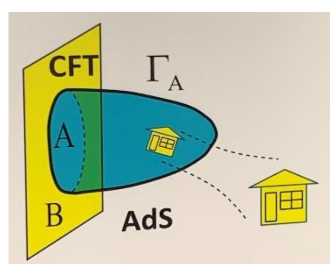
Quantum Gravity



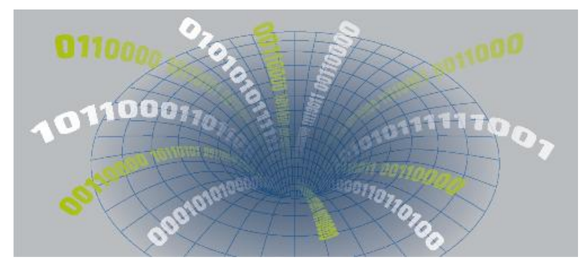
Universe

Thought experiment

Microscope



Holography



Collections of Qubits
(with entanglement)

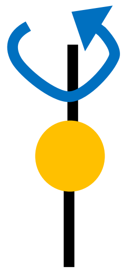
Contents

② Quantum Entanglement

Qubit

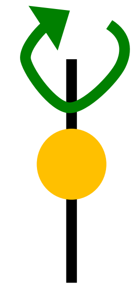
As an example of quantum state consider **electron's spin**.

Right rotation
(Up Spin)



$|0\rangle$

Left rotation
(Down Spin)



$|1\rangle$

One qubit state: $|\Psi\rangle = a|0\rangle + b|1\rangle$

Classical Computer

Classical Information

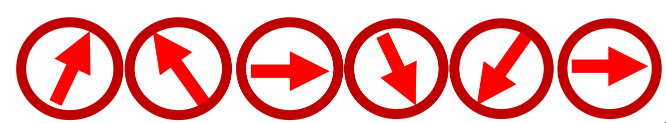
# of C info. = Bits

0 1 0 1 1 0

Quantum Computer

Quantum Information

# of Q info. = Qubits



Quantum Entanglement

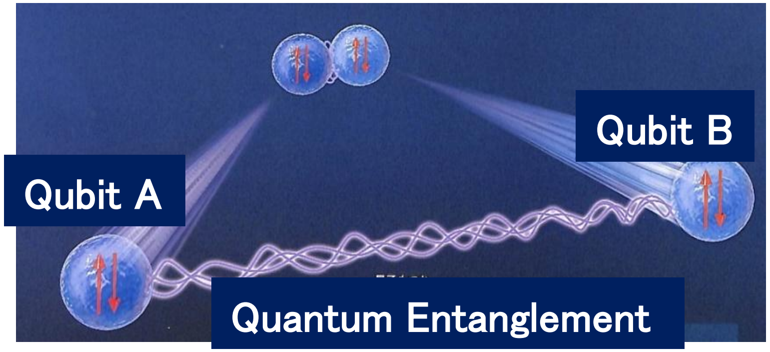
We start with a two qubit system: A and B.

Consider the Bell state :

$$|\Psi_{Bell}\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|1\rangle_B \; + \; |1\rangle_A|0\rangle_B\right)$$

If we measure A spin, then that tells us B spin at the same time !

This correlation between A and B is Quantum Entanglement !



Though we know the state for the total system AB, the state for a subsystem is not definite.

Entanglement Entropy (EE)

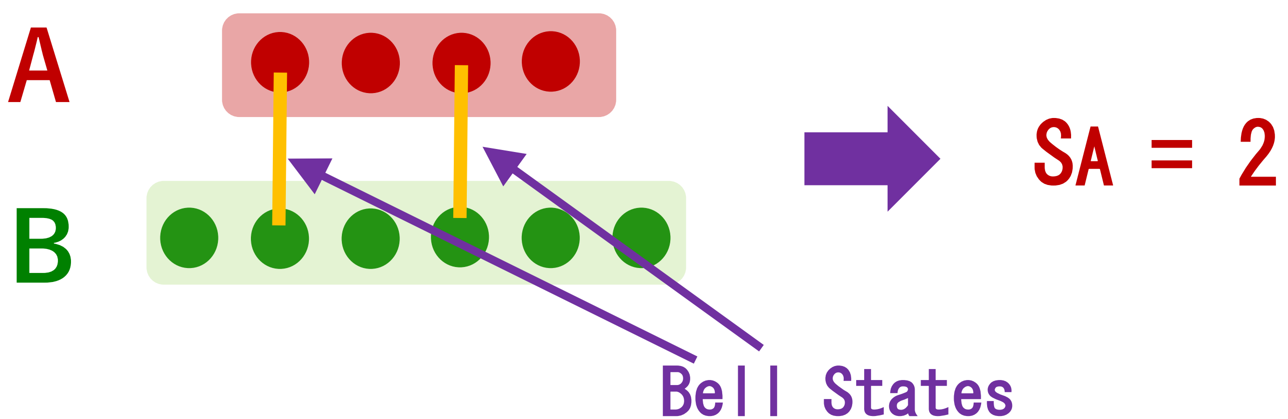
A measure of the amount of quantum entanglement

→ Entanglement Entropy (EE)

Entanglement Entropy between A and B:

S_A = # of Bell states between A and B

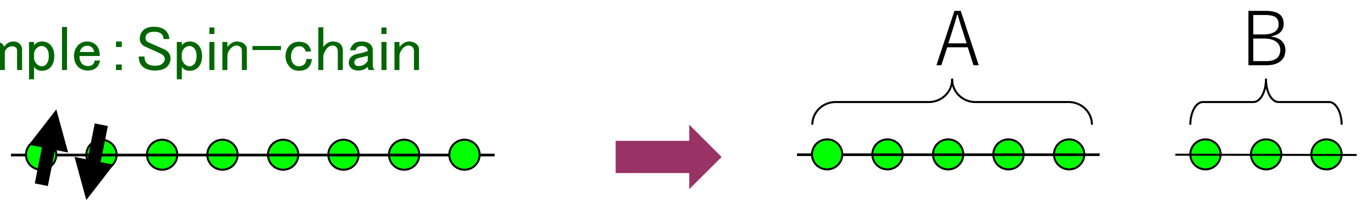
= the amount of "lost information" if an observer can only be accessible to A.



Definition of Entanglement Entropy (EE)

First we decompose the Hilbert space: $H_{tot} = H_A \otimes H_B$.

Example : Spin-chain



We introduce the reduced density matrix ρ_A

by tracing out B: $\rho_A = \mathrm{Tr}_B\left[|\Psi_{tot}\rangle\langle\Psi_{tot}|\right].$

The entanglement entropy (EE) S_A is defined by

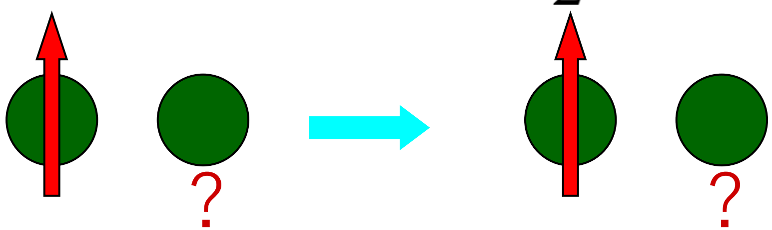
$$\boxed{S_A = -\mathrm{Tr}[\rho_A \log \rho_A]}$$

$\propto$ # of Bell Pairs between A and B

The Simplest Example: two spins (2 qubits)

$$\text{(i)}\quad |\Psi\rangle = \frac{1}{2}\left[|0\rangle_A + |1\rangle_A\right] \otimes \left[|0\rangle_B + |1\rangle_B\right]$$

$$\Rightarrow \rho_{\mathrm{A}} = \mathrm{Tr_B}\left[|\Psi\rangle\langle\Psi|\right] = \frac{1}{2}\left[|0\rangle_A + |1\rangle_A\right]\cdot\left[\langle 0|_A + \langle 1|_A\right] \cong \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.$$



Not Entangled

$$\mathrm{S_A} = 0$$

$$\text{(ii)}\quad |\Psi\rangle = \left[|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_{\mathrm{B}}\right] / \sqrt{2}$$

$$\Rightarrow \rho_A = \mathrm{Tr_B}\left[|\Psi\rangle\langle\Psi|\right] = \frac{1}{2}\left[|0\rangle_A\langle 0|_A + |1\rangle_A\langle 1|_A\right] \cong \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}.$$

Entangled

$$\mathrm{S_A} = -2\cdot\log\frac{1}{2} = \log 2.$$

Measurement of EE in Experiments

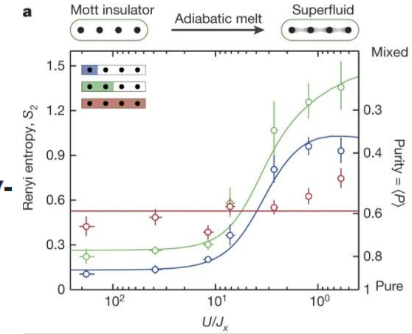
Ex.1: Ultracold bosonic atoms in optical lattices

Published: 02 December 2015

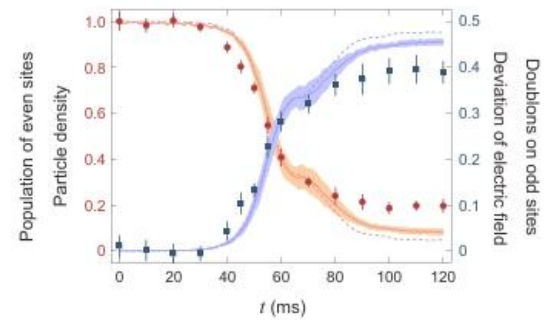
Measuring entanglement entropy in a quantum many-body system

Rajibul Islam, Ruichao Ma, Philipp M. Preiss, M. Eric Tai, Alexander Lukin, Matthew Rispoli & Markus Greiner

Nature **528**, 77–83 (2015) | Cite this article



$$H = -J\sum_{\langle i,j\rangle} a_i^\dagger a_j + \frac{U}{2}\sum_i n_i(n_i-1) \qquad (4)$$



Ex2: Trapped-ion quantum simulator

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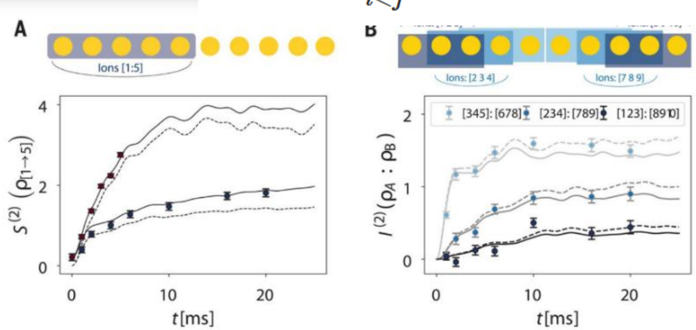
REPORT

Probing Rényi entanglement entropy via randomized measurements

TIFF BRYDGES, ANDREAS ELBEN, PETAR JURCEVIC, BENOÎT VERMERSCH, CHRISTINE MAIER, BEN P. LANYON, PETER ZOLLER, RAINER BLATT, AND CHRISTIAN F. ROOS Authors Info & Affiliations

SCIENCE • 19 Apr 2019 • Vol 364, Issue 6437 • pp. 260-263

$$H_{\mathrm{XY}} = \hbar\sum_{i<j} J_{ij}\left(\sigma_i^+\sigma_j^- + \sigma_i^-\sigma_j^+\right) + \hbar B\sum_j \sigma_j^z$$



Ex3. Topological EE in superconducting qubits

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SCIENCE • 2 Dec 2021 • Vol 374, Issue 6572 • pp. 1237-1241 • DOI: 10.1126/science.abi8378

RESEARCH ARTICLE | TOPOLOGICAL MATTER

Realizing topologically ordered states on a quantum processor

K. J. SATZINGER, Y.-J LIU, A. SMITH, C. KNAPP, M. NEWMAN, C. JONES, Z. CHEN, C. QUINTANA, X. MI, [...], AND P. ROUSHAN +88 authors

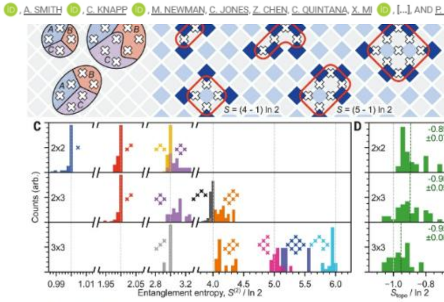
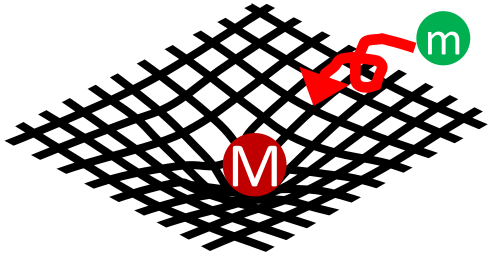


Fig. 2. Topological entanglement entropy.

③ Black hole and Entropy

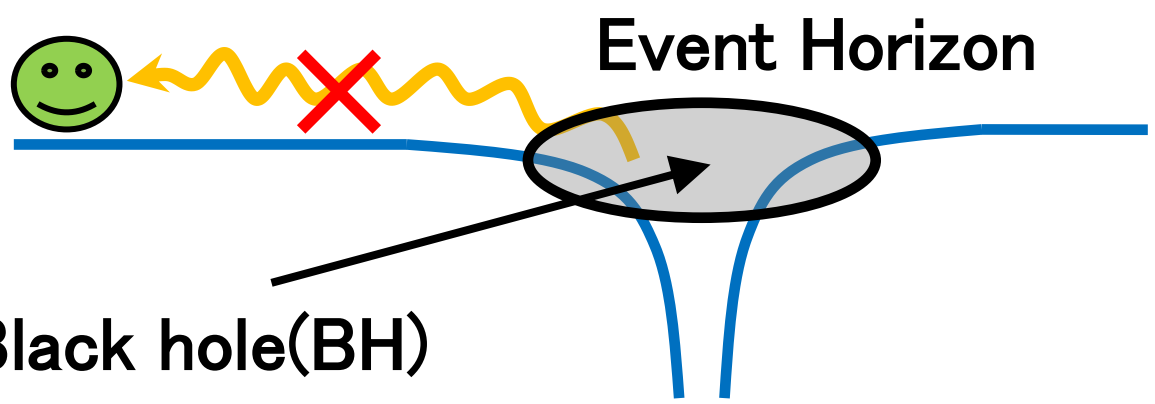
Following general relativity,
the spacetime gets distorted !



Black hole (BH)

An extremely dense astrophysical object.
Due to its strong gravitational force, even light rays cannot come out from black hole.

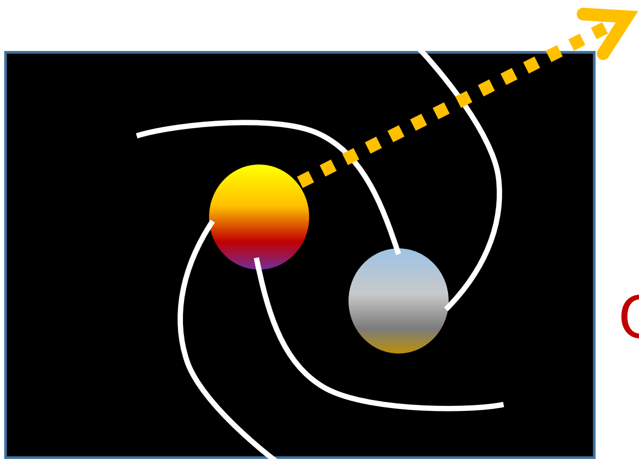
➡ Characteristic object in Einstein's general relativity



BH Entropy

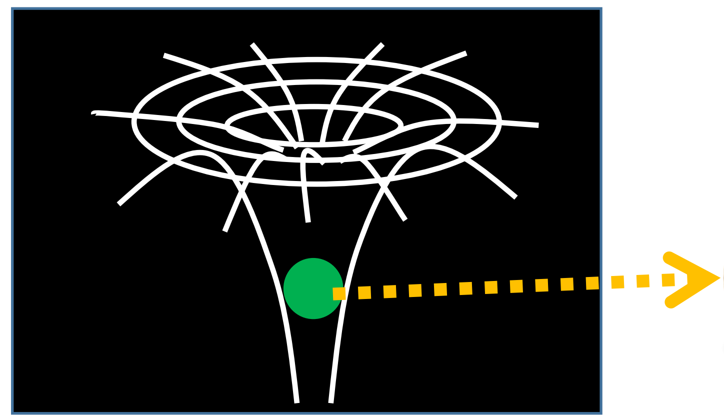
After stars collapsed into a BH, outside observers cannot access the information inside the BH.

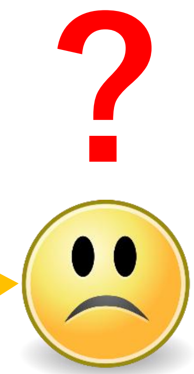
Stars



Gravitational collapse

BH





A lot of Information can be obtained !

Hidden Information !

BH entropy !

Bekentein-Hawking Formula of BH Entropy [1972-1976]

Calculations in general relativity show that a BH has the following entropy:

⇒Still mysterious !

$$\boxed{S_{BH} = \frac{k_B c^3}{\hbar} \times \frac{A_{BH}}{4G_N}}$$

⇒ BH thermodynamics !

A_{BH}= Surface Area of Black hole ⇒ **Geometry**
G_N=Newton constant ⇒ **Gravity**
$\hbar$=Planck constant ⇒ **Quantum Mechanics**
k_B=Boltzmann const.⇒ **Stat. Mech. , Quantum Info.**

$\Big\}$ **Quantum Gravity!**

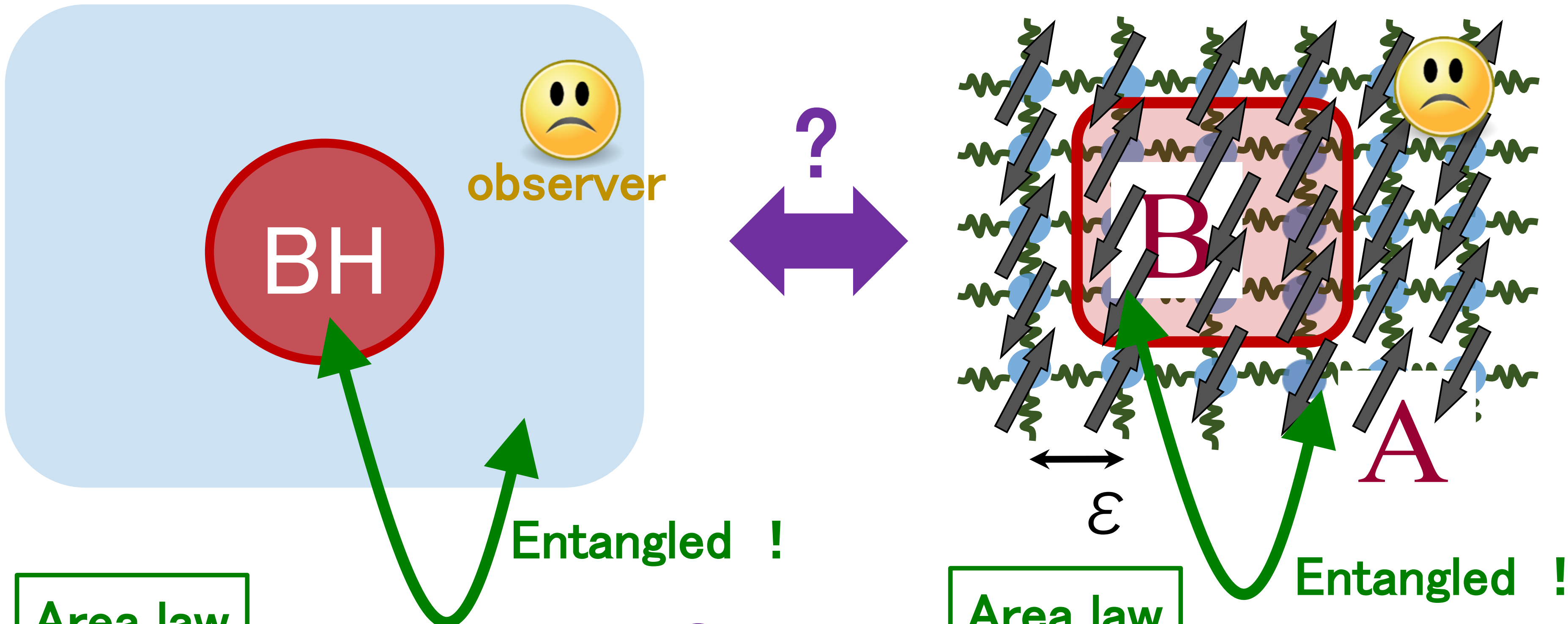
[1] BH Entropy is proportional to the **area**, not to the volume !

[2] BH has the entropy even in the **classical theory** of Gravity !

Analogy between BH and Qubits

[Original motivation of studying EE in QFTs, Bombelli et.al. 1986, Srednicki 1993]

Blackhole Spacetime **Quantum Spin System**



?

BH entropy S_{BH} | Entanglement entropy S_A

Spacetime | Matter

④ Holography and Quantum Entanglement

BH Entropy Formula

$$\boxed{S_{BH} = \frac{A_{BH}}{4G_N}}$$

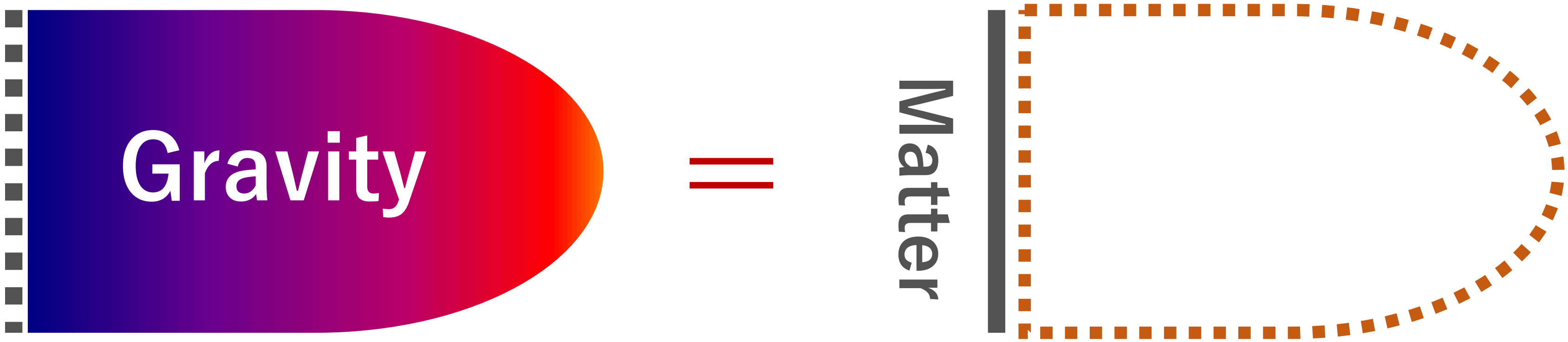
Degrees of freedom in Gravity ∝ Area

['t Hooft 1993, Susskind 1994]

Holography

Boundary of M

Gravity on M = Quantum Matter on ∂M



BH entropy(∝Area)= Thermal Entropy of Matter (∝Volume)

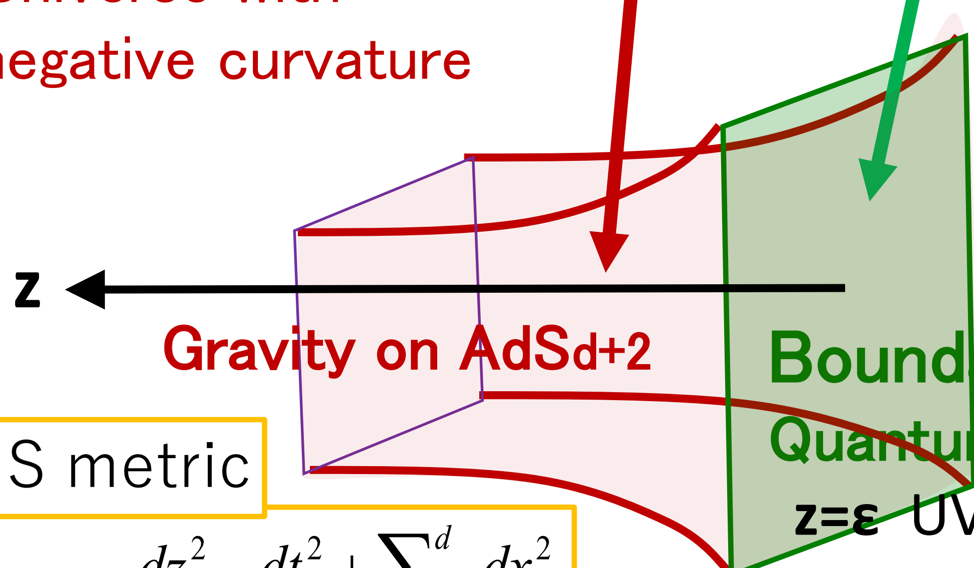
Gauge/Gravity Duality: best known example of holography

Gauge/Gravity Duality (AdS/CFT) – [Maldacena 1997]

(Quantum) Gravity on d+2 dim. Anti-de Sitter Space = **d+1 dim. Gauge theory (or Conformal field theory)**

Anti-de Sitter space (AdS)
→ Universe with negative curvature

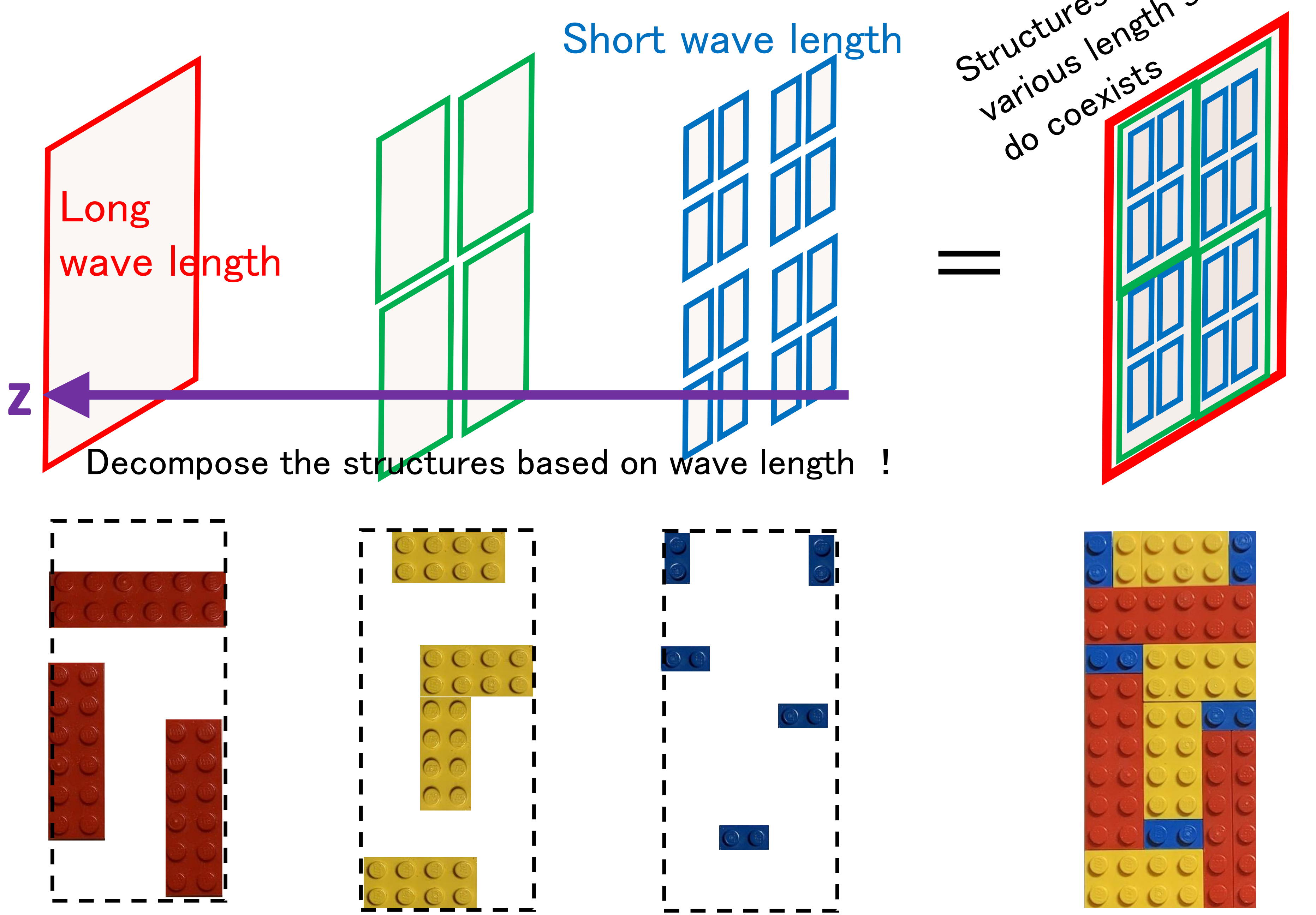
Conformal Field Theory(CFT)
→ d dim. Gapless matter at quantum critical point
→ Theory of massless particles



AdS metric

$$ds^2 = R^2 \cdot \frac{dz^2 - dt^2 + \sum_{i=1}^{d} dx_i^2}{z^2}$$

A sketch on how AdS/CFT works

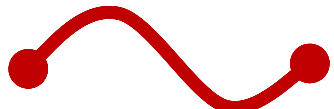


String Theory ➡ The best candidate of quantum gravity

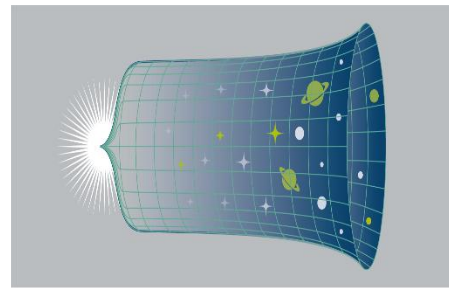
In string theory, the most microscopic constituents are strings ! [Nambu, Goto 1970, . . Yoneya, Scherk-Schwarz 1974, . .]

magnify

Open string



Matter (gauge theory)



Closed string

magnify

Universe (gravity)

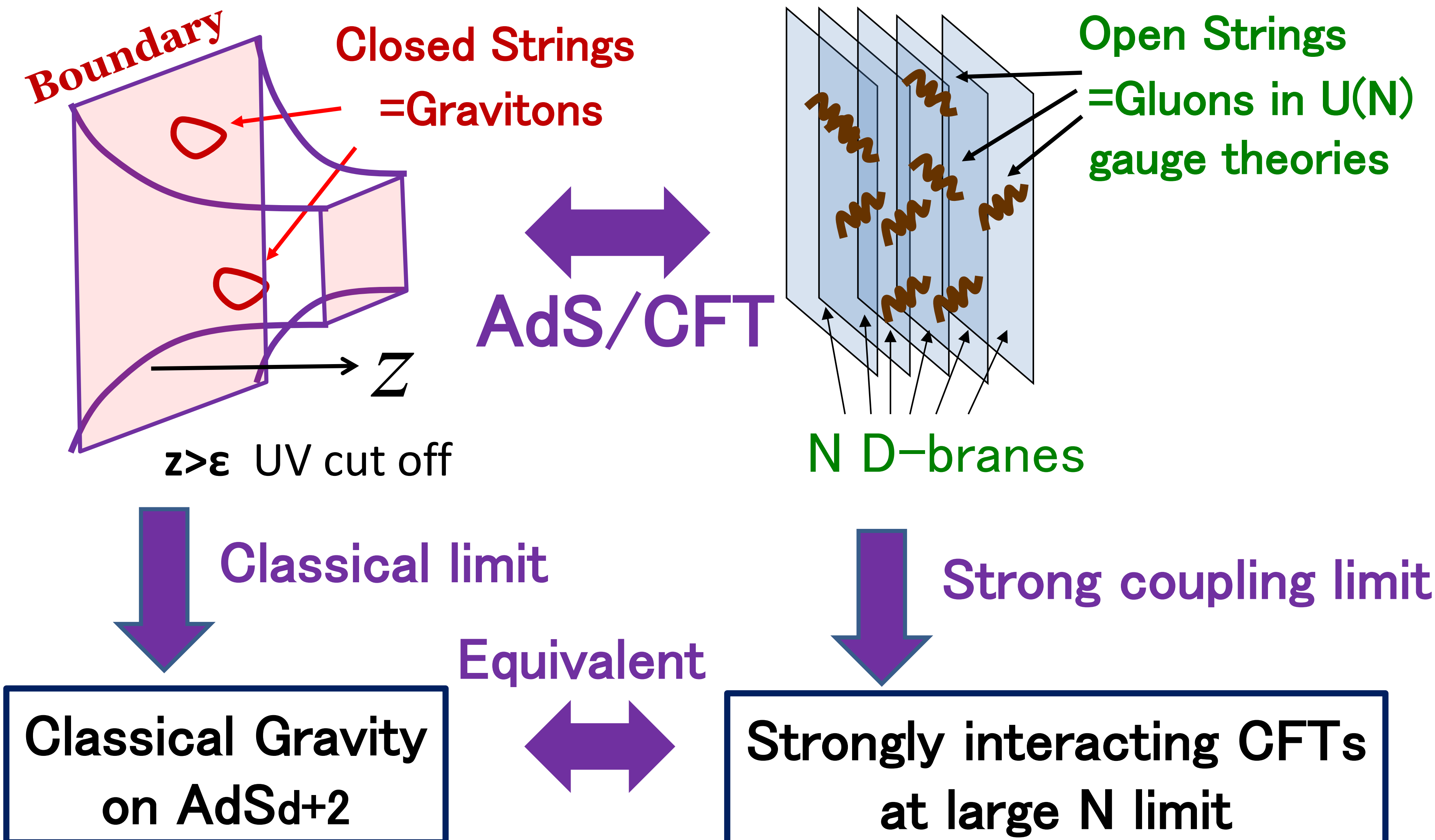
Open string (matter) and closed string (gravity) are equivalent !

Closed string

Open string

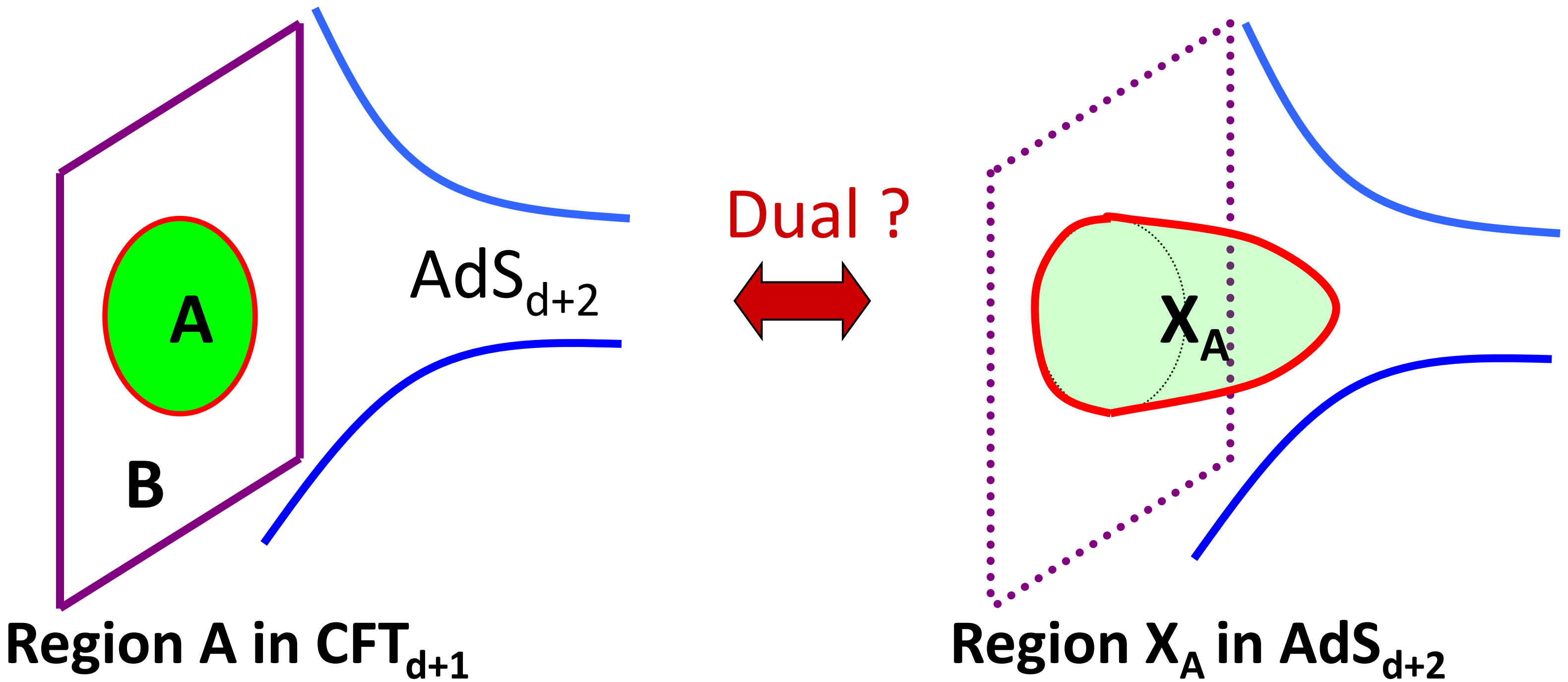
➡ String theory beautifully unifies gravity and matter !

AdS/CFT from String theory



A Basic Question of AdS/CFT:

Which region in the AdS does encode the `information in a specific region' of the CFT ?



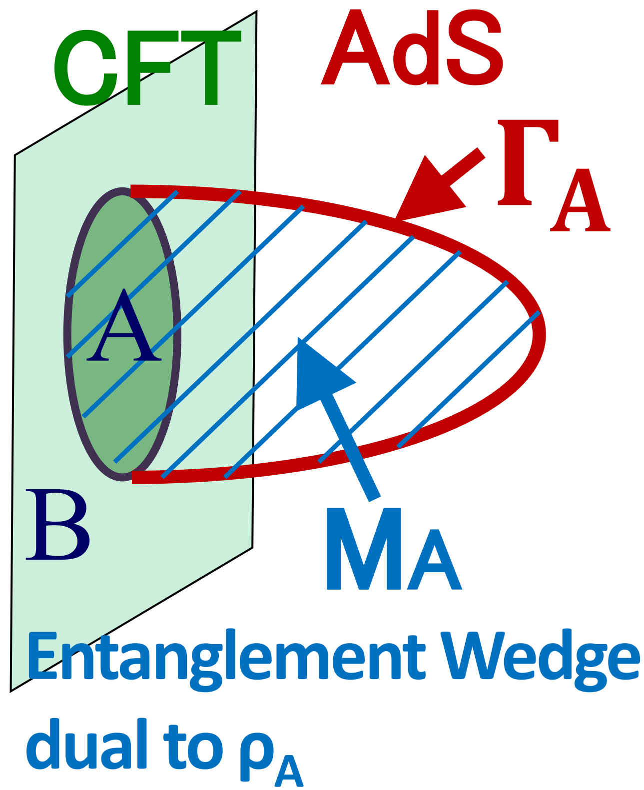
➡ Consider the entanglement entropy S_A which measures the amount of information !

Holographic Entanglement Entropy [Ver.1:Static]

[Ryu-TT 2006]

Γ_A = Surface which surrounds A in AdS

$$S_A = \mathrm{Min}\left[\frac{\mathrm{Area}(\Gamma_A)}{4G_N}\right]$$

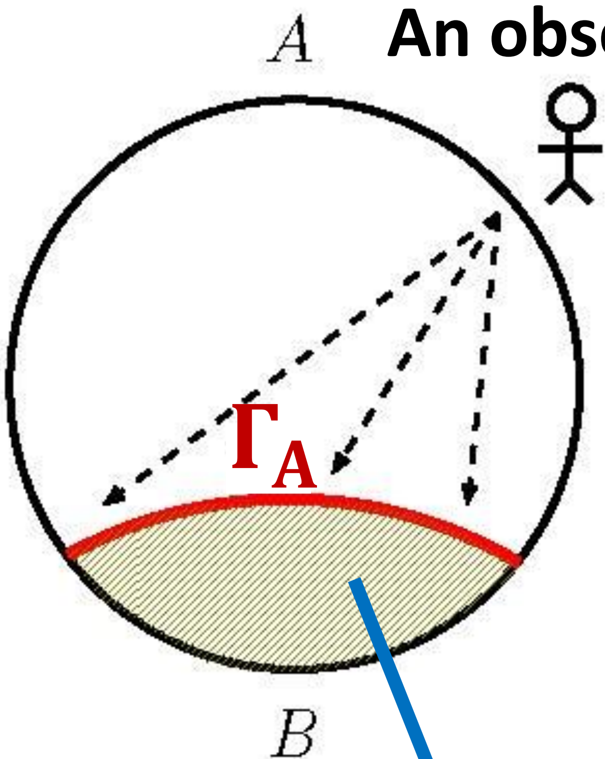


➡ A major generalization of black hole entropy formula

➡ Information in A is encoded in the entanglement wedge MA

How did we reach this formula ?

An observer in A



An observer in A, who is not accessible to B, probably thinks there is a “black hole” at Γ_A.

$\Rightarrow$This BH entropy is S_A !

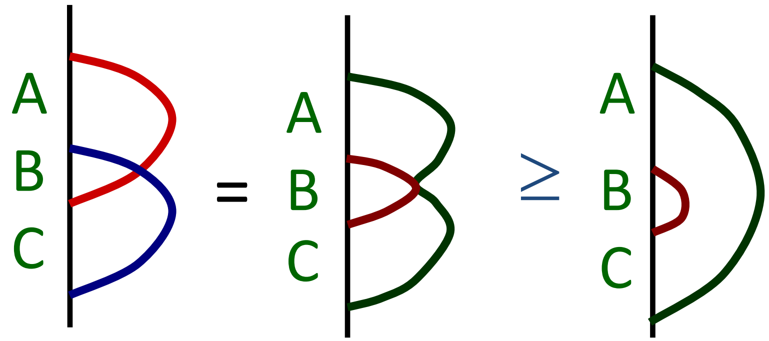
A more sophisticated interpretation is given by the **entanglement wedge dual to ρ_B.**

[Czech-Karczmarek-Nogueira-Raamsdonk, Wall 2012,
Headrick-Hubeny-Lawrence-Rangamani 2014….]

Algebraic properties in Quantum Information
⇔ **Geometric properties** in Gravity

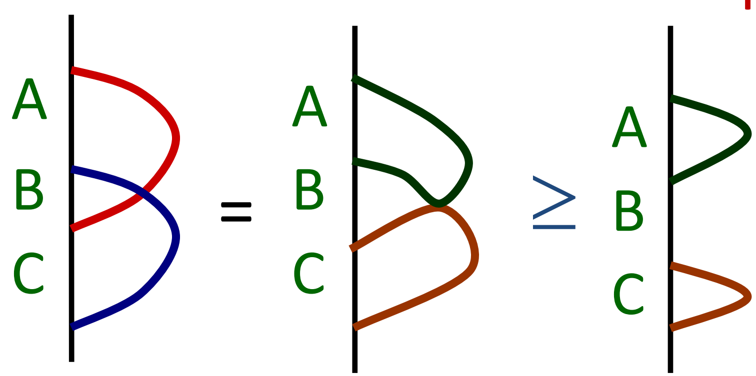
Holographic Proof of Strong Subadditivity(SSA)

[Headrick–TT 07]



$$\Rightarrow\ S_{AB}+S_{BC}\geq S_{ABC}+S_{B}$$

"Triangle inequalities in Geometry = SSA"

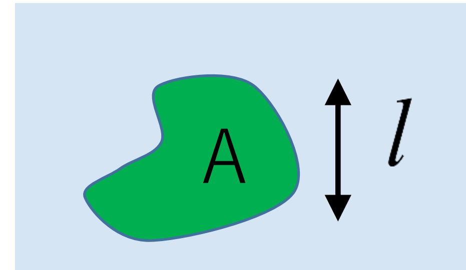


$$\Rightarrow\ S_{AB}+S_{BC}\geq S_{A}+S_{C}$$

$$(\text{Note}:\quad AB\equiv A\cup B)$$

General Behavior of HEE (=EE in CFT_{d+1}) [Ryu−TT 06,⋯]

HEE predicts the behavior of EE in strongly coupled CFTs.



$$S_A = \frac{\pi^{d/2}R^d}{2G_N^{(d+2)}\Gamma(d/2)}\left[p_1\left(\frac{l}{\varepsilon}\right)^{d-1} + p_3\left(\frac{l}{\varepsilon}\right)^{d-3} + \cdots \right.$$

$$\left. \cdots + \begin{cases} p_{d-1}\left(\frac{l}{\varepsilon}\right) + p_d & (\text{if } d+1=\text{odd}) \\ p_{d-2}\left(\frac{l}{\varepsilon}\right)^2 + q\log\left(\frac{l}{\varepsilon}\right) & (\text{if } d+1=\text{even}) \end{cases} \right],$$

Area law divergence

where $p_1 = (d-1)^{-1},\ p_3 = -(d-2)/[2(d-3)],....$

..... $q = (-1)^{(d-1)/2}(d-2)!!/(d-1)!!$.

A universal quantity (**F**) which characterizes **odd dim. CFT**.

Agrees with **conformal anomaly** (central charge) in **even dim. CFT**

Holographic Entanglement Entropy [Ver.2:Time-dependent]

[Hubeny-Rangamani-TT 2007]

A generic Lorentzian asymptotic AdS spacetime is dual to a time dependent state $|\Psi(t)\rangle$ in the dual CFT.

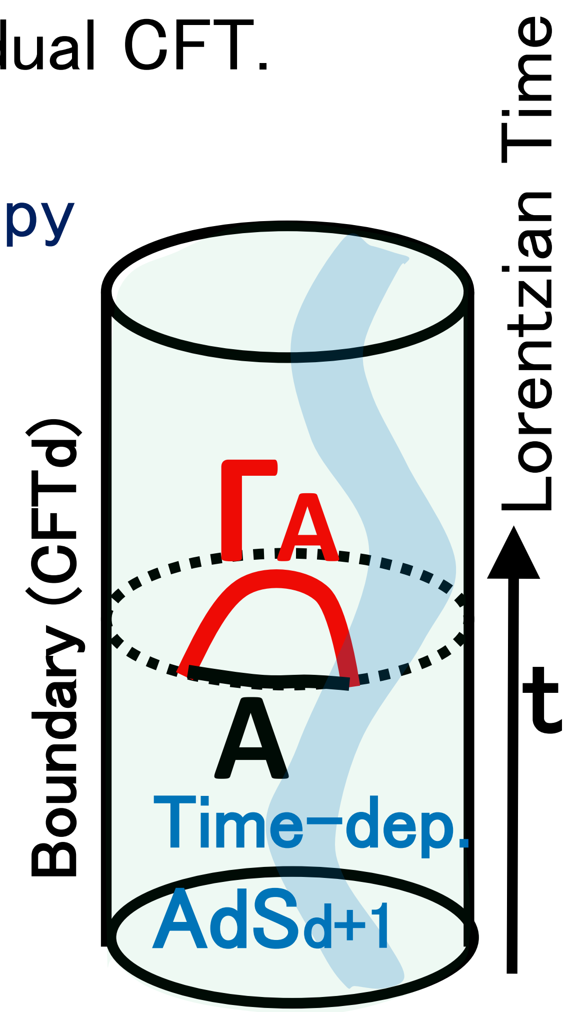
The time−dependent entanglement entropy

$$\rho_A(t) = \mathrm{Tr}_B[|\Psi(t)\rangle\langle\Psi(t)|] \Rightarrow S_A(t).$$

is computed from an extremal surface area:

$$\boxed{S_A(t) = \mathrm{Min}_{\Gamma_A}\mathrm{Ext}_{\Gamma_A}\left[\frac{A(\Gamma_A)}{4G_N}\right]}$$

$$\partial A = \partial\Gamma_A \ \text{ and } \ A \sim \Gamma_A \ .$$



Einstein Equation from Quantum Entanglement

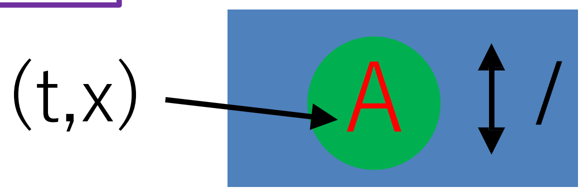
First Law of EE

First law of thermodynamics $\boxed{T\Delta S = \Delta E}$

$$\boxed{\Delta S_A \cong \Delta E_A}$$

[$E_A = -\log \rho_A$: Modular Hamiltonian]

[Casini–Huerta–Myers 13, Bhattachrrya–Nozaki–Ugajin–TT 13]



$$\boxed{\left(\partial_l^2 - \partial_l - \partial_x^2 - \frac{3}{l^2}\right)\Delta S_A(t,\vec{x},l) = \langle O \rangle \langle O \rangle}$$

[Nozaki–Numasawa–Prudenziati–TT 13]

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$$

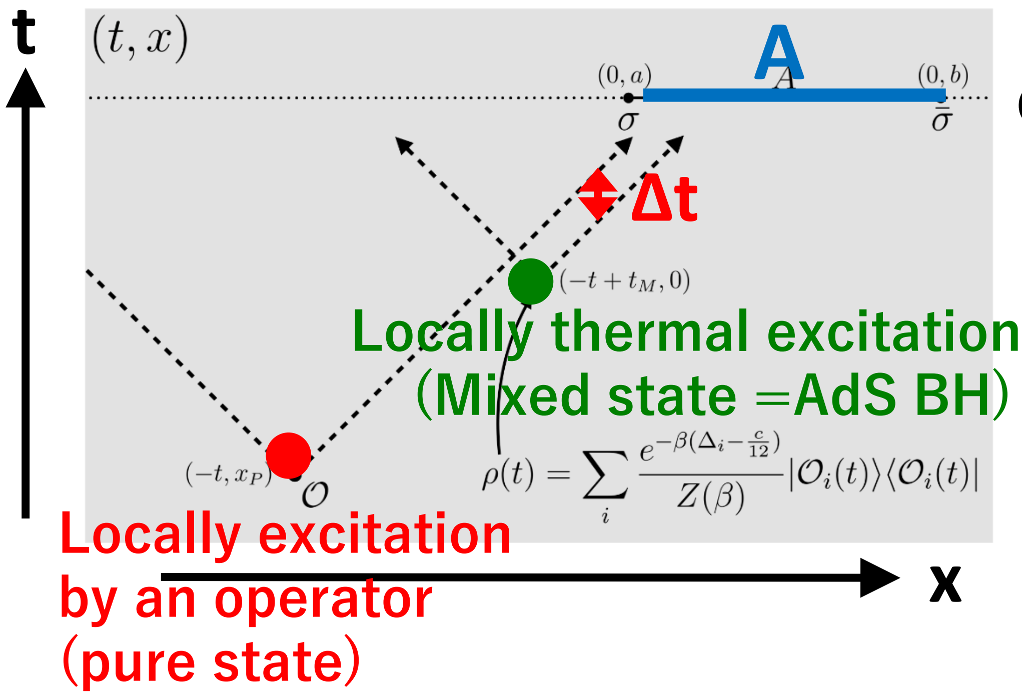
Kinetic term **C.C.** **Matter**

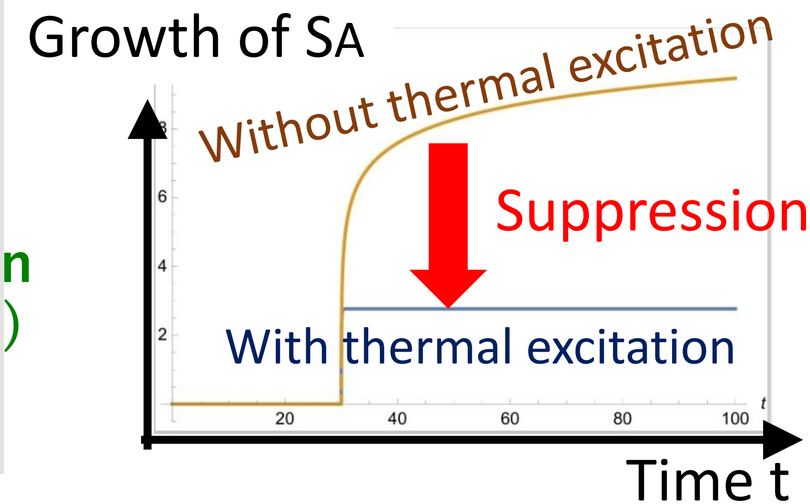
➡ The 1st law of EE explains the perturbative Einstein eq.

[Linear: Lashkari–McDermott–Raamsdonk, Faulkner–Guica–Hartman–Myers–Raamsdonk 13, Non–linear: Faulkner–Haehl–Hijano–Parrikar–Rabideau–Raamsdonk 17, Sarosi–Ugajin 17]

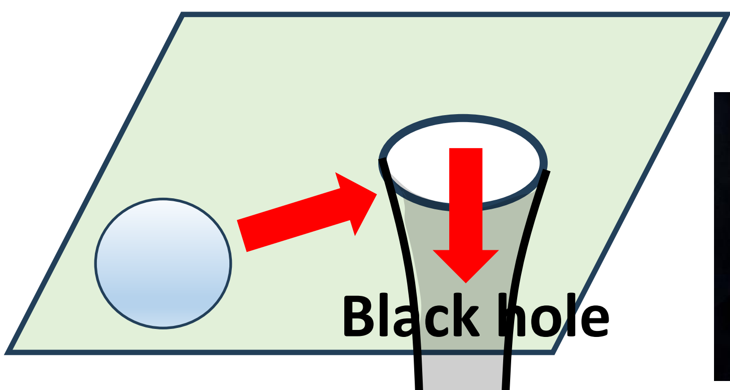
Entanglement scattering in strongly interacting 2d CFT

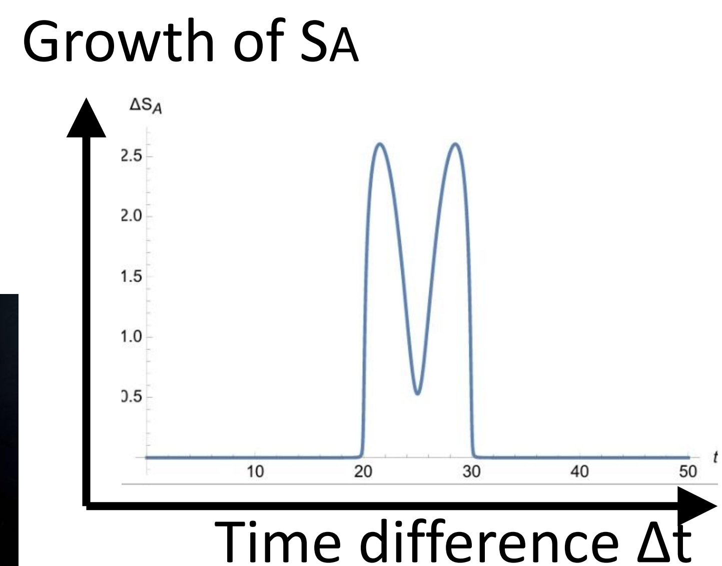
[Doi-TT 2025]





▶This is much like a Pinball game !



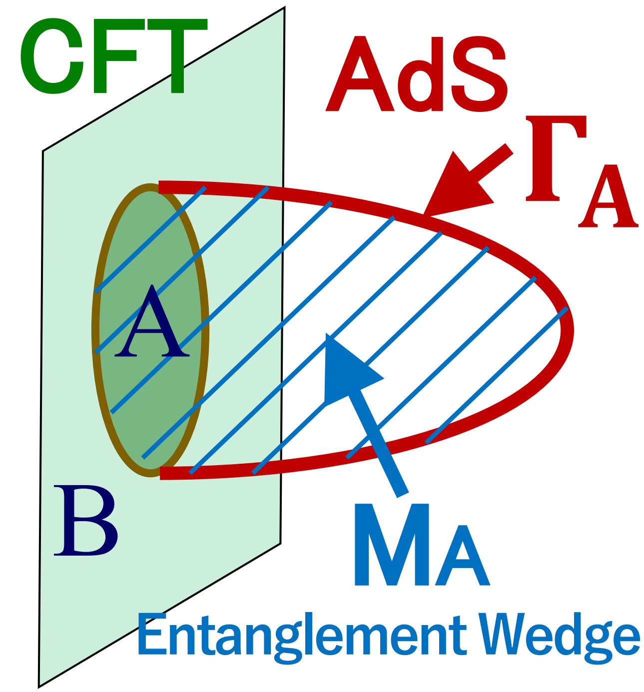


Quantum Corrections to HEE formula

HEE for classical gravity

[Ryu–TT 2006, Hubeny–Rangamani–TT 2007]

$$S_A = \underset{\Gamma_A}{\mathrm{Min\,Ext}}\left[\frac{\mathrm{Area}(\Gamma_A)}{4G_N}\right]$$



Loop corrections $1/G_N + 1 + G_N + \cdots$

↔ 1/N corrections in CFT

HEE with quantum corrections

$$S_A = \underset{\Gamma_A}{\mathrm{Min\,Ext}}\left[\frac{\mathrm{Area}(\Gamma_A)}{4G_N} + \underbrace{S_{bulk}(M_A)}_{\text{Bulk EE}}\right]$$

Quantum Extremal Surface

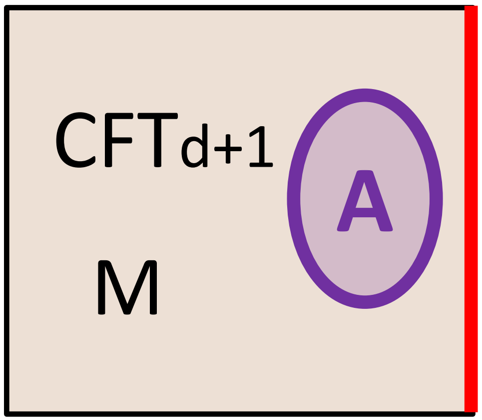
[Faulkner–Lewkowycz–Maldacena 2013,

Engelhardt–Wall 2014]

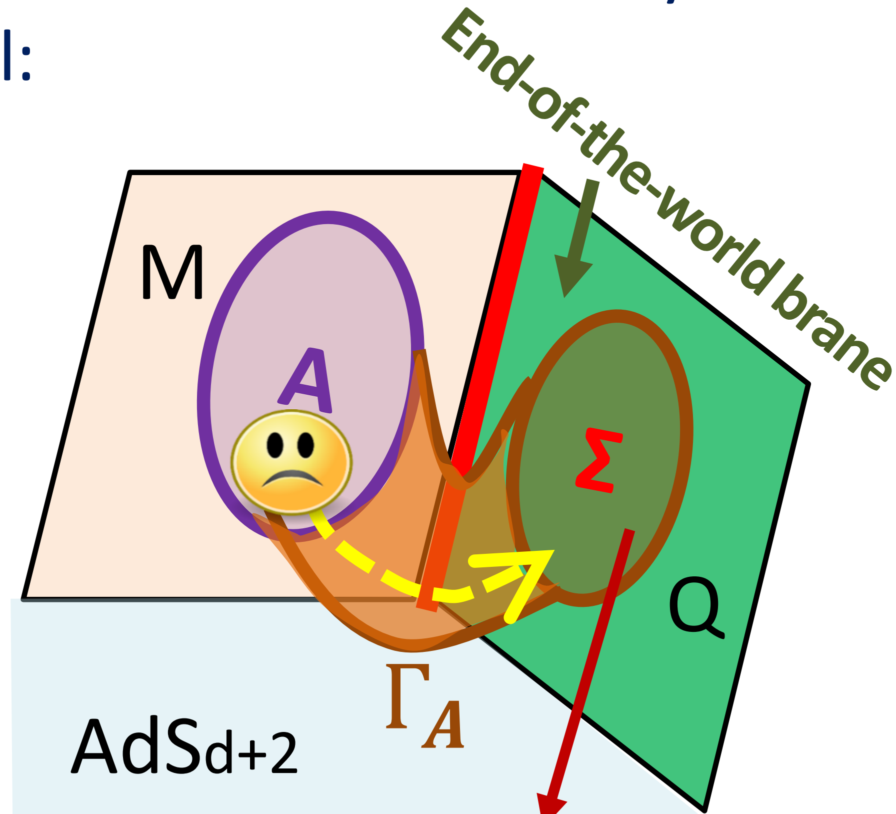
HEE in AdS/BCFT

[TT 2011, Fujita-Tonni-TT 2011]

A gravity dual of a CFT on a manifold with a boundary is given by the AdS/BCFT model:



AdS/BCFT
=



The extremal surface ΓA can end on EOW brane !

HEE formula in AdS/BCFT

$$\boxed{S_A = \underset{\Gamma_A,\,\Sigma}{\mathrm{Min\,Ext}}\left[\frac{\mathrm{Area}(\Gamma_A)}{4G_N}\right]}$$

$\partial\Gamma_A = \partial A \cup \partial\Sigma$

The region **Σ** is now known as an **Island** !

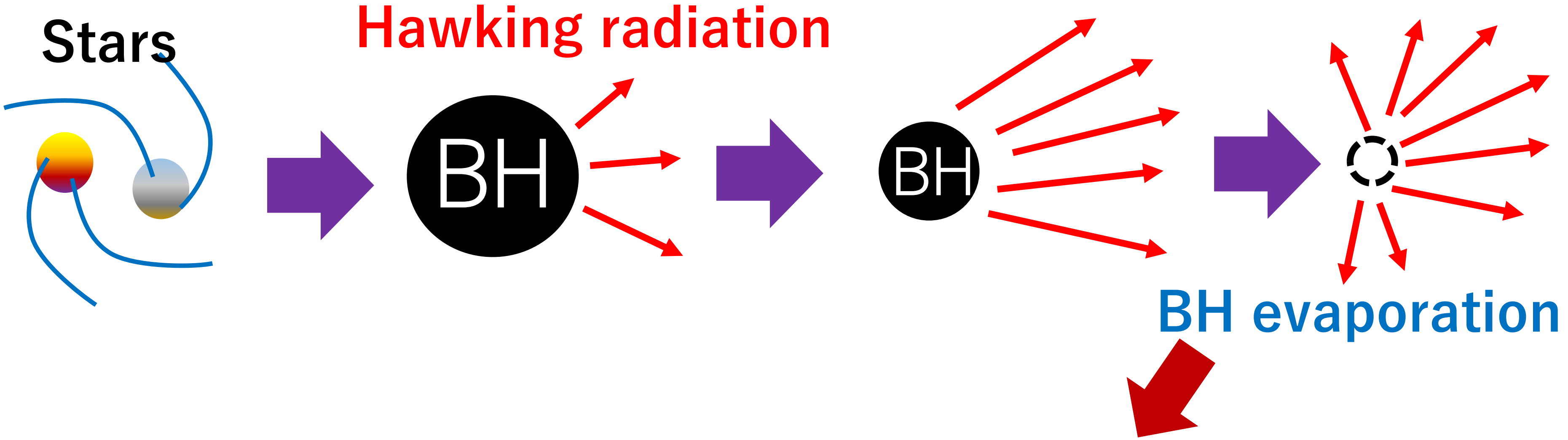
⑤ Applications to Black hole Information Problem

Black hole (BH) information Problem

A BH has a temperature and is a thermal object.

Thus, a BH radiates (Hawking radiation) and loses its energy.

Eventually it evaporates and disappears, called BH evaporation.



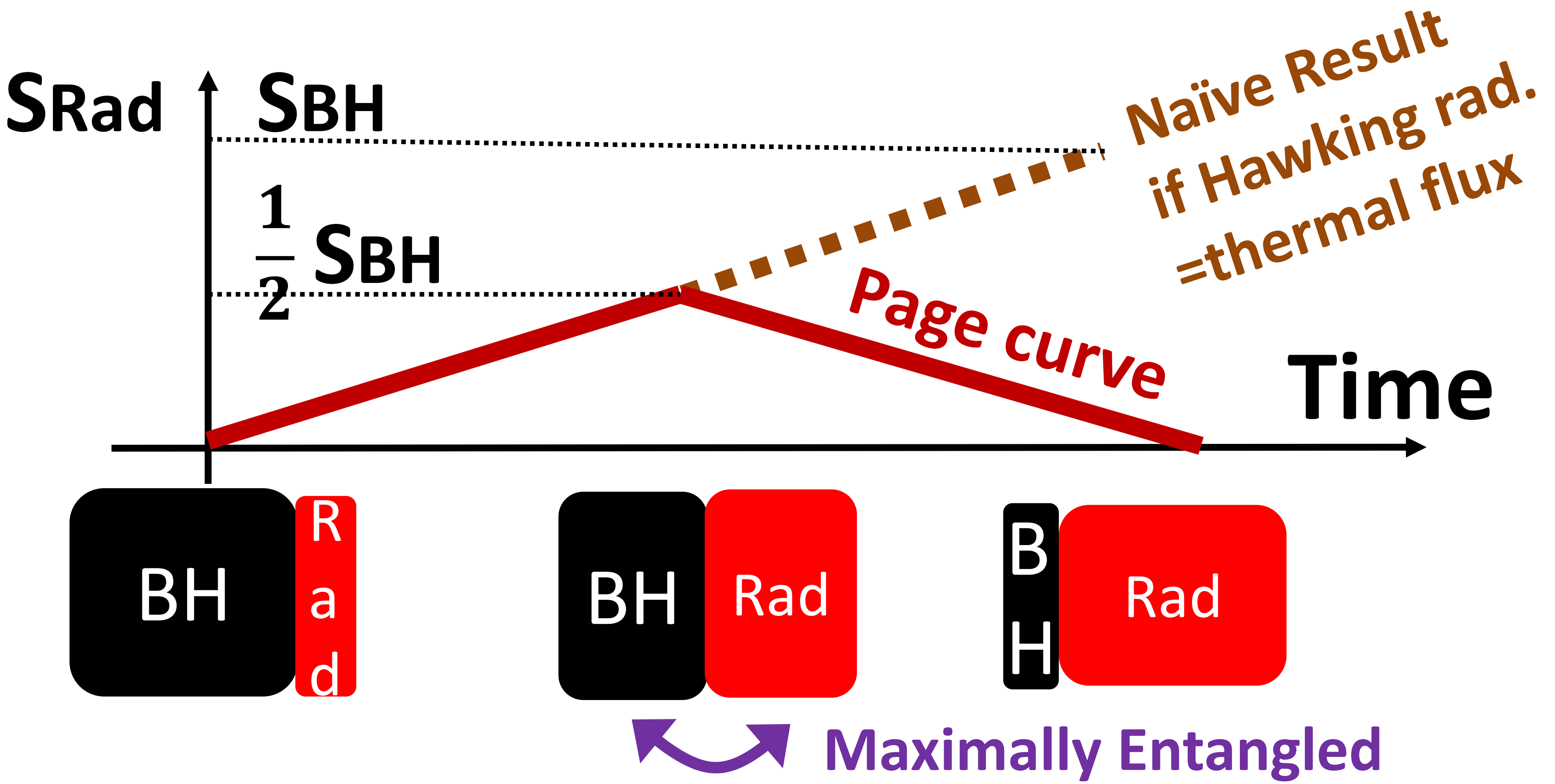
The information inside BH seems to disappear !

→If so, this contradicts with the unitarity of quantum mechanics !

Page curve [A quantification of BH information problem]

Model: $H_{tot} = H_{BH} \otimes H_{Rad}$ $\quad |\psi(t)\rangle = e^{-iHt}|\psi_0\rangle \in H_{tot}$

$$\rho_{\mathrm{Rad}}(\mathrm{t}) = \mathrm{Tr}_{\mathrm{BH}}\left[|\psi(t)\rangle\langle\psi(t)|\right] \Rightarrow \boxed{S_{Rad}(t) = -\mathrm{Tr}\left[\rho_{\mathrm{Rad}}(\mathrm{t})\log\rho_{\mathrm{Rad}}(\mathrm{t})\right].}$$



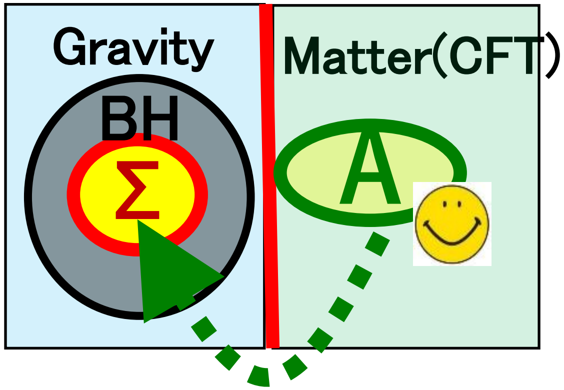
Black hole information problem and Island Formula

The Page curve was derived by generalizing the HEE formula to CFTs which is coupled to gravity, so called the Island formula ！

Island Formula: [Penington 2019, Almheiri et.al. 2019]

$$S_A = \underset{\Sigma}{\mathrm{Min}}\left[\frac{\mathrm{Area}(\partial\Sigma)}{4G_N} + S_{A\cup\Sigma}\right]$$

EE for Radiations BH entropy Bulk EE



Accessible to the Island Σ

This explains the Page curve !
➔ Unitarity of BH evaporation!

As the BH evaporation proceeds,
a secret hole (called Island) appears inside BH.
Via this secret tunnel, we recover BH information.

Mechanism of Resolving the BH information Problem

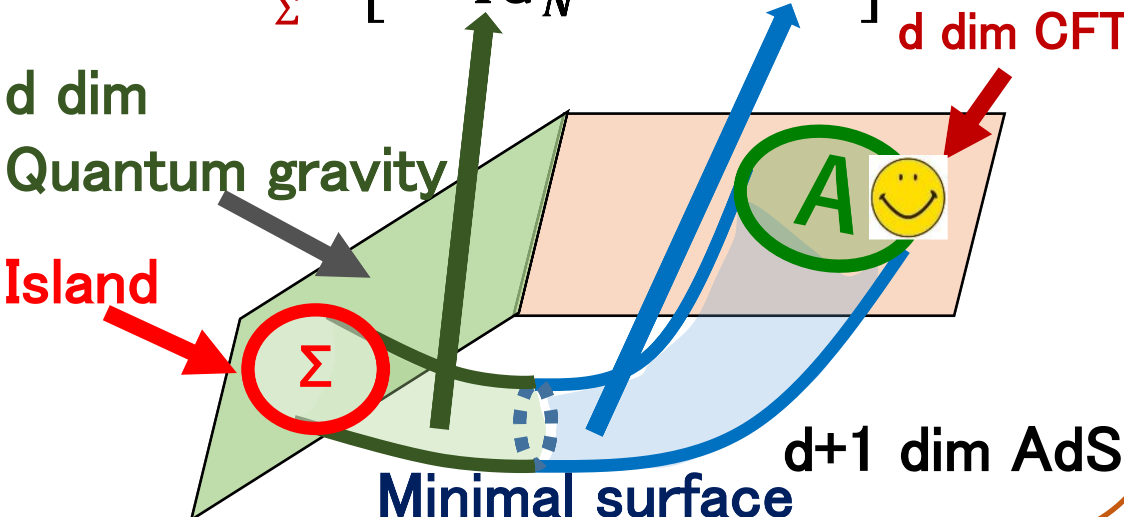
♦ A “wormhole” which connects the outside observer and the BH interior emerges in the middle of evaporation !

♠ However, the decoding of BH information from radiation is extremely hard (computational complexity is exponential) !

Higher dim. picture via AdS/BCFT

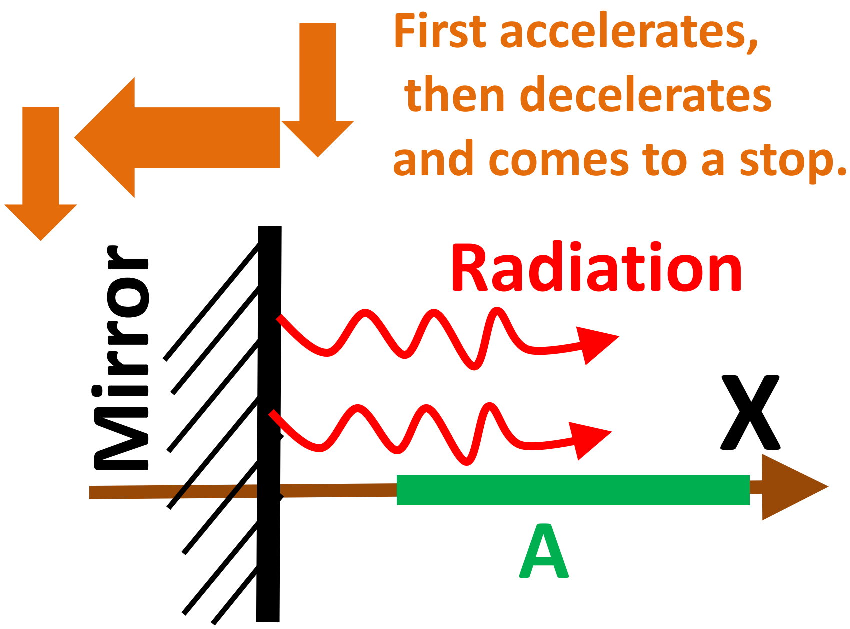
[AdS/BCFT, Double holography]

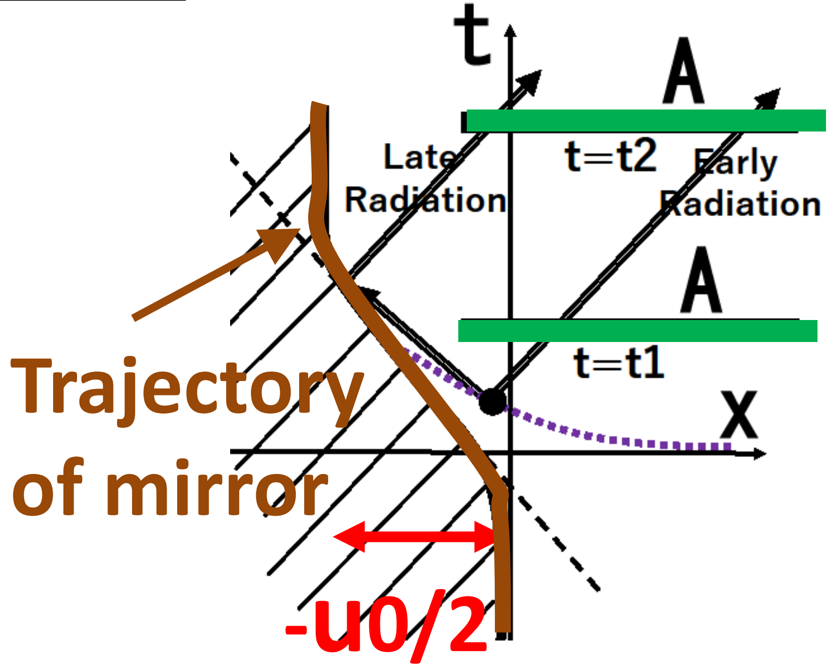
$$S_A = \underset{\Sigma}{\mathrm{Min}}\left[\frac{\mathrm{Area}(\partial\Sigma)}{4G_N} + S_{A\cup\Sigma}\right]$$



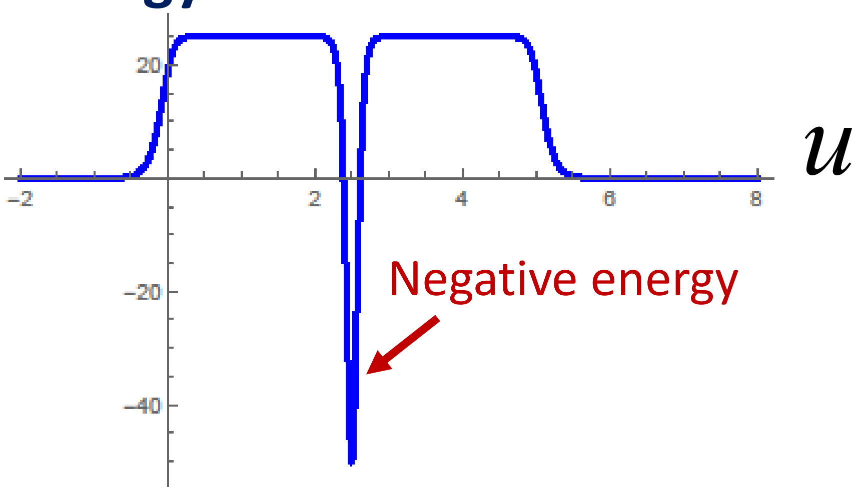
Moving-Mirror Model for BH evaporation

[Akal-Mollabashi-Shiba-Wei-TT 2021]

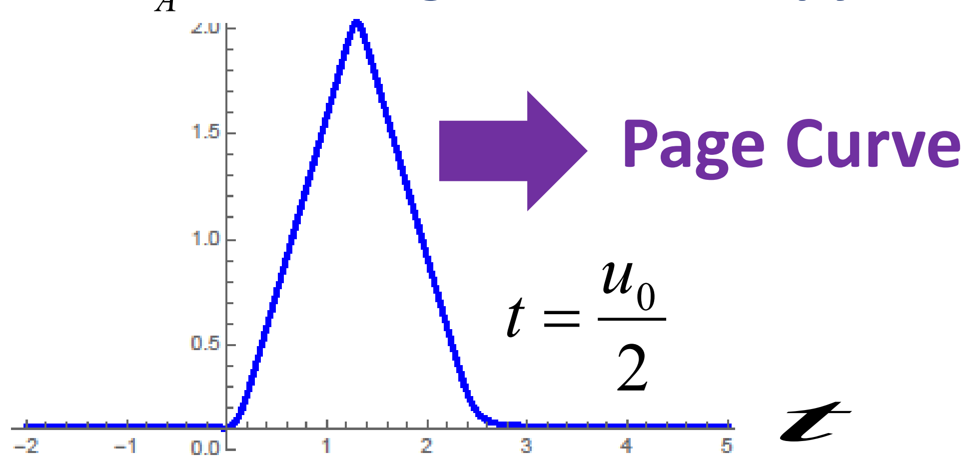




Energy flux



S_A/c **Entanglement entropy**

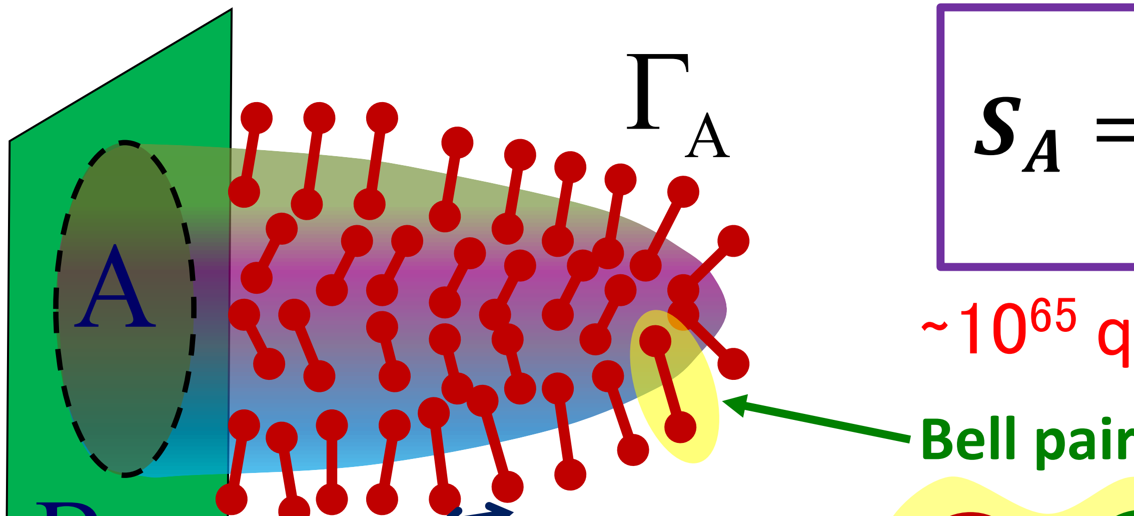


$$t = \frac{u_0}{2}$$

$$\beta = 1, \ \ \varepsilon = 0.1, \ \ u_0 = 5$$

⑥ Emergence of Universe from Quantum Entanglement

The HEE suggests that there is one qubit of entanglement for each Planck length area !



$$l_P = \sqrt{\frac{\hbar G}{c^3}} = 1.6 \times 10^{-35} m$$

$$S_A = \frac{\mathbf{Area}(\Gamma_A)}{4 l_{pl}^{D-1}}$$

~10^{65} qubits per 1cm^2 !

= Planck scale mini Universe

Spacetime may emerge from entangled Qubits !

➔ Tensor Network (TN) realizes this idea !

Tensor Network (TN)

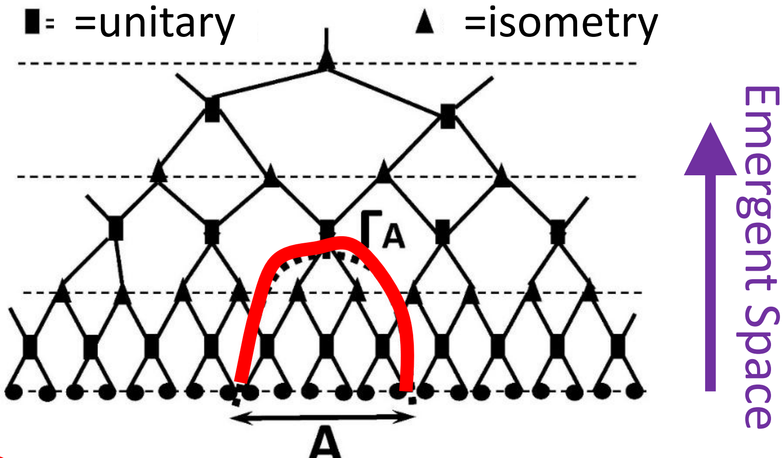
[DMRG: White 92,.. CTM: Nishino-Okunishi 96, PEPS: Verstraete-Cirac 04, ⋯.]

TN = Graphical description of quantum states

Quantum State = Network of quantum entanglement

[Ex.1] MERA TN [Vidal 2005]

➔ Describe CFT vacuum

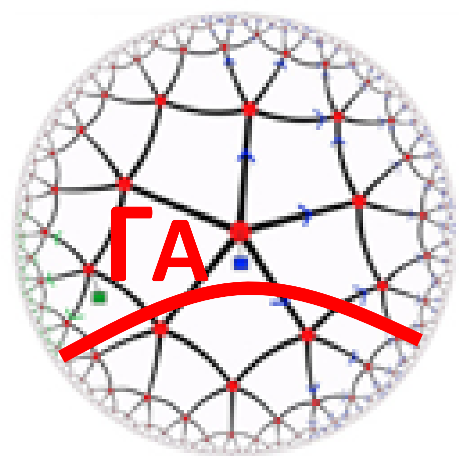


S_A = Minimal Cross Section of TN !

[Ex.2] HaPPY/RTN model

[Patawski-Yoshida-Harlow-Preskill 2015]
[Hayden-Nezami-Qi-Thomas-Walter 2016]

➔ Use quantum error correcting code



Tensor Networks = AdS

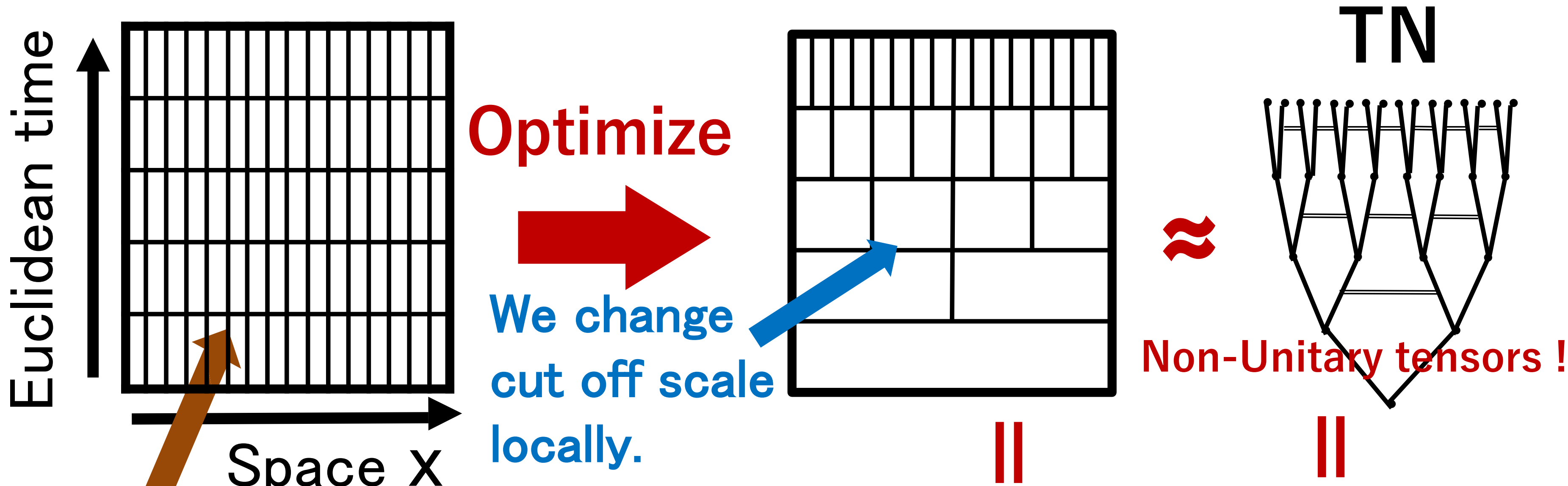
[Swingle 2009, Nozaki-Ryu-TT 2012, Czech-Lamprou-McCandlish-Sully 2015,⋯]

[Ex.3 Path-integral Optimization] [Caputa−Kundu−Miyaji−Watanabe−TT 2017]

Q. Can we describe CFT as a tensor network ? ⇒ **Path−integral**

Basic Principle

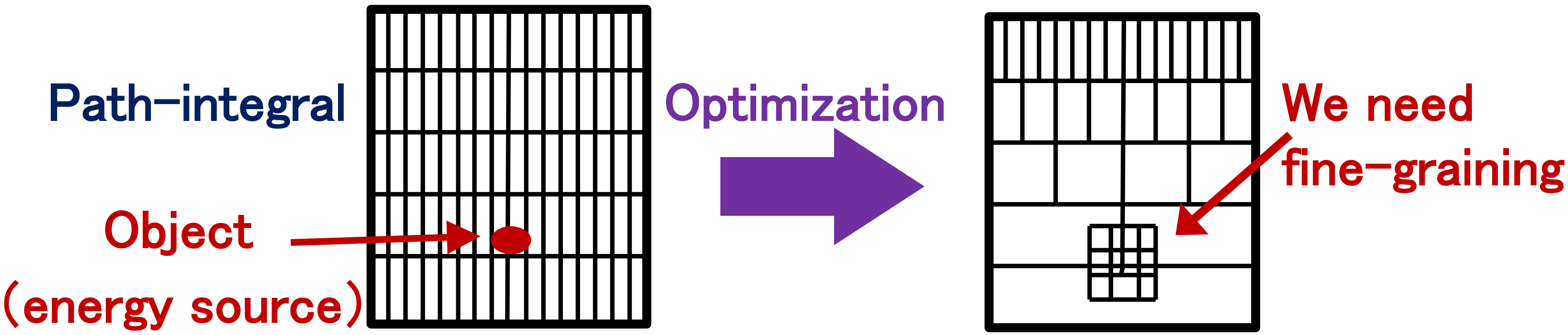
Minimize the computational cost of (discretized) path−integral.



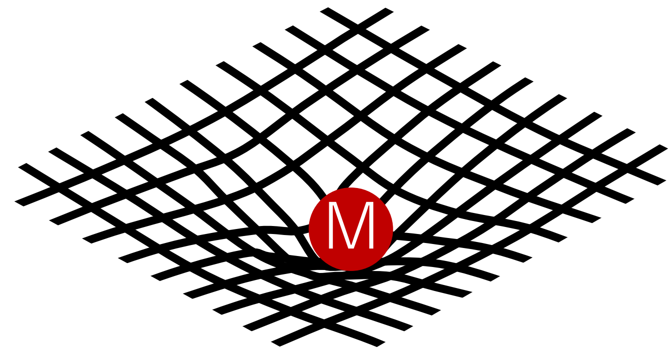
Initially, short wave length modes
can be neglected.

A time slice of AdS
emerges !

Upshot : Minimizing computational costs leads to gravity !



Energetic source (=information source)
distorts the spacetime
→ The essence of general relativity !



Holographic Perspective [Boruch-Caputa-Ge-TT 2021]

> **Path-integral Optimization**
> **= Maximization of Hartle-Hawking wave function for *AdS Universe***

Gravity might be an ideal "quantum computer" ?

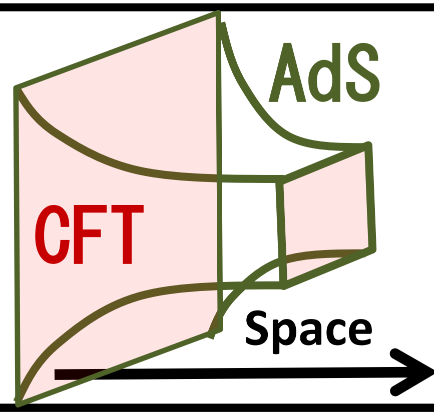
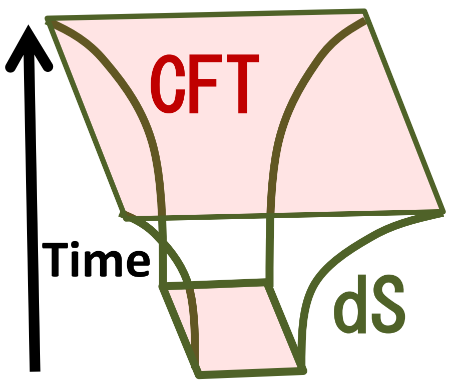
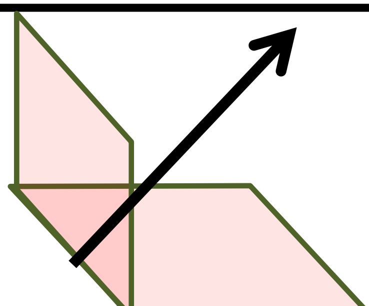
How our universe emerges from quantum information ?

How about de Sitter space instead of AdS ?

↓

Try to find a solvable example of dS/CFT !

Classification of Maximally Symmetric Spacetimes and Holography

Type	Geometry	Holography	Central charge
Anti de Sitter Space AdS $\Lambda<0$		**AdS/CFT** [Maldacena 1997] Gravity in d+2 dim. AdS = d+1 dim. CFT on $R^{1,d}$ ▶Emergent Space	In 3d, $C = \frac{3R_{AdS}}{2G_N}$
de Sitter Space dS $\Lambda>0$ "The Universe"		**dS/CFT** [Strominger 2001] Gravity in d+2 dim. dS $\stackrel{?}{=}$ d+1 dim. Euclid CFT on S^{d+1} ▶Emergent Time ?	In 3d, $C = i\frac{3R_{dS}}{2G_N}$
Flat $\Lambda=0$		String theory can describe quantum gravity.	$C = i\infty$?

Thermodynamics

AdS3 BH $$S_{AdS} = 2\pi\sqrt{\frac{cE}{3}}$$

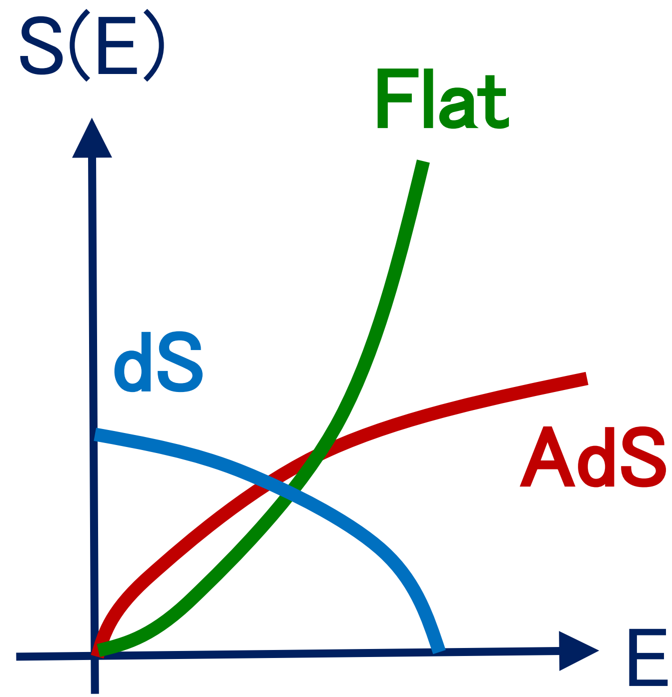
→It has a positive specific heat
and is thermodynamically stable.

dS3 BH $$S_{dS} = \frac{\pi R_{dS}}{2G_N}\sqrt{1-8G_N E}$$

→The vacuum E= 0, the state
is maximally entangled !

4D flat BH $$S_{Flat} = 4\pi G_N E^2$$

→ This leads to a negative specific heat.
It is thermodynamically, unstable.



This is one of
the main reasons
why holography
in dS/flat space
is very difficult !

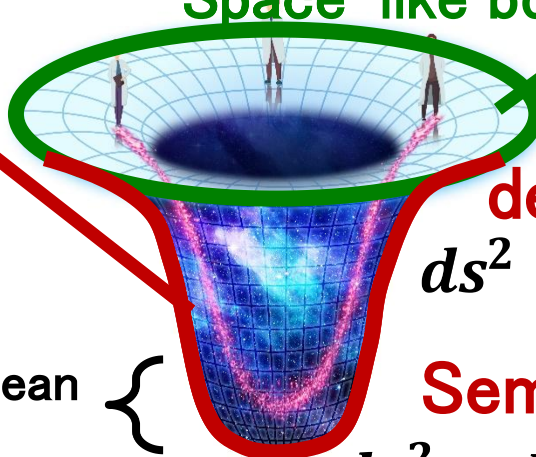
A Sketch of dS/CFT [Strominger 2001, Witten 2001, Maldacena 2002,⋯.]

Dual

d+1 dim. Lorentzian de−Sitter spacetime

Euclidean d. dim CFT on S^d

Space−like bdy



$t=\infty$

Time

Time emerges from CFT !

Lorentzian time

Euclidean time

de Sitter

$$ds^2 = R_{ds}^2(-dt^2 + \mathrm{Cosh}^2 t\, d\Omega^2)$$

Semi sphere

$$ds^2 = R_{ds}^2(d\theta^2 + \mathrm{Sin}^2\theta\, d\Omega^2)$$

$$\theta = it + \frac{\pi}{2}$$

Central charge

$$\boxed{c \propto \frac{R_{AdS}^{d-1}}{G_N} = i^{d-1} \cdot \frac{R_{dS}^{d-1}}{G_N}}$$

For dS3/CFT2 (d=2), c is imaginary !

Non-unitary CFT dual of 3 dim. dS

[Hikida–Nishioka–Taki–TT, 2021,22, Chen–Hikida–Taki–Uetoko 2022,23,24,..]

Large c limit of SU(2)$_k$ × SU(2)$_{-k}$ WZW model (a 2dim. CFT)
= Einstein Gravity on 3 dim. de Sitter (radius L_{dS})

Level

$$k \approx -2 + \frac{4iG_N}{L_{dS}}$$

Central charge

$$c = \frac{3k}{k+2} \approx i\frac{3L_{dS}}{2G_N}$$

$$Z[S^3, R_j] = |S_j^0|^2 \approx e^{\frac{\pi L_{dS}}{2G_N}\sqrt{1-8G_N E}}$$

CFT partition function

De Sitter Entropy

This non-unitary CFT is essentially equivalent to the Liouville CFT

at $b^{-2} \approx \pm\frac{i}{4G_N}$

$$I_{CFT}[\phi] = \int d^2x \left[\frac{1}{4\pi}(\partial_a\varphi\partial_a\varphi) + \underline{\mu e^{2b\varphi}}\right].$$

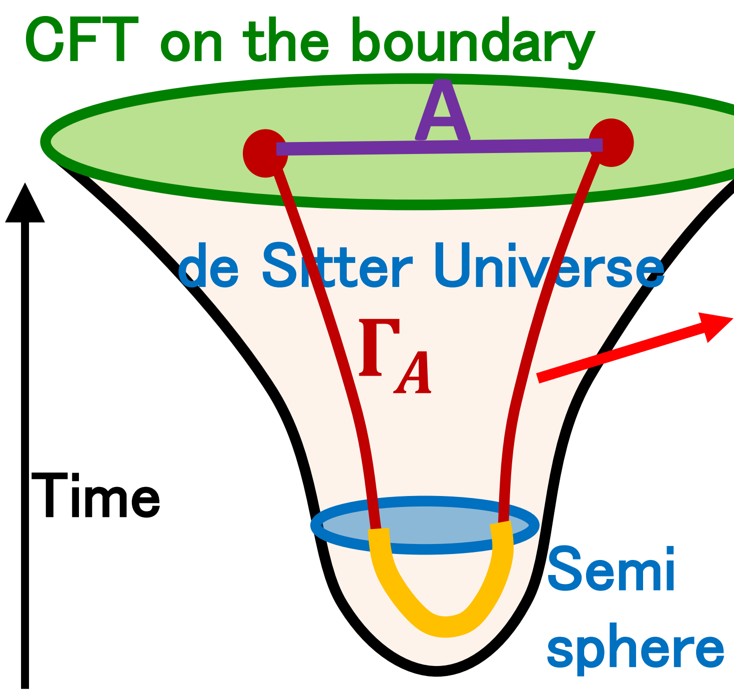
complex !

[Hikida–Nishioka–Taki–TT, 2022]

[→The same Liouvile CFT appears in Verlinde–Zhang 2024 via the Double Scaled SYK]

Does TIME emerge from quantum information ?

[Doi–Harper–Mollabashi–Taki–TT 2022]



The geodesic Γ_A which gives holographic entanglement entropy S_A includes the **time–like part** !

S_A becomes complex valued !

[This is more properly regared as the **pseudo entropy** !]

Real part of S_A → Emergence of space coordinate

Imaginary part of S_A → Emergence of time coordinate

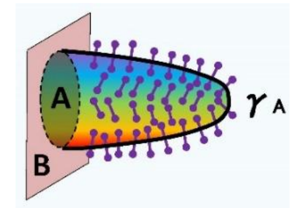
⑦ Conclusions

The formulas "Entropy (Information) = Area" are opening up

Microscope

Is gravity the fastest "quantum computer" ?

➔New insights into quantum matter, quantum computation and quantum cryptography



Universe = Entangled Qubits

Emergence

Does gravitational spacetime emerge from qubits ?

➔New approach to quantum gravity

Some list of future problems

- Holographic cosmology and "non-Hermitian" quantum information ?
- Multi-partite entanglement and holography ?
- Quantum entanglement and internal spaces of AdS/CFT ?
- Table top experiments of emergent spacetimes ?
- Explanation of the creation of the Universe from QI viewpoint ?

Thank you very much !

Pseudo Entropy and Holography

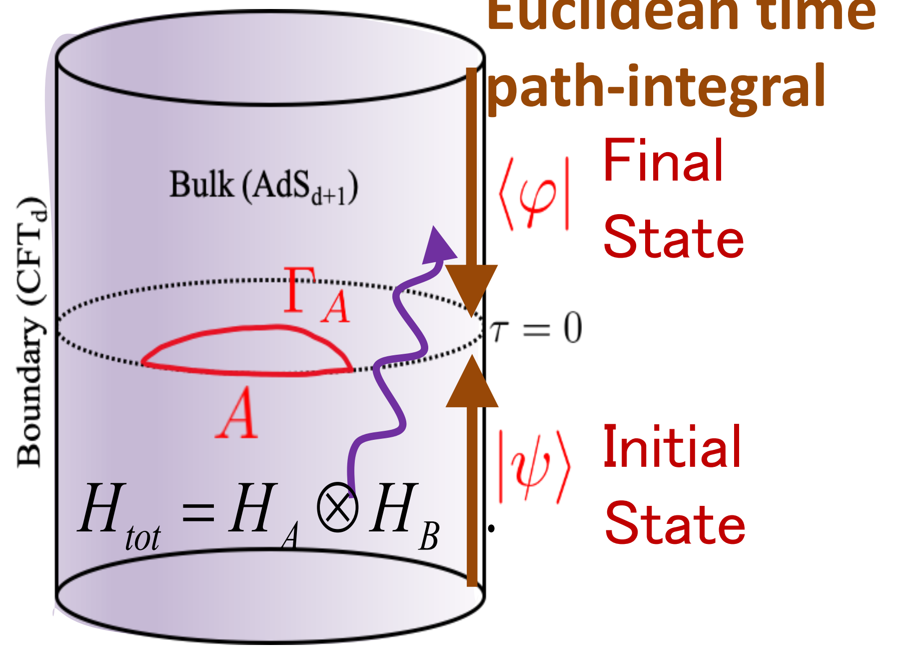
I will talk on more details
in my workshop talk tomorrow.

Q. What is "HEE" in a *Euclidean time-dependent AdS* ?

Holographic Pseudo Entropy

[Nakata–Taki–Tamaoka–Wei–TT, 2020]

$$\boxed{S(\mathcal{T}_A^{\psi|\varphi}) = \min_{\Gamma_A} \frac{\text{Area}(\Gamma_A)}{4G_N}}$$



Transition matrix:
(Not Hermitian !)

$$\tau^{\psi|\varphi} = \frac{|\psi\rangle\langle\varphi|}{\langle\varphi|\psi\rangle}.$$

$$\tau_A^{\psi|\varphi} = \text{Tr}_B\left[\tau^{\psi|\varphi}\right]$$

Pseudo entropy (PE):

$$\boxed{S\left(\tau_A^{\psi|\varphi}\right) = -\text{Tr}\left[\tau_A^{\psi|\varphi}\log\tau_A^{\psi|\varphi}\right].}$$

In general, complex valued.

How to optimize path-integral (in 2 dim. CFT)

Idea: Local change of UV cut off scale = Metric change

$$\boxed{ds^2 = e^{2\omega(x,z)}(dx^2 + dz^2).}$$

Owing to conformal symmetry, the wave function behaves as

$$\boxed{\Psi[\phi, \omega] = e^{C[\omega]} \cdot \Psi[\phi, \omega = 0]}$$

Optimization ⇒ Minimize the cost $C[\omega]$!

[$C[\omega]$≈Computational Complexity]

In two dim. CFT, $C[\omega]$ is given by Liouville action:

$$\boxed{C_{2D}[\omega] = \frac{c}{24\pi}\int dxdz\left[(\partial_x\omega)^2 + (\partial_z\omega)^2 + e^{2\omega}\right]}$$

⇒ **Minimization leads to AdS metric !**

What we expect for dS/CFT

→Let us assume dS Einstein gravity and extract general expectations.

d+1 dim. (Lorentzian) de-Sitter $ds^2 = L_{dS}^2(-dt^2 + \mathrm{Cosh}^2 t\, d\Omega^2)$

S^{d+1} (Euclidean de-Sitter) $ds^2 = L_{dS}^2(d\theta^2 + \mathrm{Sin}^2\theta d\Omega^2)$

$L_{AdS} = iL_{dS},\ \rho = i\theta$

Euclidean AdS (H^{d+1}) $ds^2 = L_{AdS}^2(d\rho^2 + \mathrm{Sinh}^2\rho d\Omega^2)$

Central charge: $\boxed{c \sim \frac{L_{AdS}^{d-1}}{G_N} = i^{d-1} \cdot \frac{L_{dS}^{d-1}}{G_N}}$

We are interested in d=2 case in this talk !

➡ (i) Central charge becomes imaginary for d=even !
(ii) Central charge gets larger in classical gravity limit.