An updated global fit to the Georgi-Machacek model

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Overview

Standard Model: Electroweak Gauge Symmetry

The electroweak gauge symmetry in the Standard Model is $SU(2)_L \times U(1)_Y$, and the corresponding covariant derivative takes the form:

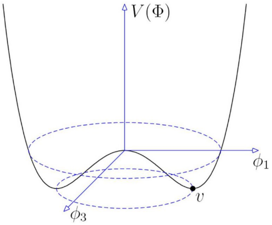
$$D_\mu \equiv \partial_\mu - ig\, W_\mu^a t^a - ig' B_\mu Y \tag{1}$$

, where t^a are the generators of $SU(2)_L$ and Y is the hypercharge.

Standard Model: Spontaneous Symmetry Breaking

The scalar sector in SM consists of a single isospin doublet field ϕ with hypercharge $Y = 1/2$. The kinetic term of the scalar field is

$$\mathcal{L} \supset (D^\mu \phi)^\dagger (D_\mu \phi) \quad (2)$$



If the scalar potential induces a non-vanishing VEV, then the EW gauge symmetry is spontaneously broken.

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Standard Model: Massive Vector Boson

If we expand the scalar field around the VEV, we obtain the mass terms for the weak vector bosons

$$\mathcal{L} \supset \frac{g^2\, v^2}{4} W_\mu^+ W^{-\mu} + \frac{(g^2 + g'^2) v^2}{8} Z_\mu Z^\mu \tag{3}$$

, from which we can clearly read-off

$$m_W = \frac{g\, v}{2} \quad \text{and} \quad m_Z = \frac{\sqrt{g^2 + g'^2}\, v}{2} = \frac{m_W}{\cos\theta_W} \tag{4}$$

, where θ_W is the weak mixing angle.

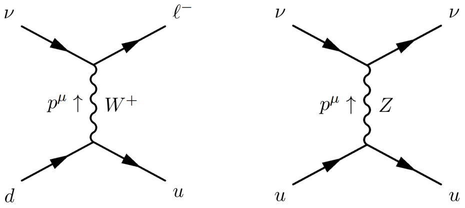
Standard Model: Electroweak ρ Parameter

To examine SM, the electroweak ρ parameter is often measured, which is defined as the ratio of strengths of the neutral to charged currents.

Definition (Electroweak ρ Parameter)

$$\rho \equiv \left(\frac{g^2}{c_W^2 m_Z^2}\right) \times \left(\frac{g^2}{m_W^2}\right)^{-1} = \frac{m_W^2}{c_W^2 m_Z^2}$$

In SM, $\rho = 1$ at tree level; However, this is a consequence of an approximate global symmetry in SM, known as *custodial symmetry*.



Custodial Symmetry: General Case

In general case, if the model consists of a scalar field X with total isospin T and hypercharge Y, the mass matrix in the basis $(W^+, W^-, W^3, B)^T$ is given by

$$M^2 = \begin{pmatrix} \begin{pmatrix} g^2/2 & 0 \\ 0 & g^2/2 \end{pmatrix} \times \left[T(T+1) - Y^2\right] & O \\ O & \begin{pmatrix} g^2 & -g\,g' \\ -g\,g' & g'^2 \end{pmatrix} \times Y^2 \end{pmatrix} v_X^2$$

Custodial Symmetry: Specific Cases

Below, the mass matrix for SM and a triplet with $Y = 1$ is provided. It is clear that in the limit $g' \to 0$, there is a symmetry between $W^1 \leftrightarrow W^2 \leftrightarrow W^3$ only for the case of SM.

ϕ (doublet, $Y = 1/2$)

$$M_\phi^2 = \frac{v_\phi^2}{4} \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & -gg' \\ 0 & 0 & -gg' & g'^2 \end{pmatrix}$$

χ (triplet, $Y = 1$)

$$M_\chi^2 = v_\chi^2 \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & 2g^2 & -2gg' \\ 0 & 0 & -2gg' & 2g'^2 \end{pmatrix}$$

Custodial Symmetry: Triplet VEV Alignment

However, one can see that the combination of two triplet fields χ and ξ is able to regenerate custodial symmetry, if the triplet VEV's are properly aligned, $v_\chi = v_\xi$.

χ (triplet, $Y = 1$)

$$M_\chi^2 = v_\chi^2 \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & 2g^2 & -2gg' \\ 0 & 0 & -2gg' & 2g'^2 \end{pmatrix}$$

ξ (triplet, $Y = 0$)

$$M_\xi^2 = v_\xi^2 \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Georgi-Machacek Model: $SU(2)_L \times SU(2)_R$-covariant Form

In the GM Model, the fields are expressed in the $SU(2)_L \times SU(2)_R$-covariant forms. The benefit is that, it is easier to write down a potential that aligns triplet VEV's correctly.

$$\begin{aligned} \Phi &\equiv \begin{pmatrix} (\phi^0)^* & \phi^+ \\ -(\phi^+)^* & \phi^0 \end{pmatrix} \\ \Delta &\equiv \begin{pmatrix} (\chi^0)^* & \xi^+ & \chi^{++} \\ -(\chi^+)^* & \xi^0 & \chi^+ \\ (\chi^{++})^* & -(\xi^+)^* & \chi^0 \end{pmatrix} \end{aligned} \tag{5}$$

Georgi-Machacek Model: Scalar Potential

The most general potential [Chen et al., 2022] is

$SU(2)_L \times SU(2)_R$-covariant Potential

$$
\begin{aligned}
V(\Phi, \Delta) = &\ \tfrac{1}{2} m_1^2 \operatorname{tr}\left[\Phi^\dagger \Phi\right] + \tfrac{1}{2} m_2^2 \operatorname{tr}\left[\Delta^\dagger \Delta\right] + \lambda_1 \left(\operatorname{tr}\left[\Phi^\dagger \Phi\right]\right)^2 + \lambda_2 \left(\operatorname{tr}\left[\Delta^\dagger \Delta\right]\right)^2 \\
&+ \lambda_3 \operatorname{tr}\left[(\Delta^\dagger \Delta)^2\right] + \lambda_4 \operatorname{tr}\left[\Phi^\dagger \Phi\right] \operatorname{tr}\left[\Delta^\dagger \Delta\right] + \lambda_5 \operatorname{tr}\left[\Phi^\dagger \tfrac{\sigma^a}{2} \Phi \tfrac{\sigma^b}{2}\right] \operatorname{tr}\left[\Delta^\dagger T^a \Delta T^b\right] \\
&+ \mu_1 \operatorname{tr}\left[\Phi^\dagger \tfrac{\sigma^a}{2} \Phi \tfrac{\sigma^b}{2}\right] (P^\dagger \Delta P)_{ab} + \mu_2 \operatorname{tr}\left[\Delta^\dagger T^a \Delta T^b\right] (P^\dagger \Delta P)_{ab}
\end{aligned}
$$

, which can be determined by 9 model parameters: $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, m_1^2, m_2^2, \mu_1, \mu_2$.

Georgi-Machacek Model: Mass Eigenstates

After diagonalizing the potential, the physical fields are

$$\text{one quintet } \begin{pmatrix} H_5^{\pm\pm} \\ H_5^{\pm} \\ H_5^0 \end{pmatrix}, \text{ one triplet } \begin{pmatrix} H_3^{\pm} \\ H_3^0 \end{pmatrix} \text{ and two singlets } H_1, h$$

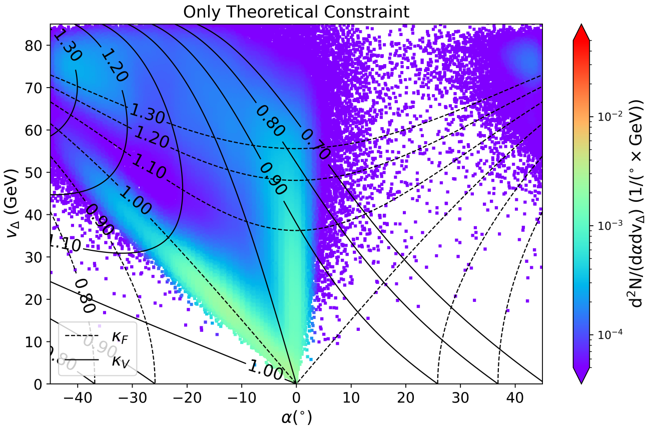
, where h is identified as the 125-GeV SM-like Higgs.

Global Scan: Theoretical Constraints Imposed

We consider three different sets of constraints at tree level.

- **Vacuum Stability**: Ensuring a stable vacuum exists in the potential.
- **Perturbative Unitarity Condition**: Restricting the largest zeroth partial-wave mode
- **Unique Vacuum Condition**: Ensuring there is no alternative global minimum in the potential

Global Scan: Bayesian MCMC fits



We use the HEPfit package to determine the posterior distribution of the model parameters.

In the plot, v_Δ represents the triplet VEV, and α stands for the mixing angle between H_1 and h.

Experimental Constraints: Higgs Signal Strength Constraint

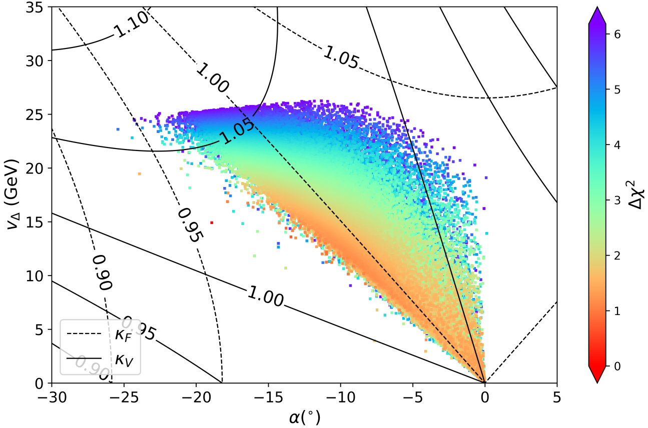
In addition to theoretical constraints, the model must also be consistent with experimental measurements from the LHC. First we evaluate the compatibility with the measurement of the Higgs boson detected at around **125 GeV**.

Usually, the comparison is performed using χ^2 values.

$$\chi^2 = \sum_i \frac{(O_i^{\text{exp}} - O_i^{\text{th}})^2}{\sigma_i^2} \tag{6}$$

, where O_i^{exp} and O_i^{th} are experimental and theory-predicted observable values in a measurement i, and σ_i is the uncertainty of the measurement.

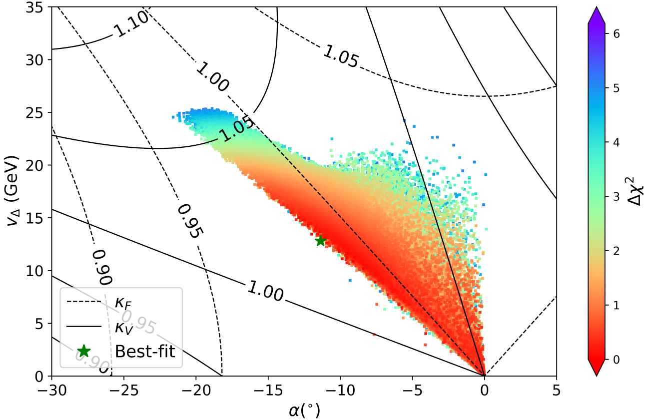
Experimental Constraints: Implementing `HiggsTools`



We use the state-of-the-art package `HiggsTools` to compute the χ^2 values for each parameter point.

In this plot, points with $\Delta\chi^2 > 6.18$ are excluded, as they are outside the 95 % C.L. from the experiment result.

Experimental Constraints: Direct Search Constraint



Finally, we exclude parameter points that are unbounded by direct searches for exotic scalars at the LHC.

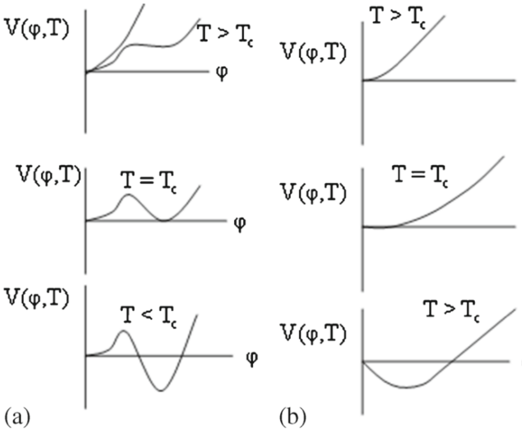
Most of the points lie in the region with $\kappa_V > 1$, as previous experimental results indicate a preference for an **enhancement in the vector boson coupling**.

Electroweak Phase Transitions: Motivations

If we consider thermal correction in the potential:

$$V = V_{\text{tree}} + V_T$$

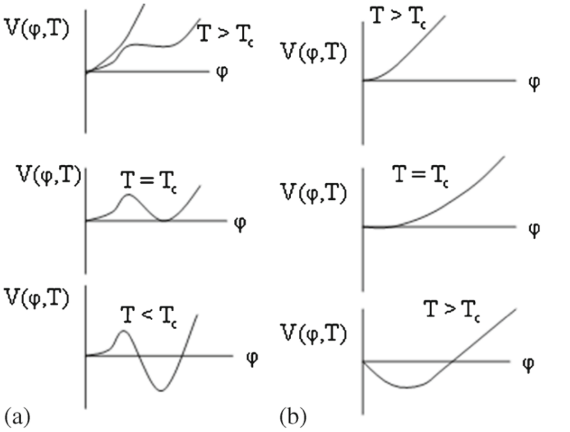
At high temperature, EW gauge symmetry is recovered (VEV's too small compared to thermal agitation)



Electroweak Phase Transitions: Motivations

As the temperature decreases after the Big Bang, at a critical temperature T_C, the symmetry-breaking VEV is induced.

The electroweak phase transition proceeds as the system evolves from a local minimum to the global minimum. This process is called the **electroweak phase transition**



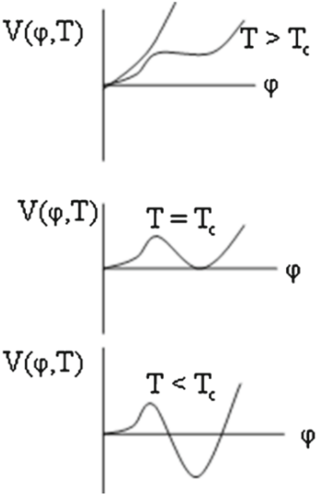
Electroweak Phase Transitions: The Baryon Asymmetry Problem

The three ingredients [Quiros, 1999] necessary for baryogenesis are

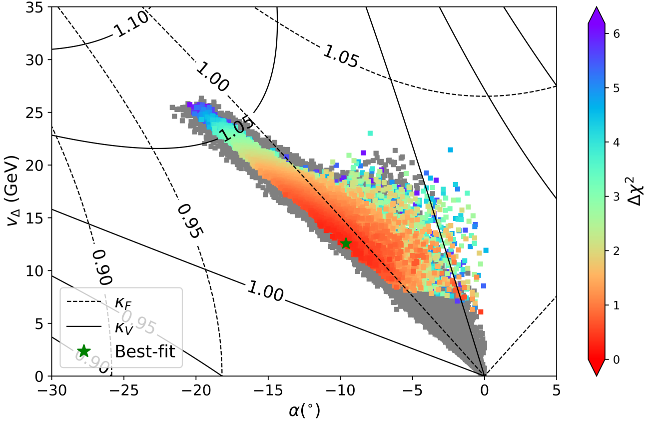
Sakharov's conditions

- B-nonconserving interactions
- C and CP violations
- Departure from thermal equilibrium

The Standard Model satisfies the first two conditions; However, the third condition, which can be induced by a **strong, first-order** EWPT, is missing in SM.



SFOEWPT: Exclusion via CosmoTransitions



We take the data points that pass through all the aforementioned constraints, and analyze their EWPT properties.

Currently, we are able to identify all the points corresponds to a **first-order** EWPT

Outlooks

The below are our future outlooks:

- Further restrict the parameter space to strong, first-order EWPTs
- Analyze the GW backgrounds induced by gravitational bubbles
- Consider extension to the Georgi-Machacek model

References

Chen, T.-K., Chiang, C.-W., Huang, C.-T., and Lu, B.-Q. (2022).

Updated constraints on the georgi-machacek model and its electroweak phase transition and associated gravitational waves.

Physical Review D, 106(5).

Quiros, M. (1999).

Finite temperature field theory and phase transitions.

Thanks for Listening