Quantum advantages in nonequilibrium thermodynamics

Department of Applied Physics, The University of Tokyo

Ken Funo

Collaborator: Hiroyasu Tajima (UEC, Japan)

Based on: H. Tajima, **KF**, PRL **127,** 190604 (2021)
KF & H. Tajima arXiv:2408.04280 (*accepted in PRL*)

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ERATO Sagawa Information-to-Energy Interconversion Project

QUANTUM ENERGY

Outline of this talk

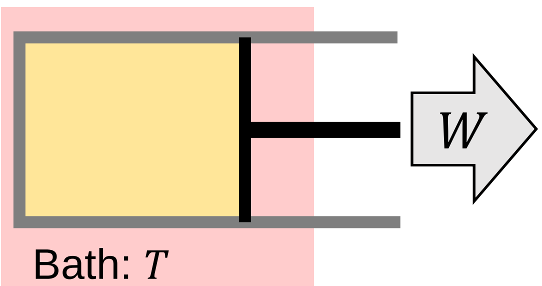
- Review: Thermodynamics in small systems
- Review: Quantum master equation and laws of thermodynamics
- Main result 1: Finite-time thermodynamic trade-off relations and quantum advantages
- Main result 2: Symmetry-based framework and fundamental limit of quantum advantages
- Application to quantum heat engines
- Conclusion

Thermodynamics in small systems

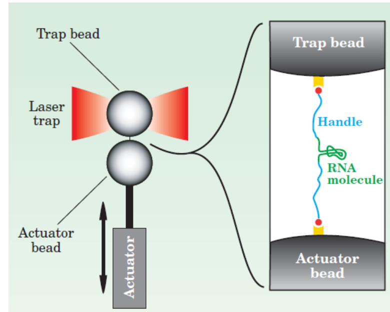
Classical thermodynamics

Steam engine

Wikipedia



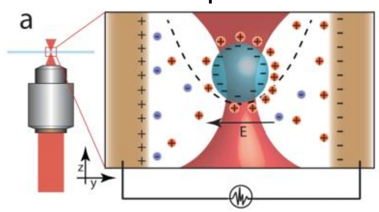
Stochastic thermodynamics



RNA molecules

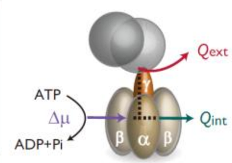
C. Bustamante *et al.*, Physics Today (2015)

Colloidal particles



S. Krishnamurthy, *et al.*, *Nat. Commun.* (2023)

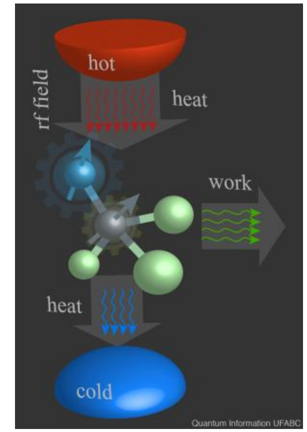
F1-ATPase



S. Toyabe *et al.*, PRL (2010)

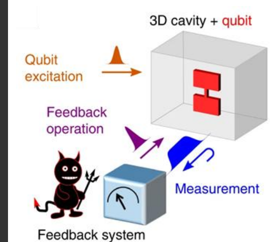
Quantum thermodynamics

NMR systems

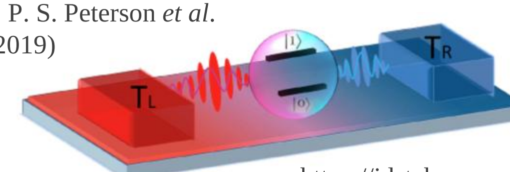


J. P. S. Peterson *et al.* (2019)

Superconducting qubits



Y. Masuyama, *et al.*, *Nat. Commun.* (2018)



https://idstch.com

~m

Thermal fluctuation ~μm

Quantum effects μm~nm

Macroscopic **Microscopic**

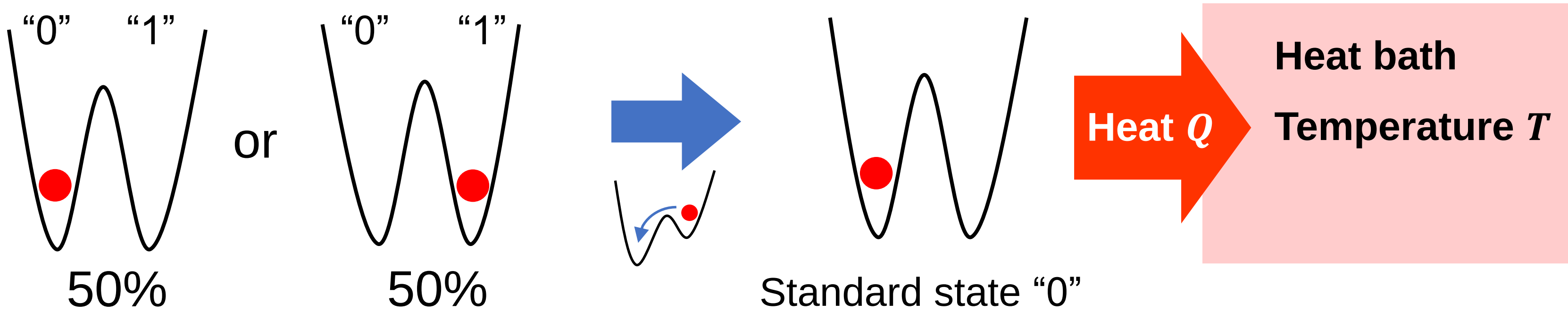
Equilibrium theory (established in 19th century)

Extended to **microscopic, nonequilibrium,** and **fluctuating** systems (late 1990s ~)

Thermodynamic laws out of equilibrium?

Fundamental cost of information erasure

■ Information erasure (Memory reset)



entropy of the system

$$S = \ln 2$$

→

entropy of the system

$$S = 0$$

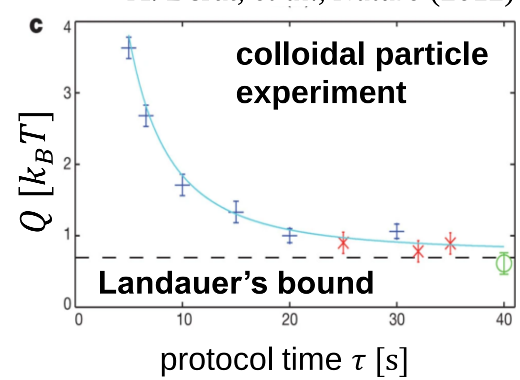
S: Shannon entropy (classical)
von Neuman entropy (quantum)

■ Landauer’s bound

R. Landauer (1961)

$$Q \geq k_B T \ln 2$$

A. Bérut, *et al.*, *Nature* (2012)



- Information erasure (decreasing the system entropy)
  → **Heat dissipation is inevitable**
- **Minimal energetic costs** required for ***information processing***

□ Information thermodynamics

Parrondo, Horowitz, & Sagawa, *Nat. Phys.* (2015)

□ Thermodynamics of computation

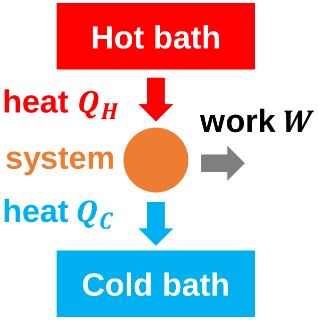
D. H. Wolpert, *et al.*, PNAS (2024)

Thermodynamics and energetic costs

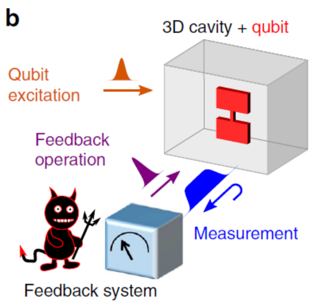
Stochastic thermodynamics & Quantum thermodynamics

✓ **Fundamental energetic costs**

Heat engine

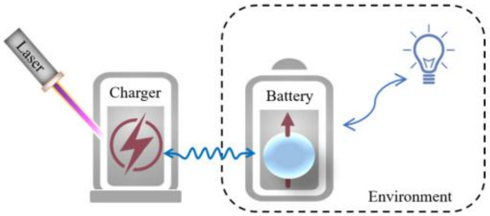


Information engine



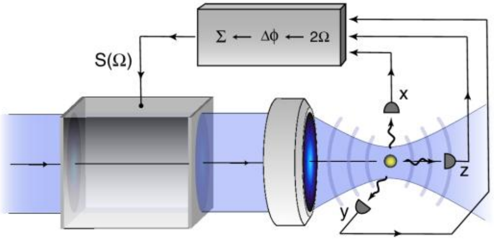
Masuyama, *et al.*, (2018)

Quantum batteries



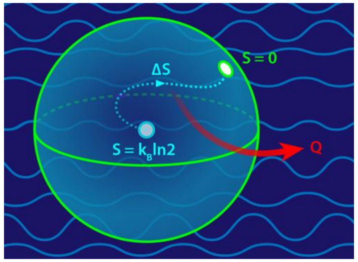
Zakavati, *et al.*, (2021)

Feedback cooling



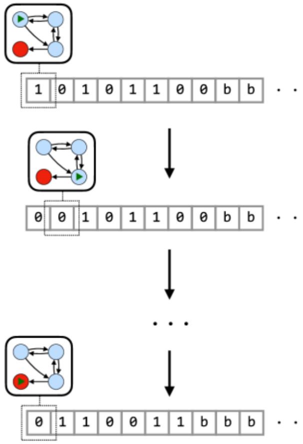
Gieseler, *et al.*, (2012)

Information erasure



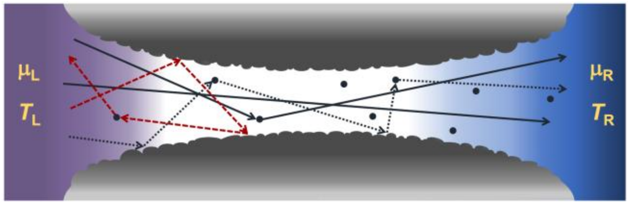
Physics, 11, 49, (2018)

Thermodynamics of computation



Kolchinsky and Wolpert (2020)

Heat transport



Pekola and Karimi (2021)

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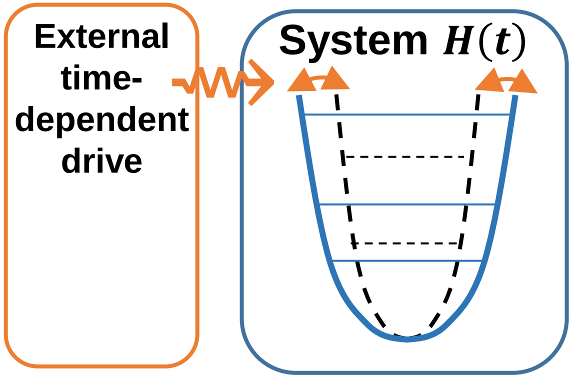
Quantum master equation and detailed balance

■ **Quantum master equation**

$$\partial_t \rho = \mathcal{L}_t[\rho] = \underbrace{-\frac{i}{\hbar}[H(t), \rho]}_{\text{unitary time-evolution (including external drive)}} + \underbrace{\mathcal{D}_t[\rho]}_{\text{effect of the bath}}$$



- **Dissipator**

$$\mathcal{D}_t[\rho] = \sum_{\omega_t} \gamma(\omega_t)\left(L_{\omega_t}\rho L_{\omega_t}^\dagger - \frac{1}{2}\left\{L_{\omega_t}^\dagger L_{\omega_t}, \rho\right\}\right)$$

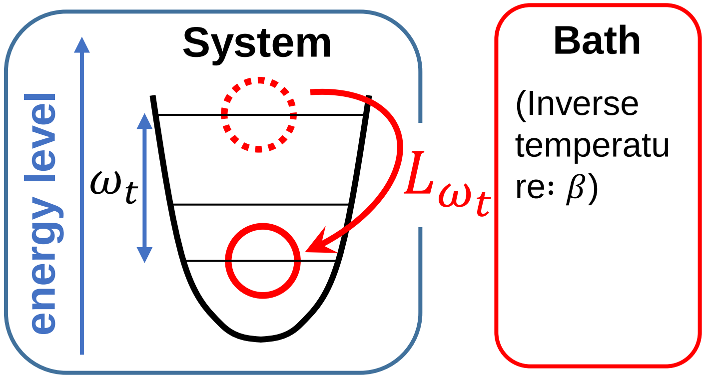
  - L_{ω_t}: jump operator

    decay, thermal excitation, decoherence

  - $\gamma(\omega_t)$: decay/excitation rate

    local detailed balance: $\gamma(\omega_t)/\gamma(-\omega_t) = \exp[\beta\omega_t]$

    → steady-state = thermal equilibrium state i.e., $\mathcal{L}_t[\rho_t^{\mathrm{th}}] = 0$, $\rho_t^{\mathrm{th}} = e^{-\beta H(t)}/Z(t)$



✓ provides thermodynamically consistent dynamics

→ allows us to study **nonequilibrium thermodynamics**

Work, heat and 1st law out of equilibrium

■ 1st law of thermodynamics (energy conservation)

$$\underbrace{\partial_t(\mathrm{Tr}[H\rho])}_{\text{internal energy change } \dot{E}} = \underbrace{\mathrm{Tr}[(\partial_t H)\rho]}_{\text{work flux } \dot{W}} + \underbrace{\mathrm{Tr}[H\partial_t\rho]}_{\text{heat flux } J}$$

■ Physical interpretation

Work flux: $\dot{W} = \mathrm{Tr}[(\partial_t H)\rho]$

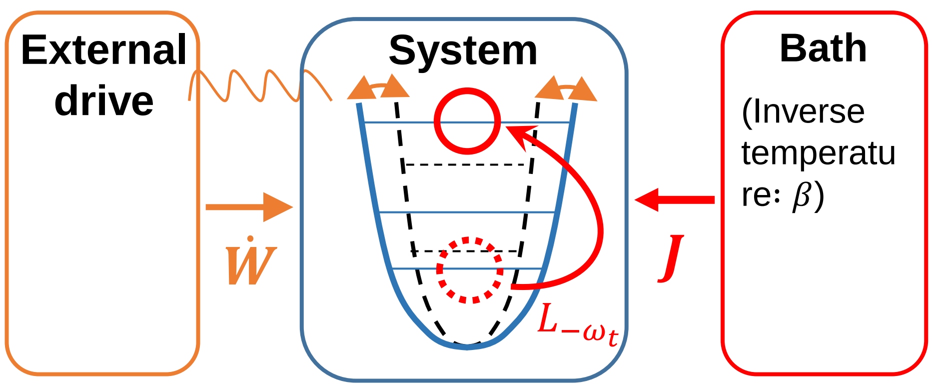
energy from the **external time-dependent driving**

Heat flux: $J = \mathrm{Tr}[H\partial_t\rho]$

$$= -\sum_\omega \omega\gamma(\omega)\mathrm{Tr}\left[L_\omega^\dagger L_\omega \rho\right]$$

substitute ME: $\partial_t\rho = -\frac{i}{\hbar}[H,\rho] + \mathcal{D}[\rho]$

energy exchange with the **bath via quantum jumps L_ω**



Entropy production and 2nd law out of equilibrium

■ Entropy production rate

$$\dot{\sigma}(\rho) = \underline{\dot{S}(\rho)} - \underline{\beta J(\rho)} \geq 0$$

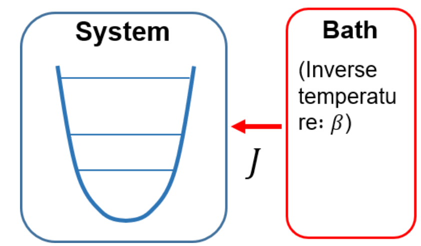
2nd law

von Neumann entropy change of the system ($S(\rho) = -\mathrm{Tr}[\rho \ln \rho]$)

entropy change of the bath ($\dot{S}_B = -\beta J$)

entropy of the entire system

$\dot{\sigma}$ = $\dot{S}$ + $\dot{S}_B$

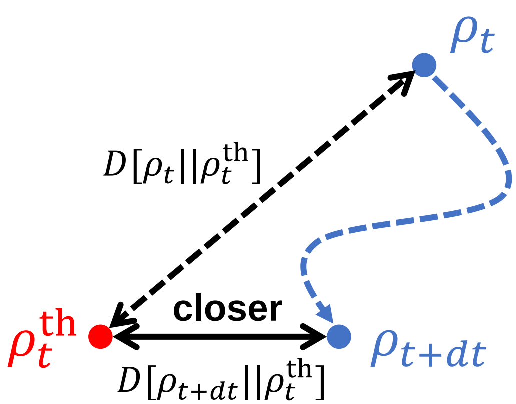


■ 2nd law of thermodynamics

- Spohn's representation [J. Math. Phys. **19,** 1227, (1978)]

$$\dot{\sigma} = \lim_{dt \to 0} \frac{1}{dt} \left(D[\rho_t || \rho_t^{\mathrm{th}}] - D[\rho_{t+dt} || \rho_t^{\mathrm{th}}] \right) \geq 0$$

- $\rho_t^{\mathrm{th}} = e^{-\beta H(t)}/Z(t)$: thermal equilibrium state
- $D[\rho||\sigma] = \mathrm{Tr}[\rho(\ln \rho - \ln \sigma)]$: relative entropy



quantifies how a nonequilibrium state ρ_t becomes ***closer*** to the thermal equilibrium state ρ_t^{th}

- Measure of **thermodynamic irreversibility** (approach to equilibrium)
- Quantifies **energetic costs** (c.f. dissipated work $\sigma = \beta W_{\mathrm{dis}}$)

$W_{\mathrm{dis}} = -\Delta\mathcal{F} - W_{\mathrm{ext}}$

$\mathcal{F}$: nonequilibrium free-energy

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Fundamental thermodynamic costs in finite-time

Second law of thermodynamics

$$\dot{\sigma} \geq 0$$

equality is achieved by **quasi-static** (infinitely slow) processes

- However, we want to achieve **high-speed operation** with **low energetic costs**

→ *fundamental limit in finite-time?*

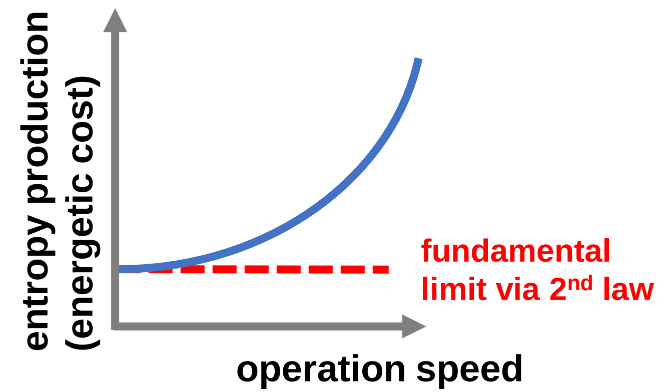
Thermodynamic speed limits

L1 distance **entropy production**

$$\frac{L(p_0, p_\tau)}{\tau} \leq \sqrt{2\sigma \bar{A}_{\mathrm{act}}}$$

time duration **activity (average transition rate)**

Thermodynamic speed limit



- fundamental *trade-off relation* between speed and energetic costs

- Classical master equation: N. Shiraishi, **KF**, K. Saito, PRL (2018)
- Quantum master equation: **KF**, N. Shiraishi, K. Saito, NJP (2019)
- Recent development using optimal transport theory: T. Van Vu, K. Saito, PRX (2023), and many more

Thermodynamic trade-off relations

■ Nonequilibrium and finite-time thermodynamic trade-off relations

current (speed) ⇔ energetic costs

- Thermodynamic uncertainty relations (TUR)

Barato, Seifert PRL (2015)
Gingrich, *et al*., PRL (2016)

precision of general current ⇔ entropy production

- Current-dissipation trade-off relation

N. Shiraishi, K. Saito, H. Tasaki, PRL (2016)
H. Tajima & **KF**, PRL (2021)

heat current ⇔ entropy production

- Thermodynamic speed limits

N. Shiraishi, **KF**, K. Saito, PRL (2018)
KF, N. Shiraishi, K. Saito, NJP (2019)

probability current (speed) ⇔ entropy production

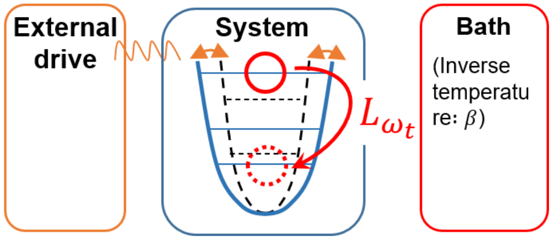
Quantum effects (coherence, entanglement)

→ **Enhancing the operation speed** while **suppressing energetic costs**?

Effect of coherence in open quantum systems

Setup: quantum master equation with detailed balance

$$\partial_t \rho = -i[H,\rho] + \sum_{a,\omega} \gamma_{a,\omega} \left[L_{a,\omega} \rho L_{a,\omega}^\dagger - \frac{1}{2}\left\{ L_{a,\omega}^\dagger L_{a,\omega}, \rho \right\} \right]$$

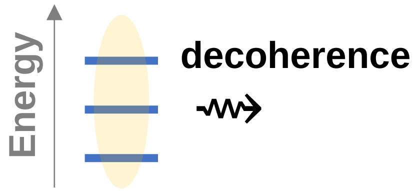


■ Bath ***destroys*** coherence between different energy levels

$$\underbrace{\dot{\sigma}(\rho_{\mathrm{diag}})}_{\text{No energy level coherence}} \leq \underbrace{\dot{\sigma}(\rho)}_{\text{With coherence}}$$

$\rho_{\mathrm{diag}} = \sum_i \Pi_i \rho \Pi_i$

Π_i :projection to i-th energy eigenspace

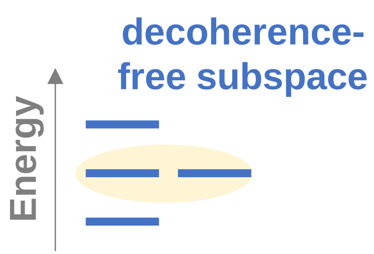


- **energy level coherence → larger $\dot{\sigma}$ (*disadvantage*)**

J. P. Santos, *et al.*, *npj Quantum Information* (2019); H. Tajima & **KF**, PRL (2021)

■ Coherence between degenerate states ***may not be destroyed***

c.f. decoherence-free subspace, Review: D. A. Lidar, Adv. Chem. Phys. (2014)



- We show that **coherence between degenerate states** is ***useful*** based on **thermodynamic trade-off relations**

H. Tajima & **KF**, PRL (2021)

Thermodynamic trade-off: Quantum advantage

H. Tajima, **KF**, PRL **127,** 190604 (2021)
[*Editors' suggestion, featured in Physics*]

■ Current-dissipation trade-off relation

□ Without degeneracy coherence

$$\frac{J^2(\rho_{\mathrm{inc}})}{\dot{\sigma}(\rho_{\mathrm{inc}})} \le \frac{A_{\mathrm{cl}}}{2}$$

□ With degeneracy coherence

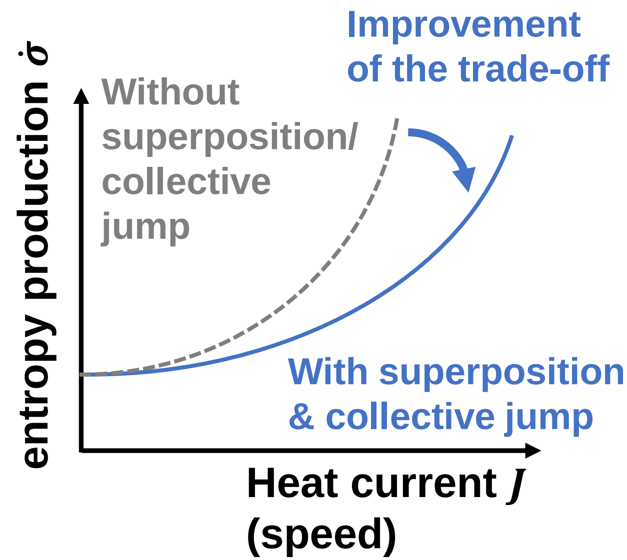
Quantum effect!

$$\frac{J^2(\rho_{\mathrm{coh}})}{\dot{\sigma}(\rho_{\mathrm{coh}})} \le \frac{A_{\mathrm{cl}} + \boxed{A_{\mathrm{qm}}}}{2}$$

- J: heat current, $\dot{\sigma}$: entropy production rate
- Average transition rate (Activity-like quantity)

$$A = \sum_\omega \underbrace{\omega^2}_{\text{transition energy}} \sum_a \underbrace{\gamma_{a,\omega} \mathrm{Tr}\left[L_{a,\omega}^\dagger L_{a,\omega}\rho\right]}_{\text{transition rate}} = A_{\mathrm{cl}} + \underline{A_{\mathrm{qm}}}$$

(superposition of degenerate states) × *(collective jumps)*



■ ***Quantum advantages*** based on universal thermodynamic trade-off relation

✓ **Quantum superposition + collective jumps** → ***higher speed & lower energetic cost***

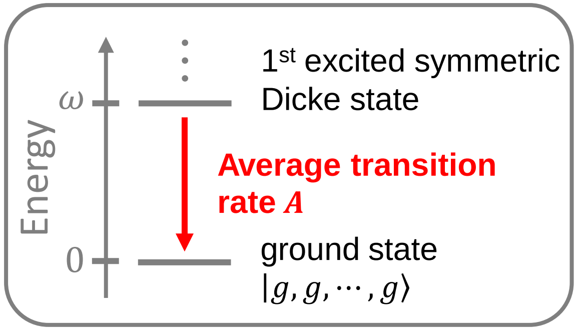
Example: Super-radiance

■ Identical N qubit system

- Hamiltonian: $H = \frac{\omega}{2}\sum_i \sigma_i^z$
- Initial state: symmetric Dicke state:

$$|\psi\rangle = N^{-\frac{1}{2}}\underbrace{(|e,g,\cdots,g\rangle + \cdots + |g,\cdots g,,e\rangle)}_{}$$

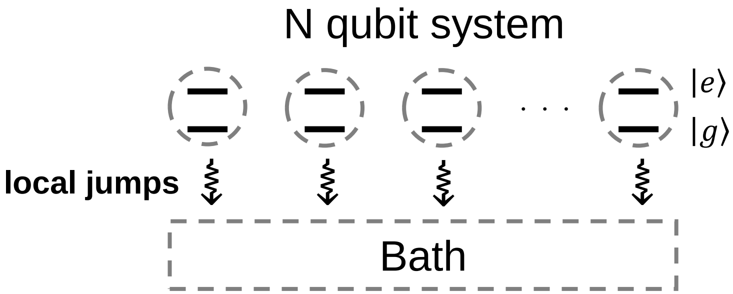
superposition of N states with one excitation



□ Local jumps: $\{\sigma_i^-\}_{i=1}^N$

$$\partial_t\rho = -i[H,\rho] + \Gamma_\downarrow \sum_i \left(\sigma_i^-\rho\sigma_i^+ - \frac{1}{2}\{\sigma_i^+\sigma_i^-,\rho\}\right)$$

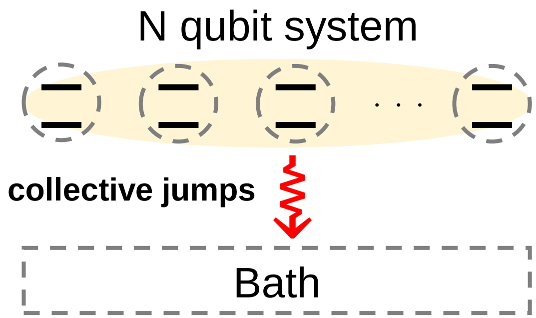
- Average transition rate: $A = \omega^2\Gamma_\downarrow$



□ Collective jumps: $J^- = \sum_i \sigma_i^-$

$$\partial_t\rho = -i[H,\rho] + \Gamma_\downarrow\left(J^-\rho J^+ - \frac{1}{2}\{J^+J^-,\rho\}\right)$$

- Average transition rate: $A = \omega^2 N\Gamma_\downarrow$ **(super-radiance)**

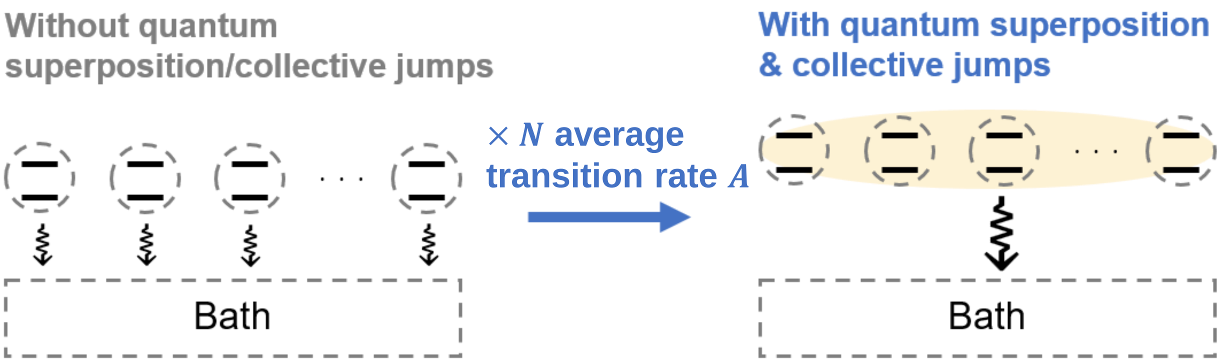


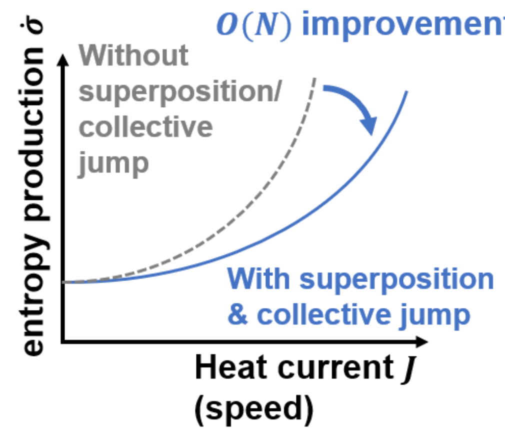
Dicke, Phys. Rev. (1954)

superposition + collective jumps → $O(N)$ enhancement of average transition rate

Summary 1: Quantum advantages in quantum systems

- Dicke super-radiance model and $O(N)$ enhancement of A





- We have shown ***quantum advantages*** based on universal thermodynamic trade-off relation

$$\frac{J^2(\rho_{\mathrm{coh}})}{\dot{\sigma}(\rho_{\mathrm{coh}})} \leq \frac{A_{\mathrm{cl}} + \boxed{A_{\mathrm{qm}}}}{2}$$

H. Tajima, **KF**, PRL **127,** 190604 (2021)

Superposition & collective jumps → **Enhancement of A** → **higher speed & lower energetic cost**

***fundamental limit* of the average jump rate A ?**

→ *Main result 2*

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Fundamental limit and symmetry-based framework

Superposition & collective jumps → **Enhancement of A** → **higher speed & lower energetic cost**

***fundamental limit* of the effective jump rate A ?**

Symmetry and decoherence-free subspace

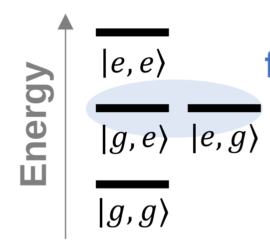
D. A. Lidar, Adv. Chem. Phys. (2014)

- Symmetric noise (same random phase ϕ)

$$|g\rangle_j \to |g\rangle_j, \ \ |e\rangle_j \to \underline{e^{i\phi}}|e\rangle_j$$

$$\begin{aligned} &|g,g\rangle \to |g,g\rangle \\ &\boxed{\begin{aligned} &|g,e\rangle \to e^{i\phi}|g,e\rangle \\ &|e,g\rangle \to e^{i\phi}|e,g\rangle \end{aligned}} \\ &|e,e\rangle \to e^{2i\phi}|e,e\rangle \end{aligned}$$

acquire the same phase →



decoherence-free subspace

Symmetry may be the key to construct a general theory

<u>**Symmetry-based framework**</u>

KF & H. Tajima arXiv:2408.04280 (*accepted in PRL*)

✓ ***General upper bound*** **on the effective jump rate A**

Setup

■ Quantum master equation

$$\partial_t \rho = \mathcal{L}(\rho) = -i[H,\rho] + \sum_{a,\omega} \gamma_{a,\omega} \left[L_{a,\omega} \rho L_{a,\omega}^\dagger - \frac{1}{2}\left\{ L_{a,\omega}^\dagger L_{a,\omega}, \rho \right\} \right]$$

- $H = \sum_k \epsilon_k \Pi_k$

  ϵ_k: energy eigenvalues, Π_k: projection to k-th energy eigenspace

  $\mathcal{N}_k = \dim(\Pi_k)$: number of degeneracy for k-th energy level

□ Symmetry of the Hamiltonian

$[H, V_g] = 0$ for all $g \in G$

□ Symmetry of Liouville superoperator

B. Buča, T. Prosen, *NJP* (2012)

Weak symmetry condition

$$\mathcal{L}(V_g x V_g^\dagger) = V_g \mathcal{L}(x) V_g^\dagger$$

for any operator x and for all $g \in G$

Time-evolution preserves the symmetry of quantum states ρ

(We assume this weaker condition)

←

Strong symmetry condition

$[V_g, L_{a,\omega}] = 0, \qquad [H, V_g] = 0$

for all $g \in G$

Time-evolution is block-decomposed into different symmetry sectors

→ analyze the average jump rate $A = \sum_\omega \omega^2 \sum_a \gamma_{a,\omega} \mathrm{Tr}\left[L_{a,\omega}^\dagger L_{a,\omega} \rho \right]$

*We choose a "natural" V_g for given H for the sake of simplicity in this presentation. For general V_g, please see arXiv:2408.04280

General upper bound on A and optimal condition

KF & H. Tajima arXiv:2408.04280 (*accepted in PRL*)

■ For ***any*** density matrices ρ and jump operators $\{L_{a,\omega}\}$, we obtain

$$A(\rho,\{L_{a,\omega}\}) \leq \sum_\omega \omega^2 \sum_k p_k \mathcal{N}_k c_k(L_{a,\omega})$$

- A can be enhanced up to $\mathcal{N}_k$ times the norm of jump operators c_k for each k-th subspace

- **$\mathcal{N}_k$: number of degeneracy**
- $p_k = \mathrm{Tr}[\Pi_k\rho]$: occupation probability of k-th state
- $c_k(L_{a,\omega}) := \mathcal{N}_k^{-1}\sum_a \gamma_{a,\omega}\mathrm{Tr}\left[L_{a,\omega}^\dagger L_{a,\omega}\Pi_k\right]$ (**Hilbert-Schmidt norm** of $L_{a,\omega}$ in the k-th subspace divided by its dimension $\mathcal{N}_k$)

*Trivial rescaling $\sqrt{\gamma_{a,\omega}}L_{a,\omega} \to \sqrt{C\gamma_{a,\omega}}L_{a,\omega}$ leads to $A \to CA$.
To exclude such effects, we introduce the norm c_k

■ Optimal condition

□ **Symmetric state ρ^{sym}**

$V_g\rho^{\mathrm{sym}} = \rho^{\mathrm{sym}}V_g^\dagger = \rho^{\mathrm{sym}}$ for all $g \in G$

□ **Symmetric jump operator $\{L_{a,\omega}^{\mathrm{sym}}\}$**

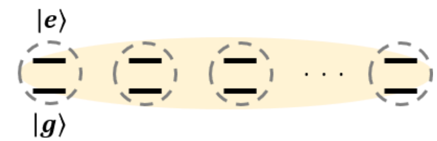
$V_g L_{a,\omega}^{\mathrm{sym}} = L_{a,\omega}^{\mathrm{sym}}V_g^\dagger = L_{a,\omega}^{\mathrm{sym}}$ for all $g \in G$

unchanged by the action of V_g

✓ Number of degeneracy $\mathcal{N}_k$ sets the fundamental limit of A

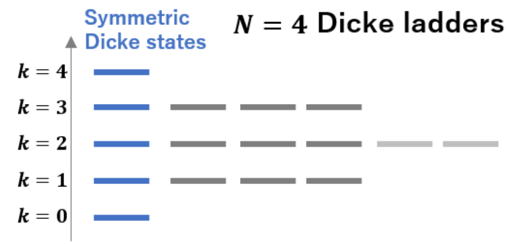
✓ Optimal symmetry condition is identified

Example: Permutation-invariant N qubit systems



Permutation symmetry

- Hamiltonian $H = \frac{\omega}{2}\sum_i \sigma_i^z$
- Invariant under permutation of subsystem labels i
- $\mathcal{N}_k = {}_N C_k$: number of degeneracy (k-th energy level)



Optimal condition

Symmetric state $\rho_k^{\mathrm{sym}} = |\psi_k^{\mathrm{sym}}\rangle\langle\psi_k^{\mathrm{sym}}|$

$$|\psi_k^{\mathrm{sym}}\rangle = \mathcal{N}_k^{-1/2} \underbrace{\sum_g V_g \, |e\rangle^{\otimes k} \otimes |g\rangle^{\otimes N-k}}_{\text{superposition of all states with } k\text{-excitation}}$$

- **symmetric Dicke states**

Symmetric jump operator L_ω^{sym}

$$L_\omega^{\mathrm{sym}} = \sum_{n=0}^{[N/2]-1} \sum_{g \in S_N} \underbrace{V_g \sigma_1^- \cdots \sigma_{n+1}^- \sigma_{n+2}^+ \cdots \sigma_{2n+1}^+ V_g^\dagger}_{(2n+1)\text{-body jump operators}}$$

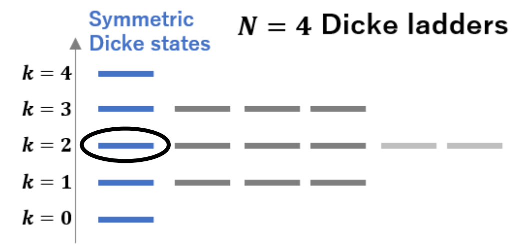
- requires multi-qubit nonlinear and collective jump

Scaling of A for permutation-invariant system

Scaling of A for different jump operators

We consider a symmetric Dicke state with $k = N/2$
(i.e., half of the qubits are excited)

$$\left|\psi_{N/2}^{\mathrm{sym}}\right\rangle = \mathcal{N}_{N/2}^{-1/2} \sum_{g\in S_N} V_g \, |e\rangle^{\otimes N/2} \otimes |g\rangle^{\otimes N/2}$$



■ Symmetric jump operator

$$L_\omega^{\mathrm{sym}} = \sum_{m=0}^{N/2} \underbrace{\sum_{g\in S_N} V_g \sigma_1^- \cdots \sigma_{m+1}^- \sigma_{m+2}^+ \cdots \sigma_{2m+1}^+ V_g^\dagger}_{\text{multi-qubit nonlinear and collective jumps}}$$

$\rightarrow \boldsymbol{A/\omega^2 c_{N/2} = \mathcal{N}_{N/2} \sim 2^N/\sqrt{N}}$ **(optimal)**

■ Three-body approximation

$$L_\omega^{3-ap} = \sum_i \sigma_i^- + \underbrace{\sum_{(i<j)\neq l} \sigma_i^- \sigma_j^- \sigma_l^+}_{\text{3-body jumps}} \rightarrow \boldsymbol{A/\omega^2 c_{N/2} = O(N^3)}$$

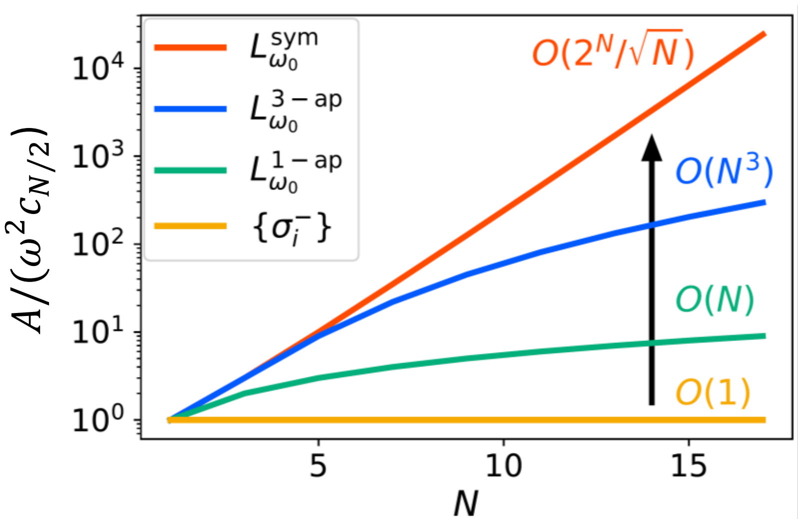
■ One-body approximation

$$L_\omega^{1-ap} = \sum_i \sigma_i^- \rightarrow \boldsymbol{A/\omega^2 c_{N/2} = O(N)}$$ **(super-radiance)**

■ Local jump operator

$$\{L_{a,\omega}^{loc}\} = \{\sigma_i^-\} \rightarrow \boldsymbol{A/\omega^2 c_{N/2} = 1}$$

Plot of $A\left(|\psi_{N/2}^{\mathrm{sym}}\rangle\langle\psi_{N/2}^{\mathrm{sym}}|, \{L_{a,\omega}\}\right)$

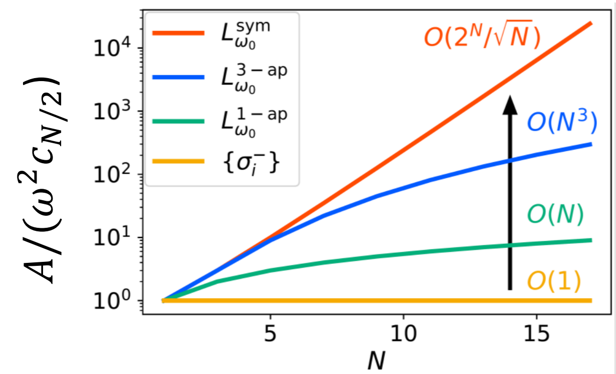


Scaling of A and symmetry condition

Scaling of A for different jump operators

We consider a symmetric Dicke state with $k = N/2$
(i.e., half of the qubits are excited)

$$\left|\psi_{N/2}^{\mathrm{sym}}\right\rangle = \mathcal{N}_{N/2}^{-1/2} \sum_{g \in S_N} V_g \, |e\rangle^{\otimes N/2} \otimes |g\rangle^{\otimes N/2}$$



- **Symmetric jump operator**

$$L_\omega^{\mathrm{sym}} = \sum_{m=0}^{N/2} \underbrace{\sum_{g \in S_N} V_g \sigma_1^- \cdots \sigma_{m+1}^- \sigma_{m+2}^+ \cdots \sigma_{2m+1}^+ V_g^\dagger}_{\text{multi-qubit nonlinear and collective jumps}}$$

$\rightarrow A/\omega^2 c_{N/2} = \mathcal{N}_{N/2} \sim 2^N/\sqrt{N}$ **(optimal)**

- **Three-body approximation**

$$L_\omega^{3-ap} = \sum_i \sigma_i^- + \sum_{(i<j)\neq l} \underbrace{\sigma_i^- \sigma_j^- \sigma_l^+}_{\text{3-body jumps}} \rightarrow A/\omega^2 c_{N/2} = O(N^3)$$

- **One-body approximation**

$L_\omega^{1-ap} = \sum_i \sigma_i^- \rightarrow A/\omega^2 c_{N/2} = O(N)$ **(super-radiance)**

- **Local jump operator**

$\{L_{a,\omega}^{loc}\} = \{\sigma_i^-\} \rightarrow A/\omega^2 c_{N/2} = 1$

Optimal symmetry condition

$$V_g L_\omega^{sym} = L_\omega^{sym} V_g^\dagger = L_\omega^{sym}$$

Strong symmetry

$$[V_g, L_\omega^{n-ap}] = 0$$

Weak symmetry

$$\mathcal{L}(V_g x V_g^\dagger) = V_g \mathcal{L}(x) V_g^\dagger$$

✓ Larger scaling when $\{L_{a,\omega}\}$ better respects the optimal symmetry condition

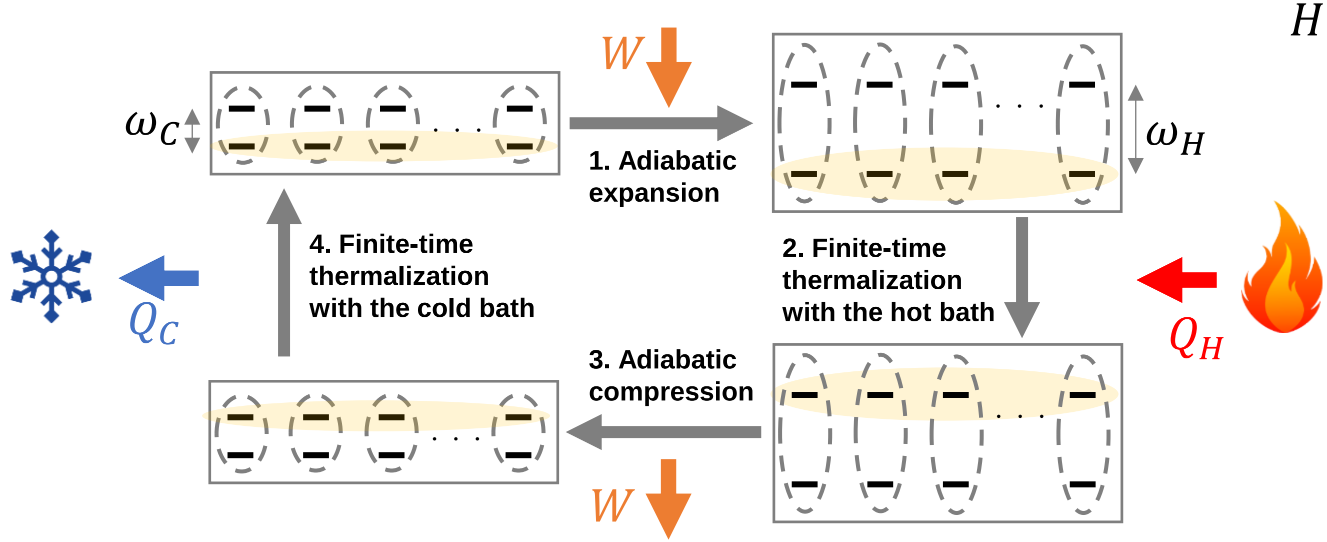
Outline of this talk

- Review: Thermodynamics in small systems
- Review: Quantum master equation and laws of thermodynamics
- Main result 1: Finite-time thermodynamic trade-off relations and quantum advantages
- Main result 2: Symmetry-based framework and fundamental limit of quantum advantages
- Application to quantum heat engines
- Conclusion

Application to heat engines

■ Finite-time quantum Otto engine

$$H = \frac{\omega}{2}\sum_i \sigma_i^z$$



■ Performance of heat engines

- **Efficiency** : $\eta = W/Q_H$
  Heat-to-work conversion efficiency
- **Power** : $P = W/\tau$
  Extracted work per unit time

τ: time duration to complete one cycle

■ Otto efficiency and Carnot efficiency

$$\eta = \underbrace{\eta_{\mathrm{Otto}} = 1 - \frac{\omega_C}{\omega_H}}_{\text{Otto efficiency}} \leq \underbrace{\eta_{\mathrm{Car}} = 1 - \frac{\beta_H}{\beta_C}}_{\text{Carnot efficiency (ideal efficiency)}}$$

Otto efficiency

Carnot efficiency (ideal efficiency)

Note: The efficiency of *finite-time* quantum Otto engine is given by η_{Otto} if the adiabatic strokes (1 and 3) are ideal

Power-efficiency trade-off and quantum coherence

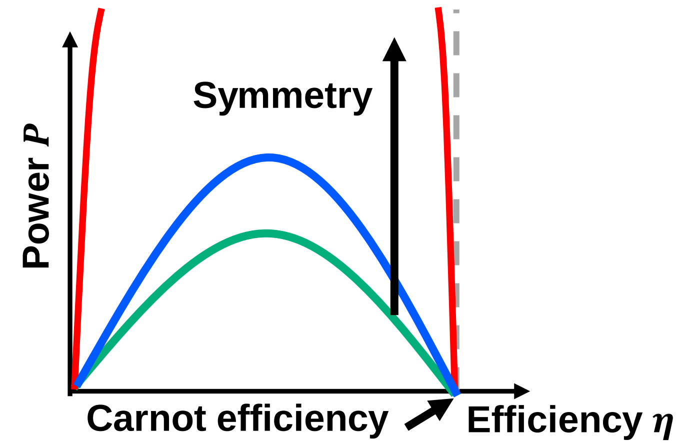
■ Power-efficiency trade-off relation

Shiraishi, Saito, Tasaki, PRL (2016);
Shiraishi, Saito, J. Stat. Phys (2019);
Tajima & **KF**, PRL (2021)

$$P \leq b\bar{A}\eta(\eta_{\mathrm{Car}} - \eta)$$

- constant factor: $b = 2(2-\eta_{Car})^2/\beta_C$
- $\bar{A} = \underbrace{\tau^{-1}\int_0^\tau dt}_{\text{time-average}}\ \underbrace{A(\rho(t),\{L_{a,\omega}\})}_{\text{average jump rate}}$

Power-efficiency trade-off relation

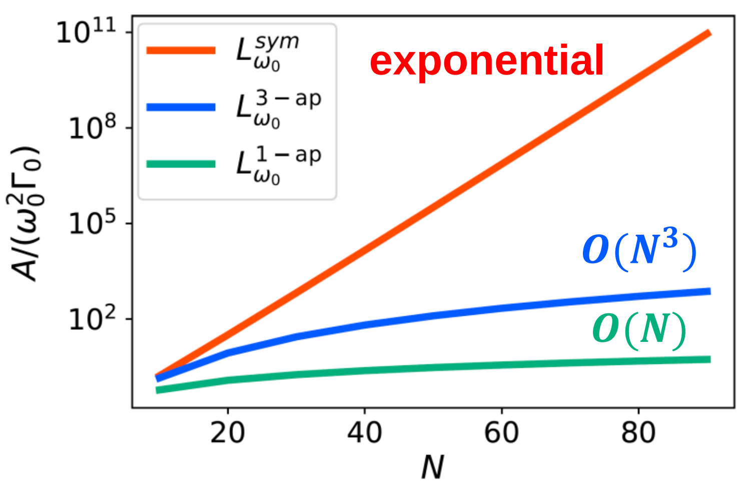


■ Scaling analysis

- $\bar{A}$ can be **enhanced** by considering the symmetry condition on the jump operator

⇒ Allows approaching **close to the Carnot efficiency** while **maintaining high power**

Scaling of A for thermal state and jump operators with **detailed balance**



Scaling of the power and efficiency

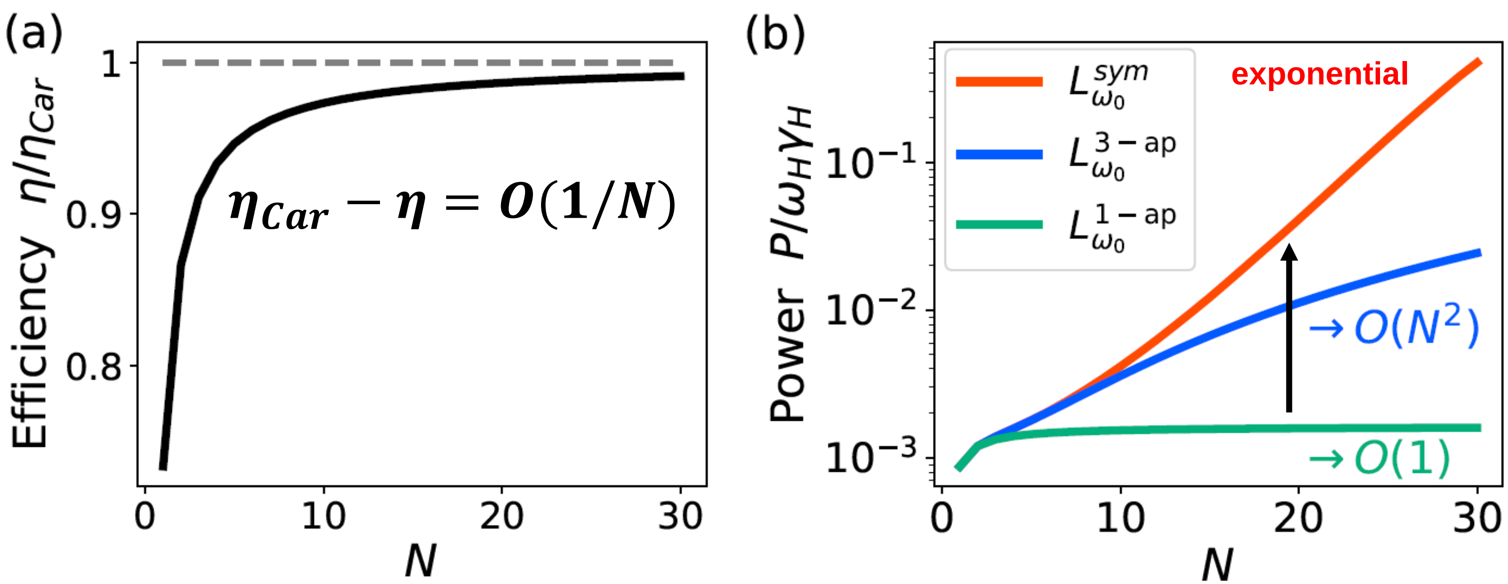
Numerical calculation of N-qubit heat engine model

- Consider a protocol that asymptotically reaches the Carnot efficiency: $\eta_{\rm Car} - \eta = O(1/N)$

Specifically, we choose (δ is a constant)

$$\omega_C = \left(\frac{\beta_H}{\beta_C} + \frac{\delta}{N}\right)\omega_H \to \eta = 1 - \frac{\omega_C}{\omega_H} = \eta_{Car} - \frac{\delta}{N}$$

- Super-radiant heat engine: $L^{1-ap} = \sum_i \sigma_i^-$ $\bar{A} = O(N) \to P = O(1)$ ***power saturates***
- 3-body jump approximation: L^{3-ap} $\bar{A} = O(N^3) \to P = O(N^2)$ ***beyond super-radiant scaling***
- Symmetric jump operator: L_ω^{sym} $\bar{A}$ & P $\to$ ***exponential scaling***



✓ $O(N^2)$ **to exponential power enhancement compared to super-radiant heat engines**

Outline of this talk

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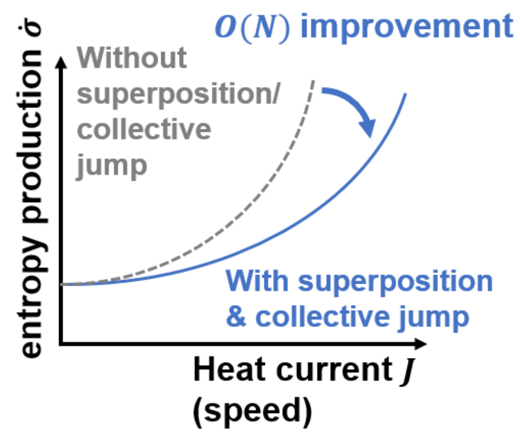
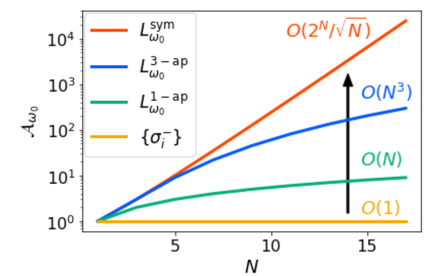
Summary

- We have shown ***quantum advantages*** based on universal thermodynamic trade-off relation

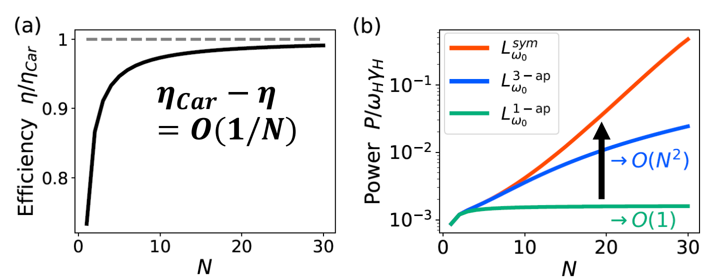
Superposition & collective jumps → **Enhancement of A** → ***higher speed & lower energetic cost***



- We have introduced a ***symmetry-based framework*** to show the **fundamental limit of the average jump rate A**



- Heat engine model asymptotically reaching the **Carnot efficiency** while **exponential scaling of power**



H. Tajima, **KF**, PRL **127,** 190604 (2021)

KF & H. Tajima arXiv:2408.04280 (*accepted in PRL*)