Entanglement-efficient distributed quantum computing (DQC)[a,b]

with embedding-enhanced distributing processes

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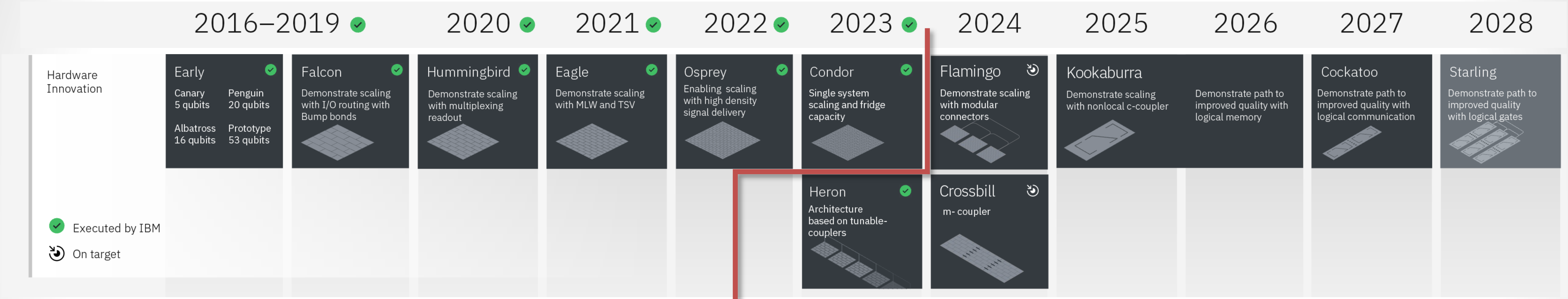
[4] National Institute of Informatics, 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan

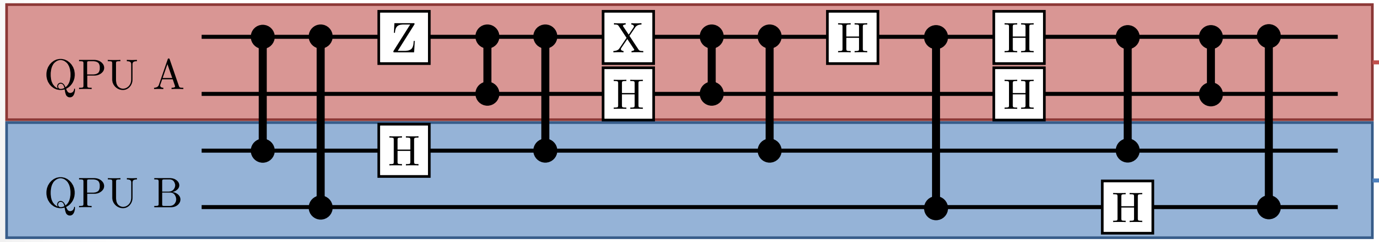
[a] J.-Y. Wu, K. Matsui, T. Forrer, A. Soeda, P. Andrés-Martínez, D. Mills, L. Henaut, and M. Murao, Quantum, 7:1196, 2023

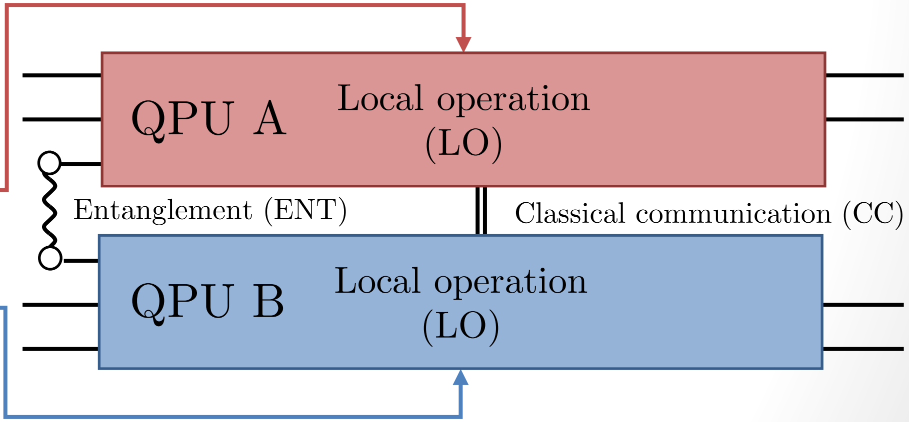
[b] P. A.-Martinez, T. Forrer, D. Mills, J.-Y. Wu, L. Henaut, K. Yamamoto, M. Murao, M. and R. Duncan, arXiv:2305.14148, 2023 (to appear in QST)

It is time to distribute quantum computing



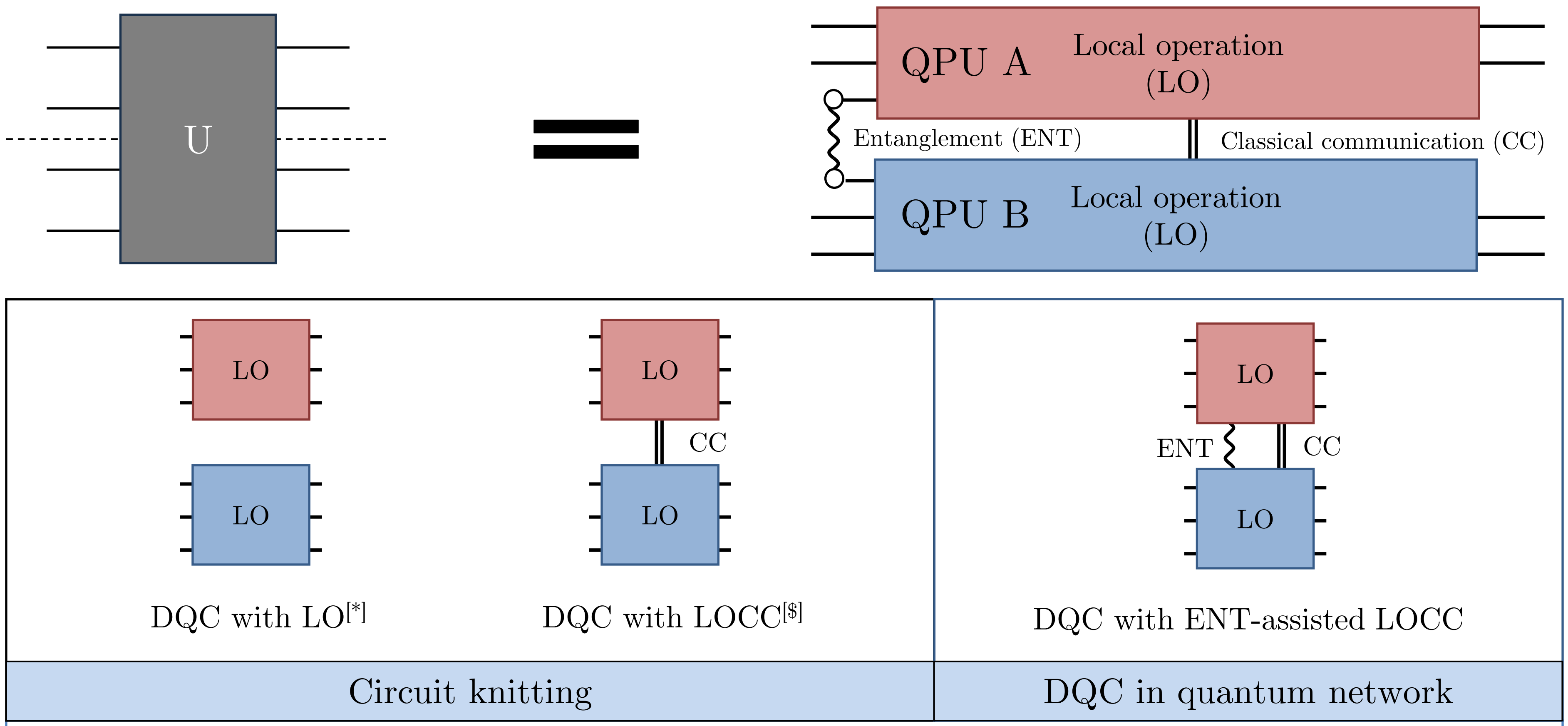
Scaling up with DQC





Implement N-qubit circuit with multiple N'-qubit QPUs with $N > N'$

Different approaches for distributed quantum computation

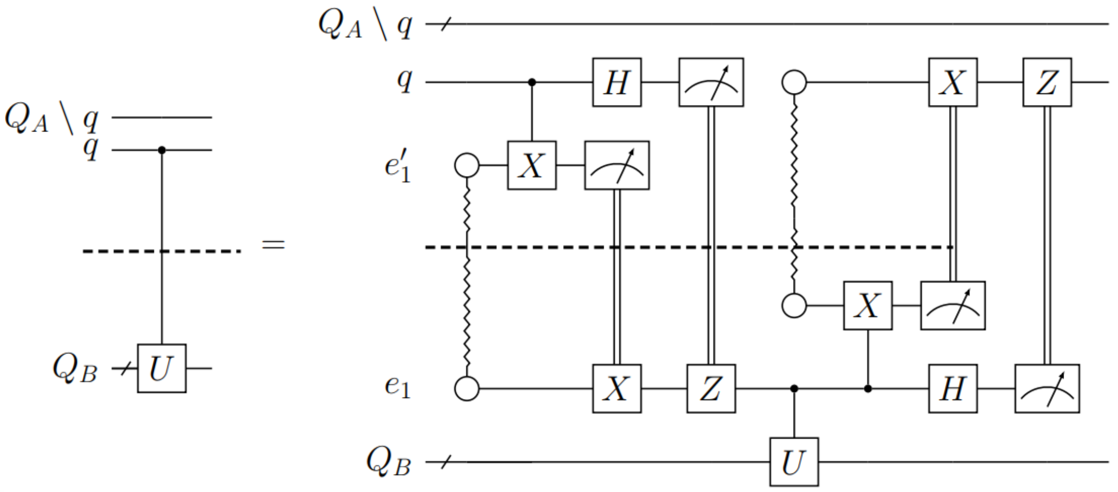


The overheads γ of different schemes: $\gamma_{LO} \geq \gamma_{LOCC} \geq \gamma_{ENT-LOCC}$

[*] K. Mitarai, and K. Fujii, Quantum 5, 388 (2021), K. Mitarai, and K. Fujii, NJP, 23, 2, 023021 (2021);
[$] C. Piveteau, and D. Sutter, IEEE Transactions on Information Theory, 70, 4, 2734 - 2745 (2024);

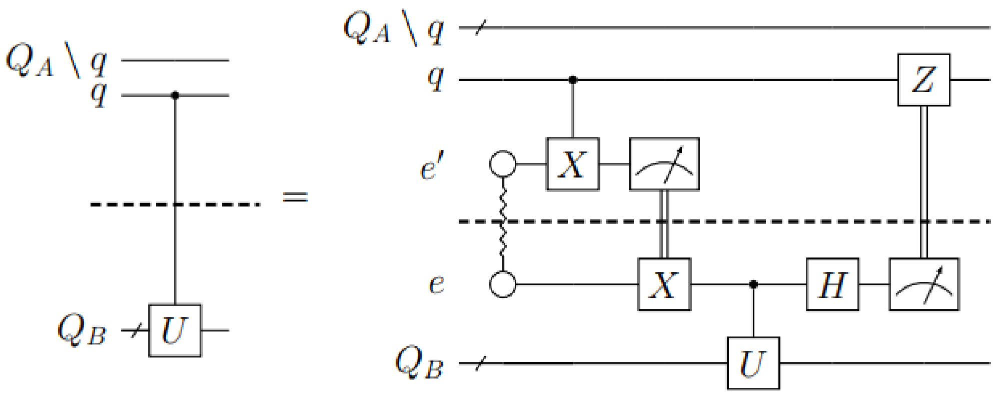
Entanglement cost in entanglement-assisted distributed quantum computing

Quantum state teleportation[1] (teledata[2])



Two entanglement pairs = 2 ebits

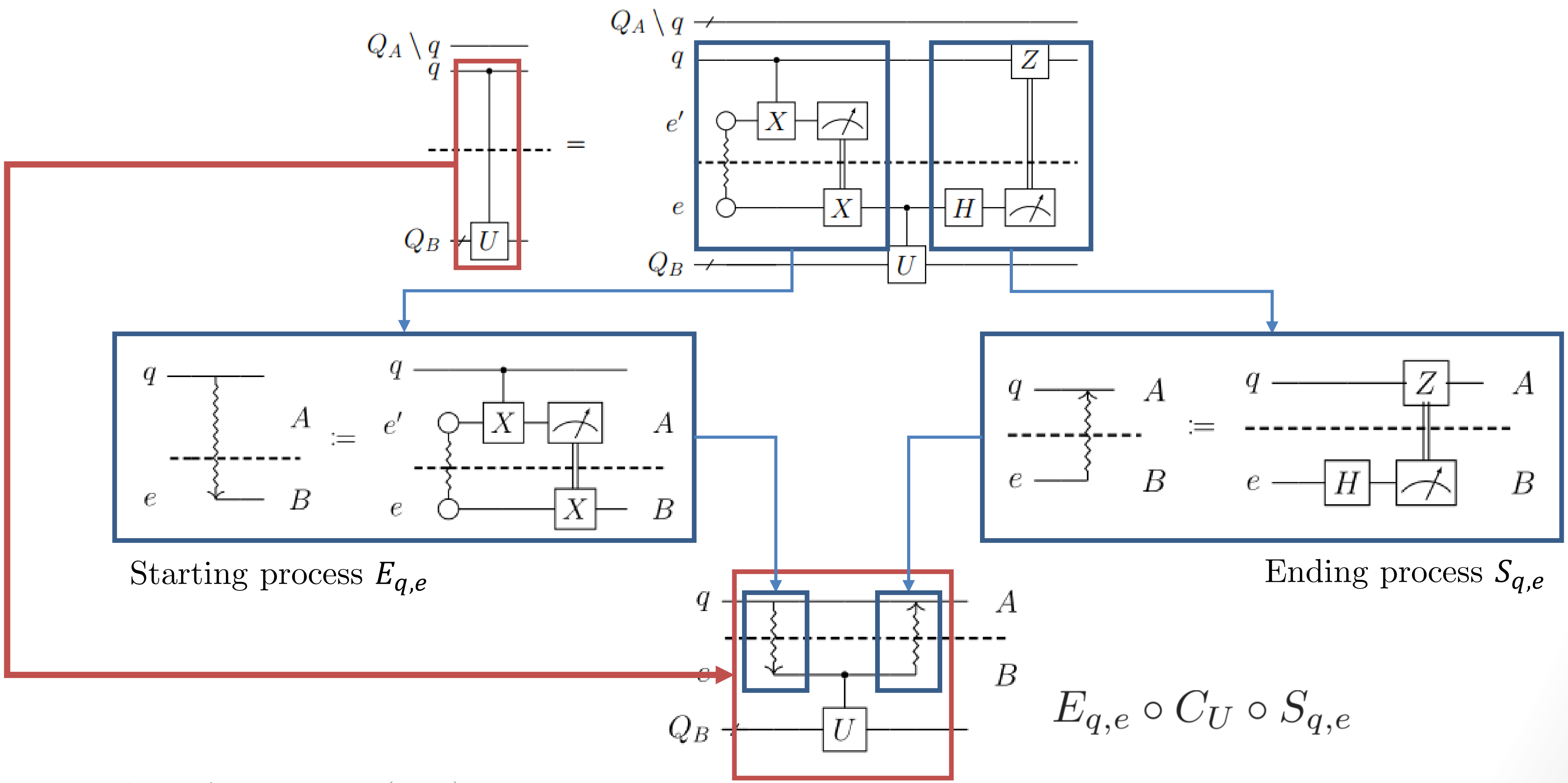
Quantum telegating[2] : e.g. EJPP protocol[3]



One entanglement pairs = 1 ebit

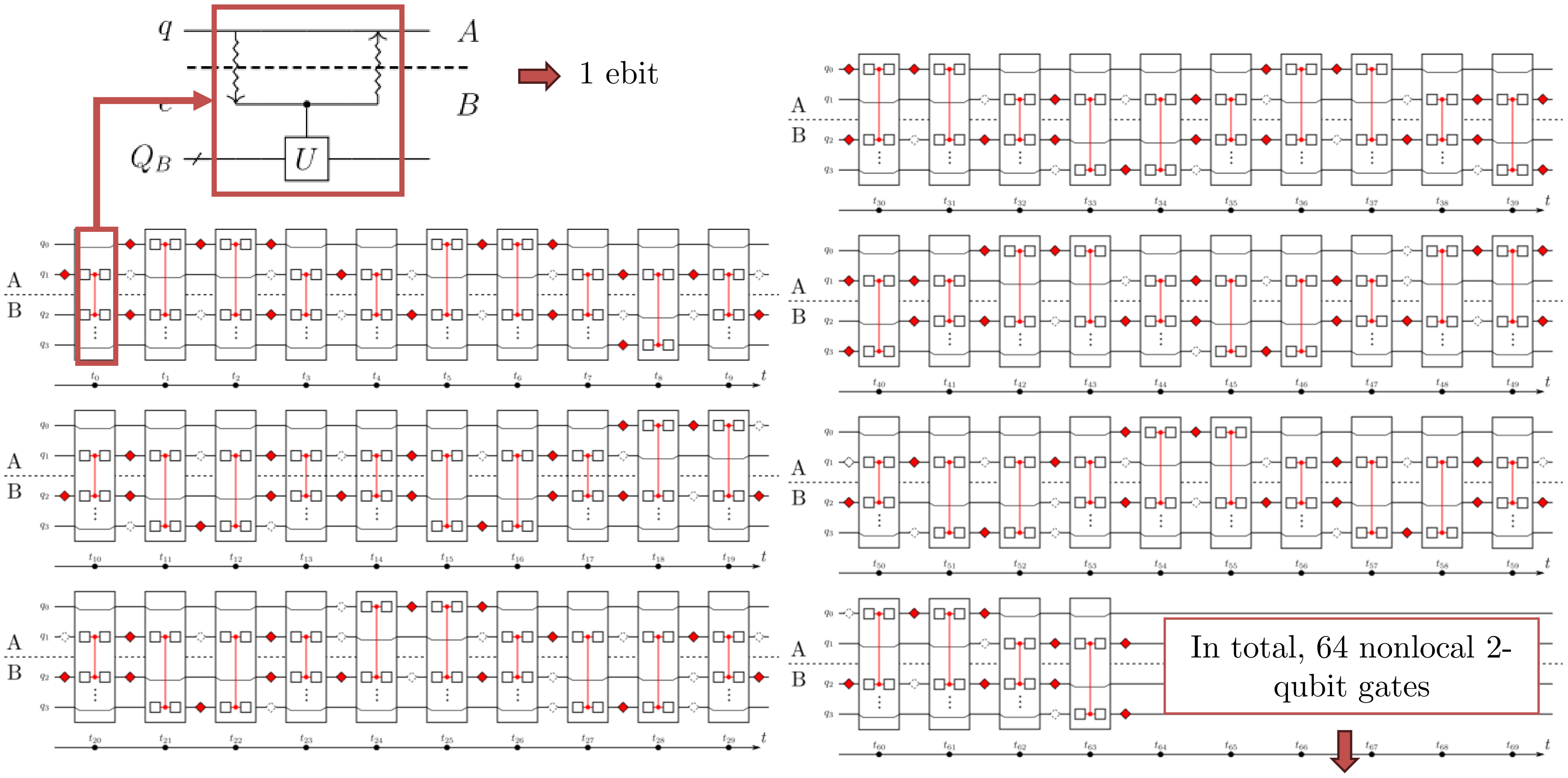
[1] C. Bennett, et. al., PRL, 70, 1895-1899 (1993)
[2] "Teledata" and "Telegate" were coined by R. Meter et al, ACM (JETC) , 3, 4, 1-23 (2008)
[3] J. Eisert, et al., PRA, 62, 052317 (2000)

DQC with quantum telegating (EJPP protocol) [1]



[1] Jens Eisert, et al., PRA, 62, 052317 (2000)

4-qubit UCC circuit for variational eigenvalue solver [1,2]



17 ebits ⇐ embedding-enhanced distribution ⇐

[1] A. G. Taube and R. J. Bartlett, International Journal of Quantum Chemistry, 106(15):3393–3401, (2006);
[2] A. Peruzzo, J. McClean, et al., Nature Communications, 5(1), (2014)

- EJPP protocol[1] ⇒ Packing process

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- Embedding-enhanced distributing processes[1]

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- Packing graph over multiple QPUs[1,2]

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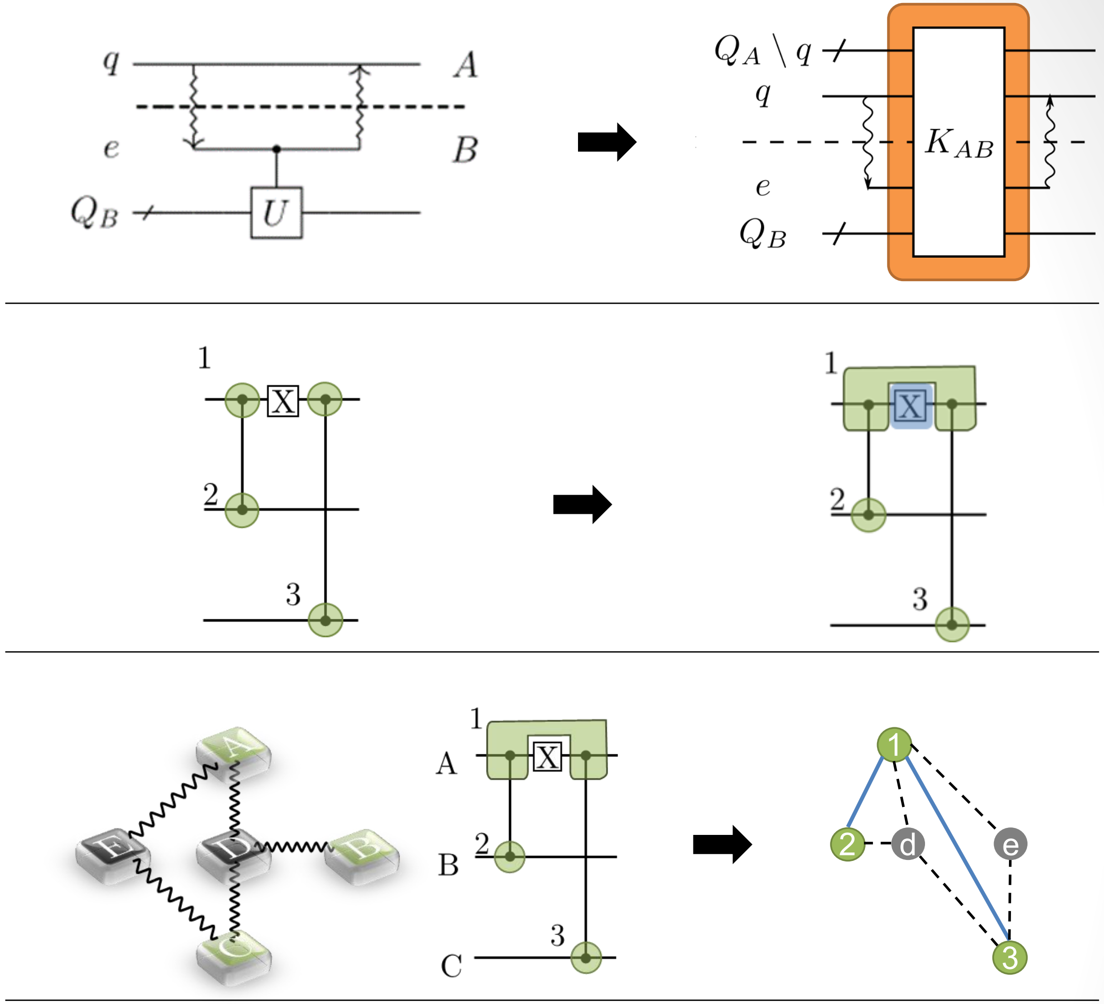
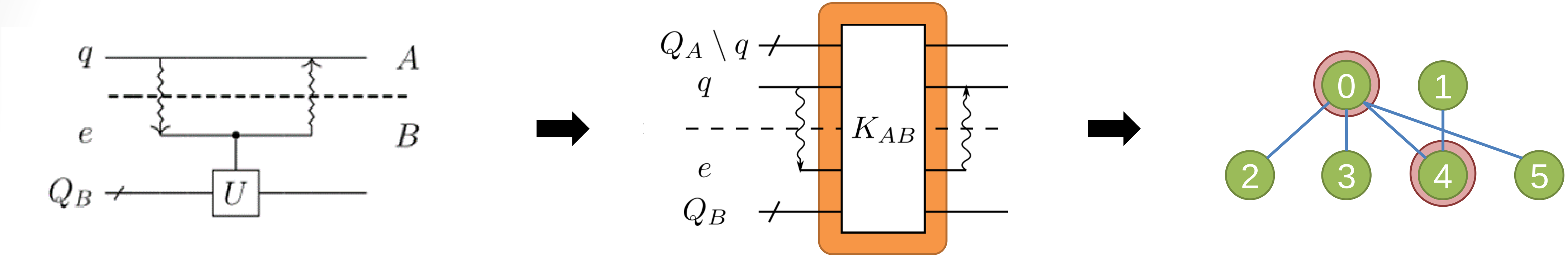


Entanglement-efficient distributed quantum computation

[1] J.-Y. Wu, K. Matsui, T. Forrer, A. Soeda, P. Andrés-Martínez, D. Mills, L. Henaut, and M. Murao, Quantum, 7:1196, 2023
[2] P. A.-Martinez, T. Forrer, D. Mills, J.-Y. Wu, L. Henaut, K. Yamamoto, M. Murao, M. and R. Duncan, arXiv:2305.14148, 2023



Embedding-enhanced distributing processes as the building blocks of DQC

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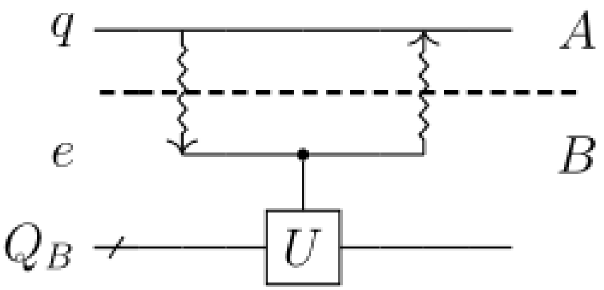
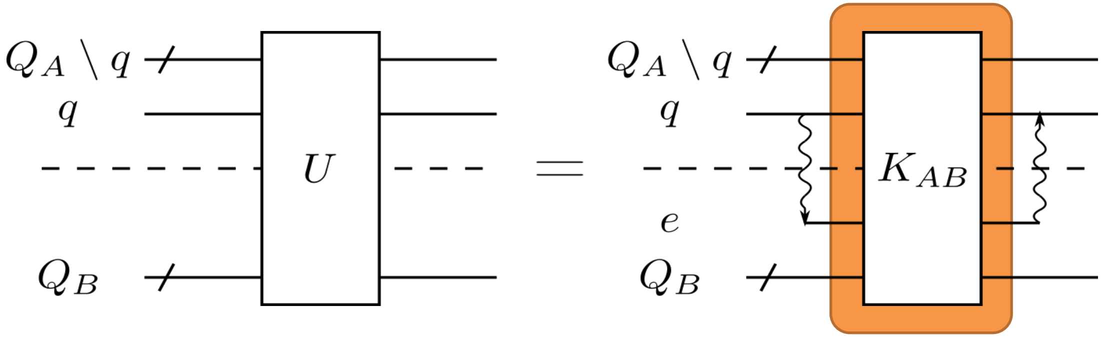
Packing graph

Extension of EJPP protocol: packing process

EJPP protocol

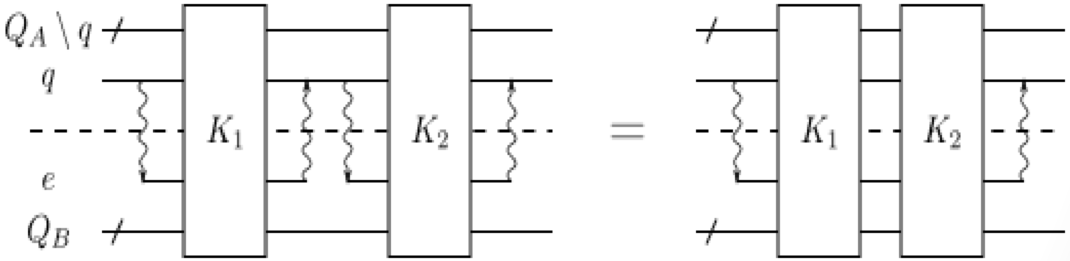


Definition (Packing process) $\mathcal{P}_{q,e}[K_{AB}] \coloneqq E_{q,e} \circ K_{AB} \circ S_{q,e}$



Theorem (Merging of packing processes). *Two sequential* entanglement-assisted packing process $\mathcal{P}_{q,e}[K_1]$ *and* $\mathcal{P}_{q,e}[K_2]$ *rooted on the same qubit q assisted by an auxiliary qubit e can be merged into one packing process.*

$$\mathcal{P}_{q,e}[K_2]\mathcal{P}_{q,e}[K_1] = \mathcal{P}_{q,e}[K_2K_1]$$

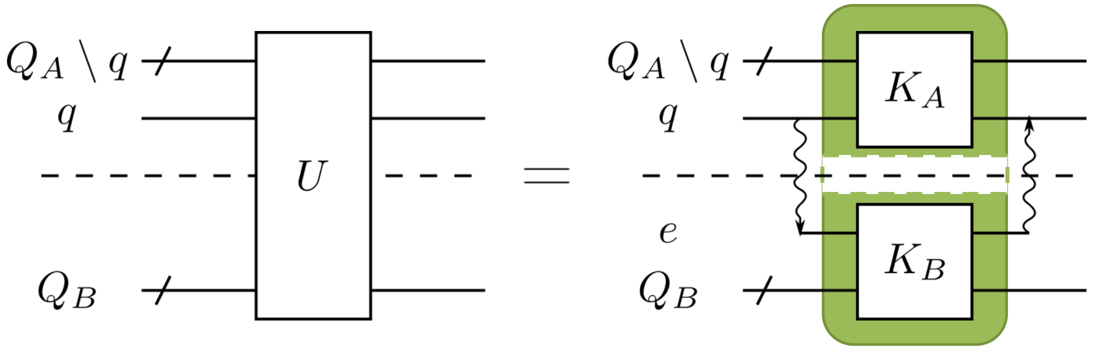


Distributing processes

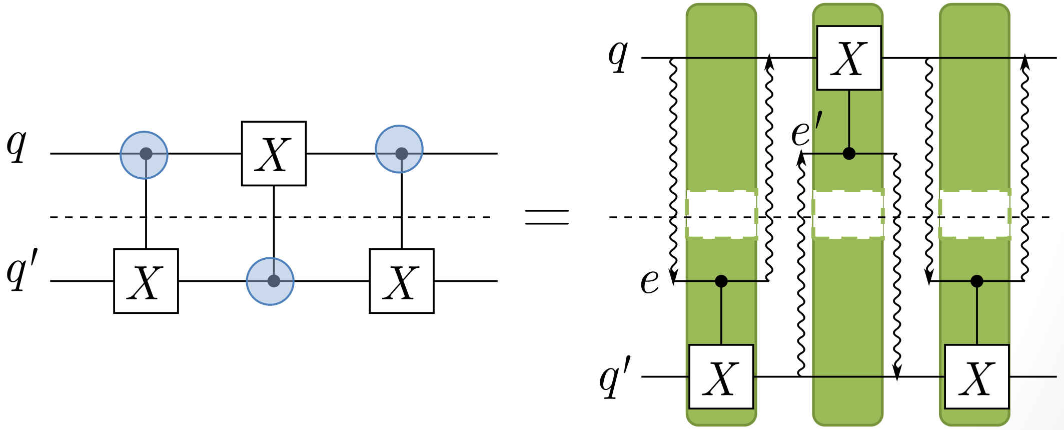
Definition (Distributing process). *A unitary U is q-rooted* distributable (or distributable on q) *over two local systems $A|B$, if there exists a q-rooted entanglement-assisted process with a local kernel with respect to $A|B$,*

$$U = \mathcal{P}_{q,e}[K_A \otimes K_B].$$

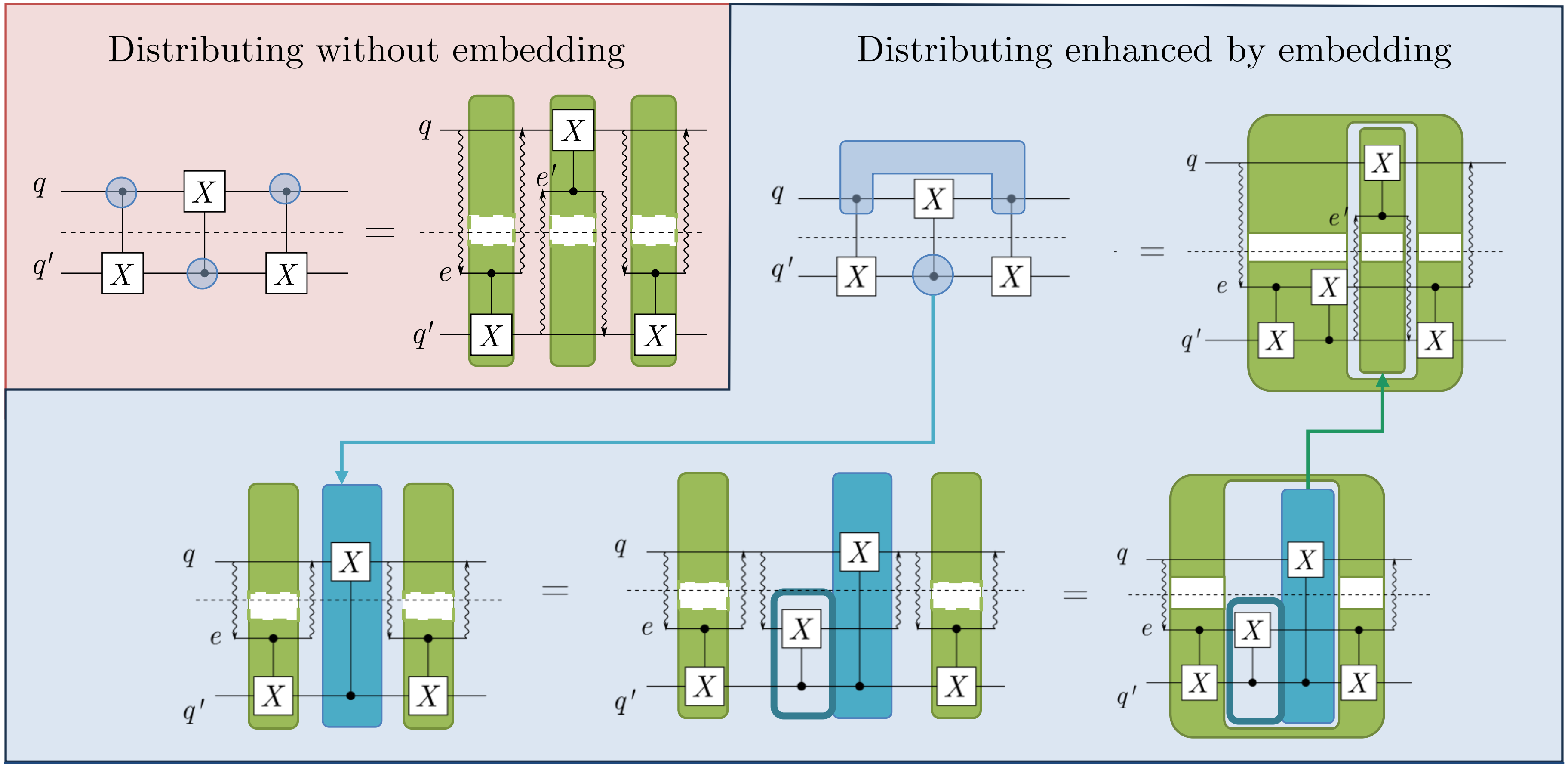
The process $\mathcal{P}_{q,e}[K_A \otimes K_B]$ is called a q-rooted distributing process *of U.*



Distributing processes are the building blocks of ENT-assisted DQC

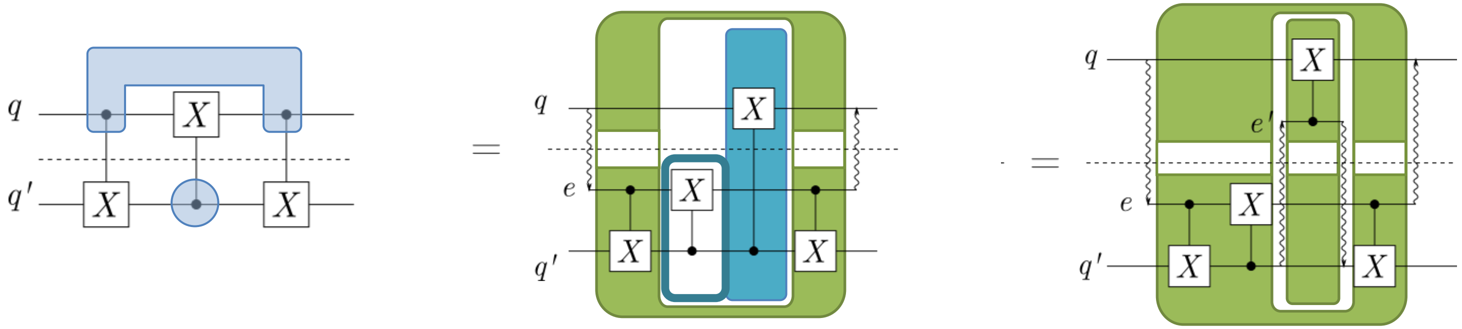


Nested distributing processes through embedding



Embedding ⇒ save 1 ebit

Distributing enhanced by embedding



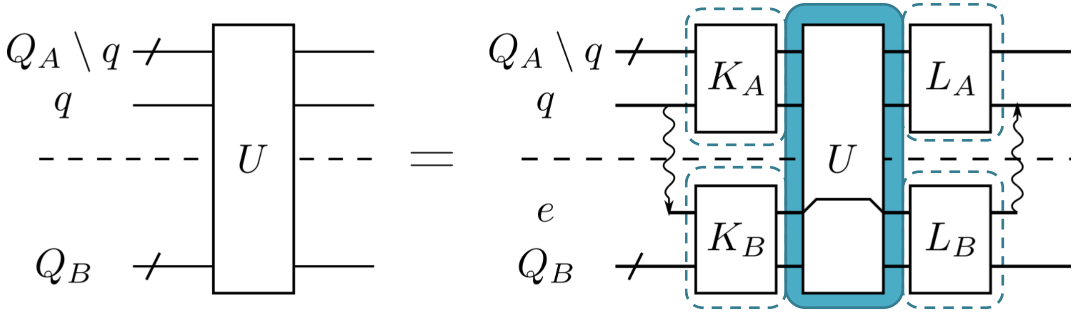
Definition (Embedding process).
A unitary U is q-rooted embeddable *with respect to a bipartition $A|B$, if there exists a corresponding kernels $\mathcal{B}_q(U)$,*

$$\mathcal{B}_q(U) = (L_{i,A} \otimes L_{i,B})\, V_i\, (K_{i,A} \otimes K_{i,B}),$$

where $L_{i,A}$ and $K_{i,A}$ and $L_{i,B}$ and $K_{i,B}$ are local operations, such that

$$U = \mathcal{P}_{q,e}[\mathcal{B}_q(U)].$$

The kernel $\mathcal{B}_q(U)$ describes a q-rooted embedding rule. *The process $\mathcal{P}_{q,e}[\mathcal{B}_q(U)]$ is a* q-rooted e-assisted embedding process *of U.*

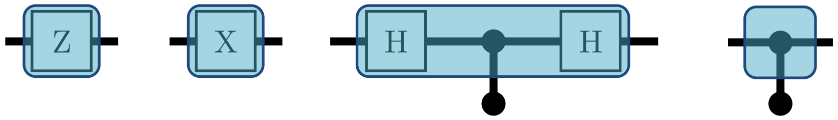


One embedding process merges
two non-sequential distributing process
⇓
saves 1 ebit.

Distributable packets:
the architecture of embedding-enhanced distribution

Let $\mathcal{V}_{q,T} = \{(q,t): t \in T\}$ be a set of gate nodes associated with nonlocal gates.

Theorem (Embedding rules)
Let $\mathbb{B}$ be the set of embedding rules.
$\mathbb{B} = \{$



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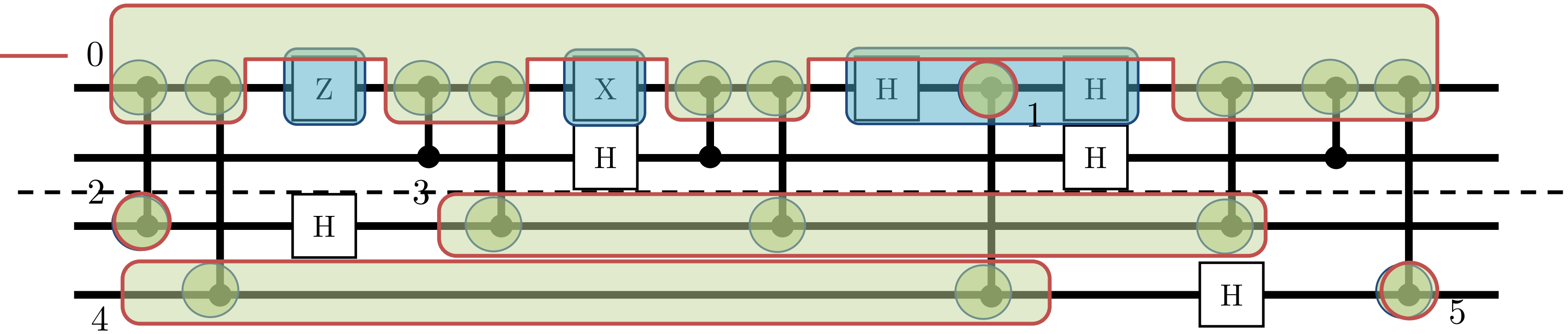
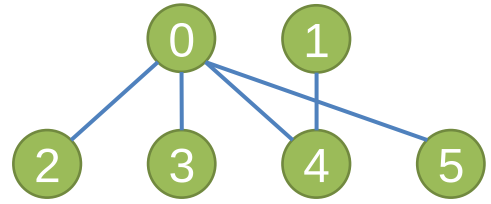


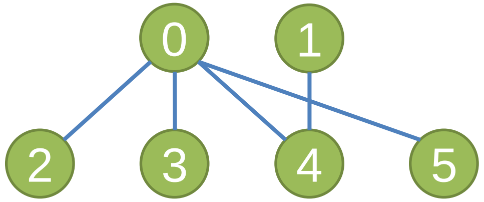
Definition (distributable packets).
The set $\mathcal{V}_{q,T}$ *is a* distributable $\mathbb{B}$-packet, *if*

1. *the* U_t *is q-rooted* distributable *for all* $t \in T$,
2. *the unitary between each pair of depths* $\{t_1, t_2\} \subseteq T$ *is* $\mathbb{B}$-embeddable.

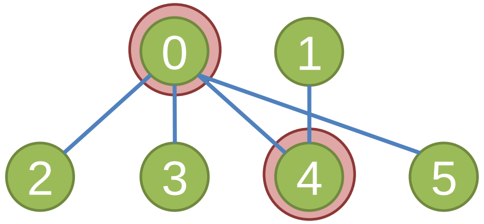
Packing graph



Packing graph



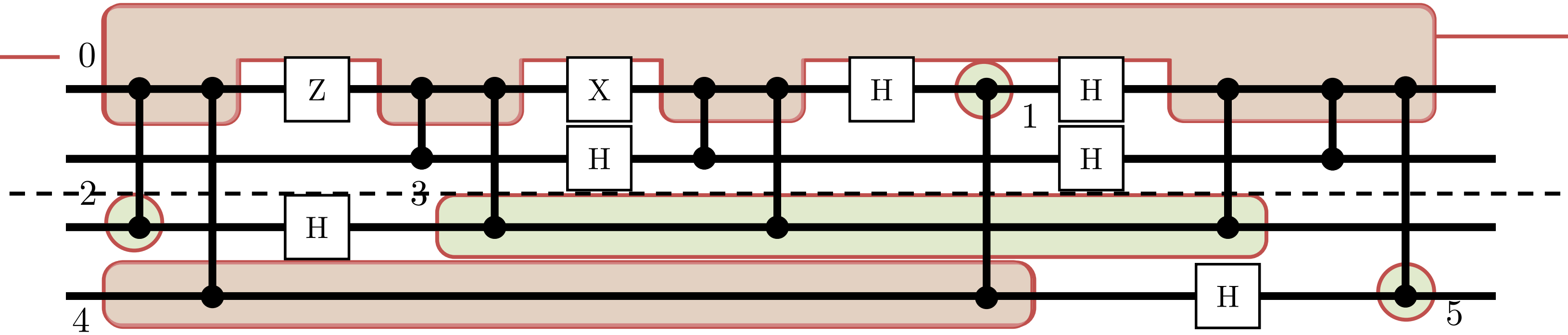
Which packets to distribute?



The algorithm

for embedding-enhanced distribution of quantum computing

Embedding-enhanced distributed quantum computing



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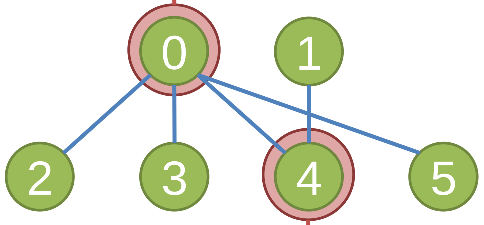
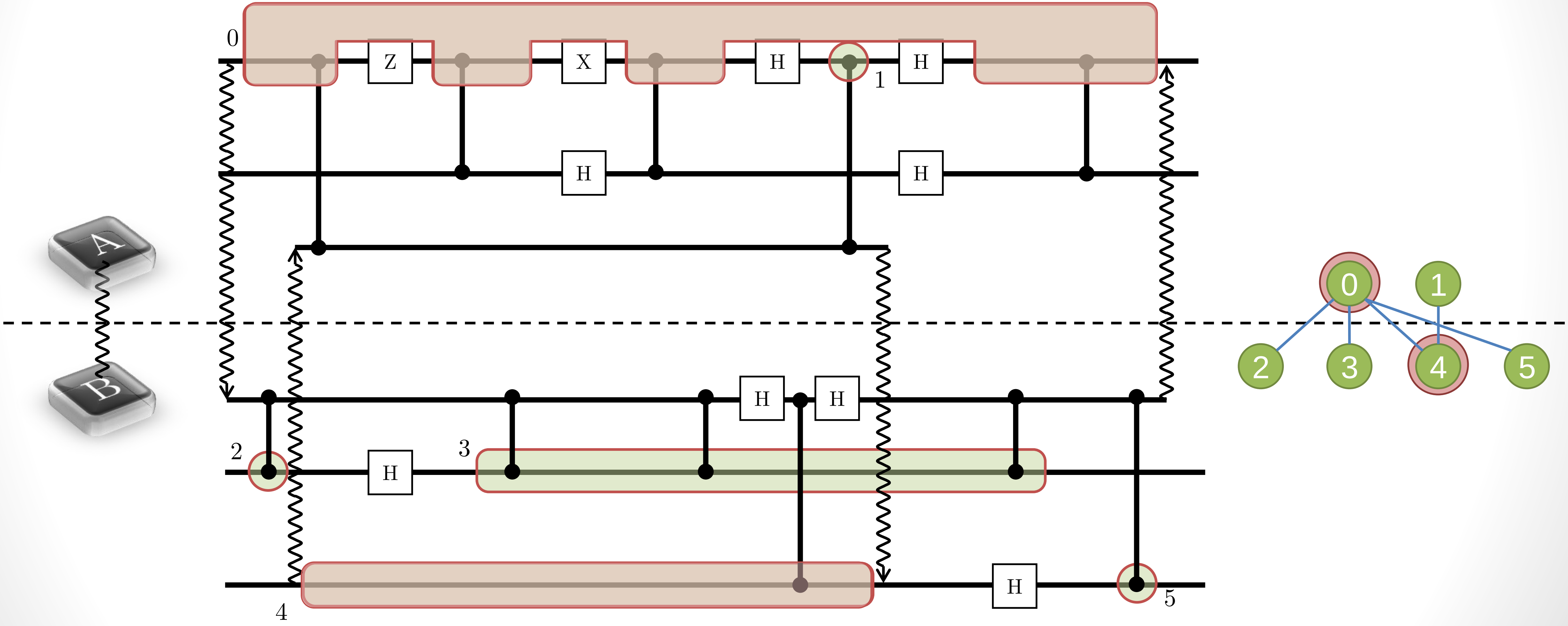
Packing graph



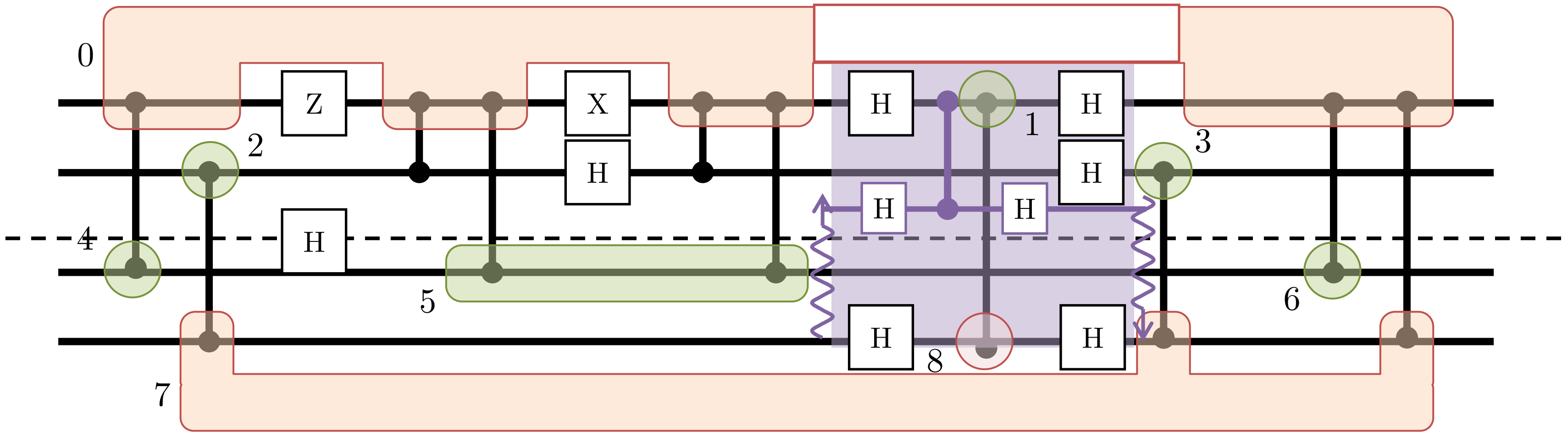
Algorithm: embedding-enhanced distributing

An optimum packing strategy = the minimum vertex covering of the packing graph.

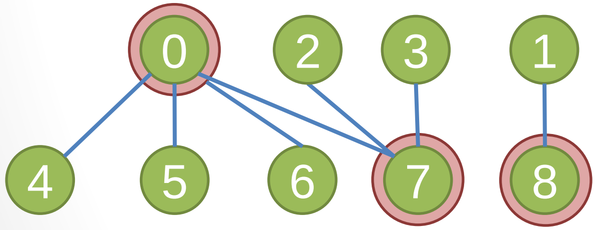
Implementation of embedding-enhanced DQC



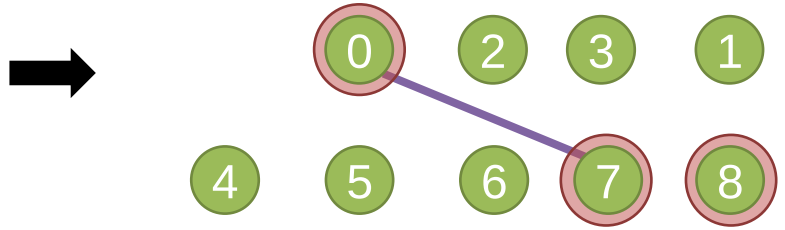
Incompatible packing and conflict edges



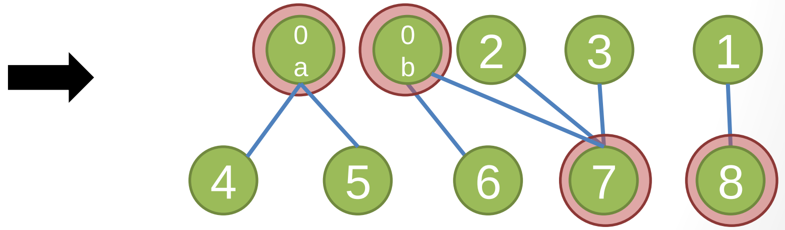
Packing graph:



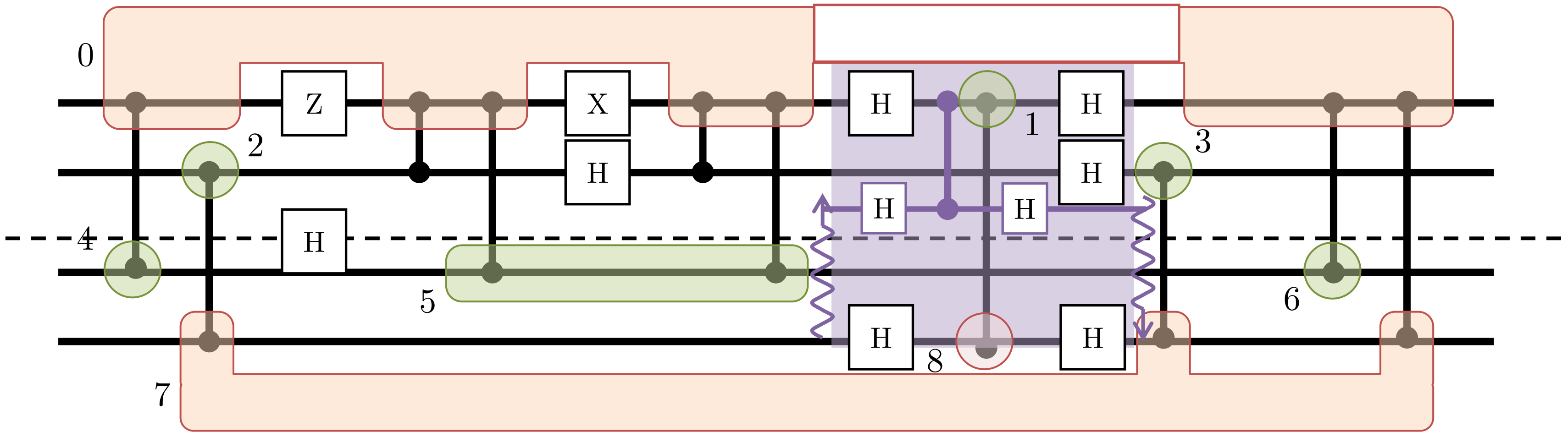
Conflict edge:



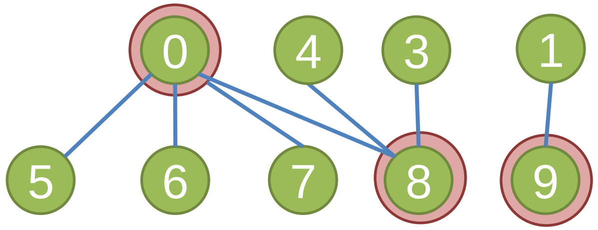
Updated packing graph:



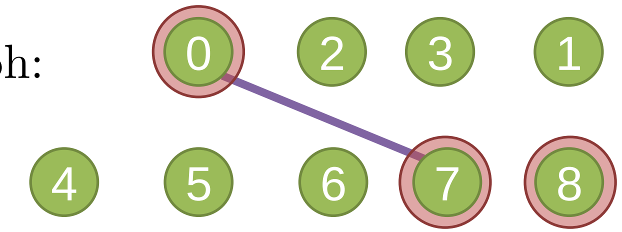
Incompatible packing and conflict edges



Packing graph

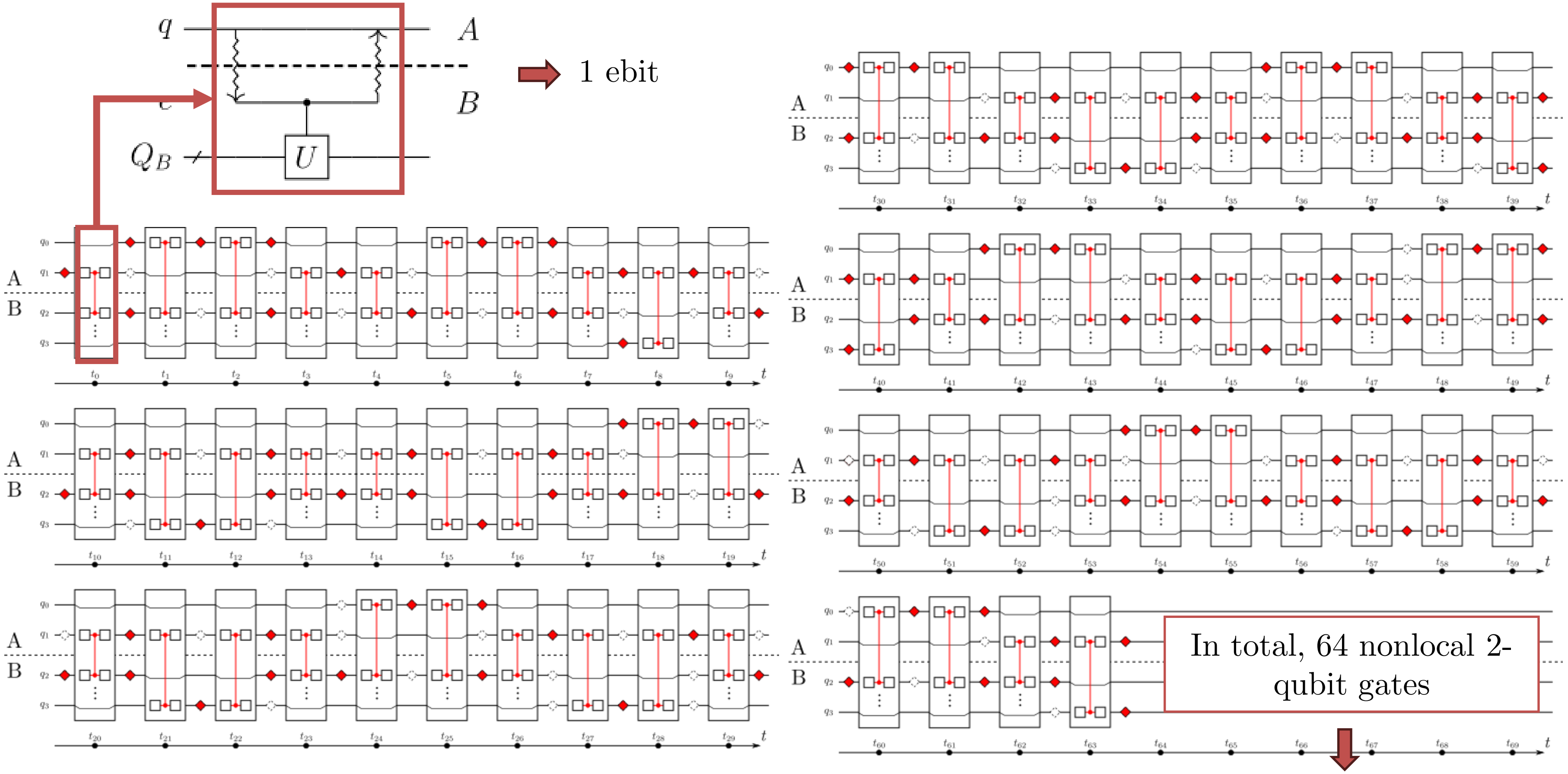


Conflict graph:



The packing graph and conflict graph of a circuit fully describe its distributability, embeddability and their compatibility

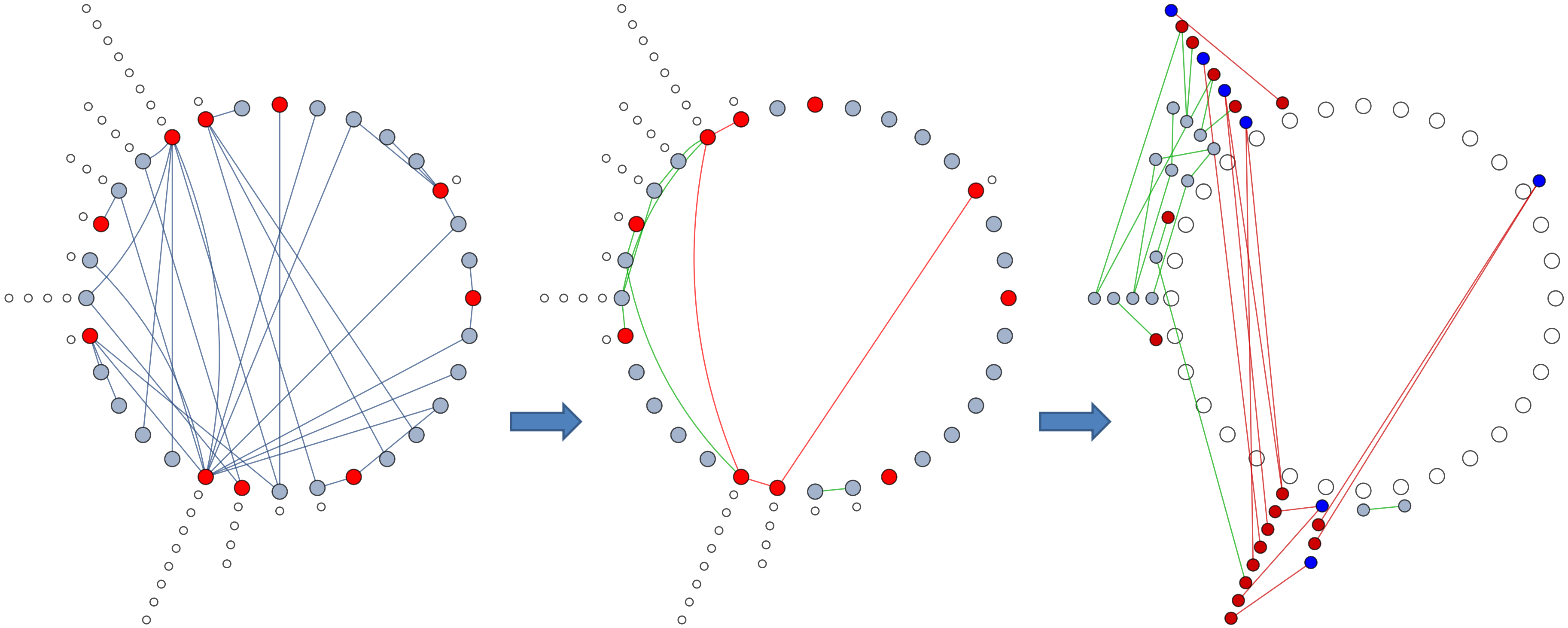
4-qubit UCC circuit for variational eigenvalue solver [1,2]



17 ebits ⇐ embedding-enhanced distribution[3] ⇐ 64 ebits?

[1] A. G. Taube and R. J. Bartlett, Int. J. of Quantum Chemistry, 106(15):3393–3401, (2006); A. Peruzzo, J. McClean, et al., Nature Communications, 5(1), (2014)
[2] J.-Y. Wu, K. Matsui, T. Forrer, A. Soeda, P. Andrés-Martínez, D. Mills, L. Henaut, and M. Murao, Quantum, 7:1196, 2023

Application in a 4-qubit unitary-coupled-cluster (UCC) circuits



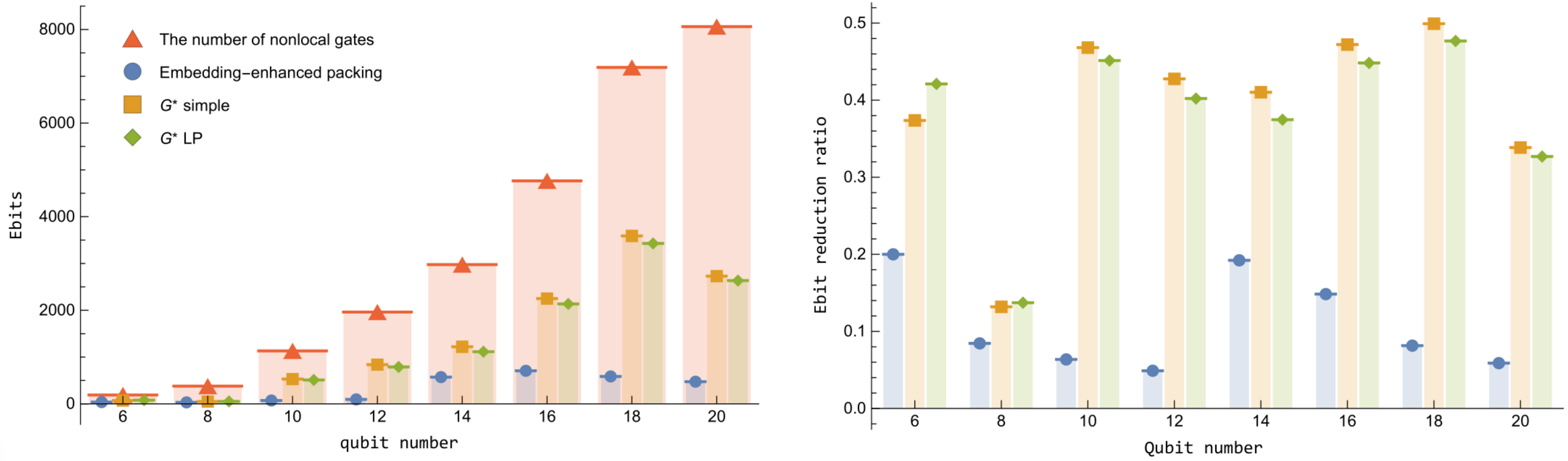
Packing graph
$|\mathbb{P}_{MVC}| = 10$

Conflict graph

Refined conflict graph
$|\mathbb{K}_{MVC}| = 7$

The 4-qubit UCC circuit has 64 nonlocal 2-qubit gates.
The entanglement cost of the UCC circuit is only $|\mathbb{P}_{MVC}| + |\mathbb{K}_{MVC}| = 17$ ebits, which saves 47 ebits

Comparison with other protocols for DQC implementation of UCC circuits

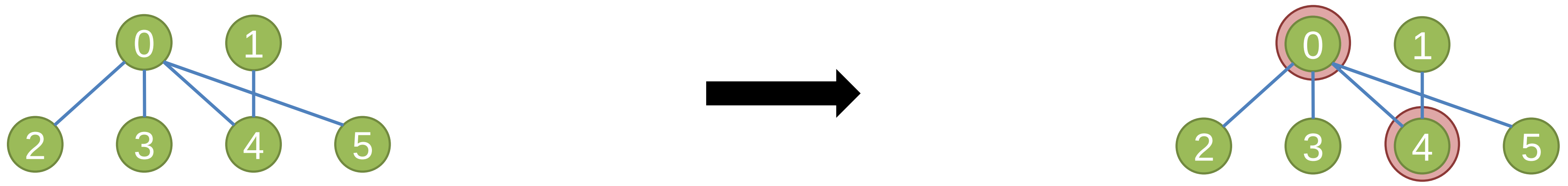


For all N-qubit UCC circuits, the embedding-enhance distributing shows significant saving of entanglement.

[*] G* simple and G* LP are the algorithms provided in:
Sundaram, Gupta, and Ramakrishnan, Efficient Distribution of Quantum Circuits, 35th International Symposium on Distributed Computing (DISC 2021) , Vol. 209

Packing graph

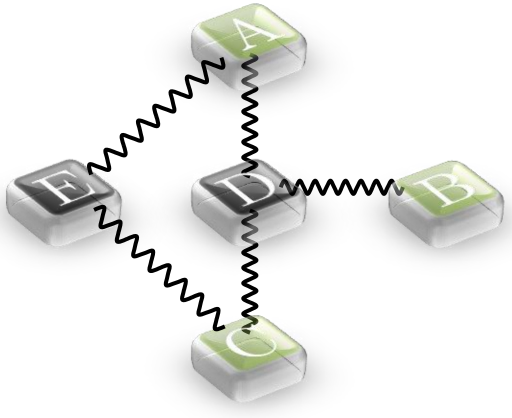
Which packets to distribute?

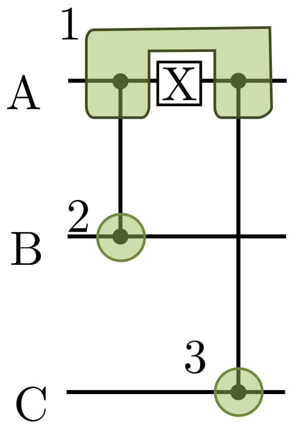


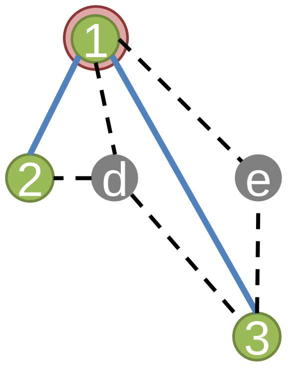
Embedding-enhanced DQC

over heterogeneous multi-QPU networks

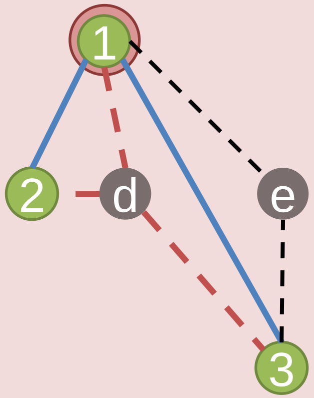
Embedding-enhanced DQC over multiple QPUs

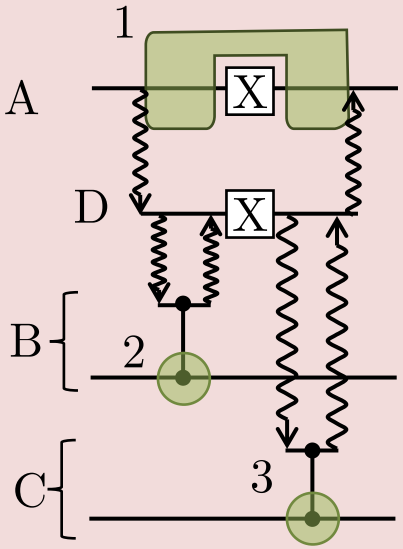




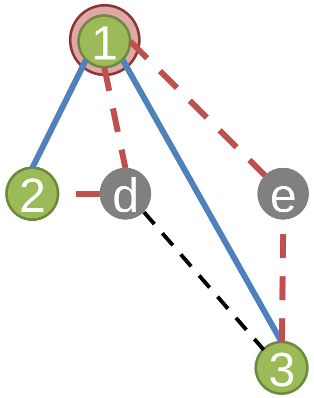


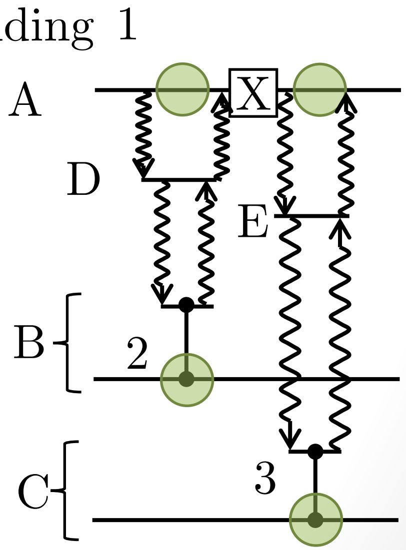
Steiner tree & Embedding



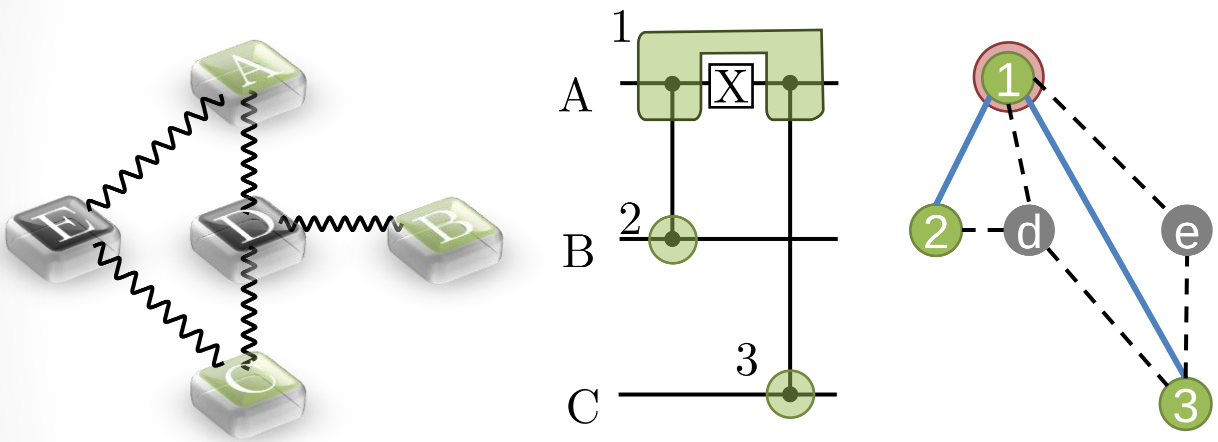


Non-Steiner tree & No embedding

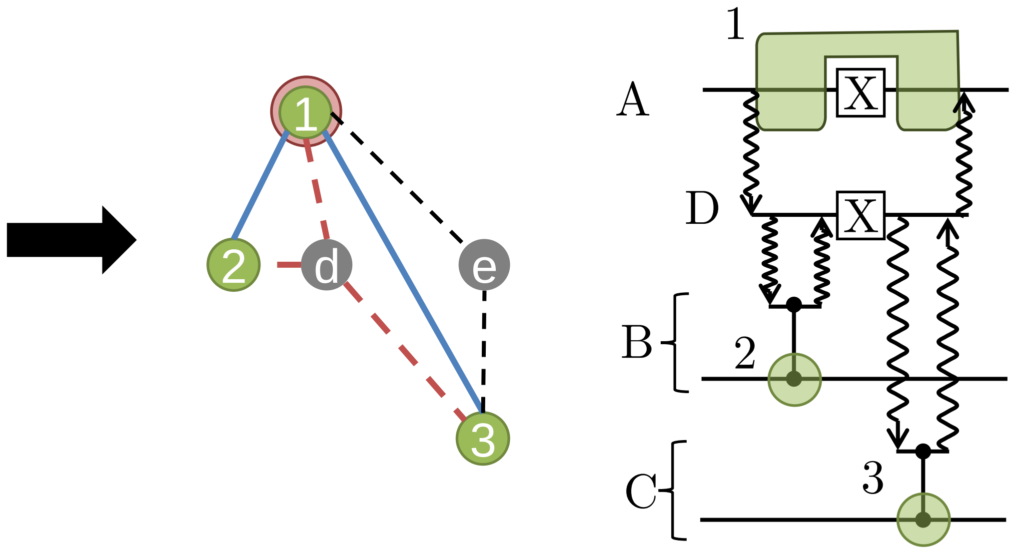




Embedding-enhanced DQC over multiple QPUs[*]



Embedding-enhanced distributing ⇒ Packing graph



Packing graph ⇒
Minimum vertex cover & Steiner tree

[*] P. A.-Martinez, T. Forrer, D. Mills, J.-Y. Wu, L. Henaut, K. Yamamoto, M. Murao, M. and R. Duncan, arXiv:2305.14148, 2023

- Embedding-enhanced distributing processes[1]

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- Packing graph over multiple QPUs[1,2]

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- Minimum vertex cover & Steiner tree of packing graph[2]

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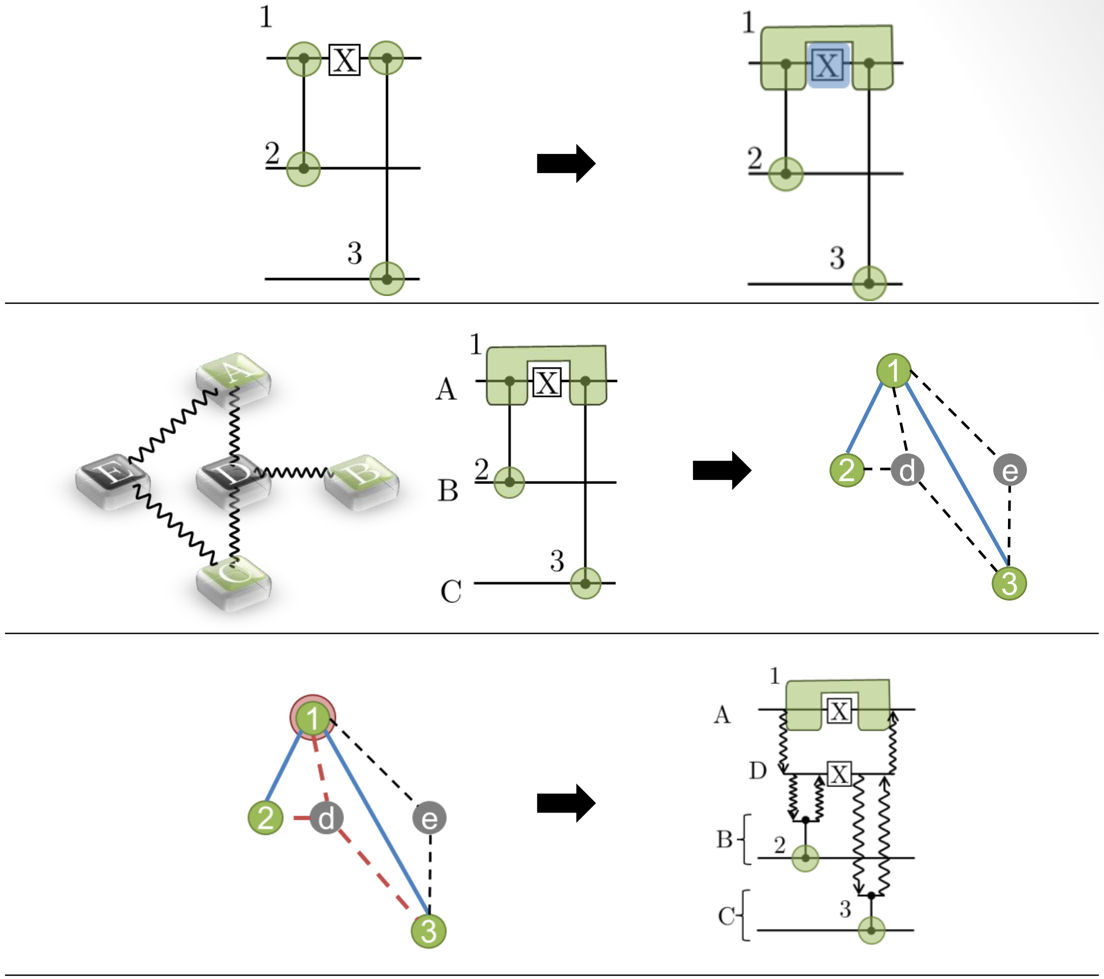

Entanglement-efficient distributed quantum computation

[1] J.-Y. Wu, K. Matsui, T. Forrer, A. Soeda, P. Andrés-Martínez, D. Mills, L. Henaut, and M. Murao, Quantum, 7:1196, 2023
[2] P. A.-Martinez, T. Forrer, D. Mills, J.-Y. Wu, L. Henaut, K. Yamamoto, M. Murao, M. and R. Duncan, arXiv:2305.14148, 2023 (to appear in QST)

Outlooks

Optimality of embedding-enhanced distribution
- Embedding with detached gate
- Case study for optimality

Embedding-enhanced distribution of multi-qubit gates (e.g. CCZ)
- Embedding rules for general n-qubit gates

Scalability enhancement of entanglement-assisted DQC
- for homogeneous networks[3]
- for heterogeneous networks

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Entanglement-efficient distributed quantum computation

[1] J.-Y. Wu, K. Matsui, T. Forrer, A. Soeda, P. Andrés-Martínez, D. Mills, L. Henaut, and M. Murao, Quantum, 7:1196, 2023
[2] P. A.-Martinez, T. Forrer, D. Mills, J.-Y. Wu, L. Henaut, K. Yamamoto, M. Murao, M. and R. Duncan, arXiv:2305.14148, 2023 (to appear in QST)
[3] S.-H. Hu, G. Biswas, and J.-Y. Wu, arXiv:2405.10942

Consortium in Photonic Quantum Computing

Open positions:

- PhDs
- Postdocs

淡江大學 Tamkang University

Jun-Yi Wu

國立清華大學 NATIONAL TSING HUA UNIVERSITY

Rui-Kuang Lee, Ming-Chang Lee

國立中央大學 National Central University

Yen-Hung Chen

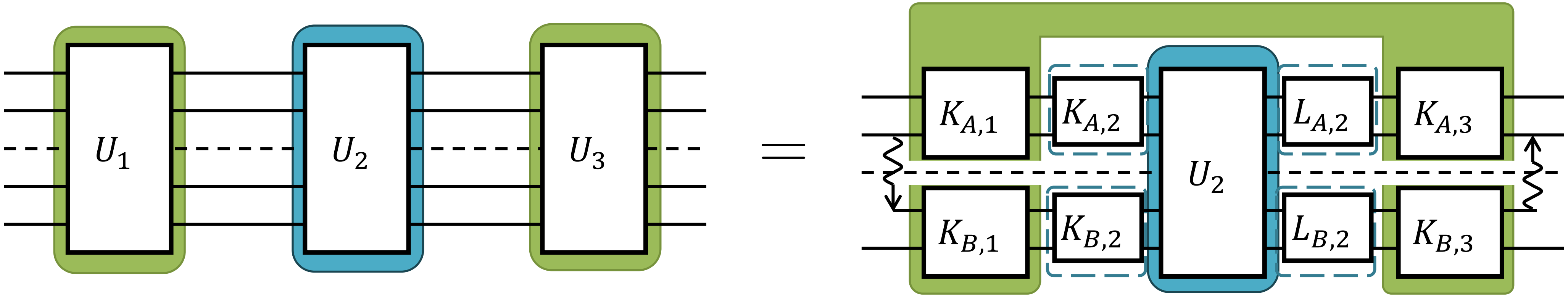
Projects:

- Verification theory in photonic quantum computing.
- Experiment in integrated photonic quantum computing.

Thank you!

Distributing quantum computation enhanced by embedding, Quantum, 7:1196 (2023)

$$U_3.U_2.U_1 = \mathcal{P}_{q,e}[(K_{A,3} \otimes K_{B,3})\ (L_{A,2} \otimes L_{B,2})\, U_2\, (K_{A,2} \otimes K_{B,2})\ (K_{A,1} \otimes K_{B,1})]$$



Embedding-enhanced multipartite DQC, arXiv: 2305.14148 (to appear in QST)