Diquarks in lattice QCD

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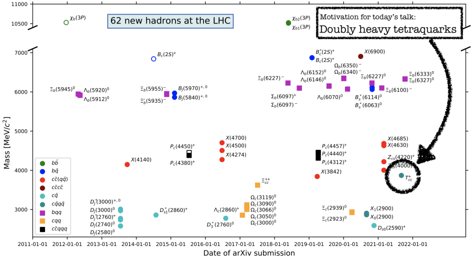
see also the review: AF., Prog.Part.Nucl.Phys. 140 (2025) 104143

Heavy spectrum today - a success story turned challenge to theory

Multi-decade-long theory-success-streak broken

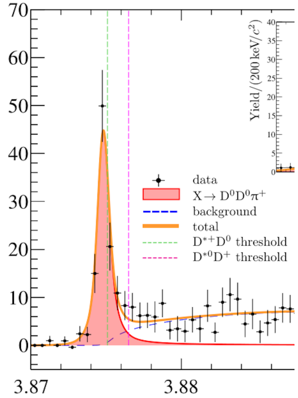
- Many new, unexpected, states observed.
- esp. 4-/5-quark states not expected before ($\sim$ 12).
- Also, many predicted quark model states not seen.



(LHCb collaboration, 2021, 62 new hadrons, in 2023: 65)

E.g.: A new family of doubly-heavy tetraquarks

T_{cc}, $\bar{u}\bar{d}cc$, $E_B = 0.3$MeV



⇝ LHCb-PAPER-2021-031

Limited explanations in theory

- QCD often approximated in models
- many extensions possible, many interpretations, some times contradictory statements
- insights through lattice QCD calculations? What role could effective QCD degrees of freedom play?

Diquarks - an attractive concept

"The concept of diquarks is almost as old as the quark model, and actually predates QCD [1]" $\rightsquigarrow$ arXiv:2203.16583; [1] PR 155, 1601 (1967)

- **Diquarks in QCD:** Formally the diquark interpolating operator may be written as

$$\boxed{D_\Gamma = q^c C\Gamma q'}$$

$\rightsquigarrow$ c, C=charge conjugation, Γ acts on Dirac space

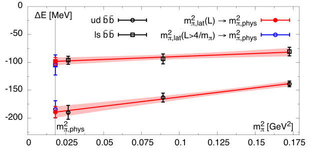
Consequences for their properties:

- "good" $(\bar{3}_F, \bar{3}_c, J^P = 0^+)$ configuration
- quarks in "good" diquarks attract each other
- large mass splitting in good, bad and not-even-bad channels
- HQSS-limit: When $q \to Q = \infty$ a diquark acts as an antiquark $[QQ] \leftrightarrow \bar{Q}$.

3 types of diquark:
good, bad and not-even bad

J^P	C	F	Op: Γ
0^+	$\bar{3}$	$\bar{3}$	γ_5, $\gamma_0\gamma_5$
1^+	$\bar{3}$	6	γ_i, σ_{i0}
0^-	$\bar{3}$	6	$\mathbb{1}$, γ_0
1^-	$\bar{3}$	$\bar{3}$	$\gamma_i\gamma_5$, σ_{ij}

- Well founded in QCD with successful applications in many hadrons.
  - Combination of good diquark and HQSS was motivation for study of T_{bb} on the lattice.
  - In this case: All model predictions observed.
  - But: What really is the role of the diquarks? What are their properties and details?



Diquarks on the lattice - a gauge invariant probe

- A problem for the lattice - and others, incl. experiment - is that diquarks are colored.
  - They are not-gauge invariant and not directly accessible as QCD observables.
  - Could fix a gauge, but then properties are gauge-dependent (masses, sizes,...)

$\rightsquigarrow$ `lattice and Dyson-Schwinger, see e.g. [15-20] in 2106.09080`

- **Alternative:** Exploit "effective" mass-decomposition of a hadron

$$\boxed{C_\Gamma(t) \sim \exp\left[-t\left(m_{D_\Gamma} + m_Q + \mathcal{O}(m_Q^{-1})\right)\right]}$$

$\rightsquigarrow$ $t \rightarrow$ large, $m_Q \rightarrow$ large

  - With a static spectator quark Q ($m_Q \rightarrow \infty$), it cancels in **mass differences**.
  - Diquark properties can be exposed in a gauge-invariant way.

$\rightsquigarrow$ `hep-lat/0510082, hep-lat/0509113, hep-lat/0609004, arxiv:1012.2353`

- **Lattice implementation:**
  - Embed the diquark in a static-light-light baryon correlation function

$$C_\Gamma(t) = \sum_{\vec{x}} \left\langle [D_\Gamma Q](\vec{x}, t)\ [D_\Gamma Q]^\dagger(\vec{0}, 0) \right\rangle$$

$\rightsquigarrow$ static quark=Q and $D_\Gamma = q^c C\Gamma q$

$\rightsquigarrow$ here: flavor combinations ud, ℓs, ss'

  - Static-light mesons may also be used for diquark-quark differences

$$C^M_\Gamma(t) = \sum_{\vec{x}} \left\langle [\bar{Q}\Gamma q](\vec{x}, t)\ [\bar{Q}\Gamma q]^\dagger(\vec{0}, 0) \right\rangle$$

Diquark spectroscopy

Mass difference 1

Mass differences of two $qq'Q$ baryons (one with good diquark):

$$\boxed{C_{\Gamma}^{qq'Q}(t) - C_{\gamma_5}^{qq'Q}(t)}$$

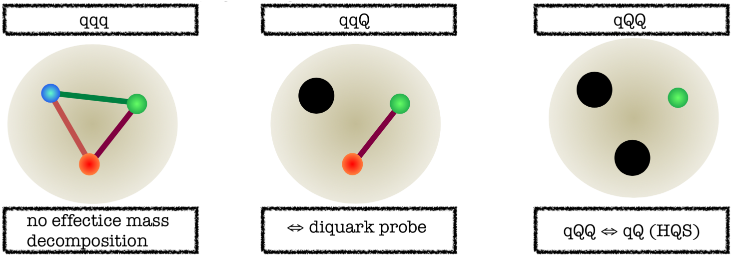
$\rightsquigarrow$ Q drops out
$\rightsquigarrow$ diquark-diquark mass difference

Mass difference 2

Mass differences of a $qq'Q$ baryon and a light-static meson:

$$\boxed{C_{\Gamma=\gamma_5}^{qq'Q}(t) - C_{\gamma_5}^{q'\bar{Q}}(t)}$$

$\rightsquigarrow$ Q drops out
$\rightsquigarrow$ diquark-quark mass difference



$\rightsquigarrow$baryon pictures from Hosaka, 2013

"These mass differences are fundamental characteristics of QCD, which should be measured carefully on the lattice." $\rightsquigarrow$ Jaffe, arXiv:hep-ph/0409065; Phys. Rept. 409 (2005)

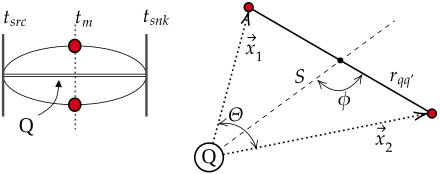
A step further: Diquark structure

We can access (good) diquark structure information through density-density correlations:

$$C_\Gamma^{dd}(\vec{x}_1, \vec{x}_2, t) = \left\langle \mathcal{O}_\Gamma(\vec{0}, 2t)\ \rho(\vec{x}_1, t)\rho(\vec{x}_2, t)\ \mathcal{O}_\Gamma^\dagger(\vec{0}, 0) \right\rangle$$

$\rightsquigarrow \mathcal{O}_\Gamma = q^c C\Gamma q$ and $\rho(\vec{x}, t) = \bar{q}(\vec{x}, t)\gamma_0 q(\vec{x}, t)$
$\rightsquigarrow t_m = (t_{snk} + t_{src})/2$ to minimize excited states

Main tool: Correlations between two light quarks' relative positions to the static quark



$\rightsquigarrow \vec{r}_{ud} = \vec{x}_2 - \vec{x}_1$ and $\vec{S} = (\vec{x}_1 - \vec{x}_2)/2$

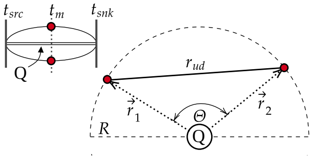
$$\rho_2(r_{ud}, S, \phi; \Gamma) = C_\Gamma^{dd}(\vec{x}_1, \vec{x}_2, t_m)$$

Note, when S and r_{ud} fixed, distance between static quark Q and light quarks q, q' is

- Minimized for $\phi = \pi$, possible disruption due to Q is largest
- Maximized for $\phi = \pi/2$, possible disruption due to Q is smallest

Good diquarks - Attraction, radius and spatial properties

Mode 1: Attraction and radius



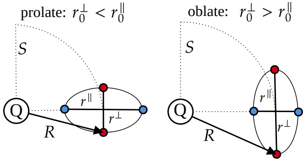
Setting $\phi = \pi/2$:

- $|\vec{x}_1| = |\vec{x}_2| = R$, use R, Θ:
  $$\rho_2^\perp(R, \Theta) = \rho_2(r_{ud}, S, \pi/2)$$
- Distance between quarks:
  $$r_{ud} = R\sqrt{2\,(1 - \cos(\Theta))}$$

Goals:

- Probe diquark attraction effect.
  $\Rightarrow$ Visible increase in $\rho_2^\perp$ for small Θ at any fixed R.
- $\rho_2^\perp(R, r_{ud}) \sim \exp(-r_{ud}/r_0)$
  $\Rightarrow$ "characteristic diquark size" r_0.

Mode 2: Radial/Tangential shape



As opposed to before $R \neq$ *fixed*:

- $\phi = \pi$: radial correlation, size $\rightsquigarrow r_0^\parallel$
- $\phi = \pi/2$: tangential, size $\rightsquigarrow r_0^\perp$
- $r_0^\perp / r_0^\parallel$ gives information on shape:
  - $= 1$, spherical
  - $\neq 1$, prolate/oblate

Goals:

- Probe $J = 0$ nature of good diquark (spherical, S-wave expectation)
- Diquark polarisation through heavy quark?

Results

Research and calculation roadmap:

1. **spectrum:** *probe mass differences*
2. **spatial correlations:** *study attraction and special status of the good diquark*
3. **structure:** *estimate size and shape of the good diquark*

Lattice setup (public PACS-CS gaugefields)

- $n_f = 2 + 1$ full QCD, $32^3 \times 64$, $a = 0.090$fm, $a^{-1} = 2.194$GeV
- $m_\pi = 164, 299, 415, 575$ and $707\,$MeV, $m_s \simeq m_s^{\rm phys}$
- Quenched gaugefields $a \simeq 0.1$fm, $m_\pi^{\rm valence} = 909\,$MeV, to match `hep-lat/0509113`

Lattice spectroscopy - diquark-(di)quark differences

We consider $qq'Q$ baryon differences:

$$C_{\Gamma}^{qq'Q}(t) - C_{\gamma_5}^{qq'Q}(t)$$

Special status of good diquark observed

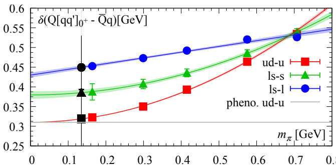
- Good *ud* diquark lowest in spectrum
- Pattern repeated in ℓs and ss'

... and $qq'Q$ baryon qQ meson differences

$$C_{\Gamma=\gamma_5}^{qq'Q}(t) - C_{\gamma_5}^{q'\bar{Q}}(t)$$

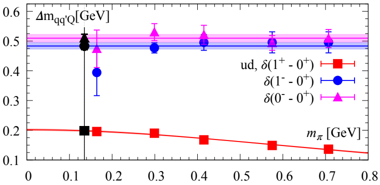
$Qqq' - \bar{Q}q'$ splittings

⇝ useful for 4-/5-quark states.

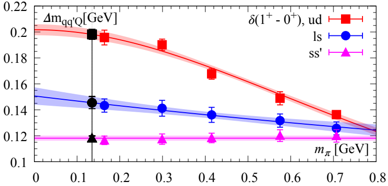


$$\delta(Q[q_1 q_2]_{0^+} - \bar{Q}q_2) = C\ \left[1 + (m_\pi/D)^{n\in 0,1,2}\right]$$

ud 0^+ versus 1^+, 0^- and 1^-



$(1^+ - 0^+)_{qq'}$ splitting



$$\delta(1^+ - 0^+)_{q_1 q_2} = A/\left[1 + (m_\pi/B)^{n\in 0,1,2}\right]$$

Diquark spectroscopy - comparing results

- We want to compare our results with phenomenology

  $\rightsquigarrow$ more details in extra info slides

  - Key resource: (Jaffe '05, arXiv:hep-ph/0409065), updated with PDG 2021 input
  - For pheno estimates combine charm and bottom hadron masses such that leading $\mathcal{O}(1/m_Q)$ ($Q = c, b$) cancel

- The main spectroscopy results are summarised as:

All in [MeV]	$\delta E_{\text{lat}}(m_\pi^{\text{phys}})$	δE_{pheno}	$\delta E_{\text{pheno}}^{\text{bottom}}$	$\delta E_{\text{pheno}}^{\text{charm}}$
$\delta(1^+ - 0^+)_{ud}$	198(4)	206(4)	206	210
$\delta(1^+ - 0^+)_{\ell s}$	145(5)	145(3)	145	148
$\delta(1^+ - 0^+)_{ss'}$	118(2)			
$\delta(Q[ud]_{0^+} - \bar{Q}u)$	319(1)	306(7)	306	313
$\delta(Q[\ell s]_{0^+} - \bar{Q}s)$	385(9)	397(1)	397	398
$\delta(Q[\ell s]_{0^+} - \bar{Q}\ell)$	450(6)			

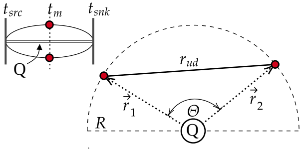
$\rightsquigarrow$ use the bottom estimate for static

$\rightsquigarrow$ use charm-bottom difference as estimate for deviation from static

$\Rightarrow \lesssim \mathcal{O}(7)$MeV deviation

- Overall, very good agreement observed.

Diquark attraction effect



Two limiting cases for the two quarks:

- $\cos(\Theta) = 1$ on top of each other
- $\cos(\Theta) = -1$ opposite each other

"Lift" as qualitative criterion:

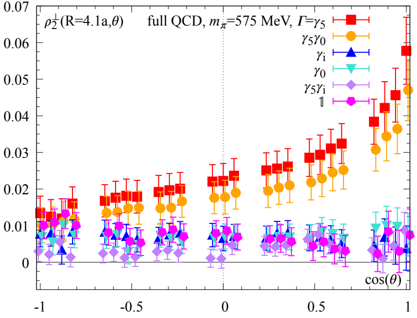
$$\frac{\rho_2^{\perp}(R, \Theta = 0, \Gamma)}{\rho_2^{\perp}(R, \Theta = \pi/2, \gamma_5)}$$

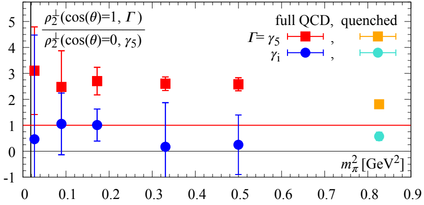
Increase observed in good diquark only

In the good diquark channel

⇒ Quarks spatially correlated.
⇒ Indication of attraction.

Spatial correlation over Θ

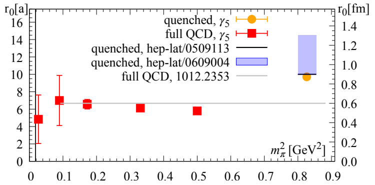




Good diquark size

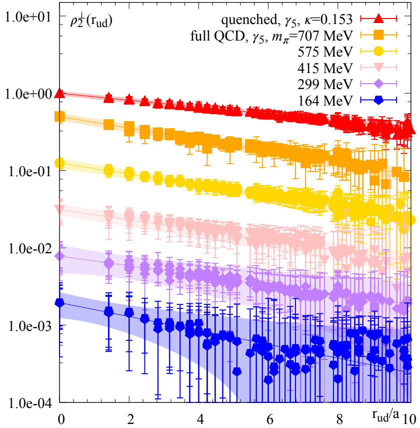
- Expectation:
  $\rho_2^{\perp}(R, r_{ud}) \sim \exp(-r_{ud}/r_0)$
- Need to control:
  - Q-interference $\rightsquigarrow$ keep $r_{ud} < R$
  - periodicity effects $\rightsquigarrow$ $L = 5r_0$ ok
- Fit form:
  $A(R, r_{ud} = 0) \sim \exp(-R/r_0)$
  $\Rightarrow$ Data well described by (single) exponential Ansatz

Size dependence $r_0(m_\pi)$



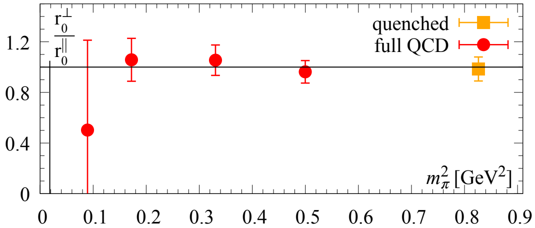
$r_0 \simeq \mathcal{O}(0.6)$fm $\sim r_{\text{meson, baryon}}$ (1604.02891)

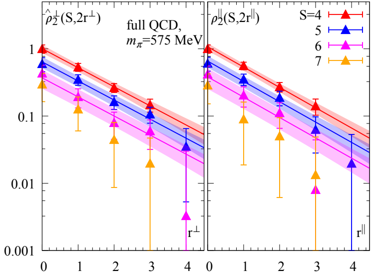
Spatial correlation over r_{ud}



- $r_{ud} = 0$ normalised, offset for each m_π
- all R shown simultaneously
- combined fits over $\forall R$ with shared r_0

Diquark shape





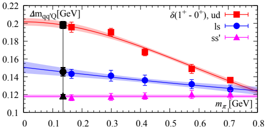
Radial/Tangential size results at $m_\pi = 575$MeV.

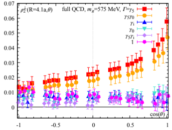
- Get radial and tangential radii $r_0^{||}$, $r_0^\perp$
- Ratio $r_0^\perp / r_0^{||}$ sensitive to distortions
  - $= 1$, spherical
  - $\neq 1$, prolate/oblate
- Ratio $\simeq 1$ for all $m_\pi \Rightarrow$ spherical
- Consistent w/ scalar, $J = 0$, shape
- No diquark polarization through Q observed

Going further

Good diquarks? - Static spectator study establishes diquarks in isolation

- Observe special GDQ status in spectrum
- Access GDQ attraction, radius and shape

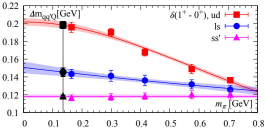


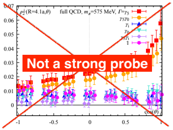


Heavy diquarks? - Study HQSS through spectrum with static spectator

- Study HQSS in hadrons
- Use $qqQ - qQ$ as probe
- Attraction not expected

first, preliminary results

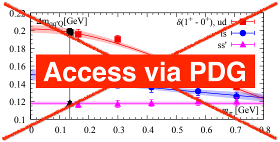


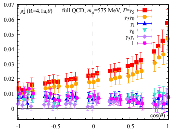


Heavy quark - diquark interaction? - Study attraction with non-static heavy quark

- Study diquark polarization
- Reduce m_Q, here $m_Q \simeq m_s$
- Weaker attraction expected

new work in progress





Testing HQSS

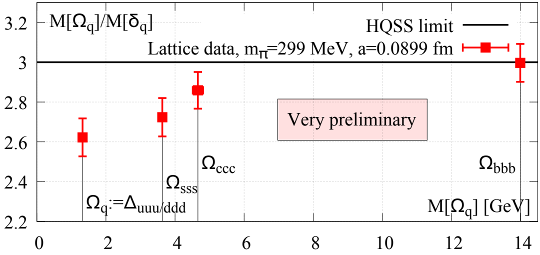
Very preliminary

- Consider the difference between the $qq'Q$ baryon and the $q\bar{Q}$ meson

$$\delta_q(t) = C^{qqQ}_{\gamma_{ii}}(t) - C^{q\bar{Q}}_{\gamma_i}(t) \quad \rightsquigarrow \text{ determine on the lattice: } M[\delta_q]$$

where $q \in l, s, c$. Then relate it to the corresponding Ω-type baryon:

$$M[\Omega_q]/M[\delta_q] \sim M[qqq]/(M[qqQ] - M[q\bar{Q}]) \quad \rightsquigarrow \text{ in the HQSS limit: } M[\Omega_q]/M[\delta_q] = 3$$

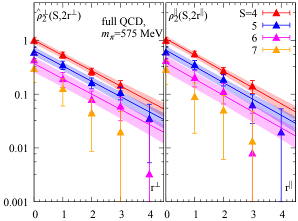


- At $m_\pi = 299$ MeV we find (low statistics)
  - HQSS expectation well fulfilled for *b*-type quark masses
  - Deviation for *c*-type quark masses and lower

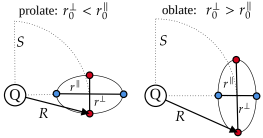
Technical note: *b*-type calculation performed using NRQCD, masses determined via dispersion in $\vec{p}^2$.

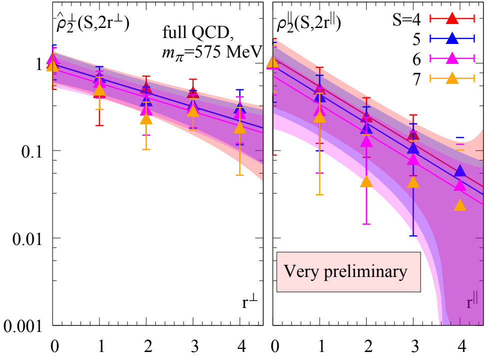
Testing diquark shape with lighter spectator quarks

Very preliminary



- Recall: Ratio $r_0^{\perp}/r_0^{||}$ sensitive to distortions
- Q =static quark: $r_0^{\perp} \simeq r_0^{||}$





Radial/Tangential size results at m_π = 575MeV where Q=strange quark.

- Replace static quark with strange quark
- Low statistics calculation at m_π = 575 MeV
- Preliminary finding: $r_0^{\perp} \neq r_0^{||}$
- Indeed: $r_0^{||} < r_0^{\perp}$ could indicate "screening" or suppression of diquark attraction(?)
  $\Rightarrow$ More work required!

see also [hep-lat/9208025], [0902.4046]

Summary - Diquarks on the lattice

Gauge invariant approach to diquarks in $n_f = 2 + 1$ lattice QCD

- Lattice setup with short chiral extrapolations, continuum limit still required

Diquark spectroscopy

- Special status of "good" diquark confirmed, attraction of 198(4)MeV over "bad"
- Chiral and flavor dependence modelled through simple Ansatz
- Very good agreement with phenomenological estimates

Diquark structure

- $q - q$ attraction in good diquark induces compact spatial correlation
- Good diquark size $r_0 \simeq \mathcal{O}(0.6)\text{fm} \sim r_{\text{meson, baryon}}$, weakly m_π dependent
- Good diquark shape appears nearly spherical

New work and outlook

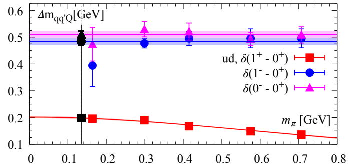
- First results on testing HQSS and its dependence on the quark mass.
- Effort to understand diquark-quark interactions via spatial correlations.
- Insights for studies of e.g. exotic tetraquarks (esp. doubly heavy)?
- Perhaps a new glimpse of the internal workings of a baryon?

Thank you for your attention.

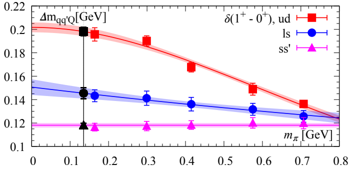
Further material

Lattice spectroscopy - diquark-diquark differences

ud 0^+ versus 1^+, 0^- and 1^-



$(1^+ - 0^+)_{qq'}$ splitting



We consider mass differences of $qq'Q$ baryons:

$$\boxed{C_\Gamma^{qq'Q}(t) - C_{\gamma_5}^{qq'Q}(t)}$$

$\rightsquigarrow$ Q drops out

$\rightsquigarrow$ measures diquark-diquark mass difference

Bad-good diquark splitting:

- Special status of good diquark observed
- Good 0^+ *ud* diquark lies lowest in the spectrum
- Bad 1^+ *ud* diquark 100-200 MeV above
- 0^- and 1^- *ud* diquarks $\sim$ 0.5 GeV above
- Pattern repeated in ℓs and ss'

$\Delta m_{qq'Q}(m_\pi)$ dependence:

- Chiral limit: $\sim$ const
- Heavy-quark limit: decreases $\sim 1/(m_{q_1} m_{q_2})$, with $m_\pi \sim (m_{q_1} + m_{q_2})$

$$\boxed{\delta(1^+ - 0^+)_{q_1 q_2} = A / \left[1 + (m_\pi/B)^{n \in 0,1,2}\right]}$$

We consider mass differences of a $qq'Q$ baryon and a light-static meson:

$$C^{qq'Q}_{\Gamma=\gamma_5}(t) - C^{q'\bar{Q}}_{\gamma_5}(t)$$

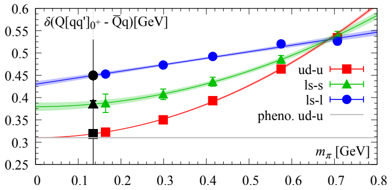
$\rightsquigarrow$ Q drops out
$\rightsquigarrow$ diquark-quark mass difference

$\Delta m_{qq'Q}(m_\pi)$ dependence:

- Chiral vs. heavy-quark limiting behaviours, as before

$$\delta(Q[q_1 q_2]_{0^+} - \bar{Q}q_2) = C\ \left[1 + (m_\pi/D)^{n\in 0,1,2}\right]$$

$Qqq' - \bar{Q}q'$ splittings



Diquark-quark splitting:

- Established mass differences between a good diquark and an [anti]quark
- May prove useful in identifying favorable tetra-, pentaquark channels
- Omits possible distortions through additional light quarks, Pauli-blocking, spin-spin interactions ...

Diquark spectroscopy - phenomenological estimates

We want to compare our results with phenomenology

- Key resource: (Jaffe '05, arXiv:hep-ph/0409065), updated with PDG 2021 input
- For pheno estimates use charm and bottom hadron masses where leading $\mathcal{O}(1/m_Q)$ $(Q = c, b)$ can be cancelled

Four estimates considered:

- $\delta(1^+ - 0^+)_{ud}$: $\boxed{\frac{1}{3}\left(2M(\Sigma_Q^*) + M(\Sigma_Q)\right) - M(\Lambda_Q)}$
- $\delta(1^+ - 0^+)_{us}$: $\boxed{\frac{2}{3}\left(M(\Xi_Q^*) + M(\Sigma_Q) + M(\Omega_Q)\right) - M(\Xi_Q) - M(\Xi'_Q)}$
- $\delta(Q[ud]_{0^+} - \bar{Q}u)$: $\boxed{M(\Lambda_Q) - \frac{1}{4}\left(M(P_{Qu}) + 3M(V_{Qu})\right)}$

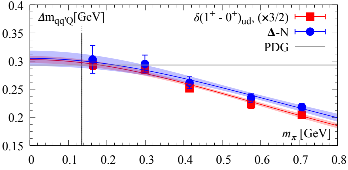
  $\rightsquigarrow$ P_{Qu}, V_{Qu} are the ground-state, heavy-light mesons
- $\delta(Q[us]_{0^+} - \bar{Q}s)$:

$$\boxed{M(\Xi_Q) + M(\Xi'_Q) - \frac{1}{2}(M(\Sigma_Q) + M(\Omega_Q)) - \frac{1}{4}(M(P_{Qs}) + 3M(V_{Qs}))}$$

$\rightsquigarrow$ P_{Qs}, V_{Qs} are the ground-state, heavy-strange mesons

Δ-Nucleon mass difference

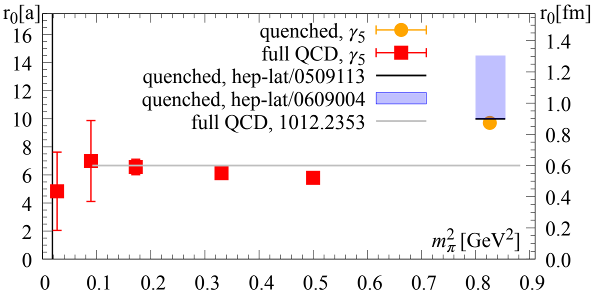
$[\Delta - N](m_\pi)$



Measured the mass difference of $\Delta - N$

- Prediction: $\delta(\Delta - N) = 3/2 \times \delta(1^+ - 0^+)_{ud}$
- Same Ansatz as before
- Prediction holds well, even at fairly large m_π

Size dependence $r_0(m_\pi)$



Good diquark size:

- Agreement w/ prev. quenched and dynamical
- Refinement through our results
- $r_0 \simeq \mathcal{O}(0.6)$fm weak m_π dependence
  $\rightsquigarrow \sim r_{\text{meson, baryon}}$, arXiv:1604.02891

$r_0(m_\pi)$ dependence:

- $m_{q,q'} \uparrow$ should produce more compact object
- But, diquark attraction$\downarrow$ works opposite
- Former effect dominates at large m_π?
- But, in quenched diquarks definitely larger...

Review of doubly heavy tetraquarks in lattice QCD

Confirm and predict doubly heavy tetraquarks non-perturbatively

Tetraquarks as ground states? What would their binding mechanism/properties be?

HQS-GDQ picture, consequences for $qq'\bar{Q}'\bar{Q}$ tetraquarks:

- $J^P = 1^+$ *ground state* tetraquark below meson-meson threshold
- Deeper binding with heavier quarks in the $\bar{Q}'\bar{Q}$ diquark
- Deeper binding for lighter quarks in the qq' diquark

Ideal for lattice: Diquark dynamics and HQS could enable $J^P = 1^+$ *ground state doubly heavy tetraquarks* with flavor content $qq'\bar{Q}\bar{Q}'$.

Goal: $\Delta E = E_{\mathrm{tetra}} - E_{\mathrm{meson-meson}}$, e.g. in $bb\bar{u}\bar{d}$, $bb\bar{\ell}\bar{s}$ and others
$\Rightarrow$ Verify, quantify predictions of binding mechanism in mind.

Lattice point of view

- Hidden flavor $qQ\bar{q}'\bar{Q}$ are tetraquark candidates as excitations of $Q\bar{Q}'$.
  $\rightsquigarrow$ technical difficulty for lattice calculations, need to resolve many f.vol states.
  $\rightsquigarrow$ $qq'\bar{Q}\bar{Q}'$, i.e. ground state candidates would be better to handle.

In the following

- Tetraquarks with two heavy (c, b) and two light (ℓ, s) quarks.
- Lattice evidence for $bb\bar{u}\bar{d}$, $bb\bar{\ell}\bar{s}$.
- Recent updates on systematics.
- Survey of candidates status.

Lattice tetraquarks - 4 main approaches

1. Static quarks ($m_Q = \infty$)
Fitted potentials used to predict bound states and resonances.
∘ Allows for potential formulation.
∘ Ansatz fitted to lattice data.
∘ Plug into Schrödinger Eq. for E_n.

$\rightsquigarrow b b \bar{u} \bar{d}$, Bicudo et al. ('17,'19)

3. Finite volume energy levels
Lattice energies equated to (un)observed states.
∘ Operator matrix (GEVP) gives $\lambda_i \propto E_i$
$\Rightarrow$ Finite volume states.
∘ Binding? Get $\Delta E = E_0 - E_{thresh}$.
∘ Mechanism? Vary quark masses.

$\rightsquigarrow$AF et al. ('17,'18, '20), Hughes et al. ('17), Junnarkar et al. ('18), Leskovec et al. ('19), Mohanta et al. ('20)

2. HAL QCD method
Lattice potentials studied for scattering properties.
∘ Expansion of energy dependent potential (systematics?).
∘ Method under debate, best motivated for heavy systems.

$\rightsquigarrow$HAL QCD ('16,'18)

4. Scattering analysis
Lattice energies studied in terms of scattering phase shifts.
∘ Excited state energies via GEVP.
∘ Analyse fvol spectrum $\Rightarrow$ Resonant, bound, virtual bound, free.

$\rightsquigarrow$Hadron Spectrum Coll. ('18,'20)

Lattice tetraquarks - 4 step recipe

The main tool is to adopt a variational approach

Lattice GEVP gives access to finite volume energy states (masses, overlaps).

Beware: Operator overlaps do not necessarily connect to the naively expected structures. Be careful when equating lattice correlators with trial-wave functions.

Step I: Set up a basis of operators, here $J^P = 1^+$

Diquark-Antidiquark:

$$D = \left((q_a)^T (C\gamma_5) q'_b \right) \times \left[\bar{Q}_a (C\gamma_i)(\bar{Q}'_b)^T - a \leftrightarrow b \right]$$

Dimeson:

$$M = (\bar{b}_a \gamma_5 u_a)\,(\bar{b}_b \gamma_i d_b) - (\bar{b}_a \gamma_5 d_a)\,(\bar{b}_b \gamma_i u_b)$$

Step II: Solve the GEVP and fit the energies

$$F(t) = \begin{pmatrix} G_{DD}(t) & G_{DM}(t) \\ G_{MD}(t) & G_{MM}(t) \end{pmatrix}, \quad F(t)\nu = \lambda(t) F(t_0)\nu \ ,$$

$$G_{\mathcal{O}_1\mathcal{O}_2} = \frac{C_{\mathcal{O}_1\mathcal{O}_2}(t)}{C_{PP}(t)C_{VV}(t)} \ , \ \lambda(t) = A e^{-\Delta E (t-t_0)} \ .$$

$\rightsquigarrow \Delta E = E_{\text{tetra}} - E_{\text{thresh}}$ in case of binding correlator $(C_{\mathcal{O}_1\mathcal{O}_2}(t))/(C_{PP}(t)C_{VV}(t))$.

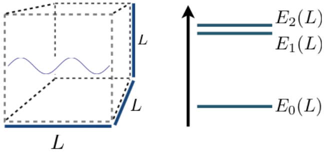
Most use these operators, but a larger basis has been worked out.
$\Rightarrow$ Need to be used by more groups. $\rightsquigarrow$ HadronSpectrum Coll. ('17)

Step III: Finite volume corrections

Large energy shifts are possible due to the finite lattice volume.

Scenario I: Scattering state
The finite volume energy belongs to a scattering state, the corrections go as

$$E_{b,L} \sim E_{b,\infty} \cdot \left[1 + \frac{a}{L^3} + \mathcal{O}(\frac{1}{L^4})\right]$$



⇝ M. Hansen

Scenario II: Stable state
The corrections are exponentially suppressed with $\kappa = \sqrt{E_{b,\infty}^2 + p^2}$

$$E_{b,L} \sim E_{b,\infty} \cdot \left[1 + Ae^{-\kappa L}\right]$$

With a single volume available:

- In a bound state corrections are $\sim$ exp(binding momentum) ⇝ strong supp. m_{had} =heavy
- In a scattering state expect large deviation around threshold

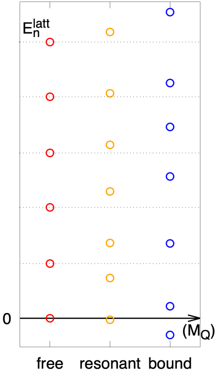
With multiple volumes available:

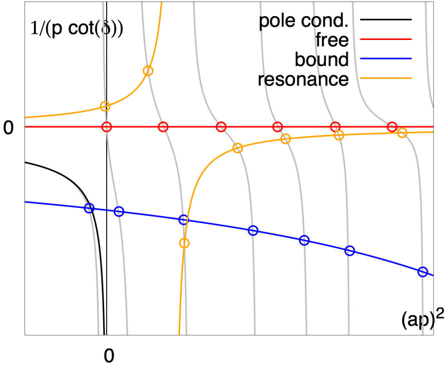
- Track mass dependence ⇝ decide bound/scatt. state
- Power law corrections might be too small to resolve

Step IV: Finite volume / Scattering analysis

Limitation: Small GEVP without f.vol analysis ok for deeply bound states.
Insufficient to tell apart free, resonant or virtual bd. states.

Extension: Connect energies to scattering phase shifts via finite volume quantisation conditions (Lüscher-formalism).



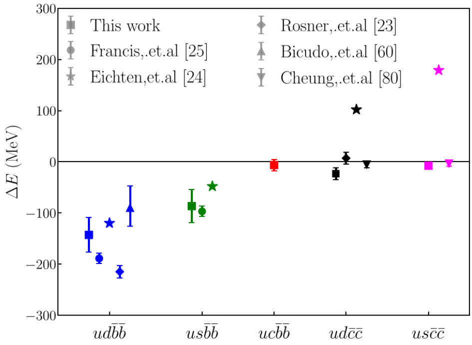


- ◦ connect (many) f.vol states to scattering parameters (sketch: BW)
- ◦ resonance: extra state(s) appear, lowest state close to threshold

What we know: A review of recent lattice studies

What we know: Deeply bound $J^P = 1^+$ $bb\bar{u}\bar{d}$ and $bb\bar{\ell}\bar{s}$ tetraquarks

Community overview



⇝Mathur et al. ('19)

Qualitative agreement with pheno

○ All three predictions met:
→ $J^P = 1^+$ bound ground state.
→ deeper binding with $m_Q \uparrow$.
→ deeper binding with $m_q \downarrow$.

○ $bb\bar{q}\bar{q}'$ are a focal point → All efforts observe deeply bound $bb\bar{u}\bar{d}$

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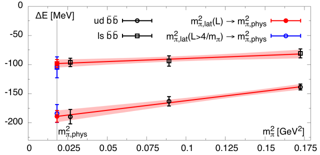
AF et al. ('17)



· Junnarkar, Mathur, Padmanath ('18)
· Leskovec, Meinel, Plaumer, Wagner ('19)
· HadronSpectrum Coll. ('17)
· Mohanta, Basak ('20)
· Colquhoun, AF, Hudspith, Lewis, Maltman ('17, '18, '20)

Overview -possible doubly heavy tetraquark candidates

Surveying candidates

observed (>1 group)
no deep binding
observed (1 group)
not confirmed (>1 group)

channel	deeply bound
$J^P = 1^+$	$bb\bar{u}\bar{d}$ $bc\bar{u}\bar{d}$
	$bb\bar{\ell}\bar{s}$ $bc\bar{\ell}\bar{s}$
	$bs\bar{u}\bar{d}$ $cs\bar{u}\bar{d}$
	$bb\bar{u}\bar{c}$ $bb\bar{s}\bar{c}$
	$cc\bar{u}\bar{d}$ $cc\bar{\ell}\bar{s}$
	$bb\bar{b}\bar{b}$
$J^P = 0^+$	$bb\bar{u}\bar{u}$ $cc\bar{u}\bar{u}$
	$bb\bar{u}\bar{d}$ $bc\bar{u}\bar{d}$
	$bb\bar{\ell}\bar{s}$ $bc\bar{\ell}\bar{s}$
	$bb\bar{s}\bar{s}$ $cc\bar{s}\bar{s}$
	$bs\bar{u}\bar{d}$ $cs\bar{u}\bar{d}$
	$bb\bar{u}\bar{c}$ $bb\bar{s}\bar{c}$
	$bb\bar{c}\bar{c}$ $cc\bar{u}\bar{d}$
	$bb\bar{b}\bar{b}$

Deeply bound states

Focus: strong interaction stable

→ $bb\bar{u}\bar{d}$ and $bb\bar{\ell}\bar{s}$ in $J^P = 1^+$.
→ $cc\bar{q}\bar{q}'$ not deep.
→ $bc\bar{q}\bar{q}'$ not clear.
→ further candidates not observed.
→ none observed in $J^P = 0^+$.

⇝Bicudo et al. ('17), AF et al. ('17,'18, '20), HadSpec Coll. ('18), Hughes et al. ('17), Junnarkar et al. ('18), Leskovec et al. ('19), Mohanta et al. ('20)

States above threshold, resonances?

→ $bb\bar{u}\bar{d}$ in $J^P = 1^+$ /w static quarks find a resonance just above threshold. ⇝Bicudo et al. ('19)
→ No results from other approaches.
→ What about $cs\bar{u}\bar{d}$?
⇝ under investigation Hudspith, AF et al.('20), HadSpec ('20)

Shallow binding?

∘ $cc\bar{u}\bar{d}$ now observed by LHCb, robust lattice post-diction?
→ Work to remove current limitations.