High-Fidelity Controlled-Phase Quantum Gate with Rydberg Atoms

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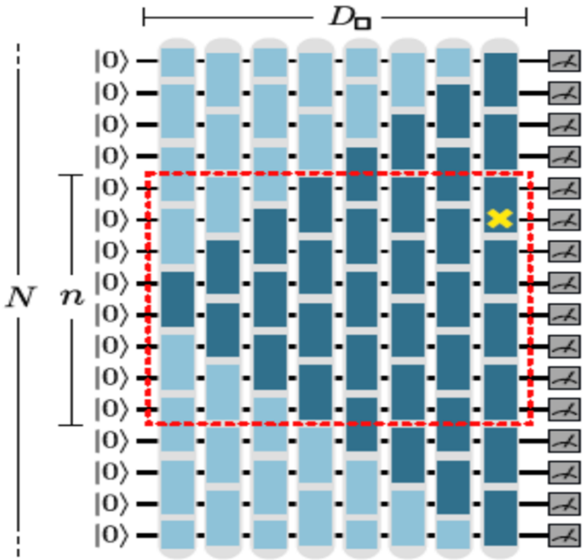
Some Metrics of Quantum Computation

Three parameters

Circuit Depth D ($\propto T_{coherence}/T_{gate}$)

Circuit Width n (Qubit number)



- Gate fidelity F

(gate error ε=1-F)

Three stages

Scalable, Universal Quantum Computers

- More qubits (e.g. 10^6)
- Better gate fidelity
- Error correction

Noisy, Intermediate-Scale Quantum (NISQ) Devices (n=20-200, $\varepsilon\approx0.01$)

- Individual qubit control
- Multi-qubit processing

Analog Quantum Devices (Hamiltonian engineering)

Cirac, Nanophotonics 10, 453(2021)

Three Milestones

Demonstrating large-scale quantum computing algorithms (e. g. Shor, Grover)

Demonstrating Quantum Error Correction & Fault-tolerant QC: reaches a better fidelity with QEC than without QEC

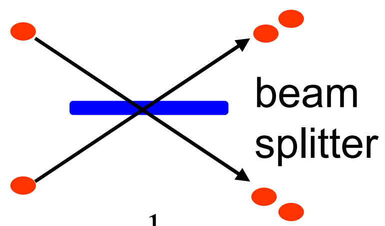
Demonstrating Quantum Supremacy (computational advantage) for not useful and then **useful** problem

About Quantum Computational Advantage (Supremacy)

- Experimental realization of a computational task, which might not be useful, that cannot be solved in a reasonable amount of time by any classical means (e.g. classical supercomputer).

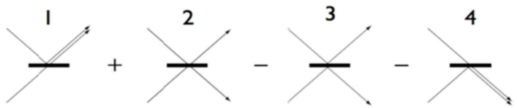
Ingredient 1: $|\psi\rangle = (|0\rangle + |1\rangle)^n$ n qubits $\Rightarrow$ 2^n degree of freedom , 2^{300}~$2\text{x}10^{900}$ ~ atoms in the universe

Ingredient 2: quantum entanglement, beam splitter in linear optics or two-qubit gates
Sampling problems: the raw output of a quantum processor is a sample from a probability distribution resulting from a measurement.

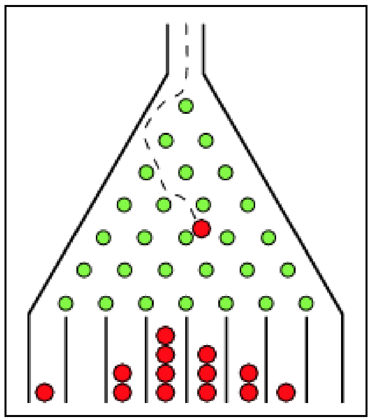


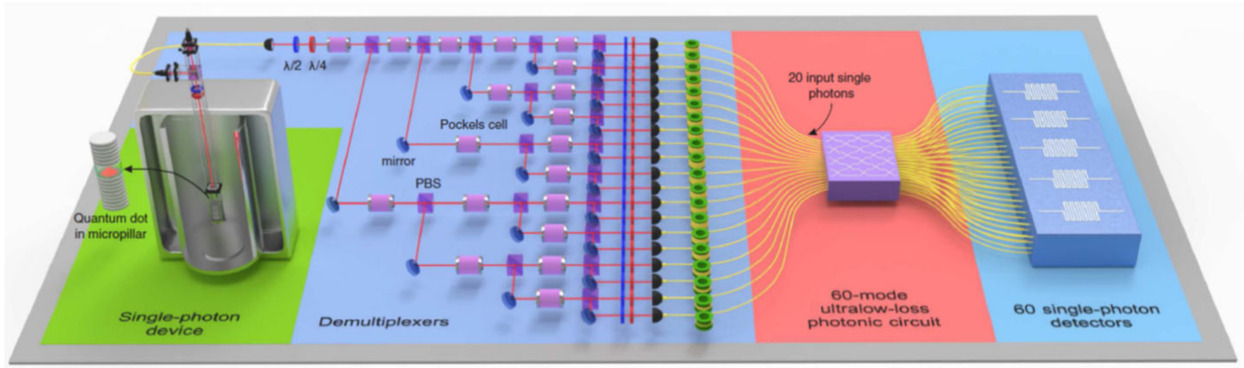
$$|\Psi\rangle_{out} = \frac{1}{\sqrt{2}}(|2,0\rangle - |0,2\rangle)$$

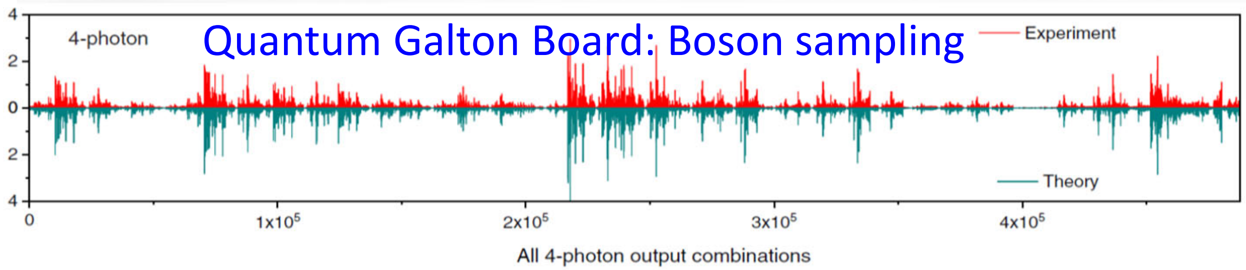
Hong-Ou-Mandel Interference



Galton Board



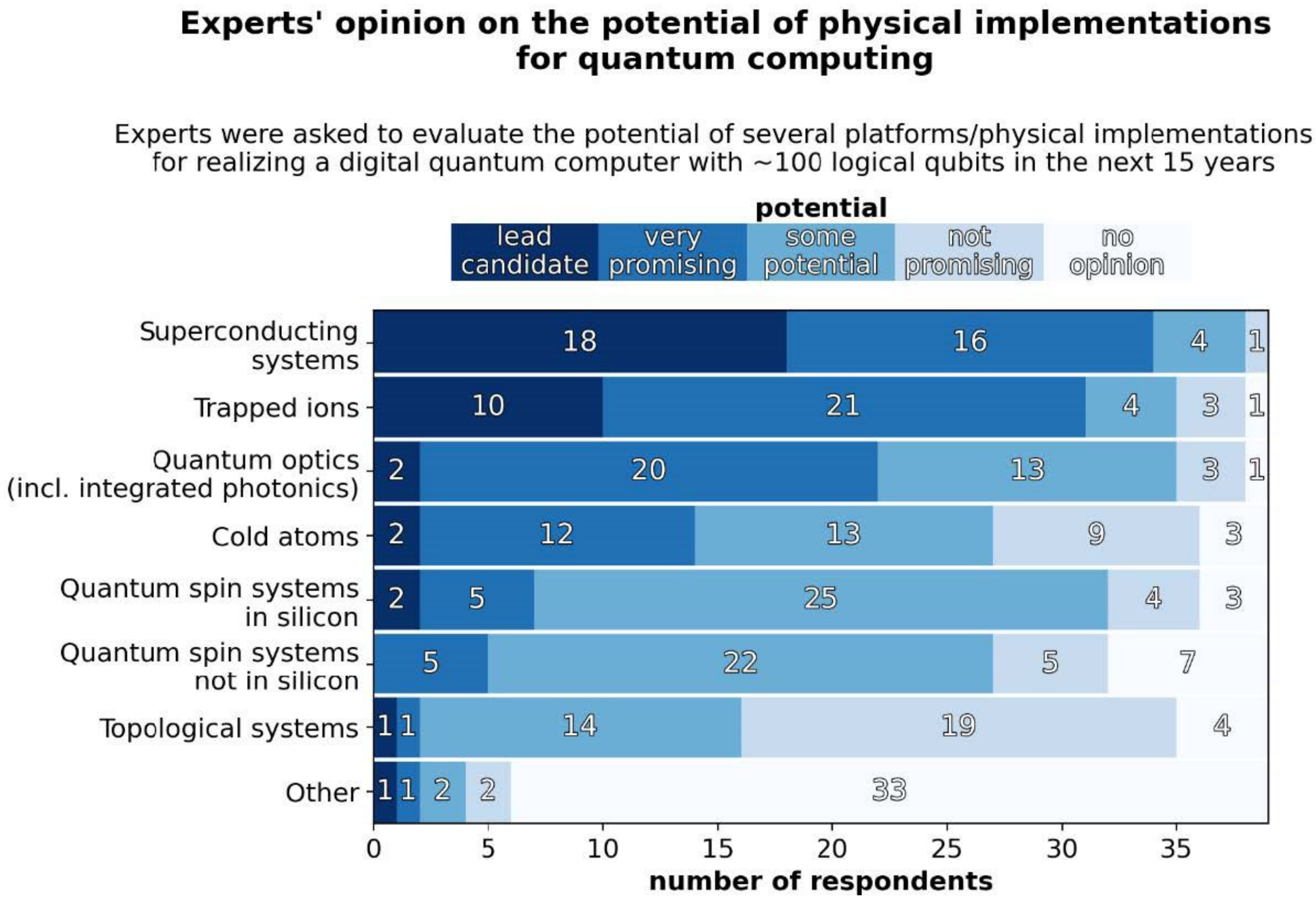




Quantum Galton Board: Boson sampling

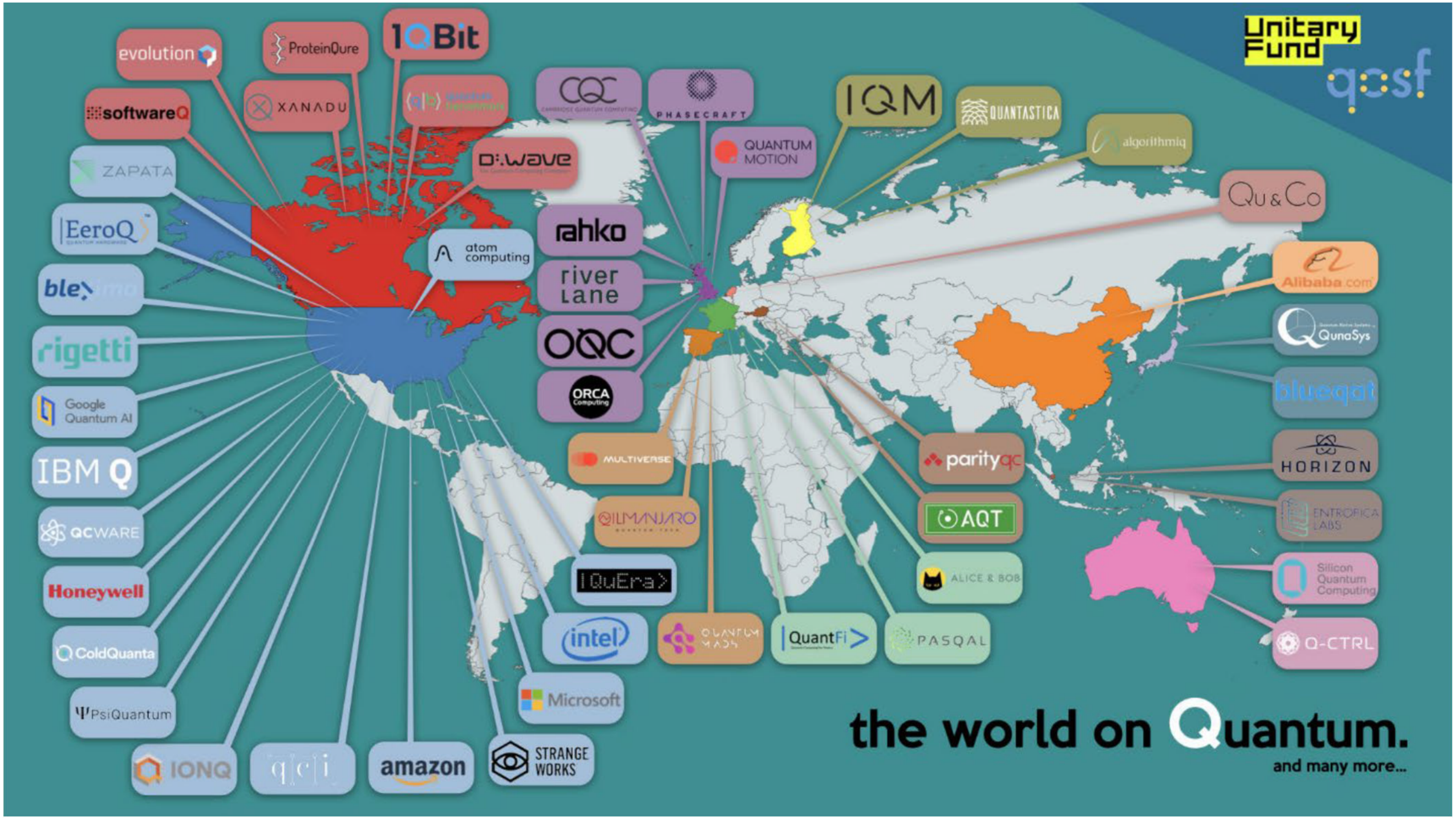
J. Preskill, arXiv: 1203.5813; S. Aaronson et al., arXiv 1011.3245; H. Wang et al., PRL 123,250503(2019); H.S. Zhong et al., Science 370, 1460(2020)

Quantum Processor : Different Physical Platforms



Quantum Threat Timeline Report 2021, Global Risk Inc.

Quantum Technology Companies



Quantum Threat Timeline Report 2021, Global Risk Inc.

Personal Viewpoint on "Second Quantum Revolution"

To gain Quantum Advantage in : Sensing/Metrology, Computation, Simulation, Communication (Security)…

Comes from

Quantum Properties : Entanglement, Coherent Superposition, Squeezing, Quantum Correlation, Nonlocality…

Requires active efforts on …

Strong & controlled atom-atom/atom-photon Interactions,
Precise state preparation and detection…

Fight against

Noise, loss, decoherence, cross-talk…
absorption, spontaneous decay,
environmental perturbations,
finite temperature…

Pursuit for Perfection, which will benefit all fields!

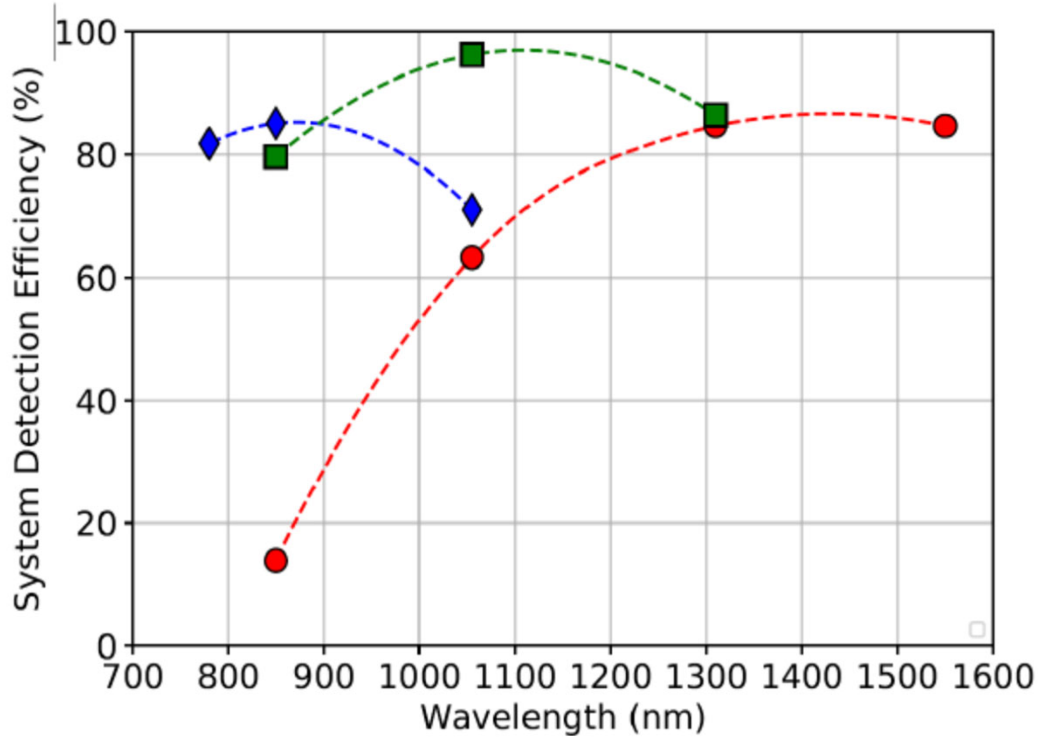
A driving force on advanced technologies and matter control.

Superconducting Nanowire Single Photon Detector

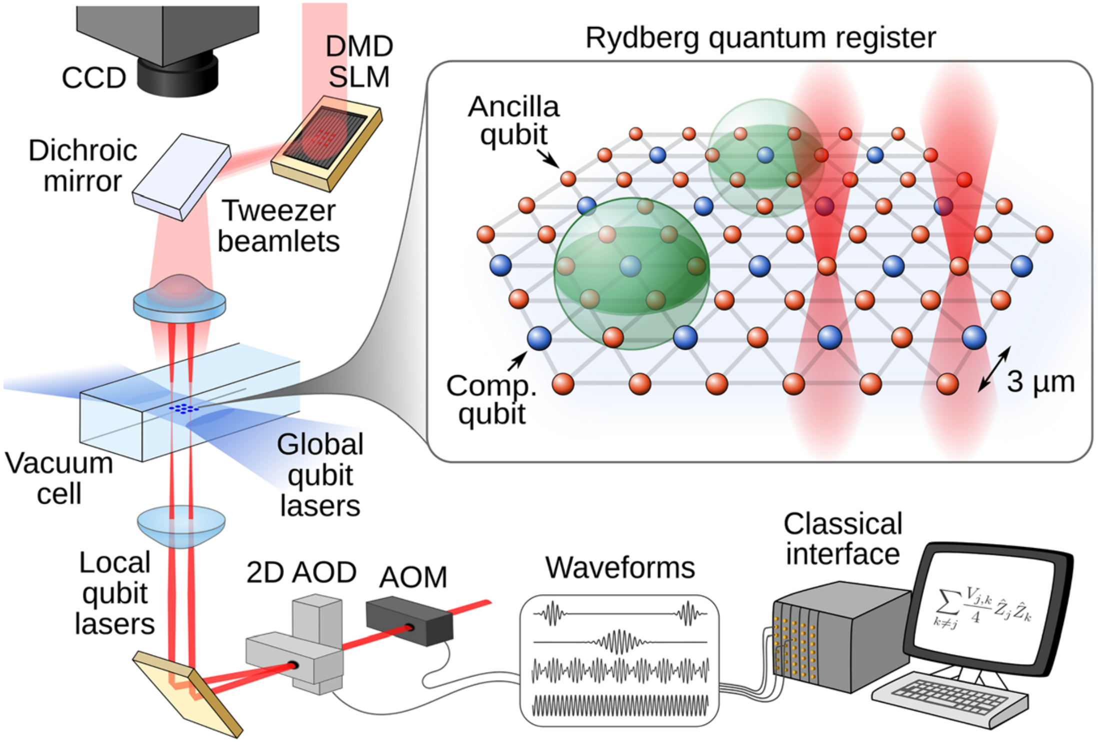
Pursuit for perfection:

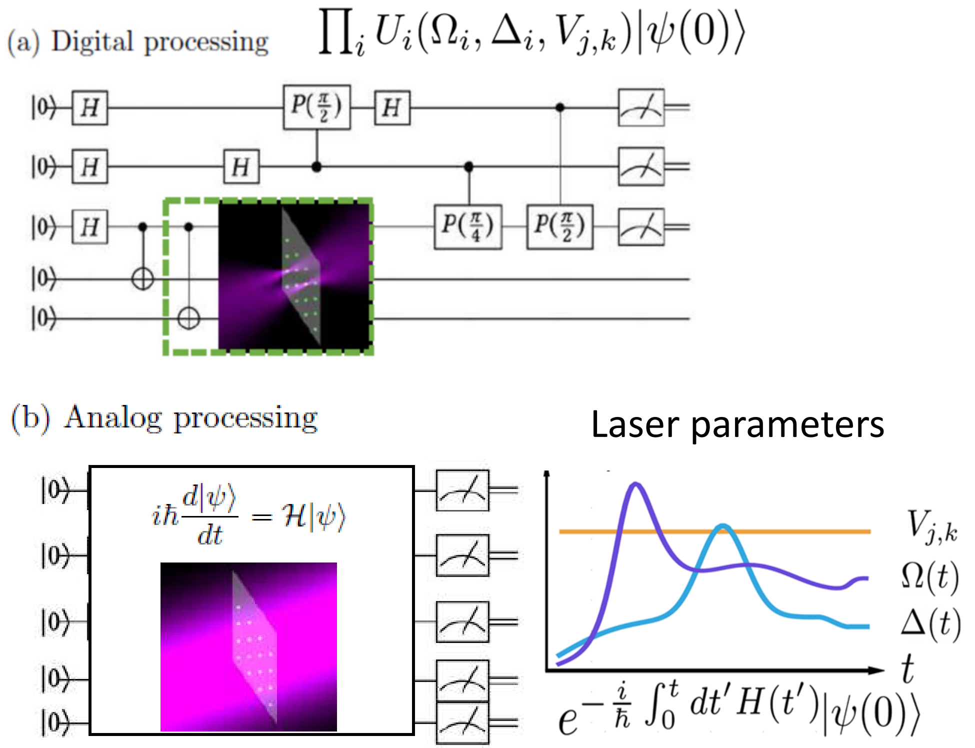
good for quantum technology,
but also benefit biology, chemistry …etc.

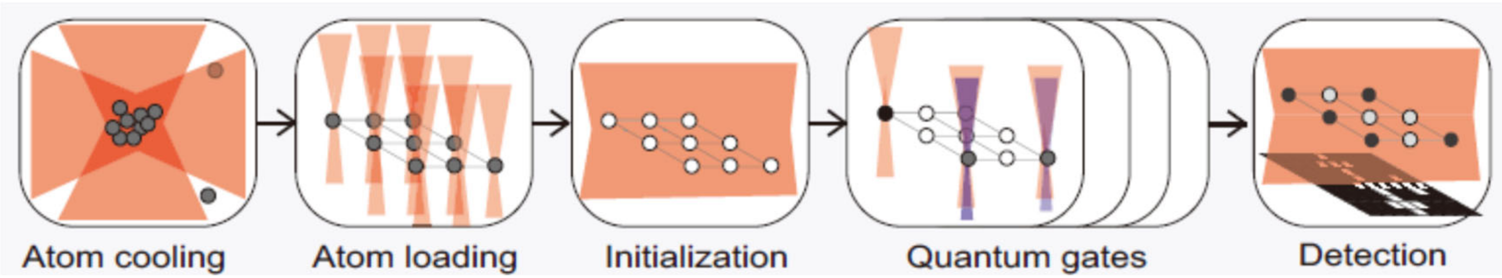
Quantum
Efficiency



Overview of a Atom-Based Quantum Processor



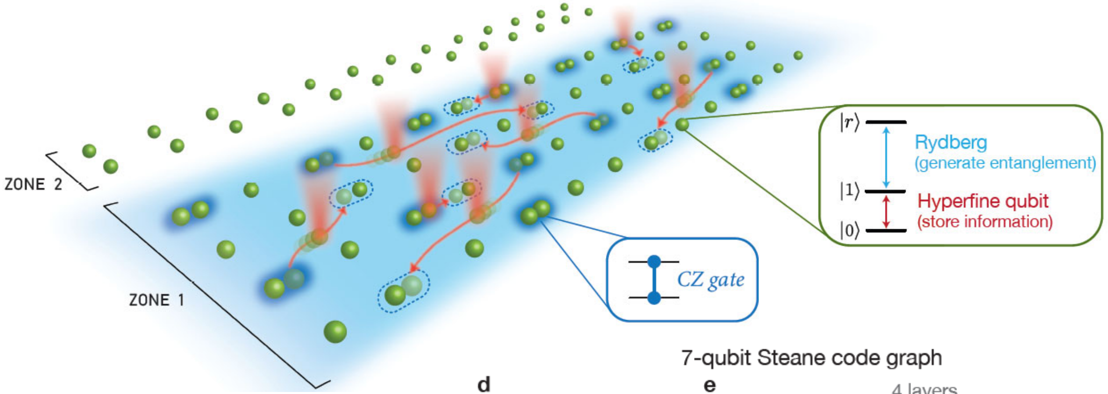


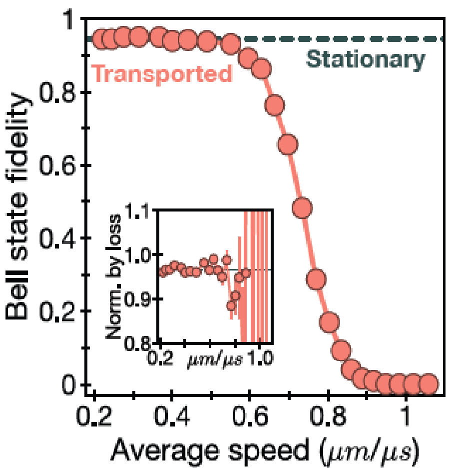


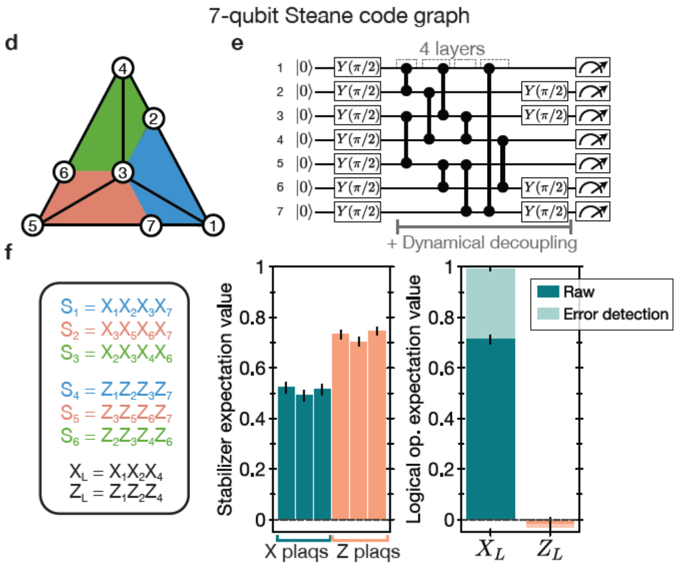
L. Henriet et al, Quantum 4, 327(2020)
M. Mrgado et al., AVS Quantum Sci. 3, 023501(2021)
X. Wu et al., Chin Phys B, 30, 020305(2021)

Recent Demo on Quantum Processor based on Neutral Atoms

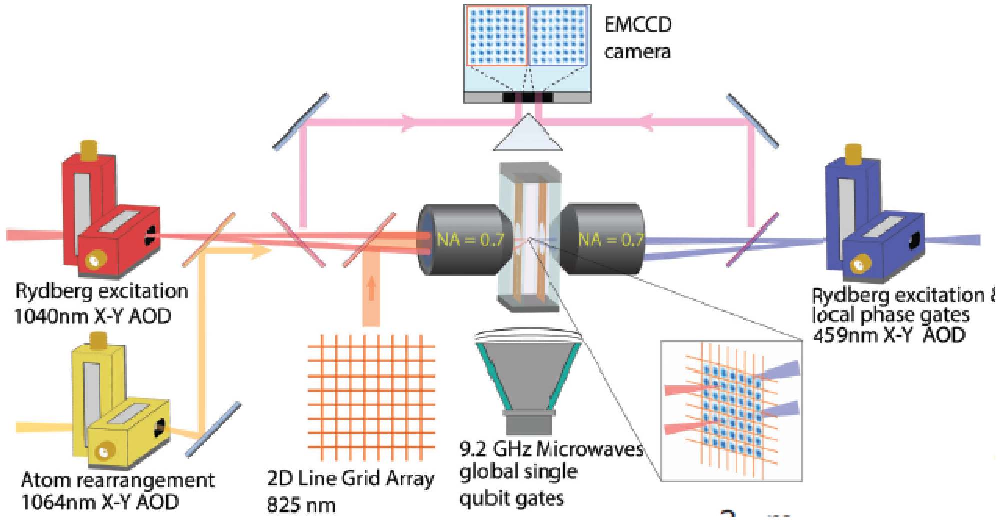
Transportable qubits, Nonlocal connectivity



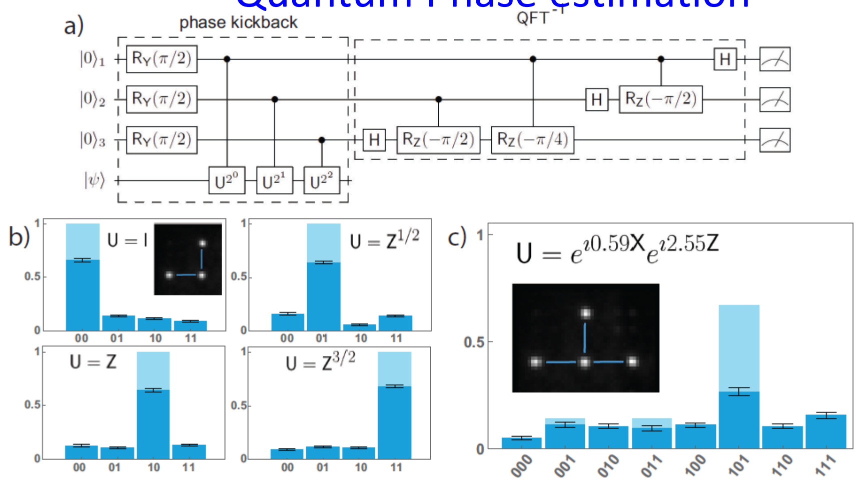




Lukin's group (Harvard), Nature 604, 451(2022)



Quantum Phase estimation



Saffman's group (U. Wisconsin-Madison), Nature 604, 457(2022)

Companies of Quantum Computer based on Neutral Atoms

Alan Aspect, Antoine Browaeys and Thierry Lahaye
at the Institut d'Optique (IOGS, CNRS)

Scientific Advisors: M Lukin, V. Vuletic, M Greiner
at Harvard/MIT University

Scientific Advisors: Jun Ye, Alexey Gorshkov at JILA/JQI

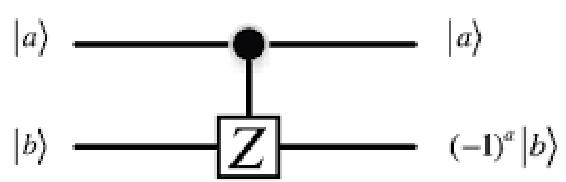
Scientific Advisors: Mark Saffman (U. Wisconsin-Madison)

Universal Quantum Gate Set for Quantum Computing

- Any quantum operation can be realized by a universal set of quantum gates.
- Arbitrary single-qubit rotation gate and phase gate, plus certain two-qubit gate (e.g. CZ or CNOT) gate form a universal gate set.
- Single-qubit gate requires precise control on atom-electromagnetic wave interaction; two-qubit gate requires precise control on atom-atom interactions

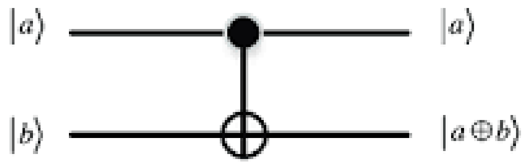
$$\mathrm{U}_{xy}(\theta,\varphi)=\begin{pmatrix}\cos(\theta/2) & -i\sin(\theta/2)e^{i\varphi}\\ -i\sin(\theta/2)e^{-i\varphi} & \cos(\theta/2)\end{pmatrix}$$

$$CZ=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{bmatrix}$$



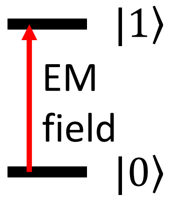
$$R_\phi=\begin{bmatrix}1&0\\0&e^{i\phi}\end{bmatrix}$$

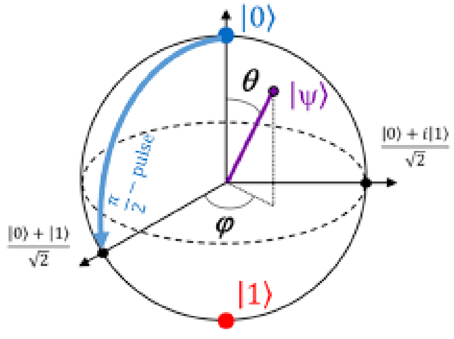
$$CNOT=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$$



Qubit and Single-Qubit Gates

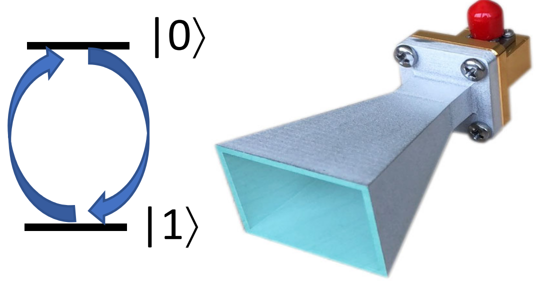
Quantum bit (qubit)





$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

Single Qubit Gates



$\Delta=0$

$$U_{xy}(\theta,\varphi) = \begin{pmatrix} \cos(\theta/2) & -i\sin(\theta/2)e^{i\varphi} \\ -i\sin(\theta/2)e^{-i\varphi} & \cos(\theta/2) \end{pmatrix}$$

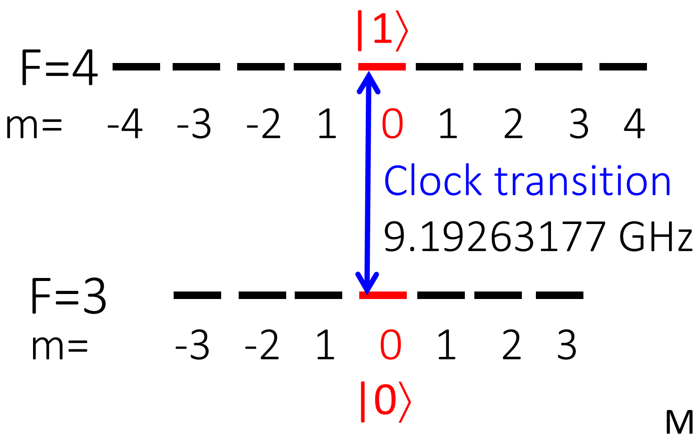
controlled by pulse area ($\theta = \Omega\tau_g$), phase (φ) and detuning (Δ)

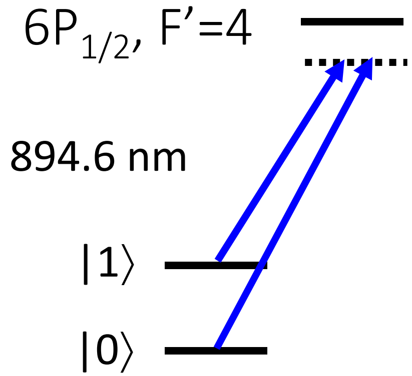
$$R_x(\theta) = U_{xy}(\theta, 0),$$

$$R_y(\theta) = U_{xy}(\theta, -\pi/2),$$

$$R_z(\theta) = U_{xy}(\pi/2, \pi/2)U_{xy}(\theta, 0)U_{xy}(\pi/2, -\pi/2).$$

^{133}Cs, $6S_{1/2}$





Raman transition with laser

Note 1: Hadamard gate

$$U_{xy}(\pi, 0)U_{xy}\left(\frac{\pi}{2}, -\frac{\pi}{2}\right) = \frac{-i}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = -i \text{ (Hadamard)}$$

Note 2: wavefunction changes a minus sign for a 2π Rabi oscillation ($\Delta=0$ case)

$$U_{xy}(2\pi, 0) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

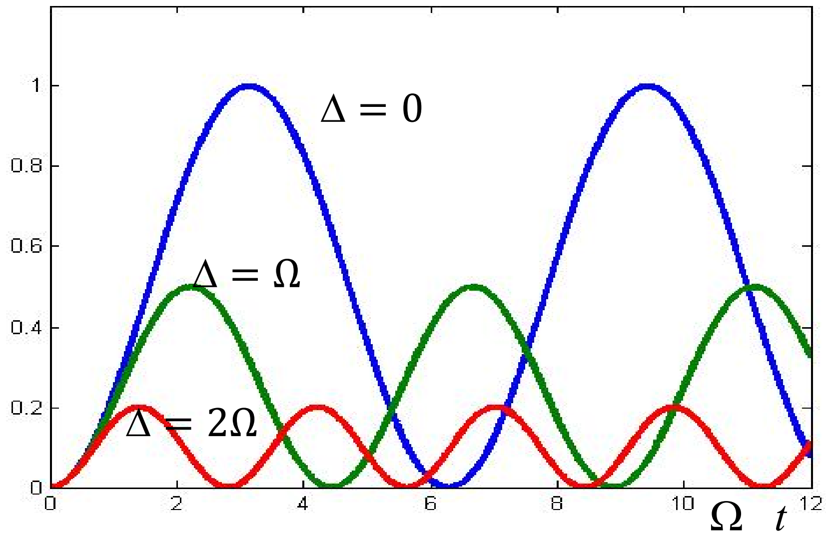
M. Mrgado et al., AVS Quantum Sci. 3, 023501(2021)

Notes on Detuned Rabi Oscillation

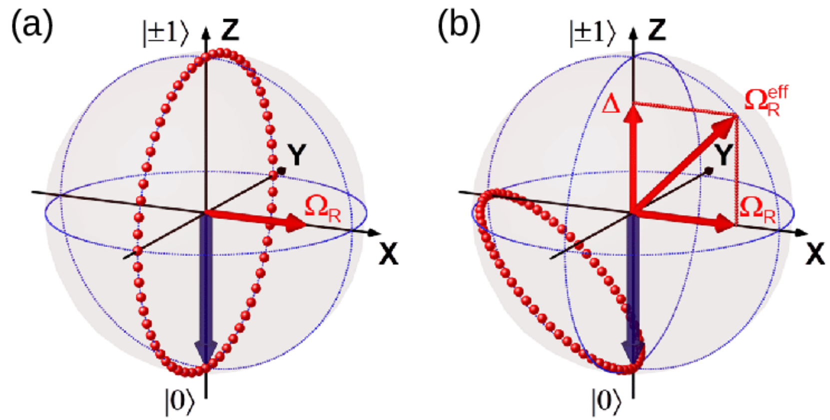
$c_1(0)=1,\ c_2(0)=0$

$$c_1(t) = \frac{e^{i\Delta\frac{t}{2}}}{\widetilde{\Omega}}[-i\Delta\sin(\frac{\widetilde{\Omega}t}{2}) + \widetilde{\Omega}\cos(\frac{\widetilde{\Omega}t}{2})]$$

$$c_2(t) = \frac{-i\Omega}{\widetilde{\Omega}}e^{-i\Delta\frac{t}{2}}\sin[\frac{\widetilde{\Omega}t}{2}]; \quad \widetilde{\Omega}=\sqrt{\Omega^2+\Delta^2}$$



M. Mrgado et al., AVS Quantum Sci. 3, 023501(2021)



$\Delta = 0$: $\Omega_R^{eff} = \Omega_R$ $\quad$ $\Delta \neq 0$: $\Omega_R^{eff} = \sqrt{\Omega_R^2 + \Delta^2}$

$$U = \exp(-i\hat{H}_j^{01}\tau_g)$$

$$= \exp\left(\frac{i\Delta\tau_g}{2}\right) \times \exp\left(-\frac{i\tilde{\Omega}\tau_g}{2}\vec{v}\cdot\vec{\sigma}\right)$$

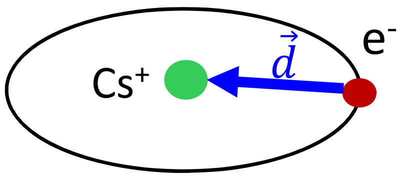
$\vec{v} = \widetilde{\Omega}^{-1}[\Omega cos\varphi, -\Omega sin\varphi, \Delta]$

$\vec{\sigma} = [\vec{X}, \vec{Y}, \vec{Z}]$ Pauli matrices

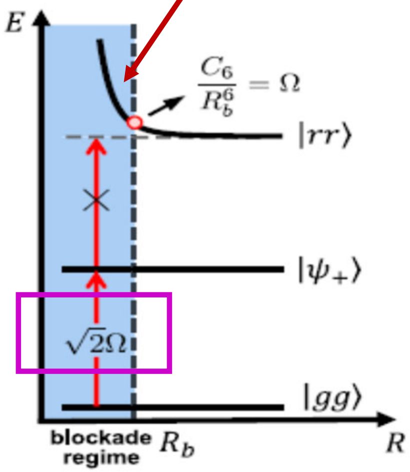
For $\Delta\neq 0$, Bloch vector is rotated by $\theta=\widetilde{\Omega}\tau_g$ around $\vec{v}$.

Two-Qubit Gates with Rydberg Interactions

Rydberg state, n≈ 60-100

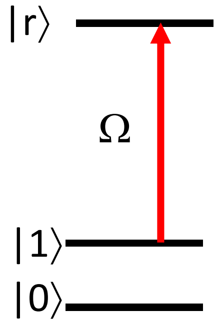


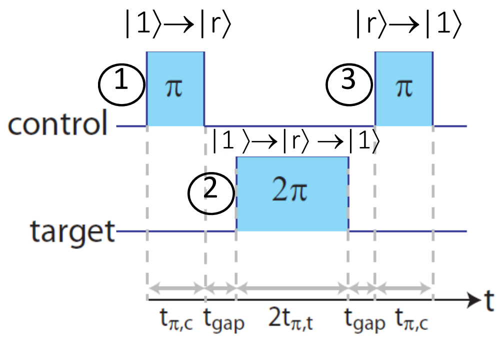
Rydberg Dipole Blockade

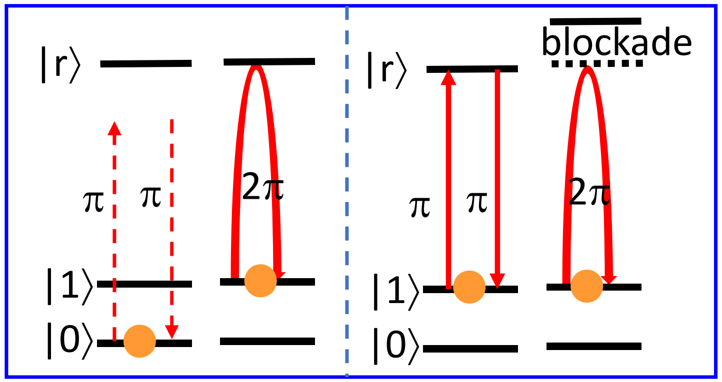


R_b: Blockade radius

$$|\psi_+\rangle = \frac{1}{\sqrt{2}}(|gr\rangle + |rg\rangle)$$







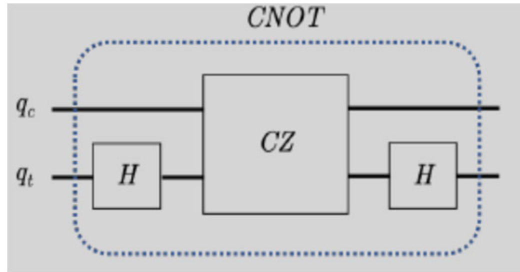
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} |0\rangle \Leftrightarrow |1\rangle \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

= (−1)CZ

Controlled-Phase (CZ) Gate

$|00\rangle \rightarrow |00\rangle$
$|01\rangle \rightarrow |01\rangle$
$|10\rangle \rightarrow |10\rangle$
$|11\rangle \rightarrow -|11\rangle = e^{i\pi}|11\rangle$

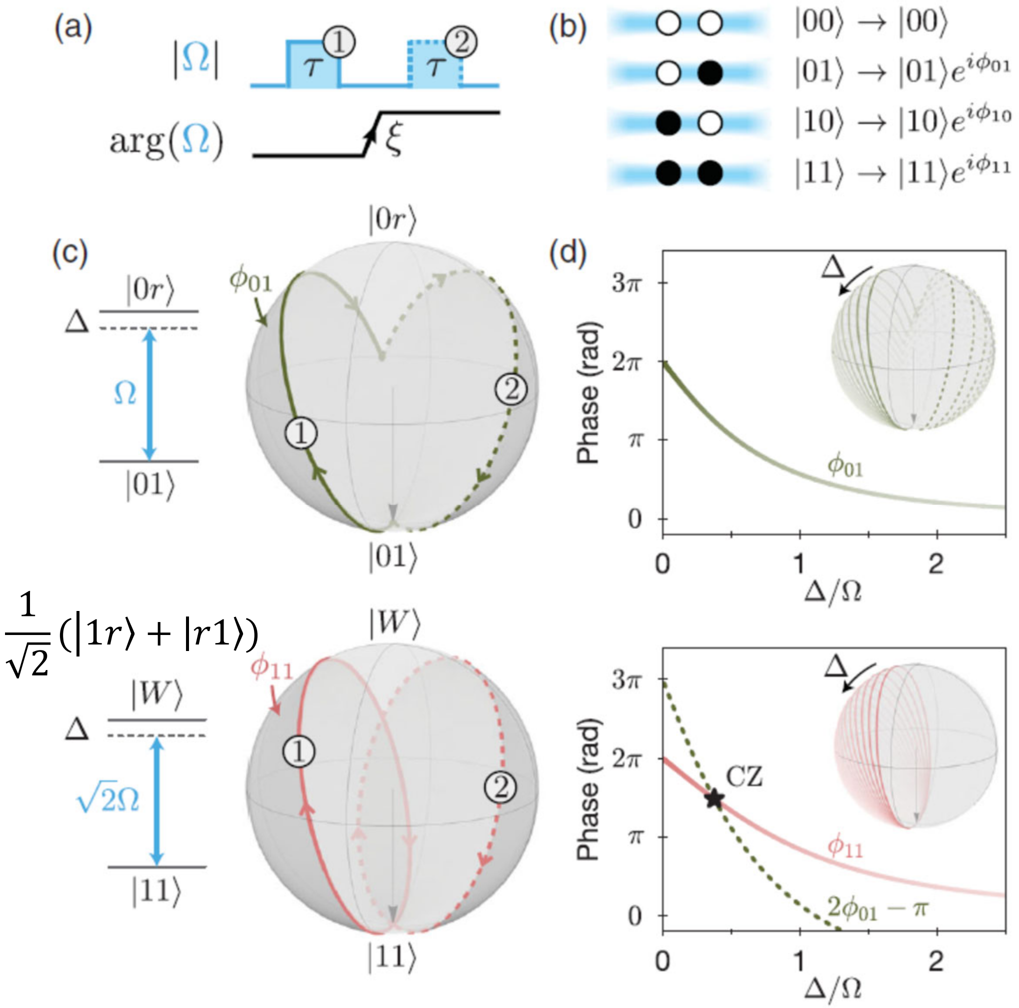
Controlled-NOT (CNOT) Gate



- Need individual addressing !
- ΩT= 4π = 12.56636

D. Jaksch et al. PRL 85, 2208(2000) ; L. Henriet et al, Quantum 4, 327(2020)

Symmetric CZ Gate with a Global Laser Pulse



H. Levine et al., PRL. 123, 170503(2019)

$|00\rangle \rightarrow |00\rangle$

$|01\rangle \rightarrow |01\rangle\, e^{i\phi_1}$

$|10\rangle \rightarrow |10\rangle\, e^{i\phi_1}$

$|11\rangle \rightarrow |11\rangle e^{i(2\phi_1+\pi)}$

CZ gate up to single-qubit phase ϕ_1

Assume perfect Rydberg blockade

- Choose τ such that $|11\rangle \rightarrow |11\rangle$ for the first pulse.
- Choose ξ such that $|01\rangle \rightarrow |01\rangle$ after two pulses.

τ constraint

$$\tau\sqrt{\Delta^2 + 2\Omega^2} = 2\pi \Rightarrow \tau = \frac{2\pi}{\sqrt{\Delta^2 + 2\Omega^2}}$$

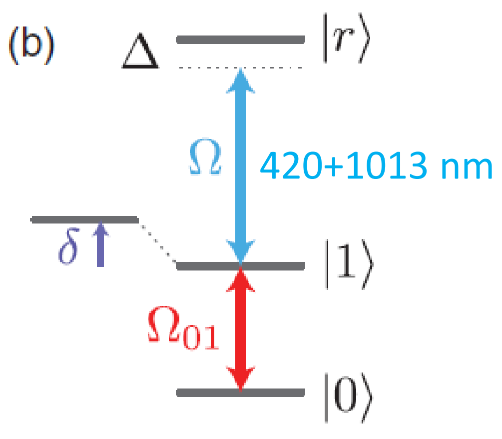
$$\phi_{11} = 2\left(\pi + \frac{\Delta\tau}{2}\right) = 2\,\pi + \frac{2\pi\Delta}{\sqrt{\Delta^2+2\Omega^2}}$$

ξ constraint $\xi(\Delta/\Omega, \Omega\tau)$ can be determined
Then $\phi_{01}(\Delta/\Omega, \Omega\tau)$ can be determined

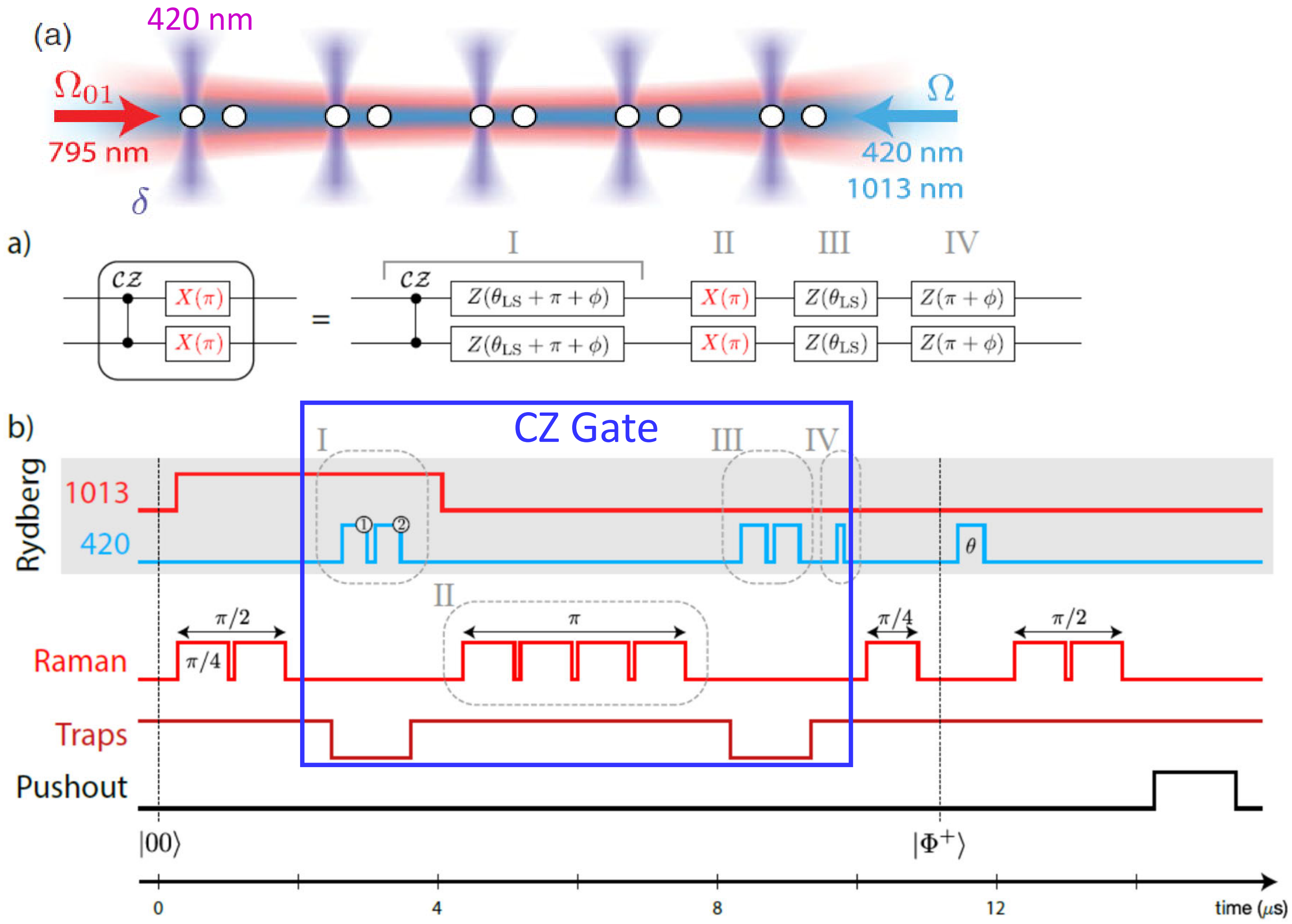
CZ constraint $\phi_{11}=2\phi_{01}+\pi \Rightarrow \Delta/\Omega$ can be determined

Δ/Ω=0.377371; ξ= 3.90242;
$(2\tau)\Omega$= T Ω = 8.58536

Actual Experimental Implementation of a CZ Gate

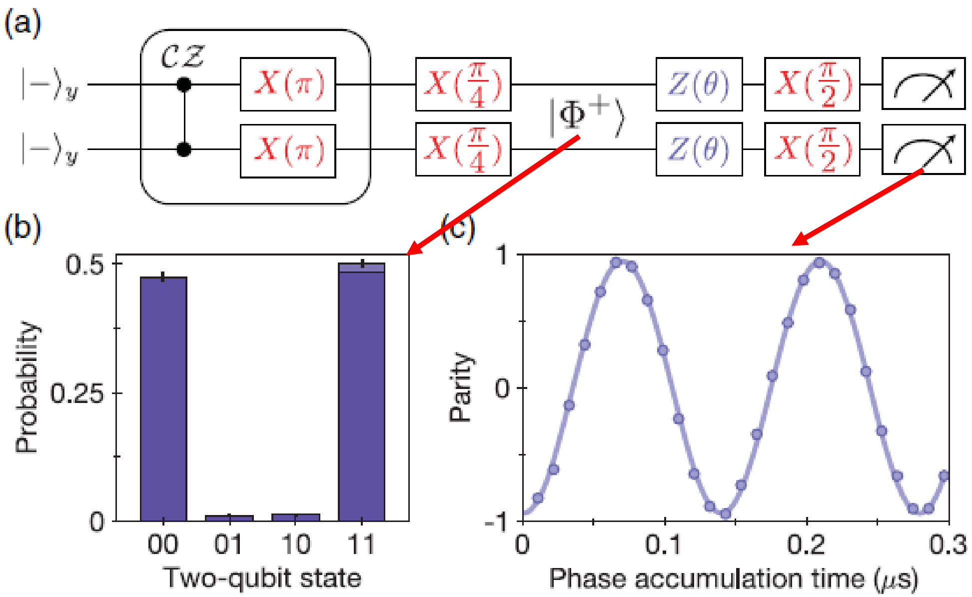


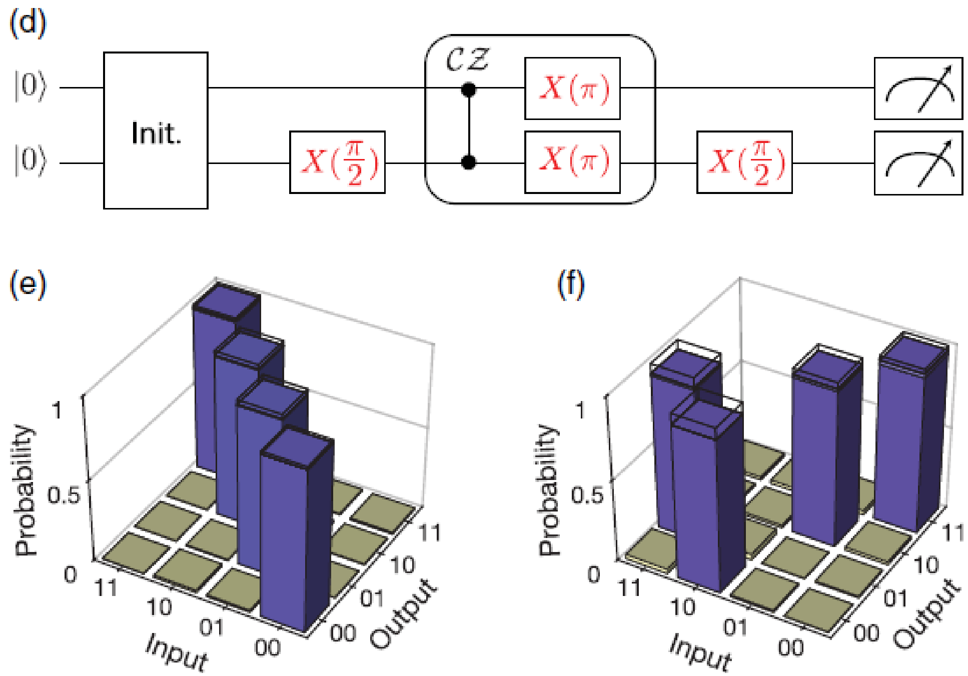
420+1013: Rydberg transition for two-qubit gate
795: Raman transition for global single-qubit rotation
420: Global single-qubit phase (Z) gate
420: Local single qubit phase (Z) gate



H. Levine et al., PRL. 123, 170503(2019)

Bell State and CNOT Gate Implementation





F=0.968

F=0.941 →F=0.974
After SPAM correction

- Bell-state fidelity evaluates the performance of a CZ gate.
- Easy to diagnose experimentally compared to tomography.

$$X\left(\frac{\pi}{2}\right)\left|00\right\rangle = \frac{1}{\sqrt{2}}\left|\begin{bmatrix}1 & -i\\ -i & 1\end{bmatrix}\right|00\rangle = \left(\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ -i\end{bmatrix}\right)_1\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ -i\end{bmatrix}\right)_2 = \frac{1}{2}(|00\rangle - i|01\rangle - i|10\rangle - |11\rangle) = |\psi_1\rangle$$

$$|\psi_2\rangle = (CZ)\psi_1\rangle = \frac{1}{2}(|00\rangle + i|01\rangle + i|10\rangle + |11\rangle) = \frac{1}{\sqrt{2}}(|\Phi^+\rangle + i(|\Psi^+\rangle)$$

Bell states $|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$; $|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$

$$X(\frac{\pi}{4})|\psi_2\rangle = |\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

H. Levine et al., PRL. 123, 170503(2019)

Notes on some relations

$$X(\theta)|00\rangle = \begin{bmatrix} \cos(\theta/2) & -isin(\theta/2) \\ -isin(\theta/2) & \cos(\theta/2) \end{bmatrix} \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)_1 \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)_2 = \left(\begin{bmatrix} \cos(\theta/2) \\ -isin(\theta/2) \end{bmatrix}\right)_1 \left(\begin{bmatrix} \cos(\theta/2) \\ -isin(\theta/2) \end{bmatrix}\right)_2$$

$$X(\theta)|01\rangle = \left(\begin{bmatrix} \cos(\theta/2) \\ -isin(\theta/2) \end{bmatrix}\right)_1 \left(\begin{bmatrix} -isin(\theta/2) \\ \cos(\theta/2) \end{bmatrix}\right)_2$$

$$X(\theta)|10\rangle = \left(\begin{bmatrix} -isin(\theta/2) \\ \cos(\theta/2) \end{bmatrix}\right)_1 \left(\begin{bmatrix} \cos(\theta/2) \\ -isin(\theta/2) \end{bmatrix}\right)_2$$

$$X(\theta)|11\rangle = \left(\begin{bmatrix} -isin(\theta/2) \\ \cos(\theta/2) \end{bmatrix}\right)_1 \left(\begin{bmatrix} -isin(\theta/2) \\ \cos(\theta/2) \end{bmatrix}\right)_2$$

$X(\theta)|\psi_2\rangle = \frac{1}{2} X(\quad)(|00\rangle + i|01\rangle + i|10\rangle + |11\rangle) = 1/2\{(\cos(\theta)+\sin(\theta))(|00\rangle+|11\rangle)+i(\cos(\theta)-\sin(\theta))(|01\rangle+|10\rangle)\}$

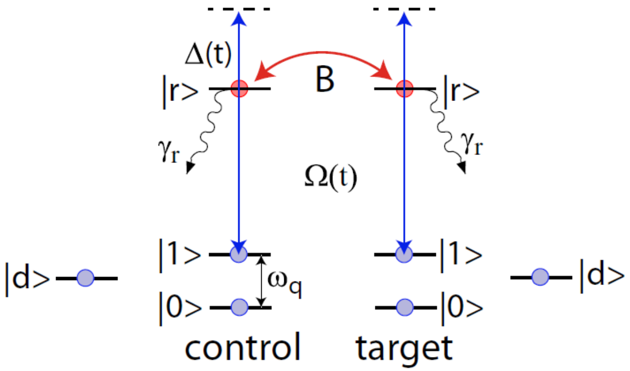
$X(\pi/4)|\psi_2\rangle = \frac{1}{\sqrt{2}}(|00\rangle+|11\rangle) = |\Phi^+\rangle$

$$Z(\theta)|\Phi^+\rangle = \frac{e^{i\frac{\theta}{2}}}{\sqrt{2}}\left(e^{-i\frac{\theta}{2}}|00\rangle + e^{i\frac{\theta}{2}}|11\rangle\right)$$

$X(\pi/2)\, Z(\theta)|\Phi^+\rangle = e^{i\frac{\theta}{2}}\{-isin(\theta/2)(|00\rangle-|11\rangle+\cos(\theta/2)(|01\rangle+|10\rangle) = \sqrt{2}e^{i\frac{\theta}{2}}\{-isin(\theta/2)|\Phi^-\rangle+\cos(\theta/2)(|\Psi^+\rangle)$

Parity Oscillation

Adiabatic Passage Pulses for a Symmetric CZ Gate



$$\frac{d\rho}{dt} = i[H,\rho] + \mathcal{L}[\rho],$$

$$\rho(0) = \rho_c(0) \otimes \rho_t(0)$$

$$H = H_c \otimes I + I \otimes H_t + B|rr\rangle\langle rr|$$

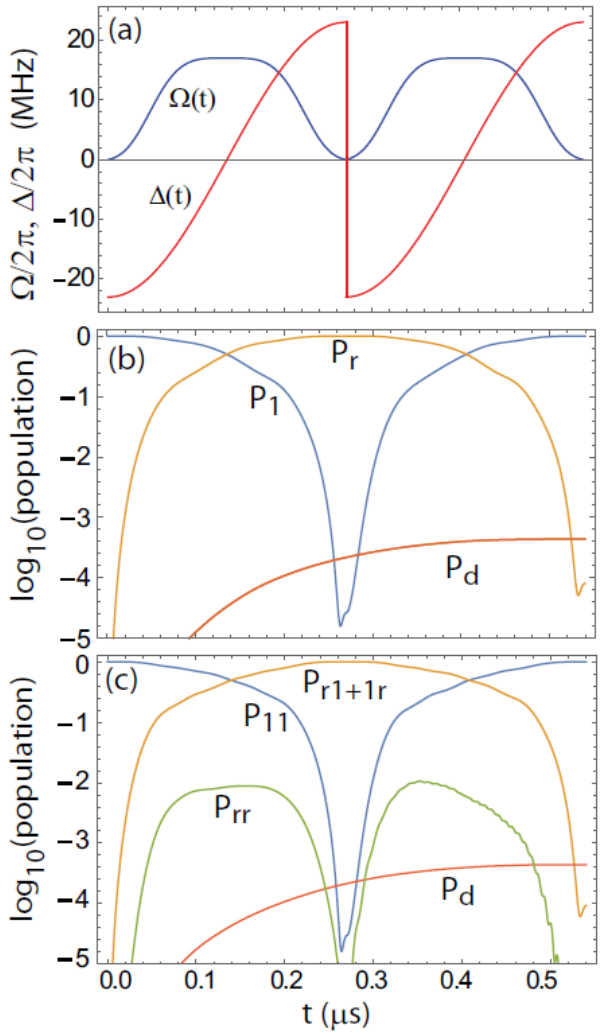
$$\mathcal{H}_{c/t} = \left[\frac{\Omega(t)}{2}|r\rangle_{c/t}\langle 1| + \text{H.c.}\right] + \Delta(t)|r\rangle_{c/t}\langle r$$

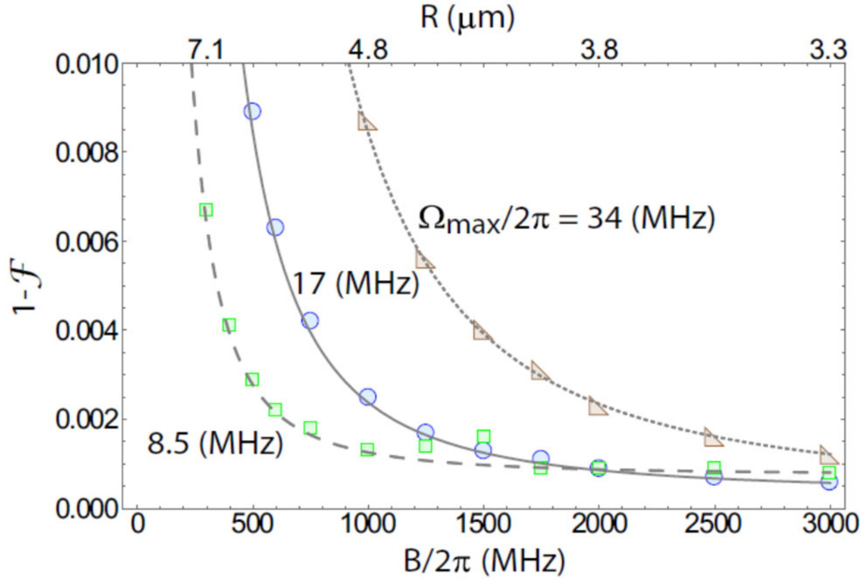
$$\mathcal{L}[\rho] = \sum_{\ell=c,t}\sum_{j=0,1,d} L_j^{(\ell)}\rho L_j^{(\ell)\dagger} - \frac{1}{2}L_j^{(\ell)\dagger}L_j^{(\ell)}\rho - \frac{1}{2}\rho L_j^{(\ell)\dagger}L_j^{(\ell}$$

$|r\rangle$=Cs n=107 $P_{3/2}$

M Saffman et al., PRA 101, 062309(2020)

Adiabatic Passage Pulse

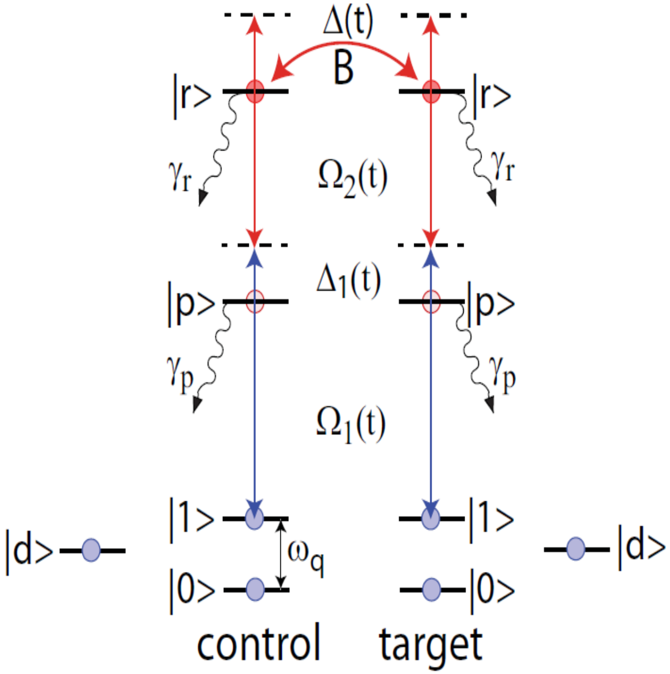


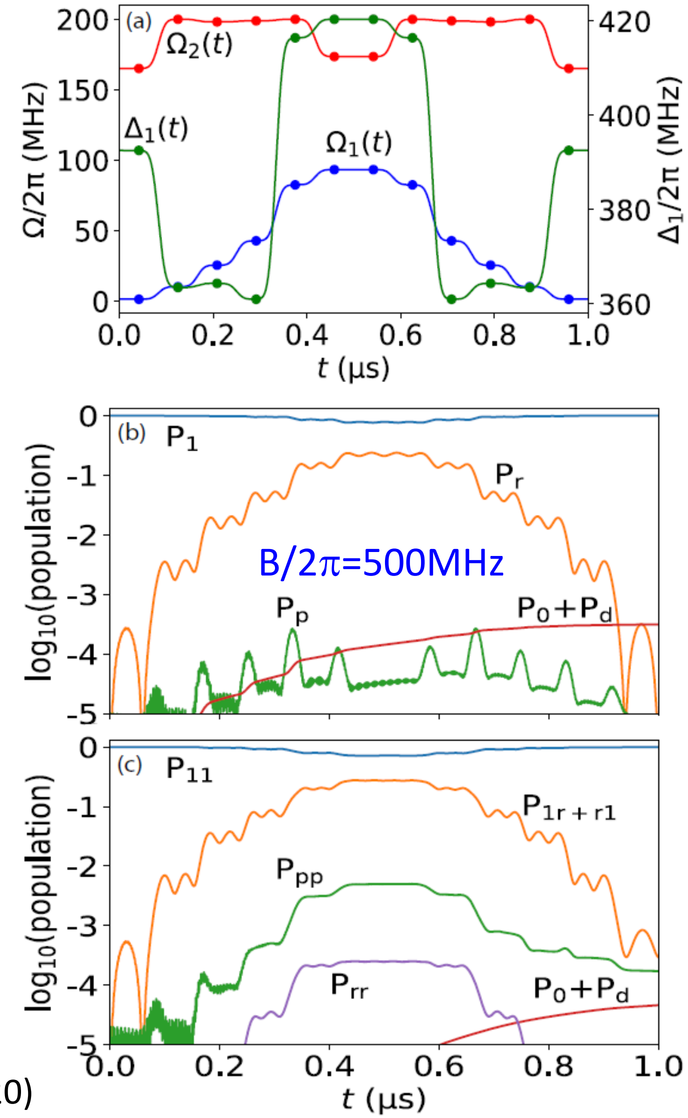


$\Omega_{max}/2\pi$ = {8.5, 17, 34} MHz
$\Delta_{max}/2\pi$ = {11.5, 23. 46} MHz
T = {1.08, 0.54. 0.24} μs
1-*F* = $b+(\Omega_{max}/B)^2$
B = {7.5, 3.5, 3.2}× 10^{-4}

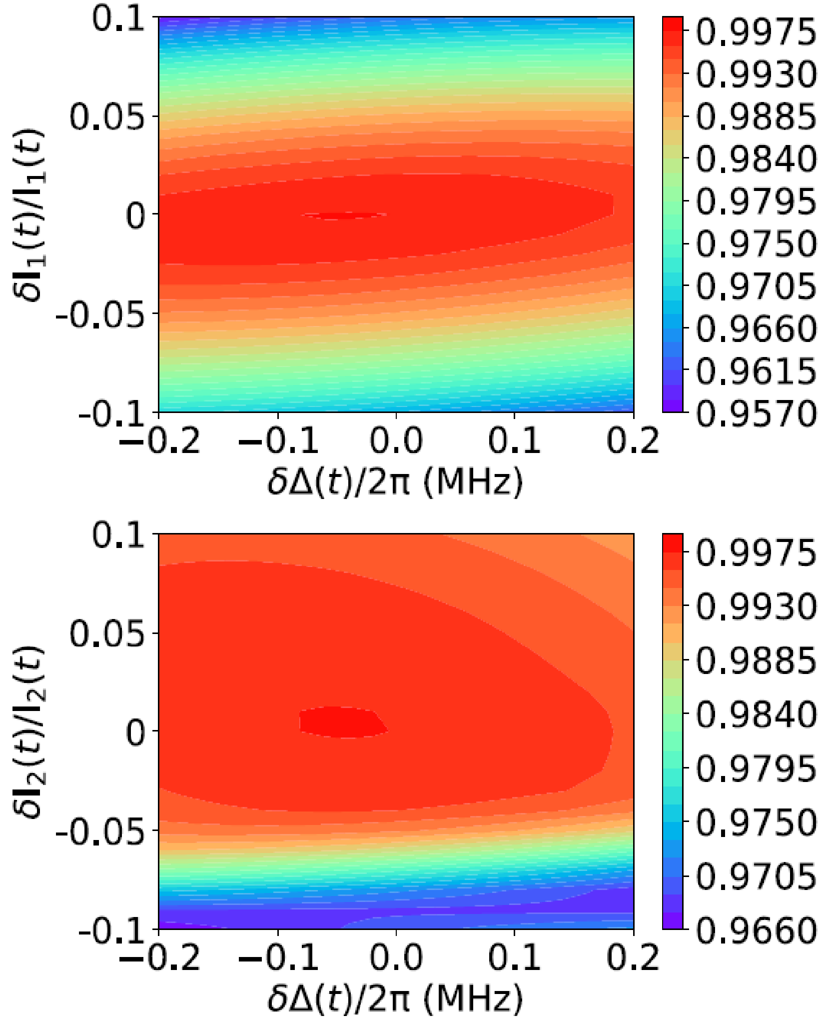
- Finite Rydberg lifetime vs Finite blockade strength.
- F> 0.999 is feasible.

Two-Photon Rydberg Excitation & Numerical Optimized Pulses





F dependence on laser parameters

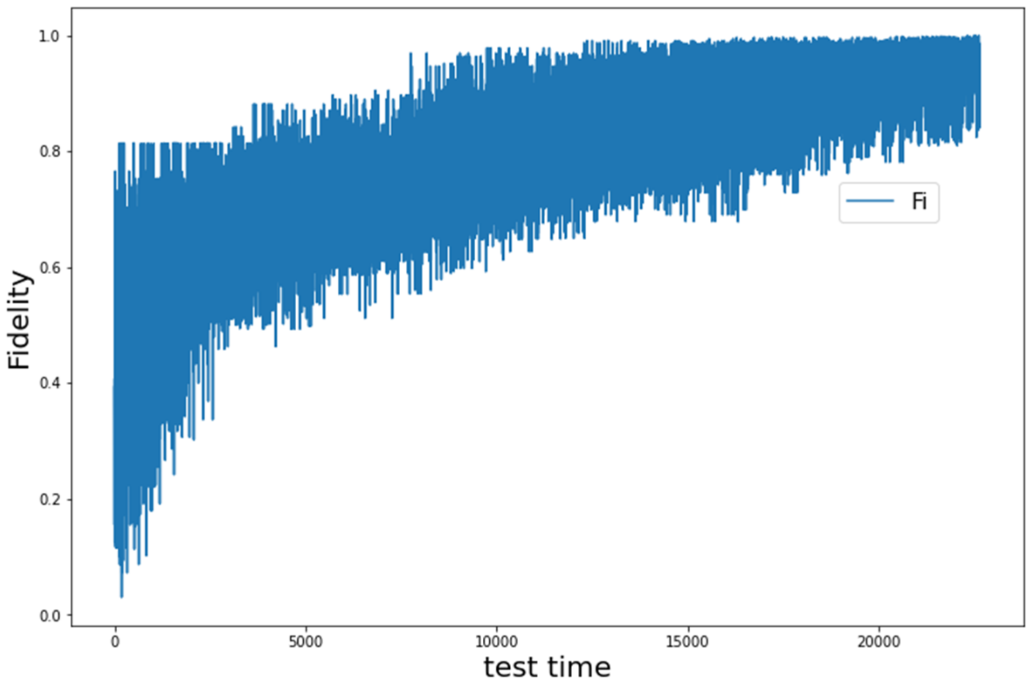


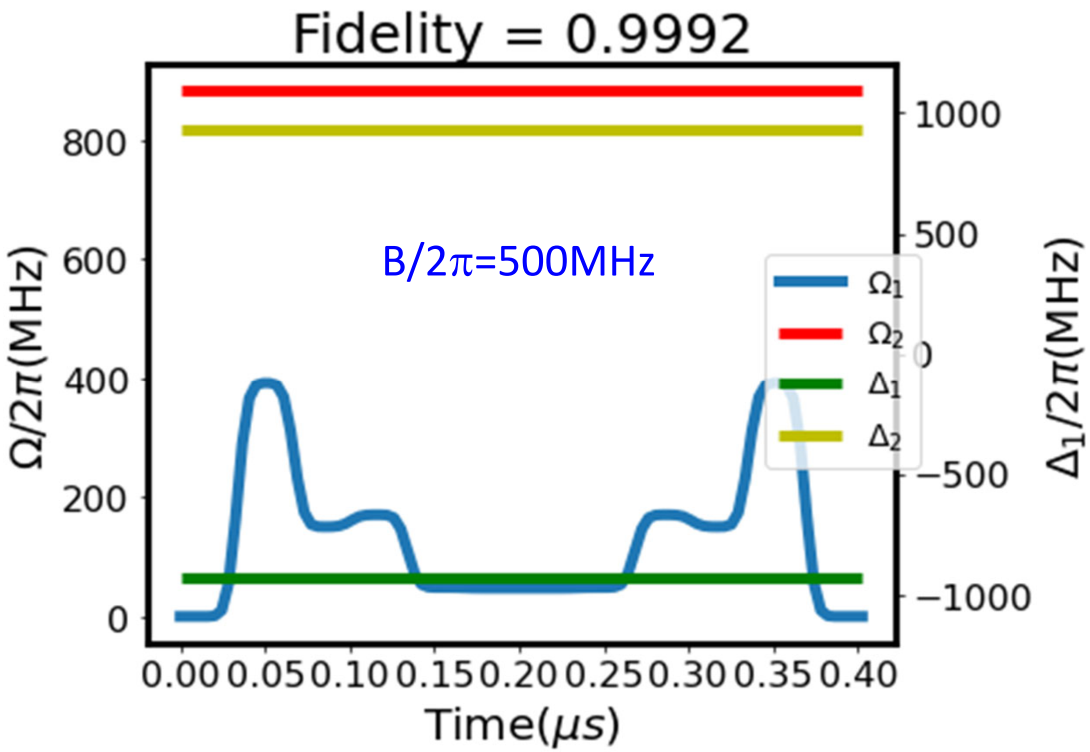
M Saffman et al., PRA 101, 062309(2020)

Our Studies

By 張子軒、王彩妮、任祥華

- Obtain optimal waveform by differential evolution.
- Fidelity > 0.999 is feasible with larger one-photon detuning and Rabi frequency.
- In general, larger Ω, larger $\Delta_{1\text{-}photon}$, larger B, smaller Γ will increase gate fidelity.





Time-Optimal CZ Gate

Time-Optimal Two- and Three-Qubit Gates for Rydberg Atoms

Sven Jandura and Guido Pupillo

University of Strasbourg and CNRS, CESQ and ISIS (UMR 7006), aQCess, 67000 Strasbourg, France

Quantum 6, 712 (2022) or arXiv: 2202.00903

Error budgeting for a controlled-phase gate with strontium-88 Rydberg atoms

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Simone Montangero ,[1,3,5] and Hans Peter Büchler [2]
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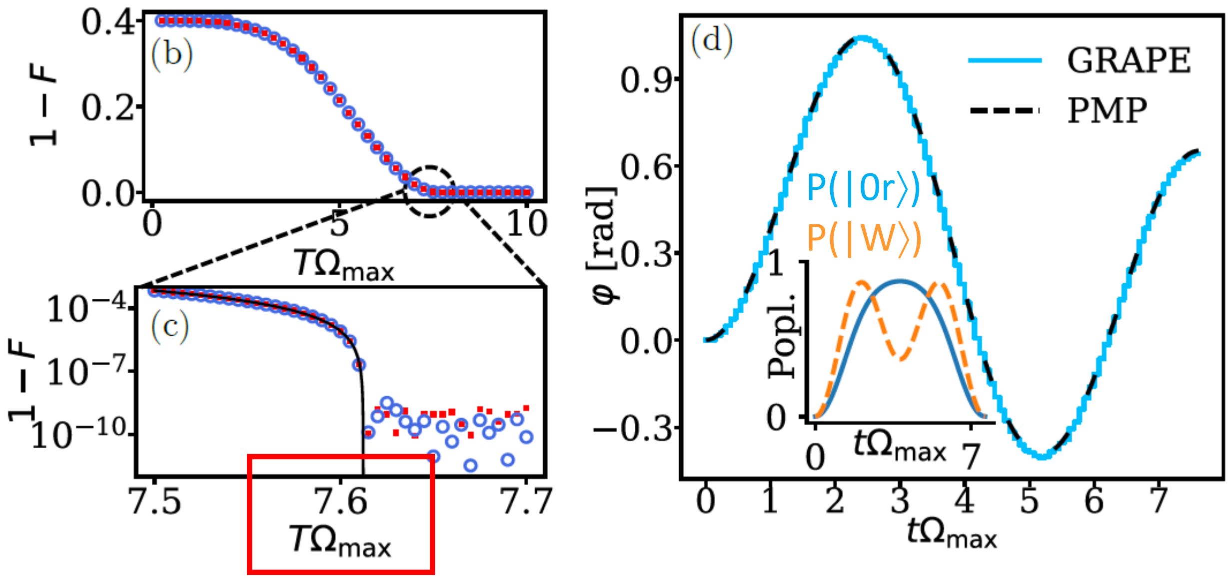
Phys. Rev. Res. 4, 033019(2022) or arXiv: 2202.13849

Vary Laser Phase, Infinite Blockade strength



$$\Omega_j(t) = |\Omega_j(t)|e^{i\varphi_j(t)}, \; |\Omega(t)| = \Omega_{\max}$$

$$\Delta_j = d\varphi_j/dt. \qquad \Delta_0=0$$

$$H_{01} + H_{11} = \begin{pmatrix} 0 & \frac{\Omega}{2} & 0 & 0 \\ \frac{\Omega^*}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}\Omega}{2} \\ 0 & 0 & \frac{\sqrt{2}\Omega^*}{2} & 0 \end{pmatrix}.$$

$|01\rangle, |0r\rangle, |11\rangle, |W\rangle$ basis.

$|\psi(0)\rangle = |10\rangle + |11\rangle$

- Existing a shortest time-optimal pulse satisfying $T^*\Omega_{max}$=7.612, 10% shorter than Lukin's scheme.
- Including finite Rydberg lifetime, the optimal pulse remains except a degradation in fidelity.

$$H_{decay} = -\frac{i\Gamma}{2}|r\rangle\langle r|$$

$T^*\Omega_{max}=7.612 \Rightarrow T_R\,\Omega_{max}=2.957$

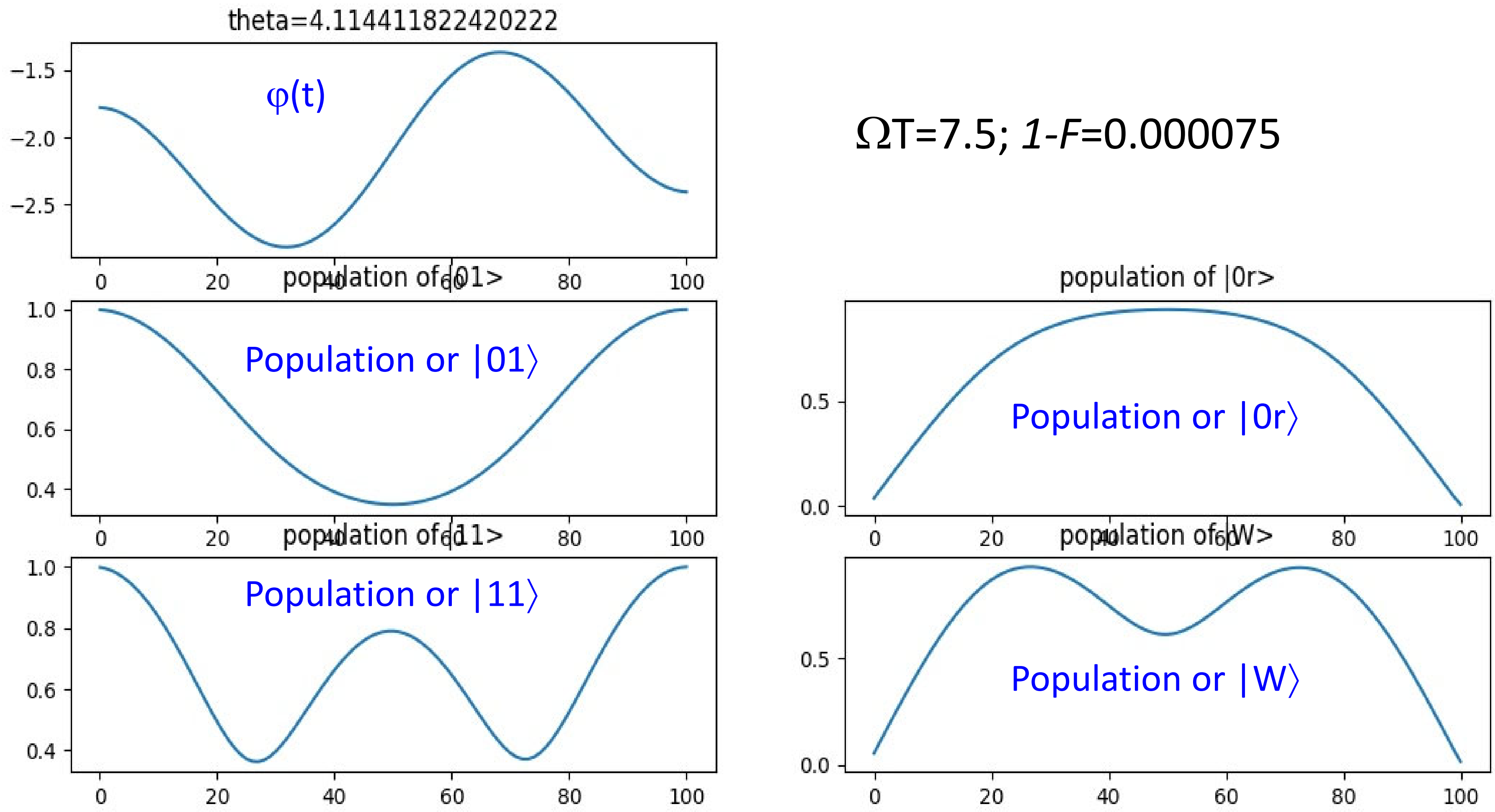
$$1 - F = \Gamma T_R \text{ for } \Gamma T_R \ll 1$$

Minimize TR $\Rightarrow T_R\,\Omega_{max}=2.947$ but $T\Omega_{max}= 30$

T_R: average time in Rydberg state

S. Jandura et al., Quantum 6, 712 (2022)

Our Simulation : Vary Laser Phase, Infinite Blockade Strength

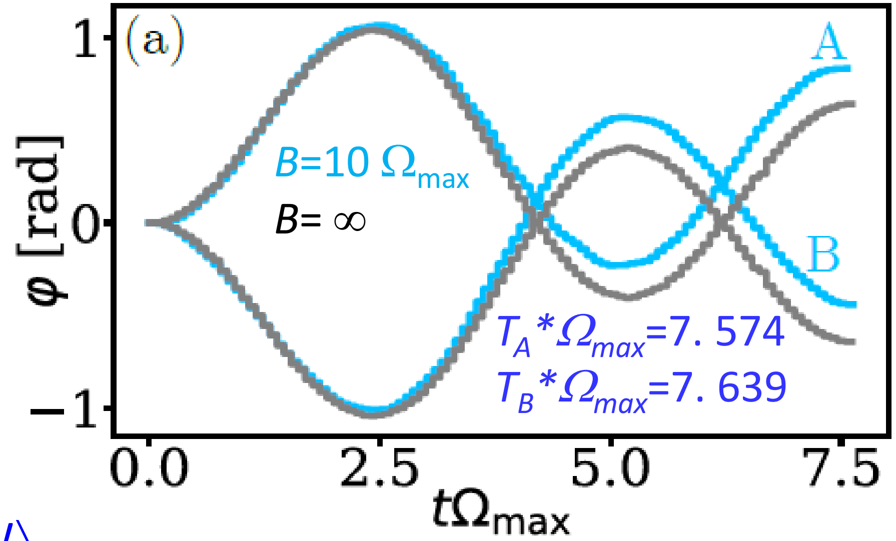


$\Omega T=7.5$; $1\text{-}F=0.000075$

Finite Blockade Strength

$$H_{01}+H_{11}=\begin{pmatrix} 0 & \frac{\Omega}{2} & 0 & 0 & 0 \\ \frac{\Omega^*}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}\Omega}{2} & 0 \\ 0 & 0 & \frac{\sqrt{2}\Omega^*}{2} & 0 & \frac{\sqrt{2}\Omega}{2} \\ 0 & 0 & 0 & \frac{\sqrt{2}\Omega^*}{2} & B \end{pmatrix}$$

$|01\rangle, |0r\rangle, |11\rangle, |W\rangle, |rr\rangle$ basis



Perturbation w.r.t $1/B$ $\Rightarrow$ effect of $|r\rangle\langle r|$ a shift on $|W\rangle$

$$H_{11}=\underbrace{\frac{\sqrt{2}\Omega}{2}(|11\rangle\langle W|+\text{h.c.})}_{H_{11}^{(0)}}\underbrace{-\frac{|\Omega|^2}{2B}|W\rangle\langle W|}_{H_{11}^{(1)}/B}$$

$$|\psi_{11}(t)\rangle = |\psi_{11}^{(0)}(t)\rangle+\frac{1}{B}|\psi_{11}^{(1)}(t)\rangle+\mathcal{O}(B^{-2}).$$

$$F=1-\frac{1}{B^2}\left(\frac{1}{4}\langle\psi_{11}^{(1)}|\psi_{11}^{(1)}\rangle-\frac{1}{10}\left|\langle 11|\psi_{11}^{(1)}\rangle\right|^2\right)+\mathcal{O}(B^{-3})$$

$$(1-F)B^2 = \frac{1}{4}\langle\psi_{11}^{(1)}(T)|\psi_{11}^{(1)}(T)\rangle - \frac{1}{10}\left|\langle 11|\psi_{11}^{(1)}(T)\rangle\right|^2+\mathcal{O}\left(B^{-1}\right) \quad (3$$

S. Jandura et al., Quantum 6, 712 (2022)

Gate error $\propto 1/B^2$

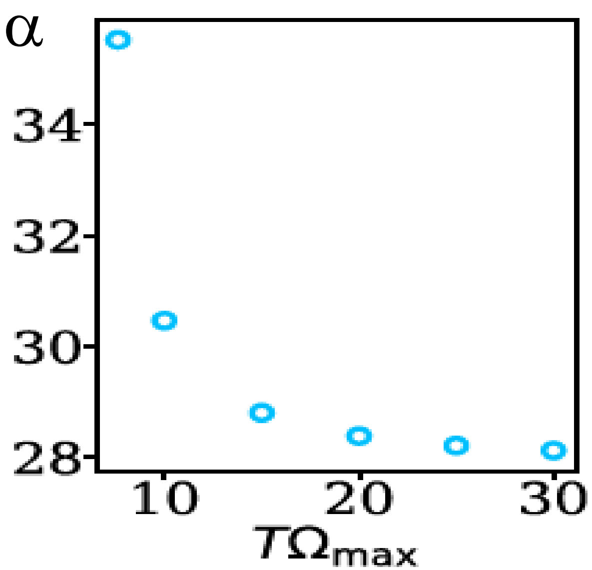
Vary Both Laser Intensity and Phase

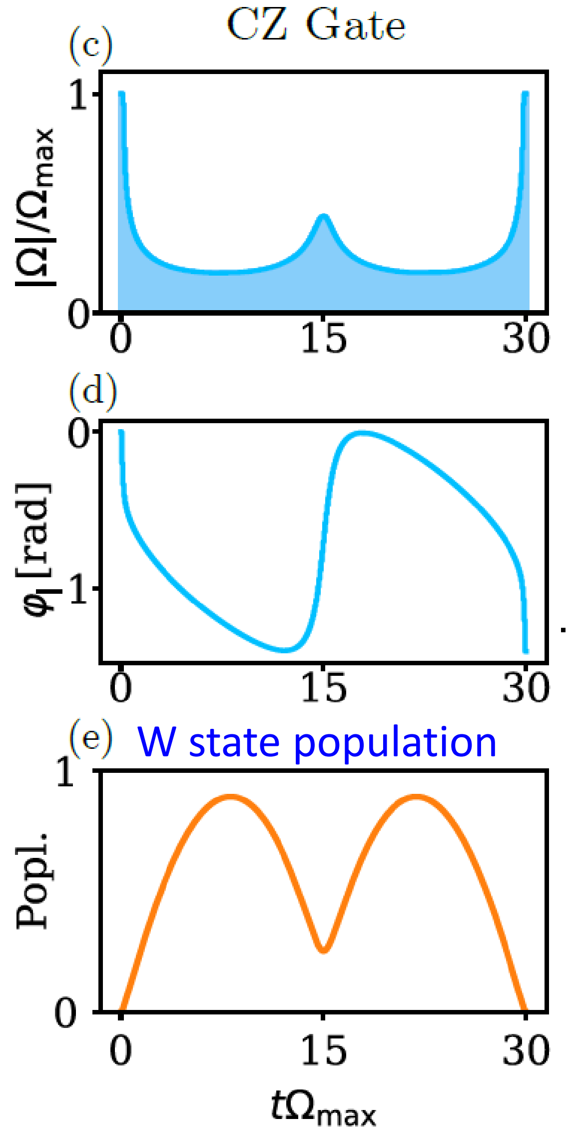
- Introduce dimensionless α=(1-F)B^2T^2 to evaluate gate performance for varying Ω case.
- Optimize both amplitude & phase waveform.
- Objective function *J,* optimize *w.r.t* all states with *F*=1 in the B=∞ cases.
- α decreases for longer pulses, 20% improvement w.r.t. time-optimal pulse.

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}|\psi_{11}^{(0)}\rangle\\|\psi_{11}^{(1)}\rangle\end{pmatrix}=-i\begin{pmatrix}H_{11}^{(0)} & 0\\H_{11}^{(1)} & H_{11}^{(0)}\end{pmatrix}\begin{pmatrix}|\psi_{11}^{(0)}\rangle\\|\psi_{11}^{(1)}\rangle\end{pmatrix}$$

$$J=CF(|\psi_{01}\rangle,|\psi_{11}^{(0)}\rangle,\theta)+\frac{1}{2}\frac{\mathrm{d}^2(1-F)}{\mathrm{d}(1/B)^2}\bigg|_{B=\infty}$$

α=35.9 for T*Ω_{max}=7.612





S. Jandura et al., Quantum 6, 712 (2022)

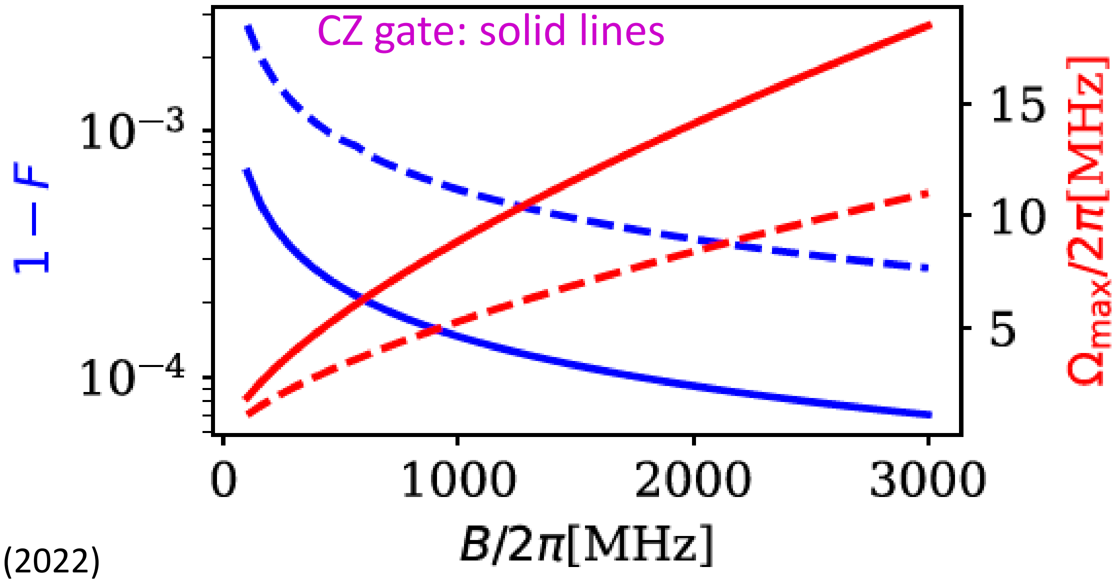
Optimal Gate Error

- Given Rydberg parameters Γ and B, there exists an optimal Ωmax that optimize gate fidelity.
- For B/2π $\cong$180 MHz for realistic case of Cs, a gate error of < 0.001 is feasible.

$$1 - F = \frac{\Gamma(T_R\Omega_{\max})}{\Omega_{\max}} + \frac{\Omega^2_{\max}\alpha}{B^2(T\Omega_{\max})^2}.$$

Minimal gate errors

Optimal Ω_{max}



$T\ \Omega_{max}$=7.612
$T_R\ \Omega_{max}$=2.957
α=35.9

S. Jandura et al., Quantum 6, 712 (2022)

M Saffman's Previous Discussions

Intrinsic Gate Error

$$E \simeq \frac{7\pi}{4\Omega\tau}\left(1+\frac{\Omega^2}{\omega_{10}^2}+\frac{\Omega^2}{7\mathrm{B}^2}\right)+\frac{\Omega^2}{8\mathrm{B}^2}\left(1+6\frac{\mathrm{B}^2}{\omega_{10}^2}\right).$$

Finite lifetime of Rydberg states

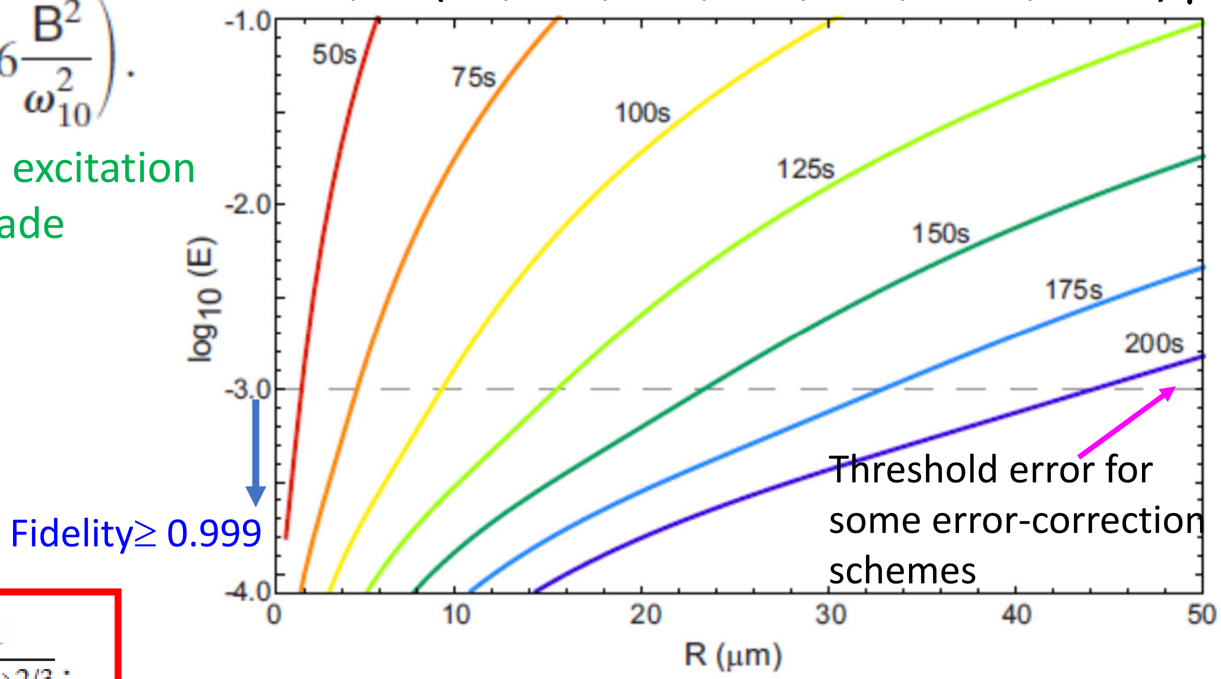
Double Rydberg excitation
Imperfect blockade

Ω: Rabi frequency of the laser pulse
τ: lifetime of the Rydberg state
B: mean blockade shift
ω_{10}: spacing of the two qubit states

$$\omega_{10} \gg (\mathrm{B},\Omega)$$

$$\boxed{\Omega_{\mathrm{opt}}=(7\pi)^{1/3}\frac{\mathrm{B}^{2/3}}{\tau^{1/3}}, \quad E_{\mathrm{min}}=\frac{3(7\pi)^{2/3}}{8}\frac{1}{(\mathrm{B}\tau)^{2/3}}.}$$

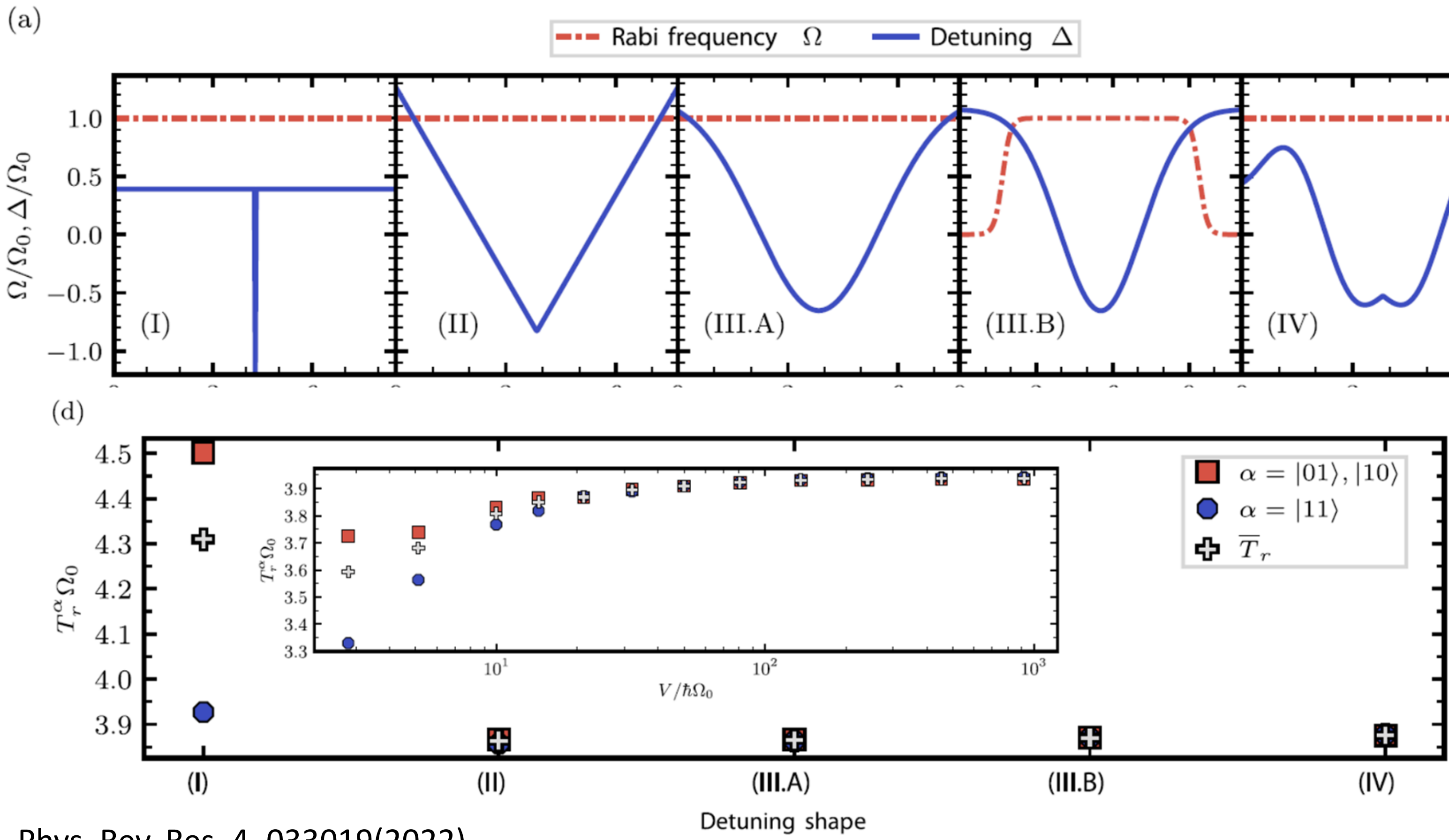
^{87}Rb, τ=(70,180,340,570,860,1200,1600) μs



M Saffman et al., Rev Mod Phys 82, 2313(2010)
P. Aliferis et al. PRA 79, 012332(2009)

Vary Laser Detuning for Optimization

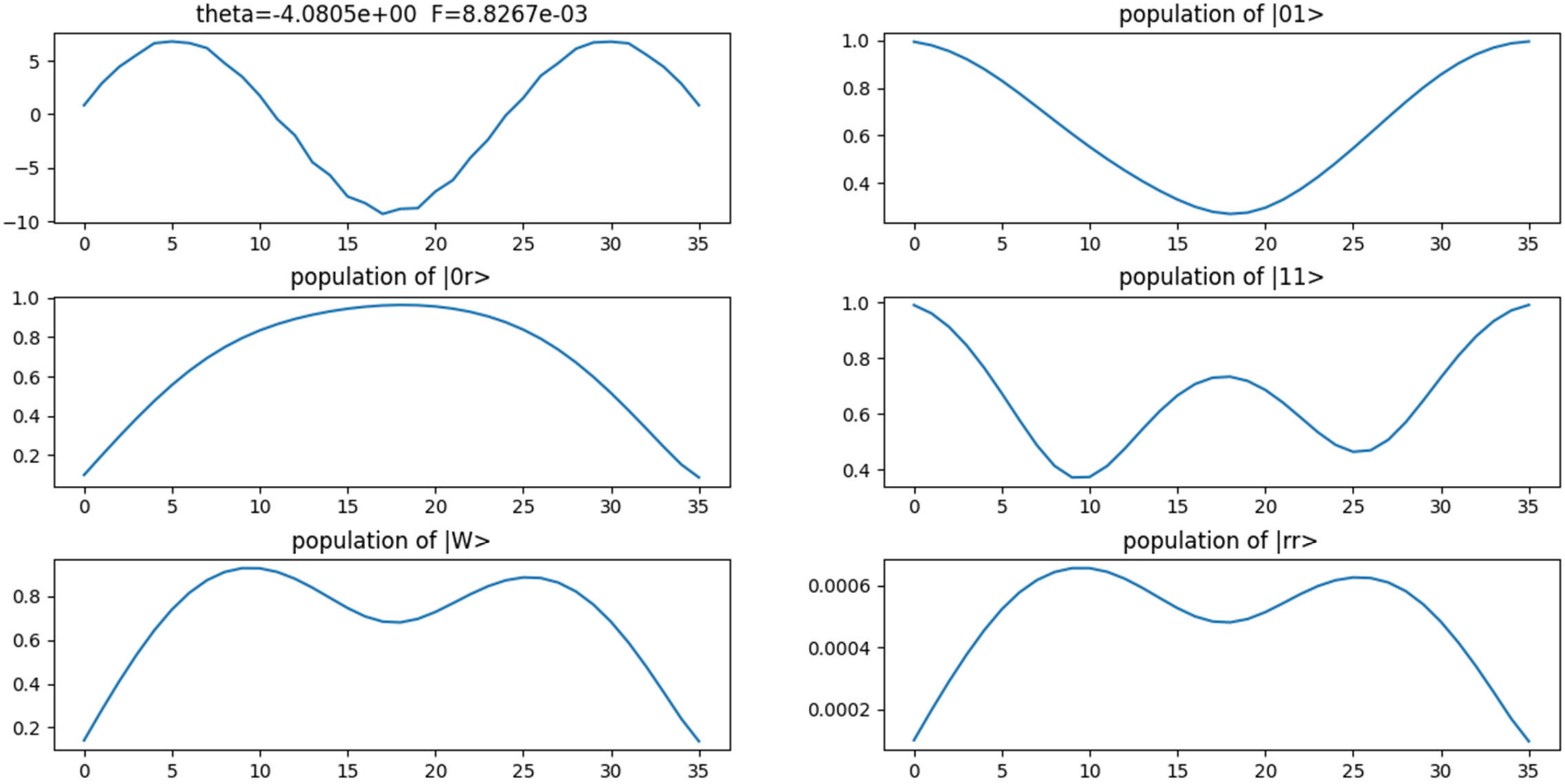
$$H_0 = \hbar \sum_{i=1}^{2} \left[\frac{\Omega(t)}{2}(\sigma_i^+ + \sigma_i^-) - \Delta(t) n_i \right],$$



A. Pagano et al., Phys. Rev. Res. 4, 033019(2022)

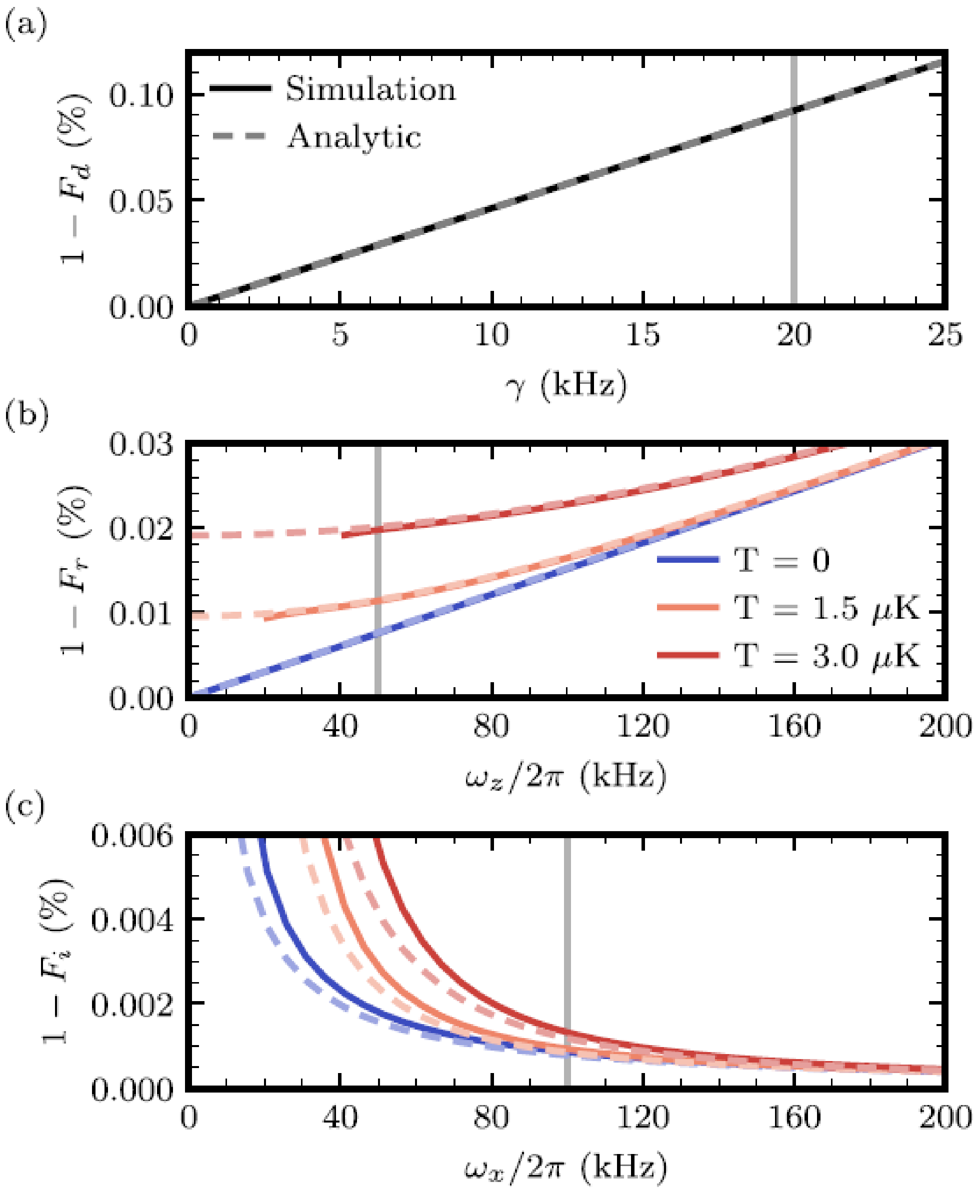
Our Simulation Results

Ω=10, T=0.7, B=10000, Γ=0



Error Sources for Two-Qubit Gates

- Intrinsic errors
  - Finite Rydberg lifetime
  - Finite blockade strength
  - Spontaneous decay of the intermediate state in two-photon excitation scheme
- Other errors
  - Laser phase, intensity, pointing noises
  - Doppler shifts due to finite atomic temperature
  - Photon recoil in Rydberg excitations
  - Inhomogeneity of the trap intensity, ac Stark shift
  - Environment electric and magnetic noises
  - …



$$H_0 = \hbar \sum_{i=1}^{2} \left[\frac{\Omega(t)}{2} \left(\sigma_i^+ e^{i\mathbf{k}\mathbf{r}_i} + \sigma_i^- e^{-i\mathbf{k}\mathbf{r}_i} \right) - \Delta(t) n_i \right],$$

$$H_{\text{trap}} = \sum_{i=1}^{2} \left[\frac{\mathbf{p}_i^2}{2m} + \frac{m\omega_x^2}{2} x_i^2 + \frac{m\omega_y^2}{2} y_i^2 + \frac{m\omega_z^2}{2} z_i^2 \right],$$

$$H_{\text{int}} = -\frac{C_6}{|\mathbf{R} + \mathbf{r}_1 - \mathbf{r}_2|^6} n_1 n_2,$$

	Bell state infidelity		Average gate infidelity	
	0 μK	1.5 μK	0 μK	1.5 μK
Rydberg decay	0.092%	0.092%	0.074%	0.074%
Photon recoil	0.008%	0.011%	0.006%	0.009%
VdW force	0.001%	0.001%	0.001%	0.001%
Summed	0.101%	0.105%	0.081%	0.084%
Full simulation	0.101%		0.081%	

A. Pagano et al., Phys. Rev. Res. 4, 033019(2022)

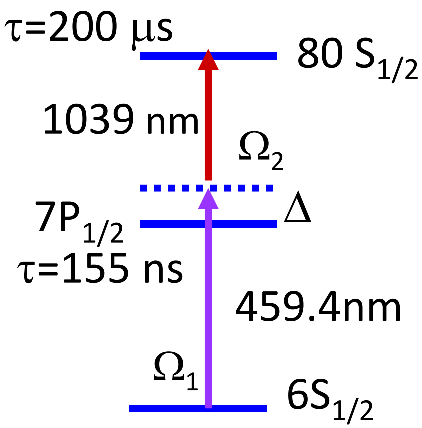
TABLE I. Experimental parameters for strontium-88. The van der Waals (VdW) coefficient C_6, the lifetime of the Rydberg state $|r\rangle$ $1/\gamma$, and the wavelength λ of the transition $|1\rangle \rightarrow |r\rangle$ are specific to our choice of the qubit and Rydberg state.

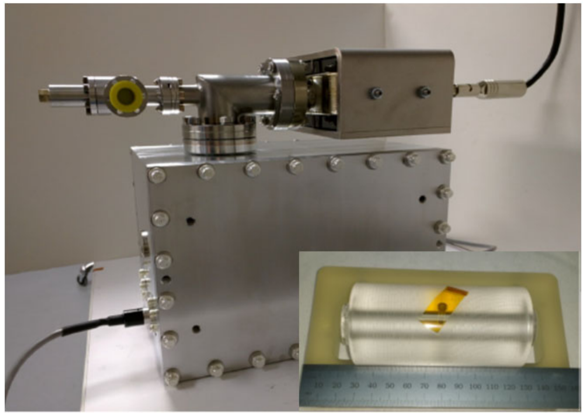
Max. Rabi frequency	$\Omega_0/2\pi$	10	MHz
Trap frequency along x, y	$\omega_x/2\pi = \omega_y/2\pi$	100	kHz
Trap frequency along z	$\omega_z/2\pi$	50	kHz
Rydberg lifetime	$1/\gamma$	50	μs
Transition wavelength	λ	323	nm
VdW coefficient	C_6/h	-154	GHz μm^6
Interatomic distance	R	3	μm

Outlook

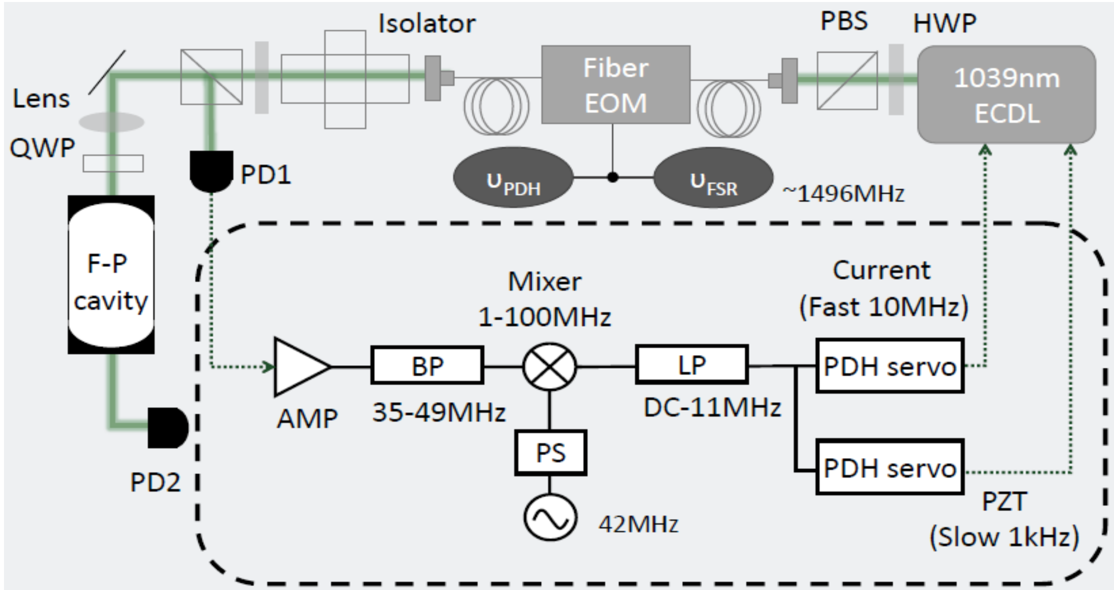
- Consider the scheme to vary detuning and Rabi frequency.
- Model with the realistic two-photon excitation scheme.
- Figure out realistic requirements (which Rydberg states, atomic temperature, trap parameters...) for Cs and the expected Gate fidelity for guiding the experiment.

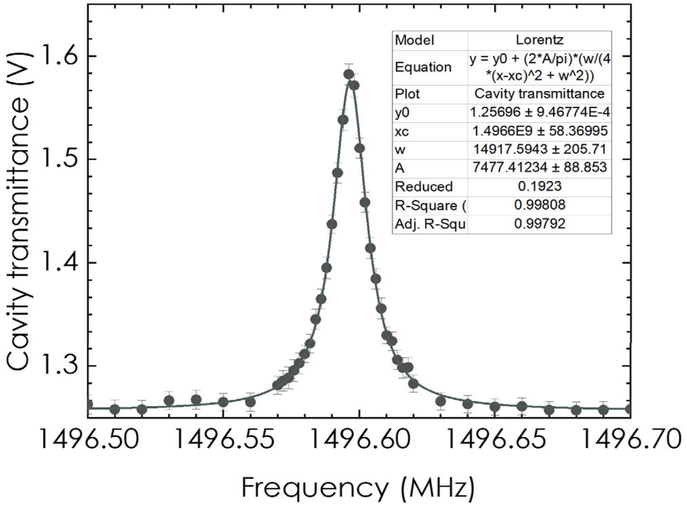
Progress : Laser Frequency Stabilization to a High-Finesse Cavity



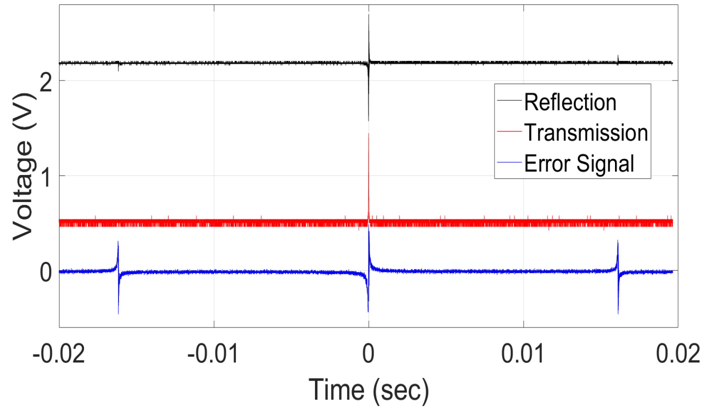


Pound-Drever-Hall Locking Scheme with free spectral range & cavity linewidth determination

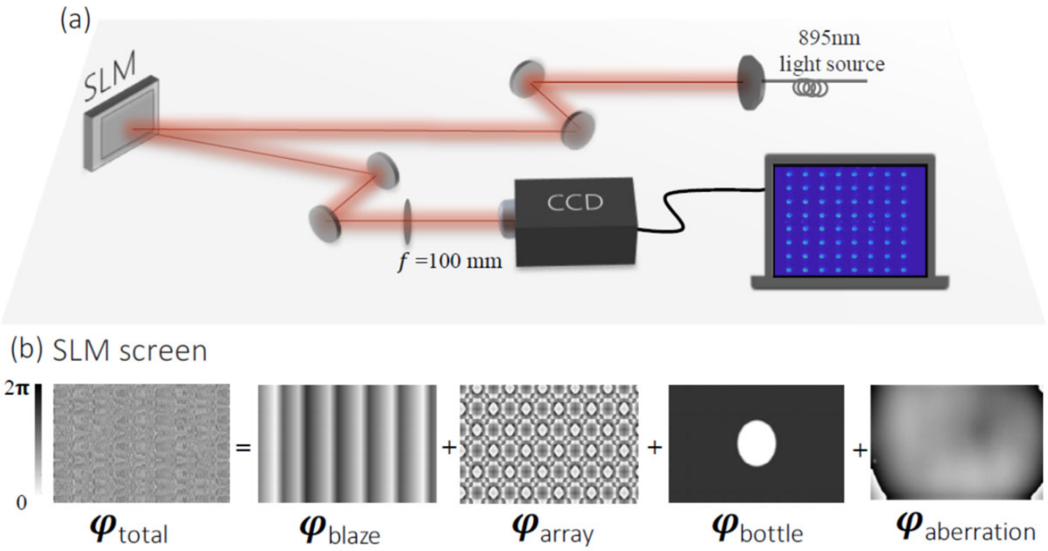




FSR= 1496.596511(58) MHz
Cavity Δν=14.918(206) kHz
Finesse= 100288(1385)
Laser linewidth sub-kHz



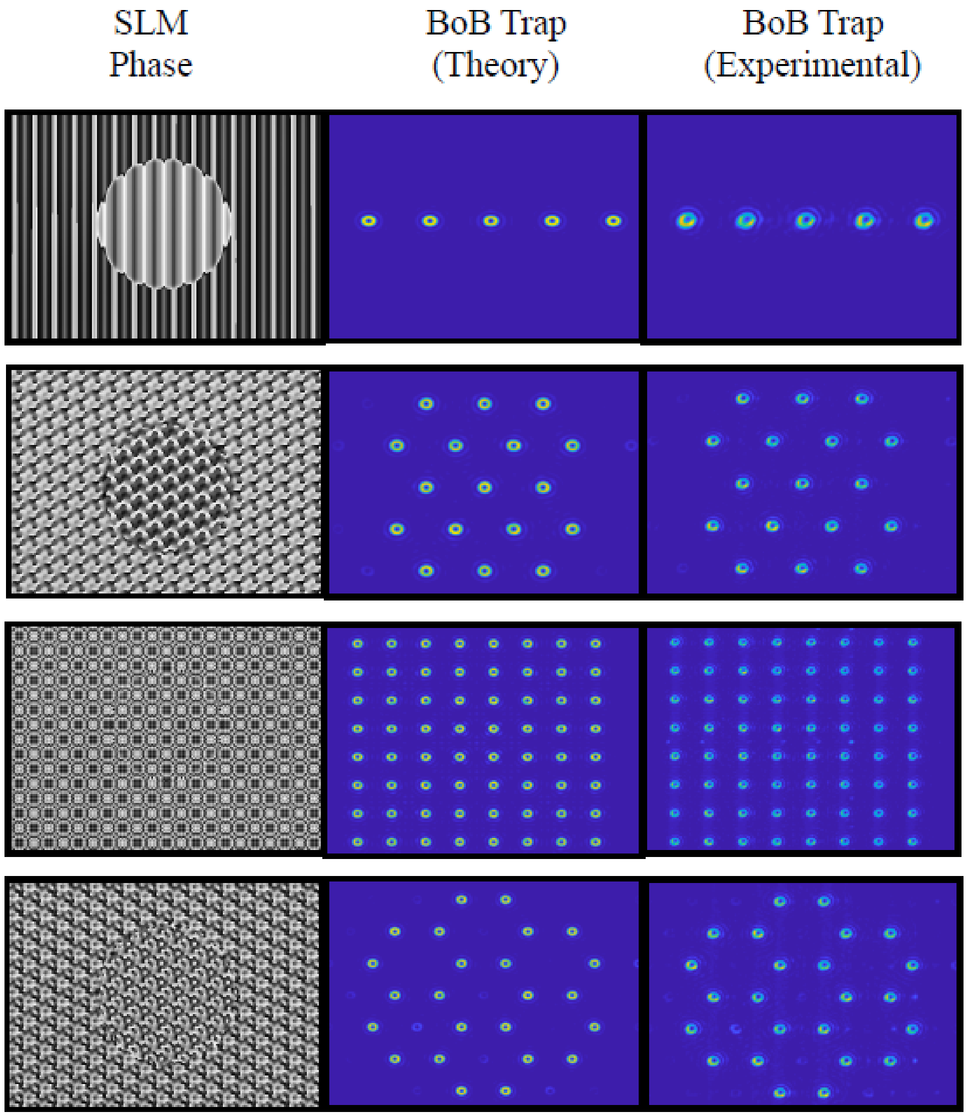
Progress : 2D Arbitrary Bottle-Beam Arrays for Atom Trapping



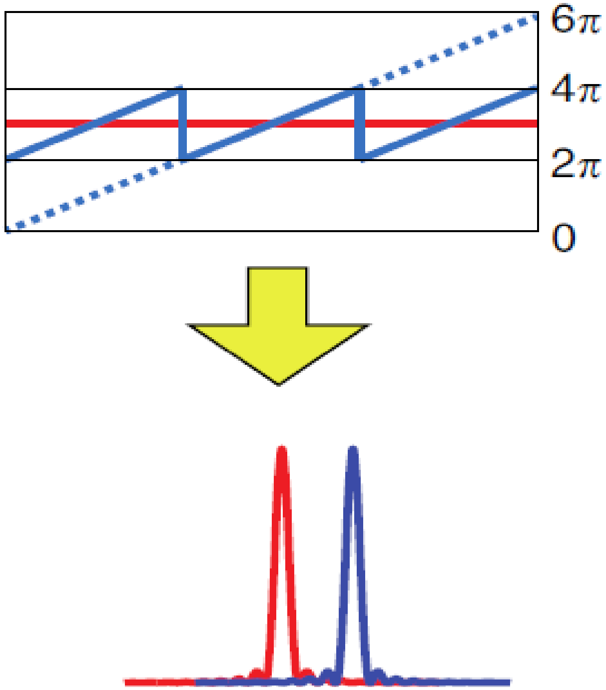
- Optimal beam splitting method, better than traditional Gerchberg-Saxton algorithm.
- Bottle beams can trap Rydberg atoms for deeper quantum circuits.

$$\Psi = \tan^{-1}\left(\frac{\sum_{m=-M}^{M}\gamma_m \sin\left[(2\pi s/\lambda)mx + \alpha_m\right]}{\sum_{m=-M}^{M}\gamma_m \cos\left[(2\pi s/\lambda)mx + \alpha_m\right]}\right) + \tan^{-1}\left(\frac{\sum_{n=-N}^{N}\gamma_n \sin\left[(2\pi s/\lambda)ny + \alpha_n\right]}{\sum_{n=-N}^{N}\gamma_n \cos\left[(2\pi s/\lambda)ny + \alpha_n\right]}\right)$$

L. A. Romero et al., J. Opt. Soc. Am A 24, 2280(2007)

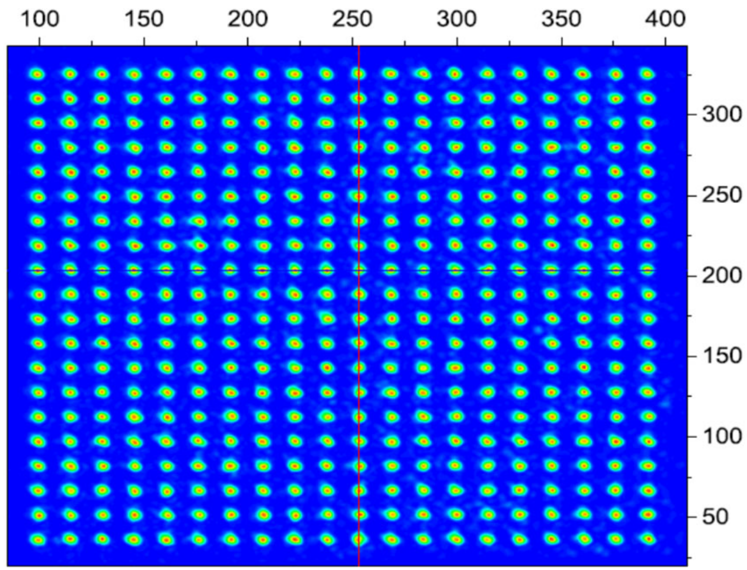


- Phase pattern of different period diffract light into corresponding diffraction order.
- Generate optimal phase pattern by optimizing amplitudes $\gamma_{m(n)}$ and phases $\alpha_{m(n)}$ that lead to the most uniform intensity distribution.

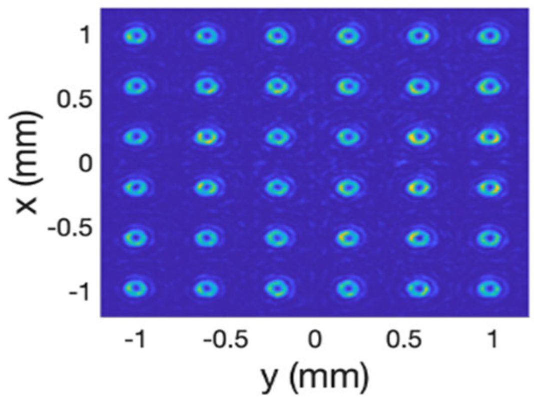


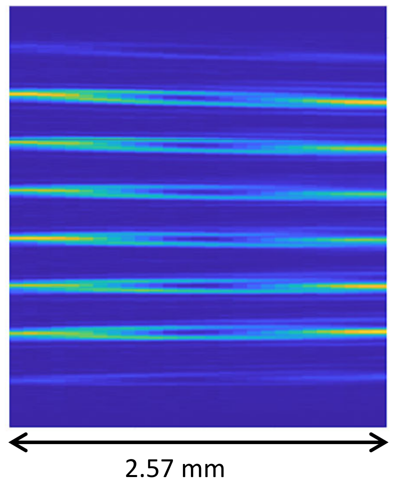
M. Mirhosseini et al., Nat. Commun 4:2781(2013)

20 x 20 Gaussian beam arrays



6 x 6 Bottle beam arrays





Keep walking and
Thank you for your attention !

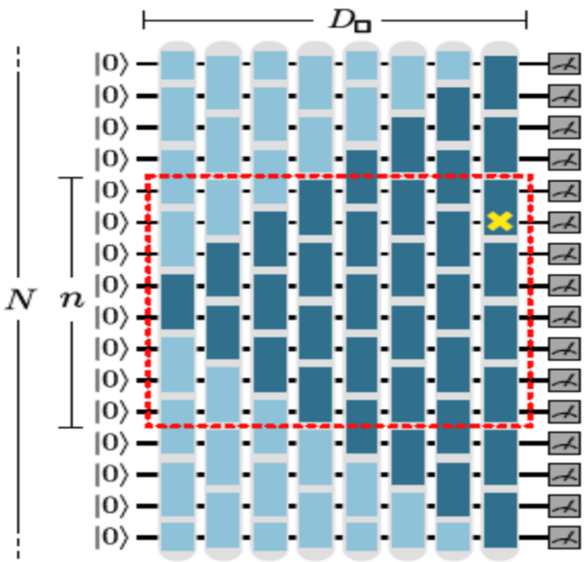
Welcome to join us!

Some Metrics of Quantum Computation

Three parameters

Circuit Depth D ($\propto T_{coherence}/T_{gate}$)

Circuit Width n (Qubit number)



- Gate fidelity F

  (gate error ε=1-F)

Three stages

Scalable, Universal Quantum Computers

- More qubits (e.g. 10^6)
- Better gate fidelity
- Error correction

Noisy, Intermediate-Scale Quantum (NISQ) Devices (n=20-200, $\varepsilon\approx0.01$)

- Individual qubit control
- Multi-qubit processing

Analog Quantum Devices (Hamiltonian engineering)

Cirac, Nanophotonics 10, 453(2021)

Three Milestones

Demonstrating large-scale quantum computing algorithms (e. g. Shor, Grover)

Demonstrating Quantum Error Correction & Fault-tolerant QC: reaches a better fidelity with QEC than without QEC

Demonstrating Quantum Supremacy (computational advantage): with high Hilbert space + good enough gate fidelity

Notes on Rydberg-Rydberg Interactions

$$\hat{V} = \frac{1}{4\pi\epsilon_0}\frac{\hat{\mu}_j \cdot \hat{\mu}_k - 3(\hat{\mu}_j \cdot \vec{n})(\hat{\mu}_k \cdot \vec{n})}{R^3_{j,k}},$$

$$\hat{H}^{abcd}_{j,k} = \frac{V_{j,k}}{2}|ab\rangle\langle cd| + \text{h.c.},$$

In the pair state basis

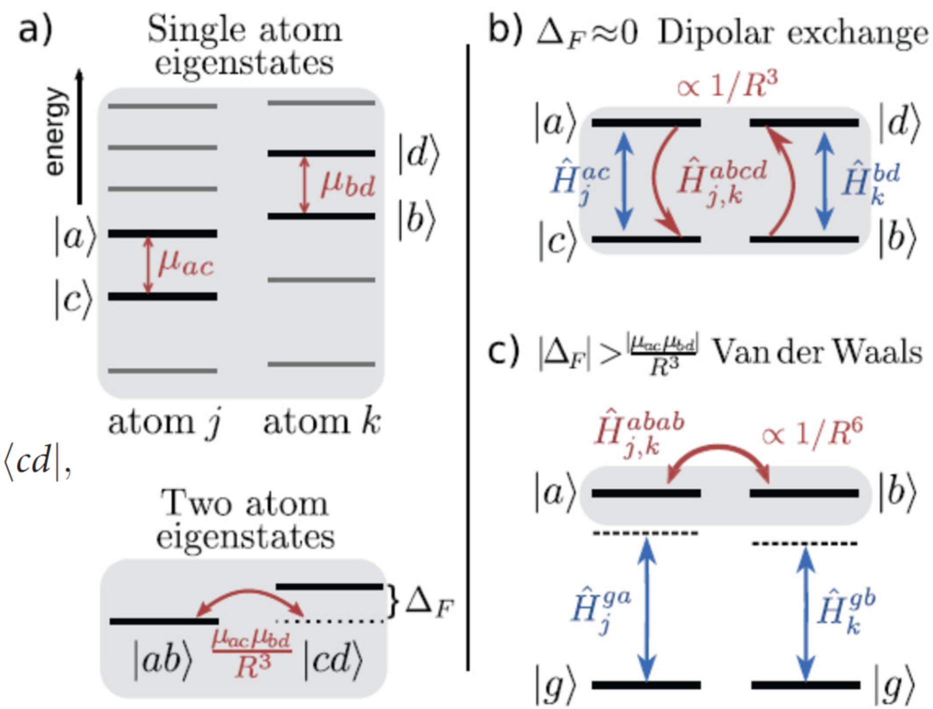
$$\hat{H} = \left(\frac{\mu_{ac}\mu_{bd}}{R^3_{j,k}}|ab\rangle\langle cd| + \text{h.c.}\right) + \Delta_F|cd\rangle\langle cd|,$$

where

$$\mu_{\alpha\beta} = \langle\alpha|\hat{\mu}|\beta\rangle/\sqrt{4\pi\epsilon_0}$$

$$\Delta_F = E_c + E_d - (E_a + E_b)$$

Förster detuning



Resonant interactions

$$\hat{H} = \frac{\mu_{ac}\mu_{bd}}{R^3_{j,k}}|ab\rangle\langle cd| + \text{h.c.},$$

$$= \hat{H}^{abcd}_{j,k}\left(V_{j,k} = \frac{2\mu_{ac}\mu_{bd}}{R^3_{j,k}}\right).$$

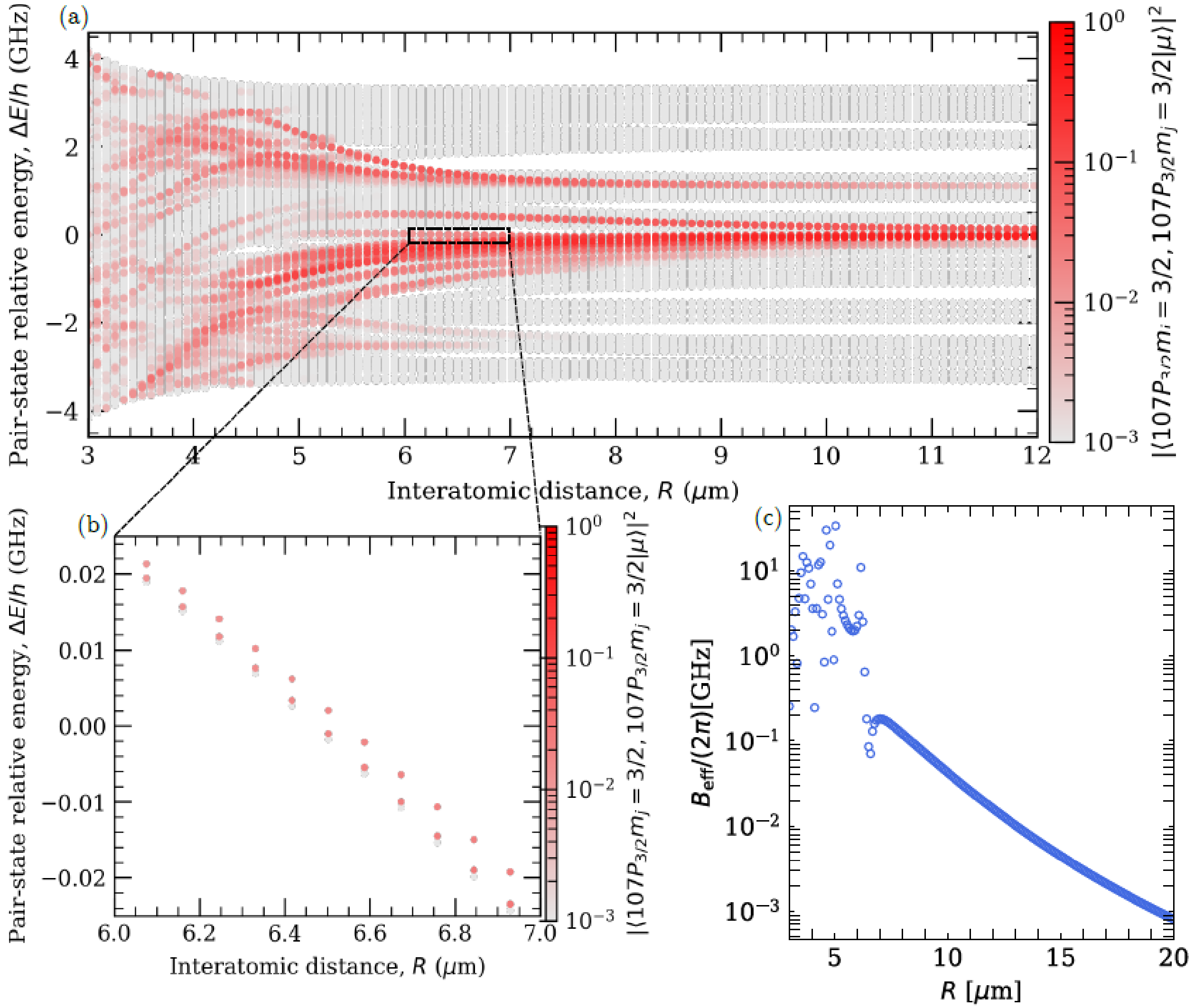
Nonresonant interactions

$$\hat{H} = -\frac{C_6}{R^6_{j,k}}|ab\rangle\langle ab| = \hat{H}^{abab}_{j,k}\left(V_{j,k} = -\frac{C_6}{R^6_{j,k}}\right),$$

$$C_6 = \frac{|\mu_{ac}|^2|\mu_{bd}|^2}{\Delta_F}.$$

M. Mrgado et al., AVS Quantum Sci. 3, 023501(2021)

Effective Blockade Strength for Cs



- Using Alkali Rydberg Calculator

atoms, $|r\rangle = |107p_{3/2}\, m = 3/2\rangle$ and $|1\rangle = |6S_{1/2} F = 4\, m_F = 4\rangle$

$$V_{dd} = \frac{e^2}{4\pi\epsilon_0}\frac{\hat{x}_1 \cdot \hat{x}_2 - 3(\hat{x}_1 \cdot \vec{n})(\hat{x}_2 \cdot \vec{n})}{R^3}$$

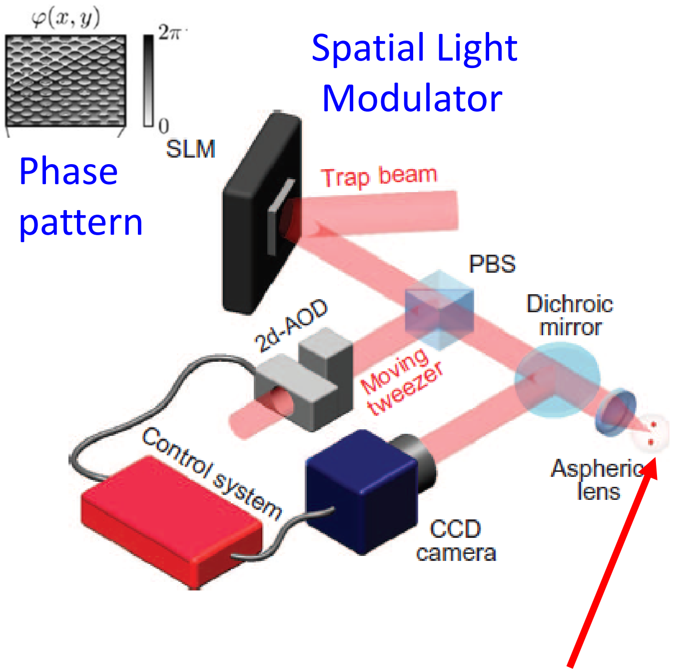
$$H = \sum_{ab}(E_a + E_b)\,|ab\rangle\,\langle ab| + \sum_{abcd}\langle ab|V_{dd}|cd\rangle\,|ab\rangle\,\langle cd|$$

104≤n≤110, L≤3

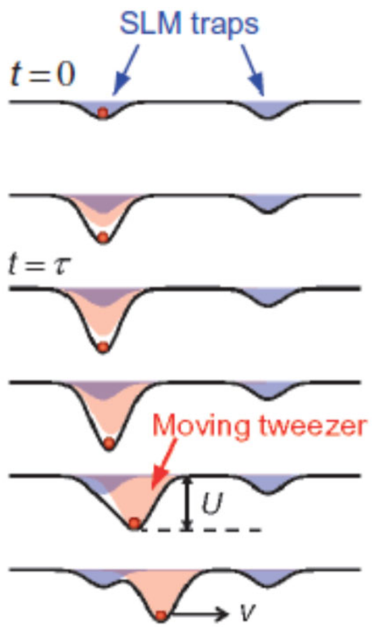
$$\frac{1}{B_{\mathrm{eff}}} = \frac{2}{\Omega^2}\sum_i \frac{|\,\langle W|H_{\mathrm{laser}}|\varphi_i\rangle\,|^2}{\Delta_i}$$

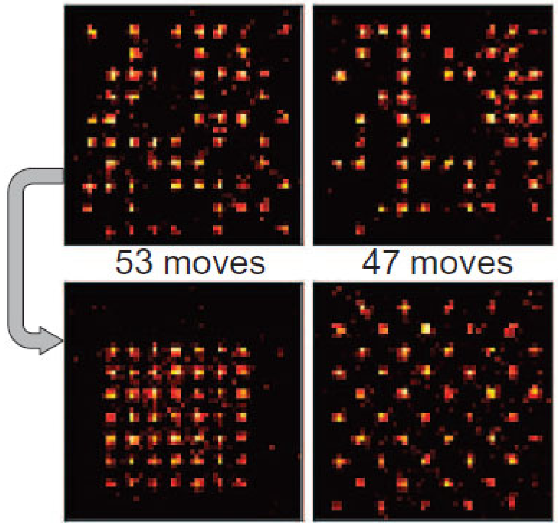
$B_{eff}/(2\pi)$=180 MHz @ R= 7μm

Atom-by-Atom Assembly of Defect-free Single-Atom Arrays

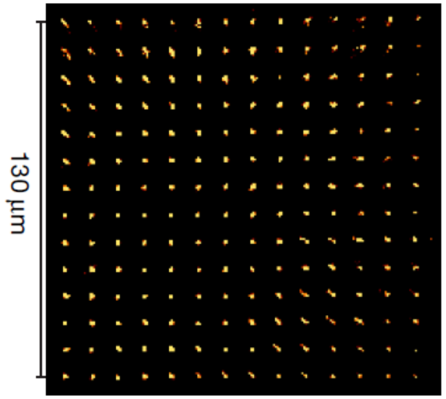


Atom Rearrangement

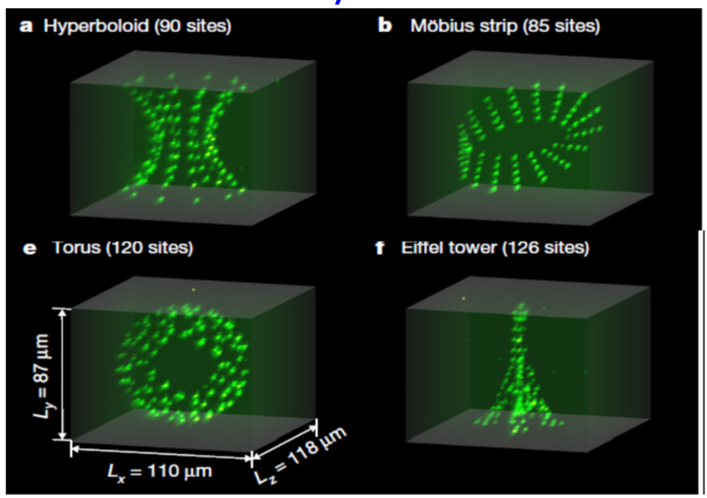




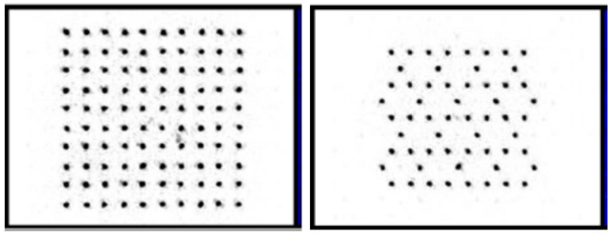
N~300 defect-free atom arrays



3D Arbitrary Patterns



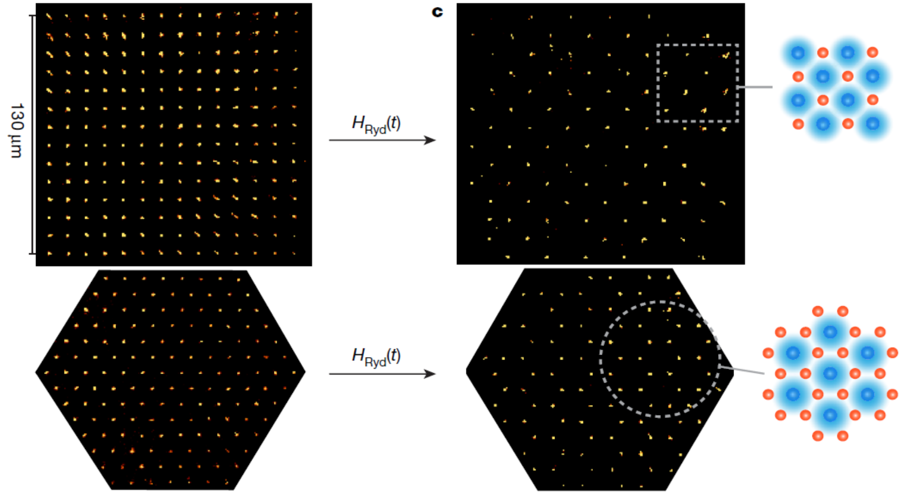
Arbitrary Trap Intensity Pattern



F. Nogrette et al., PRX 4, 021034(2014)
D. Barredo et al., Science 354, 1021(2016)
K. Schymik et al., PRA 102, 063107(2020)
E. Ebad et al., Nature 595, 227(2021)
P. Scholl et al., Nature 595, 233(2021)

Rydberg-Interacting Atom Arrays as Analog Quantum Simulator

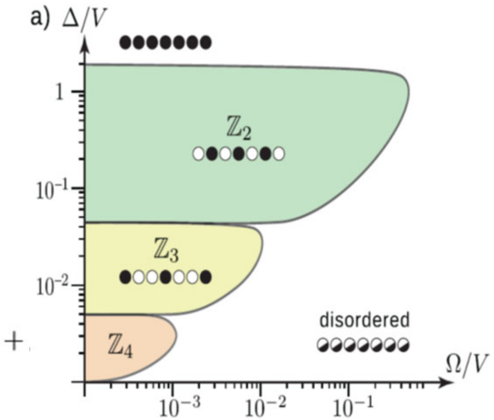
- Lattice geometry, laser parameters, Rydberg states (L=0,1,2…), blockade radius …can be tuned to simulate various quantum spin models!
- Ground states, collective excited states, dynamics, spin-frustration states….rich physics.
- State dimension beyond supercomputer capability!



Quantum Ising Models

$$H = \sum_{i=1}^{N} \frac{\hbar\Omega}{2}\sigma_i^x - \sum_{i=1}^{N} \frac{\hbar\delta}{2}\sigma_i^z + \sum_{j<i} \frac{C_6}{|\mathbf{r}_i - \mathbf{r}_j|^6} n_i n_j.$$

Phase diagram for 1D

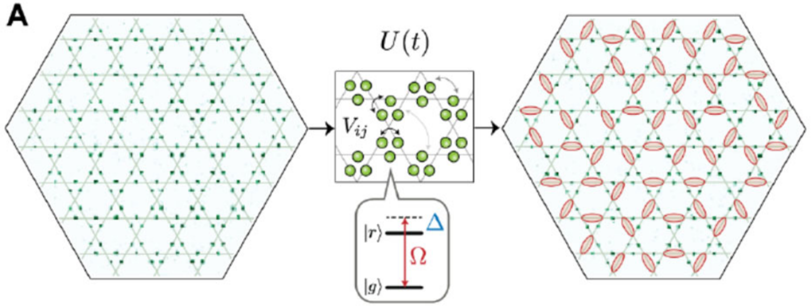


Quantum Spin Liquid State (Resonant Valance Bond)

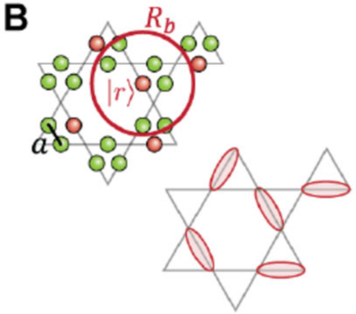
Kagome Lattice

$R_b/a = 2.4$

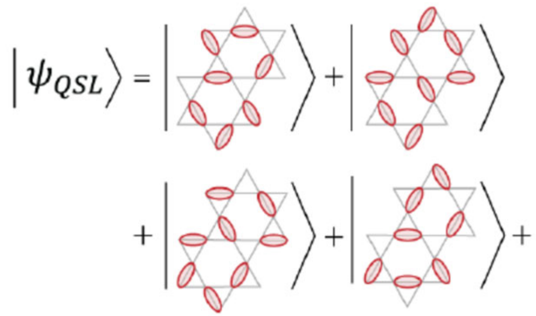
A



B

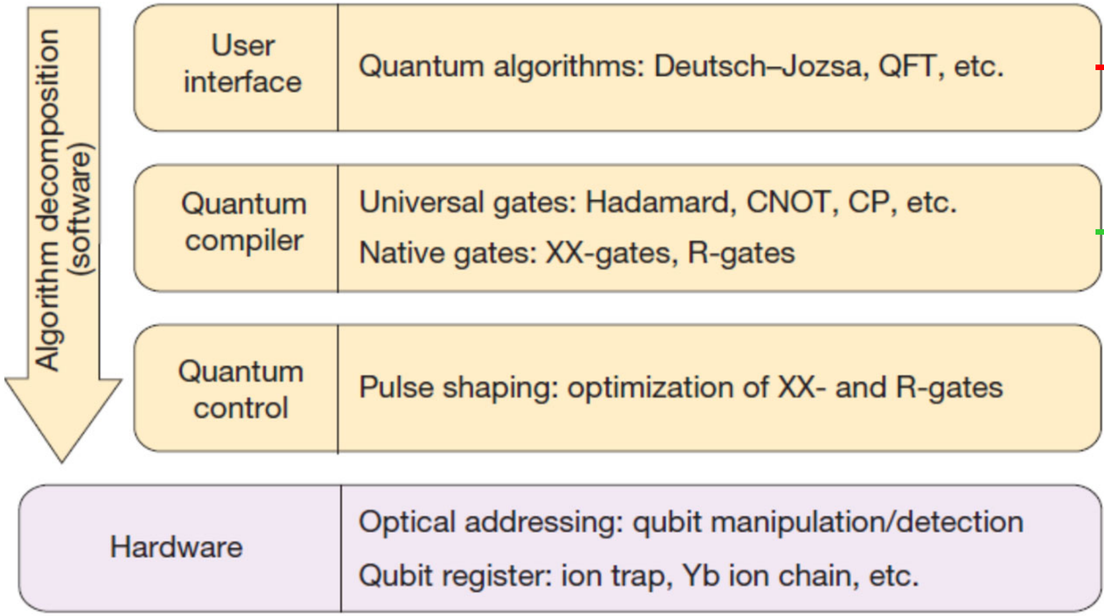


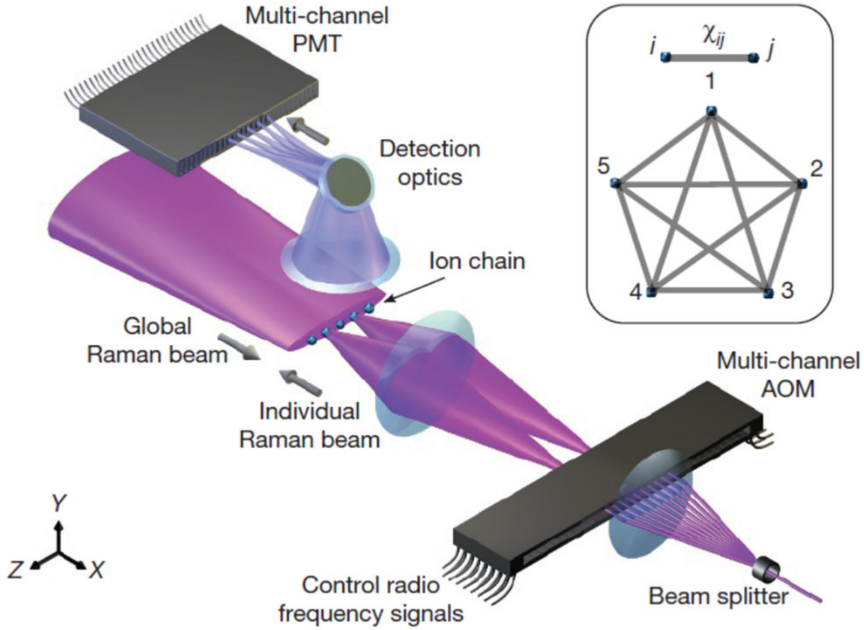
C



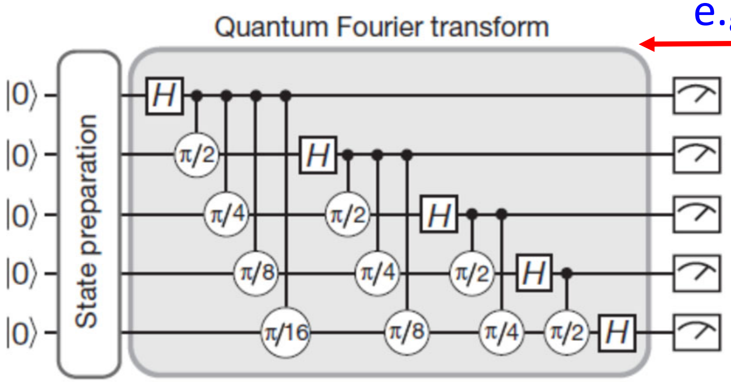
E. Ebad et al., Nature 595, 227(2021);P. Scholl et al., Nature 595, 233(2021); G. Semeghini et al. Science 374, 1242(2021)

Architecture of a Quantum Computer: Trapped Ions case

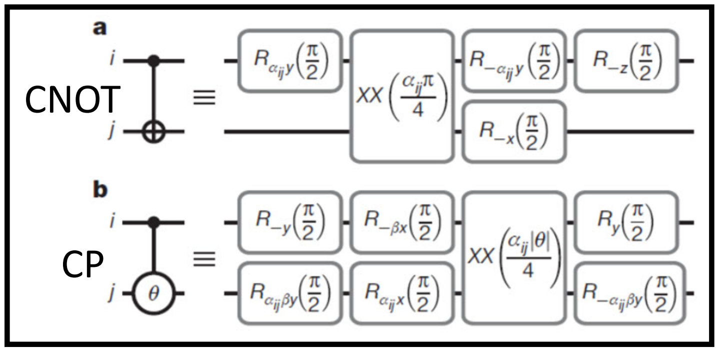




e.g. IBM Qiskit



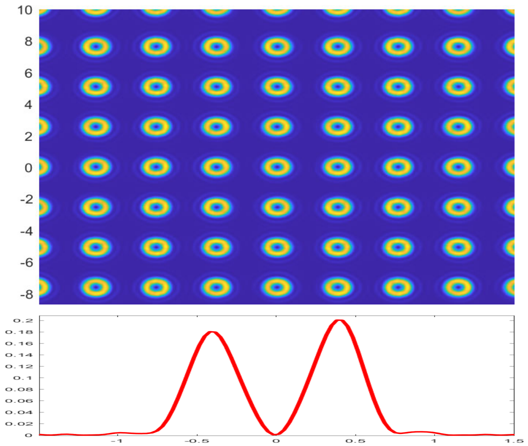
Decompose into native gates



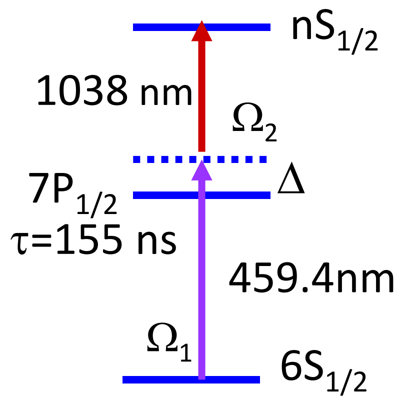
S. Debnath et al., Nature 536, 63(2016)

To Improve the Quantum Gate Performance

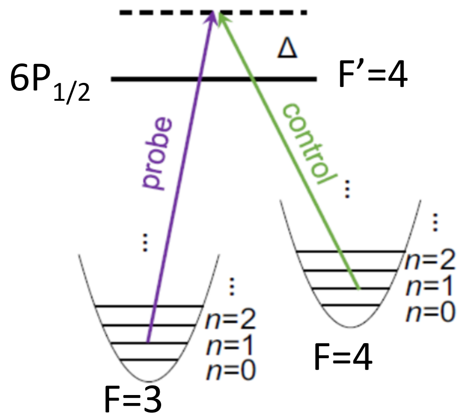
Trapping of Rydberg atoms by bottle traps for longer coherence time

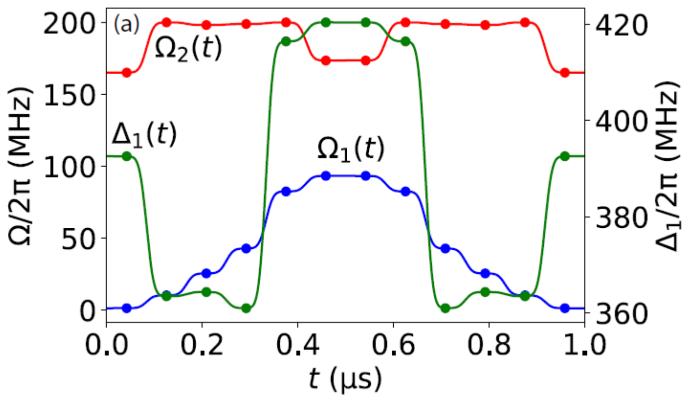


Larger laser detuning & power to reduce spontaneous decay



Dark-state sideband cooling for sub-μK temperature & better fidelity





Advanced gate scheme with adiabatic laser pulses for better fidelity

M. Saffman et al. PRA 101, 062309
X-F Shi, PR Applied 10, 034006

Target gate performance
1, Gate fidelity >0.995
2, Gate operation time <500 ns
3, Coherence time >100 μs

Intrinsic Errors of Rydberg-Atom Quantum Gate

Intrinsic Gate Error

$$E \simeq \frac{7\pi}{4\Omega\tau}\left(1+\frac{\Omega^2}{\omega_{10}^2}+\frac{\Omega^2}{7\mathrm{B}^2}\right)+\frac{\Omega^2}{8\mathrm{B}^2}\left(1+6\frac{\mathrm{B}^2}{\omega_{10}^2}\right).$$

Finite lifetime of Rydberg states

Double Rydberg excitation Imperfect blockade

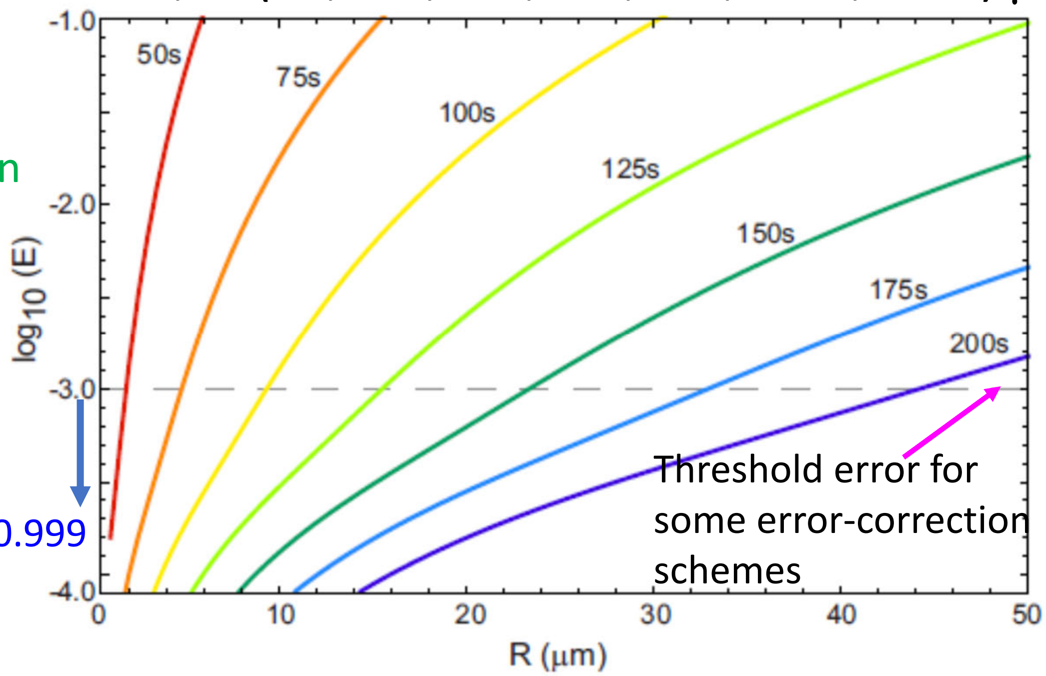
Ω: Rabi frequency of the laser pulse
τ: lifetime of the Rydberg state
B: mean blockade shift
ω_{10}: spacing of the two qubit states

$$\omega_{10} \gg (\mathrm{B},\Omega)$$

$$\boxed{\Omega_{\rm opt} = (7\pi)^{1/3}\frac{\mathrm{B}^{2/3}}{\tau^{1/3}}. \quad E_{\rm min} = \frac{3(7\pi)^{2/3}}{8}\frac{1}{(\mathrm{B}\tau)^{2/3}}.}$$

^{87}Rb, τ=(70,180,340,570,860,1200,1600) μs

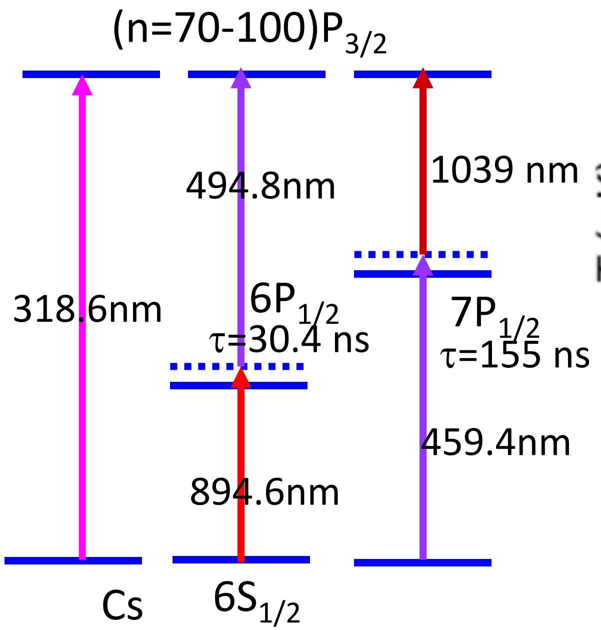
Fidelity≥ 0.999



M Saffman et al., Rev Mod Phys 82, 2313(2010)
P. Aliferis et al. PRA 79, 012332(2009)

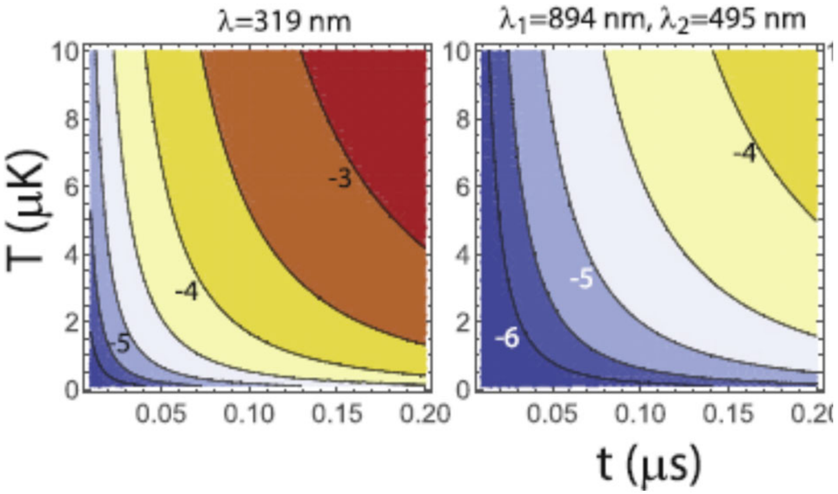
Technical Errors of Rydberg-Atom Quantum Gates

- Finite-temperature Doppler dephasing
- Laser phase, frequency, and intensity noise, pointing stability
- Spontaneous decay of the intermediate state in two-photon Rydberg excitation
- Stray magnetic and electric fields & their fluctuations
- …

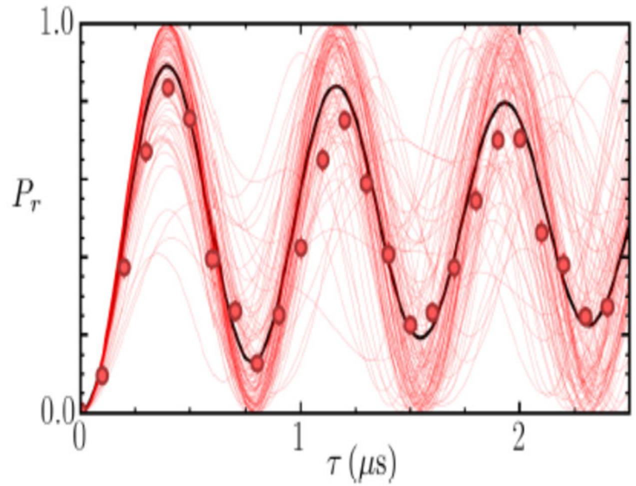


Cs Doppler-limited Bell Fidelity

$\log_{10}(1 - F_D)$.



Laser phase noise effect on Rabi oscillation of Rydberg excitation



M Saffman, J Phys B 49, 202001(2016)

S. de Leseleuc et al. PRA 97, 053803(2018)

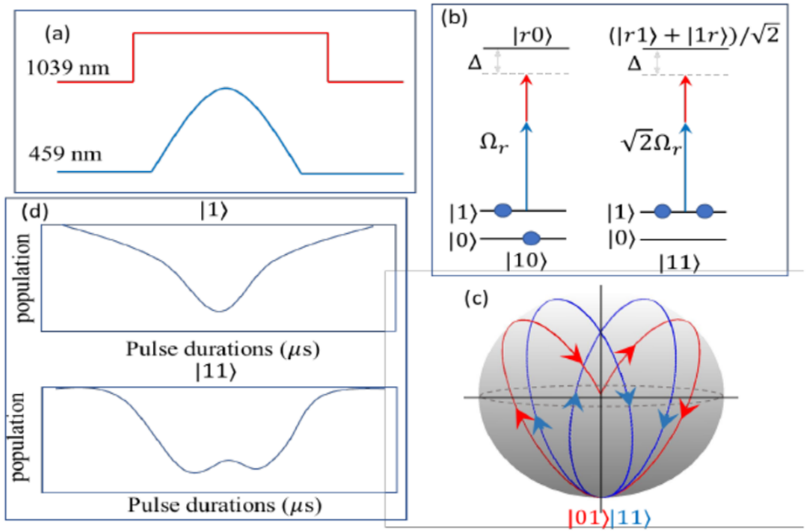
Preliminary Progresses (1/4): Theoretical Study on 2-qubit Gate

PI 任翔華
Master 張子軒
Ph.D.王彩妮

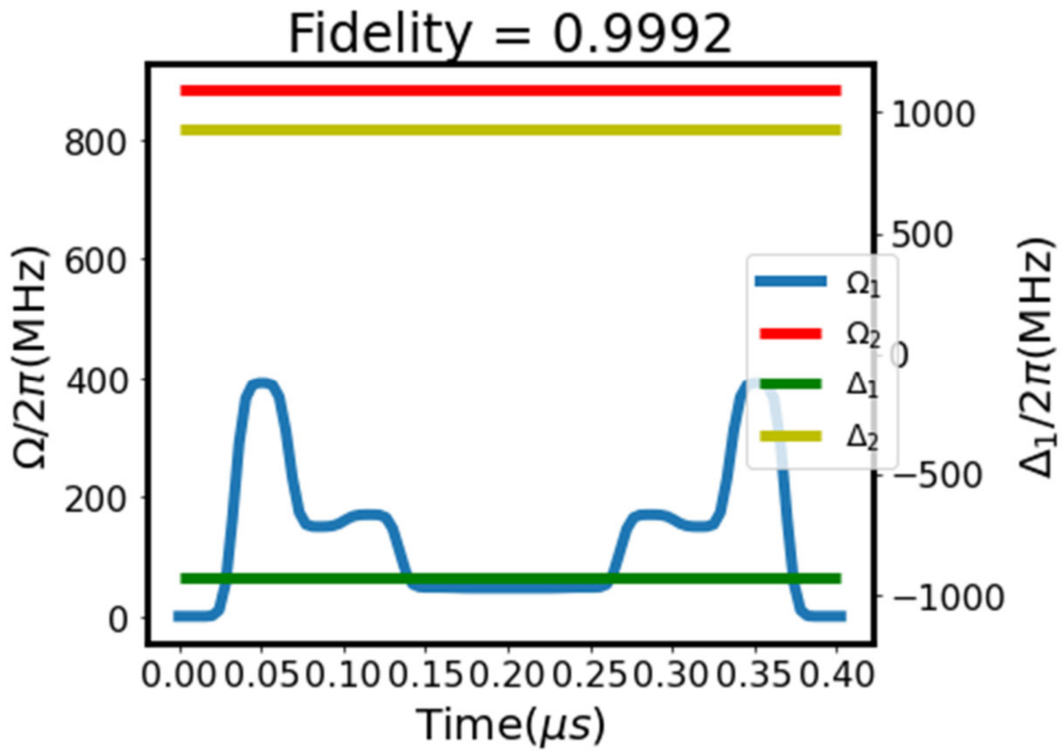
Controlled-Phase (CZ) Gate

$$|00\rangle \rightarrow |00\rangle,$$
$$|01\rangle \rightarrow |01\rangle e^{i\phi},$$
$$|10\rangle \rightarrow |10\rangle e^{i\phi},$$
$$|11\rangle \rightarrow |11\rangle e^{i(2\phi-\pi)}.$$

$$\phi_{00} - \phi_{01} - \phi_{10} + \phi_{11} = \pm\pi,$$

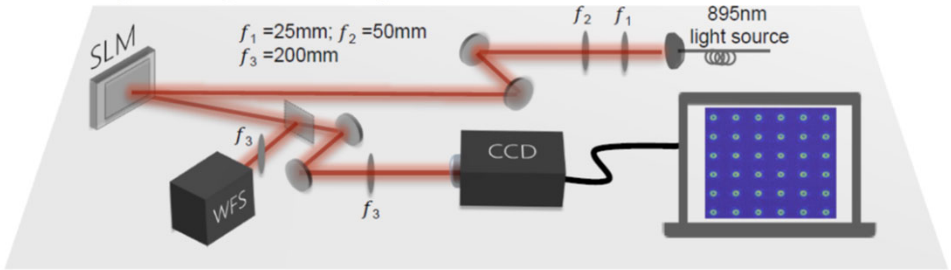


Easy to be implemented experimentally with a high gate fidelity !

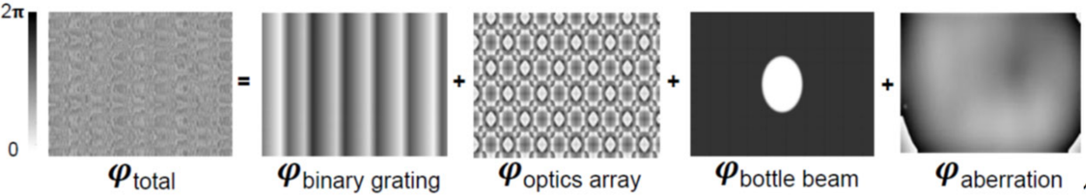


Preliminary Progresses (2/4): Generation of Bottle Beam Arrays

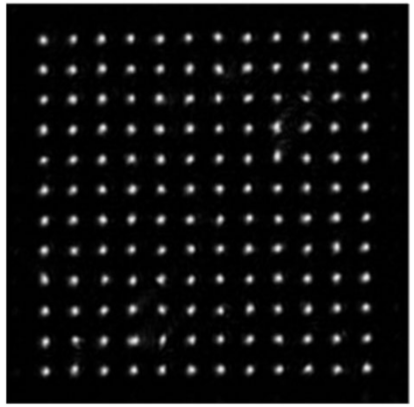
Optical System Setup



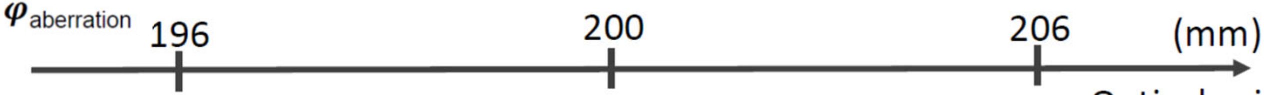
SLM screen



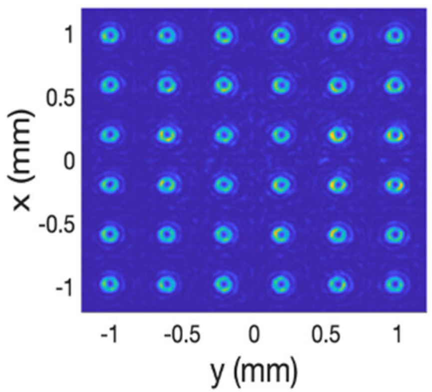
Gaussian beam arrays (12×12)



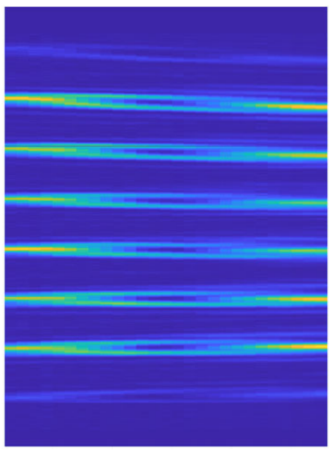
Ph.D. 王彩妮
Postdoc蕭雅萘



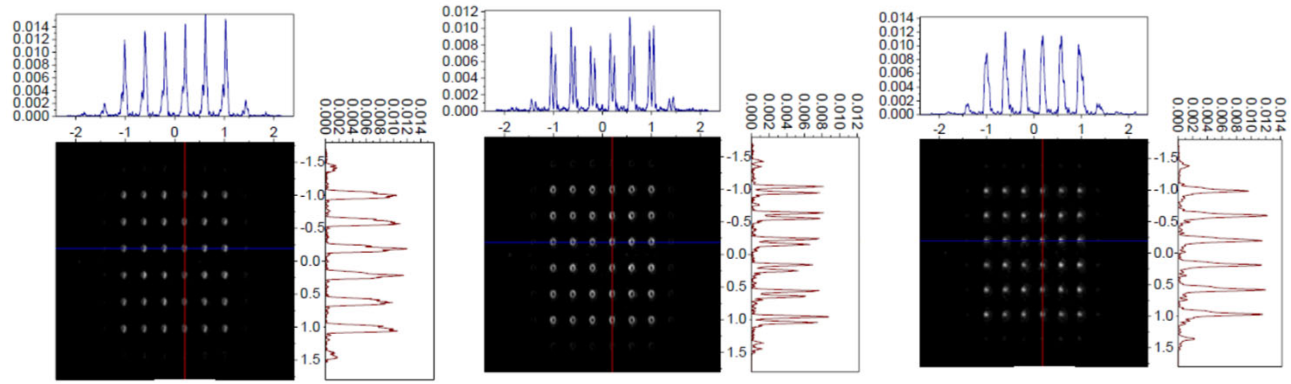
Bottle beam arrays



Trap size ~80 μm,
Trap-to-trap distance ~400 μm

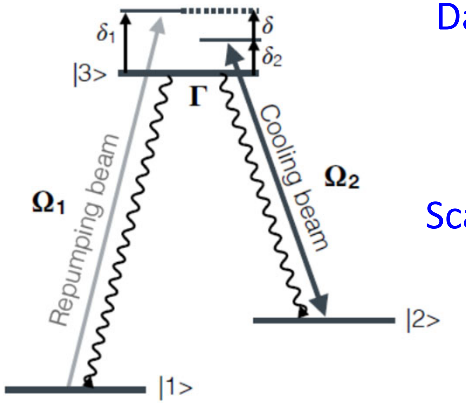


Experimental Results



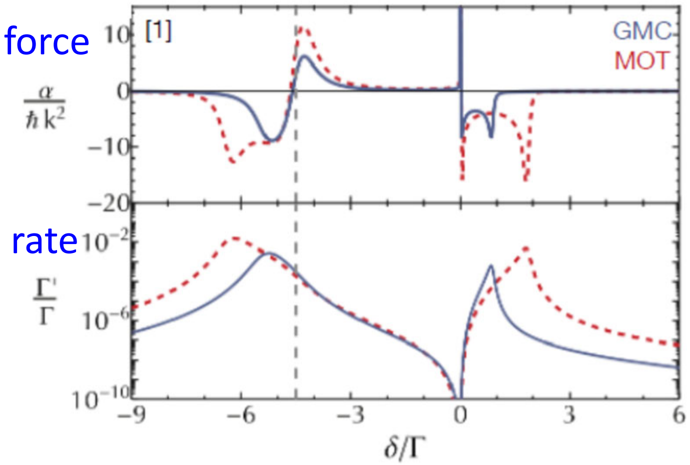
Preliminary Progresses (3/4): Λ-Enhanced Grey Molasses Cooling

Postdoc蕭雅萘
Master 陳逸璇



Damping force

Scattering rate



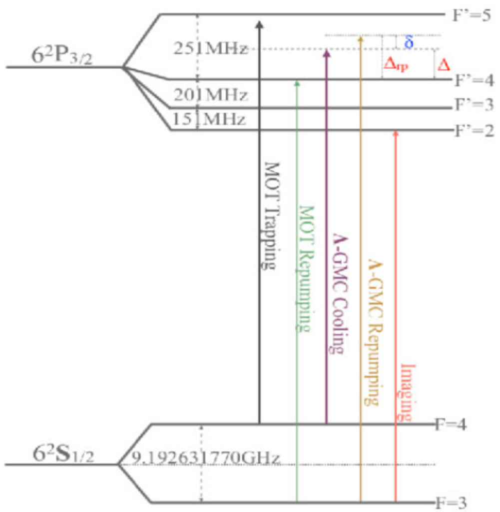
$$\mathcal{F}(v) \simeq -\alpha\, v$$

friction coefficient

$\alpha > 0$; cooling

A. T. Grier et at., PRA 87, 063411(2013)

Cs D2 line



D2 line
1.7 μk

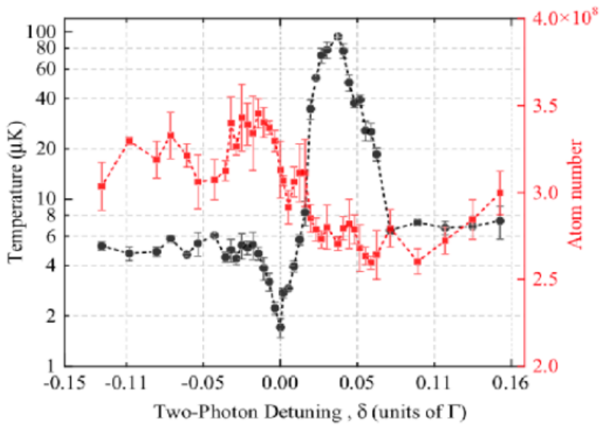
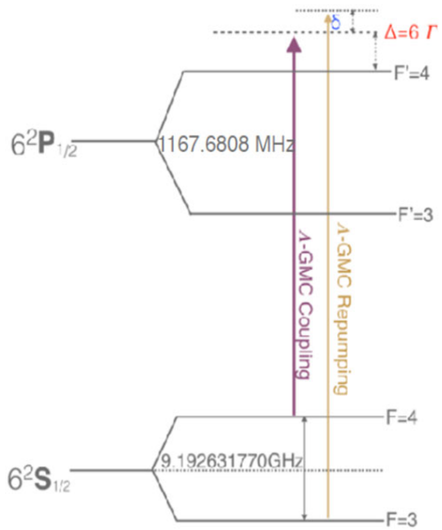
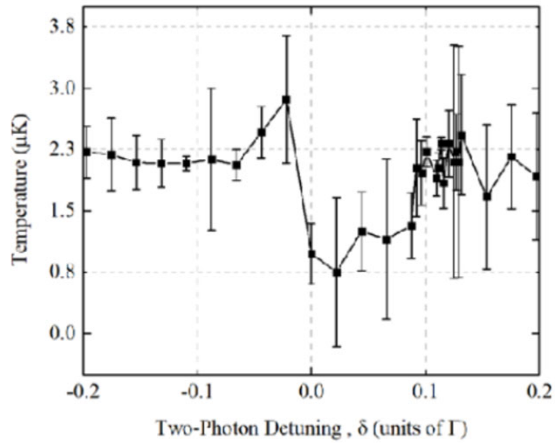


FIG. 6. Atom number (square) and temperature (circle) after 4-ms Λ-GMC versus the two-photon detuning δ. $\Delta = 5.73\,\Gamma$. $I_{\rm cool} = 0.273\, I_{\rm sat}$, and $I_{\rm rep}/I_{\rm cool} = 0.08$.

Cs D1 line



D1 line
0.7μK



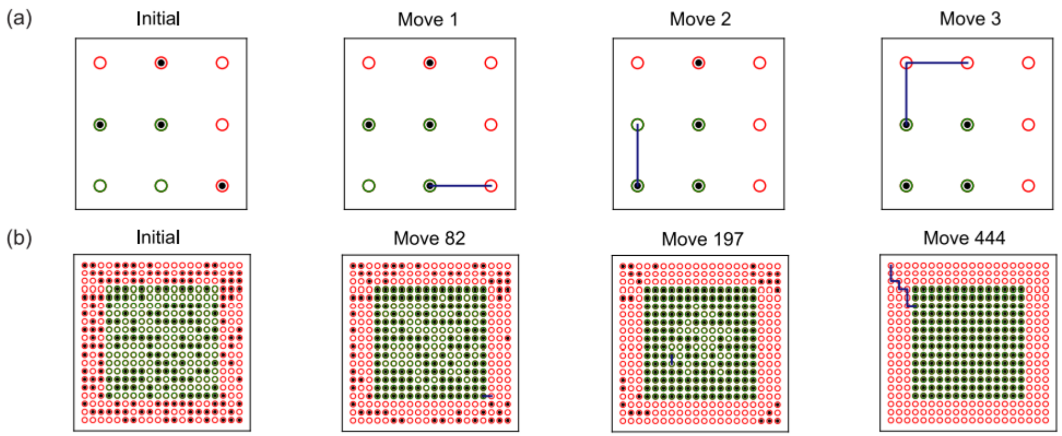
After 9ms ΛGMC versus the two photon detuning δ, $\mathbf{\Delta}$=6.57$\mathbf{\Gamma}$.

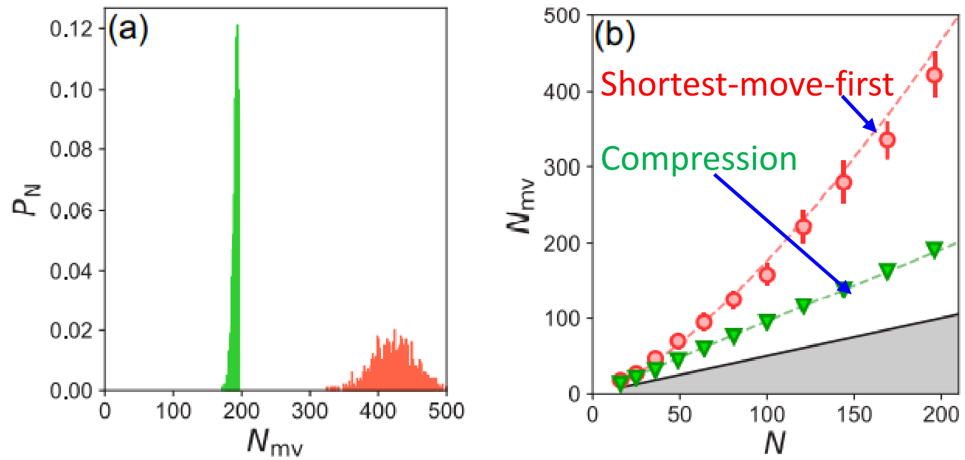
Y.-F. Hsiao et at., PRA 98, 033419(2018)

Preliminary Progresses (4/4): Simulation on Atom Rearrangement

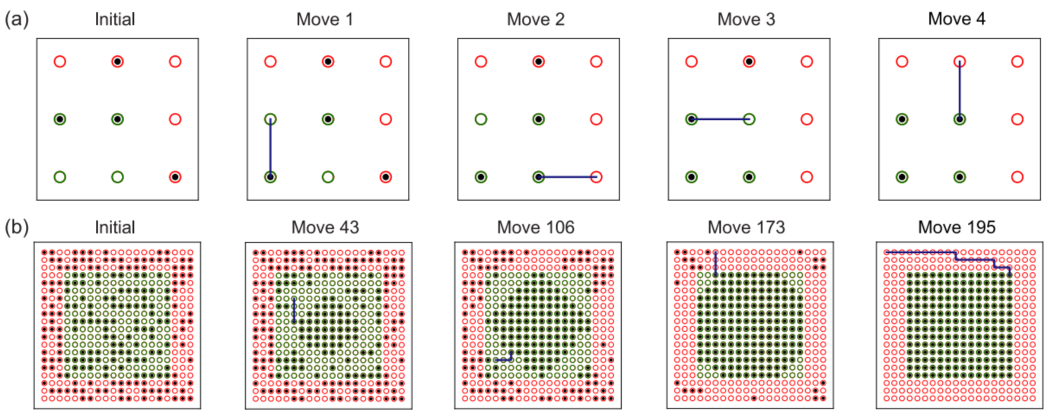
Under 曹統
RA 鄭凱鴻

Shortest-move-first algorithm





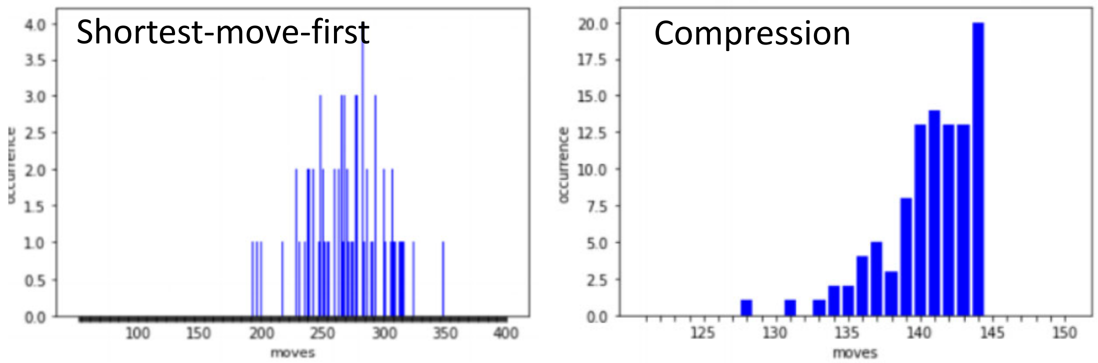
Compression algorithm



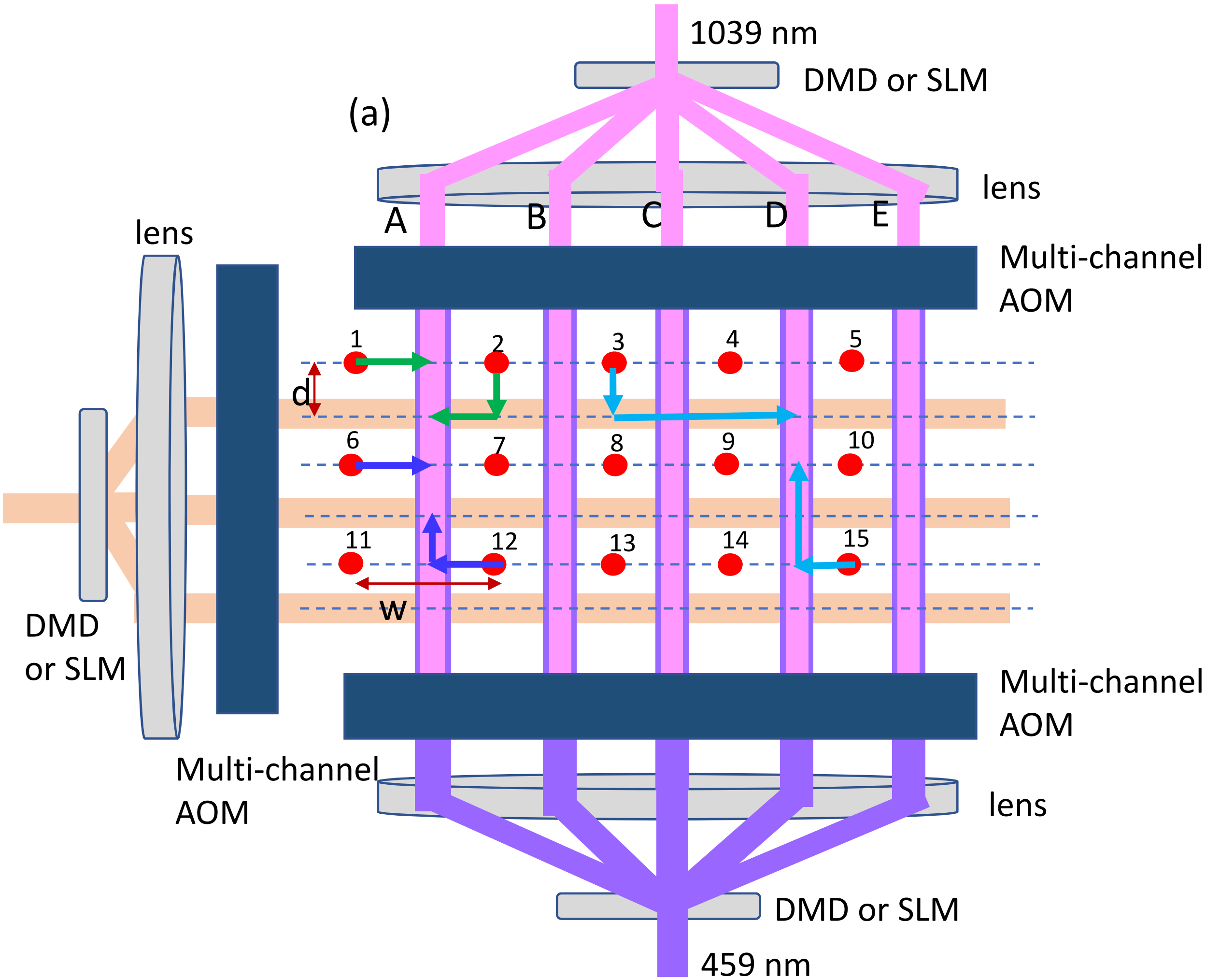
K Schymik et al, PRA102, 063107(2020)

Our simulation results

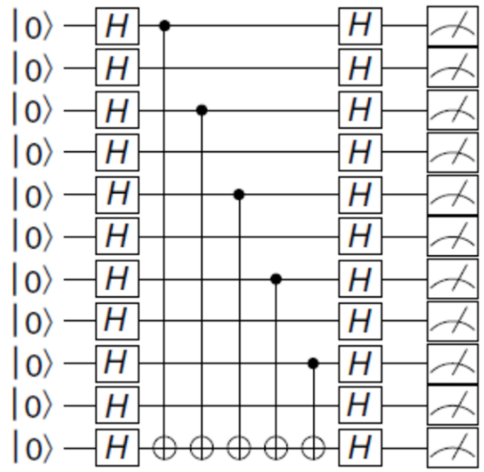
Target array 12×12, 0.6 filling rate



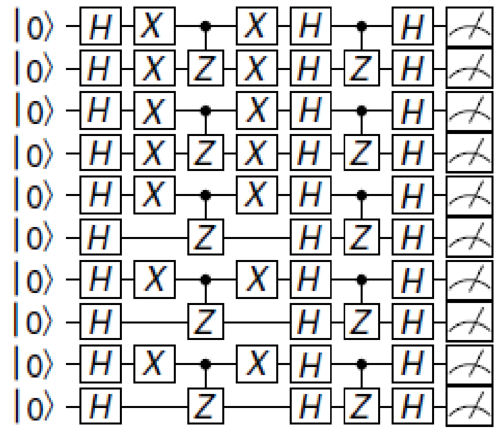
Realizing the large-scale quantum logic gate operations



Bernstein-Vazirani benchmarking algorithm

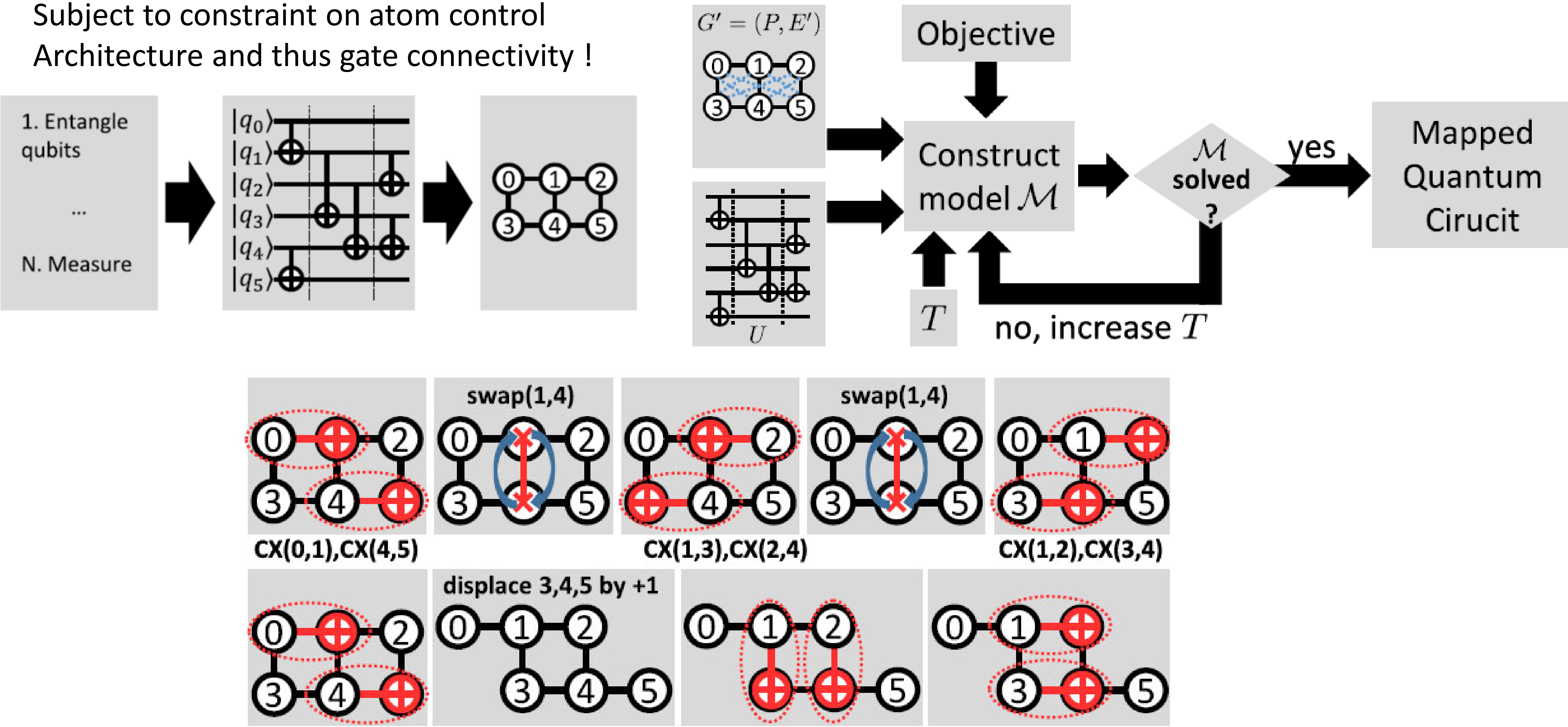


Hidden-shift algorithm



Optimal Quantum Circuit Mapping for Rydberg-Atom Platform

Subject to constraint on atom control
Architecture and thus gate connectivity !



S. Brandhofer et al., arXiv: 2109.04179

Quantum Supremacy through Quantum Approximate Optimization Algorithm (QAOA) ?

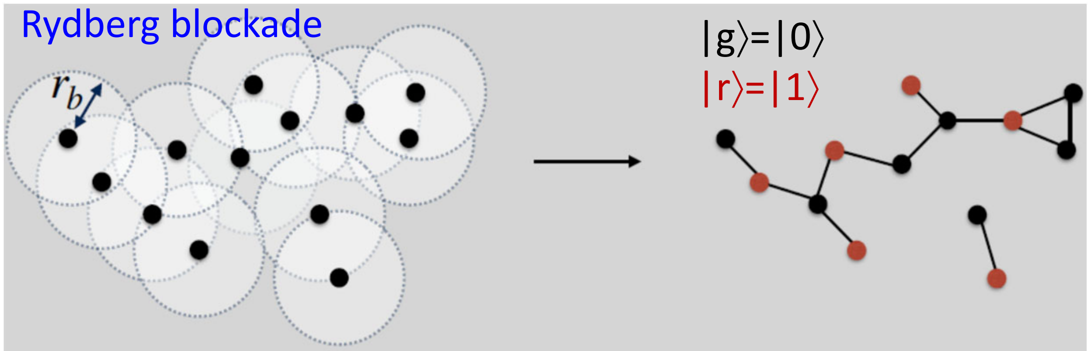
- Different quantum platform might have its own favorable computational problems.
- To steer the dynamics of quantum systems such that their final states provide solutions to optimization problems.

Cost function

$$C(z_1,\ldots,z_N) = -\sum_{i=1}^{N} z_i + U\sum_{\langle i,j\rangle} z_i z_j,$$

z_i=1 for independent set, z_i= 0 otherwise, U>>1, <i,j> adjacent vertices

$\Rightarrow$ Minimum of C(z_1,....z_N) corresponds to the maximum independent set of the graph

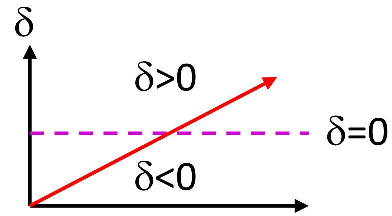


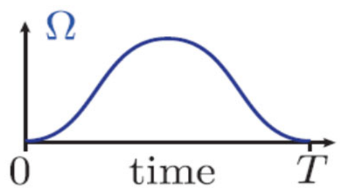
Unit-disk Maximum Independent Set Problem

$$H = \sum_{i=1}^{N} \frac{\hbar\Omega}{2}\sigma_i^x - \sum_{i=1}^{N} \frac{\hbar\delta}{2}\sigma_i^z + \sum_{j<i} \frac{C_6}{|\mathbf{r}_i - \mathbf{r}_j|^6} n_i n_j.$$

Ω<<δ, δ>0 $\Rightarrow$ UD MIS

$$n_i = (\sigma_i^z + \mathbf{1})/2$$





Quantum 4, 327(2020); arXiv: 2012.14859; 2006.11190; 1602.07674; 1808.10816& PRX 10,021067; Science 376, 1209(2022)

Physical Platforms for Quantum Computing

Platform	Qubit number	Fidelity(%) (1 qubit)	Fidelity(%) (2 qubits)	State Prep& Meas(SPAM) %	Coherence time (μs)	Gate time (μs)	Notes
Supercondu cting qubits	53(Google 2019) 65(IBM 2020)	~99.84 (Google)	~99.1-99.4 (Google)	96.2-96.9(Google 2019)	<~50	~0.025-0.05	large-scale run
Trapped ions	11, 32(IonQ full run) 79(IonQ, single qubit)	~99-99.97(IonQ) 99.9999[1] 99.99[2,3]	~98-99.3(IonQ) 99.9[2,3]	99.99[1] 99.3[4]	1-60 sec	~100 (phonon) ~0.5-2(shaped laser, 5)	large-scale run
Silicon quantum dot	2	~99.3-99.7[6]	~78[6] ~99[7]	~71[6]	~<100[6]	~0.1-0.7 [6,7]	No large-scale run
Neutral atoms	~100-300[8-11]	99.23[13] 99.5-99.67[14]	0.73[12] 98.0-99.1[14] 97.3[15] 97.4[16]	99.94[11] 99.7-99.96[14]	~7[14] ~27[15] ~100[16]	~7[12] ~4.5[15] ~0.3[16]	No large-scale run

1, PRL 113,220501(2014); 2, PRL 117,060505(2016); 3, PRL 117,060504; 4, Nat Com. 10, 5464(2019);5, Nature 555,75(2018); 6, Nature 526, 410 (2015)
7, arxiv:2007.09034;8, PRL122, 203601(2019);9, PRL123,230501(2019);10, Axiv1912.04200;11, Nat Phys 15,538(2019);12, PRL104, 010503(2010);
13, PRL114, 100503(2015); 14, Nat Phys 16, 857(2020);15, PRL121, 123603(2018);16, PRL 123, 170503(2019)

Improvements on Gate Fidelity & Coherence Time

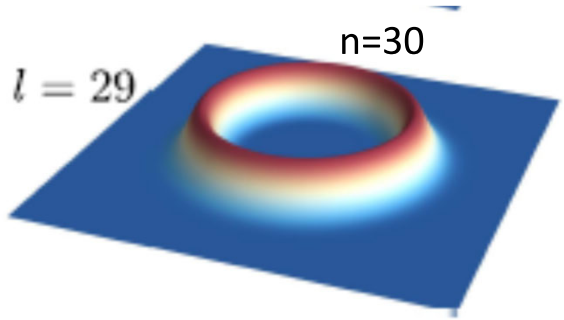
- Short-term improvements:
  - Colder atom temperature: Raman sideband cooling
  - More stable lasers (sub-kHz linewidth locked to cavity)
  - More intense laser (larger one-photon detuning for intermediate state)
  - Blue-detune/magic-wavelength traps (that can trap Rydberg atoms for longer coherence time)
- Dramatic improvement
  - Cryogenic traps (longer lifetime)
  - Circular Rydberg state (longer lifetime)

	T=0 K			T=77 K			T=300 K		
n	$S_{1/2}$	$P_{1/2}$ / $P_{3/2}$	$D_{3/2}$ / $D_{5/2}$	$S_{1/2}$	$P_{1/2}$ / $P_{3/2}$	$D_{3/2}$ / $D_{5/2}$	$S_{1/2}$	$P_{1/2}$ / $P_{3/2}$	$D_{3/2}$ / $D_{5/2}$
70	371.58	838.67	197.83	266.13	480.61	167.46	143.28	206.88	113.71
		912.81	200.70		501.50	169.54		209.24	114.76
75	462.69	1042.0	245.06	323.71	575.10	204.75	170.01	241.62	136.14
		1134.1	248.60		598.87	207.25		244.03	137.36
80	567.60	1275.9	299.26	388.13	679.12	246.83	199.18	279.08	160.87
		1388.3	303.57		705.74	249.80		281.50	162.26

Circular Rydberg state

$|m| = l = n-1, \tau \propto n^5 (not\ n^3\ for\ low\ l)$

n=50, τ≅ 30 ms



Problem: difficult to prepare into this state