Phase Transition and Gravitational Waves

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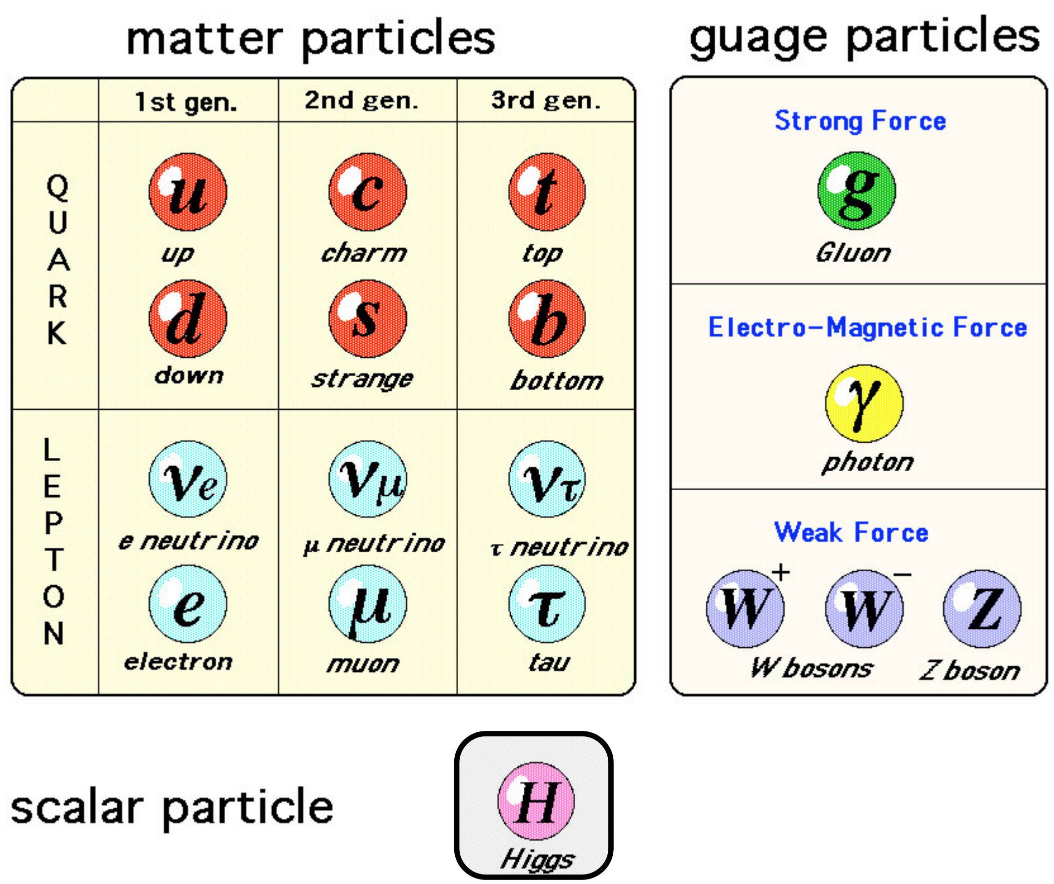
The Future is Illuminating , June 28-30, 2022

Introduction

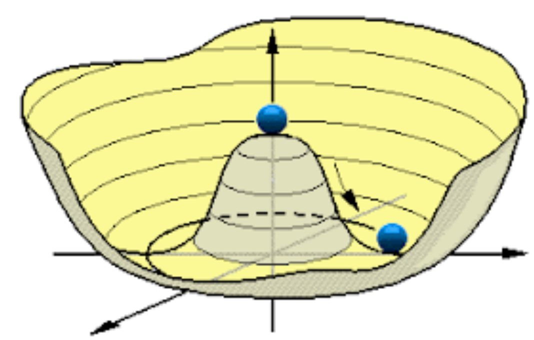
❖ Standard Model



❖ Spontaneous Electroweak Symmetry Breaking

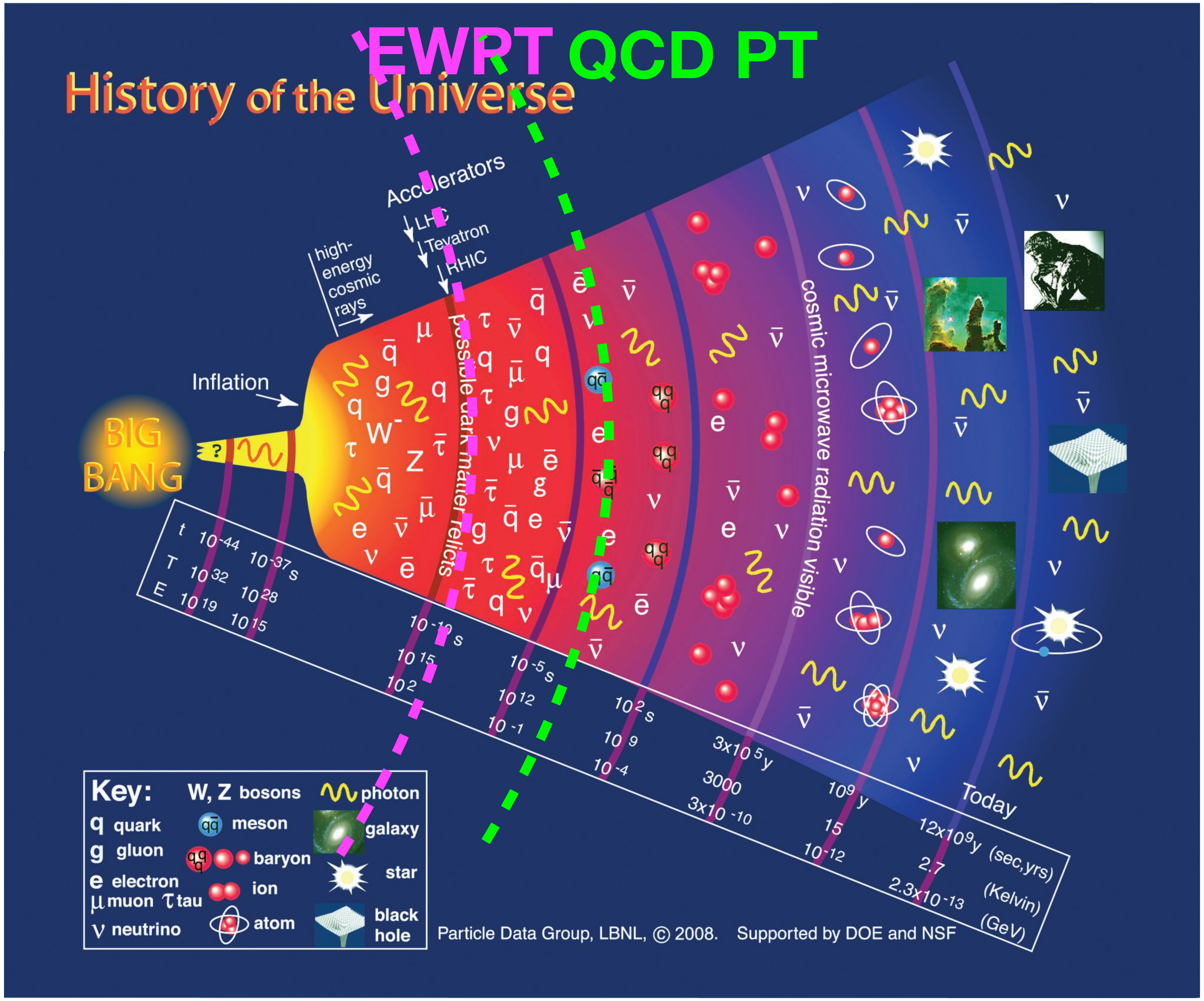
$$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_{\mathrm{em}}$$

$$\langle h \rangle = 246\ \mathrm{GeV}$$



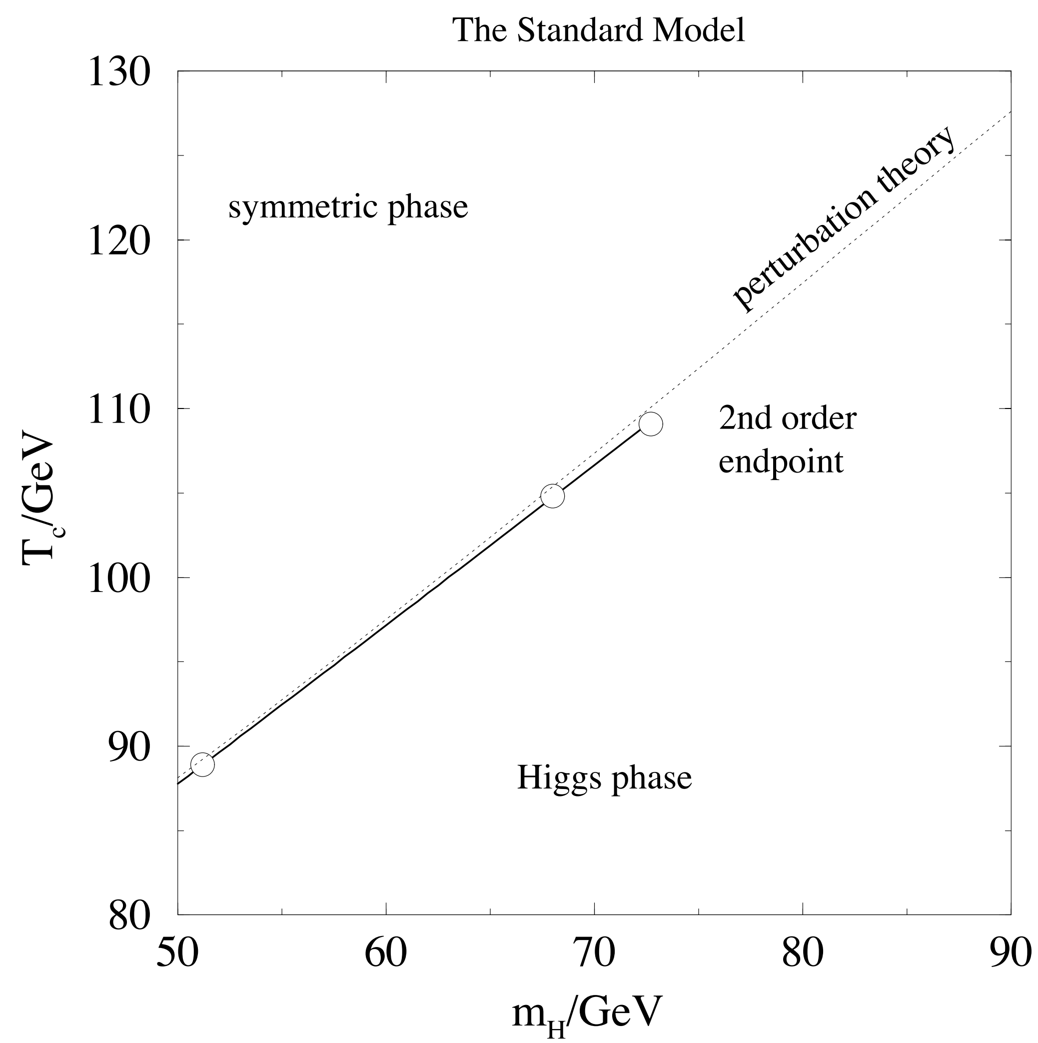
→ The SM particles get their masses.

Phase Transition



Electroweak Phase Transition

❖ **Standard Model :**



Laine, Rummukainen, hep-lat/9809045

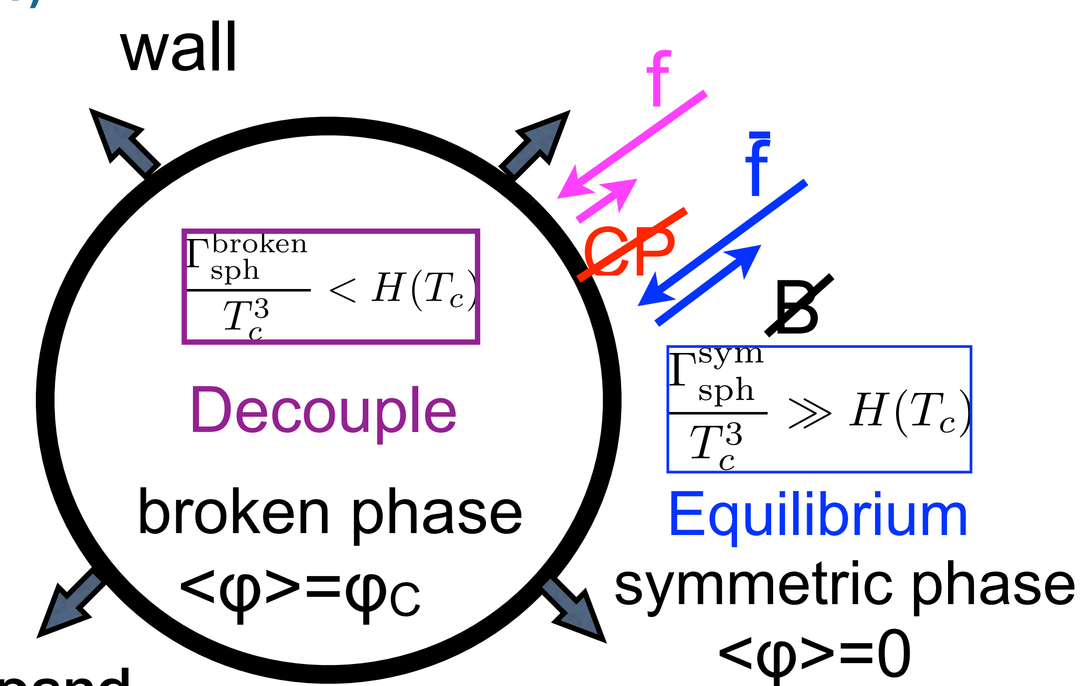
* 1st order PT gives an upper limit on m_h : $m_h \lesssim 70$ GeV
* $m_h \simeq 125$ GeV → crossover

Electroweak Baryogenesis

❖ **Baryon Asymmetry** $\frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq 6 \times 10^{-10}$

❖ **The three Sakharov's conditions** Sakharov (1967)

- B violation
- C, CP violation
- out of equilibrium



❖ **EW Baryogenesis**

✱ **The strong 1st order PT is required.**

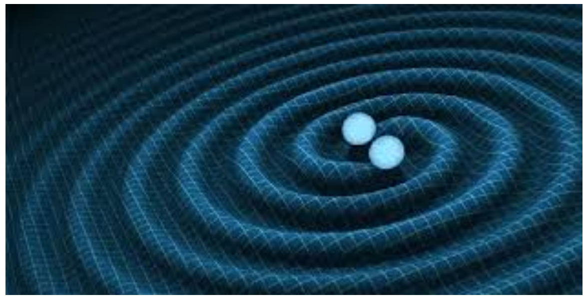
→ Bubbles of broken phase are nucleated and expand.

✱ **The sphaleron process should be decoupled in the broken phase.**

$$\frac{\Gamma_{\rm sph}^{\rm broken}}{T_c^3} < H(T_c) \quad \rightarrow \quad \frac{\varphi_c}{T_c} > 1$$

The criterion for the "strong" 1st order PT.

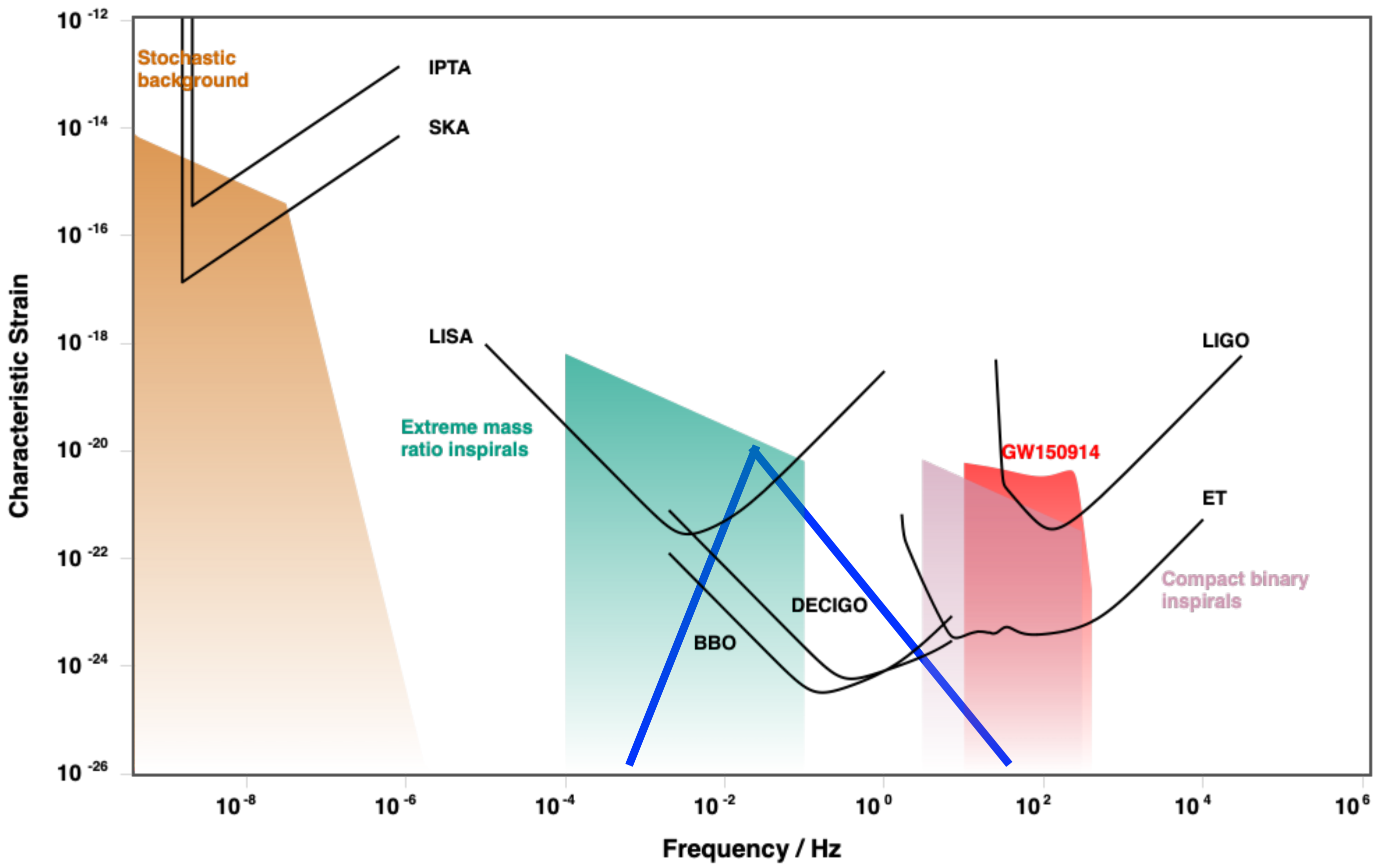
Gravitational Wave



- The detection of GWs by LIGO in 2015. → a new era

Phenomena in the early universe can be probed by the GW.

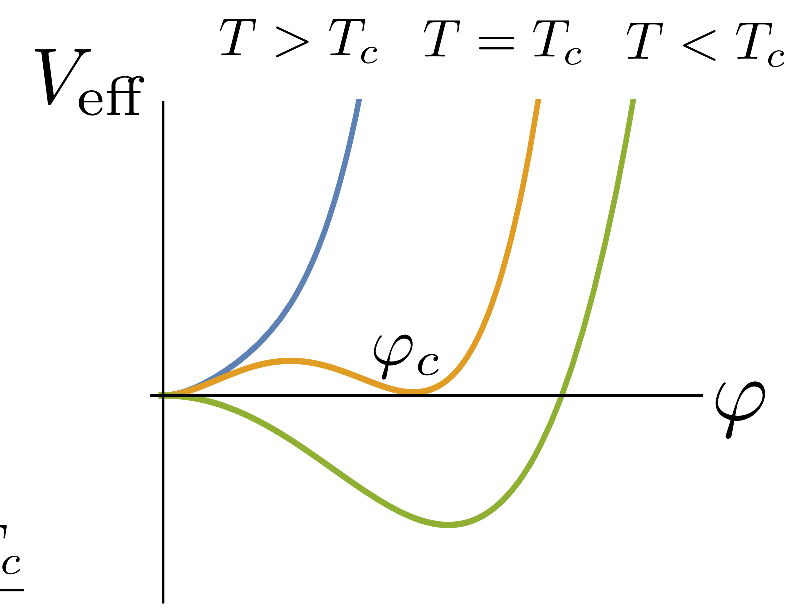
- Many new GW experiments are planned in the coming decades.
- The strongly 1st order EWPT predicts the GWs in the frequency range of 10^{-3} - 10^{-1} Hz.



- Introduction
- 1st order PT& Gravitational wave
- Extended Higgs model .. 2HDM
- Scale invariant model .. Hidden QCD sector
- Summary

First-order phase transition

- **Strong 1st order PT :**
  - **There is a sufficiently high and wide potential barrier separating the two degenerate vacua at T=Tc.**



- **High temperature expansion :**

$$V_{\rm eff}(\varphi, T) = D(T^2 - T_0^2)\varphi^2 - ET|\varphi|^3 + \frac{\lambda_T}{4}\varphi^4$$

- **Two degenerate minima :** $\varphi = 0$ and φ_c. $\quad \varphi_c = \dfrac{2ET_c}{\lambda_{T_c}}$

- **The condition for the strong 1st order PT :** $\dfrac{\varphi_c}{T_c} = \dfrac{2E}{\lambda_{T_c}} > 1$

The magnitude of *E* is crucial for the strong 1st order PT.

- **The cubic term arises from the bosonic thermal corrections.**

One-loop thermal potential :

$$V_T = \frac{T^4}{2\pi^2}\left[\sum_f n_f J_F\left(\frac{m_f^2}{T^2}\right) + \sum_B n_B J_B\left(\frac{m_B^2}{T^2}\right)\right]$$

m : field dependent mass

- Boson-loop:

$$J_B(y) \approx -\frac{\pi^4}{45} + \frac{\pi^2}{12}y - \frac{\pi}{6}y^{\frac{3}{2}} - \frac{1}{32}y^2\log\left(\frac{|y|}{a_b}\right) + \mathcal{O}(y^3),$$

- Fermion-loop:

$$J_F(y) \approx -\frac{7\pi^4}{360} + \frac{\pi^2}{24}y + \frac{1}{32}y^2\log\left(\frac{|y|}{a_f}\right) + \mathcal{O}(y^3)$$

$$a_b = 16\pi^2\exp(3/2 - 2\gamma_E), \quad a_f = \pi^2\exp(3/2 - 2\gamma_E)$$

First-order phase transition

❖ **The cubic term arises from the bosonic thermal corrections.**

SM : $$E = \frac{1}{12\pi v^3}\left(6m_W^3 + 3m_Z^3\right)$$

BSM : $$E = \frac{1}{12\pi v^3}\left(6m_W^3 + 3m_Z^3 + \cdots\right)$$

↑ extra bosonic degree of freedom

❖ **The "non-decoupling effects" is important.**

field dependent mass : $m_\Phi^2(\varphi) = M^2 + \lambda\varphi^2$

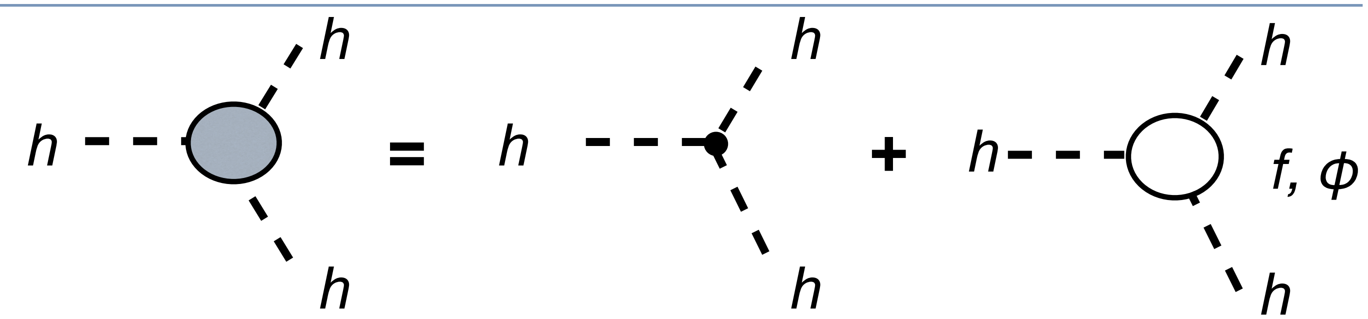
- $M^2 > \lambda\varphi^2$ $\quad V_{eff} \ni -|M|^3 T\left(1 + \frac{\lambda\varphi^2}{M^2}\right)^{3/2}$ **not contribute to E**

- $M^2 < \lambda\varphi^2$ $\quad V_{eff} \ni -\lambda^{3/2} T\varphi^3\left(1 + \frac{M^2}{\lambda\varphi^2}\right)^{3/2}$ **large λ with small M → large E**

Non-decoupling effect is required.

Higgs Self Coupling

❖ **Higgs self coupling :** $\lambda_{hhh} = \dfrac{\partial^3 V^{T=0}}{\partial h^3}$



$$\lambda_{hhh} \simeq \frac{3m_h^2}{v}\left[1 - \frac{m_t^4}{\pi^2 v^2 m_h^2} + \frac{m_\Phi^4}{12\pi^2 m_h^2}\left(1 - \frac{M^2}{m_\Phi^2}\right)^3\right]$$

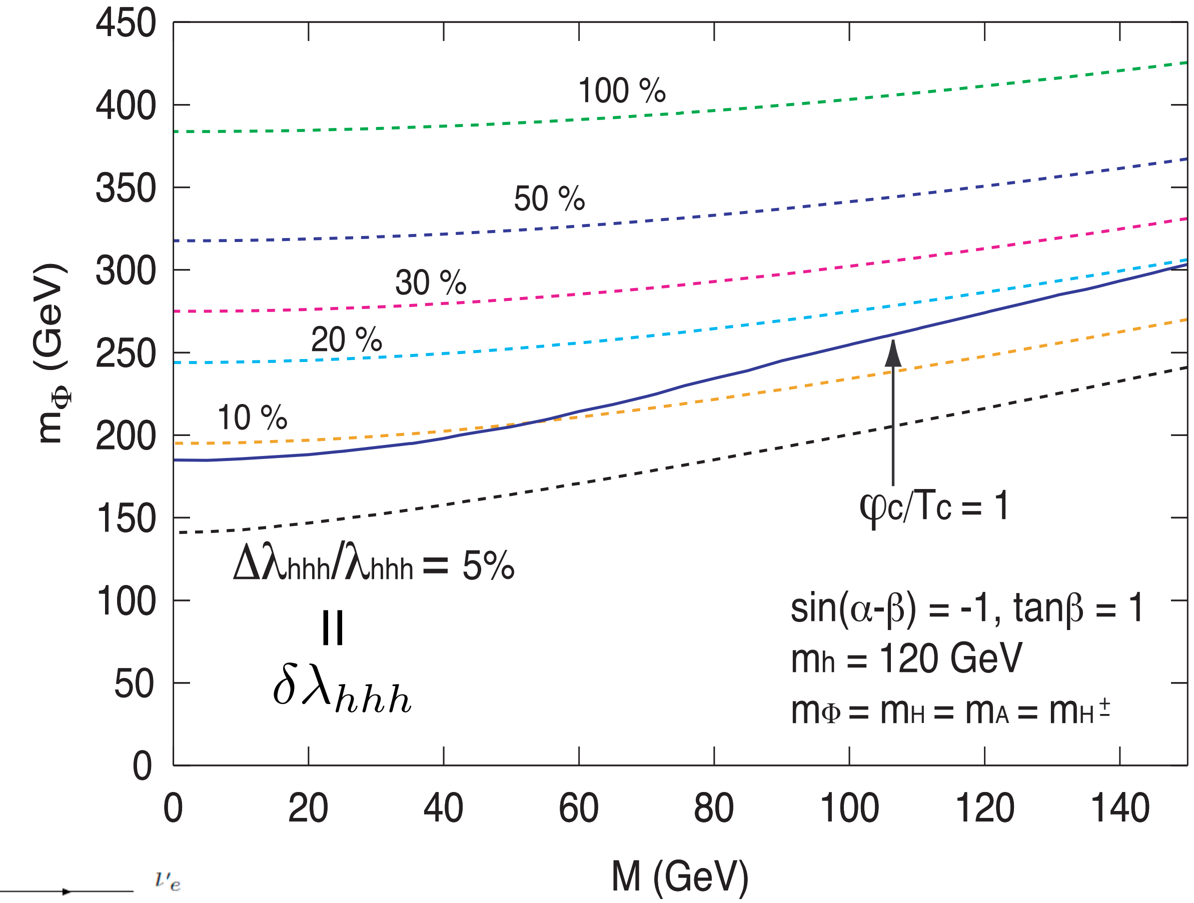
$$m_\Phi^2 = M^2 + \lambda v^2$$

$M^2 < \lambda v^2$ →**The quantum corrections grow with *m*⁴.**

❖ **cf) Two Higgs Doublet Model (2HDM)**

$$\Phi = H, A, H^\pm \qquad \delta\lambda_{hhh} \equiv \frac{\lambda_{hhh}^{\rm 2HDM} - \lambda_{hhh}^{\rm SM}}{\lambda_{hhh}^{\rm SM}}$$

Contour plot of $\Delta\lambda_{hhh}/\lambda_{hhh}$ and φ_c/T_c in the m_Φ-M plane

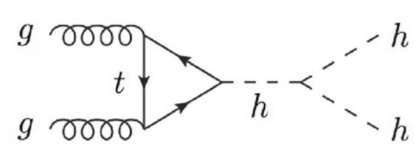


Kanemura, Okada, Senaha, PLB (2005)

❖ **Higgs pair production :**

direct probe for the Higgs self-coupling

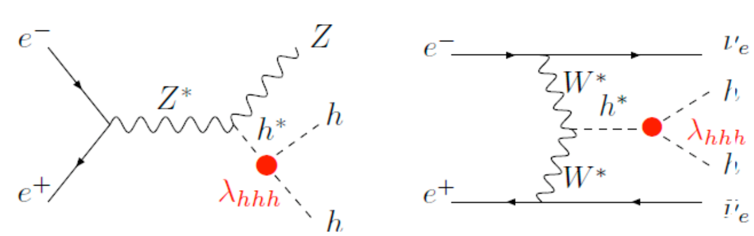
- **Current limit** $-4.2 < \delta\lambda_{hhh} < 10.9$ @95% CL
- **HL-LHC** $\delta\lambda_{hhh} \sim 50\,\%$



- **CEPC, ILC, FCC-ee**

ILC (√s = 500 GeV , L = 3 ab^-1)

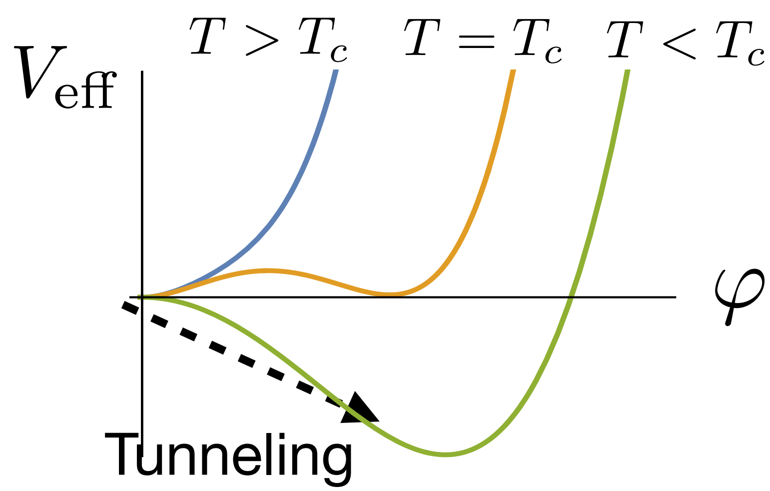
$\delta\lambda_{hhh} \sim 27\,\%$

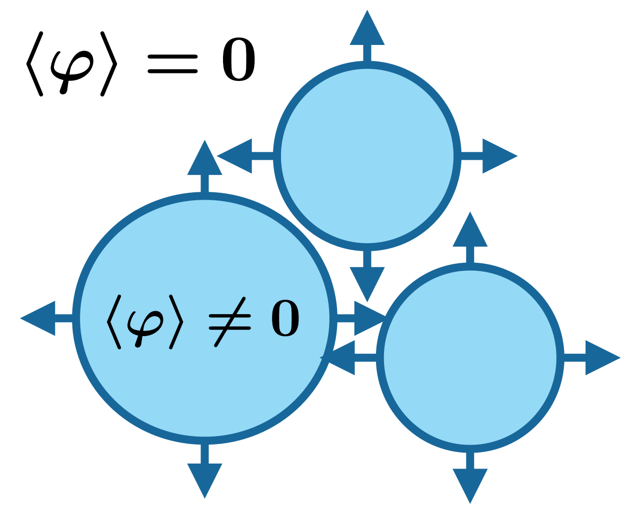


de Blas et al., arXiv:1905.03764

Gravitational Wave

- A first order phase transition is characterized by the nucleation of bubbles of the broken phase.





- Bubble nucleation:

  - Probability of the bubble nucleation :

$$\Gamma(T) \simeq T^4 \left(\frac{S_3}{2\pi T}\right)^{3/2} e^{-S_3/T}$$

S_3 : the three-dimensional Euclidean action

- Nucleation temperature : T_n

$$\left.\frac{\Gamma}{H^4}\right|_{t=t_n} \simeq 1 \quad \Rightarrow \quad \frac{S_3(T_n)}{T_n} = 4\ \ln\left(\frac{T_{\rm n}}{100{\rm GeV}}\right) + 137$$

Gravitational Wave

- **α : Released energy**

$$\alpha = -\frac{1}{\rho_{\rm rad}}\left(\Delta V(T_{\rm n}) + T_{\rm n}\frac{\partial \Delta V(T)}{\partial T}\bigg|_{T=T_{\rm n}}\right)$$

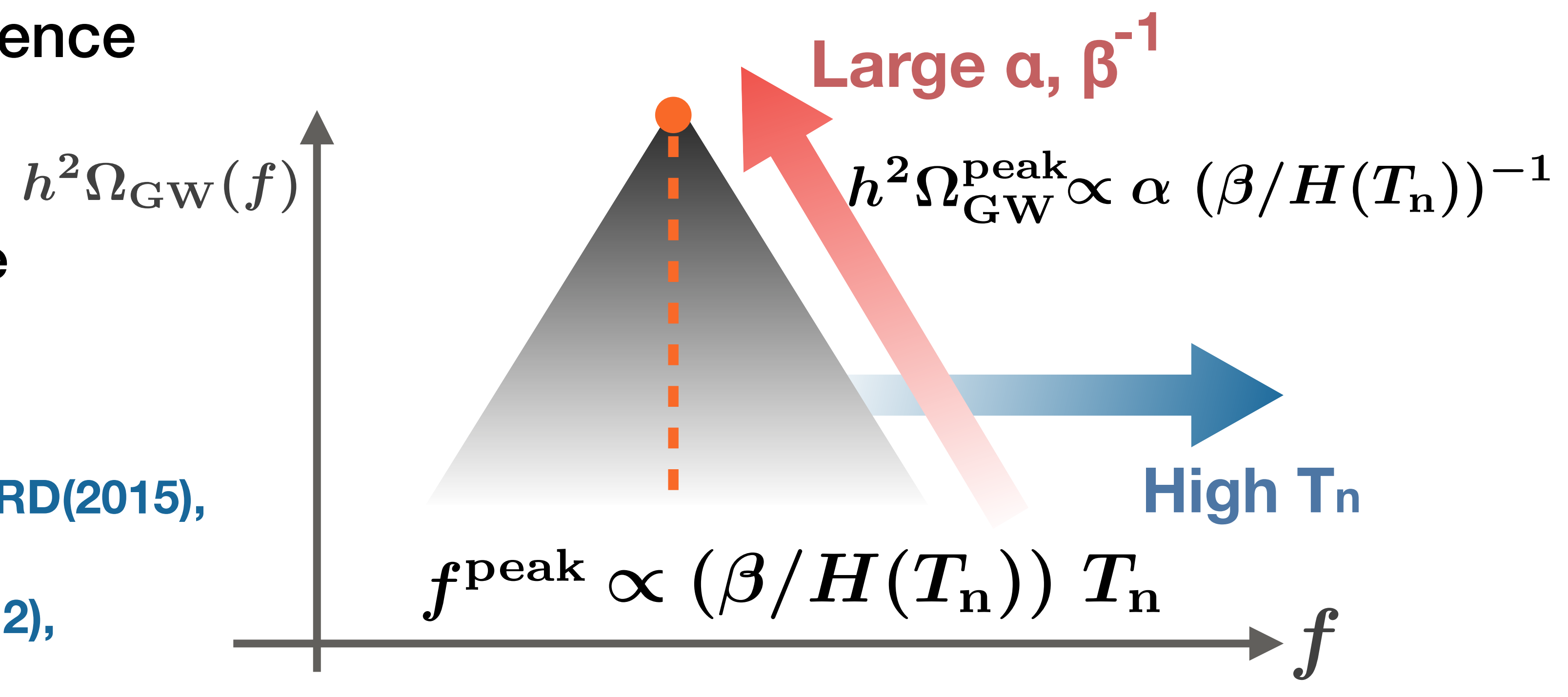
$\rho_{\rm rad}$: the thermal energy density

- **β^{-1} : Timescale of the transition**

$$\frac{\beta}{H} = T_{\rm n}\frac{d}{dT}\left(\frac{S_3(T)}{T}\right)\bigg|_{T=T_{\rm n}}$$

$$\rightarrow \frac{\beta}{H} \sim \mathcal{O}(100-1000)$$

- **Sources of GWs :** $\Omega_{\rm GW} = \Omega_\varphi + \Omega_{\rm sw} + \Omega_{\rm turb}$
  - scalar field contribution
  - sound waves
  - magnetohydrodynamic turbulence



✱ Formulas of the GW spectrum are given by numerical simulation.

Huber, Konstandin, JCAP(2008),
Hindmarsh, Huber, Rummukainen, Weir, PRD(2015),
Caprini, Durrer, Servant, JCAP(2009),
Binetruy, Bohe, Caprini, Dufaux, JCAP(2012),
:

Two Higgs Doublet Model

- **The tree-level potential :**

$$V = m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 - m_3^2(\Phi_1^\dagger\Phi_2 + \text{h.c.})$$
$$+ \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \frac{\lambda_5}{2}\left((\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right)$$

  - CP-conserving
  - Z_2 symmetry : $\Phi_1 \to \Phi_1, \quad \Phi_2 \to -\Phi_2$
  - soft-breaking mass parameter : m_3^2

- $$\Phi_1 = \begin{pmatrix} \omega_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + h_1 + iz_1) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \omega_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + h_2 + iz_2) \end{pmatrix}$$

$$v_1^2 + v_2^2 = v^2 \simeq (246 \text{ GeV})^2$$

- **Mass eigenstate**

$$\begin{pmatrix} \omega_1^\pm \\ \omega_2^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \omega^\pm \\ H^\pm \end{pmatrix}, \qquad \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} z \\ A \end{pmatrix},$$

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}, \qquad \tan\beta = \frac{v_2}{v_1}$$

Two Higgs Doublet Model

- **Scalar masses:**

$$m_h^2 = M^2\cos^2(\beta-\alpha) + (\lambda_1 \sin^2\alpha\cos^2\beta + \lambda_2\cos^2\alpha\sin^2\beta - \frac{\lambda}{2}\sin 2\alpha \sin 2\beta)v^2$$

$$m_H^2 = M^2\sin^2(\beta-\alpha) + (\lambda_1 \cos^2\alpha\cos^2\beta + \lambda_2\sin^2\alpha\sin^2\beta - \frac{\lambda}{2}\sin 2\alpha \sin 2\beta)v^2$$

$$m_A^2 = M^2 - \lambda_5 v^2$$

$$m_{H^\pm}^2 = M^2 - (\lambda_4+\lambda_5)\frac{v^2}{2}$$

$$M^2 \equiv \frac{m_3^2}{\sin\beta\cos\beta}\,, \qquad \lambda = \lambda_3+\lambda_4+\lambda_5$$

Alignment limit : $\sin(\beta-\alpha)=1$ → $m_{\Phi_i}^2 = M^2 + \lambda_i v^2$

- **ρ parameter :** $\rho^{\mathrm{exp}} \simeq 1$

→ $m_A \simeq m_{H^\pm}$ **or** $m_H \simeq m_{H^\pm}$

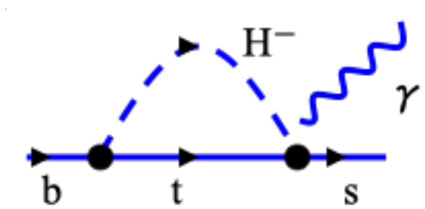
Two Higgs Doublet Model

❖ Yukawa interactions

	Φ_1	Φ_2	u_R	d_R	ℓ_R	Q_L, L_L
Type-I	$+$	$-$	$-$	$-$	$-$	$+$
Type-II	$+$	$-$	$-$	$+$	$+$	$+$
Type-X	$+$	$-$	$-$	$-$	$+$	$+$
Type-Y	$+$	$-$	$-$	$+$	$-$	$+$

* Each Yukawa type show the similar PT profile.
* Type II, Type Y : severe constraints by b→s γ.

$$m_{H^\pm} \gtrsim 600 \text{ GeV}$$



❖ Parameters in the Higgs potential

$$m_1^2, \; m_2^2, \; m_3^2, \; \lambda_{1-5}$$

$$\rightarrow \quad \tan\beta, \;\; \cos(\beta-\alpha), \; m_3^2, \; v, \; m_h, \; m_H, \; m_A, \; m_{H^\pm}$$

Two Higgs Doublet Model

Goncalves, Kaladharan, Wu, arXiv:2108.05356

- **Strength of the 1st order PT :** $\frac{\varphi_c}{T_c}$
- **Signal to Noise Ratio (SNR) for LISA :**

$$\mathrm{SNR} = \sqrt{t_{\mathrm{obs}} \int_{f_{\min}}^{f_{\max}} \mathrm{d}f \left[\frac{\Omega_{\mathrm{GW}}(f)}{\Omega_{\mathrm{noise}}(f)} \right]^2}$$

$t_{\mathrm{obs}} = 5$ years **: the observation time**

$$\Omega_{\mathrm{noise}}(f) = \frac{2\pi^2}{3H_0^2} f^3 S_n(f)$$

$S_n(f)$ **: noise spectral density**

Yagi, Int. J. Mod. Phys. D22 (2013)

Wall velocity : $\xi_w = 0.95$

- **HL-LHC sensitivity :** λ_{hhh} , Φ decay, etc

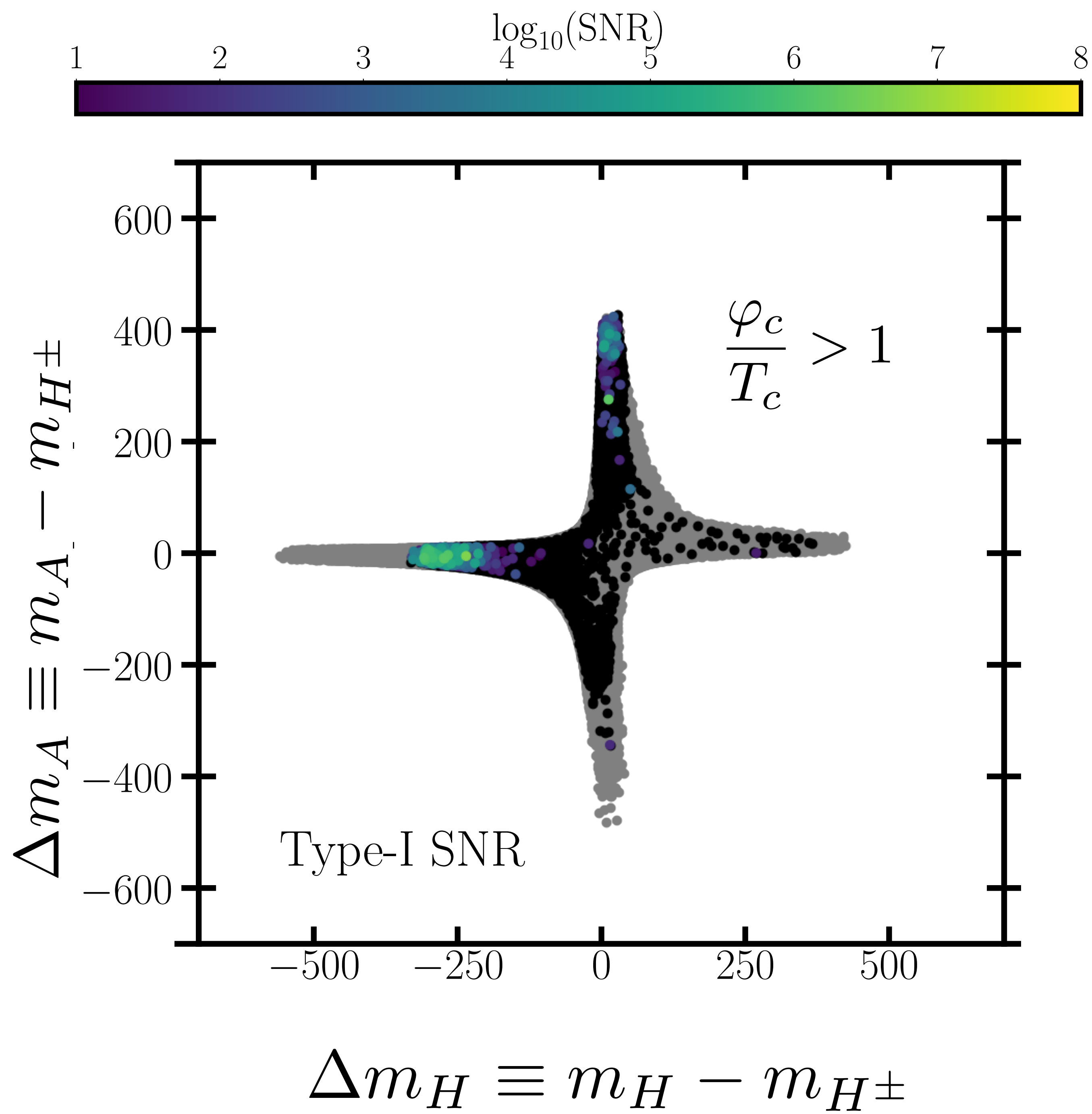
❖ **parameter scan**

$$\tan\beta \in (0.8, 25)\,, \qquad m_3^2 \in (10^{-3}, 10^5)\,\mathrm{GeV}^2\,, \qquad m_H \in (150, 1500)\,\mathrm{GeV}\,,$$
$$\cos(\beta - \alpha) \in (-0.3, 0.3)\,, \quad m_A \in (150, 1500)\,\mathrm{GeV}\,, \quad m_{H^\pm} \in (150, 1500)\,\mathrm{GeV}.$$

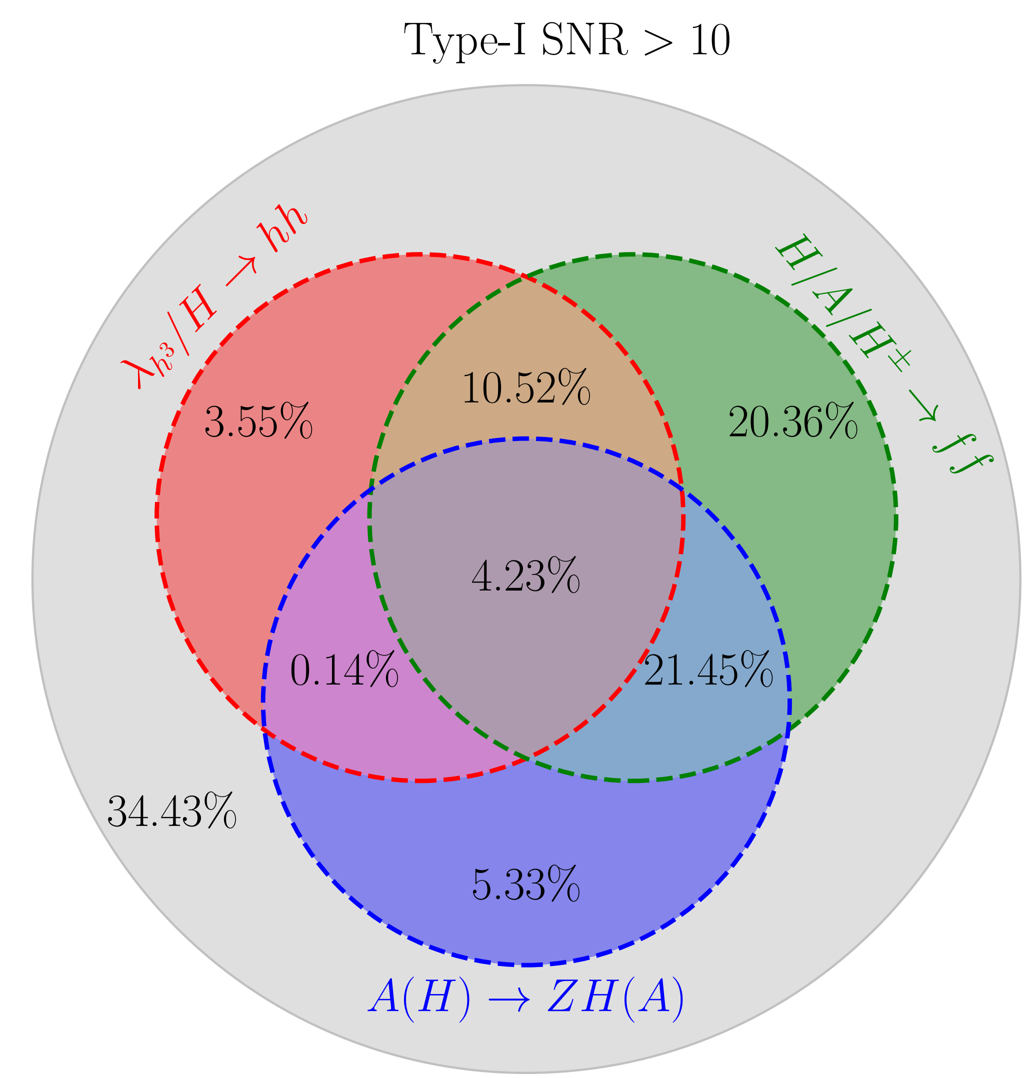
Two Higgs Doublet Model

Goncalves, Kaladharan, Wu, arXiv:12108.05356

❖ **2HDM with Type I**



❖ Factions of the parameter points that can be covered by distinct search channels



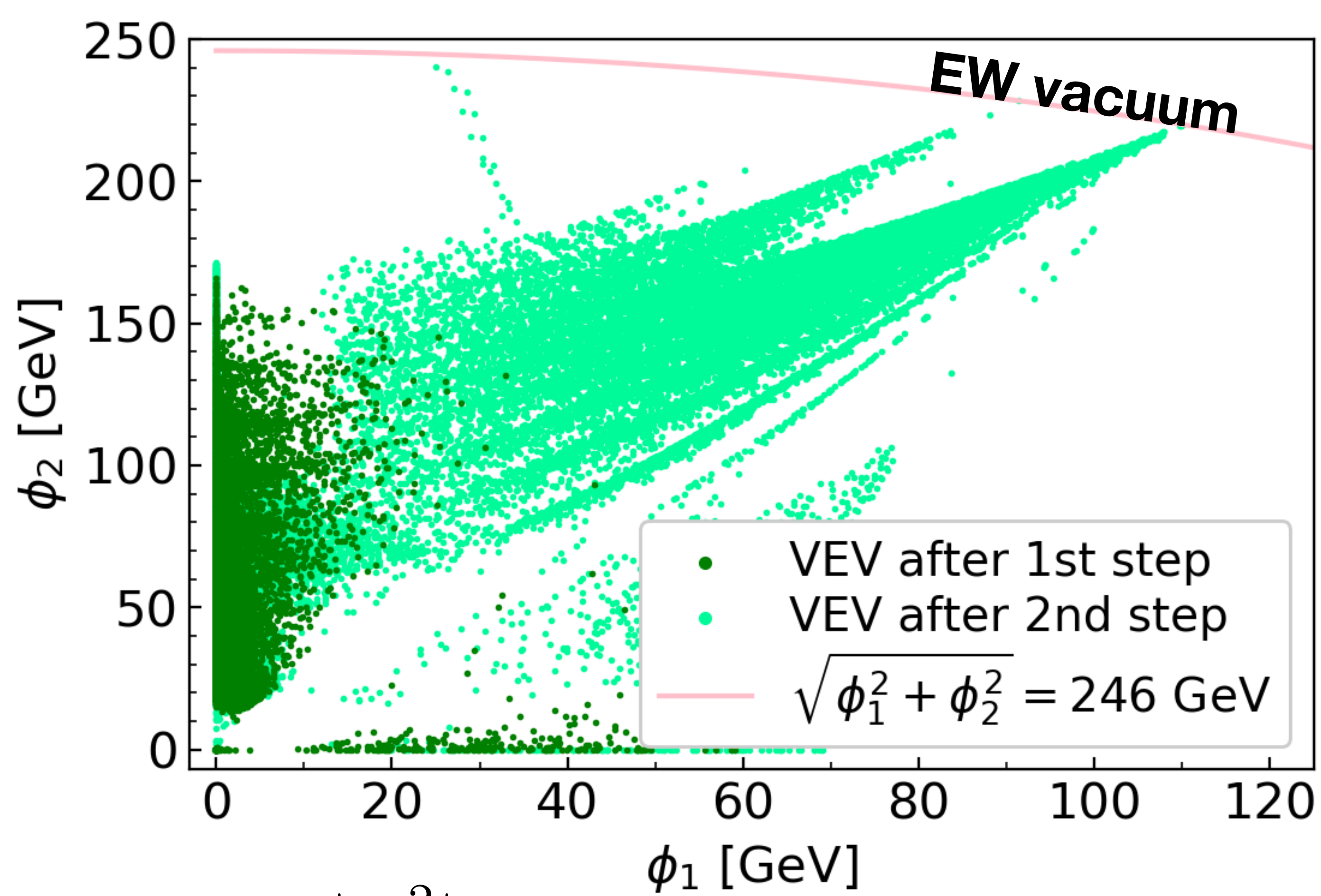
Two Higgs Doublet Model

MA, Komatsu, Shibuya, PTEP (2022)

	m_A [GeV]	m_H [GeV]	$\tan\beta$	$\cos(\beta-\alpha)$	m_3 [GeV]
Type-I ($m_A = m_{H^\pm}$)	180–1000(/10)	130–1000(/10)	2–10(/0.5)	−0.25–0.25(/0.05)	0–100(/5)

- **2-step PT**

The VEVs after each step of the 2-step PTs



- $m_2^2 < 0$ with large $|m_2^2|$

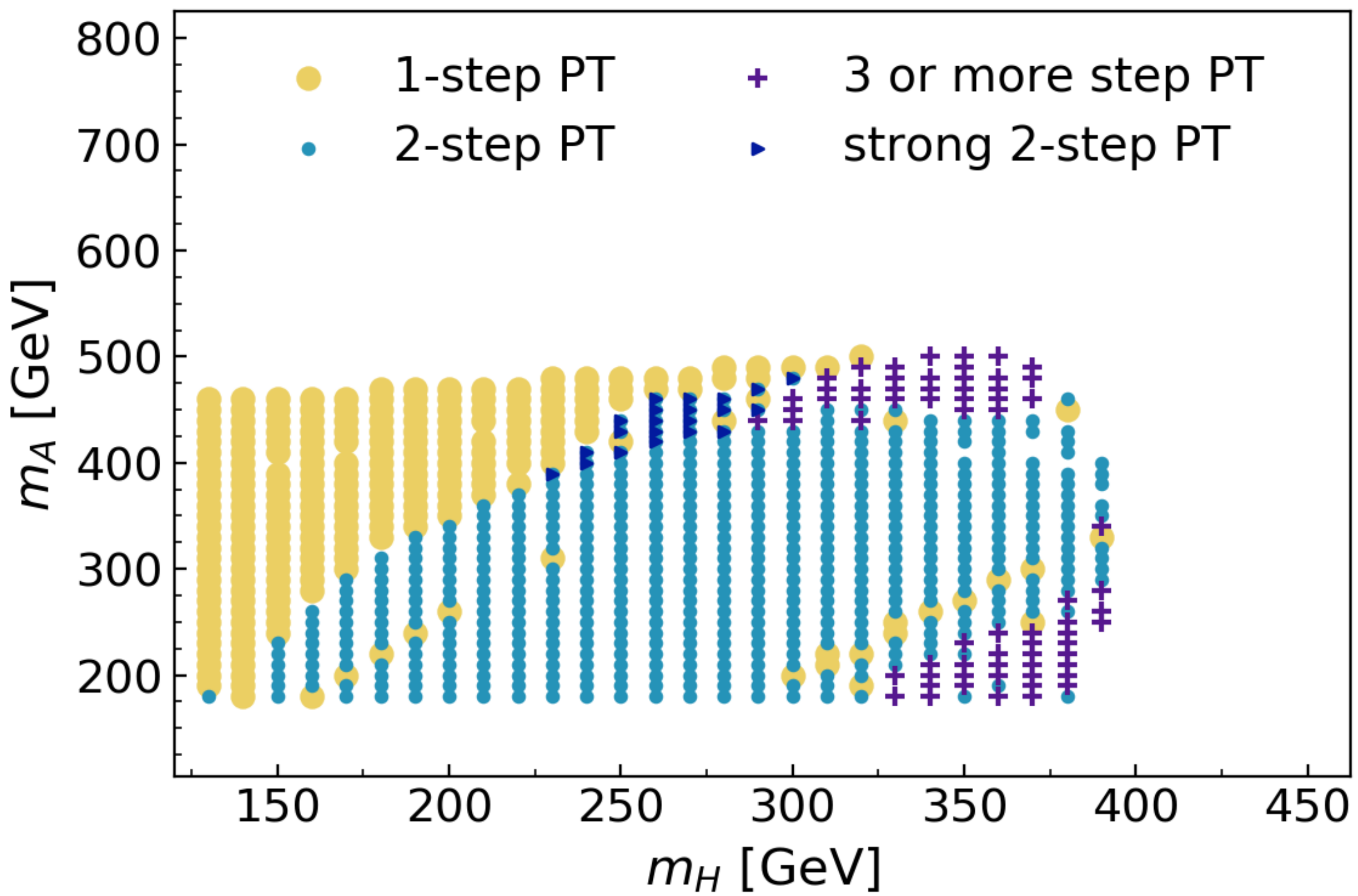
→ **The first step in the 2-step PT tends to occur along the ϕ_2 axis.**

Two Higgs Doublet Model

Type-I $(m_A = m_{H^\pm})$

MA, Komatsu, Shibuya, PTEP (2022)

$$\tan\beta = 2, \quad \cos(\beta-\alpha) = -0.2, \quad m_3 = 0$$



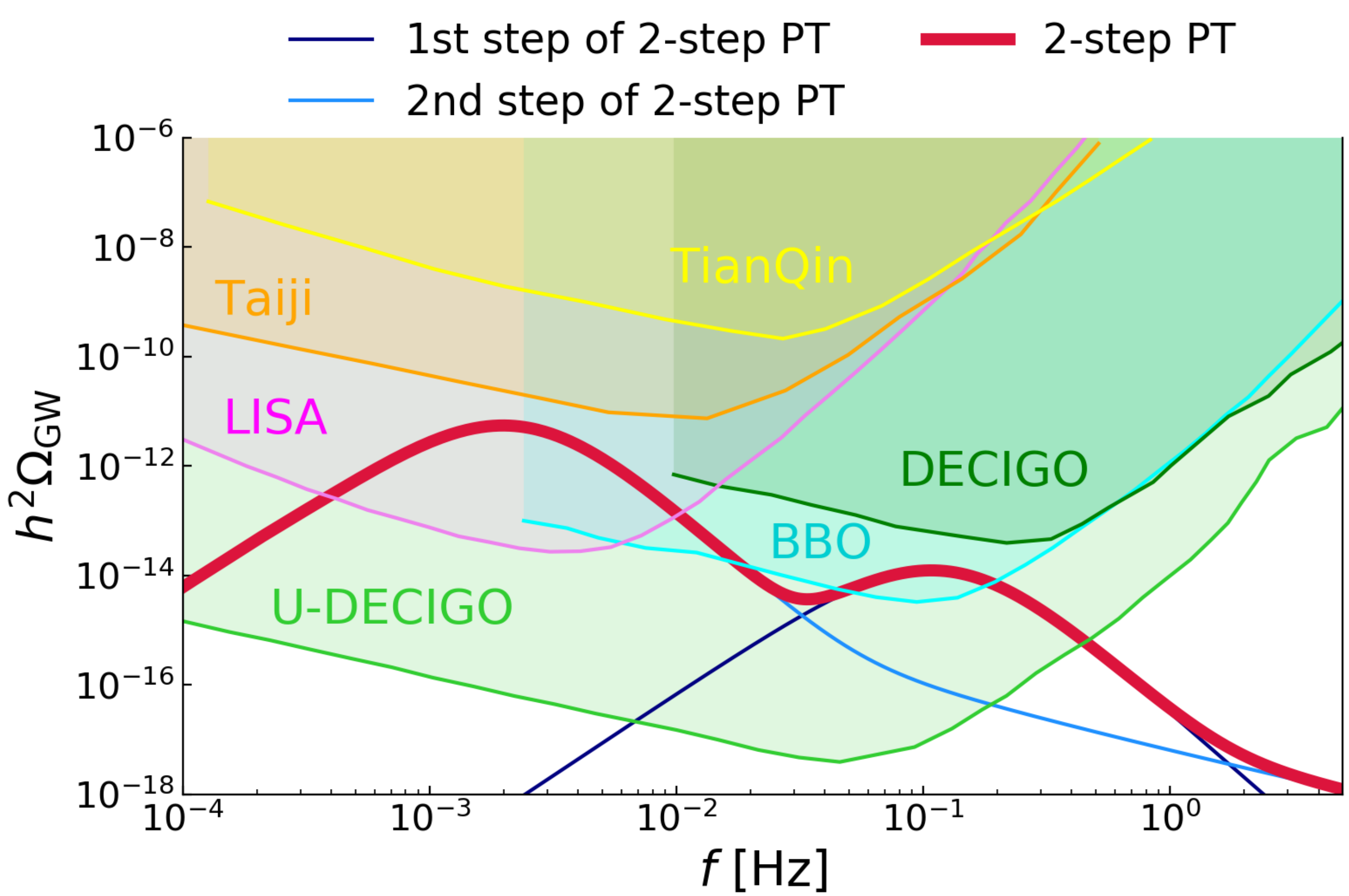
- **strong 2-step PTs : The first step PTs of the 2-step PTs are strongly first order.**
- **The strong 2-step PTs occur only with the mass hierarchy** $m_A > m_H$.

Two Higgs Doublet Model

MA, Komatsu, Shibuya, PTEP (2022)

$$m_A = m_{H^\pm} = 490 \text{ GeV},\ m_H = 300 \text{ GeV},\ \tan\beta = 2.3,\ \cos(\beta-\alpha) = -0.21,\ m_3 = 20 \text{ GeV}.$$

$\xi_w = 1$



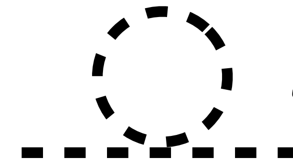
❖ **The strengths of the PT :** $\dfrac{\varphi_c}{T_c} = 2.1$ (for 1st), 4.2 (for 2nd)

❖ **hhh coupling :** $\delta\lambda_{hhh} = \dfrac{\lambda_{hhh} - \lambda_{hhh}^{\rm SM}}{\lambda_{hhh}^{\rm SM}} = 2.2$

Classical Scale Invariant Model

❖ **Hierarchy problem**

$$V_{\rm SM} = m_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2$$



$$\sim \lambda_H \Lambda^2 \sim \delta m_H^2$$

$$m_h^2 \ll \Lambda^2$$

$$m_h^2 = m_H^2 - \delta m_H^2 \simeq (10^{16}\ {\rm GeV})^2 - (10^{16}\ {\rm GeV})^2 = (125\ {\rm GeV})^2$$

❖ **Classically scale invariance**

- One of candidates for the solution of hierarchy problem.

$$V_{\rm SM} = m_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2$$

- with the hidden QCD sector

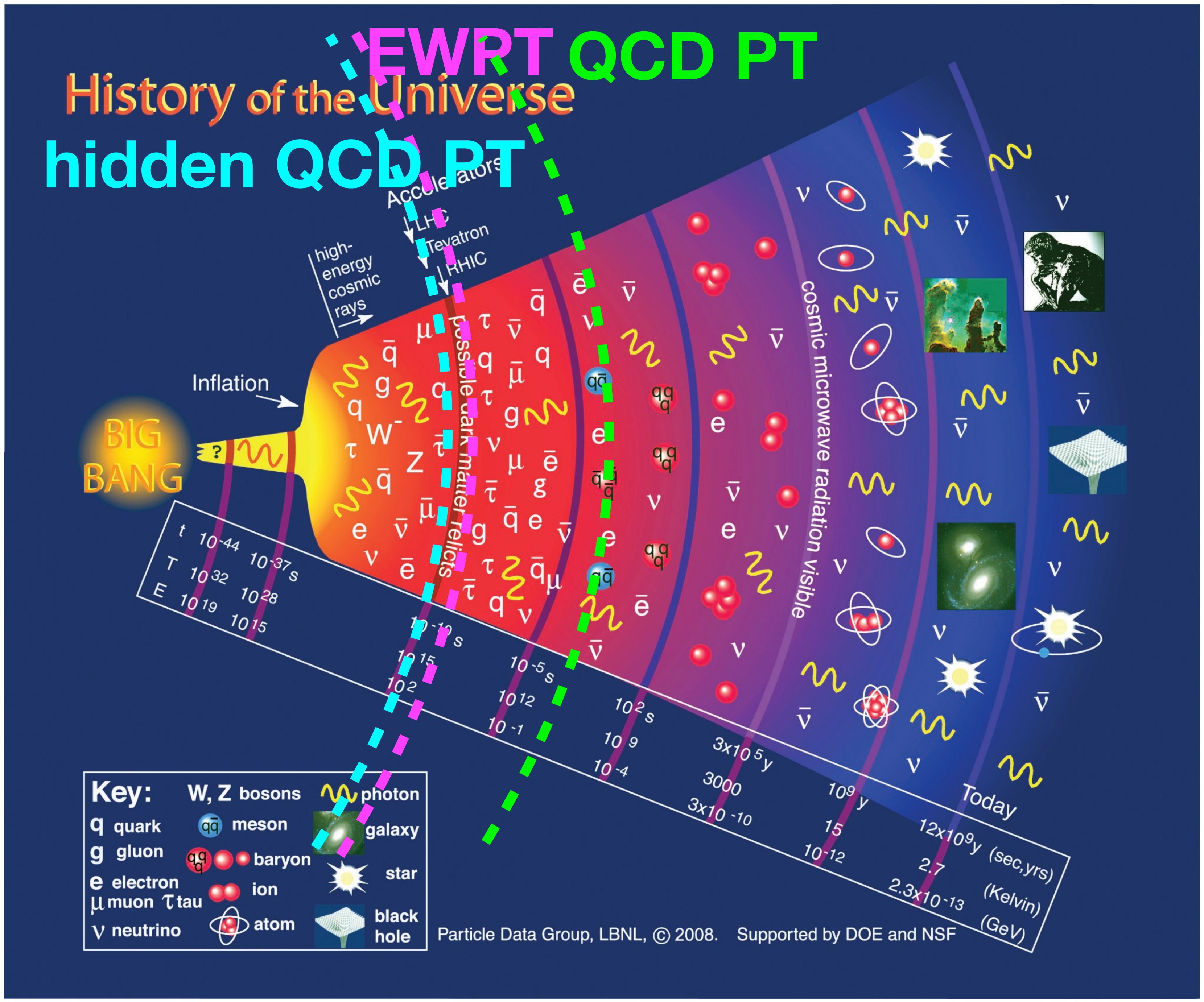
Hur et al, PLB (2011) , Holthausen et al, JHEP(2013)

Dynamical chiral symmetry breaking (DχSB) in the hidden sector triggers the EWSB.

strong 1st order?

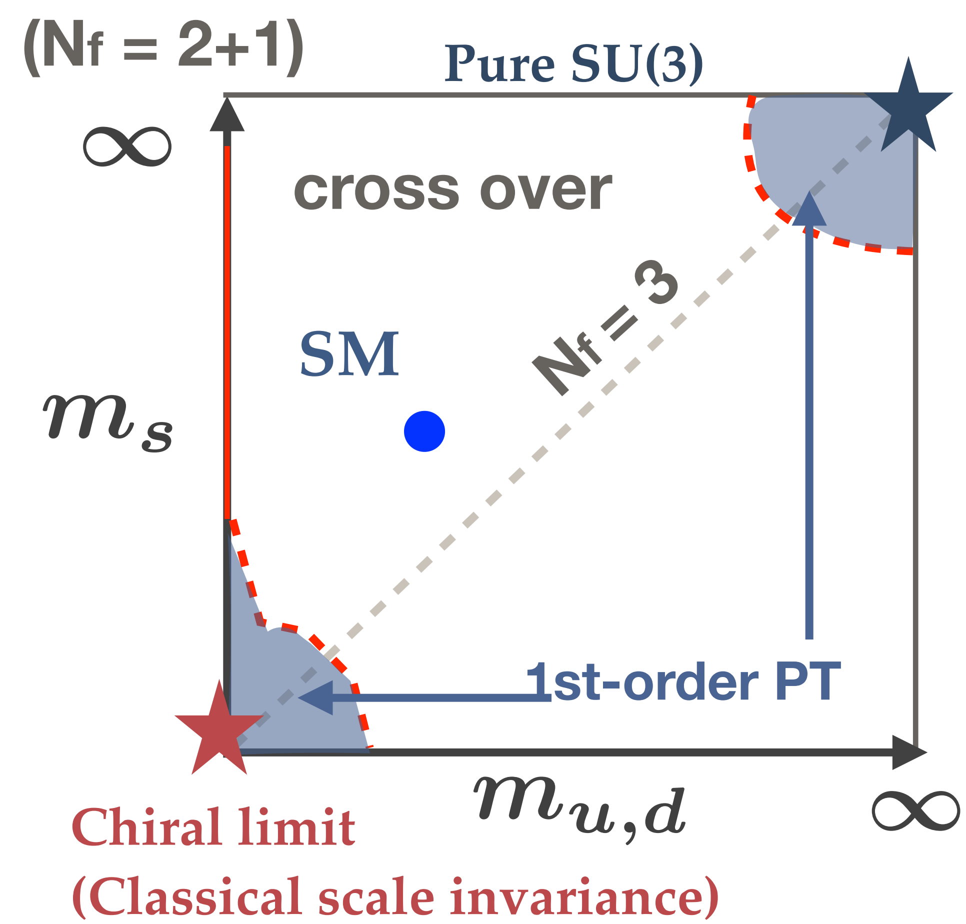
MA, Goto, Kubo, PRD (2017)
MA, Kubo JHEP(2021)

Classical Scale Invariant Model



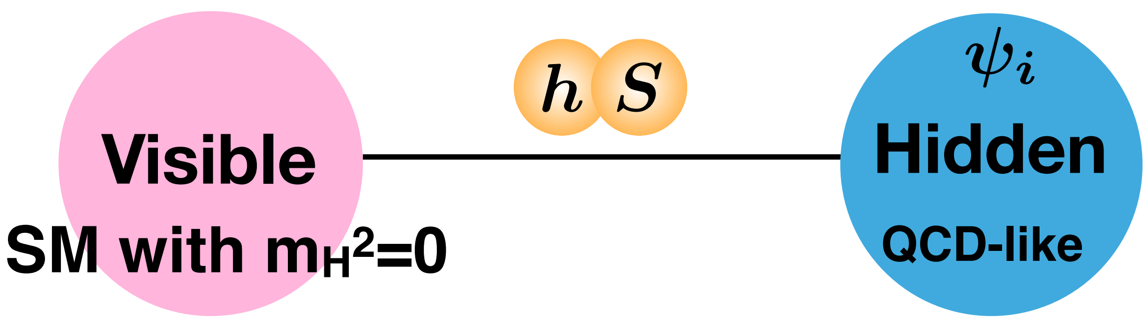
- **QCD PT**

Ding et al. arXiv: 1111.0185



Classical Scale Invariant Model

- **Gauge Symmetry :** $G_{\rm SM} \times {\rm SU}(3)_{\rm H}$



S real singlet scalar

ψ_i hidden vector-like fermions (i=1-3)

- **Scalar potential for vis. sector :** $V_{{\rm SM}+S} = \lambda_H (H^\dagger H)^2 - \frac{1}{2}\lambda_{HS} S^2 (H^\dagger H) + \frac{1}{4}\lambda_S S^4$
- **Lagrangian of the hidden sector :** $\mathcal{L}_{\rm H} = -\frac{1}{2}{\rm Tr}\ F^2 + {\rm Tr}\ \bar{\psi}\,(i\partial\!\!\!/ + g_{\rm H} G\!\!\!/ + g' Q B\!\!\!/ - yS)\,\psi$
- **The DχSB in the hidden sector triggers the EW symmetry breaking.**

$${\rm SU}(3)_L \times {\rm SU}(3)_R \ \to \ {\rm SU}(3)_V$$

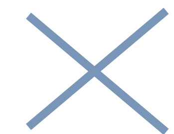
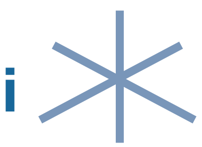
Hidden QCD scale $\Lambda_{\rm H}$

$\Lambda_{\rm H}$	Dynamical χSB	$\langle \bar{\psi}\psi \rangle \neq 0$	$\langle \bar{\psi}\psi \rangle \sim \Lambda_{\rm H}^3$
	Explicit χSB	$\langle S \rangle \neq 0$	$yS\,\langle \bar{\psi}\psi \rangle$
EW	EWSB	$\langle h \rangle \neq 0$	$\lambda_{HS}\,\langle S \rangle^2\,(H^\dagger H)$

Classical Scale Invariant Model

❖ **Nambu—Jona-Lasinio (NJL) Lagrangian**

$$\mathcal{L}_{\mathrm{H}} \longrightarrow \mathcal{L}_{\mathrm{NJL}} = \mathrm{Tr}\ \bar{\psi}(i\partial\!\!\!/ + g' Q B\!\!\!/ - yS)\psi + 2G\mathrm{Tr}\ \Phi^\dagger \Phi + G_D(\det\ \Phi + h.c)$$

cf. current mass m_c  4-Fermi  6-Fermi 

$$(\Phi)_{ij} = \bar{\psi}_i(1-\gamma_5)\psi_j$$

- ***G* and G_D are dimensional parameters.**

❖ **The mean-field approximation:**

$\mathrm{SU}(3)_V$ limit

$$\sigma = -4G\left\langle\bar{\psi}\psi\right\rangle\ , \quad \phi_a = -2iG\left\langle\bar{\psi}\gamma_5\lambda^a\psi\right\rangle \qquad (a = 1, \cdots, 8)$$

Chiral condensate

Hidden mesons
(Massive NG bosons)

Dark Matter candidates

Classical Scale Invariant Model

- the one-loop effective potential from MFA:

$$V_{\rm NJL}(\sigma, S; \Lambda_H) = \frac{3}{8G}\sigma^2 - \frac{G_D}{16G^3}\sigma^3 - 3n_c I_0(M;\Lambda_H)$$

the constituent fermion ψ mass : $M = \sigma + yS - \frac{G_D}{8G^2}\sigma^2$

  - G and G_D : determined by scaling-up the values for the real hadrons.

$$G^{QCD\,1/2}\Lambda^{QCD} = 1.82, \quad (-G_D^{QCD})^{1/5}\Lambda^{QCD} = 2.29$$

$$\rightarrow \quad G^{1/2}\Lambda_H = 1.82, \quad (-G_D)^{1/5}\Lambda_H = 2.29$$

- the free parameters of the model : $\lambda_{HS},\ \lambda_S,\ y\ ,\ \lambda_H,\ \Lambda_H$

chosen to satisfy

$m_h = 126\ \mathrm{GeV},\ v = 246\ \mathrm{GeV}$

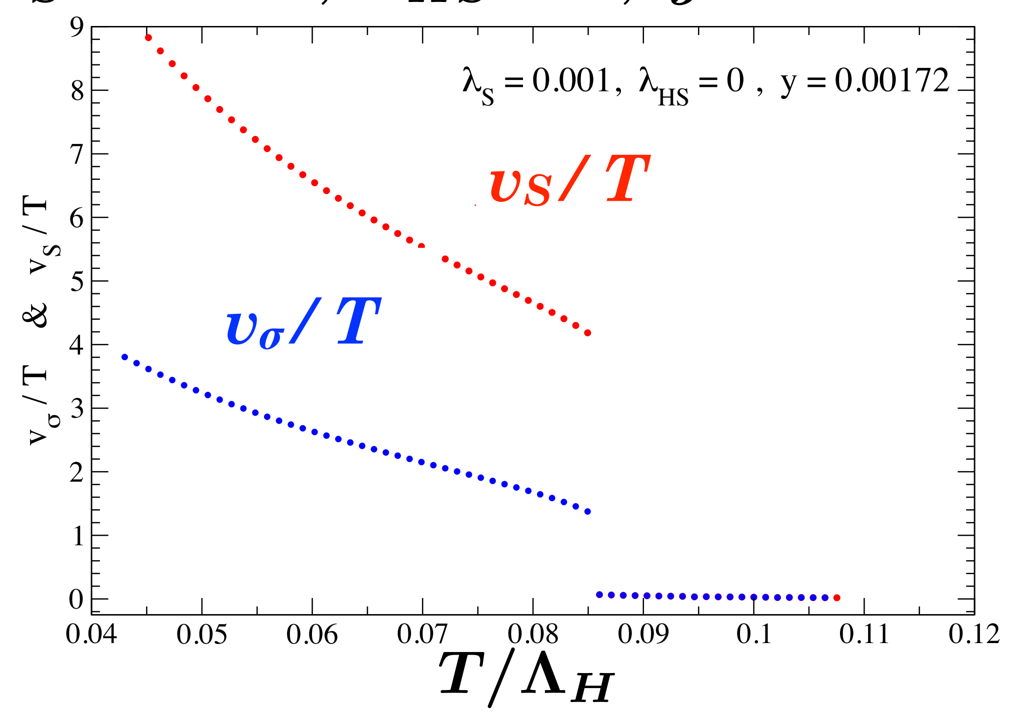
$$V_{\mathrm{SM}+S} = \lambda_H (H^\dagger H)^2 - \frac{1}{2}\lambda_{HS} S^2 (H^\dagger H) + \frac{1}{4}\lambda_S S^4$$

- the <σ> and <S> can be determined through the minimization of the effective potential at finite temperature.

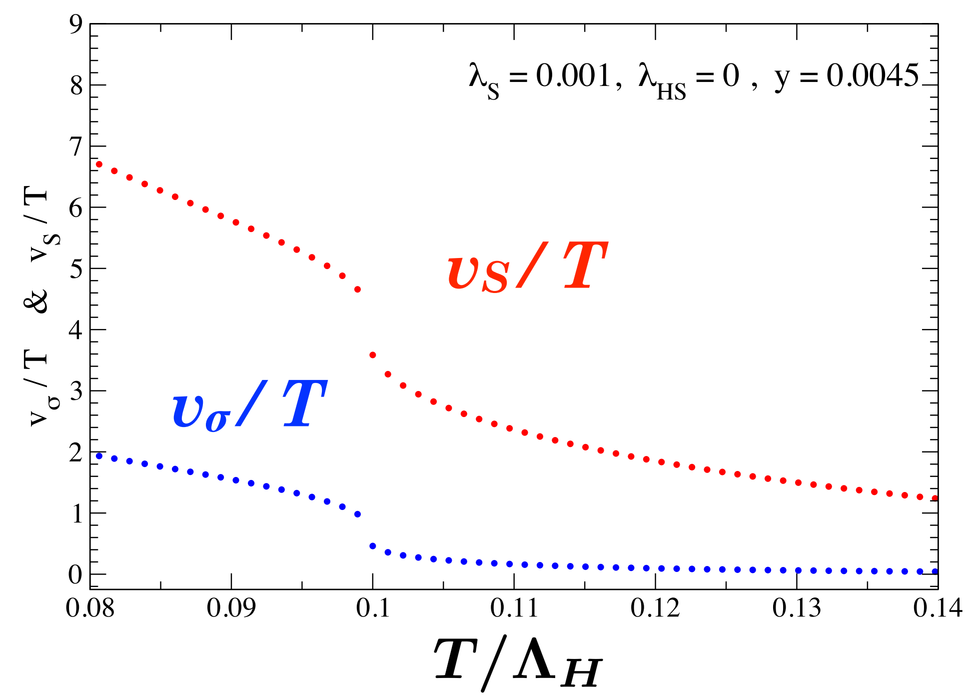
Hidden Chiral Phase Transitions

MA, Kubo JHEP(2021)

$\lambda_S = 0.001,\ \lambda_{HS} = 0,\ y = 0.00172$



$\lambda_S = 0.001,\ \lambda_{HS} = 0,\ y = 0.0045$



- **The chiral PT in the hidden sector becomes first order for** $y \lesssim \mathcal{O}(10^{-3})$.
- **α, β** $\begin{cases} \alpha \sim 0.02 \\ \beta/H \sim 4000\,(8000) \quad \text{for} \quad y \simeq 0.0017\,(0.001) \end{cases}$
- The SW as a GW source are active for less than a Hubble time.

- **The reduction factor for the SW contribution:**

Ellis, Lewicki, No, JCAP(2019)
Ellis, Lewicki, No, Vaskonen, JCAP(2019)

$$h^2\Omega_{\rm sw}(f) \to h^2\Omega_{\rm sw}(f) \times \tau_{\rm sw} H$$

τ_{SW} : the duration of the sound-wave period

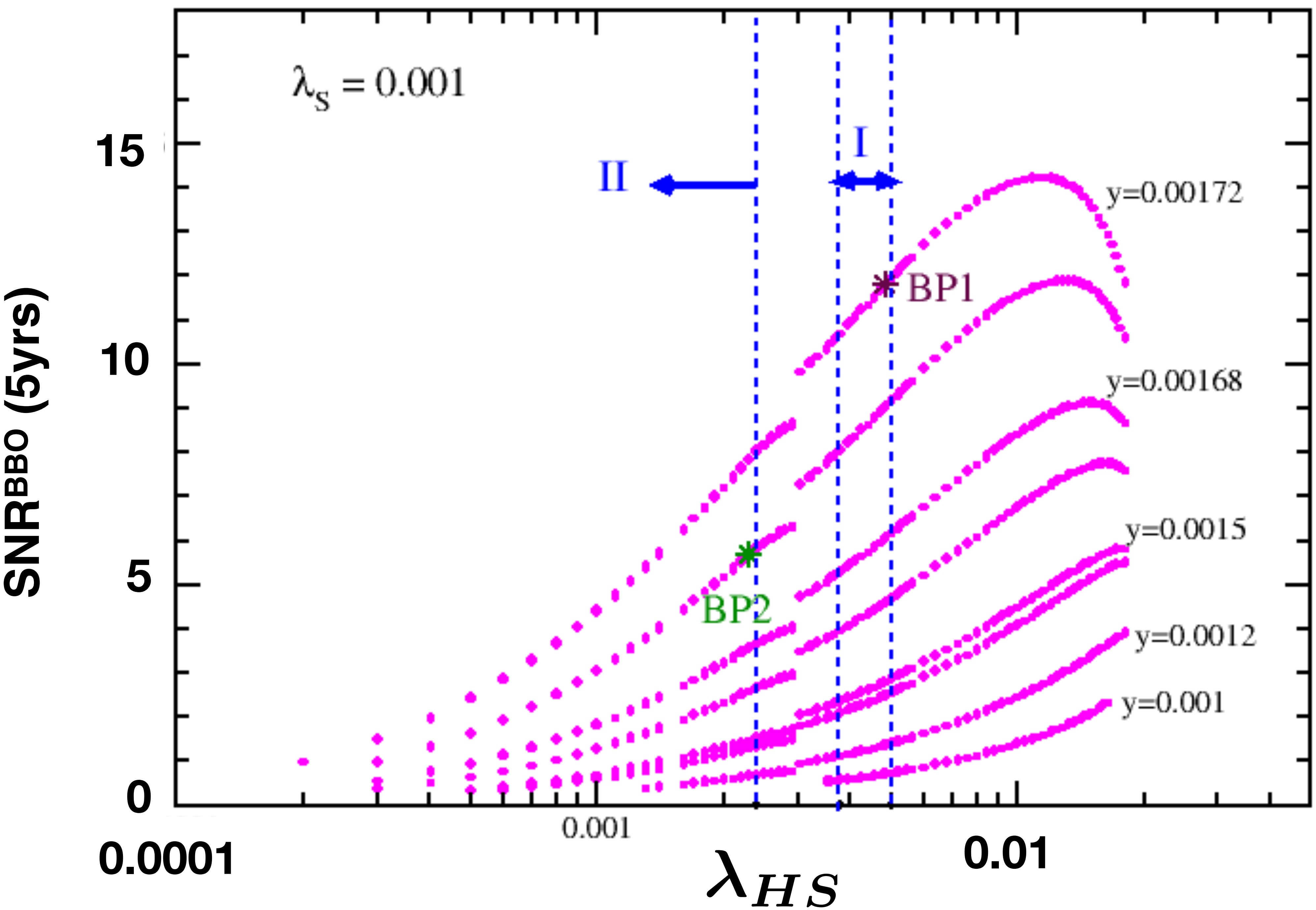
- **We find** $\tau_{\rm sw} H \sim 5 \times 10^{-3}$ for $\beta/H \sim 10^4$

Jouguet detonation : ξ_w=0.6809

SNR^{BBO} (5yrs)

MA, Kubo JHEP(2021)

$m_S > m_h$ $m_S < m_h$



BP1:

$m_s = 90$ GeV

$\Lambda_H = 4.3$ TeV

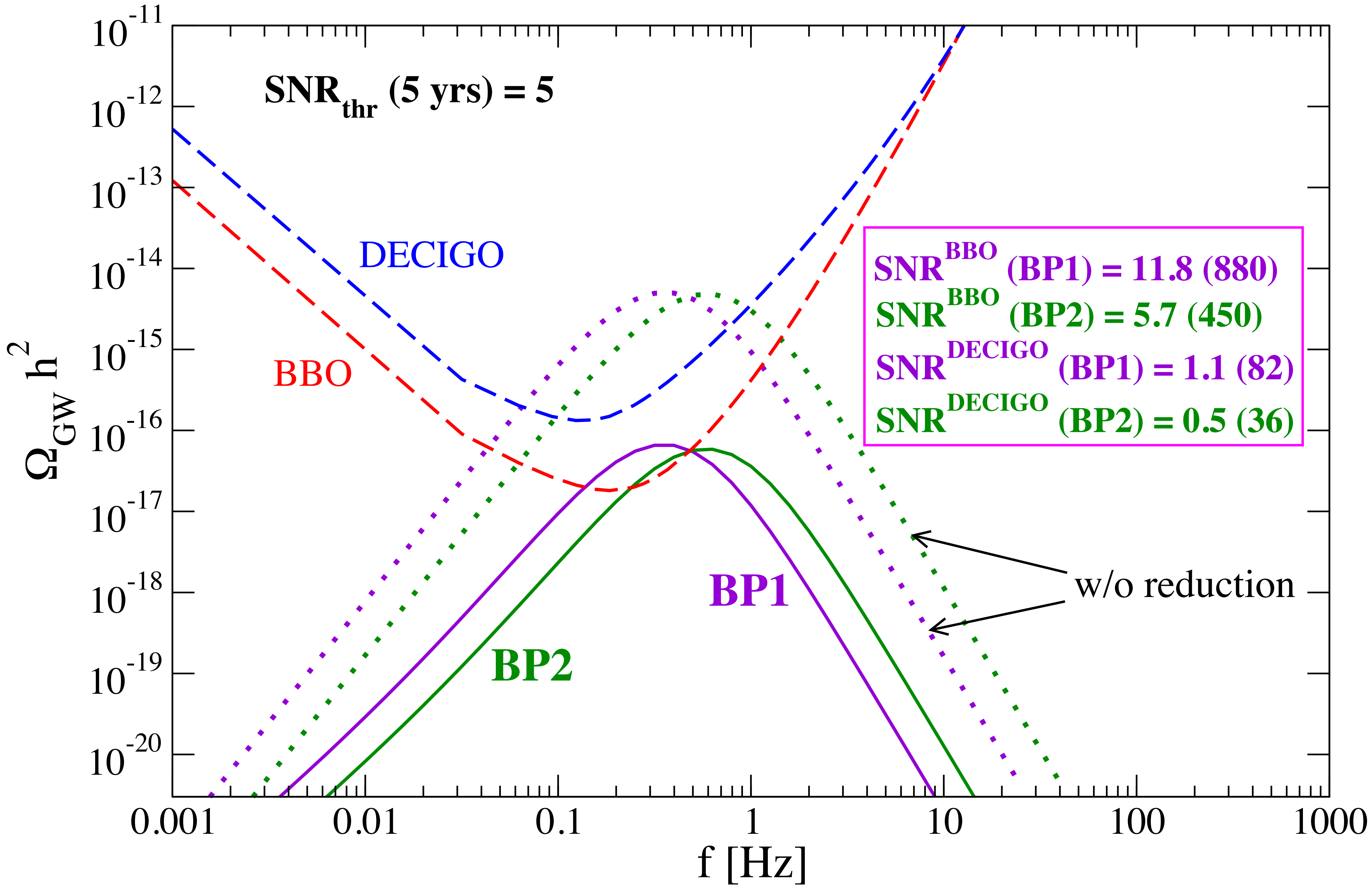
BP2:

$m_s = 150$ GeV

$\Lambda_H = 6.6$ TeV

GW Spectrum

MA, Kubo JHEP(2021)



Summary

- The 1st order PT is a promising probe for the new physics in the early Universe.

- Many models with extended scalar sector predict the strong 1st order PT.

  - 2HDM - favors low scalar masses
    - strong 2-step PTs are possible

  - a scale invariant model with the hidden QCD sector
    - Dynamical chiral PT in the hidden sector becomes the 1st order for the smaller Yukawa coupling.

- The GW signal predicted by the BSM could be detected at future space-based GW detector.

- The GW, LHC, and future collider complementarities will probe the 1st order PT.