Imprint of the chiral symmetry restoration on observables in (2+1)-flavor QCD

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(work done with HotQCD collaboration)

The Future is Illuminating
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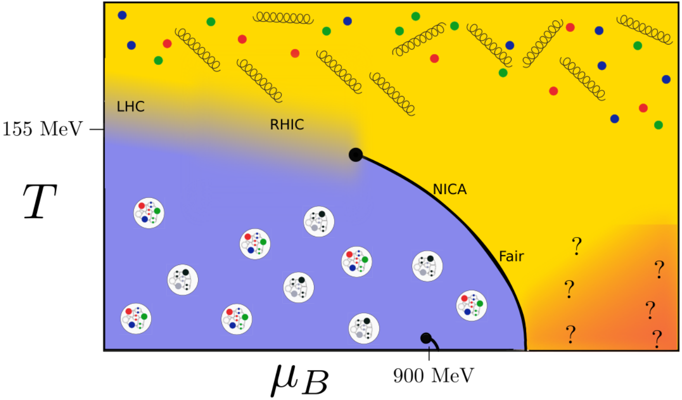
Outline

QCD phase diagram in $T - \mu_B$ plane

[Guenther, 2010.15503]

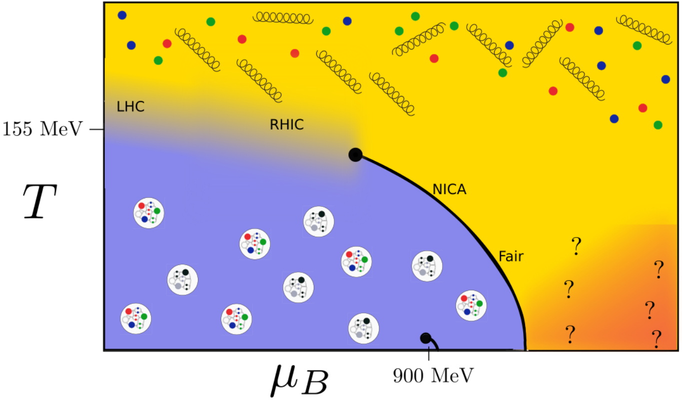


QCD phase diagram in $T - \mu_B$ plane

- first-principle non-perturbative controlled calculation with lattice

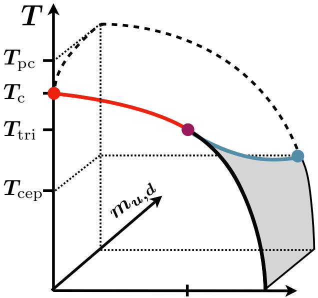
- sign problem at finite μ

[Guenther, 2010.15503]



$$T_{pc} = 156.5(1.5)\ \text{MeV}$$

[HotQCD, PLB 795 (2019) 15–21]



[Karsch, 1905.03936]

$$T_c = 132^{+3}_{-6}\ \text{MeV}$$

[HotQCD, PRL 123, 062002 (2019)]

Chiral limit of QCD with two light flavors

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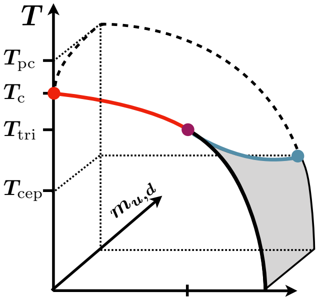

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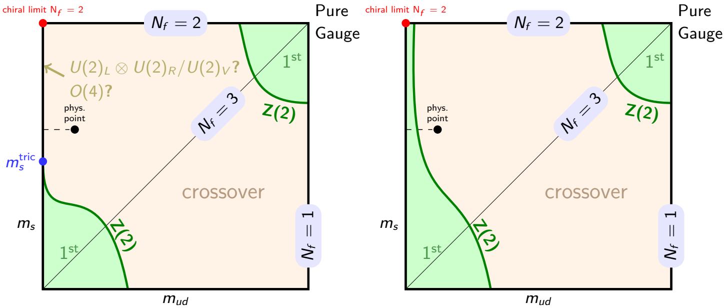
- ➔ T_c - a fundamental scale of QCD
- ➔ **Expected** to belong to the **universality class** of $\mathbf{3d}\ \boldsymbol{O(4)}$ spin model
  $[SU(2) \times SU(2) \approx O(4)]$
  [Pisarski and Wilczek, PRD 29 338 (1984)]
- ➔ Important for understanding the phase diagram at physical mass
  [Hatta and Ikeda, PRD67 014028 (2003)]

Depending on $U_A(1)$ symmetry restoration relative to chiral symmetry restoration and number of flavors, the chiral phase transition can also be first order [Pisarski and Wilczek, PRD 29 338 (1984)]



[Philipsen and Pinke, PRD93, 114507, 2016]

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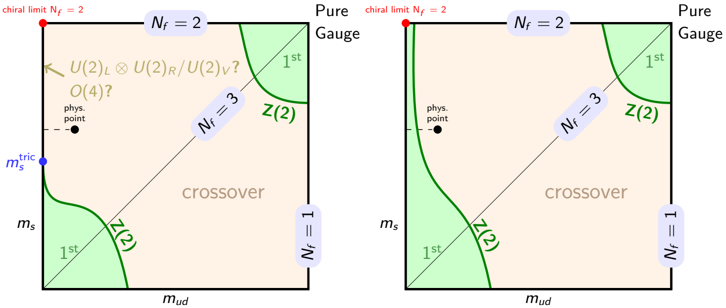

[Philipsen and Pinke, PRD93, 114507, 2016]

- ➔ Variable N_f [Cuteri, Philipsen and Sciarra, 2107.12739]
  $N_f = 3$ [Dini et al, Phys.Rev.D 105 (2022) 3, 034510]
- ➔ Effective $U_A(1)$ restoration [Ding et al, PRL 126 (2021) 8, 082001]
- ➔ Imprint on observables [Kaczmarek et al, 2010.15593]

The left scenario seems to be favored

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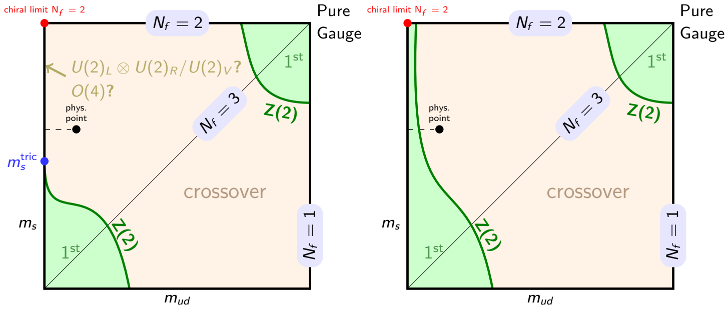

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RG Scaling

Free energy density $f(t,h) = -\frac{1}{V}\ln Z = f_s + f_r$

dimensionless couplings

$t = \frac{1}{t_0}\frac{T - T_c}{T_c}$ "reduced" temperature

$h = \frac{H}{h_0}$ magnetization

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For $3d$ $O(N)$ models
relevant fields $u_1 = u_t, u_2 = u_h$
with $y_t, y_h > 0$
infinitely many irrelevant fields
with $y_j < 0$

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Using $y_t = \frac{1}{\nu}, y_h = \frac{\beta\delta}{\nu}$
$2 - \alpha = d\nu, \quad \gamma = \beta(\delta - 1),$
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$$\boldsymbol{f_s = h^{(2-\alpha)/\beta\delta} f_f(z)}$$

Universal scaling function $f_f(z)$
Scaling variable $z = \frac{t}{h^{\beta\delta}}$

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Critical behavior in vicinity of the chiral phase transition

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(T, \vec{\mu}, m_l) = \boldsymbol{h^{(2-\alpha)/\beta\delta} f_f(z)} + f_r(T, \vec{\mu}, m_l)$$

infinite volume

singular **regular**

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$$\bar{t} \equiv t t_0 = \left(\frac{T - T_c}{T_c} + \kappa_2^X \left(\frac{\mu_X}{T} \right)^2 \right)$$

"energy-like" coupling

$$H \equiv h h_0 = \frac{m_l}{m_s}$$

"magnetic-like" coupling

Scaling variable

$$\boldsymbol{z = z_0 \bar{t} / H^{1/\beta\delta}}$$

$$z_0 = h_0^{1/\beta\delta} / t_0$$

Chiral phase transition at

$\boldsymbol{m_l \equiv m_u = m_d = 0 \; (h = 0)}$

$\boldsymbol{T = T_c \; (t = 0)}$ at $\mu = 0$

Derivatives of free energy density / pressure

magnetic-like

$$\frac{\partial^2 \ln Z}{\partial H^2}$$

$$\sim H^{1/\delta-1}$$

$$\sim H^{-0.79}$$

divergence : **strong**

critical exponents			
	α	β	δ
$O(4)$	-0.21	0.38	4.82
$O(2)$	-0.017	0.349	4.78
$Z(2)$	+0.109	0.325	4.8

Derivatives of free energy density / pressure

	magnetic-like	mixed
	$\frac{\partial^2 \ln Z}{\partial H^2}$	$\frac{\partial^2 \ln Z}{\partial H \partial t}$
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$3d$ $O(2)$ universality class
Staggered fermions at finite lattice spacing
$O(4)$ recovered in continuum limit
At our lattice spacing $T_c \sim 145$ MeV

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- Computing resources : Jülich (Germany), Piz Daint (Switzerland), JLAB (USA), Bielefeld (Germany) and Wuhan (China) supercomputing facilities.

Results I : Polyakov loop and HQFE

Clarke, Kaczmarek, Karsch, Lahiri, MS, PRD 103, L011501 (2021)

Polyakov loop

$$P_{\vec{x}} \equiv \frac{1}{3} \operatorname{tr} \prod_{\tau} U_4(\vec{x}, \tau) \ , \ P \equiv \frac{1}{N_\sigma^3} \sum_{\vec{x}} P_{\vec{x}}$$

Heavy quark free energy

$$F_q(T, H) = -T \ln \langle P \rangle = -\frac{T}{2} \lim_{|\vec{x}-\vec{y}| \to \infty} \ln \langle P_{\vec{x}} P_{\vec{y}}^{\dagger} \rangle$$

In quenched limit ($m_q \to \infty$),
$\langle P \rangle = 0$: confinement
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- However at physical masses, no inflection point observed in $\langle P\rangle$ around T_{pc}
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Is the rapid change in $\langle P \rangle$ around T_{pc} still sensitive to the confinement-deconfinement transition or something else?

Polyakov loop as energy-like observable

– purely gluonic and invariant under chiral transformation

$$F_q/T = AH^{(1-\alpha)/\beta\delta} f'_f(z_0 \frac{T-T_c}{T_c} H^{-1/\beta\delta}) + f_{\rm reg}$$

$f_{\rm reg}(T,H) = \sum_{i,j} a^r_{i,2j} t^i H^{2j} = a^r_{0,0} + a^r_{1,0} t$

– 5 parameter fit alongwith universal $O(2)$ scaling function and critical exponents

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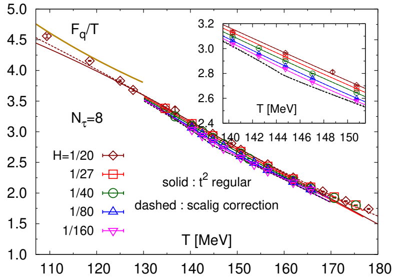

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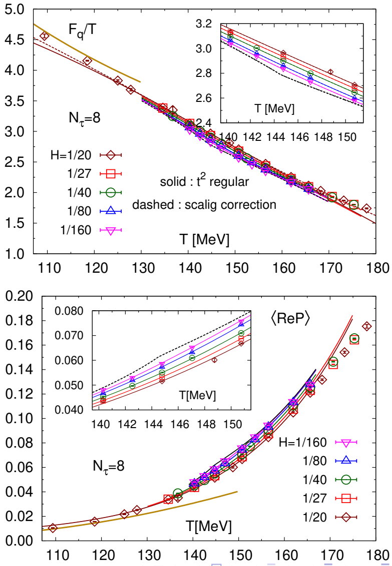
$$\langle P \rangle_{T,H} = \exp\left(-AH^{(1-\alpha)/\beta\delta} f_f'(z) - f_{\text{reg}}\right)$$



Derivative of Heavy Quark Free Energy

$$\frac{\partial F_q(T,H)/T}{\partial H} = -\frac{1}{\langle P \rangle}\frac{\partial \langle P \rangle}{\partial H} \equiv -\frac{1}{\langle P \rangle} \langle P \cdot \Psi \rangle - \langle P \rangle \langle \Psi \rangle$$

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$$\frac{\beta-1}{\beta\delta} = -0.39, O(2)$$

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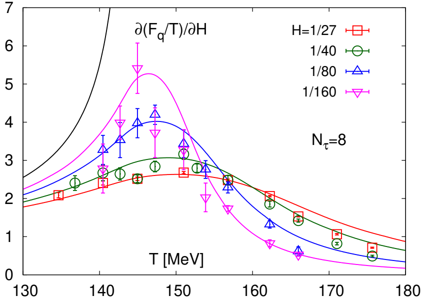

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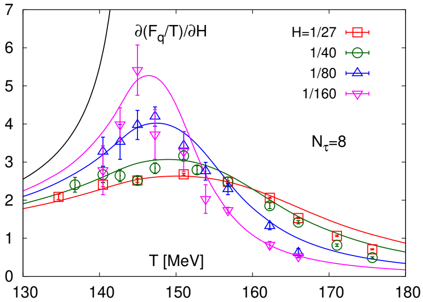

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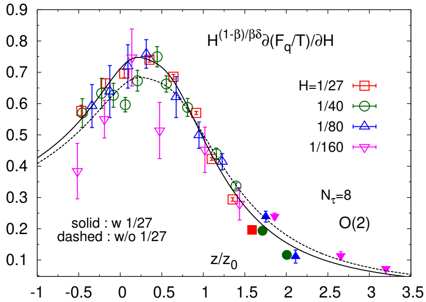

consistent with $O(2)$ scaling

Fit results and C_V-like quantity

singular part				regular part	
$H_{\max}$	A	T_c	z_0	$a^r_{0,0}$	$a^r_{1,0}$
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T_c and z_0 match with earlier results
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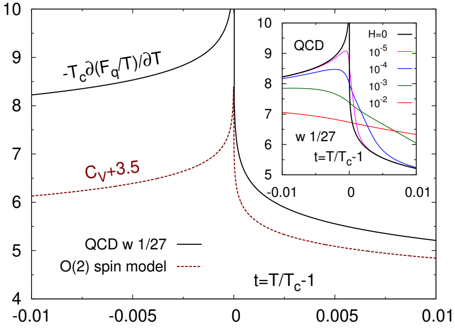
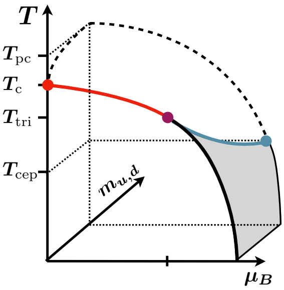
Characteristic spike develops for $H \sim 10^{-5}$ inaccessible in current lattice simulations.



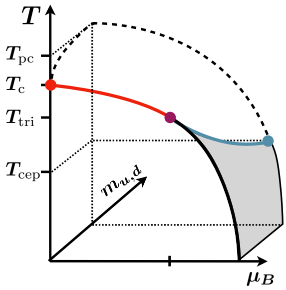
Results II : Conserved charge fluctuations



[MS, Kaczmarek, Karsch, Lahiri, Schmidt, Acta Phys. Pol. B Proc. Suppl. 14, 383 (2021)]

$$\frac{p}{T^4} = h^{(2-\alpha)/\beta\delta} f_f(z) + \text{reg.}$$

$$t = \frac{1}{t_0}\left(\frac{T-T_c}{T_c} + \kappa_2^X \left(\frac{\mu_X}{T}\right)^2\right), \; X = B, Q, S$$



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Conserved charge fluctuations at $\mu = 0$ (Singular part) :

$$\chi_{2n}^X = -\left.\frac{\partial^{2n} p/T^4}{\partial(\mu_X/T)^{2n}}\right|_{\mu_X=0} \sim \; -(2\kappa_2^X)^n\, H^{(2-\alpha-n)/\beta\delta} f_f^{(n)}(z)$$

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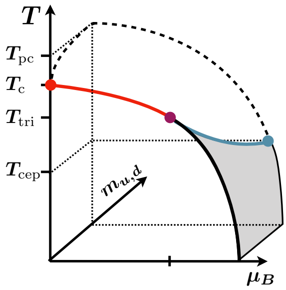

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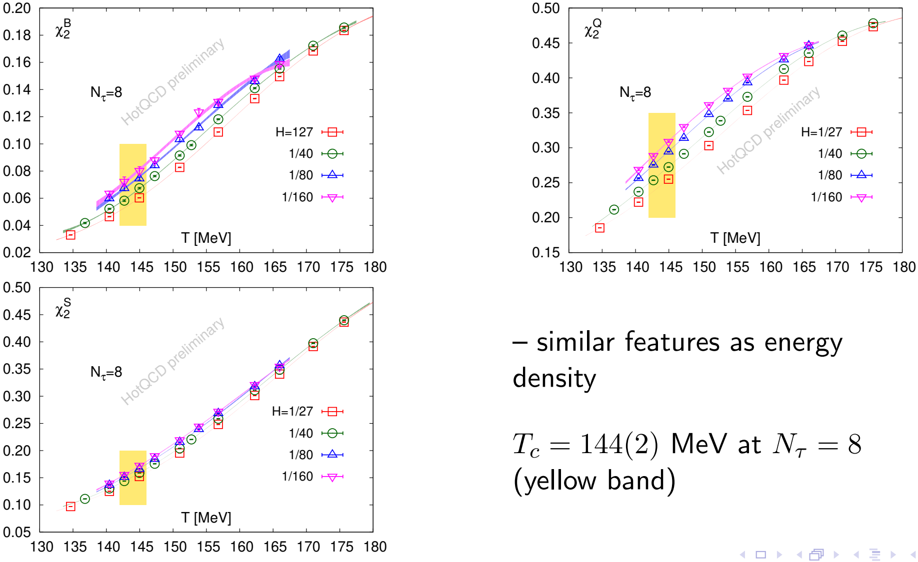
measurable in HIC experiments



– similar features as energy density

$T_c = 144(2)$ MeV at $N_\tau = 8$ (yellow band)

Estimation of singular contribution to χ_2

$$\chi_2^X(T_c, H) \sim -\kappa_2^X H^{(1-\alpha)/\beta\delta} f_f^{(1)}(0) + \text{const. reg. term} + \mathcal{O}(H^2)$$

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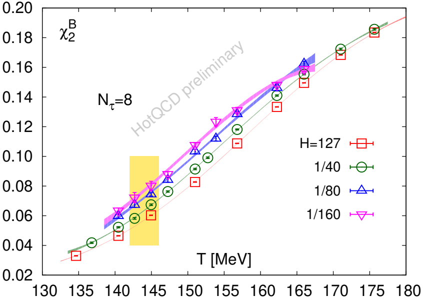

– expect straight line fit for $\chi_2(T_c, H)$ vs $H^{0.61}$ if scaling holds ($O(2)$ exponents)

– $T_c \sim 144$ MeV for $N_\tau = 8$

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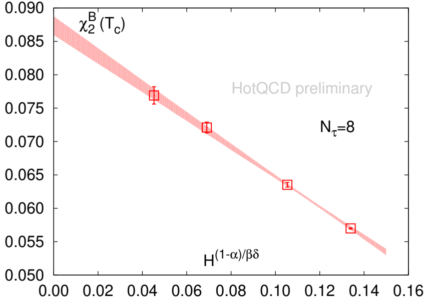

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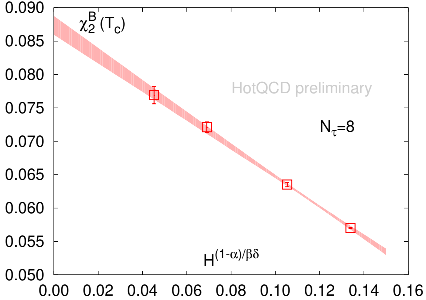

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Singular contribution to χ_2^B at physical masses $\sim$ 50%

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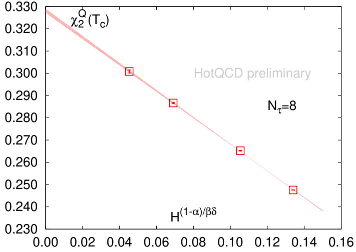

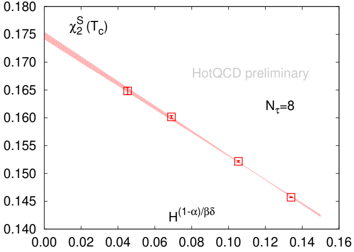

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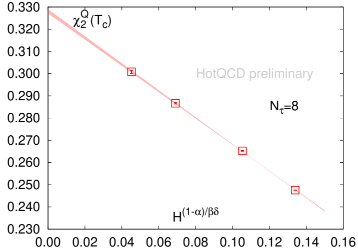

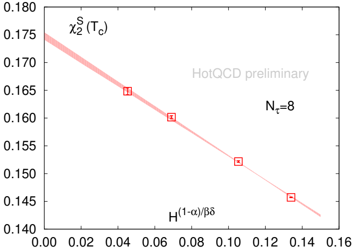

singular contribution at physical mass

$\chi_2^B(T_c)$	$\sim 50\%$
$\chi_2^Q(T_c)$	$\sim 30\%$
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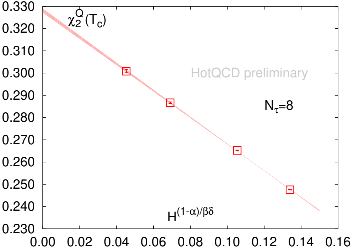

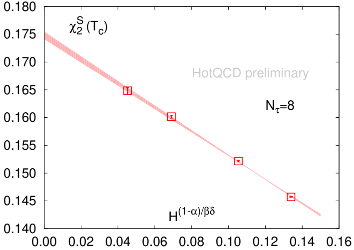

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– ratio of singular parts = ratio of κ_2

$\kappa_2^Q/\kappa_2^B \quad \sim 2.6$

$\kappa_2^B/\kappa_2^S \quad \sim 1.0$

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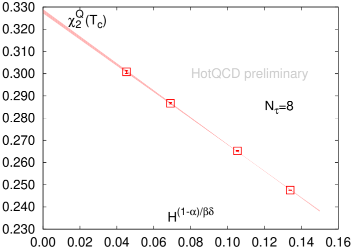

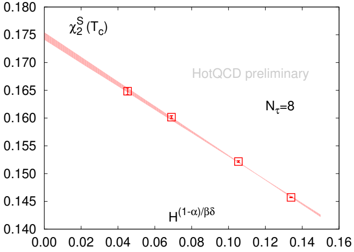

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$\kappa_2^Q/\kappa_2^B \quad \sim 2.6 \quad 1.8(8)^*$

$\kappa_2^B/\kappa_2^S \quad \sim 1.0 \quad 0.9(4)^*$

Close to results for physical mass

*[HotQCD, Phys. Lett. B 795 (2019) 15]

Mixed observables / Derivatives of observables w.r.t t and H

Quark Chiral condensate $\Sigma_u = \frac{m_s}{f_K^4} \langle \bar{u}u \rangle \Rightarrow$ *magnetic-like* observable

$$\Sigma_u \sim H^{1/\delta} f_G(z) + \text{reg.}$$

$$C_{2,B}^{\Sigma_u} \equiv \frac{\partial^2 \Sigma_u}{\partial(\mu_B/T)^2} \sim -\kappa_2^B H^{(\beta-1)/\beta\delta} f'_G(z) + \text{reg.}$$

Divergent already for 2nd order

$$\frac{\beta-1}{\beta\delta} = -0.39, O(2)$$

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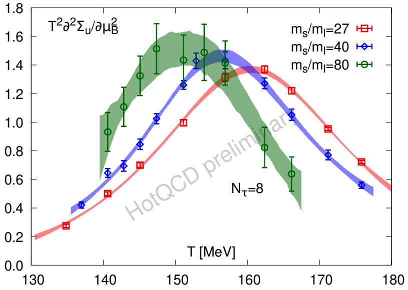
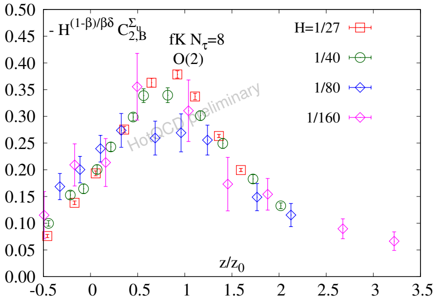
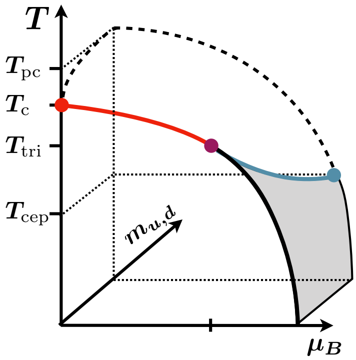




Curvature along the μ direction



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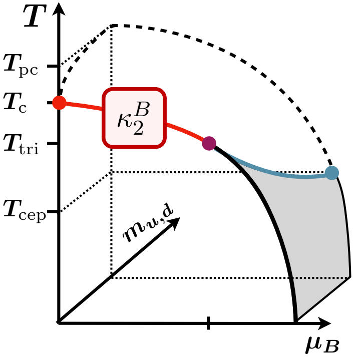

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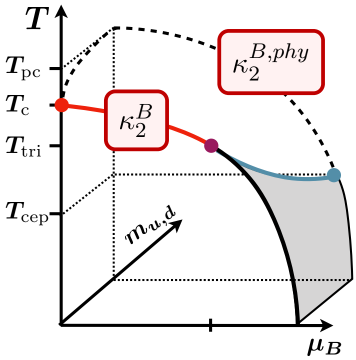

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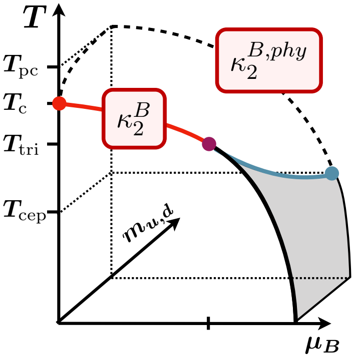
Curvature at a given mass H from Taylor expansion

$$T_{pc}(\mu_B, H) = T_{pc}(0, H)\left(1 - \kappa_2^{B,H}\left(\frac{\mu_B}{T_{pc}(0,H)}\right)^2\right)$$



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[HotQCD, PLB 795 (2019) 15]

Physical mass
$\kappa_2^{B,phy} = 0.015(4)$

Measurements from different groups in agreement

Curvature in the chiral limit

$$\boxed{\kappa_2^B \simeq \frac{T^2 \frac{\partial^2}{\partial \mu_B^2} f}{2T \frac{\partial}{\partial T} f}}$$ (Regular contributions)

$\rightarrow$ proper choice of f such that regular terms are suppressed in the ratio

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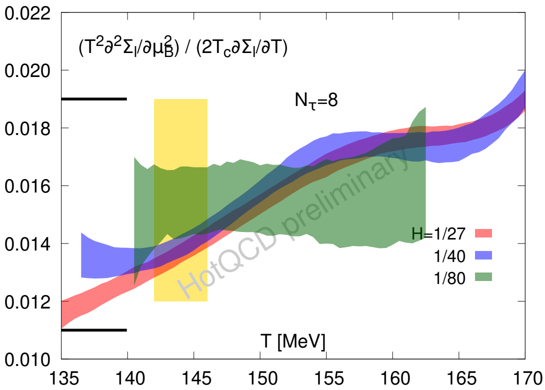

Curvature doesn't seem to change towards the chiral limit

Conclusions and Outlook

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Thank you for your attention