

Minimal dark matter model for muon g-2 with scalar lepton partners at the TeV scale

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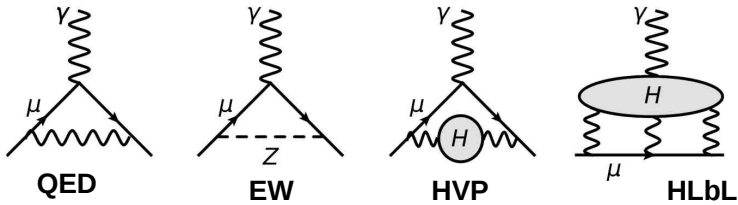
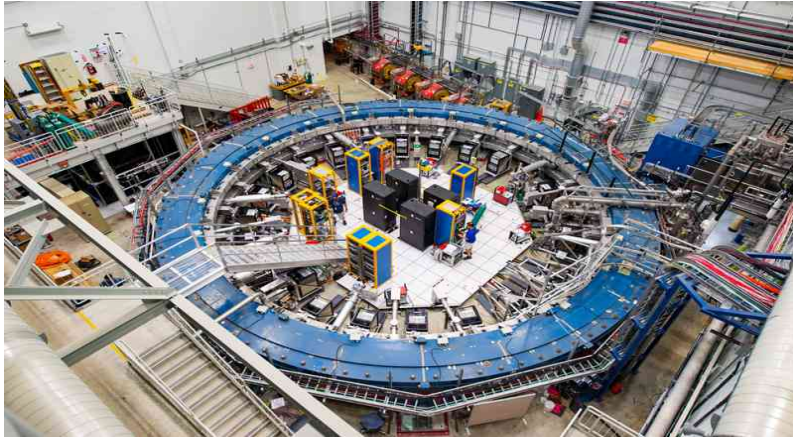
“The Future is Illuminating” Workshop, NTHU, Hsinchu, Taiwan, ROC



Outline

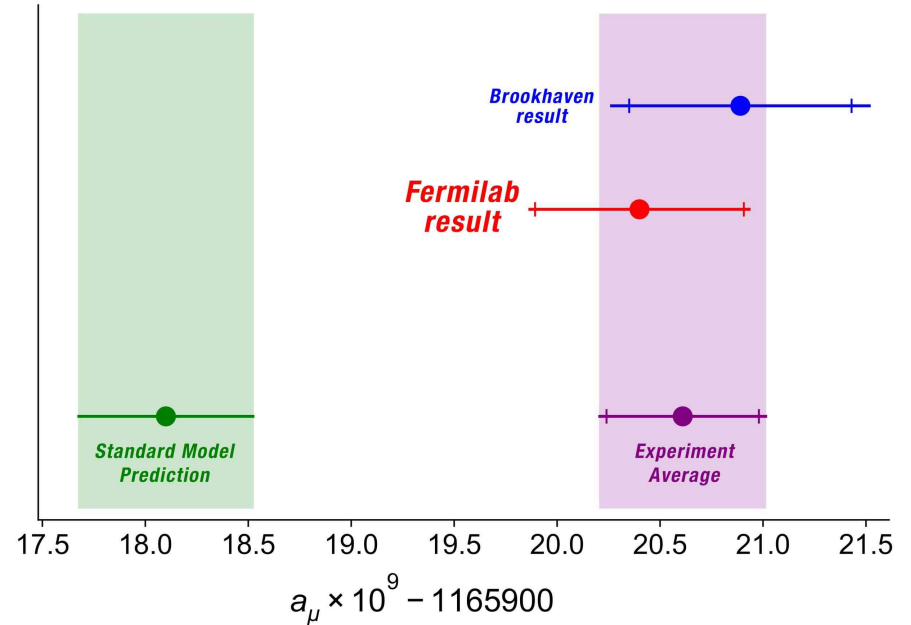
- Motivation: muon $g-2$ anomaly
- A minimal framework: two sets of muonic scalars and a Majorana
- DM production mechanism
- Theoretical consistency constraints
- Direct detection
- Conclusions

Motivation: FNAL measurement



Hadronic contribution: Data-driven (dispersion relation) vs first-principles (lattice QCD; reduction to 1.5σ tension)

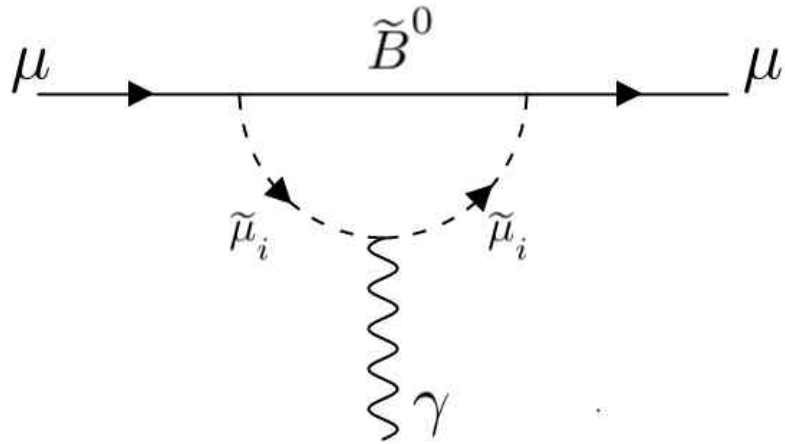
$$\begin{aligned}
 a_\mu(FNAL) &= 116592040(54) \times 10^{-11} (0.46 \text{ ppm}) \\
 a_\mu(BNL) &= 116592080(54)(33) \times 10^{-11} (0.54 \text{ ppm}) \\
 a_\mu(\text{exp}) &= 116592061(41) \times 10^{-11} (0.35 \text{ ppm}) \\
 a_\mu(SM) &= 116591810(43) \times 10^{-11} (0.37 \text{ ppm})
 \end{aligned}$$



$$a_\mu \equiv \frac{g-2}{2}$$

$$\Delta a_\mu = (25.1 \pm 5.9) \times 10^{-10}$$

Assume new physics



Particle	SU(2) _L	U(1) _Y	Z ₂
$\tilde{l}_L = (\tilde{\nu}_\mu \ \tilde{\mu}_L)^T$	2	-1/2	-1
$\tilde{\mu}_R$	1	-1	-1
$l_L = (\nu_\mu \ \mu_L)^T$	2	-1/2	1
μ_R	1	-1	1
\tilde{B}^0	1	1	-1

DM candidate ←

Generic LR mixing; origin yet unspecified ←

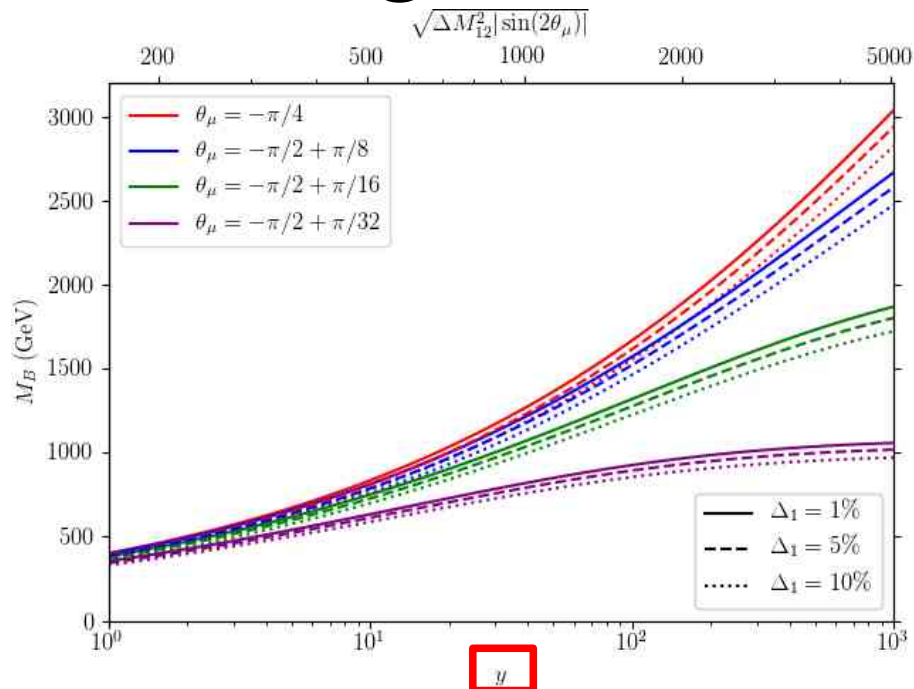
$$\mathcal{L} \supset -\lambda_{\tilde{\mu}_R} \tilde{\mu}_R^* \tilde{B}^0 P_R \mu - \lambda_{\tilde{\mu}_L} \tilde{\mu}_L^* \tilde{B}^0 P_L \mu - \lambda_{\tilde{\nu}} \tilde{\nu}_\mu^* \tilde{B}^0 P_L \nu_\mu + \text{H.c.}$$

$$\mathcal{L} \supset -(\tilde{\mu}_L^* \ \tilde{\mu}_R^*) \begin{pmatrix} m_{LL}^2 & m_{LR}^2 \\ m_{LR}^2 & m_{RR}^2 \end{pmatrix} \begin{pmatrix} \tilde{\mu}_L \\ \tilde{\mu}_R \end{pmatrix}$$

$$\begin{pmatrix} \tilde{\mu}_1 \\ \tilde{\mu}_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_{\tilde{\mu}} & -\sin \theta_{\tilde{\mu}} \\ \sin \theta_{\tilde{\mu}} & \cos \theta_{\tilde{\mu}} \end{pmatrix} \begin{pmatrix} \tilde{\mu}_L \\ \tilde{\mu}_R \end{pmatrix}$$

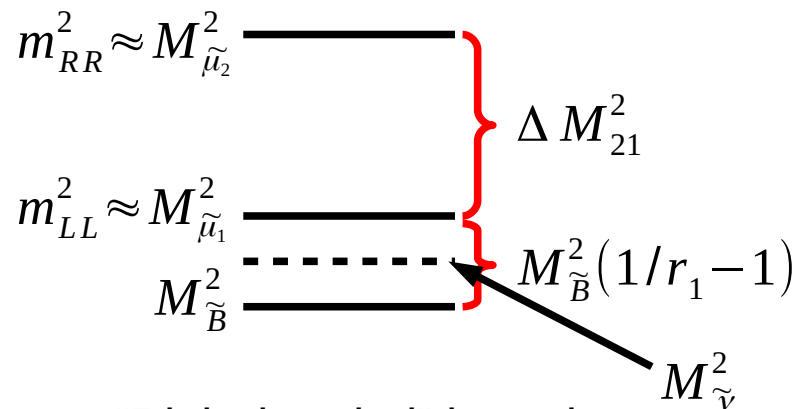
$$M_{\tilde{\nu}}^2 = M_{\tilde{\mu}_1}^2 + \Delta M_{21}^2 \sin^2 \theta_{\tilde{\mu}} - M_W^2$$

Muon g-2 contribution

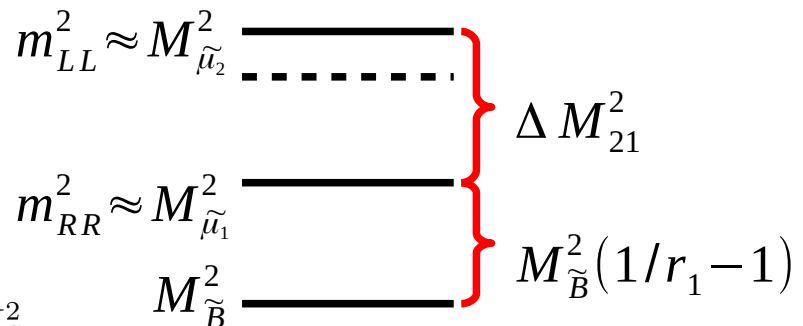


Parameter space:
 M_B, Δ_1, y, θ

“Left-handed” branch



“Right-handed” branch



$$\frac{\Delta a_\mu}{25.1 \times 10^{-10}} \simeq \left(\frac{100 \text{ GeV}}{M_{\tilde{B}}} \right) \left[-\frac{\sin(2\theta_{\tilde{\mu}})}{6.23 \times 10^{-2}} \right] \left[\frac{L_0(r_1)}{0.05} \right], \quad \Delta M_{21}^2 \simeq M_{\tilde{B}}^2$$

$$\frac{\Delta a_\mu}{25.1 \times 10^{-10}} \simeq (0.068) \left(\frac{1 \text{ TeV}}{M_{\tilde{B}}} \right)^3 \left[-\frac{\Delta M_{21}^2 \sin(2\theta_{\tilde{\mu}})}{4M_W^2} \right] \left[\frac{L_1(r_1)}{0.083} \right], \quad \Delta M_{21}^2 \ll M_{\tilde{B}}^2$$

Additional gauge invariant terms and MFV

- Interactions of extra states with SM gauge fields are implied
- Take MSSM as benchmark
- Only 1 light SM Higgs-like state
- **LR mixing comes from the A-term**

Take home message: We are working in the “large y ” regime where MFV assumption is relaxed!

$$\mathcal{L} \supset m_Z^2 \sum_j \mathcal{S}_j^\dagger \left(\hat{T}_3 - \hat{Q}_{em} s_W^2 \right) \mathcal{S}_j \left(1 + \frac{h}{v} \right)^2$$

$$- \frac{m_{LR}^2}{v} h (\tilde{\mu}_L \tilde{\mu}_R^\dagger + \text{h.c.})$$

$$A_\mu = m_{LR}^2/v = gM_W y$$

cf. Minimal flavor violation (MFV) +
weak scale-TeV dimensionful coupling

$$A_\mu \sim (\text{muon Yukawa})(\text{weak scale})$$

DM production mechanism

Standard lore:

S-wave annihilation

-need mass insertion ($\sim m_\mu^2/M^2$) to conserve angular momentum or LR mixing of scalars (killed by MFV)

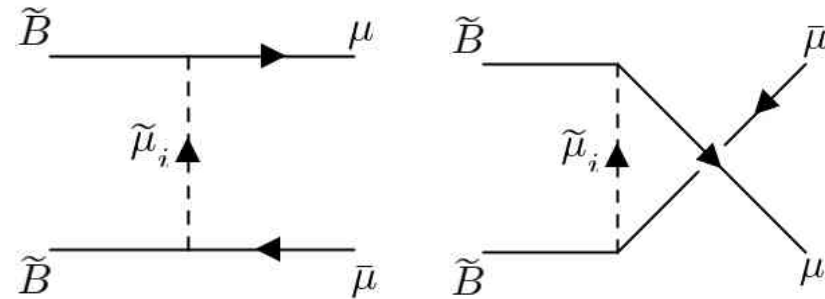
P-wave annihilation

-leads to O(1) pb cross section for ~ 100 GeV sleptons; excluded by collider searches

“Incredible bulk”^[1]

-MFV is relaxed, y can be large
-s-wave cross section is correlated with Δa_μ

WIMP thermal freeze-out?



Remark:
Could be saved by CPV phase (enters as $1/\cos^2\phi$)

$$\langle\sigma v\rangle_0 = \frac{32\pi^3}{m_\mu^2} (\Delta a_\mu)^2 \mathcal{F} \quad \Omega h^2 \sim 0.1 \left(\frac{1 \text{ pb}}{\langle\sigma v\rangle(T_f)} \right)$$

$$= (2.25 \times 10^{-4} \text{ pb}) \left(\frac{\Delta a_\mu}{25 \times 10^{-10}} \right)^2 \mathcal{F},$$

$$\mathcal{F} \equiv \frac{1}{[L(M_B^2/m_1^2) - L(M_B^2/m_2^2)]^2} \left(\frac{1}{1 + m_1^2/M_B^2} - \frac{1}{1 + m_2^2/M_B^2} \right)^2$$

[1] Fukushima, K., Kelso, C., Kumar, J., Sandick, P., & Yamamoto, T. (2014). MSSM dark matter and a light slepton sector: The incredible bulk. Physical Review D, 90(9), 095007.

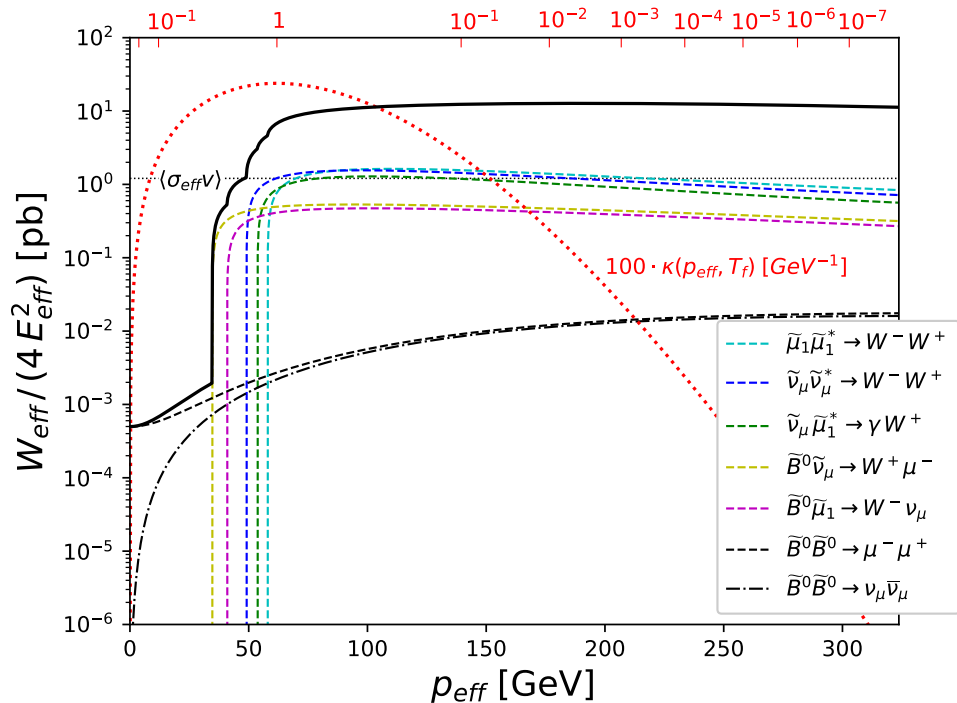
DM production mechanism

Coannihilations

$$\frac{dn}{dt} + 3Hn = -\langle\sigma v\rangle_{eff} (n^2 - n_{eq}^2)$$

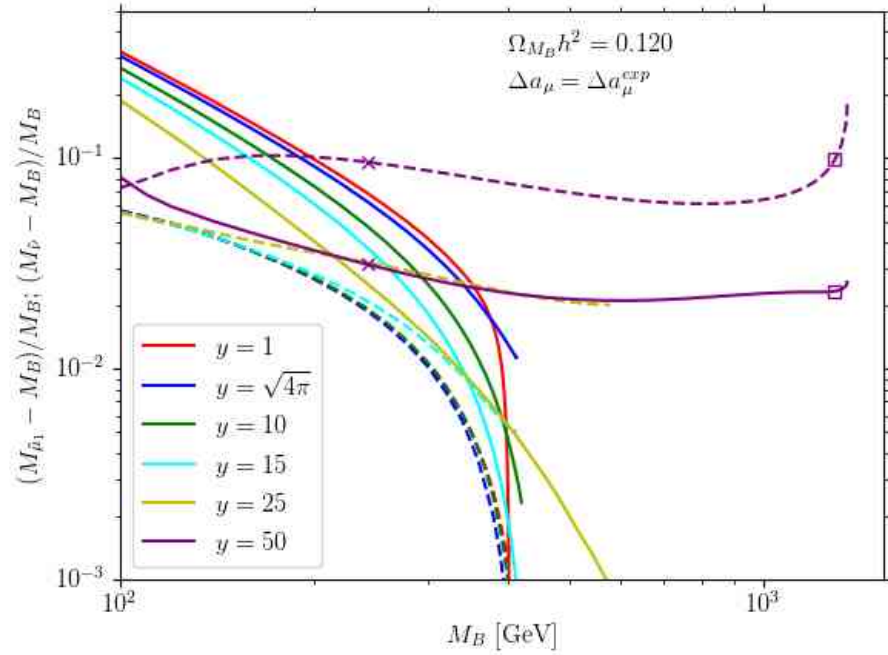
$$\begin{aligned} \langle\sigma_{eff} v\rangle(T) &= \sum_{ij} \langle\sigma_{ij} v\rangle(T) \frac{n_i^{eq}(T)n_j^{eq}(T)}{[n^{eq}(T)]^2} \\ &= \frac{\int_0^\infty dp_{eff} p_{eff}^2 W_{eff}(s) K_1(\frac{\sqrt{s}}{T})}{m_1^4 T [\sum_i \frac{g_i}{g_1} \frac{m_i^2}{m_1^2} K_2(\frac{m_i}{T})]^2} \end{aligned}$$

$$\begin{aligned} W_{eff}(s) &= \sum_{ij} \frac{p_{ij}}{p_{11}} \frac{g_i g_j}{g_1^2} W_{ij} \\ &= \sum_{ij} \sqrt{\frac{[s - (m_i - m_j)^2][s - (m_i + m_j)^2]}{s(s - 4m_1^2)}} \frac{g_i g_j}{g_1^2} W_{ij} \end{aligned}$$

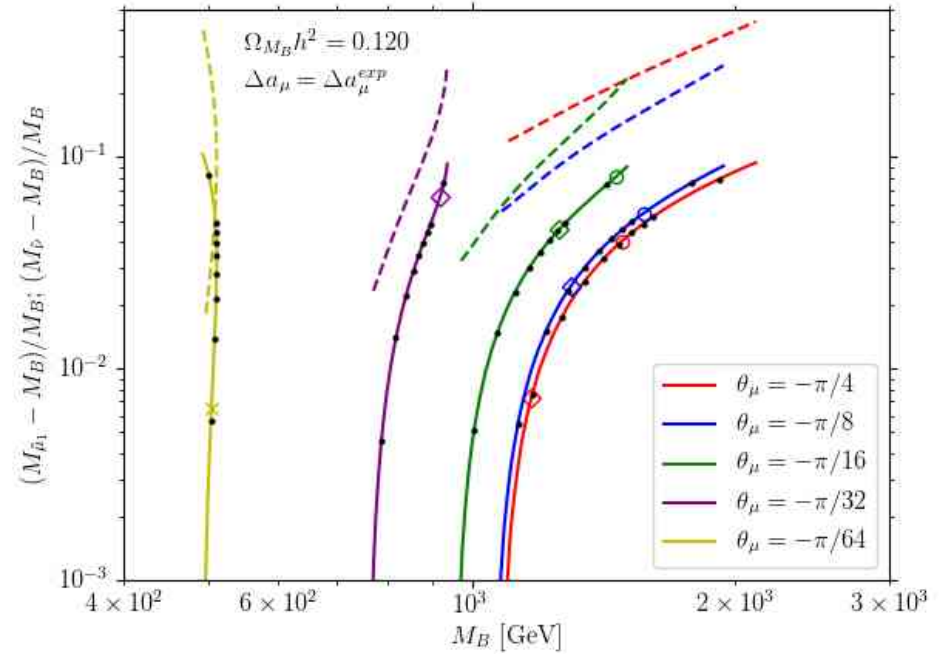


$$\langle\sigma_{eff} v\rangle = \int_0^\infty dp_{eff} \frac{W_{eff}(p_{eff})}{4E_{eff}^2} \kappa(p_{eff}, T)$$

g-2 + relic density



LHB (fixed y)
(dashed curve:
sneutrino-bino
rel. mass split.)



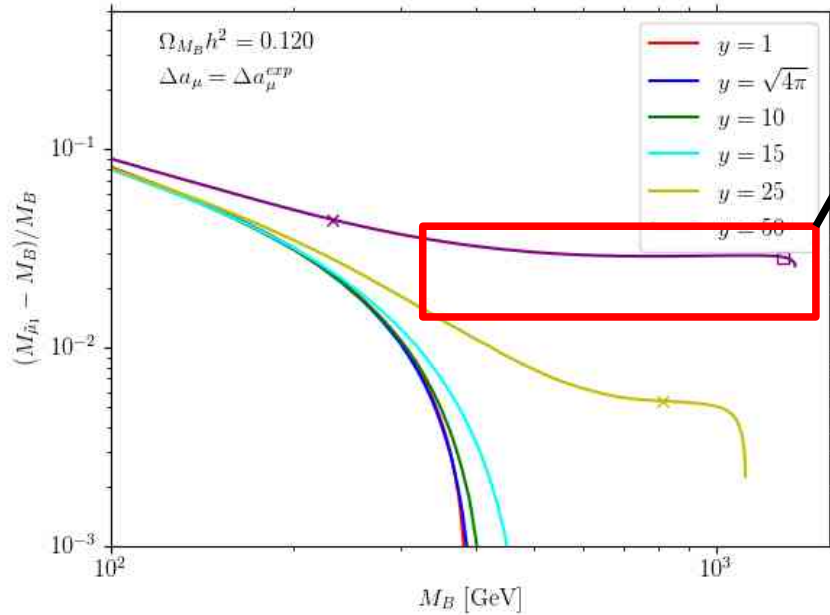
LHB (fixed θ)

$$\langle \sigma v \rangle_{eff} = \sum_{ij} \langle \sigma v \rangle_{ij} r_i r_j,$$

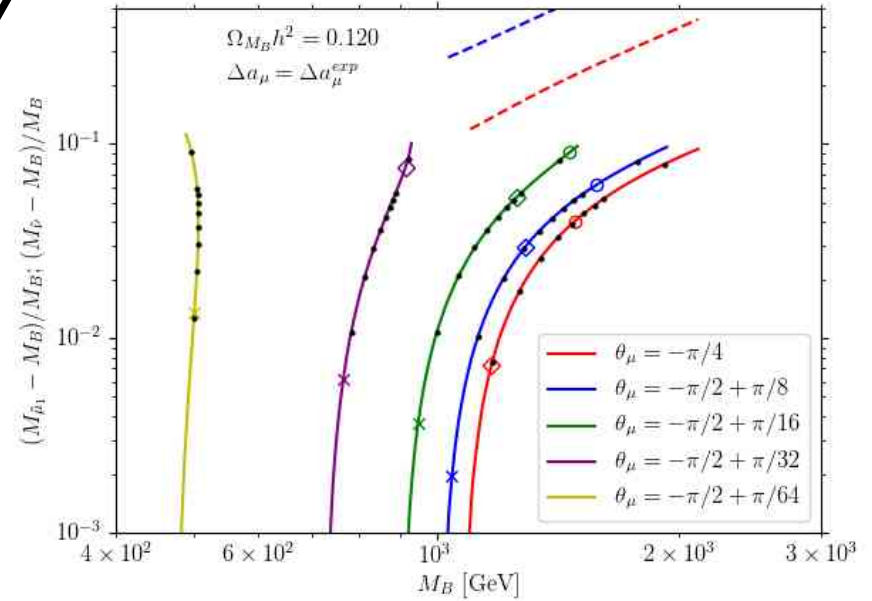
$$r_i \equiv \frac{\frac{g_i}{g_{LSP}} (1 + \Delta_i)^{3/2} \exp(-x\Delta_i)}{1 + \sum_{k \neq LSP} \frac{g_k}{g_{LSP}} (1 + \Delta_k) \exp(-x\Delta_k)}$$

g-2 + relic density

Large y : What's happening here???



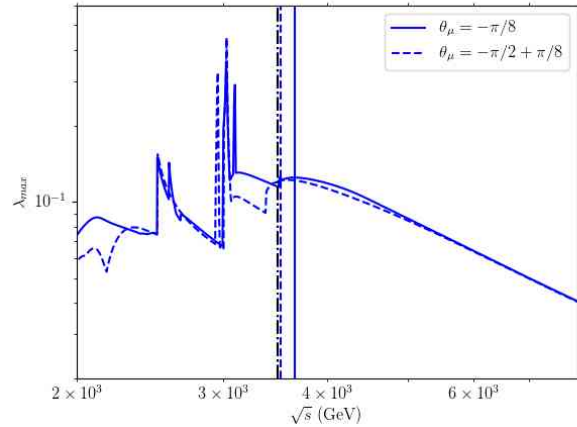
RHB (fixed y)



RHB (fixed θ)

Take home message: Low y explored in old MSSM scans; relaxing MFV and considering compressed spectra leads to TeV scale particles

Theoretical consistency: Perturbative unitarity

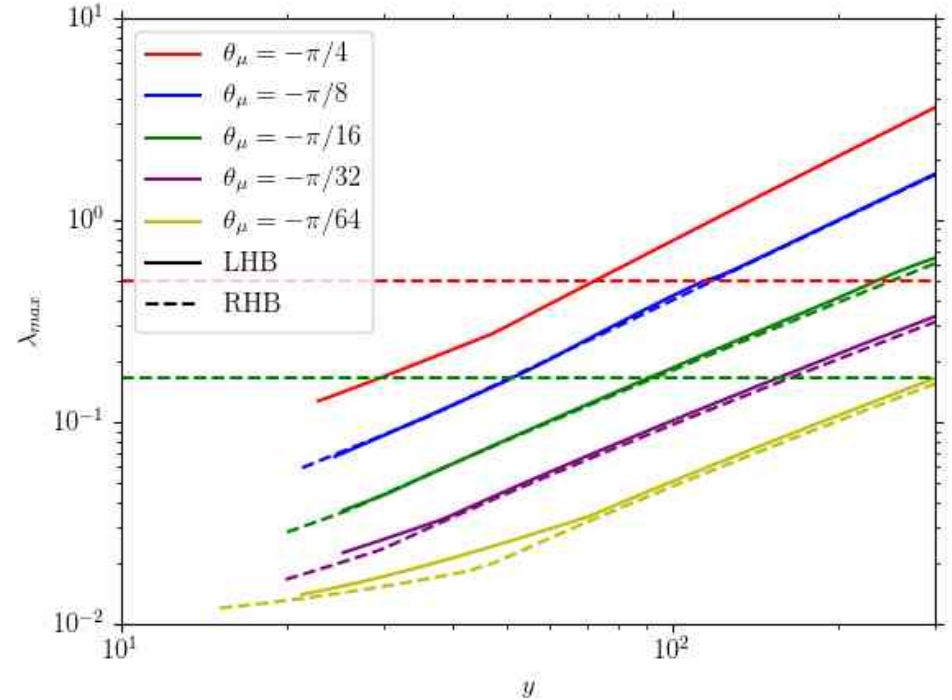


$$a_{fi,J}(s) \equiv \frac{1}{32\pi} \sqrt{\frac{4p_1 p_3}{2\delta_{12} 2\delta_{34} s}} \int_{-1}^1 d(\cos\theta) T_{fi}(s, \cos\theta) P_J(\cos\theta)$$

$$2\text{Im}\{a_{fi,J}\} \leq \sum_k a_{kf,J}^* a_{ki,J}$$

Criterion: Max eigenvalue < 0.5

Implemented using SARAH-SPheno
interfacing



Careful not to pick up the poles
and thresholds!

Theoretical consistency: EW vacuum stability

$$V_2 = \frac{m_{LL}^2}{2} (X^2 + Y^2) + \frac{m_{RR}^2}{2} Z^2 + \frac{\mu^2}{2} h^2$$

$$V_{\text{mix}} = \frac{k_s}{\sqrt{2}} hYZ$$

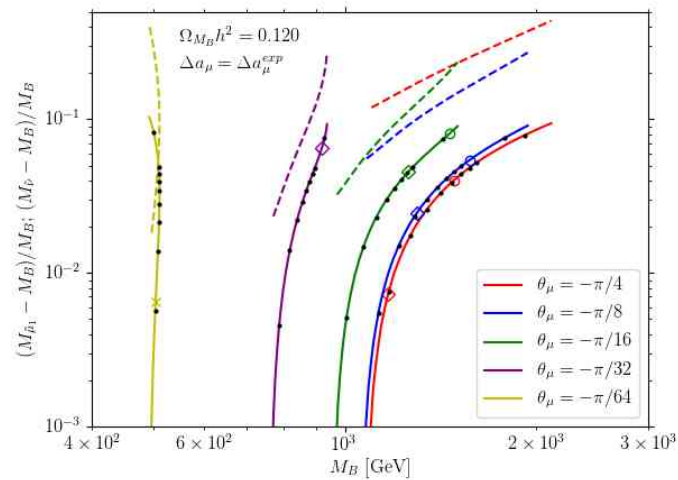
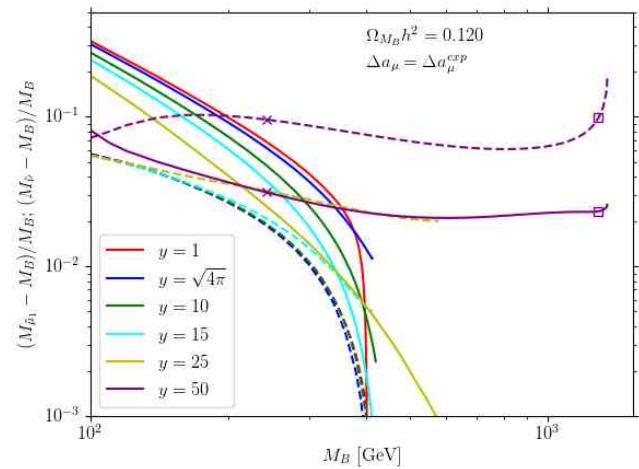
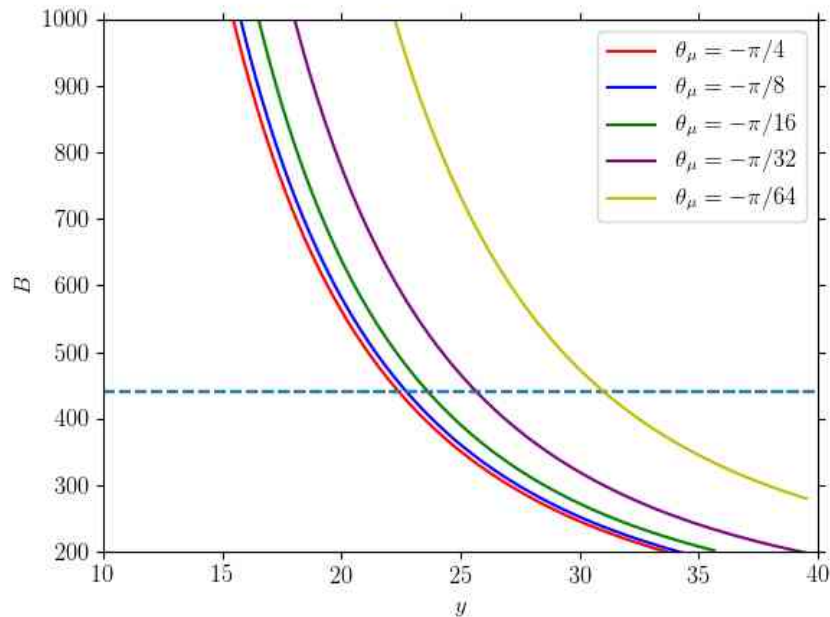
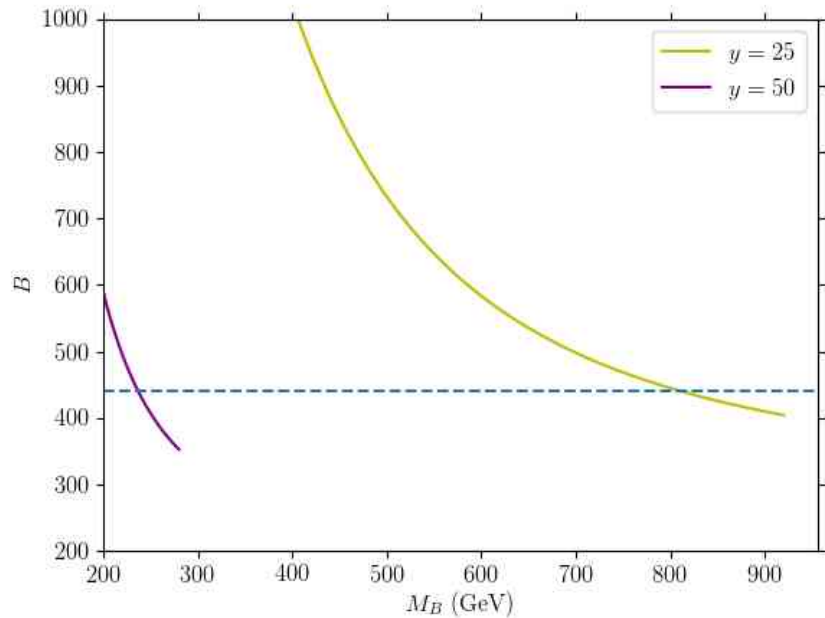
$$V_D^{(1)} = \frac{g'^2}{32} [h^2 - (X^2 + Y^2) + 2Y_R Z^2]^2$$

$$V_D^{(2)} = \frac{g^2}{32} [h^4 - 2h^2(X^2 - Y^2) + (X^2 + Y^2)^2].$$

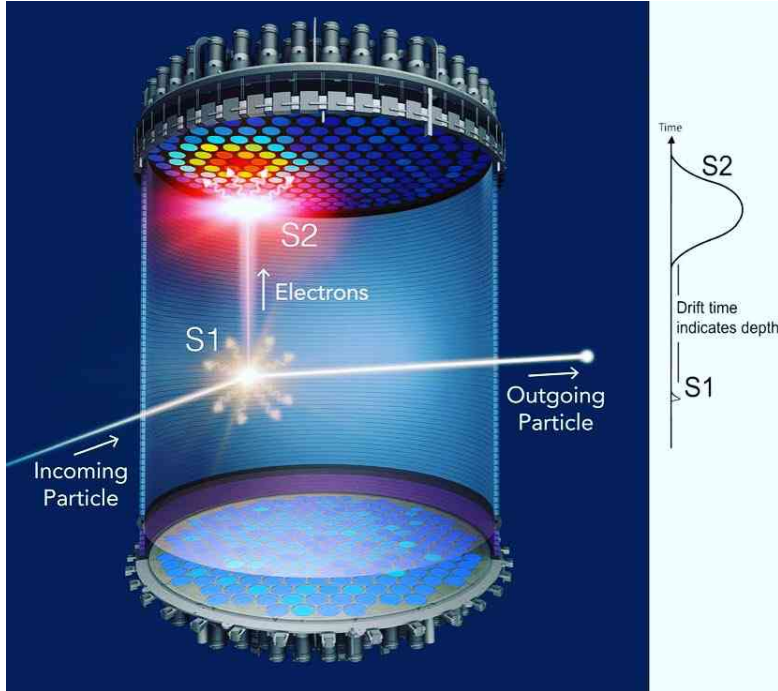
- Absolute stability
- Metastability
 - Equivalent to setting the bounce action B at least 440
- Used FindBounce to compute B

$$B = \int_0^\infty d\rho [\mathcal{T} + \mathcal{V}], \quad \mathcal{T} = \frac{\pi^2}{2} \rho^3 \left[\frac{1}{2} \sum_{\phi=h,Y,Z} \left(\frac{d\phi}{d\rho} \right)^2 \right],$$

$$\mathcal{V} = \frac{\pi^2}{2} \rho^3 V_{\text{tot}}(h, Y, Z)$$

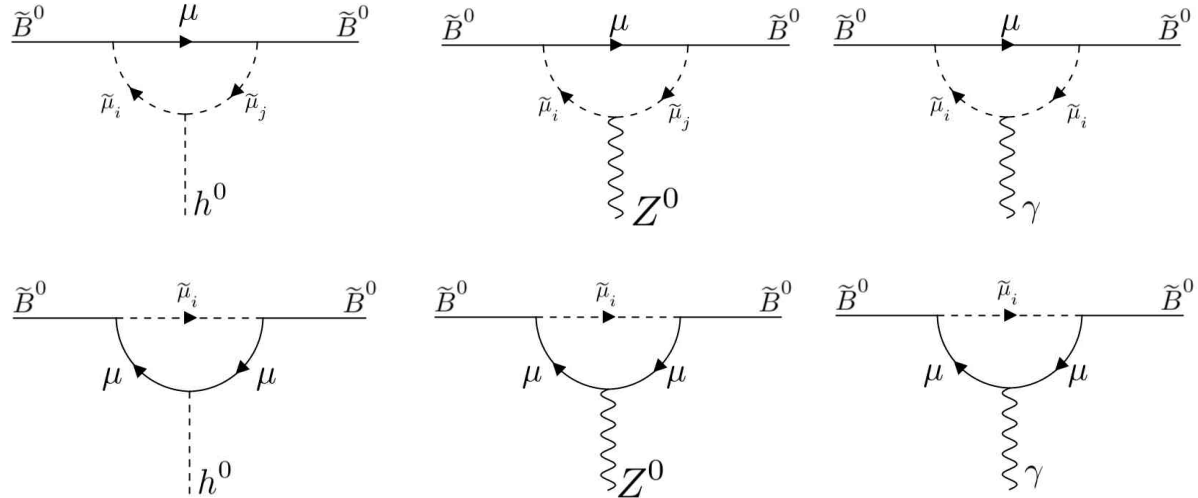


Direct detection

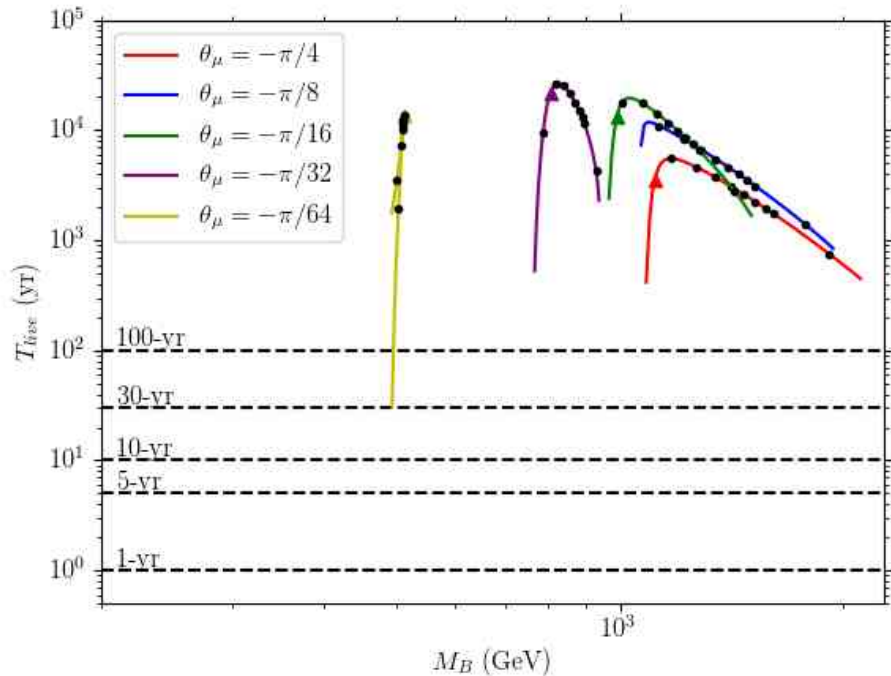
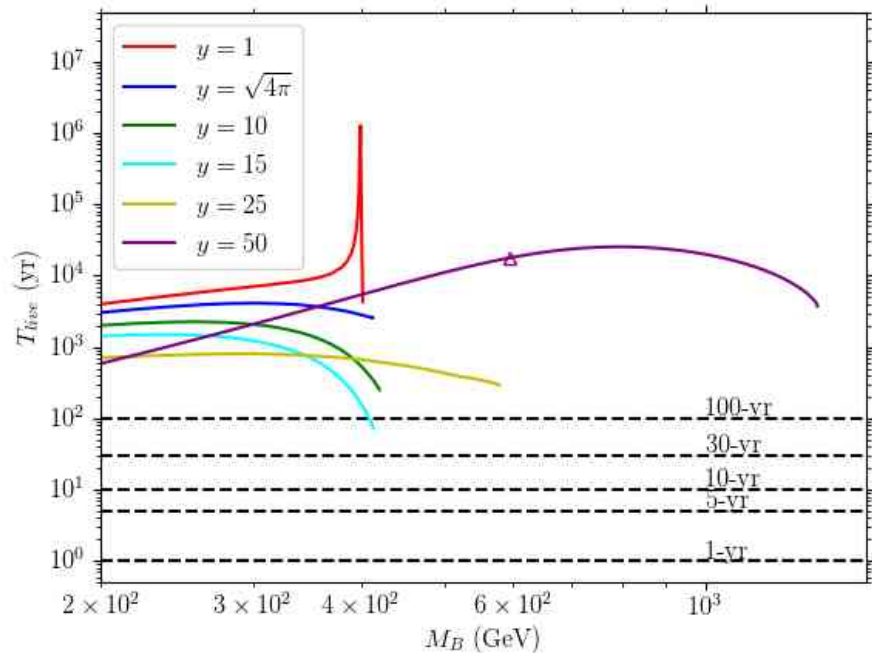


$$\mathcal{L}_{\tilde{B}^0 q} = c_q^{(0)} \tilde{B}^0 \tilde{B}^0 m_q \bar{q} q + c_q^{(1)} \tilde{B}^0 \gamma_\mu \gamma^5 \tilde{B}^0 \bar{q} \gamma^\mu \gamma^5 q + e Q_q c_A(k^2) \tilde{B}^0 \gamma_\mu \gamma^5 \tilde{B}^0 \bar{q} \gamma^\mu q$$

$$\mathcal{L}_{\text{NREFT}} = \sum_{N=p,n} [c_N^{(0)} \mathcal{O}_1^{(N)} - 4c_N^{(1)} \mathcal{O}_9^{(N)}] - e c_A(k^2) [2\mathcal{O}_8^{(p)} - 2\mathcal{O}_9^{(p)}], \quad \mathcal{O}_8 \equiv \vec{S}_\chi \cdot \vec{v}^\perp, \quad \mathcal{O}_9 \equiv i \vec{S}_\chi \cdot \left(\vec{S}_N \times \frac{\vec{k}}{m_N} \right)$$



Direct detection: bino-like DM



$$c_q^{(0)} \simeq \frac{g^2 y_{H_2^0 11}}{32\pi^2 M_{H^0}^2 M_{\tilde{B}}} (\lambda_{\tilde{\mu}R}^2 \sin^2(\theta_{\tilde{\mu}}) + \lambda_{\tilde{\mu}L}^2 \cos^2(\theta_{\tilde{\mu}})) \left[1 + \frac{1-r_1}{r_1} \ln(1-r_1) \right]$$

$$c_A \approx \frac{e}{48\pi^2} \sum_{i=1,2} \alpha_{\mu}^{(i)} \beta_{\mu}^{(i)} \int_0^1 dx \frac{3x-2}{x + (1-x)t_i - x(1-x)r_i}$$

$$L_A(r_i, t_i) \approx \begin{cases} \left(2 - \frac{2}{t_i} - 3 \ln t_i\right) + \left(4 + \frac{1}{t_i^2} - \frac{5}{t_i} - 3 \ln t_i\right) (1-r_i) + \mathcal{O}((1-r_i)^2), \\ \left(3 - 3t_i + 2 \ln t_i\right) + \left(\frac{7}{2} - 5t_i + \frac{3t_i^2}{2} + 2 \ln t_i\right) r_i + \mathcal{O}(r_i^2). \end{cases}$$

Summary

- We assessed the viability of a minimal setup involving two muonic scalars and a Majorana fermion to resolve the muon $g-2$ anomaly and provide for a DM candidate
- Relaxing MFV + compressed spectra at TeV = DM production through coannihilations
- Deviating too much from MFV may violate perturbative unitarity and EW vacuum (meta)stability
- Direct detection weakly constrains the model, warrants alternative ways of probing the model, e.g. lepton collider, NS heating

Thank you for your attention.

謝謝大家的關注。