Glueball dark matter in SU(N) lattice gauge theory

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In Collaboration with
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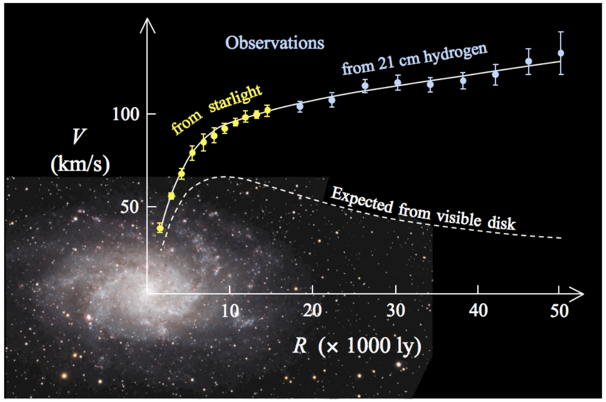
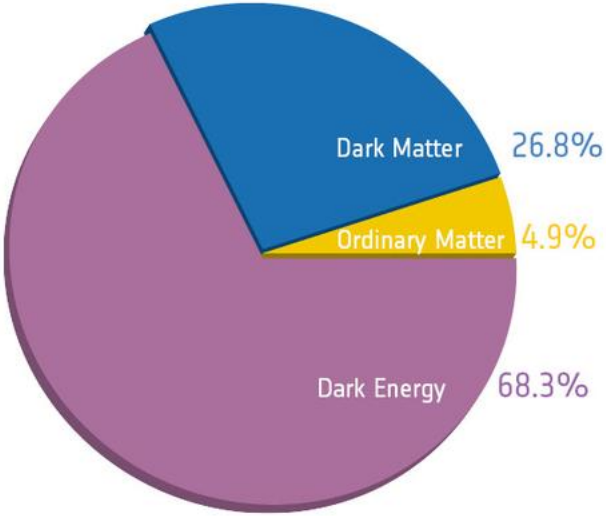
Based on
N. Yamanaka et al., Phys. Lett. B **813**, 136056 (2021)
N. Yamanaka et al., Phys. Rev. D **102**, 054507 (2020)
N. Yamanaka et al., PoS LATTICE2021 (2022) 447

2022/06/29
NCTS Workshop

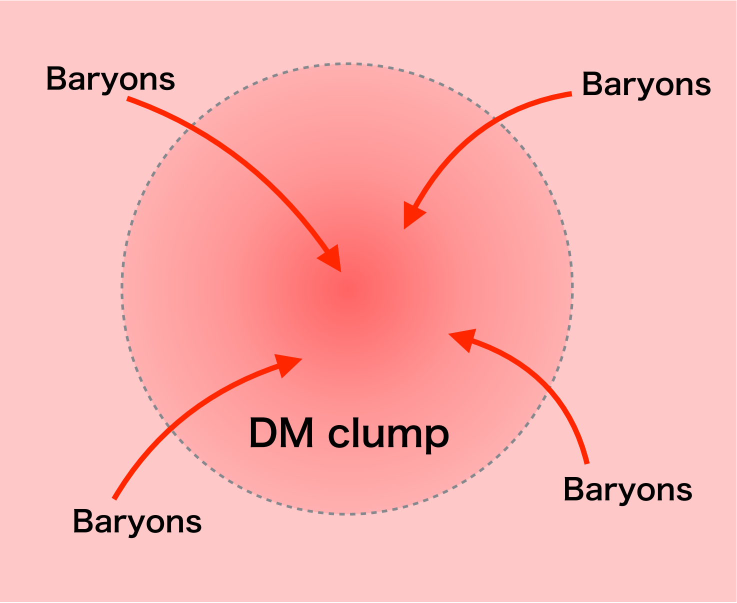
Many evidences of Dark matter



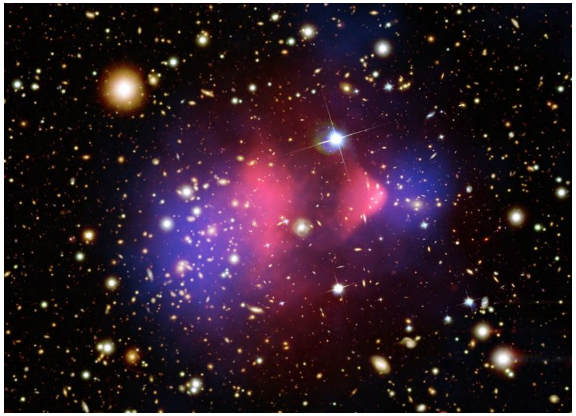
Galactic rotation curve



DM density extracted from CMB



N-body simulation : large-scale structure



Bullet cluster : collision of galaxies

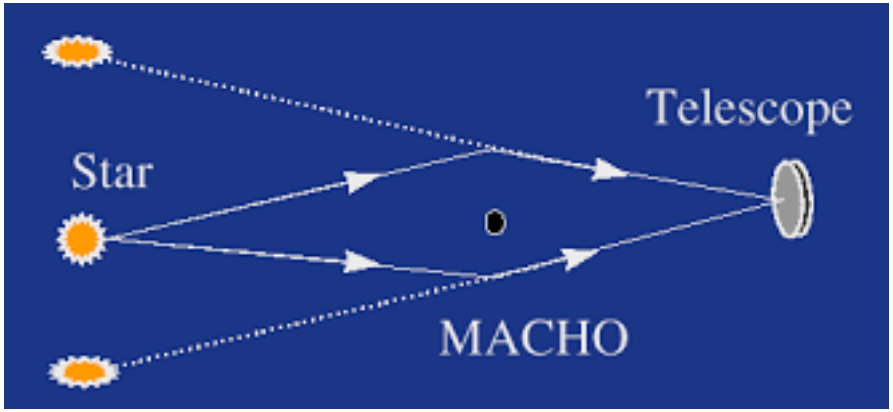
Is the dark matter a MACHO (=PBH) ??

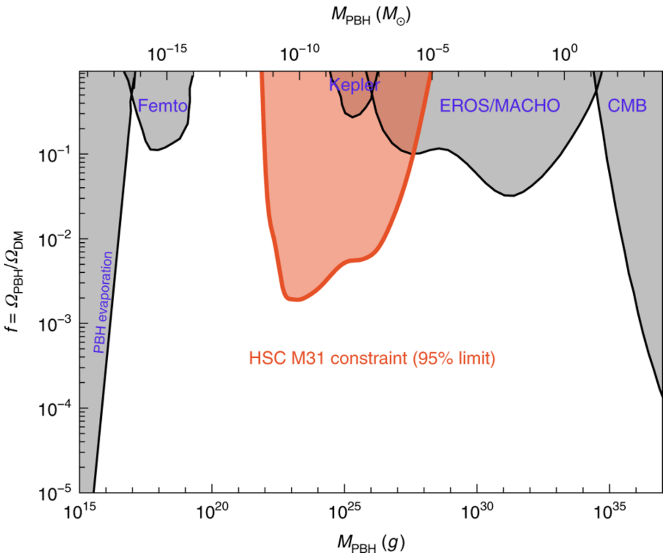
MACHO : Massive Compact Halo Object

Almost non luminous astronomical body

Example : primordial blackholes, brown dwarfs

Can be probed with gravitational lensing





H. Niikura et al., Nature Astronomy (2019) (arXiv:1701.02151 [astro-ph.CO])

MACHOs are not favored by observations,
even if a window (around $M_{PBH}/M_\odot \sim 10^{-12}$) is still left

⇒ Dark matter is likely to be particles?

WIMP dark matter

WIMP : weakly interacting massive particle

WIMP = particle physics

Property of WIMPs:

No charge, no color

Nonrelativistic (= “cold” DM , not neutrino)

No candidates in standard model of particle physics

Challenge in particle physics:

⇒ Find theory explaining dark matter!

Several WIMP scenarios

There are several classes of WIMPs

- **Elementary WIMPs with extension of the standard model**

  Often protected by discrete symmetry

  (R-parity in SUSY, KK-parity in extradimension, Z_2 symmetry in extended Higgs, …)

  Particles interact with SM particles : constraints from direct/indirect detections

  L. Roszkowski et al., Rept. Prog. Phys. **81** (2018) 066201

- **Axions (very light particle DM)**

  Solve the Strong CP problem

  May have problems with quantum gravity (axion quality problem?)

- **Additional (dark) gauge theories**

  New gauge theories introduce dark photons, pions, baryons, glueballs

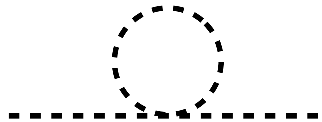
  Theories are often nonperturbative and composite DM appears

Summary of dark gauge theories and their problems (1)

Dark photons, nonabelian gauge bosons :

Photons get mass through Higgs mechanism

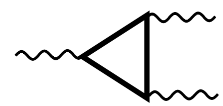
Problem : Higgs sector is ad hoc, massive parameter introduces fine-tuning pb.



$\sim \Lambda^2$

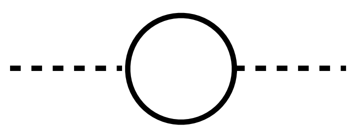
Dark nonabelian gauge theory with chiral fermions :

We have the problem of chiral anomaly



If the dark fermions are massless, then massless dark pions becomes the DM
⇒ Hot DM problem

If dark fermions have mass < scale parameter (Λ), no hot DM problem,
but Yukawa coupling and Higgs mechanism will be required ⇒ Fine-tuning!



$\sim \Lambda^2$

Summary of dark gauge theories and their problems (2)

Dark nonabelian gauge theory with vectorlike fermions :

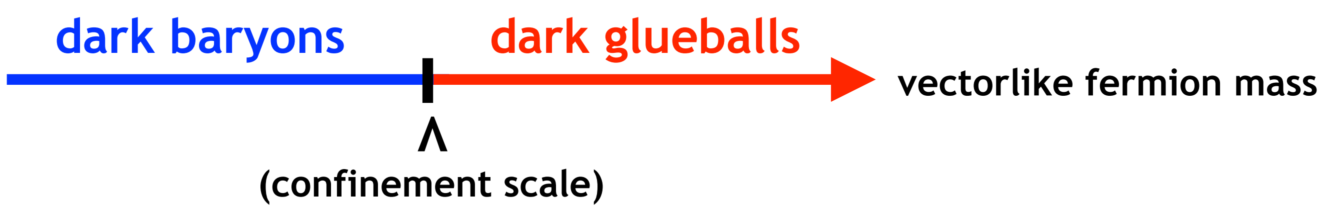
Vectorlike fermions (same gauge representation for LH and RH fermions)
$\Rightarrow$ No problem of chiral anomaly, Higgs mechanism

If the fermions have additional weak $SU(2)_L$ charge, the dark baryon number can be generated by thermal change of topological charge (sphaleron process)

T. Appelquist et al., Phys. Rev. D **92**, 075030 (2015)

Problem : vectorlike mass is ad hoc, how much is it?

If vectorlike mass < Λ $\Rightarrow$ dark baryons , else $\Rightarrow$ dark glueballs



Dark Yang-Mills theory (no coupled scalars or fermions) :

$\Rightarrow$ Dark glueballs

No apparent problem

(see next page)

SU(N) pure Yang-Mills theory

$$\mathcal{L}_{\rm YM} = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu,a}$$

($a = 1,\ldots,N_c^2-1$)

⇒ The simplest interacting theory

Important properties:

$\mathscr{L}_{\rm YM}$ does not have apparent scale, but scale is dynamically generated (dimensional transmutation)

Renormalizable theory, running coupling has logarithmic scale variation,

difference of N_c can generate $\Lambda_{\rm YM}$'s which differ by orders of magnitude

No scalars and massive fermions ⇒ Free from quadratic divergences

⇒ No important fine-tuning problem in the choice of $\Lambda_{\rm YM}$!

(Suppose a GUT which generates SM and DM,
the difference of mass scales between SM and DM is not serious)

⇒ Theory with very high naturalness

Dark matter in hidden YM theory:

Lightest particles are glueballs ! ⇒ SU(N) glueballs are candidate of DM

(summarized in the report of USQCD Collaboration : Eur. Phys. J. A 55 (2019) 198)

Self-interacting dark matter

The DM distribution can be predicted in N-body simulation with gravity only

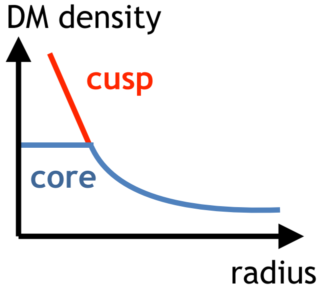
⇒ Successful in describing the large scale structure (scale > Mpc)

Introducing DM self-interaction changes the structure smaller than Mpc
(= DM-DM scattering)

There are (were?) several problems in the galactic DM distribution:

- **Core vs Cusp problem:**

  N-body simulation predicts cuspy DM distribution near the galactic center, whereas observations suggest flat ones.



- **Too-big-to-fail problem:**

  Satellite galaxies are less dense than those predicted by the N-body simulation.

- **Missing satellite problem:**

  More satellite galaxies than those predicted by the N-body simulation are observed.

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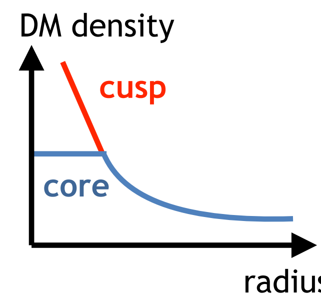

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Resolved thanks to improvement of observation?

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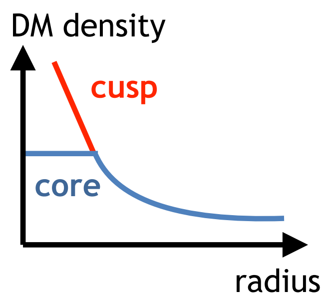

- **Too-big-to-fail problem:**

  Satellite galaxies are less dense than those predicted by the N-body simulation.

- **Missing satellite problem:**

  Resolved thanks to improvement of observation?

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➡ Still under debate, but this shows the importance of the investigation of DM-DM scattering

Object of study

Glueballs of SU(N) Yang-Mills theory are good candidates of dark matter

We need to quantify scattering between dark matter particles

In this work, we study the interglueball scattering on lattice which is the only way to quantify nonperturbative physics of nonabelian gauge theory.

The Yang-Mills theory depends only on the scale parameter Λ (for given N_c): can we determine Λ from observation?

Object:

In this work, we study the interglueball scattering of SU(N) Yang-Mills theory on lattice, and set constraint on its scale parameter Λ from observational data.

Setup

We consider the SU(2) pure Yang-Mills theory

- Standard SU(2) plaquette action :

  Lattice spacings : β = 2.1, 2.2, 2.3, 2.4, 2.5

  Volume : 10^3x12 ∼ 16^3x24

  Confs. generated with pseudo-heat-bath method (1 M confs.)

- Use SX-ACE (@RCNP, Osaka U.), vector machine

- Improvement of glueball operator : APE smearing

  We use all space-time translational and cubic rotational symmetries to effectively increase the statistics (like the all-mode average for meson and baryon observables)

  Reduction of the statistical error w/ cluster decomposition principle (CDERT)

  chiQCD Collaboration, PRD**97**, 034507 (2018)

Scale determination

We do not know the scale of the YM theory, so we leave it as a free parameter Λ

Nevertheless, all quantities calculated on lattice depend on Λ

$\Rightarrow$ We express all quantities in unit of Λ (and finally constrain Λ from other data).

Relation between Λ and string tension:

$$\frac{\Lambda_{\overline{MS}}}{\sqrt{\sigma}} = 0.503(2)(40) + \frac{0.33(3)(3)}{N^2}$$

$$= 0.586(41) \qquad \text{(for SU(2))}$$

Fitted from the analysis of the running coupling

C. Allton et al., JHEP **0807** (2008) 021
M. Teper, Acta Phys. Polon. B **40** (2009) 3249

String tension in SU(2) YM :

β	$a\sqrt{\sigma}$
2.1	0.608(16)
2.2	0.467(10)
2.3	0.3687(22)
2.4	0.2660(21)
2.5	0.1881(28)

M. Teper, Phys. Lett. B **397** (1997) 223; hep-th/9812187

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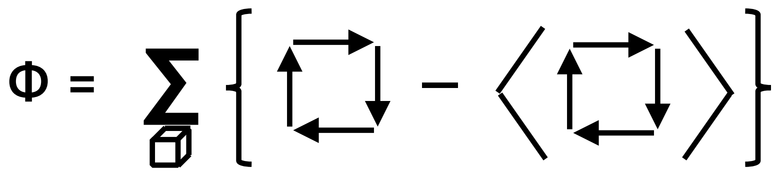
String tension in SU(2) YM :

β	$a\sqrt{\sigma}$	a (in unit of Λ^{-1})
2.1	0.608(16)	0.356(27)
2.2	0.467(10)	0.273(20)
2.3	0.3687(22)	0.216(15)
2.4	0.2660(21)	0.156(11)
2.5	0.1881(28)	0.110(8)

$\Rightarrow$ Lattice spacing is now expressed in unit of Λ

Glueball operator and operator improvement

0^{++} glueball operator:

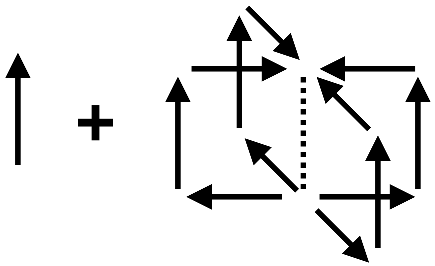


Glueball has vacuum expectation value
→ Subtract
Sum over cubic rotational invariance

APE smearing :

$U^{(n+1)}$ so as to maximize $\mathrm{Re\,Tr}\,[\,U^{(n+1)}\,V^{(n)\dagger}]$

where $V^{(n)} = \alpha \times$



⇒ Gaussian spread: $2\sqrt{\dfrac{n}{4+\alpha}}$

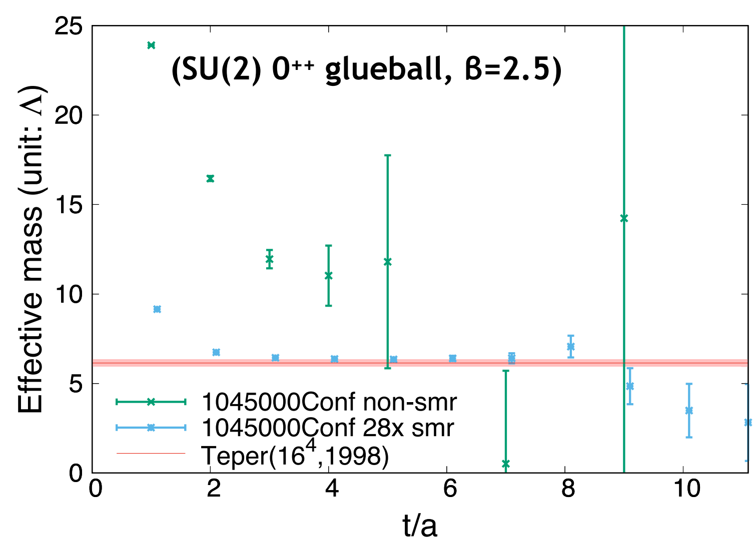
(in lattice unit)

Ape Collaboration, PLB **192** (1987) 163
N. Ishii et al., PRD **66**, 094506 (2002)

Optimal parameters
for SU(2), β=2.5:

n = 28

α = 2.0



Extract the scattering cross section on lattice

In principle, the information of the hadron-hadron scattering can be extracted by analyzing the Nambu-Bethe-Salpeter (NBS) amplitude
(n-point function with the sink having equal-time space-like correlation)

2 known methods to extract the information of scattering:

- **Direct method (Luescher):**

  Calculate the scattering phase shift directly in the momentum space, need the modulation of the energy of NBS wavefunction in momentum

  To determine the energy of the system, ground state saturation (plateau) is absolutely required.

  M. Luescher, Nucl. Phys. B **354**, 531 (1991).

- **HALQCD method:**

  Calculate the interhadron potential on lattice, scattering phase shift is obtained by solving the scattering equation with this potential.

  For E < threshold, NBS amplitude fulfills nonrelativistic Schroedinger eq.

  Crucial advantage : no need of GS saturation (see later)

  S. Aoki, T. Hatsuda, and N. Ishii, Prog. Theor. Phys. **123**, 89 (2010).

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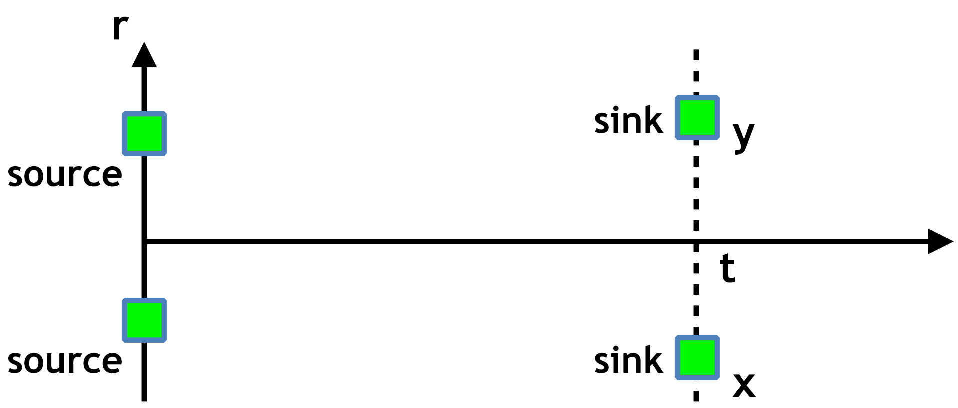
Nambu-Bethe-Salpeter amplitude

The information of the scattering is included in the following n-point correlator (Nambu-Bethe-Salpeter amplitude):

$$C_{\phi\phi}(t, \mathbf{x}-\mathbf{y}) \equiv \frac{1}{V}\sum_{\mathbf{r}} \langle 0 \,| T[\phi(\mathbf{x}+\mathbf{r}, t)\phi(\mathbf{y}+\mathbf{r}, t) \cdot \mathcal{J}(0)] |\, 0 \rangle$$

$\mathcal{J}(0)$: **source op.**



- **2-glueball (0^{++}) state mixes with all other multi-glueball states:**

  ⇒ The source may be chosen as 1-body, 2-body, etc, on convenience.

  We choose 1-body source, signal is noisier with higher-body source.

- **The NBS amplitude obeys the Schroedinger equation below inelastic threshold**

Nambu-Bethe-Salpeter amplitude

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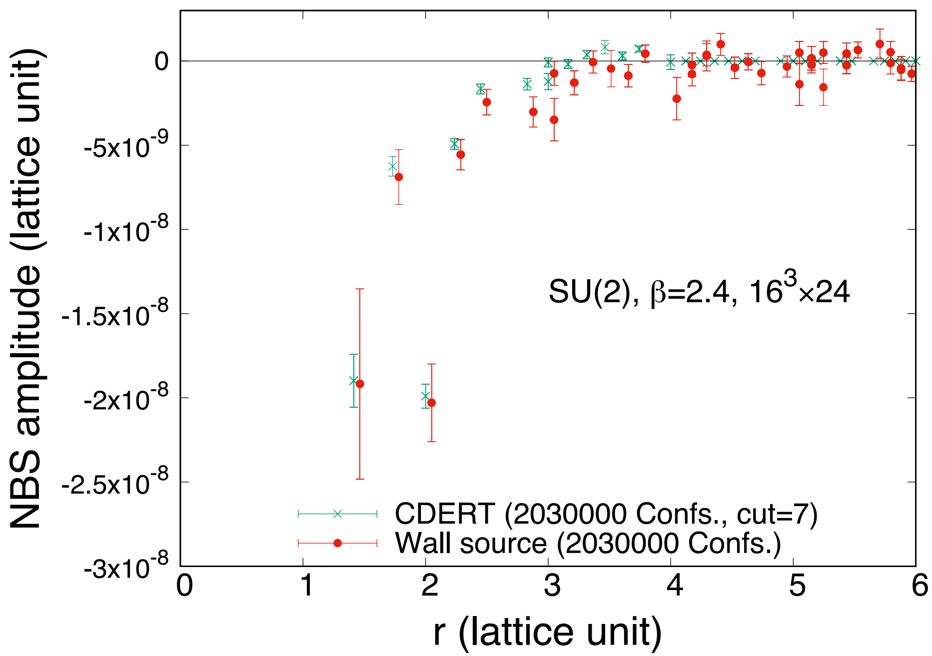
$\mathcal{J}(0)$ **: source op.**

Result of NBS amplitude calculation:



- **2-glueball** ... **ates:**

  ⇒ The sou ... **nvenience.**

  We ch ... **-body source.**

- **The NBS a** ... **inelastic threshold**

HALQCD method

Extract the interglueball potential from the NBS amplitude by inversely solving Schroedinger equation

$$\left[\frac{1}{4m_\phi}\frac{\partial^2}{\partial t^2}-\frac{\partial}{\partial t}+\frac{1}{m_\phi}\nabla^2+\frac{(\mathbf{r}\times\nabla)^2}{2m_\phi r^2}\right]R(t,\mathbf{r})=\int d^3\mathbf{r}' U(\mathbf{r},\mathbf{r}')R(t,\mathbf{r}')$$

$$R(t,\mathbf{r})\equiv\frac{C_{\phi\phi}(t,\mathbf{r})}{e^{-2m_\phi t}}$$

N. Ishii et al., PLB **712** (2012) 437.

- Crucial advantage : do not need ground state saturation

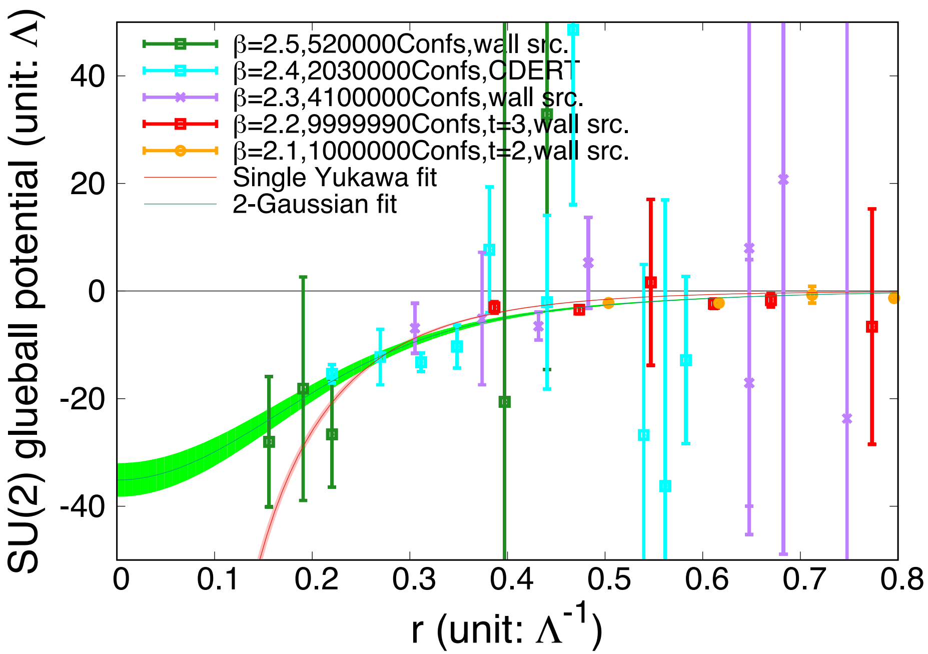
➡ **Almost mandatory to use time-dependent HAL method for the glueball analysis, since the glueball correlator becomes very noisy before ground state saturation**

- Inelastic threshold for glueball = $3m_\varphi$: high enough to use low t

- Subtract centrifugal force for removing higher angular momenta

NY, H. Iida, A. Nakamura, M. Wakayama, Phys. Lett. B **813**, 136056 (2021); Phys. Rev. D **102**, 054507 (2020).

Result of interglueball potential calculation



We test two fitting forms:

- **Yukawa fit:**

$$V_Y(r) = V_1 \frac{e^{-m_\phi r}}{4\pi r}$$

V_1 = -231 ± 8 **χ^2 d.o.f. = 1.3**

- **2-Gaussian fit:**

$$V(r) = V_1 e^{-\frac{(m_\phi r)^2}{8}} + V_2 e^{-\frac{(m_\phi r)^2}{2}}$$

V_1 = (-8.5 ± 0.5)Λ
V_2 = (-26.6 ± 2.6)Λ **χ^2 d.o.f. = 0.9**

DM cross section is derived from phase shift calculated with the potentials

(usual calculation of nonrelativistic quantum mechanism)

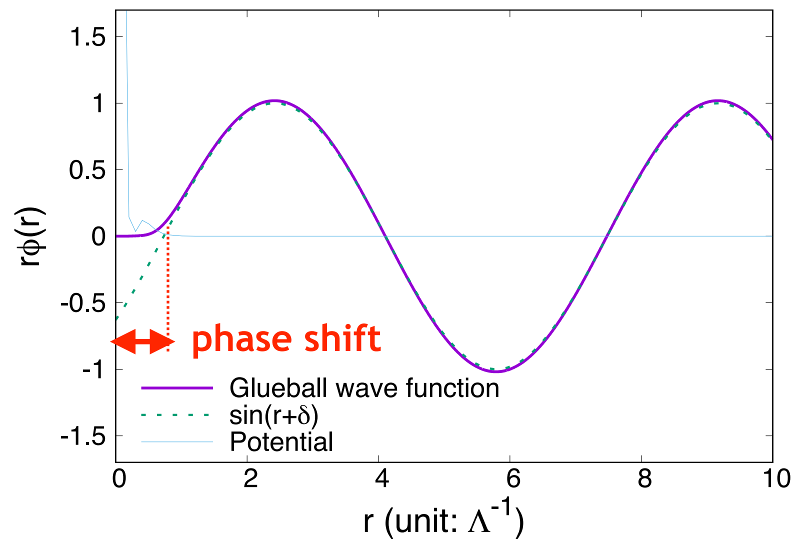
NY, H. Iida, A. Nakamura, M. Wakayama, Phys. Lett. B **813**, 136056 (2021);
Phys. Rev. D **102**, 054507 (2020).

From potential to scattering cross section

Potential ⇒ Scattering phase shift:

Solve $$\left(\frac{\partial^2}{\partial r^2}+k^2+U(r)\right)\phi(r)=0$$

$$\phi(r) \propto \sin[r+\delta(k)] \qquad (r\to\infty)$$



Scattering phase shift ⇒ Cross section:

We are interested in the low energy DM cross section, s-wave dominant :

$$\sigma_{\rm tot} = \frac{4\pi}{k^2}\sin^2[\delta(k\to 0)]$$

Yukawa: $\sigma_{\rm tot}$ = (2.5 - 4.7)Λ^{-2} (stat.)

2-Gaussian: $\sigma_{\rm tot}$ = (14 - 51)Λ^{-2} (stat.)

$\sigma_{\rm tot}$ = (2 - 51) Λ^{-2} (stat.and sys.)

(sys. due to fitting forms)

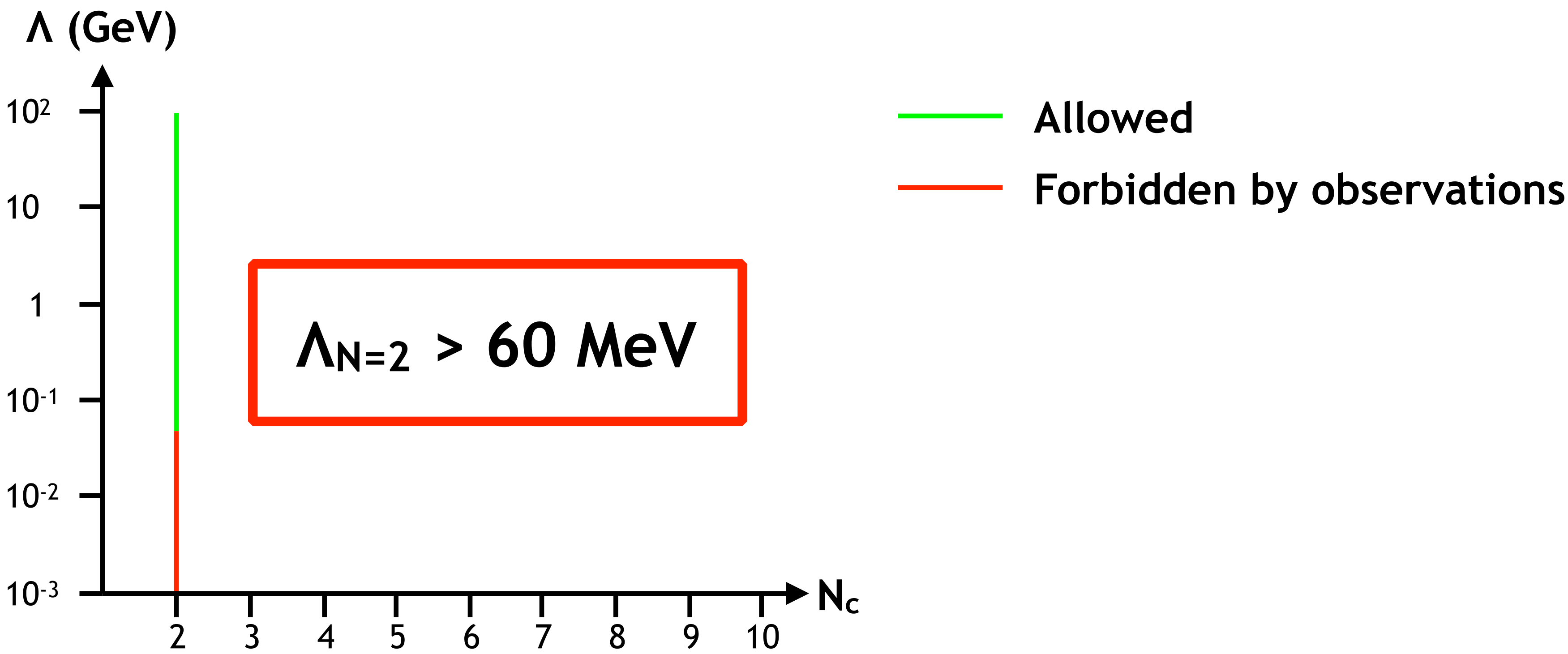
NY, H. Iida, A. Nakamura, M. Wakayama, Phys. Lett. B **813**, 136056 (2021).

Constraint on SU(N) YM scale parameter from DM X section

Observational constraints: $\frac{\sigma_{tot}}{m_\Phi} < 1.0\ \mathrm{cm}^2/\mathrm{g}$

Robust constraint from galactic cluster shape, collisions (upper limit)

A. H. Peter et al., MNRAS **430**, 81 (2013), **430**, 105 (2013); S. W. Randall et al., APJ **679**, 1173 (2008).



$\Lambda_{N=2} > 60$ MeV

N_c vs. scale parameter (Λ) diagram

NY, H. Iida, A. Nakamura, M. Wakayama, Phys. Lett. B **813**, 136056 (2021).

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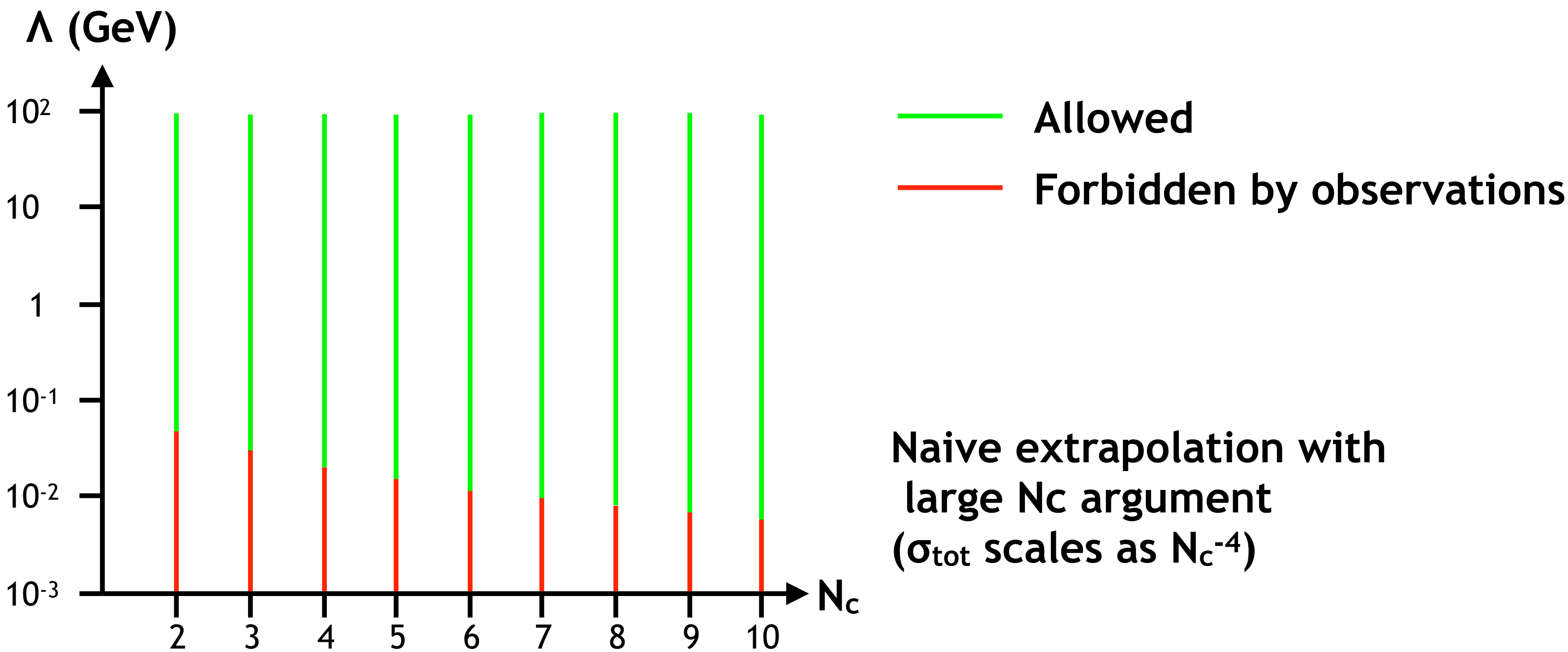


Naive extrapolation with large Nc argument (σ_{tot} scales as N_c^{-4})

N_c vs. scale parameter (Λ) diagram

NY, H. Iida, A. Nakamura, M. Wakayama, Phys. Lett. B **813**, 136056 (2021).

Constraint on SU(N) YM scale parameter from DM X section

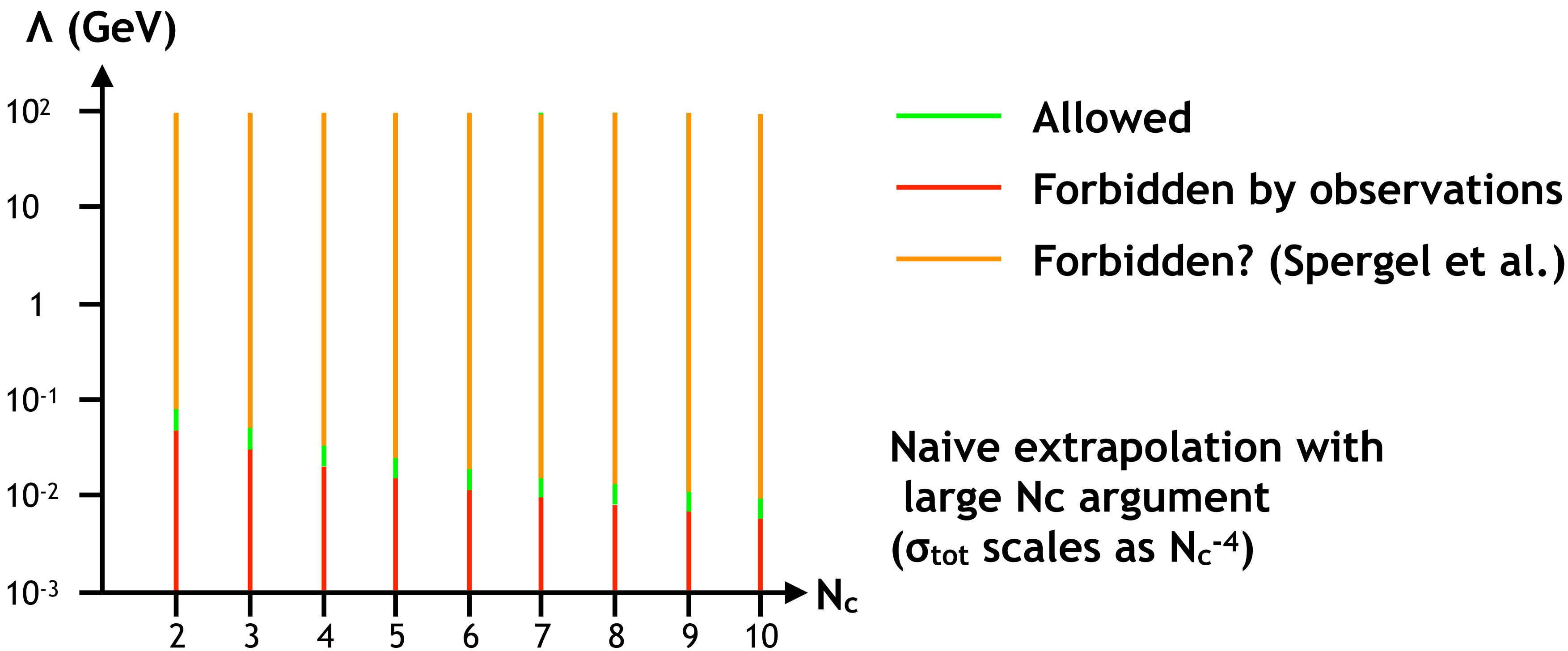
Observational constraints: $0.45\ \mathrm{cm^2/g} < \dfrac{\sigma_{tot}}{m_\Phi} < 1.0\ \mathrm{cm^2/g}$

Robust constraint from galactic cluster shape, collisions (upper limit)

A. H. Peter et al., MNRAS **430**, 81 (2013), **430**, 105 (2013); S. W. Randall et al., APJ **679**, 1173 (2008).

Constraint from Spergel et al. (lower limit), under discussion?

D. N. Spergel et al., PRL **84**, 3760 (2000).



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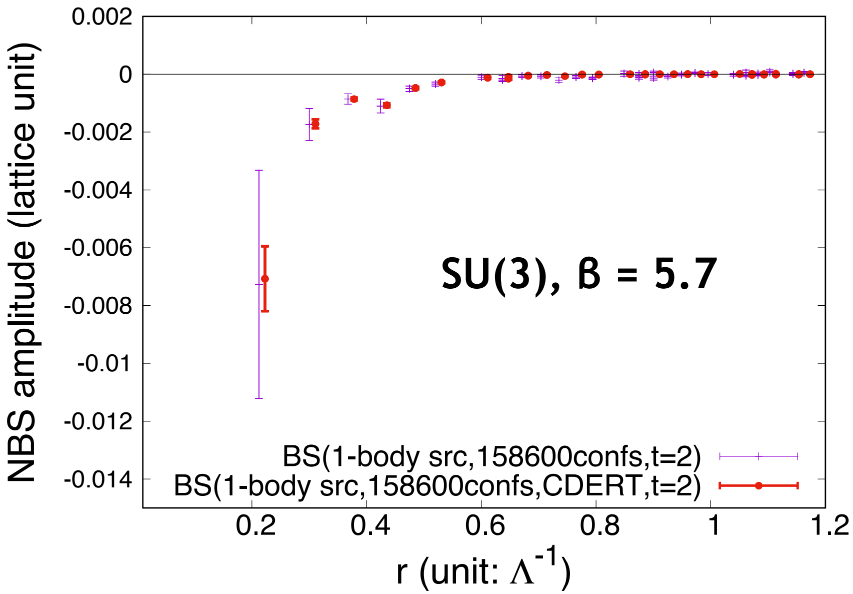
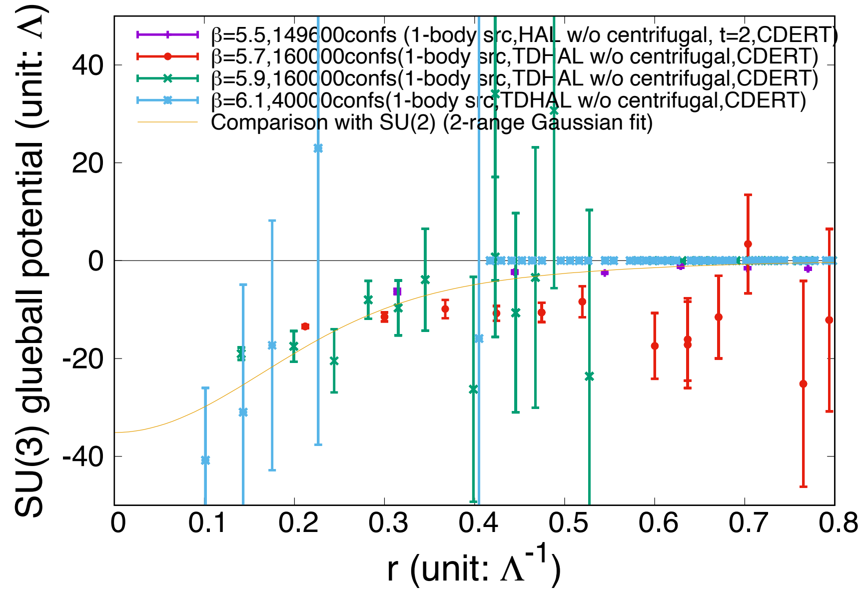
SU(3) result (preliminary)

NBS amplitude:



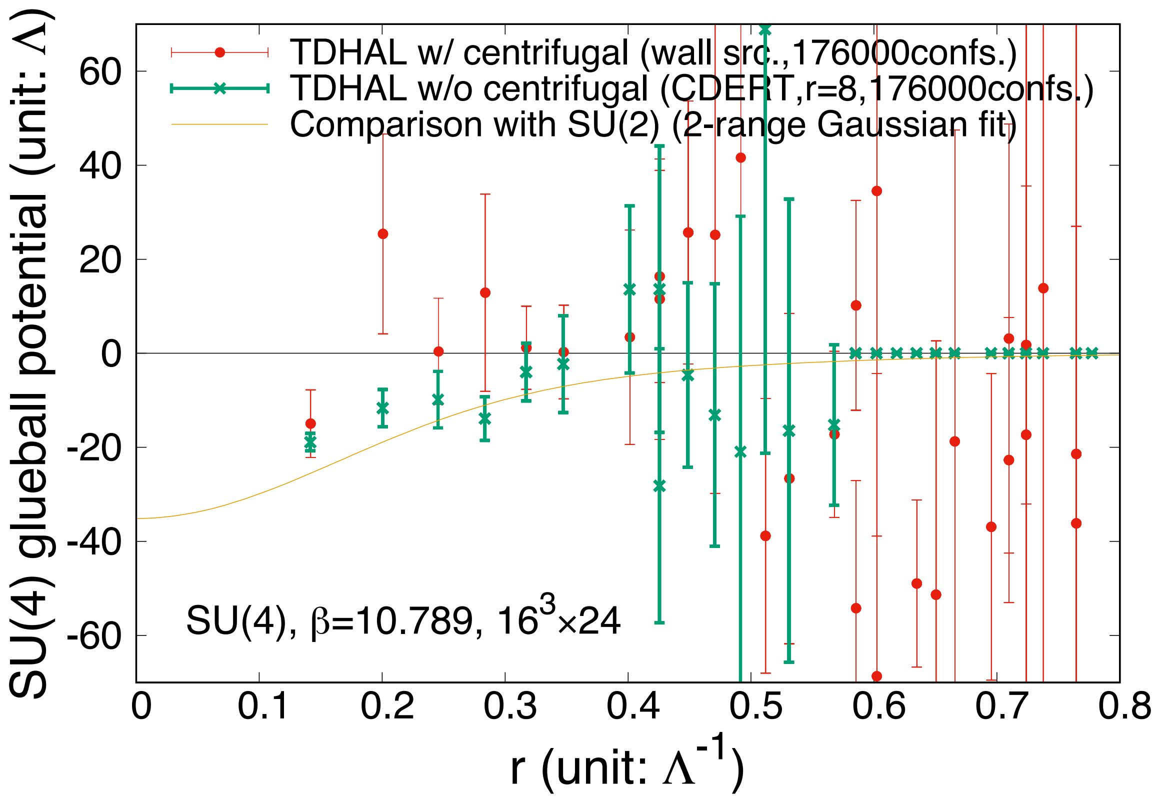
Potential:



- **The CDERT is very efficient in reducing statistical error**
- **The removal of centrifugal force makes the potential attractive, like SU(2)**
- **The value of the SU(3) interglueball potential is close to the SU(2) one**
- **Anomalous constant-like structure around r ∼ 0.4 Λ^{-1} : discretization error?**

NY et al., PoS LATTICE2021 (2022) 447

SU(4) result (preliminary)



- **The SU(4) interglueball potential is close to the SU(2), SU(3) ones**
- **The large N_c scaling is not clear (the potential scales as $1/N_c^2$)**

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Summary

- Dark matter is important to explain our existence, and it is strongly suggested by observations.
- Glueballs of the SU(N) Yang-Mills theory are good WIMP candidates of dark matter, study of self-interaction is important.
- We calculated the interglueball potential in the SU(2,3,4) Yang-Mills theory on lattice using the HALQCD method.
- We calculated the scattering phase shift and derived the interglueball cross section, and we could constrain Λ of SU(2) YMT for the 1st time from observational data : Λ > 60 MeV.
- Preliminary SU(3,4) results YMT look consistent with SU(2).

Homeworks:

- Calculations for $N_c > 2$ on-going : extrapolate to large N_c.
- Systematics due to discretization to be discussed: calculate with improved action (on-going work).

Backup slides