ResBos2 and the CDF W Mass measurement

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C.-P. Yuan
Michigan State University
Wu-Ki Tung Endowed Professor

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Rapid Response Workshop on W Boson Mass Anomaly

In collaboration with Josh Isaacson and Yao Fu arXiv:2205.02788

CTEQ – Tung et al. (TEA)
in memory of Prof. Wu-Ki Tung

Outline

- What's ResBos for?
- ResBos2 is the Version 2 of ResBos
- Higher order effects to the measurement of W mass at CDF II
- Conclusions and outlook

ResBos (Resummation for Bosons)

Initial state QCD soft gluon resummation
and
Final state QED corrections

In collaboration with
Csaba Balazs, Alexander Belyaev, Ed Berger,
Qing-Hong Cao, Chuan-Ren Chen, Yao Fu,
Josh Isaacson, Zhao Li, Steve Mrenna,
Pavel Nadolsky, Jian-Wei Qiu, Carl Schmidt,
Peng Sun, Bin Yan and Feng Yuan

hep-ph/9704258
hep-ph/0401026
hep-ph/1205.4311
arXiv:2205.02788
etc.

What’s ResBos for?

C T E Q

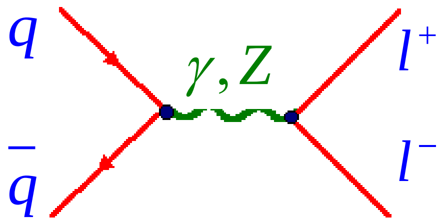
Precision Electroweak Physics at Hadron Colliders

Physics of Drell-Yan, W, Z and Higgs Bosons

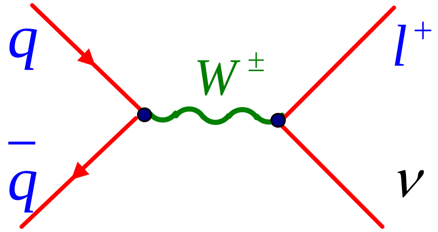
What's it for? An Example

- Transverse momentum of

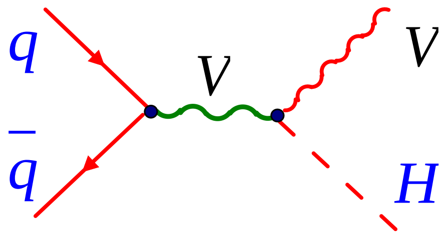
Drell-Yan



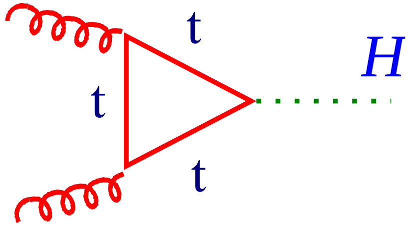
$W^\pm$



V H



$g\, g \to H$



including initial state QCD Resummation
(and final state QED radiation)

- Kinematics of Leptons from the decays
(Spin correlation included)

W-boson physics

❶ W-boson production and decay at hadron collider

❷ How to measure W-boson mass and width?

❸ High order radiative corrections:

☞ QCD (NLO, NNLO, Resummation)

☞ EW (QED-like, NLO)

❹ ResBos and ResBos-A

Resummation calculations agree well with data on transverse momentum (qT) distribution of Z boson

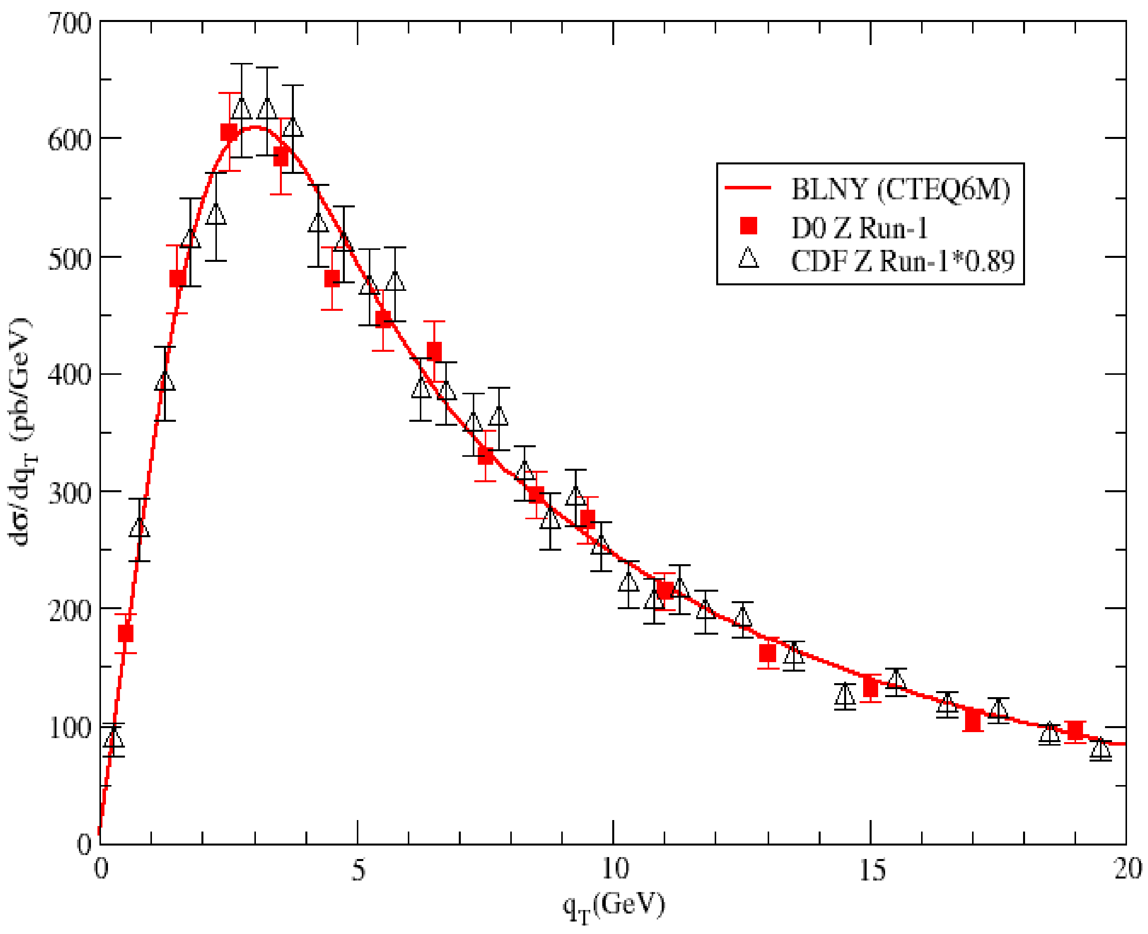
C T E Q

$P\bar{P} \rightarrow Z$ @ Tevatron

Predicted by ResBos:

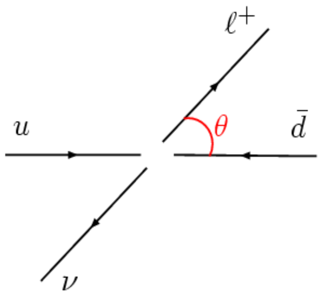
A program that
includes the effect of
multiple soft gluon emission
on the production of
W and Z bosons
in hadron collisions.

Predict qT(W)



Transverse momentum of the charged lepton

- In (ud) c.m. system,

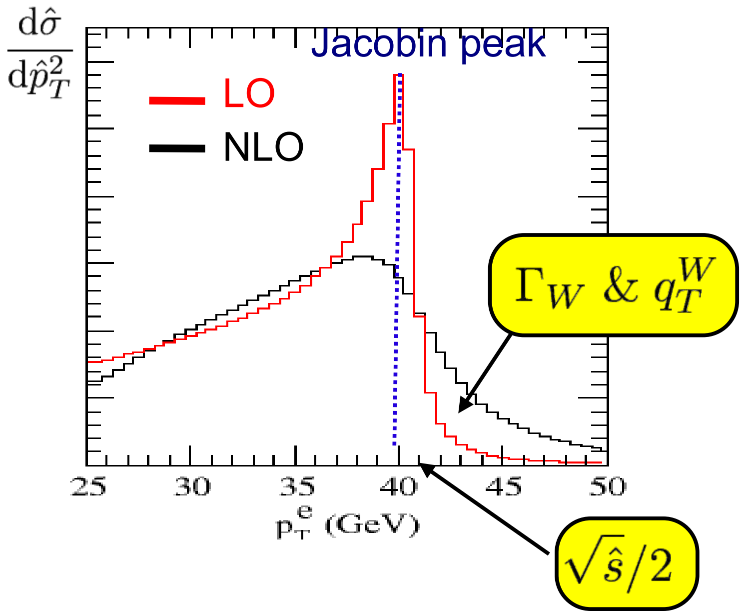


$$\hat{p}_T^2 = \frac{1}{4}\hat{s}\sin^2\theta$$

Jacobin factor

$$\frac{\mathrm{d}\cos\theta}{\mathrm{d}\hat{p}_T^2} = -\frac{2}{\hat{s}}\frac{1}{\sqrt{1-\frac{4\hat{p}_T^2}{\hat{s}}}}$$

$$\Longrightarrow \quad \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{p}_T^2} \sim \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\cos\theta} \times \frac{1}{\sqrt{1-4\hat{p}_T^2/\hat{s}}}$$



sensitive region for measuring

M_W : $p_T^e \sim 30 - 45$ GeV

Γ_W : not a good observable

Sensitive to qT(W)

Transverse mass of the W-boson

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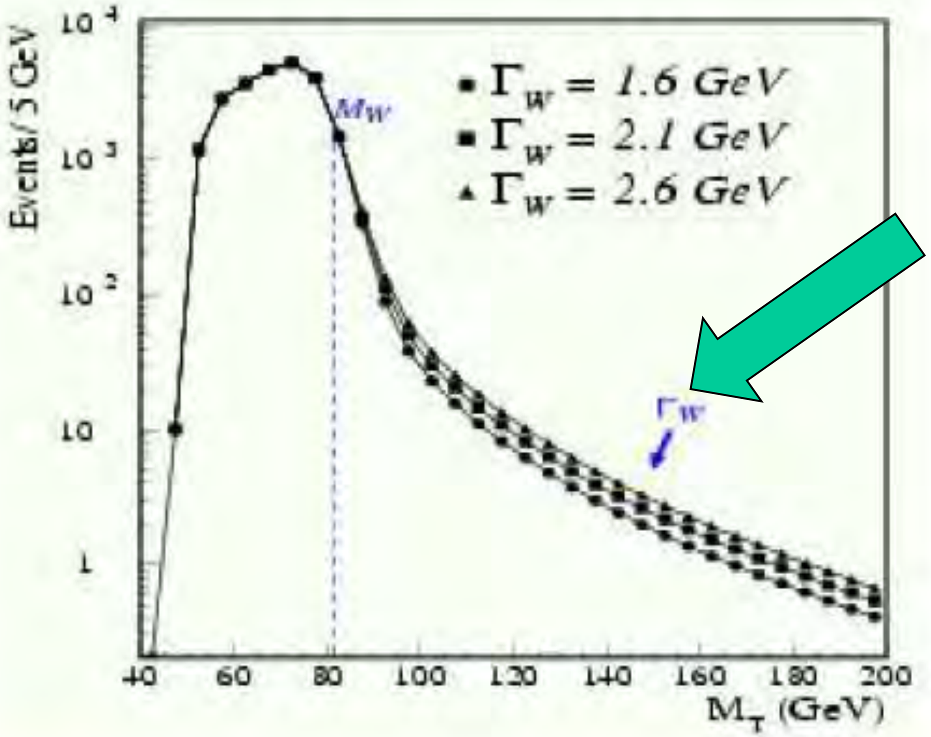
- Definition:

$$m_T^2(\ell,\nu) = 2\, p_T^\ell\, p_T^\nu (1 - \cos\phi_{\ell\nu})$$

p_T^ν: from overall p_T imbalance

$$\Longrightarrow \quad \frac{d\hat{\sigma}}{dm_T^2} \sim \frac{1}{\sqrt{1 - m_T^2/\hat{s}}}$$

- ☞ unaffected by longitudinal boosts of $\ell\nu$ system
- ☞ not sensitive to q_T^W
- ☞ tail knows about Γ_W (direct measurement)



Sensitive to width Γ_W

Less sensitive to qT(W)

sensitive region:

M_W : $M_T \sim 60 - 100$ GeV

Γ_W : $M_T > 100$ GeV

What's QCD Resummation?

- Perturbative expansion

$$\frac{d\hat{\sigma}}{dq_T^2} \sim \alpha_S \left\{ 1 + \alpha_S + \alpha_S^2 + \cdots \right\}$$

- The singular pieces, as $\frac{1}{q_T^2}$ (1 or log's)

$$\frac{d\hat{\sigma}}{dq_T^2} \sim \frac{1}{q_T^2} \sum_{n=1}^{\infty} \sum_{m=0}^{2n-1} \alpha_S^{(n)} \ln^{(m)}\left(\frac{Q^2}{q_T^2}\right)$$

$$\sim \frac{1}{q_T^2} \{ \quad \alpha_S \left(\underline{L+1}\right)$$

$$+\alpha_S^2 \left(\underline{L^3+L^2} + \underline{L+1}\right)$$

$$+\alpha_S^3 \left(\underline{L^5+L^4} + \underline{L^3+L^2} + \underline{L+1}\right)$$

$$+\cdots \quad \}$$

$$L \equiv \ln\left(\frac{Q^2}{q_T^2}\right)$$

Relevant for experimental observables with more than one large scale ($> \Lambda_{QCD}$) such as Q and qT.
E.g., measuring the qT distribution of a boson (Drell-Yan) production with mass Q.

For Q=91 and qT=4.
$L \sim 6$, with $\alpha_s = 0.12$, thus
$$\alpha_s L \sim 1$$

Resummation is to reorganize the results in terms of the large Log's.

Resummed results:

Determined by $A^{(1)}$ and $B^{(1)}$

$$\frac{d\sigma}{dq_T^2} \sim \frac{1}{q_T^2}\Big\{ [\alpha_s(L+1) \quad +\alpha_s^2(L^3+L^2)+\alpha_s^3(L^5+L^4) \quad +\cdots \;]$$

$$+[\qquad +\alpha_s^2(L+1) \quad +\alpha_s^3(L^3+L^2) \quad +\cdots \;]$$

$$+[\qquad\qquad +\alpha_s^3(L+1) \quad +\cdots \;]$$

$$+\cdots \;\Big\}$$

Determined by $A^{(2)}$ and $B^{(2)}$

Determined by $A^{(3)}$ and $B^{(3)}$

→ QCD Resummation

In the formalism by Collins-Soper-Sterman, in addition to these perturbative results, the effects from physics beyond the leading twist is also implemented as

[non-perturbative functions].

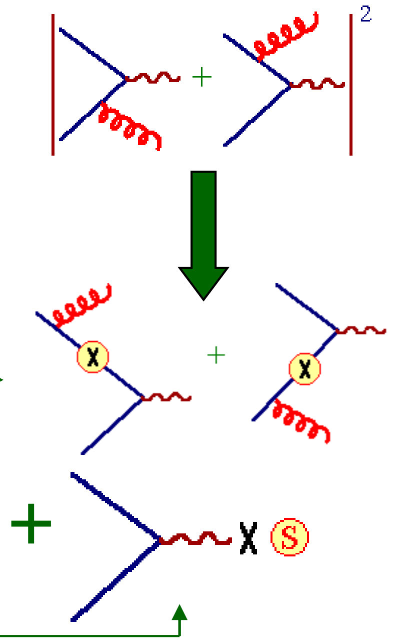
As $q_T \to 0$

$$
\left.\frac{d\sigma}{dq_T^2\, dy\, dQ^2}\right|_{q_T \to 0}
= \left(\frac{\pi}{s}\sigma_0\right)\cdot\delta\left(Q^2 - M_W^2\right)\cdot\left(\frac{1}{2\pi q_T^2}\right)\left(\frac{\alpha_s(Q)}{\pi}\right)
$$

$$
\cdot\Big\{ f_{q/A}(x_A,Q)\left[P_{\bar{q}\leftarrow\bar{q}} \otimes f_{\bar{q}}\right]_{x_B,Q} + \left[P_{q\leftarrow q} \otimes f_q\right]_{x_A,Q} f_{\bar{q}/B}(x_B,Q)
$$

$$
+ f_{q/A}(x_A,Q) f_{\bar{q}/B}(x_B,Q)\cdot\left[2\left(\frac{4}{3}\right)\ln\left(\frac{Q^2}{q_T^2}\right) + 2(-2)\right]\Big\}
$$

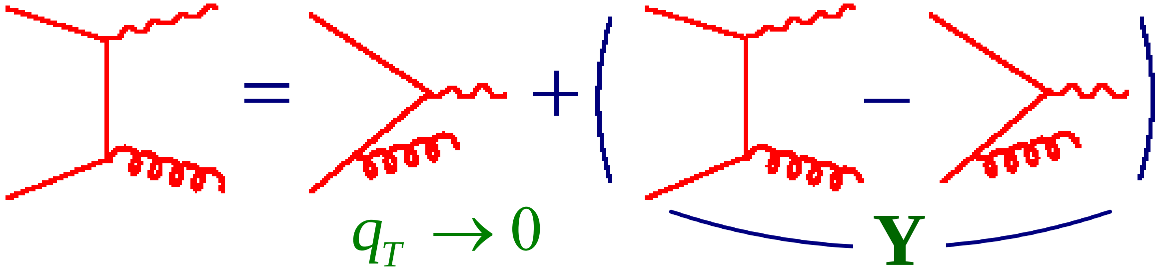
Diagramatically,



Exponentiate

To preserve transverse momentum conservation, we have to go to the impact parameter space (b-space) to perform resummation.

Fixed order calculation:



The Y-term is added to include the full fixed-order contribution.

CSS qT-resummation formalism

C T E Q

$$\frac{d\sigma}{dq_T^2\, dy\, dQ^2} = \frac{\pi}{S}\sigma_0 \delta\left(Q^2 - M_W^2\right)\cdot$$

$$\left\{\frac{1}{(2\pi)^2}\int d^2b\; e^{iq_T\cdot b}\widetilde{W}\left(b, Q, x_A, x_B\right)\cdot[\text{Non-perturbative functions}]\right.$$

$$\left. + Y\left(q_T, y, Q\right)\right\}$$

$$\widetilde{W} = e^{-S(b)}\cdot C\otimes f\left(x_A\right)\cdot C\otimes f\left(x_B\right)$$

$$C\otimes f(x_A) \to \sum_j \int_{x_A}^1 \frac{d\xi_A}{\xi_A} C_{qj}\left(\frac{x_A}{\xi_A}, b, \mu\right)\cdot f_{j/A}\left(\xi_A, \mu\right)$$

$$C\otimes f(x_B) \to \sum_k \int_{x_B}^1 \frac{d\xi_B}{\xi_B} C_{qk}\left(\frac{x_A}{\xi_A}, b, \mu\right)\cdot f_{k/B}\left(\xi_B, \mu\right)$$

$e^{-S(b)} \to$ **Sudakov form factor** $S(b) = \int_{\left(\frac{b_0}{b}\right)^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right)A\left(\bar{\mu}\right) + B\left(\bar{\mu}\right)\right]$

[Non-perturbative functions] are functions of (b, Q, x_A, x_B) which include QCD effects beyond Leading Twist.

- Example: for $W^{\pm}$

$$\sigma_0 = \frac{\pi}{3} \sqrt{2}\, M_W^2 G_F \ \left(\Sigma_{jj'} (KM)^2{}_{jj'}\right)$$

The couplings of gauge bosons to fermions are expressed in the way to include the dominant electroweak radiative corrections. The propagators of gauge bosons also contain energy-dependent width, as done in LEP precision data analysis.

Note:

$$A \equiv \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \cdot A^{(n)}, \qquad B \equiv \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \cdot B^{(n)},$$

$$C \equiv \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \cdot C^{(n)}$$

non-perturbative factor

$$S(b) = \int_{\frac{C_1^2}{b^2}}^{C_2^2 Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\ln\left(\frac{C_2^2 Q^2}{\bar{\mu}^2}\right) A(\bar{\mu}) + B(\bar{\mu}) \right]$$

- Lower limit goes to zero as b goes to infinity
- Requires evaluation of $\alpha_s(C_1/b)$ which is non-perturbative
- Need to introduce a non-perturbative cutoff (b^*-prescription):

$$b^* = \frac{b}{\sqrt{1 + \frac{b^2}{b_{\text{max}}^2}}}$$

$A(\mu)$ and $B(\mu)$ are expanded in terms of $\alpha_s(\mu)$.

As $b \to \infty$, $\alpha_s\left(\frac{C_1}{b}\right) \to \infty$. Hence, introducing b^* prescription to factorize non-perturbative (S_{NP}) and perturbative $S(b^*)$ regions.

$$S(b) = S_{NP} + S(b^*)$$

b_{max} is a parameter, of order 1/GeV.

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[non-perturbative function] is a function of (b,Q,x_A,x_B), implemented to include effects beyond Leading Twist.

Until we know how to calculate QCD non-perturbatively, (Lattice Gauge Theory?), these functions can only be parameterized. However, the same functions should describe Drell-Yan, $W^{\pm}$, Z^0 data.

- Test QCD in problems involving multiple scales.
- Measuring these non-perturbative functions may help in understanding the non-perturbative part of QCD.

[non-perturbative functions] , dependent of Q, b, x_A, x_B, is necessary to describe q_T – distribution of Drell-Yan, $W^{\pm}$,Z^0 events.

$$\exp\left[-g_1 b^2 - g_2 b^2 \ln\left(\frac{Q}{2Q_0}\right) - g_1 g_3 b^2 \ln\left(100 x_A x_B\right)\right]$$

New term with x-dependence

Q_0 is a parameter.

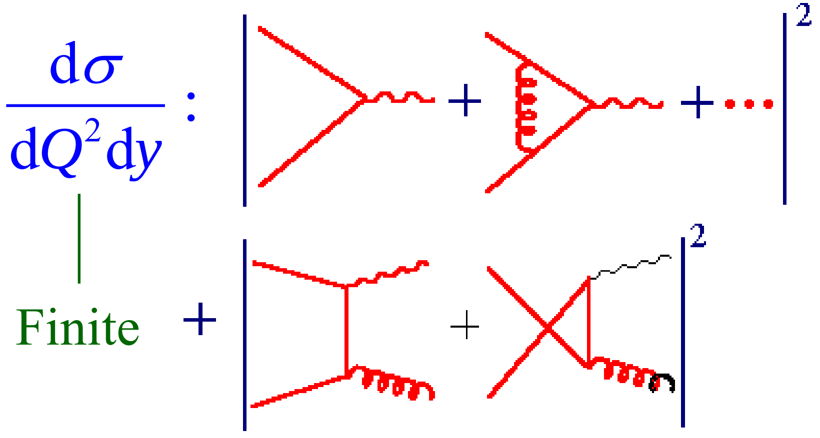
BLNY parametrization

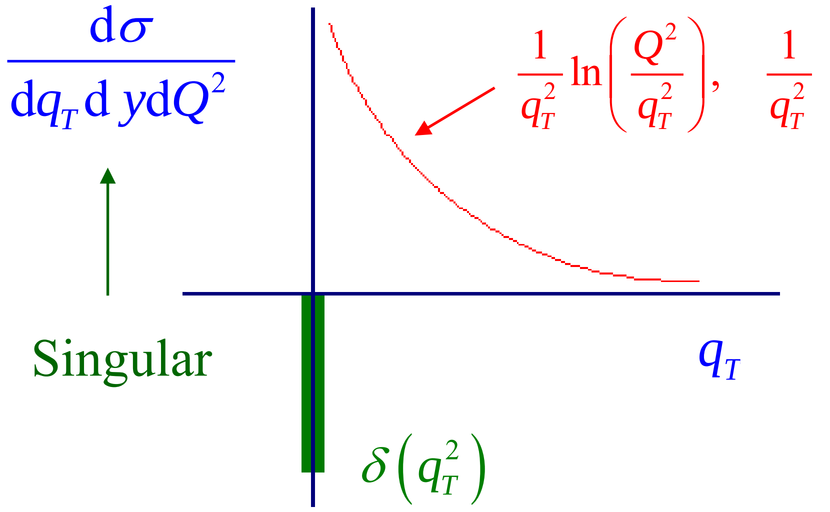
hep-ph/0212159

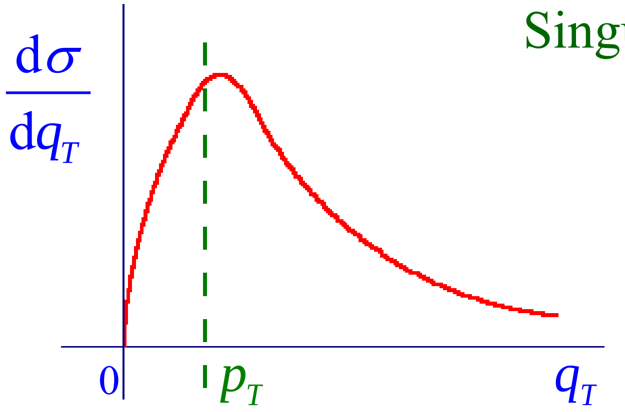
The coefficients g_1, g_2, g_3 need to be determined by existing data.

To recover the "K-factor" in the NLO total rate

→ To include the C-Functions







→ The area under the q_T – curve will reproduce the total rate at the order $\alpha_S^{(1)}$ if Y term is calculated to $\alpha_S^{(1)}$ as well.

Include NNLO in high q_T region

- To improve prediction in high q_T region
- To speed up the calculation, it is implemented through K-factor table which is a function of (Q, q_T, y) of the boson, not just a constant value.

→ ResBos predicts both rate and shape of distributions.

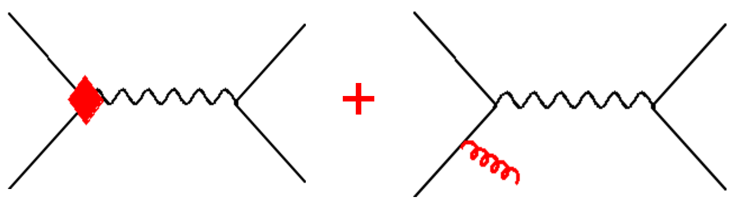
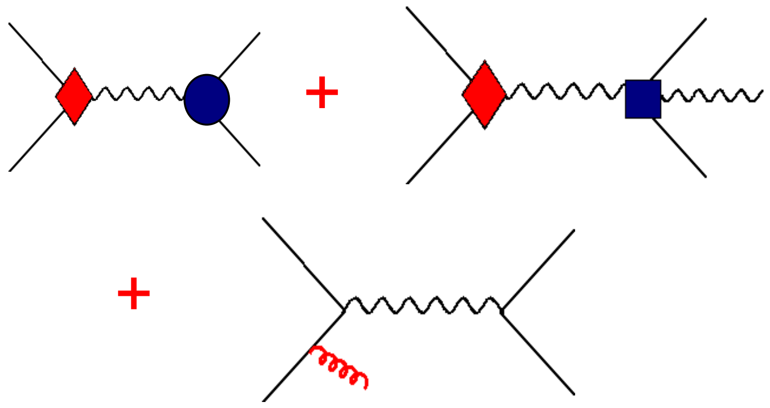
Precision measurements require accurate theoretical predictions

C T E Q

- ResBos-A: improved ResBos by including final state NLO QED corrections to W and Z production and decay

hep-ph/0401026

Qing-Hong Cao and CPY

Resum+Born	Resum+NLO
	

● and ■ denote FQED radiation corrections, which dominates the W mass shift.

Final state QED radiation has important effect on the measurement of W boson mass in the muon decay channel.

CDF used PHOTOS and HORACE for FSR effect.

ResBos2

Version 2 of ResBos

(matched to NNLO in total inclusive rate)

Josh Isaacson, Yao Fu and CPY; arXiv:2205.02788

Collins-Soper-Sterman Formalism

Resummation

$$\frac{d\sigma_{\text{res}}}{dQ^2 d^2\vec{q}_T dy d\Omega} = \sigma \int \frac{d^2b}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}}\tilde{W},$$

$$\tilde{W} = e^{-S(b)} \, C \otimes f(x_A, C_3/b) C \otimes f(x_B, C_3/b)$$

$$S(b) = \int_{\frac{C_1^2}{b^2}}^{C_2^2Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{C_2^2Q^2}{\bar{\mu}^2}\right) A(\bar{\mu}) + B(\bar{\mu})\right]$$

- Sudakov factor
- Collinear factors
- Perturbative Coefficients (A, B, C)

[Collins, Soper, Sterman, '85] [...]

ResBos vs. ResBos2

Order	Boundary Condition (C)	Anomalous Dimension γ_i (B)	Γ_{cusp} (A)	Fixed Order Matching (Y)
LL	1	-	1-loop	-
NLL	1	1-loop	2-loop	-
NLL' (+ NLO)	α_s	1-loop	2-loop	α_s
NNLL (+ NLO)	α_s	2-loop	3-loop	α_s
NNLL' (+ NNLO)	α_s^2	2-loop	3-loop	α_s^2
N^3LL (+ NNLO)	α_s^2	3-loop	4-loop	α_s^2
N^3LL' (+ N^3LO)	α_s^3	3-loop	4-loop	α_s^3
N^4LL (+ N^3LO)	α_s^3	4-loop	5-loop	α_s^3

ResBos → NNLL (+ NLO)

ResBos2 → N^3LL (+ NNLO)

TABLE I. The definitions for the accuracy of the resummation calculation. The accuracy used by CDF was NNLL + NLO, while the state-of-the-art is N^3LL + NNLO.

Josh Isaacson, Yao Fu and CPY; arXiv:2205.02788

CDF W mass measurement

C T E Q

SM prediction: 80,358.1 ± 5.2 MeV

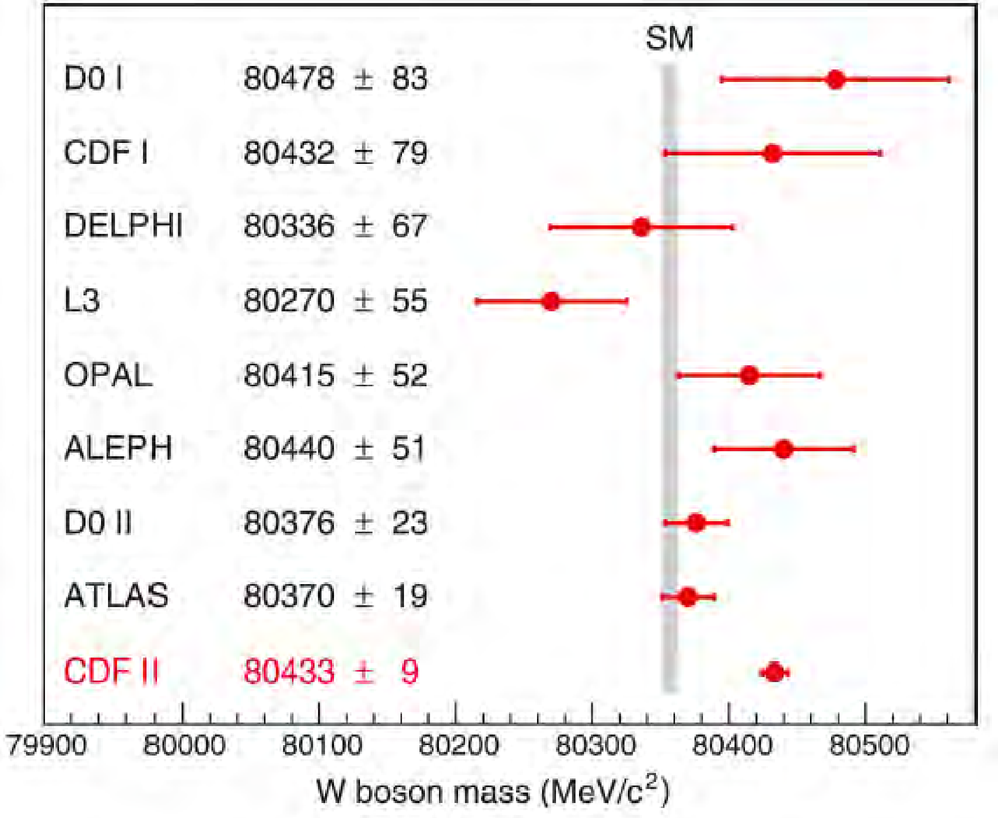


Figure reproduced from CDF-II measurement (Science 376, 170).

Also, LHCb result: 80,354 ± 32 MeV

Quoted from CDF paper (Science 367, 170)

Simulated experiments are used to evaluate the statistical correlations between fits, which are found to be 69% (68%) between m_T and p_T^ℓ (p_T^ν) fit results and 28% between p_T^ℓ and p_T^ν fit results (*43*). The six individual M_W results are combined (including correlations) by means of the best linear unbiased estimator (*66*) to obtain $M_W = 80{,}433.5 \pm 9.4$ MeV, with χ^2/dof = 7.4/5 corresponding to a probability of 20%. The m_T, p_T^ℓ, and p_T^ν fits in the electron (muon) channel contribute weights of 30.0% (34.2%), 6.7% (18.7%), and 0.9% (9.5%), respectively. The combined result is shown in Fig. 1, and its associated systematic uncertainties are shown in Table 2.

Study the impact of higher order effects: from NNLL+NLO

to NNNLL+NNLO

FROM RESBOS TO RESBOS2
FROM W(321)+Y TO W(432)+YK(R)

Shorthand notation:
W(321)=W(321)+Y, with full lepton angular correlations to $\alpha_s$order.
W(432)=W(432)+YK(R), with full lepton angular correlations to α_s^2 order.

Methodology

Our Procedure:

- Generate pseudodata using N^3LL+NNLO prediction
- Tune NNLL+NLO prediction to reproduce $p_T(Z)$ data
- Validate tuned result against $p_T(W)$ data
- Use tuned result to generate mass templates
- Extract W mass from template fit for each observable
- Calculate the mass shift from the input value for pseudodata

Details:

- Pseudodata $M_W = 80,358$ MeV
- Cuts:
  - $p_T(Z) < 15$ GeV, $p_T(W) < 15$ GeV
  - $30 < p_T(\ell) < 55GeV$, $30 < p_T(\nu) < 55$ GeV
  - $|\eta(\ell)| < 1$
  - $66 < M_{\ell\ell} < 116$ GeV (Z events), $60 < m_T < 100$ GeV (W events)
- Number of Events:
  - 1,811,700 $W \to e\nu$
  - 66,180 $Z \to ee$
  - 2,424,486 $W \to \mu\nu$
  - 238,534 $Z \to \mu\mu$

from NNLL+NLO to NNNLL+NNLO
Namely, from ResBos to ResBos2

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- Generate pseudodata, including pT(Z), pT(W), mT, pT(e), pT(nu), using W(432) and CT18 NNLO central set PDF.

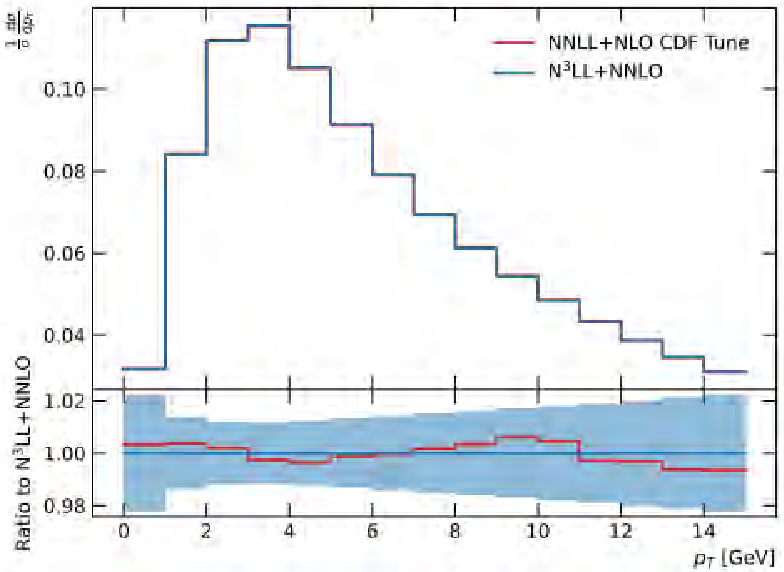
$\alpha_s = 0.118$
CT18NNLO.00 PDF set

- Fit the normalized pT(Z) pseudodata with W(321) calculation and CT18 NNLO α_s series PDFs, in which the g2 and α_s values are the fitting parameters. This is called tuned W(321) prediction.

$\alpha_s = 0.120$
CT18NNLO_as_0120 PDF set

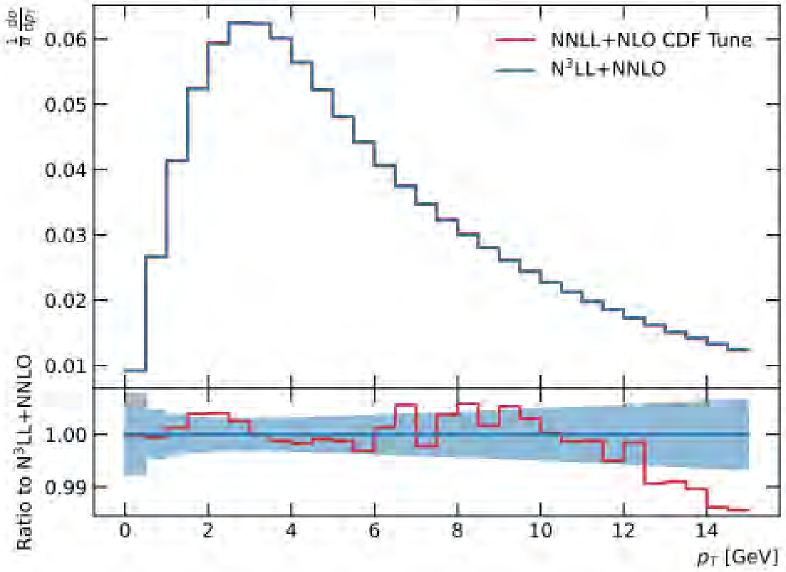
Comparison of tuned W(321) and pseudodata W(432)

$$\frac{1}{\sigma}\frac{d\sigma}{dp_T(Z)}$$



Fit to normalized pT(Z) pseudodata W(432)

$$\frac{1}{\sigma}\frac{d\sigma}{dp_T(W)}$$



Prediction of normalized pT(W) distribution from the tuned W(321)

The blue band represents the statistical uncertainty of the CDF measurement.

M_W template

- **Generate M_W template using the tuned W(321)**

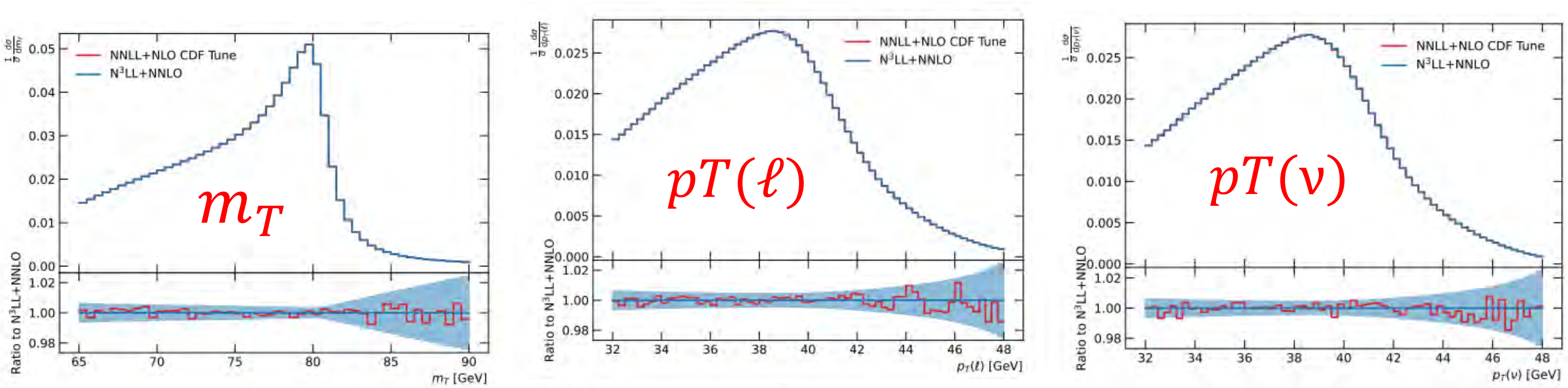
  Template: From 80.336 to 80.435 GeV; step is 1 MeV

- **Shift in M_W from W(432) to tuned W(321)**

  Do χ^2 fit to the normalized $M_T, p_T(l), p_T(v)$ distributions to find M_W

  Shift: $Fitted\ M_W - input\ M_W(80.385)$

Shift in M_W, when using the tuned W(321)



	Mass Shift [MeV]	
Observable	ResBos2	+Detector Effect+FSR
m_T	1.5 ± 0.5	$0.2 \pm 1.8 \pm 1.0$
$p_T(\ell)$	3.1 ± 2.1	$4.3 \pm 2.7 \pm 1.3$
$p_T(\nu)$	4.5 ± 2.1	$3.0 \pm 3.4 \pm 2.2$

The blue band represents the statistical uncertainty of the CDF measurement.

Unc1: statistical uncertainty of the generated samples
Unc2: uncertainty from different random seed of Gaussian smearing. It is estimated by generating 100 different smeared pseudodata with different random seed, using the mean value to determine the average shift, an the RMS to determine its uncertainty.

Another simple smearing model was also used:
5% smearing on $p_T(l)$ and 11% $on\ p_T(v)$, the main conclusion does not change.

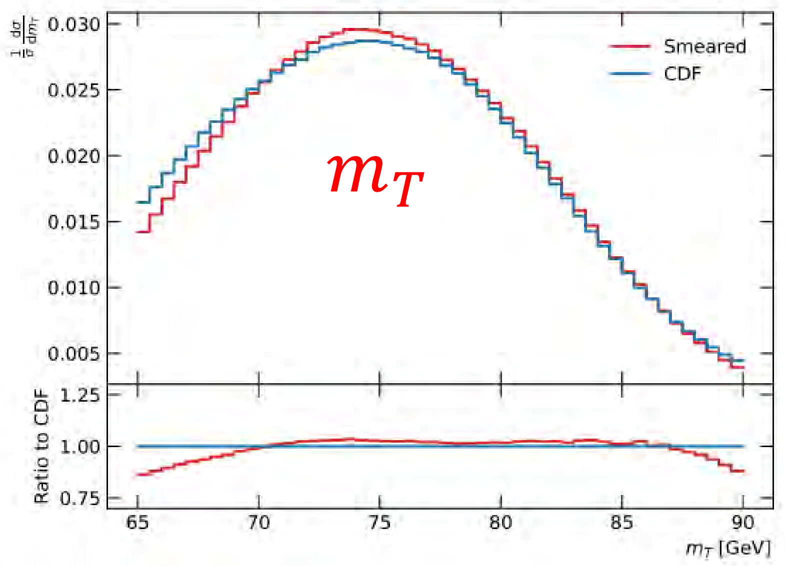
Detector Resolution effect and FSR

C T E Q

- Smearing the momentum of $p_T(l)$ and $p_T(\nu)$

$$\frac{\sigma}{E} = \frac{a}{\sqrt{E}} \oplus b \oplus \frac{c}{E},$$

Gaussian smearing effect applied on $p_T(l)$ and $p_T(\nu)$



Same smearing was applied to both M_W template and tuned W(321) predictions.

Consider the electron channel, for its smaller background and less final state QED radiation (FSR) correction, as compared to the muon channel.

Angular correlation

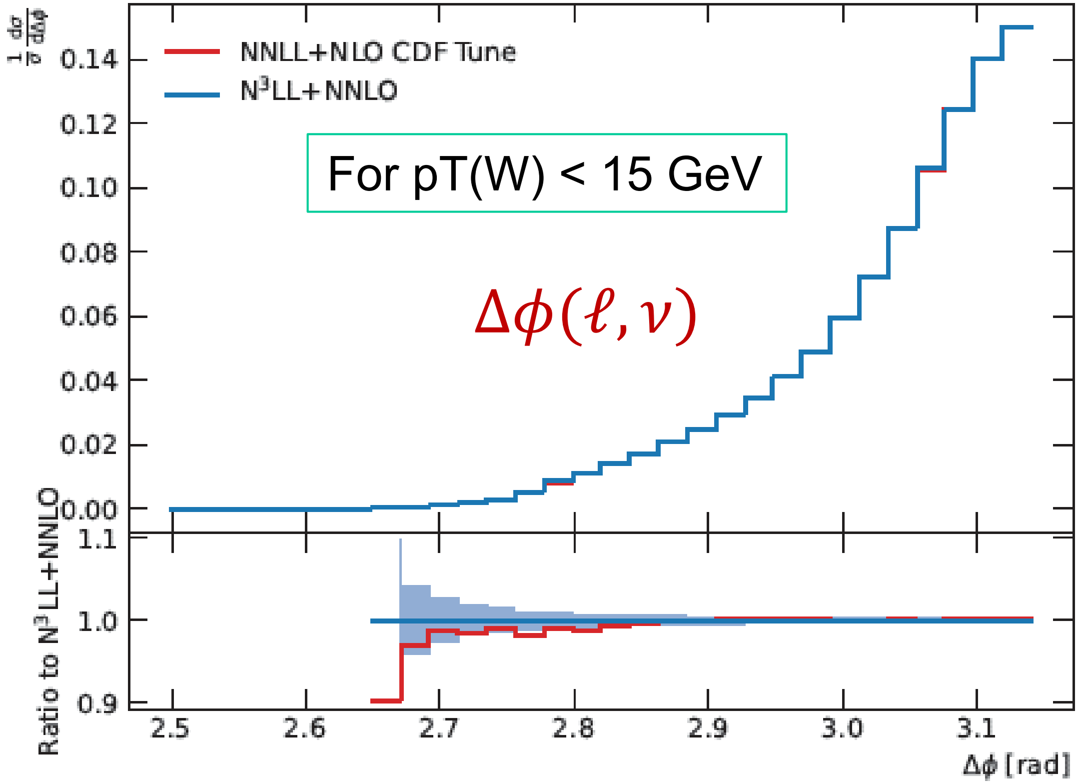


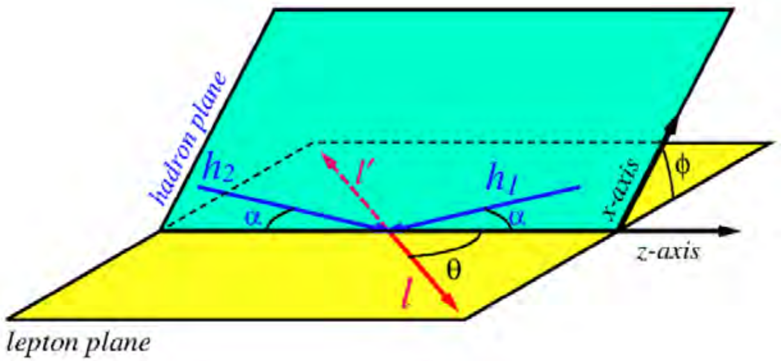
FIG. 7. Comparison of the generated pseudodata for $\Delta\phi$ using the N^{3}LL+NNLO calculation compared to the CDF tuned prediction at NNLL+NLO. The blue band represents the statistical uncertainty associated with the CDF measurement.

- W(321) has the correct lepton angular correlations at NLO.
- W(432) has the correct lepton angular correlations at NNLO.

$$m_T^2 = 2\left(p_T(\ell)p_T(\nu) - \vec{p}_T(\ell)\cdot\vec{p}_T(\nu)\right)$$

$$= 2\, p_T(\ell)p_T(\nu)\left(1 - \cos\Delta\,\phi(\ell,\nu)\right)$$

Angular Coefficients



Collins-Soper frame

$$\frac{d\sigma}{dp_T^Z\, dy^Z\, dm^Z\, d\cos\theta\, d\phi} = \frac{3}{16\pi}\frac{d\sigma^{U+L}}{dp_T^Z\, dy^Z\, dm^Z}$$
$$\Big\{(1+\cos^2\theta) + \frac{1}{2}A_0(1-3\cos^2\theta) + A_1\ \sin 2\theta\ \cos\phi$$
$$+\frac{1}{2}A_2\ \sin^2\theta\ \cos 2\phi + A_3\ \sin\theta\ \cos\phi + A_4\ \cos\theta$$
$$+A_5\ \sin^2\theta\ \sin 2\phi + A_6\ \sin 2\theta\ \sin\phi + A_7\ \sin\theta\ \sin\phi\Big\}.$$

$$\langle P(\cos\theta,\phi)\rangle = \frac{\int P(\cos\theta,\phi)d\sigma(\cos\theta,\phi)d\cos\theta d\phi}{\int d\sigma(\cos\theta,\phi)d\cos\theta d\phi}.$$

$$\langle\frac{1}{2}(1-3\cos^2\theta)\rangle = \frac{3}{20}(A_0-\frac{2}{3});\quad \langle\sin 2\theta\cos\phi\rangle = \frac{1}{5}A_1;\quad \langle\sin^2\theta\cos 2\phi\rangle = \frac{1}{10}A_2;$$
$$\langle\sin\theta\cos\phi\rangle = \frac{1}{4}A_3;\quad \langle\cos\theta\rangle = \frac{1}{4}A_4;\quad \langle\sin^2\theta\sin 2\phi\rangle = \frac{1}{5}A_5;$$
$$\langle\sin 2\theta\sin\phi\rangle = \frac{1}{5}A_6;\quad \langle\sin\theta\sin\phi\rangle = \frac{1}{4}A_7.$$

Angular Coefficients

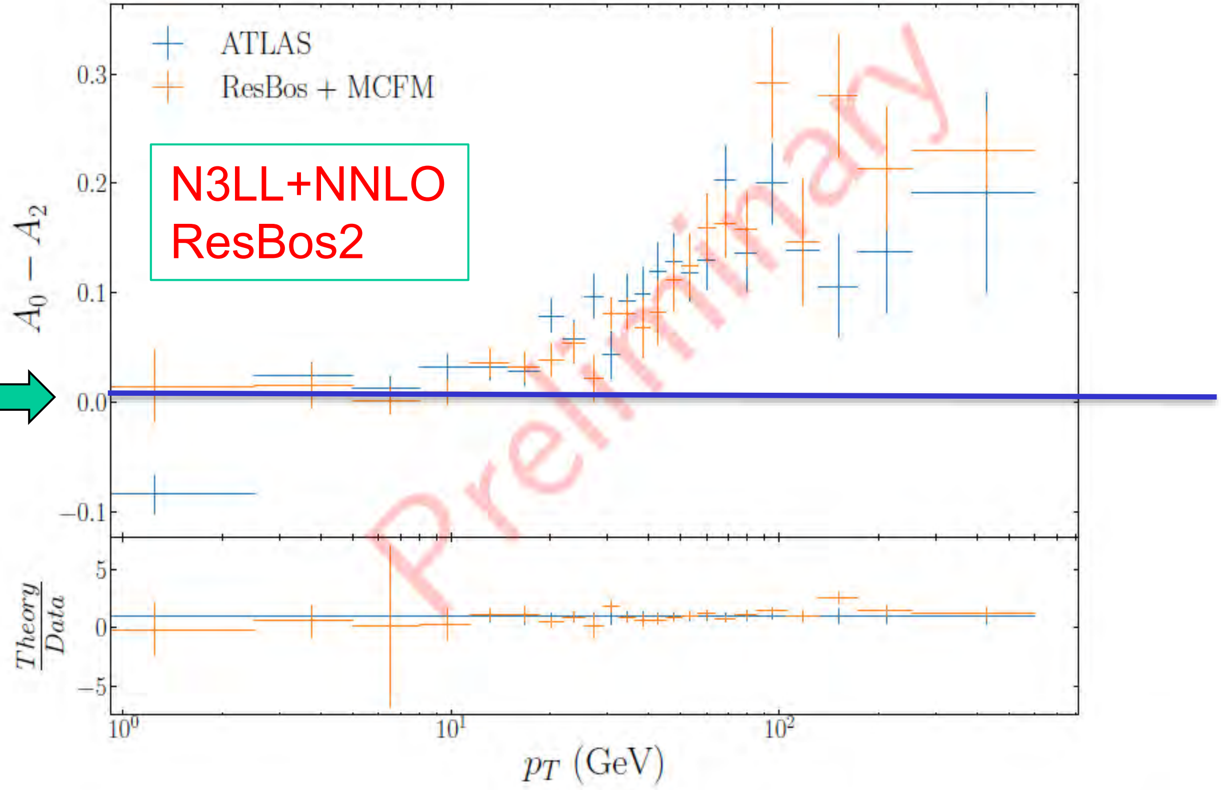
- Well known issue with angular coefficients in the ResBos code at NNLO (No issue with matching to NLO)
- CDF-II only used the NLO so the angular functions are exact to that order
- ResBos only included NNLO corrections to the total rate, but not to the angular functions
- This is an issue with matching to an incomplete NNLO calculation, and not an issue with the resummation or the matching to fixed order
- Only effects larger p_T ($p_T > 30$ GeV, CDF has a cut of $p_T < 15$ GeV)
- Has been resolved via matching to MCFM (preliminary results next slides)

Violation of Lam-Tung relation beyond NLO in QCD

CTEQ

Lam-Tung relation: $A_0 - A_2 = 0$, valid up to NLO.
Phys. Rev. D 21 (1980) 2721

Violation due to higher order (beyond NLO) QCD corrections or k_T-factorization, and higher twist effects.



NOTE: Uncertainties are purely statistical for ResBos + MCFM

M_W template for studying the width effect

C T E Q

- **Generate M_W template using the W(432)**

  Template: From 80.336 to 80.435 GeV, step is 1 MeV

  Width: 2.0895 GeV (used in the CDF paper)

- **Changing the width of W boson**

  According to the uncertainty of the W boson width reported by PDG, which is 0.042GeV

  Three pseudodata are generated for:

  $M_W = 80.385GeV, \Gamma_W = 2.0475GeV$
  $M_W = 80.385GeV, \Gamma_W = 2.1315GeV$
  $M_W = 80.385GeV, \Gamma_W$ is determined by NLO calculation, which is proportional to M_W^3

  $\Gamma_W \sim g^2 M_W$, with $g^2 \sim G_F M_W^2$

M_W template for studying the width effect

- **Generate M_W template using the W(432)**

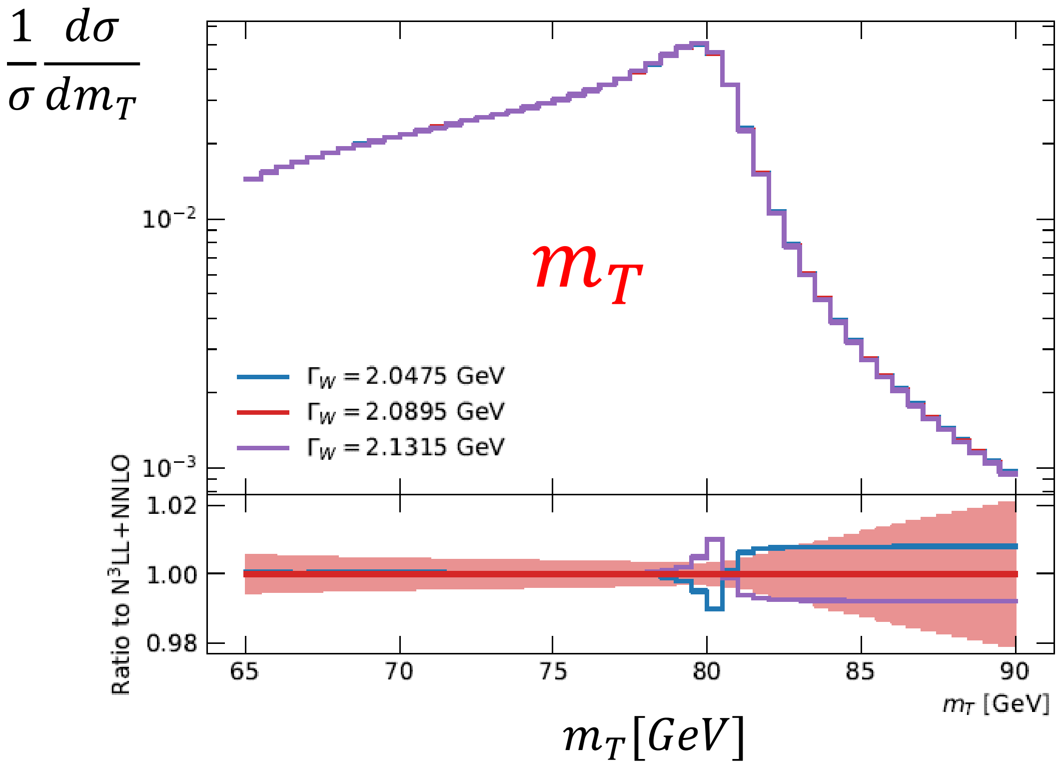
Shift in M_W due to different W boson width:

Width	Mass Shift [MeV]
2.0475 GeV	2.0 ± 0.5
2.1315 GeV	0.3 ± 0.5
NLO	1.2 ± 0.5

M_W is defined by the relativistic Breit-Wigner mass distribution -- the propagator of a resonance state with energy-dependent width Γ_W.

arXiv: 1311.0894

$$S \frac{d\sigma}{dQ^2} \sim \frac{Q^2}{(Q^2 - M_W^2)^2 + Q^4\, \Gamma_W^2 / M_W^2}$$



The red band represents the statistical uncertainty of the CDF measurement.

PDF-induced shift in W boson mass

M_W template for studying the shift due to various PDFs

- **Generate M_W template using the W(432)**

  Template: From 80.336 to 80.435 GeV; step is 1 MeV, CT18NNLO central set PDF.

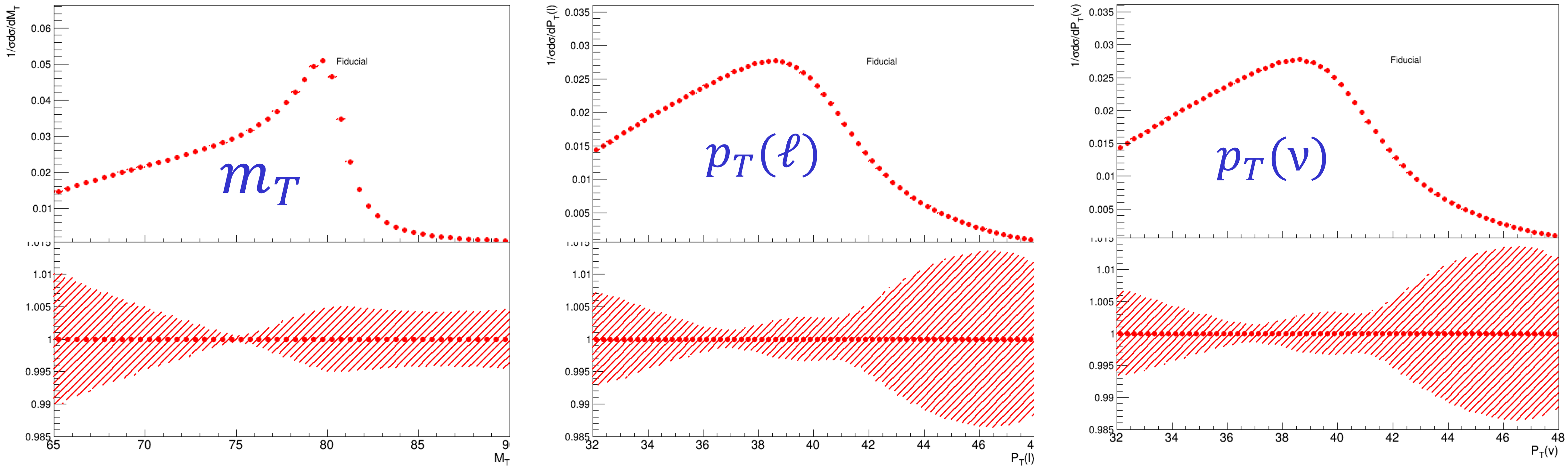
  Study the shift due to various PDFs in higher order calculation

- **Pseudodata generated by using W(432) + other PDFs**

  Do χ^2 fit to the normalized M_T, $p_T(l)$, $p_T(v)$ distributions to find M_W

  Shift: $Fitted\ M_W - input\ M_W(80.385)$

PDF-induced uncertainty in CDF W mass measurement



$$m_T^2 = 2\left(p_T(\ell)p_T(\nu) - \vec{p}_T(\ell)\cdot\vec{p}_T(\nu)\right)$$

Normalized distributions after imposing all the kinematic cuts of CDF II data.

M_W template for studying the shift in M_W due to various PDFs

C T E Q

	m_T		$p_T(\ell)$		$p_T(\nu)$	
PDF Set	NNLO	NLO	NNLO	NLO	NNLO	NLO
CT18	0.0 ± 1.3	1.8 ± 1.2	0.0 ± 15.9	2.0 ± 14.3	0.0 ± 15.5	2.9 ± 14.2
MMHT2014	1.0 ± 0.6	2.6 ± 0.6	6.2 ± 7.8	36.7 ± 7.0	3.9 ± 7.5	36.0 ± 6.7
NNPDF3.1	1.1 ± 0.3	2.1 ± 0.4	2.1 ± 3.8	13.5 ± 4.9	5.4 ± 3.7	10.0 ± 4.9
CTEQ6M	N/A	2.8 ± 0.9	N/A	19.0 ± 10.4	N/A	20.9 ± 10.2

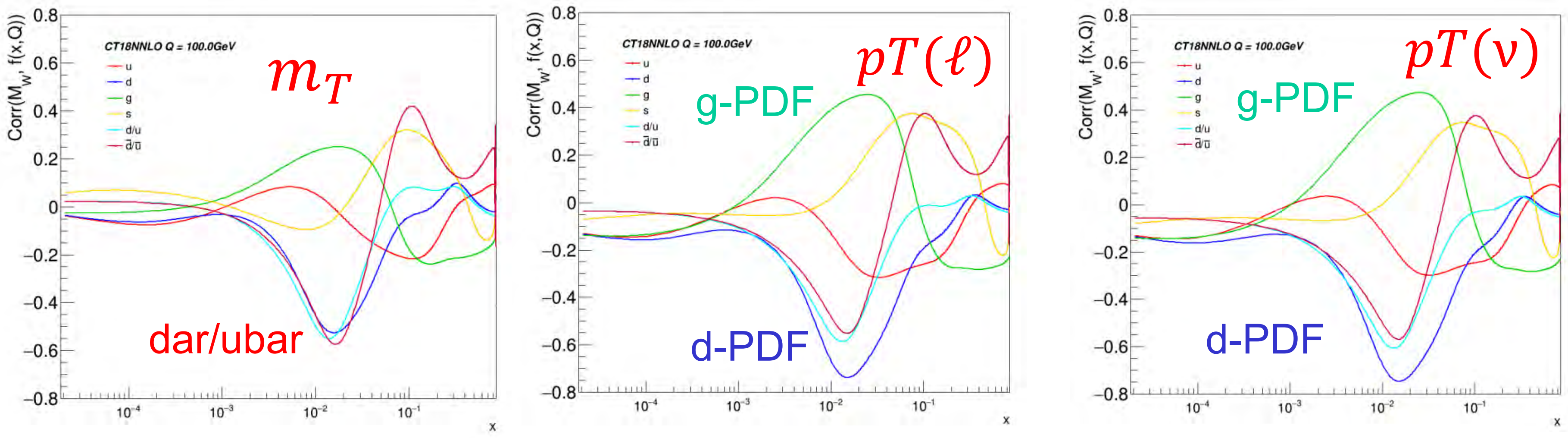
- The errors were generated by its own error PDF sets.
- Larger shifts are found when using the normalized pT(e) or pT(nu) distributions.
- Larger shifts are found when using NLO PDFs. (See correlation ellipses.)

- Gluon PDF can contribute to NLO and NNLO predictions.
- pT(e) and pT(nu) are more sensitive to gluon PDF errors than mT(e,nu), hence, generate more shift in Mw. (See correlation cosine plots.)

Correlation cine between the uncertainty of the extracted M_W and that of PDFs

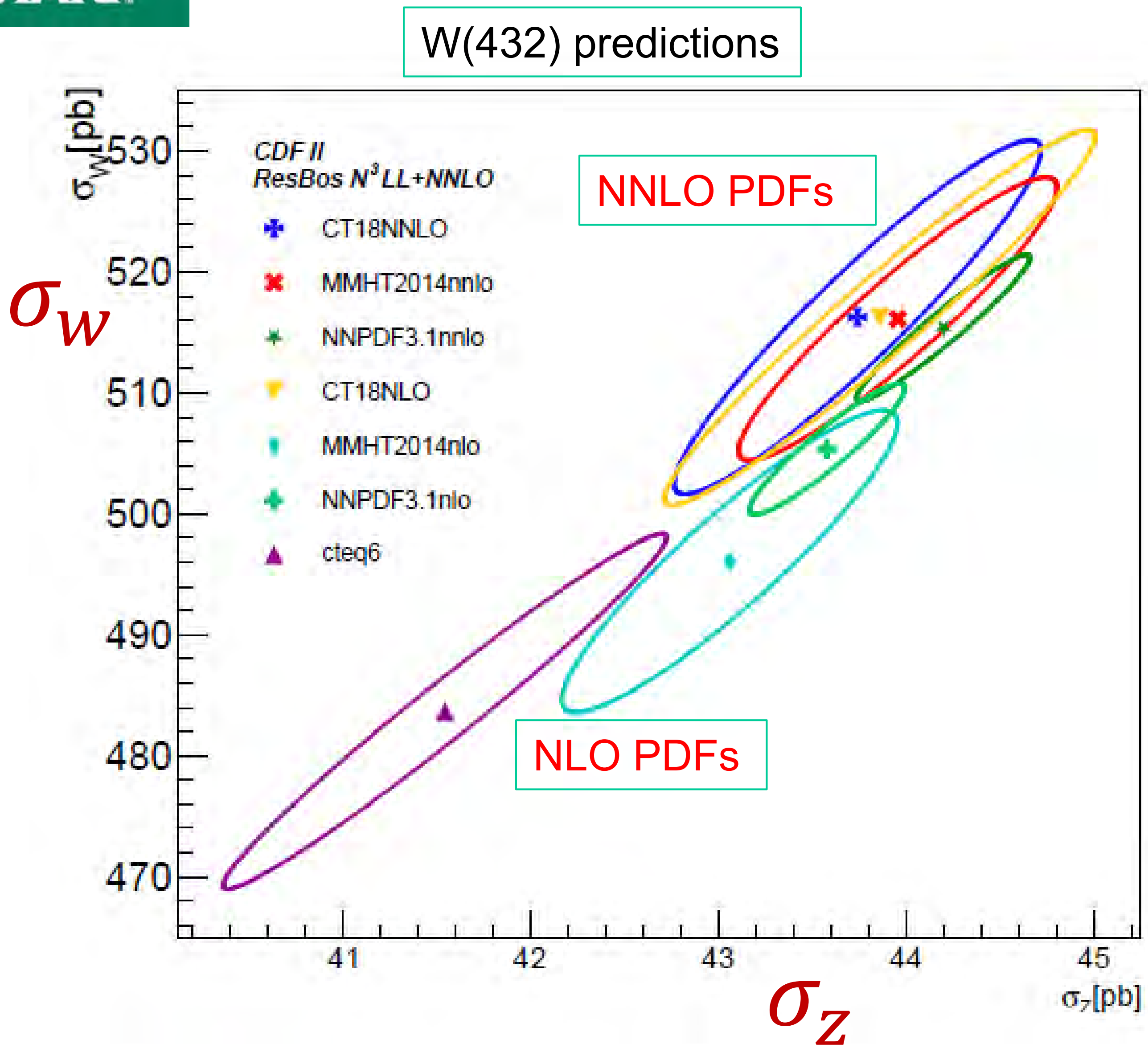
CTEQ

Extracted from $\frac{1}{\sigma}\frac{d\sigma}{dm_T}$ Extracted from $\frac{1}{\sigma}\frac{d\sigma}{dp_T(l)}$ Extracted from $\frac{1}{\sigma}\frac{d\sigma}{dp_T(\nu)}$



- Gluon PDF can contribute to NLO and NNLO predictions.
- pT(e) and pT(nu) are more sensitive to gluon PDF errors than mT(e,nu), hence, generate more shift in Mw.

PDF-induced correlation ellipses



- Gluon PDF can contribute to NLO and NNLO predictions.
- Slightly larger PDF-induced errors by NLO PDF sets than NNLO PDF sets.
- Correlation of W and Z (fiducial) cross sections, sensitive to strange quark PDF, varies with different PDF sets.

FIG. 10. PDF-induced correlation ellipses, at the 68% confidence level (C.L.), between the fiducial cross sections of W and Z boson production at the Tevatron Run II.

ePump-optimization

arXiv: 1806.07950; 1907.12177

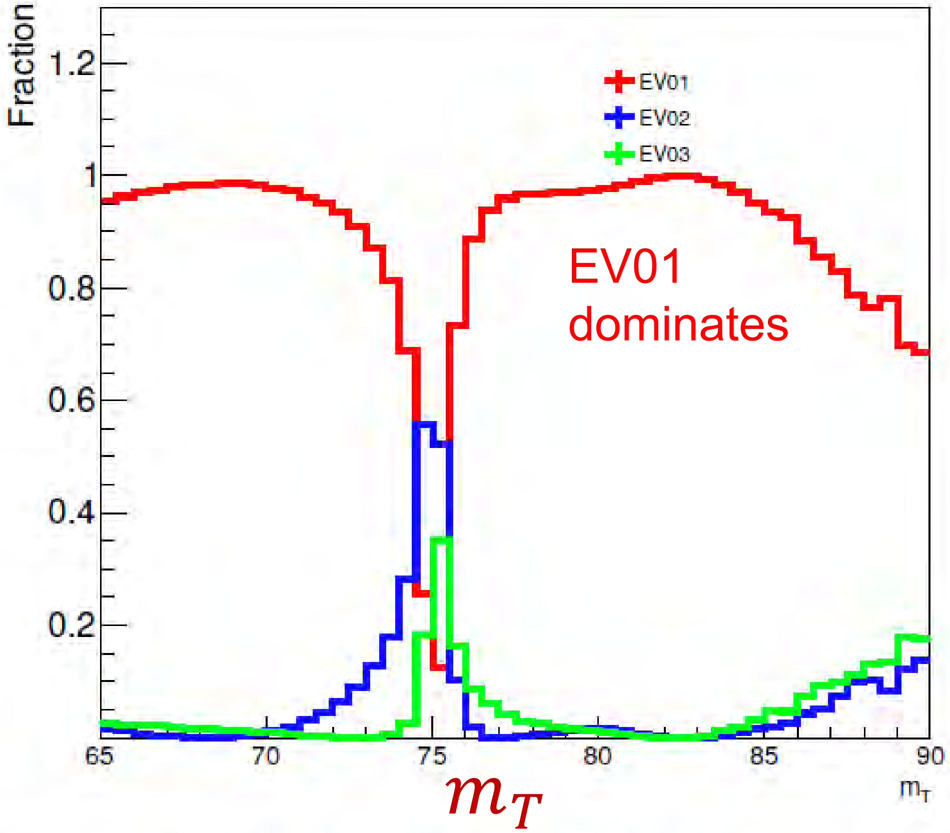


FIG. 12. Fractional contribution of the three leading optimized eigenvector PDFs (EV01, EV02 and EV03) to the variance of the m_T distribution, normalized to each bin, obtained from the ePump-optimization analysis.

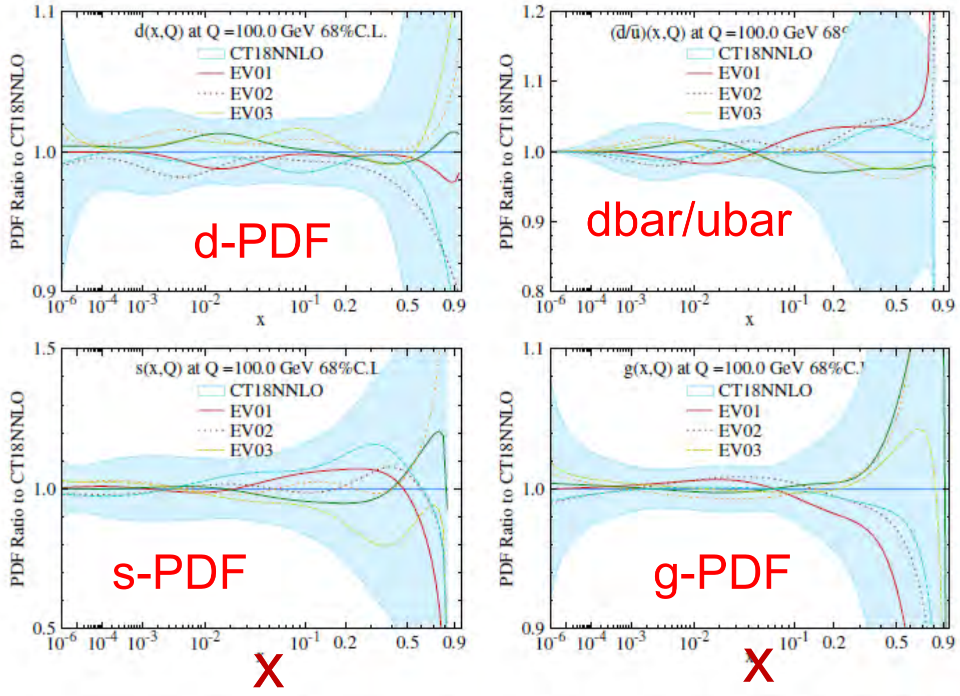


FIG. 13. Ratios of the top three pairs of eigenvector PDFs and the original CT18 NNLO error PDFs, at $Q = 100$ GeV, to the CT18 NNLO central value of d, $\bar{d}/\bar{u}$, s and g PDFs. These eigenvector PDFs were obtained after applying the ePump-optimization to the original CT18 NNLO PDFs with respect to the m_T distribution.

The three eigenvalues are 44.5, 3.0 and 2.4, respectively, with 50 bins in the m_T distribution.

ePump-optimization

arXiv: 1806.07950; 1907.12177

The three eigenvalues are 44.5, 3.0 and 2.4, respectively, with 50 bins in the m_T distribution.

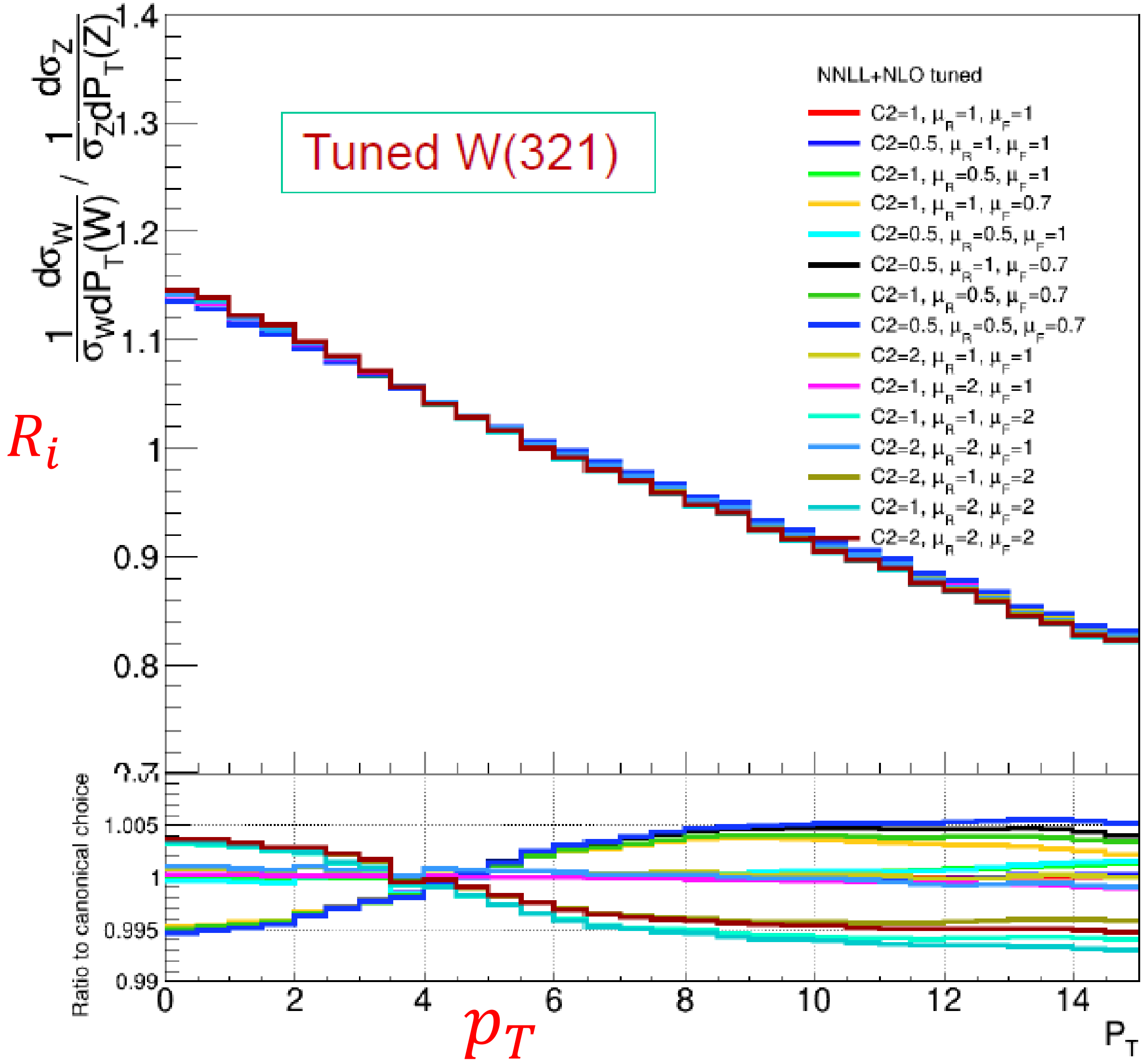
The combination of those top three optimized error PDFs contributes up to 99.6% in the total PDF variance of the 50 given data points, *i.e.*, with 50 bins in m_T distribution. This ePump-optimization allows us to conveniently use these three leading new eigenvectors (with a total of six error sets), in contrast to applying the full 58 error sets of the CT18 NNLO PDFs, to study the PDF-induced uncertainty of the m_T observable.

Only need to use these 2*3+1=7, not 2*29+1=59, PDF sets to study detector effects, etc. via Monte Carlo simulation.

Note that the first leading eigenvector set already accounts for 44.5/50= 89% of the PDF variance.

Only one EV01 set is needed to simulate mT distribution for mT > 77 GeV. This is useful for, e.g., studying detector resolution effect and detection efficiency, etc.

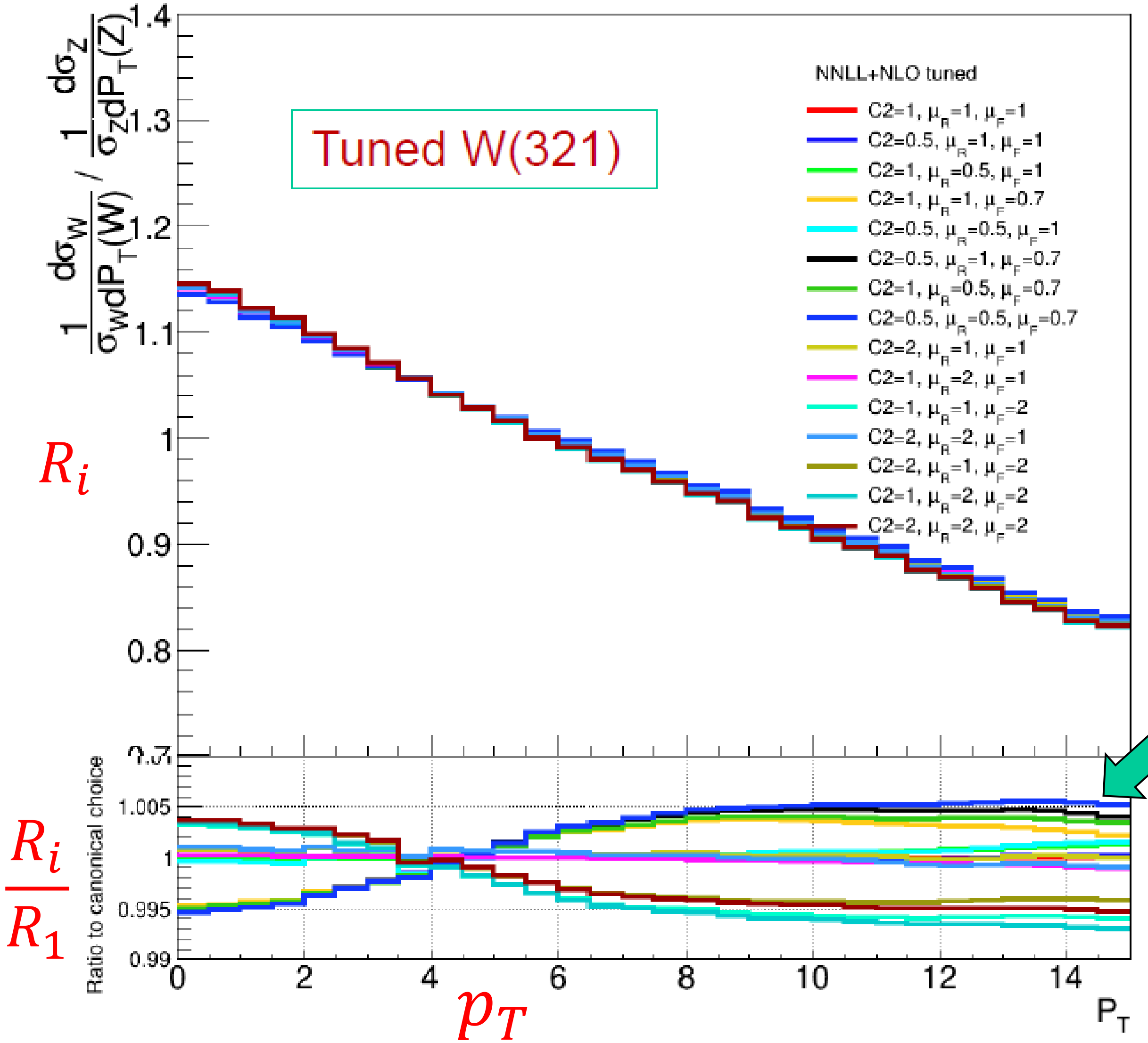
Ratios (R_i) of normalized pT(W) and pT(Z) due to QCD scale variation



- CDF did not use ResBos code to study the impact on the M_W measurement from varying the QCD scales to model the pT(W) distribution, after using their pT(Z) distribution to fix the g_2 and α_s values. Instead, they used DYQT code.
- In this study, we follow CDF approach and assume a fully correlated scale variation between pT(W) and pT(Z) when varying the QCD scales in the ResBos calculation.
- We consider 15 scale variations -- varying C_2, $C_1 = C_3 = \mu_F$ and μ_R by a factor 2 around the canonical scales, with ratios greater than 2 dropped.

Ratios of normalized pT(W) and pT(Z) due to QCD scale variation

C T E Q



- The upper panel shows

$$R_i = \frac{\left(\frac{d\sigma}{\sigma dp_T}\right)_W}{\left(\frac{d\sigma}{\sigma dp_T}\right)_Z} \quad \text{for scale choice } i = 1,2, \ldots 15$$

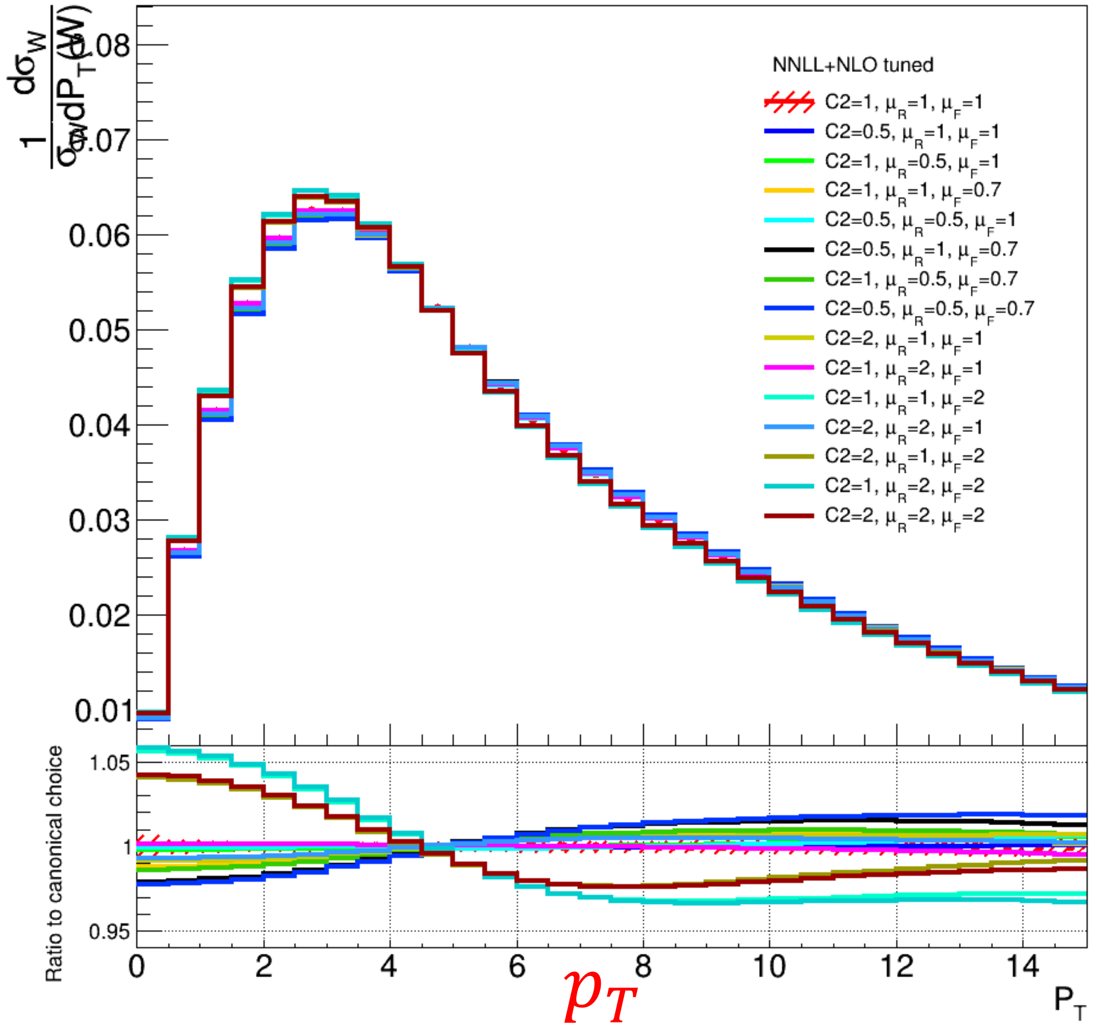
- The lower panel shows the ratio $\frac{R_i}{R_1}$

($i = 1$ is the canonical scale choice.)

The envelope of $\frac{R_i}{R_1}$ is found to be covered by the scale choice $(C_2,\ C_1 = C_3 = \mu_F,\ \mu_R) = (0.5, 0.7, 0.5)$, after symmetrizing it about 1.
Later, we shall refer this curve as $En(p_T)$.

Normalized pT(W) due to QCD scale variation

$\left(\frac{d\sigma}{\sigma dp_T}\right)_W$



p_T

- CDF use pT(W) data to constrain the range of QCD scale variation.
- The criteria is to impose the change in total χ^2 of the normalized pT(W) distribution by one unit, i.e., $\Delta\chi^2 = 1$

Fit to normalized pT(W) data, and require $\Delta\chi^2 = 1$

- Scale variation on the extracted M_W, from $m_T, p_T(e), p_T(\nu)$ distributions, derived from various $p_T(W)$.
- Almost all scale variations, other than the canonical scale choice used in generating the tuned W(321), have $\Delta\chi^2 > 1$.

	Mass Shift [MeV]						
	m_T		$p_T(\ell)$		$p_T(\nu)$		$\Delta\chi^2$
Scale	ResBos2	+Detector Effect+FSR	ResBos2	+Detector Effect+FSR	ResBos2	+Detector Effect+FSR	
C2=1,μ_R=1,μ_F=1	1.2 ± 0.5	-0.5 ± 1.8 ± 1.0	0.8 ± 2.1	-0.8 ± 2.6 ± 1.3	0.8 ± 2.1	-0.6 ± 3.4 ± 2.0	0
C2=0.5,μ_R=1,μ_F=1	1.2 ± 0.5	-0.4 ± 1.8 ± 1.0	1.2 ± 2.1	-0.3 ± 2.6 ± 1.3	1.3 ± 2.1	-0.2 ± 3.4 ± 1.9	2.5
C2=1,μ_R=0.5,μ_F=1	1.2 ± 0.5	-0.4 ± 1.8 ± 1.0	-1.2 ± 2.1	-2.6 ± 2.7 ± 1.3	-1.2 ± 2.1	-1.7 ± 3.4 ± 2.0	15.3
C2=1,μ_R=1,μ_F=0.7	0.8 ± 0.5	-2.9 ± 1.8 ± 1.0	-8.2 ± 2.1	-8.9 ± 2.7 ± 1.3	-8.2 ± 2.1	-8.1 ± 3.4 ± 2.0	222.1
C2=0.5,μ_R=0.5,μ_F=1	1.2 ± 0.5	-0.3 ± 1.8 ± 1.0	-0.7 ± 2.1	-2.2 ± 2.7 ± 1.3	-0.7 ± 2.1	-1.3 ± 3.4 ± 2.0	14.0
C2=0.5,μ_R=1,μ_F=0.7	0.7 ± 0.5	-3.1 ± 1.8 ± 1.0	-15.7 ± 2.2	-15.4 ± 2.7 ± 1.3	-15.7 ± 2.2	-12.4 ± 3.4 ± 2.0	747.4
C2=1,μ_R=0.5,μ_F=0.7	0.8 ± 0.5	-2.8 ± 1.8 ± 1.0	-10.1 ± 2.2	-10.7 ± 2.7 ± 1.3	-10.2 ± 2.1	-9.2 ± 3.4 ± 2.0	308.2
C2=0.5,μ_R=0.5,μ_F=0.7	0.7 ± 0.5	-3.0 ± 1.8 ± 1.0	-17.8 ± 2.2	-17.4 ± 2.7 ± 1.4	-17.8 ± 2.2	-13.5 ± 3.4 ± 2.0	921.8
C2=2,μ_R=1,μ_F=1	1.2 ± 0.5	-0.4 ± 1.8 ± 1.0	-4.5 ± 2.1	-5.3 ± 2.7 ± 1.3	-4.5 ± 2.1	-3.7 ± 3.4 ± 2.0	135.4
C2=1,μ_R=2,μ_F=1	1.2 ± 0.5	-0.6 ± 1.8 ± 1.0	2.3 ± 2.1	0.7 ± 2.6 ± 1.3	2.3 ± 2.1	0.3 ± 3.4 ± 1.9	9.9
C2=1,μ_R=1,μ_F=2	1.8 ± 0.5	3.0 ± 1.8 ± 1.0	30.4 ± 2.0	26.7 ± 2.5 ± 1.2	30.5 ± 2.0	21.7 ± 3.3 ± 1.8	4492.0
C2=2,μ_R=2,μ_F=1	1.2 ± 0.5	-0.5 ± 1.8 ± 1.0	-2.9 ± 2.1	-3.8 ± 2.7 ± 1.3	-2.9 ± 2.1	-2.8 ± 3.4 ± 2.0	86.6
C2=2,μ_R=1,μ_F=2	1.7 ± 0.5	2.8 ± 1.8 ± 1.0	20.5 ± 2.0	17.3 ± 2.6 ± 1.2	20.6 ± 2.0	15.0 ± 3.4 ± 1.9	2111.1
C2=1,μ_R=2,μ_F=2	1.8 ± 0.5	2.9 ± 1.8 ± 1.0	31.8 ± 2.0	28.1 ± 2.5 ± 1.2	31.8 ± 2.0	22.6 ± 3.3 ± 1.8	4833.7
C2=2,μ_R=2,μ_F=2	1.7 ± 0.5	2.7 ± 1.8 ± 1.0	21.9 ± 2.0	18.8 ± 2.6 ± 1.2	22.0 ± 2.0	15.9 ± 3.4 ± 1.9	2311.3

Ratio of normalized pT(W) and pT(Z) in the "Envelope" approach by CDF

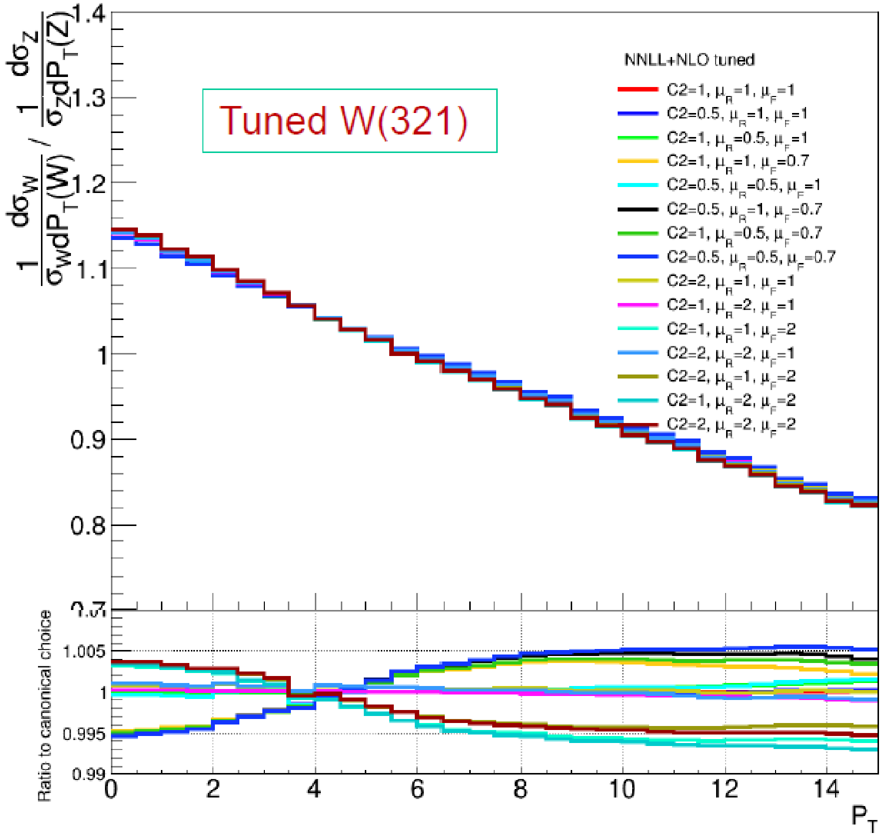
- Reweight the normalized pT(W) distribution (with i=1) by

$$a * (En(p_T) - 1) + 1$$

  with a varying from -1 to 1, for every pT bin.

- Generate the normalized pT(W) distribution after reweighting the (i=1) result by applying the weight a. The result of a=0 corresponds to the result of i=1.

- For a given $p_T(W)$, after reweighting, one can extract M_W, from the corresponding $m_T, p_T(e), p_T(\nu)$ distributions.



Fit to normalized pT(W) data, and require $\Delta\chi^2 = 1$, using CDF "Envelope" approach

- Scale variation on the extracted M_W, from $m_T, p_T(e), p_T(\nu)$ distributions, derived from various $p_T(W)$.
- Using CDF "envelope" method to constrain the allowed $p_T(W)$ distribution due to QCD scale variation in the ratio of normalized pT(W) and pT(Z).

	Mass Shift [MeV]					
	m_T		$p_T(\ell)$		$p_T(\nu)$	
Scale	ResBos2	+Detector Effect+FSR	ResBos2	+Detector Effect+FSR	ResBos2	+Detector Effect+FSR
a=0.176	1.2 ± 0.5	$0.8 \pm 1.8 \pm 1.1$	3.1 ± 2.1	$-6.5 \pm 2.7 \pm 1.3$	1.4 ± 2.1	$-4.9 \pm 3.4 \pm 2.0$
a=-0.176	1.2 ± 0.5	$-0.7 \pm 1.8 \pm 01.$	1.8 ± 2.1	$9.4 \pm 2.6 \pm 1.2$	0.0 ± 2.1	$4.8 \pm 3.4 \pm 1.9$

Conclusions and outlook

- Higher order effect in the ResBos calculation can bring the discrepancy from $7\,\sigma$ down to about $6\,\sigma$, a shift around 10 MeV toward the Standard Model (SM) prediction.
- LHC will further improve M_w measurement.
- A combined analysis of LHC and Tevatron M_w measurements will come in near future.
- If it is due to New Physics (NP), similar effect may also affect the measurement of weak-mixing angle $\sin^2\theta_w$ via the forward-backward charged asymmetry (A_{FB}) of Drell-Yan pair production at the high luminosity LHC. In this case, it is crucial to be able to factorize the effect of PDFs in the A_{FB} measurement from the genuine electroweak physics (in either SM or NP).

arXiv: 2202.13628

- More collaborations among experimentalists and theorists are needed!

Learned from Prof. Joey Huston @ MSU

==== my answers to questions from Dr. Natascia Vignaroli =====

(1)

> What do you think about the CDF anomaly?

=>

Our paper only discussed the impact of higher order contributions to the extraction of M_W, based on CDF's data-driven method. We cannot answer the question about the difference observed by CDF between their data and SM prediction.

If it is not due to new physics effect, then one could ask:

— Could there be some common systematic(s) among all six of the CDF analyses?

— Would it be worthwhile to do a W-mass analysis of Z -> ee, \mu \mu, though it will be statistics limited?

(2)

> How about the ATLAS measurements?

=>

ATLAS has a much better detector, but as compared to CDF, it suffers from being "too energetic" — most W bosons are boosted (to both longitudinal and transverse directions)! CDF has smaller PDF uncertainties, smaller QCD radiation (Sudakov) effects, and smaller pileup, etc.

===

Q: Impact of TMD PDFs on W mass measurement

- CSS qT resummation formalism is a “model” of TMD (transverse momentum dependent) factorization.
- The data-driven method done by CDF – using pT(Z) distribution to model pT(W) – would probably fix any possible “inefficiency” of the CSS qT resummation calculation for modeling TMD PDFs.
- CDF further used pT(W) data to constrain the allowed QCD scale variation in the ratio of normalized pT(W) and pT(Z).
- The only caveat is that u and d (and other flavor) quarks inside the proton might have different “intrinsic” transverse momenta, at the order of Λ_{QCD}. This has been explored in a phenomenology study of arXiv:1807.0210. However, some Lattice-QCD calculation does not seem to support this scenario. (See arXiv:1011.1213)

⇒ More study is needed.

Lessons learned from W mass measurements

C T E Q

Experimentalists + Theorists

2017 Featured Story #1: Million-dollar gift establishes endowed professorship in honor of the late Dr. Wu-Ki Tung

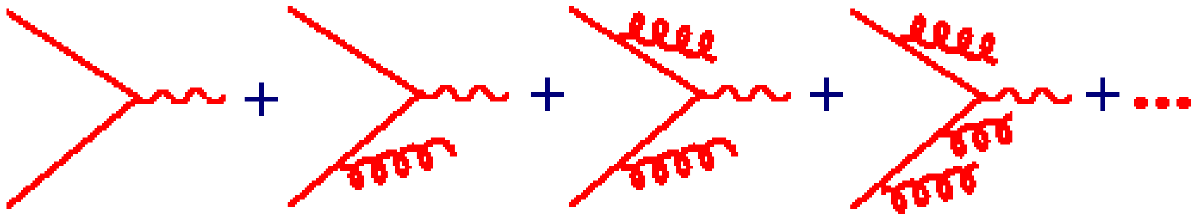
Michigan State University
(1992-2009)

http://www.pa.msu.edu/node/5921

- Co-founder of CTEQ (**The Coordinated Theoretical-Experimental Project on QCD)** in 1989 – present
- Nowadays, many, like this Workshop, are doing precisely that.

Backup slides

Diagramatically, Resummation is doing



Resum large $\alpha_S^n \ln^m\left(\frac{Q^2}{q_T^2}\right)$ terms

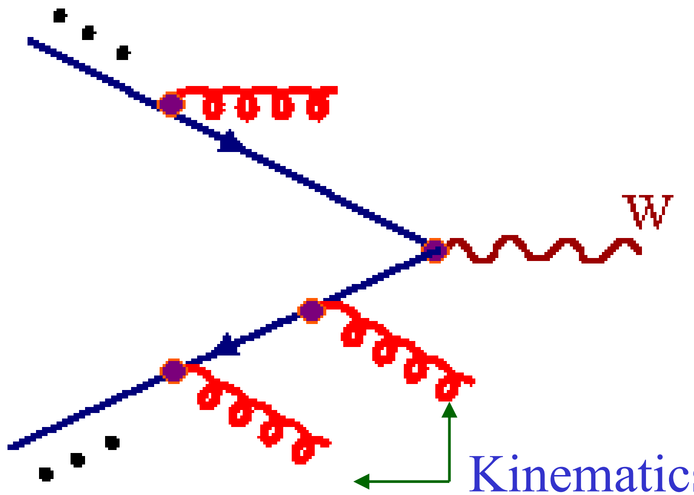
$$\left.\frac{d\sigma}{dq_T^2\,dy}\right|_{q_T\to 0} \sim \frac{1}{q_T^2}\sum_{n=1}^{\infty}\sum_{m=0}^{2n-1}\alpha_S^n \ln^m\left(\frac{Q^2}{q_T^2}\right)\cdot C_m^n$$

Monte-Carlo programs ISAJET, PYTHIA, HERWIG contain these physics.

(Note: Arbitrary cut-off scale in these programs to affect the amount of Backward radiation , i.e. Initial state radiation.)

CTEQ

Monte-Carlo Approach



Backward Radiation
(Initial State Radiation)

Kinematics of the radiated gluon, controlled by Sudakov form factor with some arbitrary cut-off. (In contrast to perform integration in impact parameter space, i.e., b space.)

➡ The shape of q_T (w) is generated. But, the integrated rate remains the same as at Born level (finite virtual correction is not included).

* Recently, there are efforts to include part of higher order effect in the event generator.

Event Generators (PYTHIA, HERWIG)

Note that the integrated rate is the same as the Born level rate ($\alpha_S^{(0)}$) even though the q_T – distribution is different (i.e., not $\delta\left(q_T^2\right)$ any more).

$$\sigma = \int \mathrm{d}q_T^2 \frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \int \mathrm{d}^2 b \underbrace{\int e^{i\vec{q}_T\cdot\vec{b}}\,\mathrm{d}^2 q_T}\, \sigma_0 e^{-S(b)} \quad \rightarrow \quad \delta^2(b)$$

$$= \int \mathrm{d}^2 b\, \delta^2(b) \cdot \sigma_0 \cdot e^{-S(b)} \quad \rightarrow \quad 1 \text{ at } b=0$$

$$= \sigma_0$$

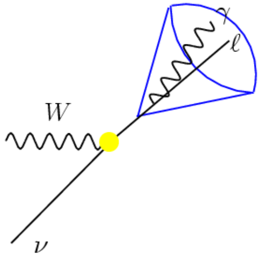
For C-Function = $\delta\left(1 - \frac{x_A}{\xi_B}\right)$

Need to consider the recombination effect

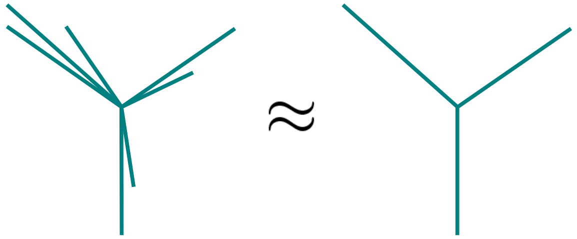
CTEQ

- Experimental: difficult to discriminate between electrons and photons with a small opening angle

- Theoretical: to define infra-safe quantities which are independent of long-distance physics



Essential feature of a general IRS physical quantity:
The observable must be such that it is insensitive to whether n or n+1 particles contributed if the n+1 particles has n-particle kinematics.



- Procedure @ Tevatron (for electron)

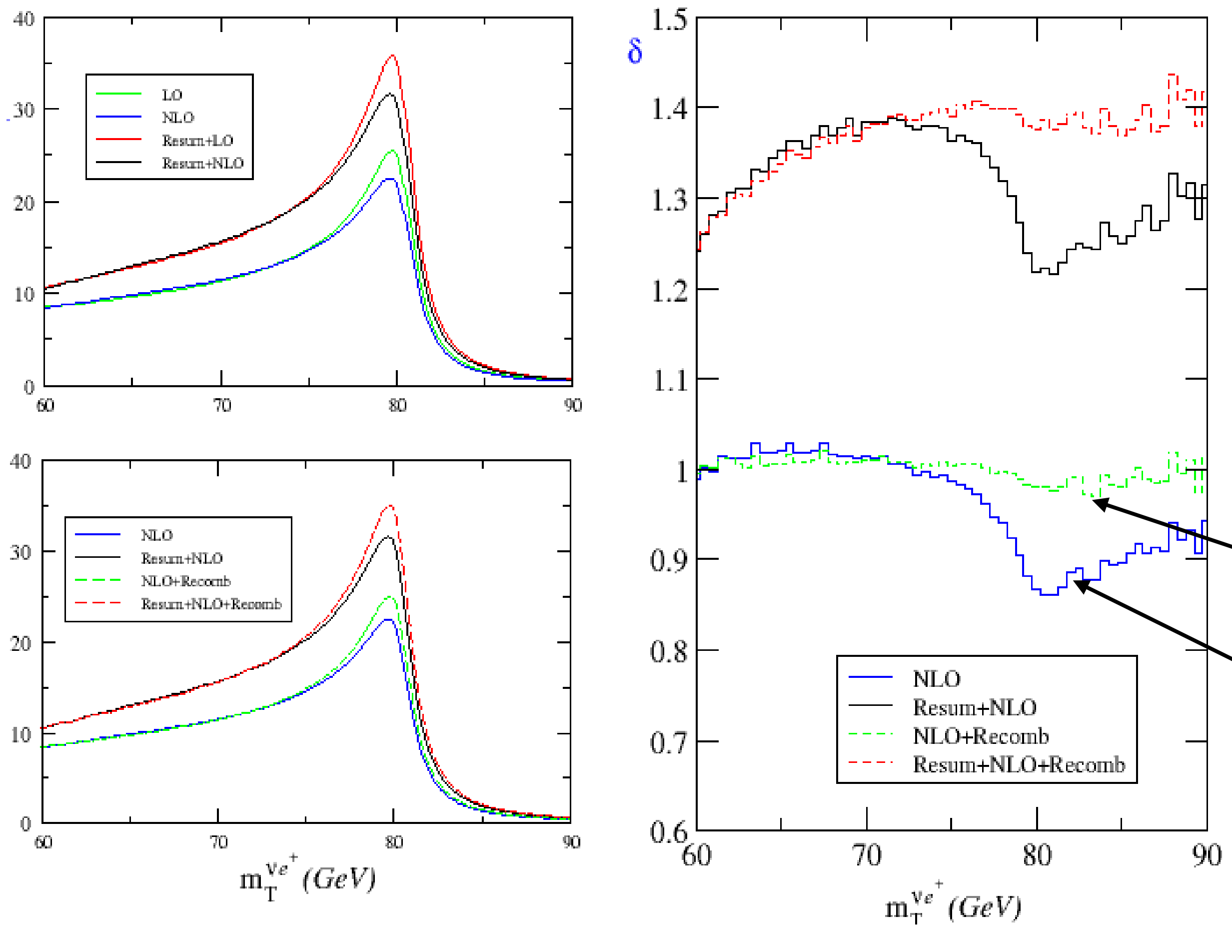
☞ $p'_e = p_e + p_\gamma$

- $\Delta R(e,\gamma) < 0.2$
- $E_\gamma < 0.15 E_e$ for $0.2 < \Delta R(e,\gamma) < 0.3$

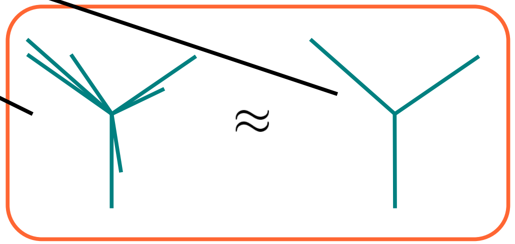
☞ rejection

- $E_\gamma > 0.15\ E_e$ for $0.2 < \Delta R(e,\gamma) < 0.4$

Recombination Effects for detecting electrons



Effects of QED correction decrease significantly after recombination.



infrared-safe

Where is it?

- **ResBos**: http://hep.pa.msu.edu/resum/
- **Plotter**: http://hep.pa.msu.edu/wwwlegacy

ResBos-A (including final state NLO QED corrections)
http://hep.pa.msu.edu/resum/code/resbosa/
has not been updated.
Why? Because it was not used for Tevatron experiments.

The plan is to include final state QED resummation inside ResBos2.

Sorry, the website is temporary down and will be restored later.

Physical processes included in ResBos

$W^{\pm}$

γ, Z — including gauge invariant set amplitude

H — Including the full NNLO contribution

$\gamma\gamma, ZZ, WW$

New physics: W', Z', H^+, A^0, H^0 …

Limitations of ResBos

- Any perturbative calculation is performed with some approximation, hence, with limitation.
- To make the best use of a theory calculation, we need to know what it is good for and what the limitations are.

It does not give any information about the hadronic activities of the event.

→ It could be used to reweight the distributions generated by (PYTHIA) event generator, by comparing the boson (and it decay products) distributions to ResBos predictions.

This has been done for W-mass analysis by CDF and D0

Conclusion

- ResBos is a useful tool for studying electroweak gauge bosons and Higgs bosons at the Tevatron and the LHC.
- It includes not only QCD resummation for low q_T region but also higher order effect in high q_T region, with spin correlations included via gauge invariant set of matrix elements.

PDF-induced uncertainty

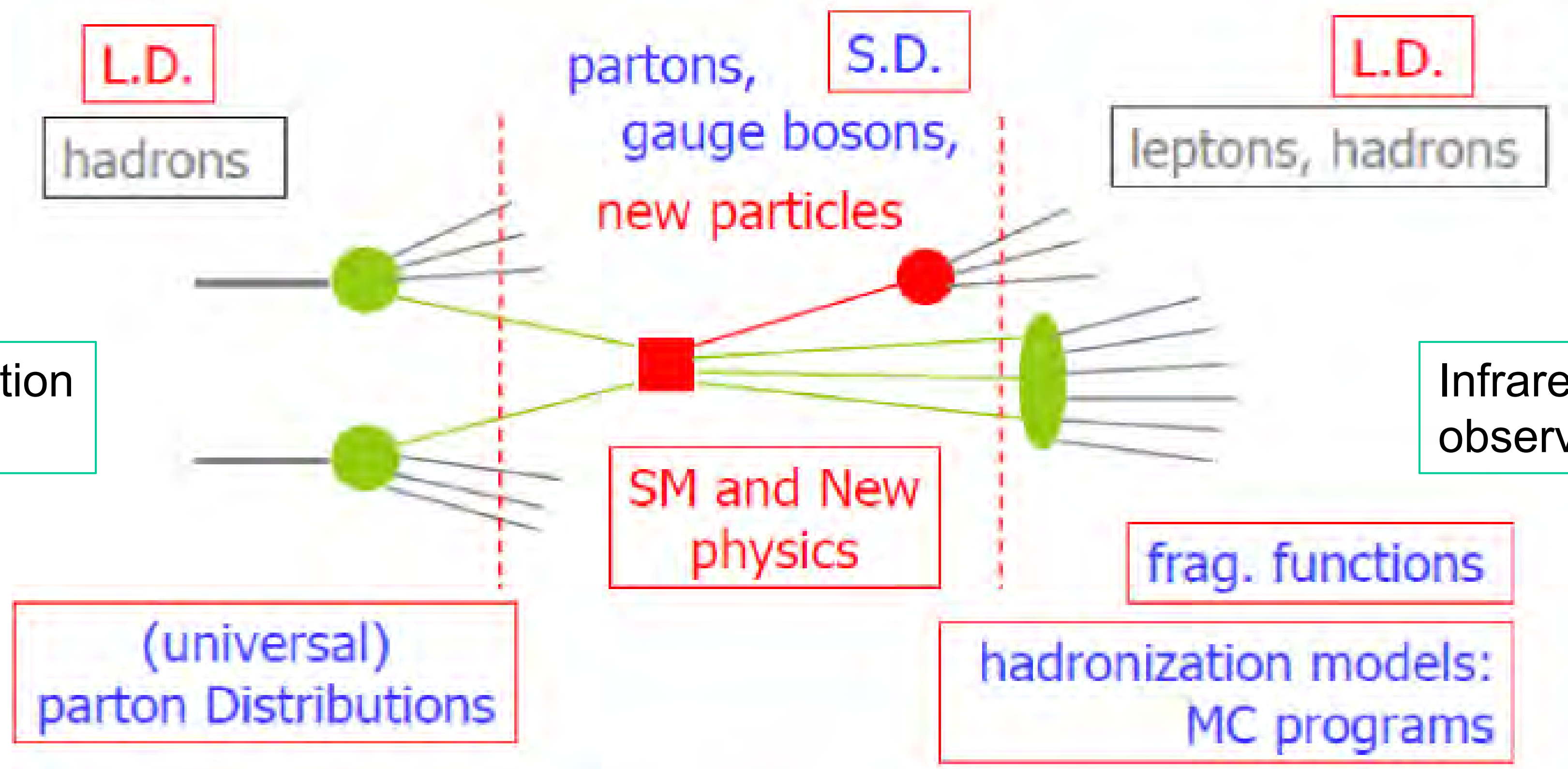
Hessian Method

Hessian error PDF sets

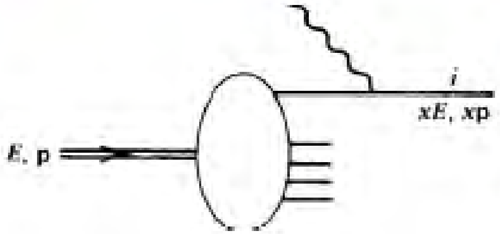
such as CT18 and MSHT20 PDFs

QCD improved parton model

Factorization Theorem



Infrared safe observables



Parton distribution function (PDF) $f_{j/A}(x, Q)$ discribe the possibility to find a parton j, i.e. quark and gluon, in a nucleaon A.

PDF-induced uncertainty

Let $X = X(\{a_i\})$ to be the observable as a function of fitting parameter. Using the linear approximation of parameter $\{z_i\}$, the symmetry uncertainty of X is,

$$\Delta X = \frac{1}{2}\left(\sum_{i=1}^{N_p}\left[X(\{z_i^+\}) - X(\{z_i^-\})\right]^2\right)^{1/2},$$

Where $\{z_1^{\pm}\} = \{\pm T, 0....\}$, $\{z_2^{\pm}\} = \{0, \pm T, 0....\}$ and so on. The asymmetry uncertainty of X is,

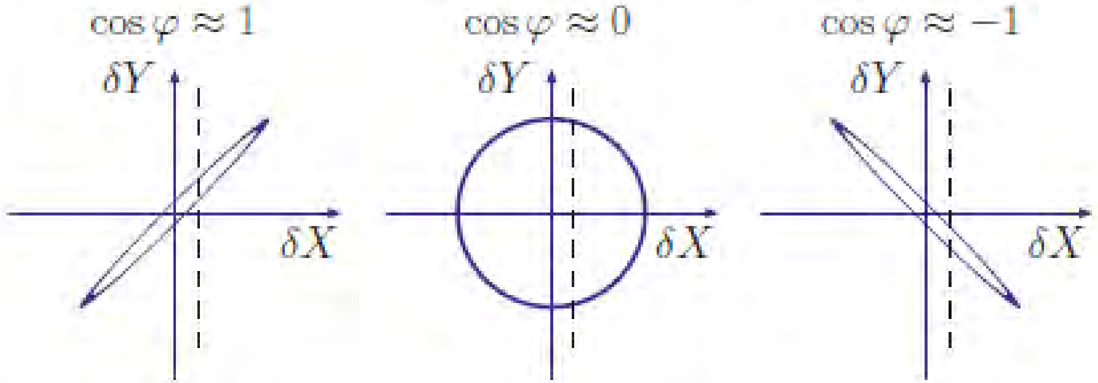
$$\delta^+ X = \sqrt{\sum_{i=1}^{N_a}\left[\max\left(X_i^{(+)} - X_0, X_i^{(-)} - X_0, 0\right)\right]^2},$$

$$\delta^- X = \sqrt{\sum_{i=1}^{N_a}\left[\max\left(X_0 - X_i^{(+)}, X_0 - X_i^{(-)}, 0\right)\right]^2},$$

PDF-induced correlations

C T E Q

Correlation ellipse for observables X and Y



In the framework of Hessian method, the correlation between two observables X and Y, which are function of PDFs, can be worked out as:

Correlation cosine

$$\cos\varphi = \frac{\vec{\nabla}X \cdot \vec{\nabla}Y}{\Delta X \Delta Y} = \frac{1}{4\Delta X\,\Delta Y}\sum_{\alpha=1}^{N}\left(X_\alpha^{(+)} - X_\alpha^{(-)}\right)\left(Y_\alpha^{(+)} - Y_\alpha^{(-)}\right)$$

Where the ΔX and ΔY are their symmetric uncertainties. By this correlation angle φ, the tolerance ellipse is defined by

$$X = X_0 + \Delta X \cos\theta,\ Y = Y_0 + \Delta Y \cos(\theta + \varphi),$$

ePump
(error PDF Updating Method Package)

- A tool to examine the impact of a new data set to further constrain the existing PDFs without using a global analysis code.
- A tool to reduce the total number of error PDF sets relevant to specific experimental observables.
- A tool to perform a simultaneously fit to the parameter of New Physics model and PDFs.

http://hep.pa.msu.edu/epump/

arXiv: 1806.07950
arXiv: 1907.12177

Sorry, the website is temporary down and will be restored later.

Motivation for ePump

- UpdatePDFs: With many data sets and NNLO calculations, global fitting can be time consuming.
  - Need for fast and efficient method to estimate effects of new data before doing global fit.
  - Can estimate effects of different data set choices in real time.
- OptimizePDFs: Experimental analyses may require many MC calculations, using PDF error sets. Again, it's time consuming.
  - Optimize Hessian error PDFs to the observables, so irrelevant error PDFs may be discarded, while PDF-dependence is still maintained to desired precision.

ePump Updating: Hessian Profiling

C T E Q

- Plenty of new data from LHC need careful study on the impact to PDFs. However, **a complete global fitting take 1 to 2 days** after the implementation of applgrid/fastNLO and parallelization.
- **Hessian reweighting/profiling method** predict the updated PDFs and observables after including new data in global analysis of given Hessian eigenvector sets.

$$\Delta\chi^2(Z) = \Delta\chi^2_{old}(Z) + (X_i^E - X_i(Z))C_{ij}^{-1}(X_j^E - X_j(Z))$$

$$\text{Updated best-fit PDF}: f^0_{new} = f^0 + \Delta f \cdot Z$$

$$\text{Updated error PDFs}: f^{\pm(r)} = f^0_{new} \pm \Delta f \cdot U^{(r)} / \sqrt{1+\lambda^{(r)}}$$

$$\text{Updated observables}: Y^0_{new} = Y^0 + \Delta Y \cdot Z$$

- The Hessian reweighting method resulting as the program of **ePump** (error PDF updating method package) (PRD98,094005(2018) Carl Schmidt *et al*, PRD100,114024(2019), T.-J. Hou *et al*)

How to use ePump

C T E Q

It could be a theory prediction of New Physics model, such as the Standard Model Effective Field Theory (SMEFT).

Theory Files:
Theory predictions for each of 56 eigenvectors

Data Files:
Standard Data file, including Correlated errors

ePump

New PDF set
56 eigenvectors

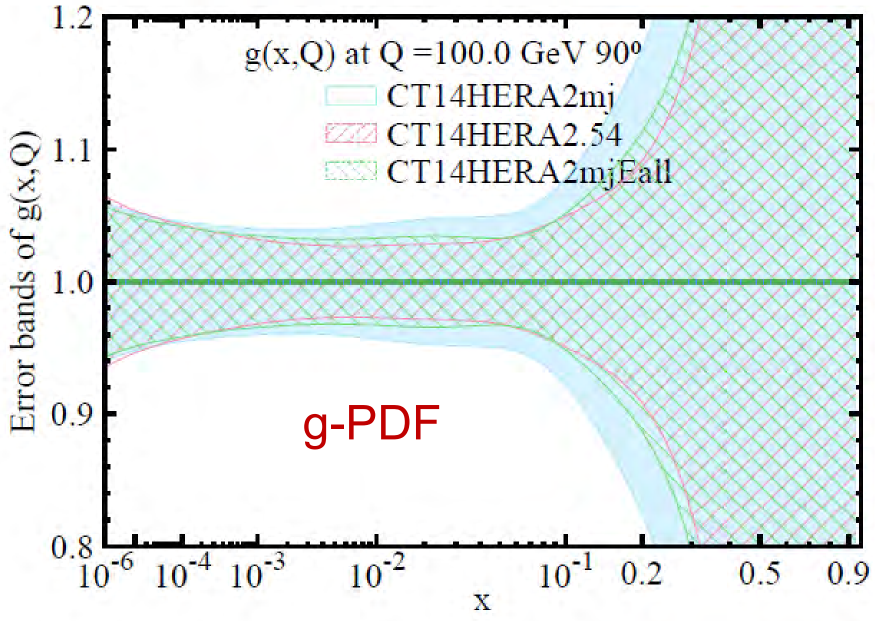
CT14 has 56 error PDF sets

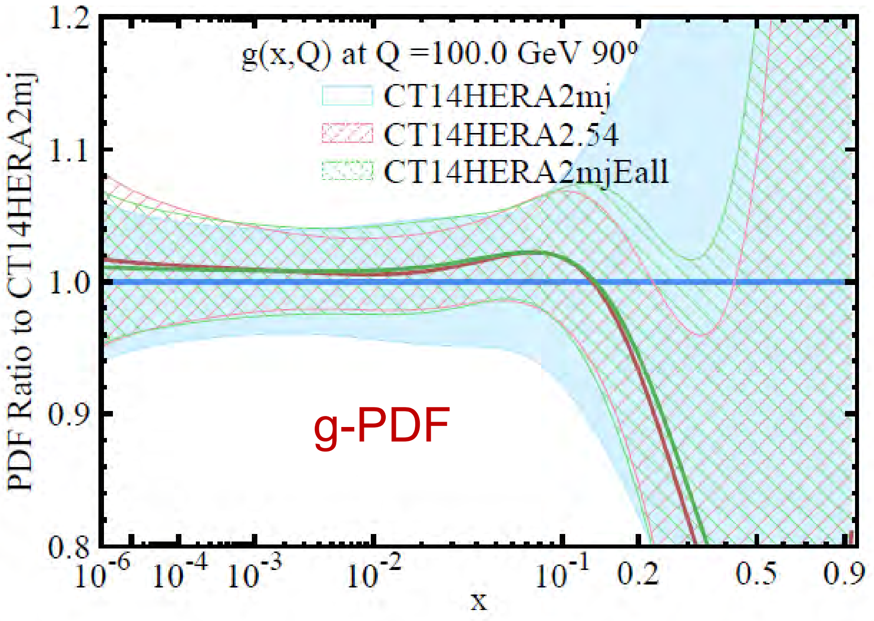
(Auxiliary Theory Files may also be included to update predictions for observables not included in fit.)

ePump-updating

An example to show the impact of jet data to constrain gluon PDF in the relevant x region.

- Remove all CDF, D0, ATLAS 7TeV, CMS TeV jet data from CT14HERA2 and refit → CT14HERA2mj.
- Add back the 4 data sets to CT14HERA2mj by ePump and compare with CT14HERA2.





Optimized PDFs

- Based on Data Set Diagonalization Pumplin, PRD 80 (2009) 034002

  Maximize: $\sum \left(X_\alpha(\mathbf{z}) - X_\alpha(\mathbf{0})\right)^2 / \left|\Delta X_\alpha\right|^2$ subject to constraint $\mathbf{z}^2 = 1$

  Using Hessian approximation, $X_\alpha(\mathbf{z}) = X_\alpha(\mathbf{0}) + \mathbf{\Delta} X_\alpha \cdot \mathbf{z}$

  leads to matrix $M^{ij} = \sum \Delta X_\alpha^i \, \Delta X_\alpha^j / \left|\Delta X_\alpha\right|^2$ with eigenvalues/vectors, $\lambda^{(r)}$ and $\mathbf{U}^{(r)}$

⇒ New error PDFs: $f^{\pm(r)} = f^0 \pm \mathbf{\Delta} \boldsymbol{f} \cdot \mathbf{U}^{(r)}$

- Order PDFs by eigenvectors (note : $\sum \lambda^{(r)} =$ number of observables)
- Full set of optimized PDFs reproduces Hessian symmetric errors
- Eigenvalue $\lambda^{(r)}$ gives (sum of) fractional contribution of $f^{\pm(r)}$ to variances
- Depending on precision required, keep reduced set of error PDFs, based on eigenvalues.

Summary

- The ePump package contains two functionalities
  - UpdatePDFs is a fast & efficient method to estimate the effect of new data on the a current set of best-fit and Hessian error PDFs.
  - OptimizePDFs can be used to find optimized set of Hessian error PDFs for specialized experimental analyses. It gives a simple method for reducing the number of optimized error PDFs, while maintaining a specified precision.

- A tool to examine the impact of a new data set to further constrain the existing PDFs without using a global analysis code.
- A tool to reduce the total number of error PDF sets relevant to specific experimental observables.
- A tool to perform a simultaneously fit to parameters of New Physics model and PDFs.