The CDF II W mass anomaly and Beyond SM from additional Higgs to dark photon models

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Rapid Response Workshop on W Boson Mass Anomaly

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Y Cheng, X-G He, Z-L Huang, M-W Li, arXiv: 2204. 05031
Y Cheng, X-G He, F Huang J Sun, Z-P Xing, arXiv: 2004.10156
And work in progress

1. The CDF II W mass anomaly

2. Beyond SM Higgs explanation

3. Dark Photon explanation

4. Conclusions

1.The CDF II W mass anomaly

T. Aaltonen et al. (CDF), Science **376**, 170 (2022).

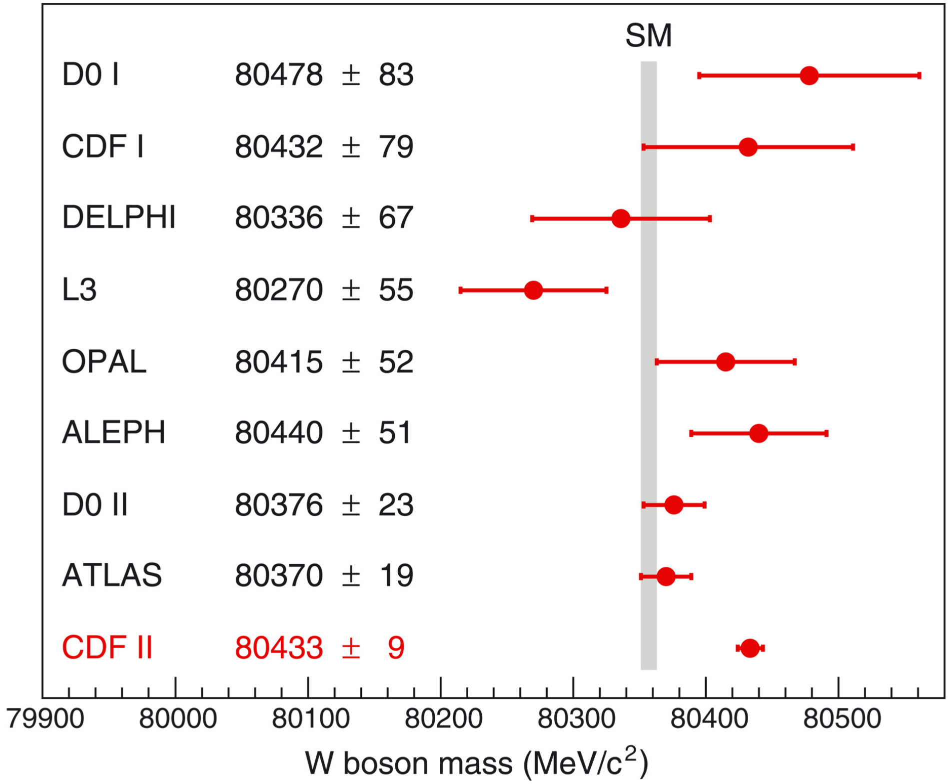
Recent CDF II analysis show that the W mass is more than SM value

m_W = 80,375 (6) MeV

The CDF II values is 7σ above SM value!

An anomaly!

A new fever? So far 125 papers cited the CDF paper.



Needs further more accurate experimental measurements.

Improved SM calculations. A hint for new physics beyond SM?

This talk: explore what new Higgs and dark photon model can do for explain the W mass anomaly.

Theoretically, the change of W mass is related to the electroweak precision oblique parameters S, T and U as

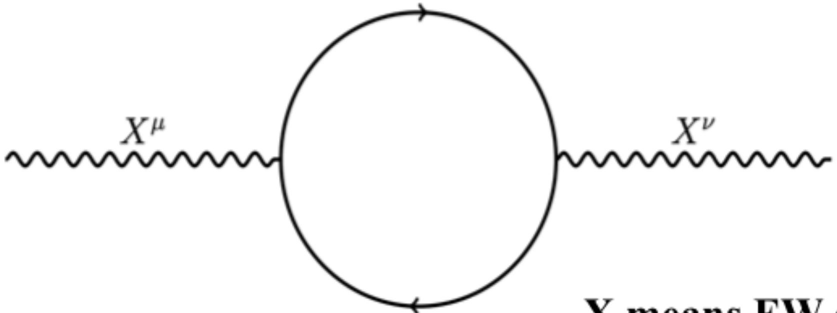
$$\Delta m_W^2 = m_Z^2 c_W^2 \left(-\frac{\alpha S}{2(c_W^2 - s_W^2)} + \frac{c_W^2 \alpha T}{(c_W^2 - s_W^2)} + \frac{\alpha U}{4 s_W^2} \right)$$

originated from tree and loop modifications beyond the SM

Tree level modification

$$\rho = \frac{m_w^2}{m_z^2 \cos^2\theta_w} = \frac{\sum_i v_i^2 \left(I_i(I_i+1) - Y_i^2\right)}{2 \sum_i Y_i^2 v_i^2}$$

Loop **Vacuum polarization**



$$= i\Pi_{XX}(p^2) g^{\mu\nu}$$

X means EW gauge bosons

ρ-1= 1/(1- αT)

$$\widehat{\alpha}(M_Z)T \equiv \frac{\Pi^{\text{new}}_{WW}(0)}{M_W^2} - \frac{\Pi^{\text{new}}_{ZZ}(0)}{M_Z^2} ,$$

Neutral - charged current

$$\frac{\widehat{\alpha}(M_Z)}{4\,\widehat{s}_Z^2\widehat{c}_Z^2} S \equiv \frac{\Pi^{\text{new}}_{ZZ}(M_Z^2) - \Pi^{\text{new}}_{ZZ}(0)}{M_Z^2} - \frac{\widehat{c}_Z^2 - \widehat{s}_Z^2}{\widehat{c}_Z\widehat{s}_Z}\frac{\Pi^{\text{new}}_{Z\gamma}(M_Z^2)}{M_Z^2} - \frac{\Pi^{\text{new}}_{\gamma\gamma}(M_Z^2)}{M_Z^2} ,$$

Different energy scales

$$\frac{\widehat{\alpha}(M_Z)}{4\,\widehat{s}_Z^2}(S+U) \equiv \frac{\Pi^{\text{new}}_{WW}(M_W^2) - \Pi^{\text{new}}_{WW}(0)}{M_W^2} - \frac{\widehat{c}_Z}{\widehat{s}_Z}\frac{\Pi^{\text{new}}_{Z\gamma}(M_Z^2)}{M_Z^2} - \frac{\Pi^{\text{new}}_{\gamma\gamma}(M_Z^2)}{M_Z^2} .$$

Global Fits to available data

Parameter	Input Value	PDG 2021				CDF 2022			
		$\chi^2_{\min}(\text{dof}) = 18.73(16)$				$\chi^2_{\min}(\text{dof}) = 64.45(16)$			
		Fit Result	Pull	Fit w/o Input	Pull	Fit Result	Pull	Fit w/o Input	Pull
m_W [GeV]	80.379(12)	80.361(6)	−1.47	80.357(6)	−1.86	–	–	–	–
	80.4335(94)	–	–	–	–	80.381(5)	−5.80	80.357(6)	−8.53
$\Delta\alpha^{(5)}_{\text{had}}$ [a]	0.02761(11)	0.02756(11)	−0.44	0.02716(38)	−4.06	0.02746(10)	−1.37	0.02603(36)	−14.37
m_h [GeV]	125.25(17)	125.25(17)	−0.02	$92^{(21)}_{(18)}$	−193.26	125.24(17)	−0.06	$42^{(10)}_{(8)}$	−489.71
m_t [GeV] [b]	172.76(58)	173.02(56)	0.45	176.2(20)	5.83	174.04(55)	2.19	184.2(16)	19.55
$\alpha_s(m_Z)$	0.1179(9)	0.1180(9)	0.14	0.1193(9)	1.53	0.1177(9)	−0.26	0.1152(29)	−0.22
Γ_W [GeV]	2.085(42)	2.0905(5)	0.13	2.0905(5)	0.13	2.0919(5)	0.16	2.919(5)	0.16
Γ_Z [GeV]	2.4952(23)	2.4942(6)	−0.45	2.4940(7)	−0.51	2.4946(6)	−0.26	2.4945(7)	−0.31
m_Z [GeV]	91.1875(21)	91.1882(20)	0.34	91.2037(90)	7.72	91.1909(20)	1.63	91.2393(77)	24.66
$A^{0,b}_{\text{FB}}$	0.0992(16)	0.1031(3)	2.44	0.1033(3)	2.54	0.1036(3)	2.72	0.1037(3)	2.83
$A^{0,c}_{\text{FB}}$	0.0707(35)	0.0737(3)	0.85	0.0737(3)	0.85	0.0740(3)	0.95	0.07404(25)	0.95
$A^{0,\ell}_{\text{FB}}$	0.0171(10)	0.01623(10)	−0.87	0.01622(10)	−0.88	0.01637(10)	−0.73	0.01636(10)	−0.74
A_b	0.923(20)	0.93462(4)	0.58	0.93462(4)	0.58	0.93464(4)	0.58	0.93464(4)	0.58
A_c	0.670(27)	0.6679(2)	−0.08	0.6679(2)	−0.08	0.6682(2)	−0.07	0.6682(2)	−0.07
A_ℓ(SLD)	0.1513(21)	0.1471(5)	−2.00	0.1469(5)	−2.10	0.1478(5)	−1.70	0.1476(5)	−1.78
A_ℓ(LEP)	0.1465(33)	0.1471(5)	0.18	0.1469(5)	0.12	0.1478(5)	0.37	0.1476(5)	0.32
R^0_b	0.21629(66)	0.21583(10)	−0.69	0.21582(10)	−0.71	0.21580(10)	−0.74	0.21579(10)	−0.76
R^0_c	0.1721(30)	0.17222(6)	0.04	0.17222(6)	0.04	0.17223(6)	0.04	0.17223(6)	0.04
R^0_ℓ	20.767(25)	20.735(8)	−1.28	20.732(8)	−1.40	20.733(8)	−1.35	20.730(8)	−1.48
σ^0_h [nb]	41.540(37)	41.491(8)	−1.34	41.489(8)	−1.39	41.490(8)	−1.35	41.488(8)	−1.39
$\sin^2\theta^\ell_{\text{eff}}(Q_{FB})$	0.2324(12)	0.23151(6)	−0.74	0.23151(6)	−0.74	0.23143(6)	−0.81	0.23143(6)	−0.81
$\sin^2\theta^\ell_{\text{eff}}$(Teva)	0.23148(33)	0.23151(6)	0.10	0.23151(6)	0.10	0.23143(6)	−0.15	0.23143(6)	−0.15
$\overline{m}_c$ [GeV]	1.27(2)	1.27(2)	0.00	–	–	1.27(2)	0.00	–	–
$\overline{m}_b$ [GeV]	$4.18^{(3)}_{(2)}$	$4.18^{(3)}_{(2)}$	0.00	–	–	$4.18^{(3)}_{(2)}$	0.00	–	–

13 dof	PDG 2021				CDF 2022			
	Result	Correlation			Result	Correlation		
	$\chi^2_{\min} = 15.42$	S	T	U	$\chi^2_{\min} = 15.44$	S	T	U
S	0.06 ± 0.10	1.00	0.90	−0.57	0.06 ± 0.10	1.00	0.90	−0.59
T	0.11 ± 0.12		1.00	−0.82	0.11 ± 0.12		1.00	−0.85
U	-0.02 ± 0.09			1.00	0.14 ± 0.09			1.00

C-T Lu, L Wu, Y-C Wu, B. Zhu, arXiv: 2204.03796

2. Beyond SM Higgs explanation

Singlet: loop contribution; Doublet: loop contribution (S Lee, K cheung, J. Kim, C-T Lu, arXiv: 2204.10338…); Triplet: tree and loop contributions (Y Cheng, X-G He, Z-L Huang, M-W Li, arXiv: 2204.05031, S. Kanemura, K. Yagyu, arXiv:2204. 07511, P. Perez, P Patel, arXiv: 2204.07144, J. Heeck, arXiv: 2204.10274 …) .

Tree level contributions

$$\rho - 1 = \frac{1}{1-\hat{\alpha}(M_Z)T} - 1 \approx \hat{\alpha}(M_Z)T$$

$$\rho = \frac{m_w^2}{m_z^2 \cos^2\theta_w} = \frac{\sum_i v_i^2 (I_i(I_i+1) - Y_i^2)}{2\sum_i Y_i^2 v_i^2}$$

Singlet: No tree level modification to $\rho=1$. Doublet: No tree level modification to $\rho=1$.

Type-II seesaw triplet Δ with Y=1: $\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$ $\rho = 1 - 2v_\Delta^2/(v^2+v_\Delta^2)$

Triplet with Σ with Y=0: $\Sigma = \frac{1}{2}\begin{pmatrix} \Sigma^0 & \sqrt{2}\Sigma^+ \\ \sqrt{2}\Sigma^- & -\Sigma^0 \end{pmatrix}$ $\rho = 1 + 4v_\Sigma^2/v^2$

If only W mass is changed from SM value to the CDF II value,
$\rho - 1 = 0.0019$ much larger than SM fits: $\rho - 1 = 0.00038$.

In Type-II Seesaw model contribution from Δ in the wrong sign!

Δ responsible for neutrino mass: $L_{Yukawa} = Y_{ij}\overline{l^c}_i L l_j \Delta$

$M_\nu = 2Y\langle\Delta_0\rangle = \sqrt{2}(Y_{ij}v_\Delta) = (m_{ij})$ $U^T M\nu U = D(m_1,m_2,m_3)$

$$\left|(Y^\dagger Y)_{ji}\right| = \frac{1}{2v_\Delta^2}\left|(m^\dagger m)_{ji}\right| = \frac{1}{2v_\Delta^2}\left|U_{j2}U_{i2}^*\Delta m_{21}^2 + U_{j3}U_{i3}^*\Delta m_{31}^2\right|$$

Combining Br(μ -> e γ) < 3.3x10-8, $v_\Delta > (6.25 - 8.39)\mathrm{eV}\left(\frac{100\mathrm{GeV}}{m_\Delta}\right)$

Only when m_Δ is less than smaller than 0.6 MeV, v_Δ needs to be larger than 1 GeV or so to have a significant effect on ρ.

One can choose small enough v_Δ to make Type-II seesaw model consistent with data, but cannot explain W mass anomaly.

Contribution from Σ can reach central value with v_Σ = 5.4 GeV to solve the W mass anomaly.   arXiv: 2204.05031

Loop contributions

L. Lavoura and L-F Li, Phys. Rev. D49(1994)1409

$$S = -\frac{Y}{3\pi}\sum_{I_3=-J}^{J} I_3 \ln\frac{m_{I_3}^2}{\mu^2} - \frac{2}{\pi}\sum_{I_3=-J}^{J}(I_3 c_W^2 - Y s_W^2)^2 \xi\left(\frac{m_{I_3}^2}{m_Z^2}, \frac{m_{I_3}^2}{m_Z^2}\right) ,$$

$$T = \frac{1}{16\pi c_W^2 s_W^2}\sum_{I_3=-J}^{J}(J^2 - I_3^2 + J + I_3)\frac{\eta\left(m_{I_3}^2, m_{I_3-1}^2\right)}{m_Z^2} ,$$

$$U = \frac{1}{6\pi}\sum_{I_3=-J}^{J}(J^2 - 3I_3^2 + J)\ln\frac{m_{I_3}^2}{\mu^2}$$

$$+\frac{1}{\pi}\sum_{I_3=-J}^{J}\left[2(I_3 c_W^2 - Y s_W^2)^2 \xi\left(\frac{m_{I_3}^2}{m_Z^2}, \frac{m_{I_3}^2}{m_Z^2}\right) - (J^2 - I_3^2 + J + I_3)\xi\left(\frac{m_{I_3}^2}{m_W^2}, \frac{m_{I_3-1}^2}{m_W^2}\right)\right] .$$

the functions are defined as

$$\xi(x,y) = \frac{4}{9} - \frac{5}{12}(x+y) + \frac{1}{6}(x-y)^2 + \frac{1}{4}\left[x^2 - y^2 - \frac{1}{3}(x-y)^3 - \frac{x^2+y^2}{x-y}\right]\ln\frac{x}{y} - \frac{1}{12}d(x,y)f(x,y) ,$$

$$\eta(x,y) = x + y - \frac{2xy}{x-y}\ln\frac{x}{y} ,$$

$$d(x,y) = -1 + 2(x+y) - (x-y)^2$$

$$f(x,y) = \begin{cases} -2\sqrt{d(x,y)}\left[\arctan\frac{x-y+1}{\sqrt{d(x,y)}} - \arctan\frac{x-y-1}{\sqrt{d(x,y)}}\right] , & d(x,y) > 0 \\ \sqrt{-d(x,y)}\ln\frac{x+y-1+\sqrt{-d(x,y)}}{x+y-1-\sqrt{-d(x,y)}} , & d(x,y) \le 0 \end{cases}$$

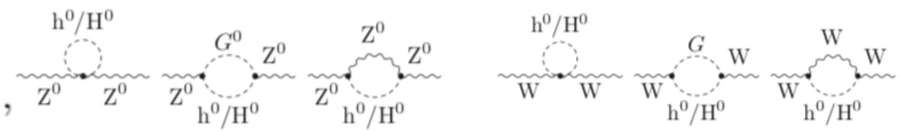
Singlet contribution

D Lopez-Val, T Robens arXiv: 1406.1043

$$V(\phi, S) = -\mu_1^2(\phi^+\phi) - \mu_2^2 S^2 + \lambda_1(\phi^+\phi)^2 + \lambda_2 S^4 + \lambda_3(\phi^+\phi)S^2$$

ϕ-SM doublet, S-singlet when both develop vev, the neutral real parts will mixing. Indicate m_h and m_H the eigen-masses, and θ mixing angle,

$$\frac{\alpha S}{4s_W^2 c_W^2} = \frac{\alpha s_\theta^2}{96\pi c_W^2 S_W^2}\left[\ln\frac{m_H^2}{m_h^2} + \tilde{G}(m_H^2, m_Z^2) - \tilde{G}(m_h^2, m_Z^2)\right],$$



$$\alpha T = \frac{3\alpha s_\theta^2}{16\pi s_W^2}\left[\frac{1}{c_W^2}\left(\frac{m_h^2}{m_h^2 - m_Z^2}\ln\frac{m_h^2}{m_Z^2} - \frac{m_H^2}{m_H^2 - m_Z^2}\ln\frac{m_H^2}{m_Z^2}\right) - \left(\frac{m_h^2}{m_h^2 - m_W^2}\ln\frac{m_h^2}{m_W^2} - \frac{m_H^2}{m_H^2 - m_W^2}\ln\frac{m_H^2}{m_W^2}\right)\right],$$

$$\frac{\alpha U}{4s_W^2} = \frac{\alpha s_\theta^2}{96\pi s_W^2}\left[(\tilde{G}(m_h^2, m_Z^2) - \tilde{G}(m_H^2, m_Z^2)) - (\tilde{G}(m_h^2, m_W^2) - \tilde{G}(m_H^2, m_W^2))\right].$$

$$\tilde{G}(x,y) = -\frac{79}{3} + 9\frac{x}{y} - 2\frac{x^2}{y^2} + \left(-10 + 18\frac{x}{y} - 6\frac{x^2}{y^2} + \frac{x^3}{y^3} - 9\frac{x+y}{x-y}\right)\ln\frac{x}{y} + \left(12 - 4\frac{x}{y} + \frac{x^2}{y^2}\right)\frac{K(x, x^2 - 4xy)}{y},$$

$$K(t,r) = \begin{cases} \sqrt{r}\ln\left|\frac{t-\sqrt{r}}{t+\sqrt{r}}\right|, & r \geq 0 \\ 2\sqrt{-r}\arctan\frac{\sqrt{-r}}{t}, & r < 0 \end{cases}$$

Identify h the SM Higgs, H needs to be below 10 GeV and sinθ >0.6 to explain the W mass anomaly which modifies Higgs properties and is ruled out!

Loop contribution from Type-II seesaw triplet

S. Mandal et al. arXiv:2203.06362

$$V(\phi,\Delta)=-m_\phi^2(\phi^+\phi)+\frac{\lambda}{4}(\phi^+\phi)^2+\lambda_1(\phi^+\phi)Tr(\Delta^+\Delta)+\lambda_2(Tr(\Delta^+\Delta))^2$$
$$+\lambda_3 Tr(\Delta^+\Delta\Delta^+\Delta)+\lambda_4\phi^+\Delta\Delta^+\phi+\tilde{M}_\Delta^2 Tr(\Delta^+\Delta)+(\mu\phi^T i\tau_2\Delta^+\phi+h.c.)$$

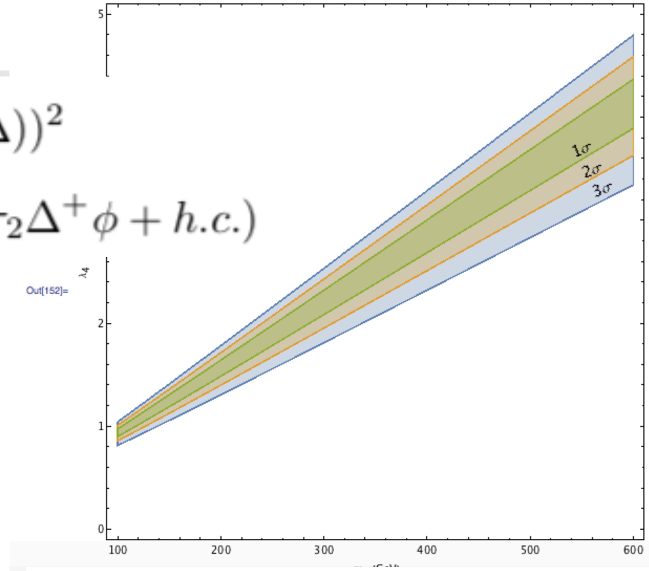
$$m_\Delta^2\approx m_{H^0}^2\approx m_{A^0}^2\approx m_{H^+}^2+\frac{\lambda_4}{4}v^2\approx m_{H^{++}}^2+\frac{\lambda_4}{2}v^2$$

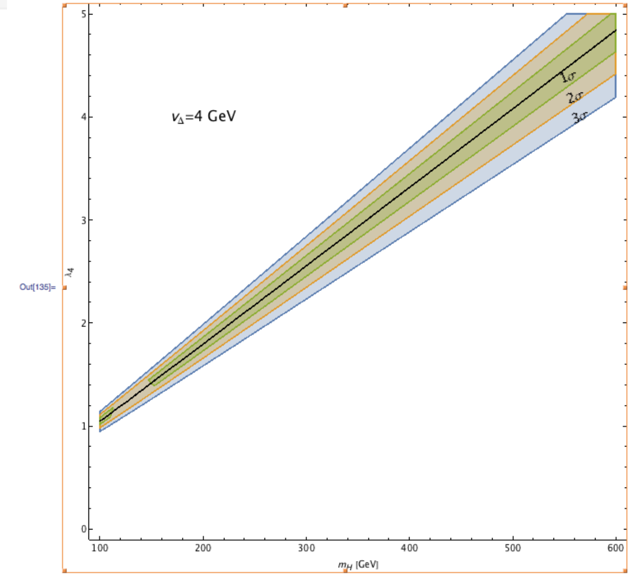
$$S=\frac{\lambda_4}{6\pi}\frac{v^2}{m_{H^+}^2}-\frac{m_Z^2}{30\pi m_{H^+}^2}(2-4s_W^2+5s_W^4)\,,$$

$$T=\frac{1}{192\pi^2\alpha_{em}}\frac{\lambda_4^2 v^2}{m_{H^+}^2}\,,$$

$$U=\frac{m_Z^2}{30\pi m_{H^+}^2}(-2s_W^2+5s_W^4)\,.$$

$$\Delta m_W^2=m_Z^2c_W^2\left(-\frac{\alpha S}{2(c_W^2-s_W^2)}+\frac{c_W^2\alpha T}{(c_W^2-s_W^2)}+\frac{\alpha U}{4s_W^2}\right)$$
$$=m_Z^2c_W^2\left[\frac{\alpha_{em}(2+s_W^2)}{120\pi(c_W^2-s_W^2)}\frac{m_Z^2}{m_{H^+}^2}+\frac{(c_W^2\lambda_4^2-16\pi\alpha_{em}\lambda_4)}{192\pi^2(c_W^2-s_W^2)}\frac{v^2}{m_{H^+}^2}\right]\,.$$





The tree level vev contribution cannot explain the W mass anomaly. The loop level contribution can explain the anomaly!

Loop contribution from Y=0 triple Σ

$$V(\phi,\Sigma)=-m_\phi^2(\phi^+\phi)+\lambda_0(\phi^+\phi)^2-M_\Sigma^2 Tr(\Sigma^2)+\lambda_1 Tr(\Sigma^4)+\lambda_2(Tr(\Sigma^2))^2$$
$$+\alpha(\phi^+\phi)Tr(\Sigma^2)+\beta\phi^+\Sigma^2\phi+a_1\phi^+\Sigma\phi\,.$$

$$S=-\frac{2}{\pi}\sum_{I_3=-1}^{1}(I_3c_W^2)^2\xi\left(\frac{m_{I_3}^2}{m_Z^2},\frac{m_{I_3}^2}{m_Z^2}\right)=-\frac{4c_W^4}{\pi}\xi\left(\frac{m_{H+}^2}{m_Z^2},\frac{m_{H+}^2}{m_Z^2}\right)\approx-\frac{c_W^4}{15\pi}\frac{m_Z^2}{m_{H+}^2}\,,$$

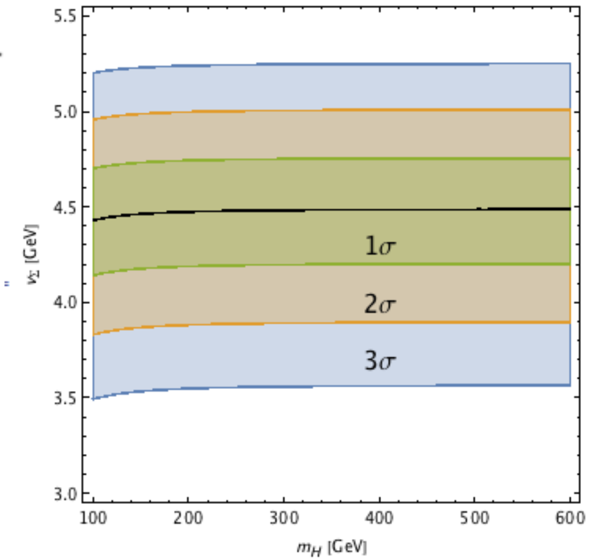
$$T=\frac{1}{16\pi c_W^2 s_W^2}\sum_{I_3=-1}^{1}(2-I_3^2+I_3)\eta\left(\frac{m_{I_3}^2}{m_Z^2},\frac{m_{I_3-1}^2}{m_Z^2}\right)=\frac{1}{12\pi c_W^2 s_W^2 m_Z^2}\frac{(m_{H^+}^2-m_{H^0}^2)^2}{m_{H^+}^2}\approx 0\,,$$

$$U=\frac{1}{6\pi}\sum_{I_3=-1}^{1}(2-3I_3^2)\ln\frac{m_{I_3}^2}{\mu^2}+\frac{1}{\pi}\sum_{I_3=-1}^{1}\left[2(I_3c_W^2)^2\xi\left(\frac{m_{I_3}^2}{m_Z^2},\frac{m_{I_3}^2}{m_Z^2}\right)-(2-I_3^2+I_3)\xi\left(\frac{m_{I_3}^2}{m_W^2},\frac{m_{I_3-1}^2}{m_W^2}\right)\right]$$
$$=\frac{1}{6\pi}\ln\frac{m_{H^0}^4}{m_{H^+}^4}+\frac{4}{\pi}\left[c_W^4\xi\left(\frac{m_{H^+}^2}{m_Z^2},\frac{m_{H^+}^2}{m_Z^2}\right)-\xi\left(\frac{m_{H^+}^2}{m_W^2},\frac{m_{H^0}^2}{m_W^2}\right)\right]\approx-\frac{c_W^2s_W^2}{15\pi}\frac{m_Z^2}{m_{H^+}^2}\,.$$

Loop: $\Delta m_W^2=m_Z^2c_W^2\frac{\alpha}{60\pi}\frac{c_W^2}{c_W^2-s_W^2}\frac{m_Z^2}{m_{H^+}^2}\,.$

Tree: $\Delta m_W^2=m_Z^2c_W^2\frac{c_W^2}{c_W^2-s_W^2}\frac{4v_\Sigma^2}{v^2}$

Both tree and loop can have significant contributions to W mass. Can explain W mass anomaly!



3. Dark photon explanation

arXiv: 2004.10156

A theory of $U(1)_{em} x U(1)_X$ gauge group

$$L = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + A_\mu j^\mu_{em} - \frac{1}{4} X_{\mu\nu}X^{\mu\nu} + X_\mu j^\mu_X$$

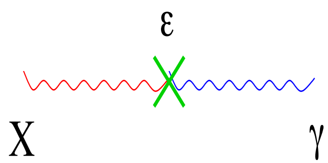
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$$

A is the usual photon field and X is a new gauge field X and $j^\mu_{em,X}$ currents
X may have or not have a finite mass $m^2_A X^\mu X_\mu/2$

If j^μ_X does not involve with SM particle, X is a photon like particle which cannot be probed using laboratory probes – a **Dark Photon**

$$\epsilon X_{\mu\nu}F^{\mu\nu}$$



Possible to add the following renormalizable and gauge invariant term.

This kinetic mixing term mixes photon and Dark Photon making dark photon to interact with SM particle, Dark Photon enlightened!

Holdom 1986, Foot and He 1991,….

Abelian kinetic mixing

Work with $SU(3)_C x SU(2)_L x U(1)_Y x U(1)_X$

Kinetic mixing can happen between $U(1)_Y$ and $U(1)_X$

$$\mathcal{L} = -\frac{1}{4}\tilde{X}_{\mu\nu}\tilde{X}^{\mu\nu} - \frac{\sigma}{2}\tilde{X}_{\mu\nu}\tilde{Y}^{\mu\nu} - \frac{1}{4}\tilde{Y}_{\mu\nu}\tilde{Y}^{\mu\nu} + j_Y^{\mu}\tilde{Y}_{\mu} + j_X^{\mu}\tilde{X}_{\mu}$$

Need to re-write in the canonical form to identify physics gauge bosons. (mixing term removed!)

This may generate dark photon to interact with SM J^{μ}_{Y}

Work with SM photon and dark photon

$$\tilde{Y}_\mu = \tilde{c}_W \tilde{A}_\mu - \tilde{s}_W \tilde{Z}_\mu \ , \ \ \tilde{W}^3_\mu = \tilde{s}_W \tilde{A}_\mu + \tilde{c}_W \tilde{Z}_\mu$$

$$\begin{aligned}\mathcal{L} = \ & -\frac{1}{4}\tilde{X}_{\mu\nu}\tilde{X}^{\mu\nu} - \frac{1}{4}\tilde{A}_{\mu\nu}\tilde{A}^{\mu\nu} - \frac{1}{4}\tilde{Z}_{\mu\nu}\tilde{Z}^{\mu\nu} - \frac{1}{2}\sigma\tilde{c}_W\tilde{X}_{\mu\nu}\tilde{A}^{\mu\nu} + \frac{1}{2}\sigma\tilde{s}_W\tilde{X}_{\mu\nu}\tilde{Z}^{\mu\nu} \\ & + j^\mu_{em}\tilde{A}_\mu + j^\mu_Z\tilde{Z}_\mu + j^\mu_X\tilde{X}_\mu + \frac{1}{2}m_Z^2\tilde{Z}_\mu\tilde{Z}^\mu \ ,\end{aligned}$$

$$j^\mu_{em} = -\sum_f \tilde{e}Q_f\bar{f}\gamma^\mu f \ , \ \ j^\mu_Z = -\frac{\tilde{e}}{2\tilde{s}_W\tilde{c}_W}\bar{f}\gamma^\mu(g^f_V - g^f_A\gamma_5)f \ ,$$

$$g^f_V = I^f_3 - 2Q_f\tilde{s}^2_W \ , \quad g^f_A = I^f_3 \ .$$

Make the above Lagrangian the canonical form by a redefinition of fields

$$\begin{pmatrix} \tilde{A} \\ \tilde{Z} \\ \tilde{X} \end{pmatrix} = \begin{pmatrix} 1 & \frac{-\sigma^2\tilde{s}_W\tilde{c}_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2\tilde{c}^2_W}} & \frac{-\sigma\tilde{c}_W}{\sqrt{1-\sigma^2\tilde{c}^2_W}} \\ 0 & \frac{\sqrt{1-\sigma^2\tilde{c}^2_W}}{\sqrt{1-\sigma^2}} & 0 \\ 0 & \frac{\sigma\tilde{s}_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2\tilde{c}^2_W}} & \frac{1}{\sqrt{1-\sigma^2\tilde{c}^2_W}} \end{pmatrix} \begin{pmatrix} \tilde{A}' \\ \tilde{Z}' \\ \tilde{X}' \end{pmatrix} \ ,$$

$$
\begin{aligned}
\mathcal{L} = \; & -\frac{1}{4}\tilde{X}'_{\mu\nu}\tilde{X}'^{\mu\nu} - \frac{1}{4}\tilde{A}'_{\mu\nu}\tilde{A}'^{\mu\nu} - \frac{1}{4}\tilde{Z}'_{\mu\nu}\tilde{Z}'^{\mu\nu} \\
& + j^{\mu}_{em}\left(\tilde{A}'_{\mu} - \frac{\sigma^2 \tilde{s}_W \tilde{c}_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2\tilde{c}_W^2}}\tilde{Z}'_{\mu} - \frac{\sigma \tilde{c}_W}{\sqrt{1-\sigma^2\tilde{c}_W^2}}\tilde{X}'_{\mu}\right) \\
& + j^{\mu}_{Z}\left(\frac{\sqrt{1-\sigma^2\tilde{c}_W^2}}{\sqrt{1-\sigma^2}}\tilde{Z}'_{\mu}\right) + j^{\mu}_{X}\left(\frac{\sigma \tilde{s}_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2\tilde{c}_W^2}}\tilde{Z}'_{\mu} + \frac{1}{\sqrt{1-\sigma^2\tilde{c}_W^2}}\tilde{X}'_{\mu}\right) \\
& + \frac{1}{2}m_Z^2\frac{1-\sigma^2\tilde{c}_W^2}{1-\sigma^2}\tilde{Z}'_{\mu}\tilde{Z}'^{\mu} + \frac{1}{2}m_X^2\left(\frac{\sigma \tilde{s}_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2\tilde{c}_W^2}}\tilde{Z}'_{\mu} + \frac{1}{\sqrt{1-\sigma^2\tilde{c}_W^2}}\tilde{X}'_{\mu}\right)^2 .
\end{aligned}
$$

Introduce a singlet S with vev to give the original X a mass m_X

$$
\begin{pmatrix}
\frac{m_Z^2(1-\sigma^2\tilde{c}_W^2)^2 + m_X^2\sigma^2\tilde{s}_W^2}{(1-\sigma^2)(1-\sigma^2\tilde{c}_W^2)} & \frac{m_X^2\sigma\tilde{s}_W}{\sqrt{1-\sigma^2}(1-\sigma^2\tilde{c}_W^2)} \\
\frac{m_X^2\sigma\tilde{s}_W}{\sqrt{1-\sigma^2}(1-\sigma^2\tilde{c}_W^2)} & \frac{m_X^2}{1-\sigma^2\tilde{c}_W^2}
\end{pmatrix}
\qquad
\begin{pmatrix} Z \\ X \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix}\begin{pmatrix} \tilde{Z}' \\ \tilde{X}' \end{pmatrix}
$$

$$
\tan(2\theta) = \frac{2m_X^2\sigma\tilde{s}_W\sqrt{1-\sigma^2}}{m_Z^2(1-\sigma^2\tilde{c}_W^2)^2 - m_X^2[1-\sigma^2(1+\tilde{s}_W^2)]}
$$

$$\bar{m}_Z^2 = \frac{m_Z^2(1-\sigma^2\tilde{c}_W^2)^2 + m_X^2\sigma^2\tilde{s}_W^2}{(1-\sigma^2)(1-\sigma^2\tilde{c}_W^2)} c_\theta^2 + \frac{m_X^2}{1-\sigma^2\tilde{c}_W^2} s_\theta^2 + 2 s_\theta c_\theta \frac{m_X^2 \sigma \tilde{s}_W}{\sqrt{1-\sigma^2}(1-\sigma^2\tilde{c}_W^2)} ,$$

$$\bar{m}_X^2 = \frac{m_Z^2(1-\sigma^2\tilde{c}_W^2)^2 + m_X^2\sigma^2\tilde{s}_W^2}{(1-\sigma^2)(1-\sigma^2\tilde{c}_W^2)} s_\theta^2 + \frac{m_X^2}{1-\sigma^2\tilde{c}_W^2} c_\theta^2 - 2 s_\theta c_\theta \frac{m_X^2 \sigma \tilde{s}_W}{\sqrt{1-\sigma^2}(1-\sigma^2\tilde{c}_W^2)} .$$

$$\bar{m}_Z^2 = m_Z^2(1+\tilde{z})$$

$$\begin{aligned}
\mathcal{L} = \; & -\frac{1}{4}A_{\mu\nu}A^{\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} + \frac{1}{2}\bar{m}_Z^2 Z^\mu Z_\mu \\
& + j^\mu_{em} A_\mu - j^\mu_{em}\left(\frac{\sigma^2\tilde{s}_W\tilde{c}_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2\tilde{c}_W^2}} c_\theta + \frac{\sigma\tilde{c}_W}{\sqrt{1-\sigma^2\tilde{c}_W^2}} s_\theta\right) Z_\mu + j^\mu_Z \frac{\sqrt{1-\sigma^2\tilde{c}_W^2}}{\sqrt{1-\sigma^2}} c_\theta Z_\mu \\
& + j^\mu_X \left(\frac{\sigma\tilde{s}_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2\tilde{c}_W^2}} c_\theta + \frac{1}{\sqrt{1-\sigma^2\tilde{c}_W^2}} s_\theta\right) Z_\mu \\
& - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \frac{1}{2}\bar{m}_X^2 X^\mu X_\mu + j^\mu_X \left(-\frac{\sigma\tilde{s}_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2\tilde{c}_W^2}} s_\theta + \frac{1}{\sqrt{1-\sigma^2\tilde{c}_W^2}} c_\theta\right) X_\mu . \\
& + j^\mu_{em}\left(\frac{\sigma^2\tilde{s}_W\tilde{c}_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2\tilde{c}_W^2}} s_\theta - \frac{\sigma\tilde{c}_W}{\sqrt{1-\sigma^2\tilde{c}_W^2}} c_\theta\right) X_\mu - j^\mu_Z \frac{\sqrt{1-\sigma^2\tilde{c}_W^2}}{\sqrt{1-\sigma^2}} s_\theta X_\mu . \qquad (11)
\end{aligned}$$

The above modifications can be recasted into S, T, U parameters

Follow procedure in

C-P Burgess et al., Phys. Rev. D 50(1994) 7011

$$L = \frac{1}{2}(1+z-C)m_Z^2 Z^\mu Z_\mu + (1+w-z)m_W^2 W^\mu W_\mu^\dagger$$
$$+ \left(1-\frac{A}{2}\right) j_{em}^\mu A_\mu + \left(1-\frac{C}{2}\right)(j_Z^\mu + G j_{em}^\mu) Z_\mu + \left(\left(1-\frac{B}{2}\right) j_W^\mu W_\mu^+ + h.c.\right)$$

$$j_W^\mu = -(\tilde{e}/\sqrt{2}\tilde{s}_W)\bar{f}^u \gamma^\mu L V_{KM} f^d$$

$$\alpha S = 4s_W^2 c_W^2 (A-C) - 4 s_W c_W (c_W^2 - s_W^2) G\ , \quad \alpha T = w - z\ ,$$
$$\alpha U = 4 s_W^2 (s_W^2 A - B + c_W^2 C - 2 s_W c_W G)\ .$$

Compare our L, we have B = 0, w=0, A=0, and

$$C = 2\left(1 - \frac{\sqrt{1-\sigma^2 c_W^2}}{\sqrt{1-\sigma^2}} c_\theta\right)\ , \quad G = -\frac{\sigma^2 s_W c_W}{1-\sigma^2 c_W^2} - \frac{\sigma c_W \sqrt{1-\sigma^2}}{1-\sigma^2 c_W^2}\frac{s_\theta}{c_\theta}\ , \quad z = C + \tilde{z}$$

S, T, U parameters due to dark photon interaction

To σ^2 order, we have

$$\alpha S = \frac{4 s_W^2 c_W^2 \sigma^2}{1 - m_X^2/m_Z^2}\left(1 - \frac{s_W^2}{1 - m_X^2/m_Z^2}\right) ,$$

$$\alpha T = -\sigma^2 s_W^2 \frac{m_X^2/m_Z^2}{(1 - m_X^2/m_Z^2)^2} ,$$

$$\alpha U = 4 s_W^4 c_W^2 \sigma^2 \left(-\frac{1 - 2m_X^2/m_Z^2}{(1 - m_X^2/m_Z^2)^2} + \frac{2}{1 - m_X^2/m_Z^2}\right) .$$

$$\begin{aligned}\Delta m_W^2 &= m_Z^2 c_W^2 \left(-\frac{\alpha S}{2(c_W^2 - s_W^2)} + \frac{c_W^2 \alpha T}{(c_W^2 - s_W^2)} + \frac{\alpha U}{4 s_W^2}\right) \\ &= -m_Z^2 c_W^2 \frac{m_Z^2 (1 - s_W^2)\sigma^2 s_W^2}{(m_X^2 - m_Z^2)(-1 + 2 s_W^2)} .\end{aligned}$$

$m_X > m_Z$ will help to explain the W mass anomaly.

Non-abelian kinetic mixing

Example: $SU(2)_L$ gauge boson W^a mixing with dark photon X from $U(1)_X$
To have gauge invariant kinetic mixing, needs to introduce the triplet Σ

$$\tilde{X}^{\mu\nu}\tilde{W}^a_{\mu\nu}\Sigma^a\ , \qquad \epsilon^{\mu\nu\alpha\beta}\tilde{X}_{\mu\nu}\tilde{W}^a_{\alpha\beta}\Sigma^a$$

First CP conserving, second CP violating

Not possible to have second one for abelian kinetic mixing: $\epsilon^{\mu\nu\alpha\beta}\tilde{X}_{\mu\nu}\tilde{Y}_{\alpha\beta} = 2\partial^\mu(\tilde{X}^\nu\tilde{Y}_{\mu\nu})$
When Σ has a non-zero vev v_Σ non-abelian kinetic mixing happen

$$\sqrt{2}\tilde{X}^{\mu\nu}\tilde{W}^3_{\mu\nu}v_\Sigma, \quad \sqrt{2}\epsilon^{\mu\nu\alpha\beta}\tilde{X}_{\mu\nu}\tilde{W}^3_{\alpha\beta}v_\Sigma$$

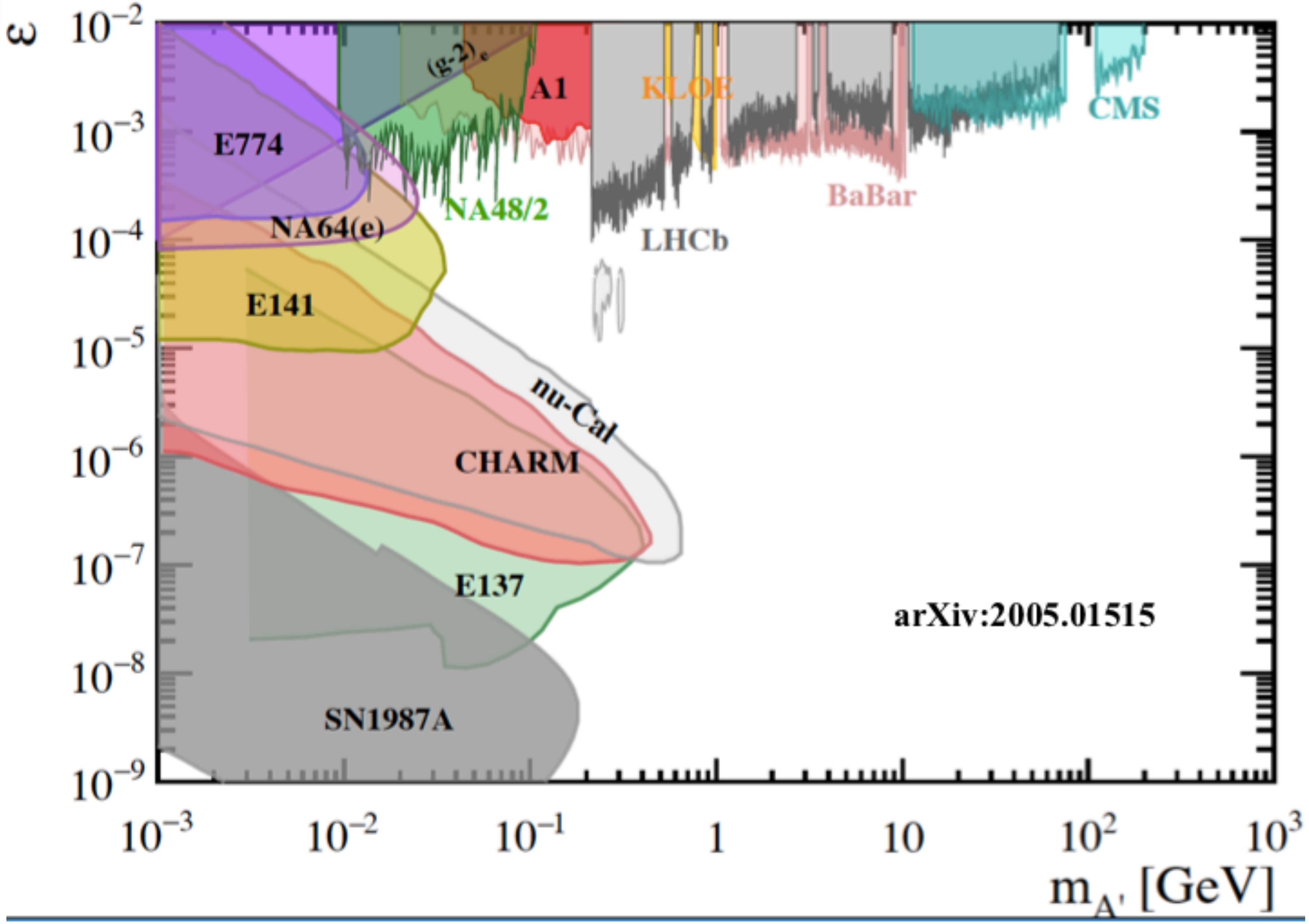
$$\tilde{W}^3_{\mu\nu} = s_W\tilde{A}_{\mu\nu} + c_W\tilde{Z}_{\mu\nu} + ig(W^-_\mu W^+_\nu - W^-_\nu W^+_\mu)$$

Can simultaneously have abelian kinetic mixing by keeping a term $-(1/2)\sigma\tilde{X}^{\mu\nu}\tilde{Y}_{\mu\nu}$

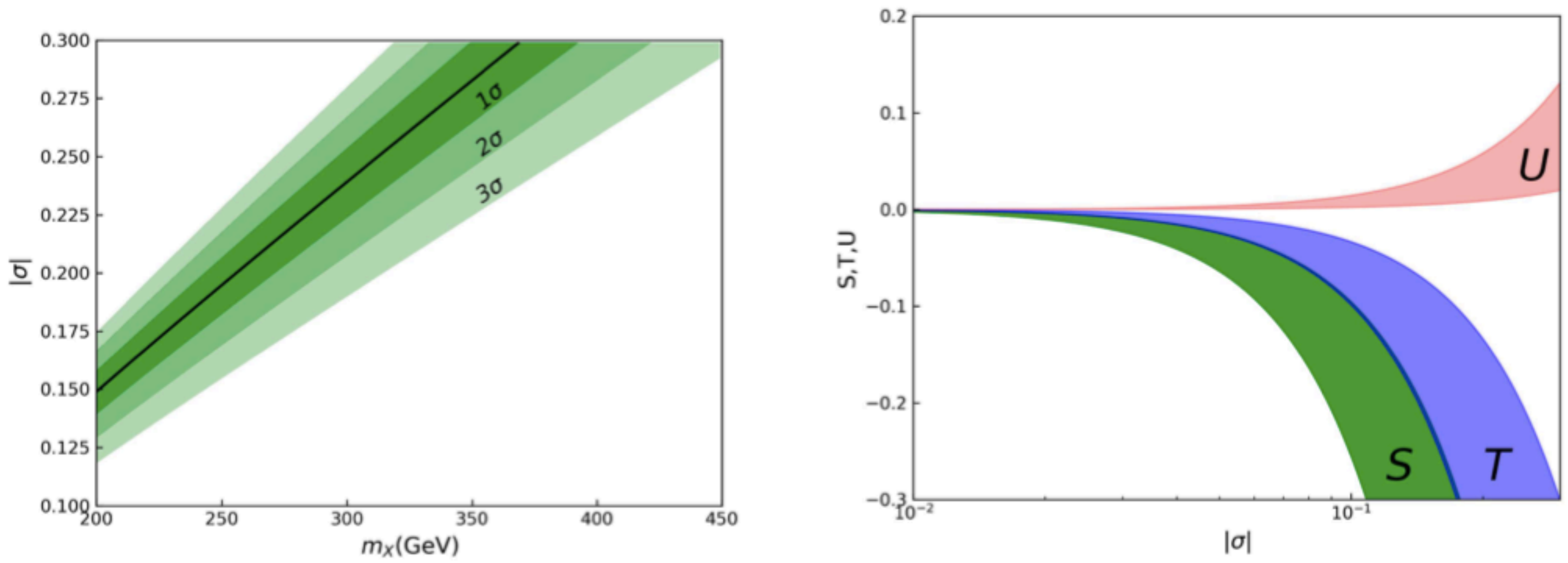
$$\begin{aligned}\Delta m^2_W &= m^2_Z c^2_W\left(-\frac{\alpha S}{2(c^2_W - s^2_W)} + \frac{c^2_W\alpha T}{(c^2_W - s^2_W)} + \frac{\alpha U}{4s^2_W}\right)\\ &= -m^2_Z c^2_W\frac{m^2_Z(1-s^2_W)\sigma^2 s^2_W}{(m^2_X - m^2_Z)(-1+2s^2_W)} + m^2_Z c^2_W\frac{c^2_W}{c^2_W - s^2_W}\frac{4v^2_\Sigma}{v^2}\ .\end{aligned}$$

Smaller contribution from non-abelian mixing contribution neglected.

Constraints on dark photon mass and mixing

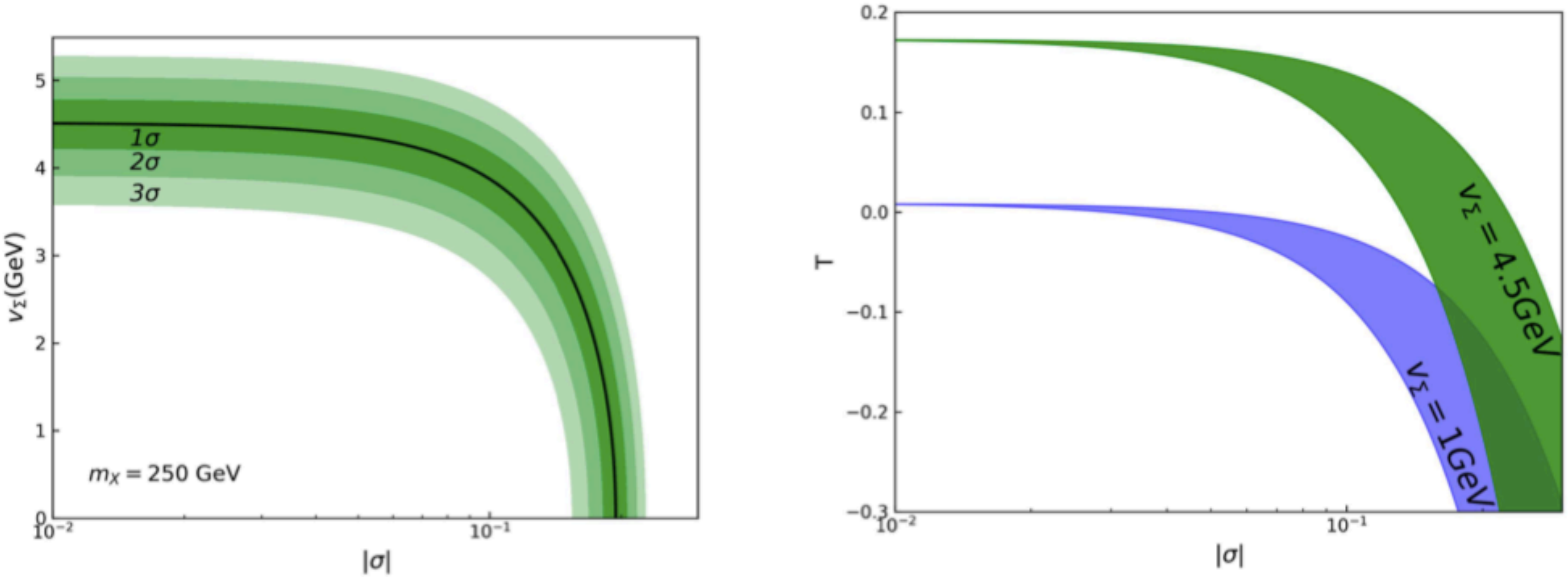


Numerical Results



(a) (b)

FIG. 1: (a) The CDF allowed regions in $m_X - |\sigma|$ plane. The allowed parameter space is shown in black line for central value, the 1σ, 2σ and 3σ ranges are also shown. (b) The S, T and U parameters as functions of $|\sigma|$ for m_X in the range of $200 - 300$ GeV. The size of observables decrease when m_X increases.



(a) (b)

FIG. 2: (a) The CDF allowed regions in $|\sigma| - v_\Sigma$ plane for some given values for m_X. (b) The T parameter for two different values of v_Σ.

4. Conclusions

CDF II data for W mass is larger than SM prediction at 7σ level. There is an anomaly!

This anomaly can be explained by some beyond SM physics.

An additional singlet Higgs does not work.

Non-trivial $SU(2)_L$ Higgs can explain the anomaly:

A Type-II seesaw model triplet needs to have a very small vev to avoid problem, its loop level contribution can explain the anomaly.

An additional Y=0 triplet can have sizable tree and loop contributions.

Dark photon kinetic mixing effects can be casted into S, T, U parameters, and can explain the anomaly.

Thank you for your attentions!