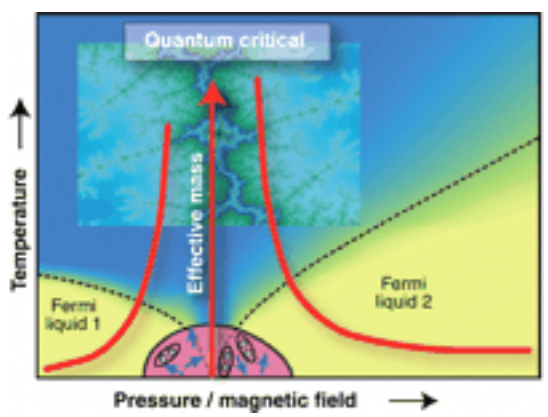
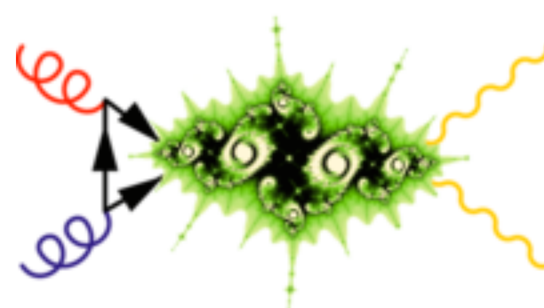
NCTS International Joint Workshop

October 14, 2021

Continuum Dark Matter





KOREA UNIVERSITY 1905

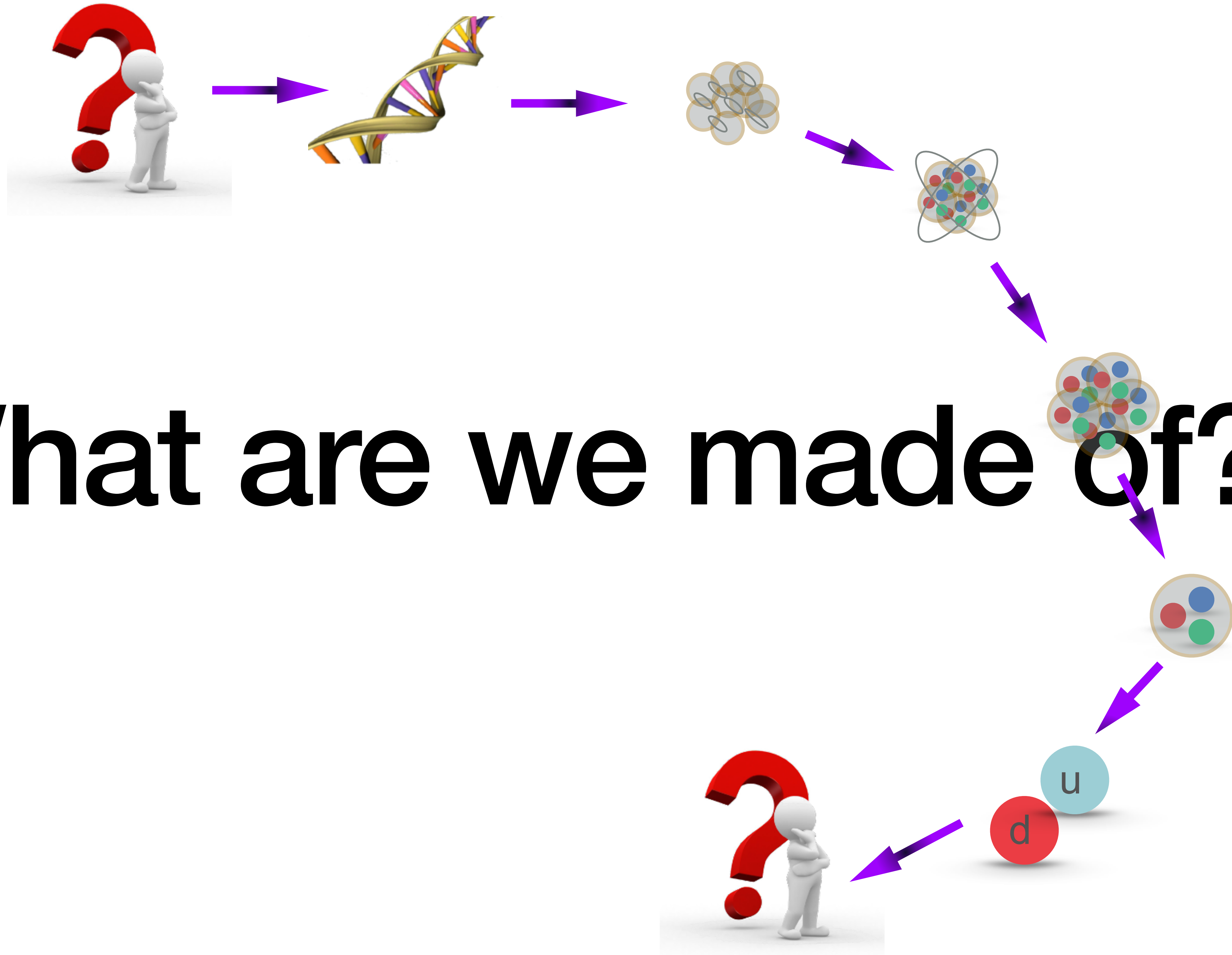
Seung J. Lee

With Csaki, Hong, Kurup, Perelstein, Xue; 2105.14023 [hep-ph]
With Csaki, Hong, Kurup, Perelstein, Xue; 2105.07035 [hep-ph]

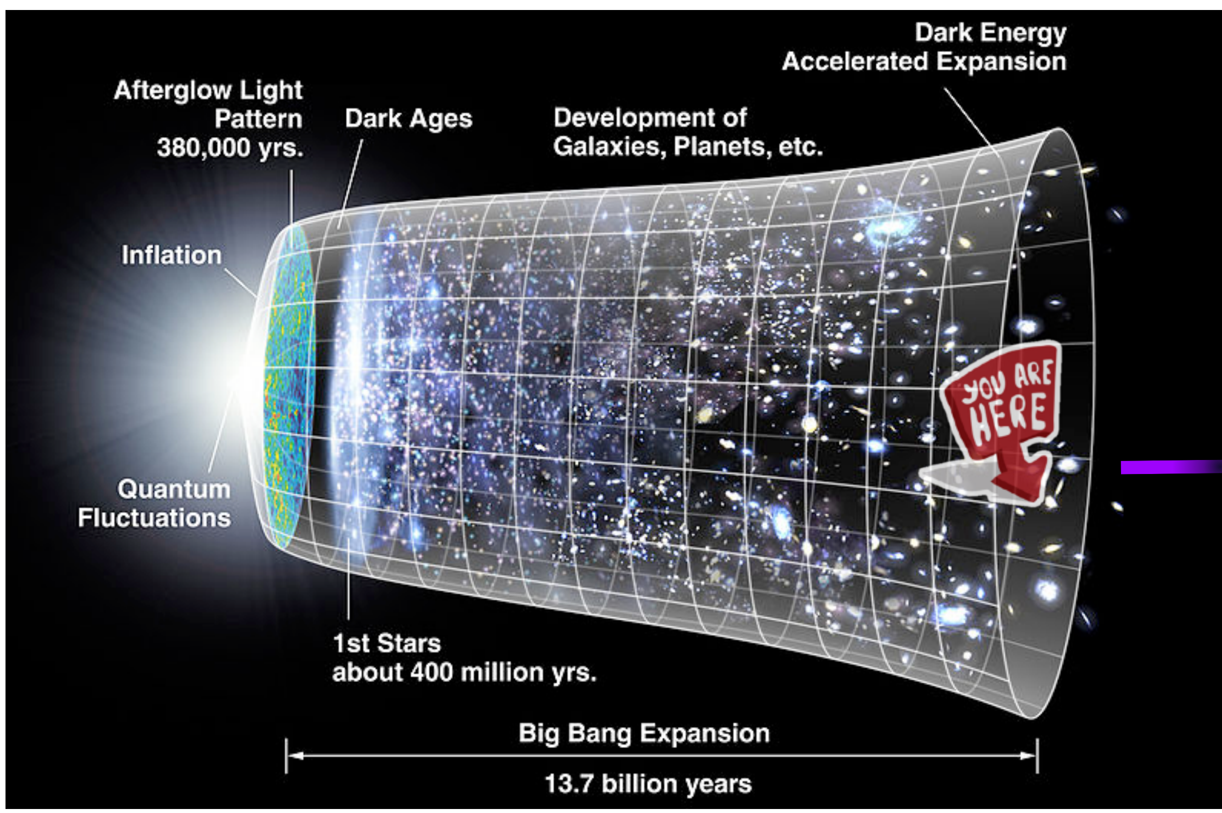
Outline

- Introduction
- Gapped Continuum
- Gapped Continuum QFT
- Equilibrium and Non-equilibrium Thermodynamics
- Z-portal Model for Gapped Continuum DM
- Summary

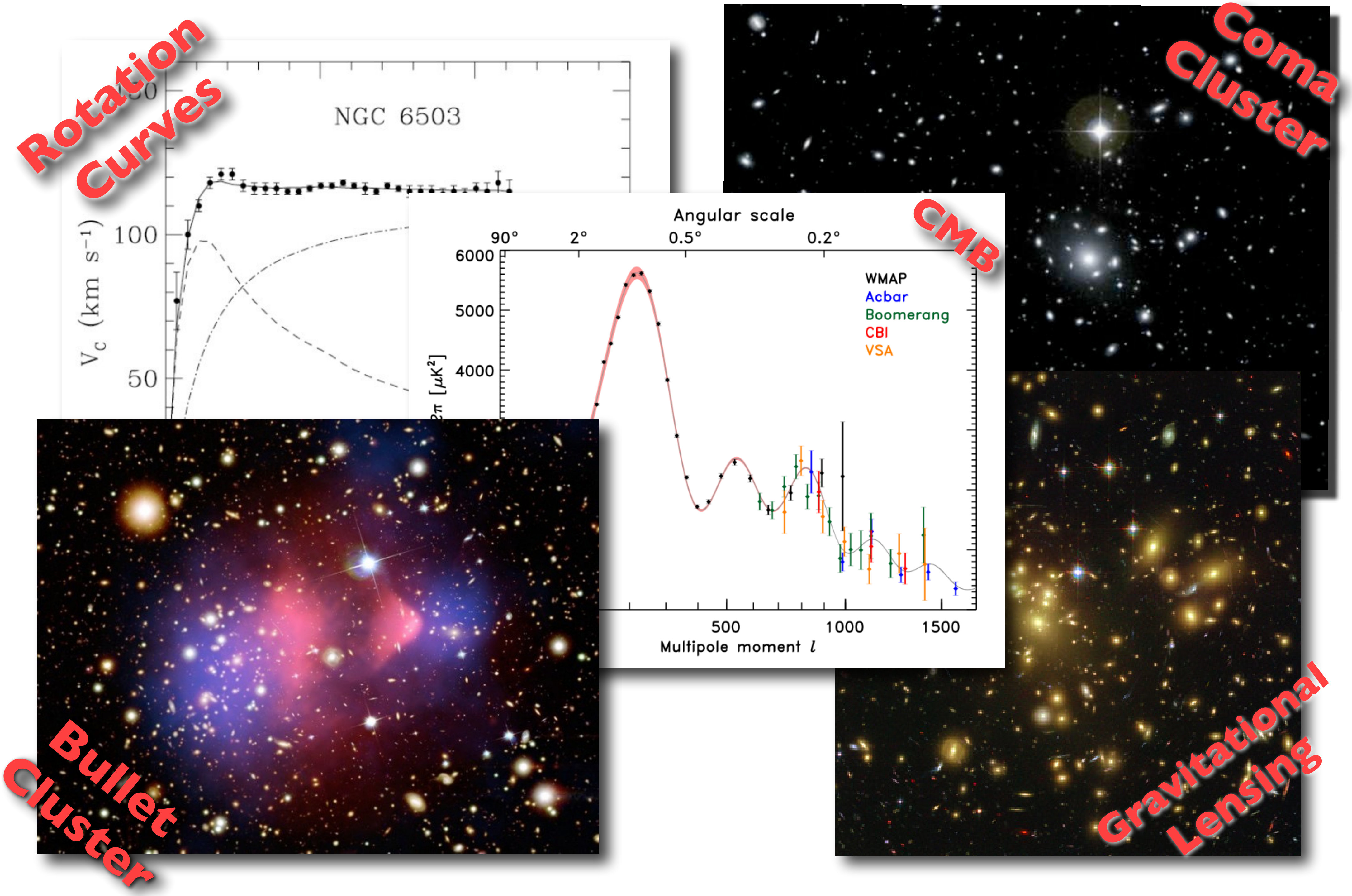
What are we made of?

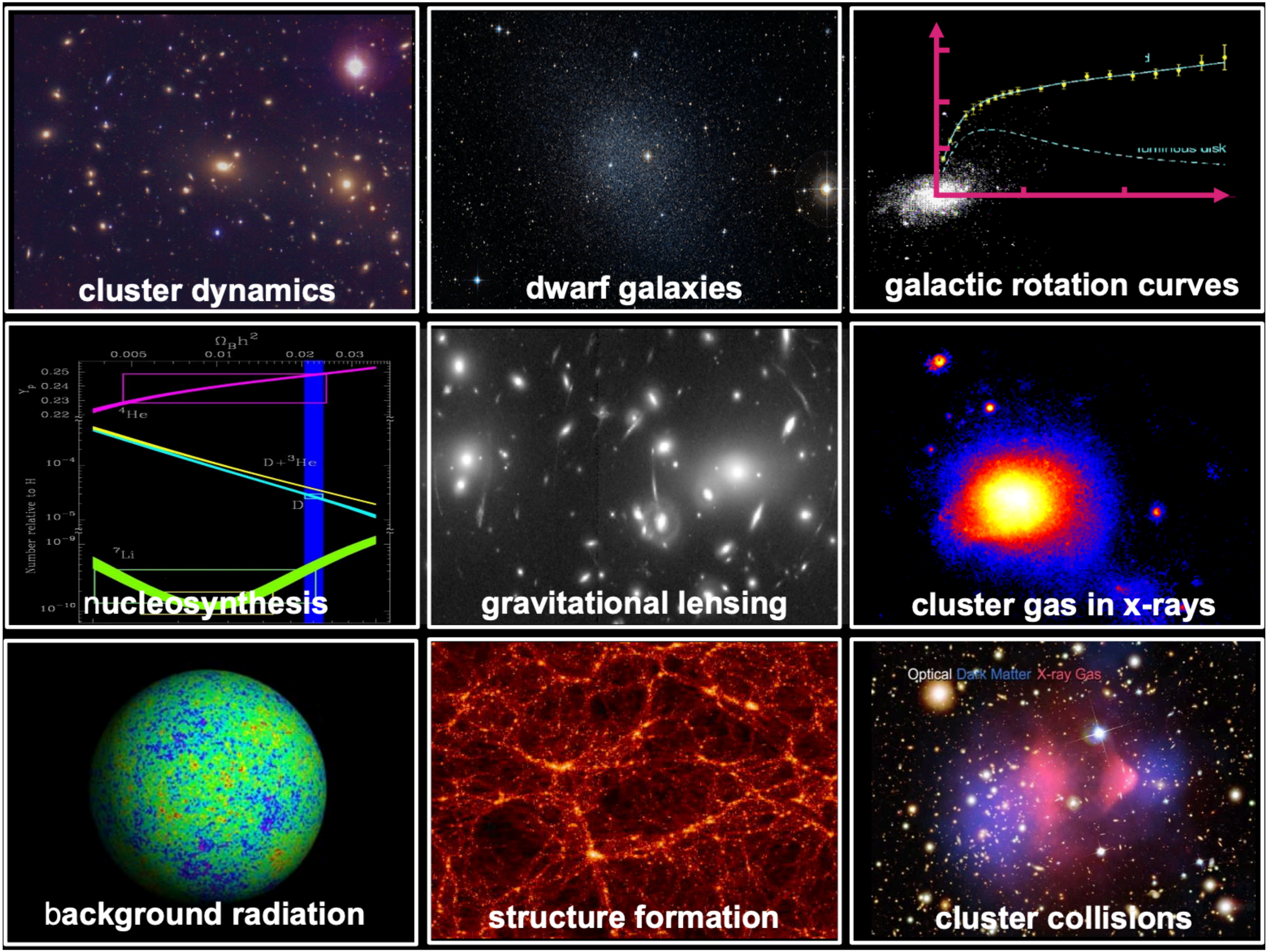


How did we get here?



EVIDENCE FOR DARK MATTER





cluster dynamics
dwarf galaxies
galactic rotation curves
$\Omega_B h^2$
0.005
0.01
0.02
0.03
Y_p
0.25
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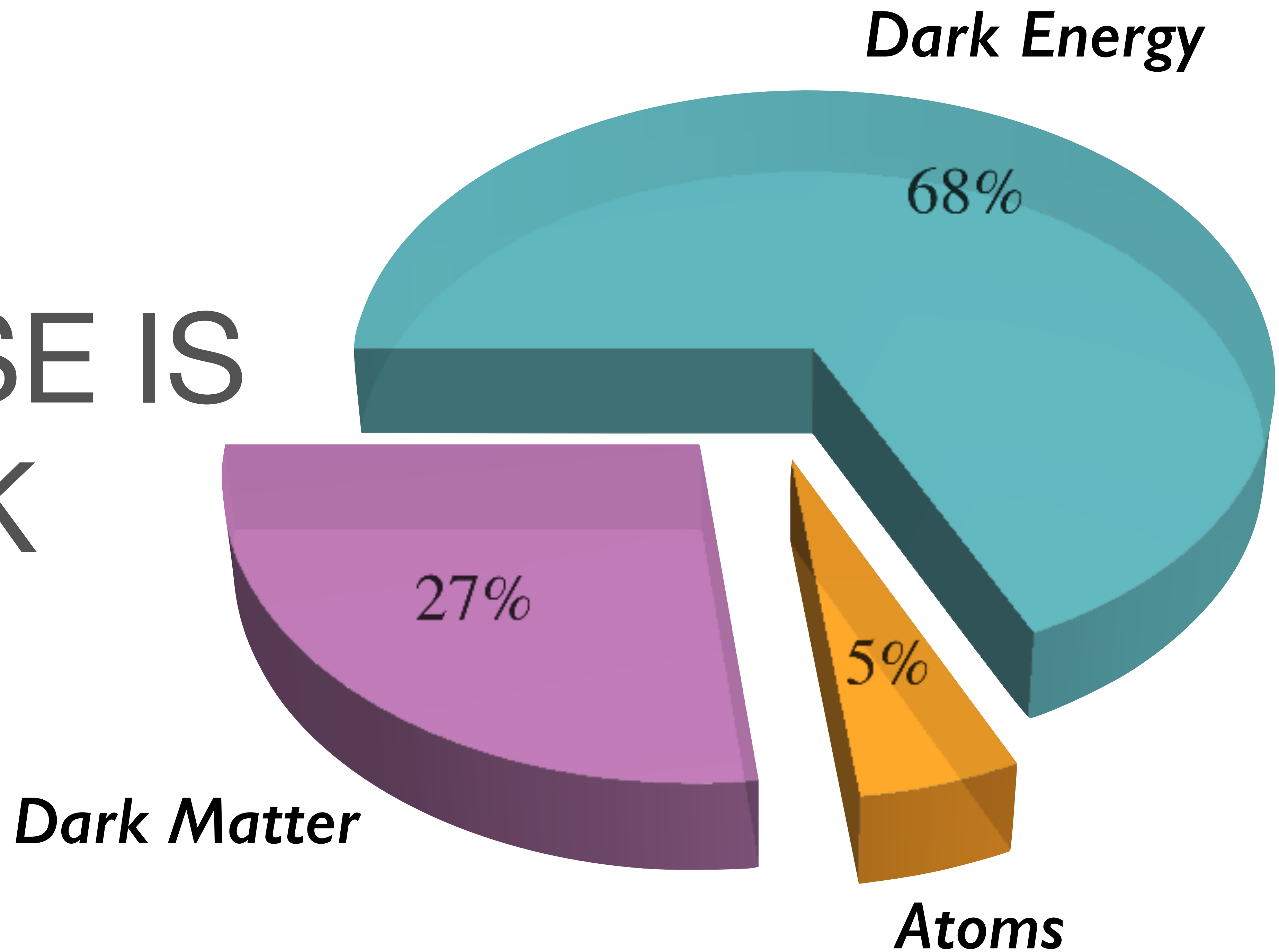
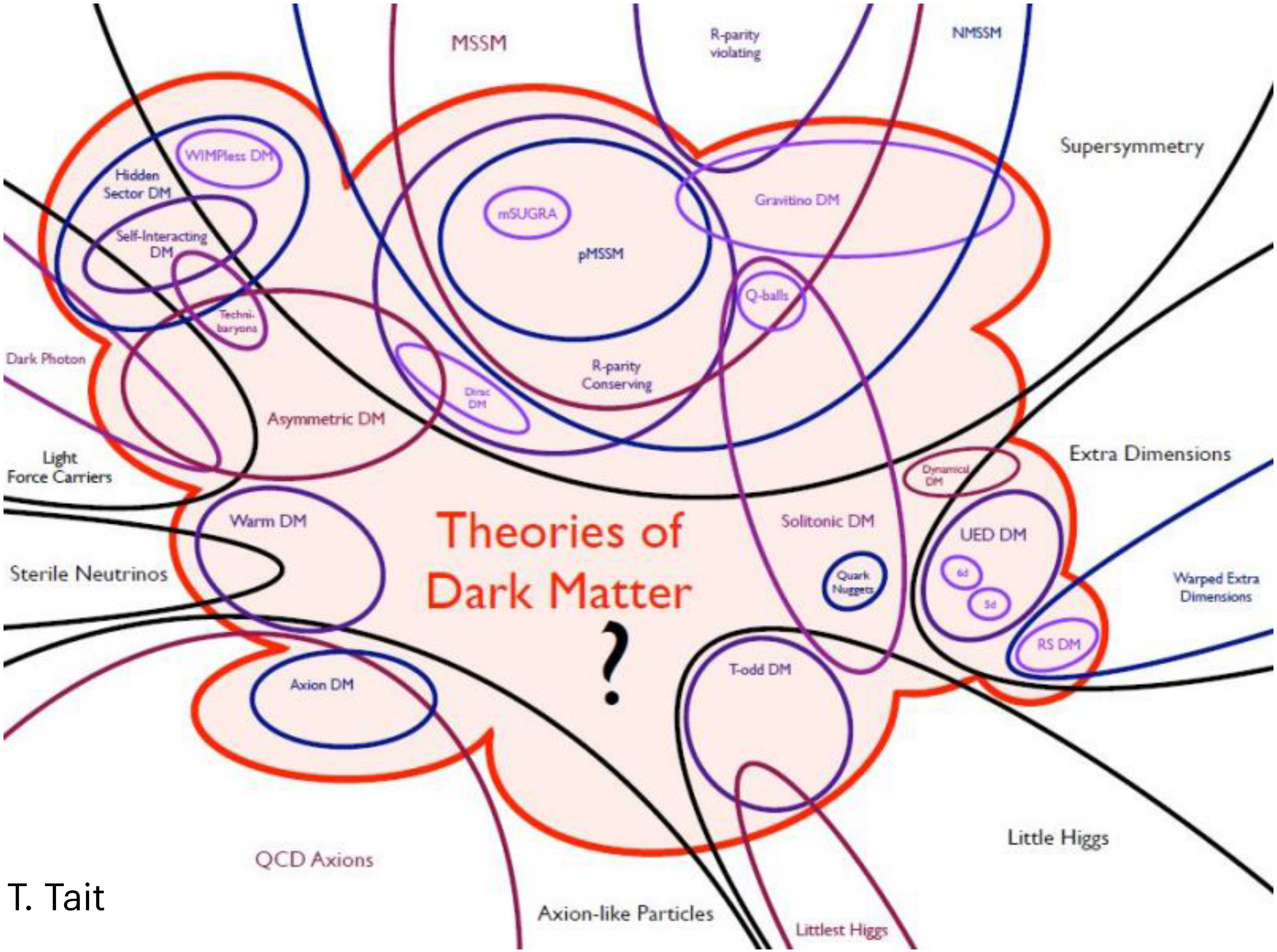
0.23
0.22
^{4}He
D+^{3}He
D
^{7}Li
Number relative to H
10^{-4}
10^{-5}
10^{-9}
10^{-10}
nucleosynthesis
gravitational lensing
cluster gas in x-rays
background radiation
structure formation
Optical Dark Matter X-ray Gas
cluster collisions

THE UNIVERSE IS DARK



One of the biggest mysteries of the universe

Theories of DM ?



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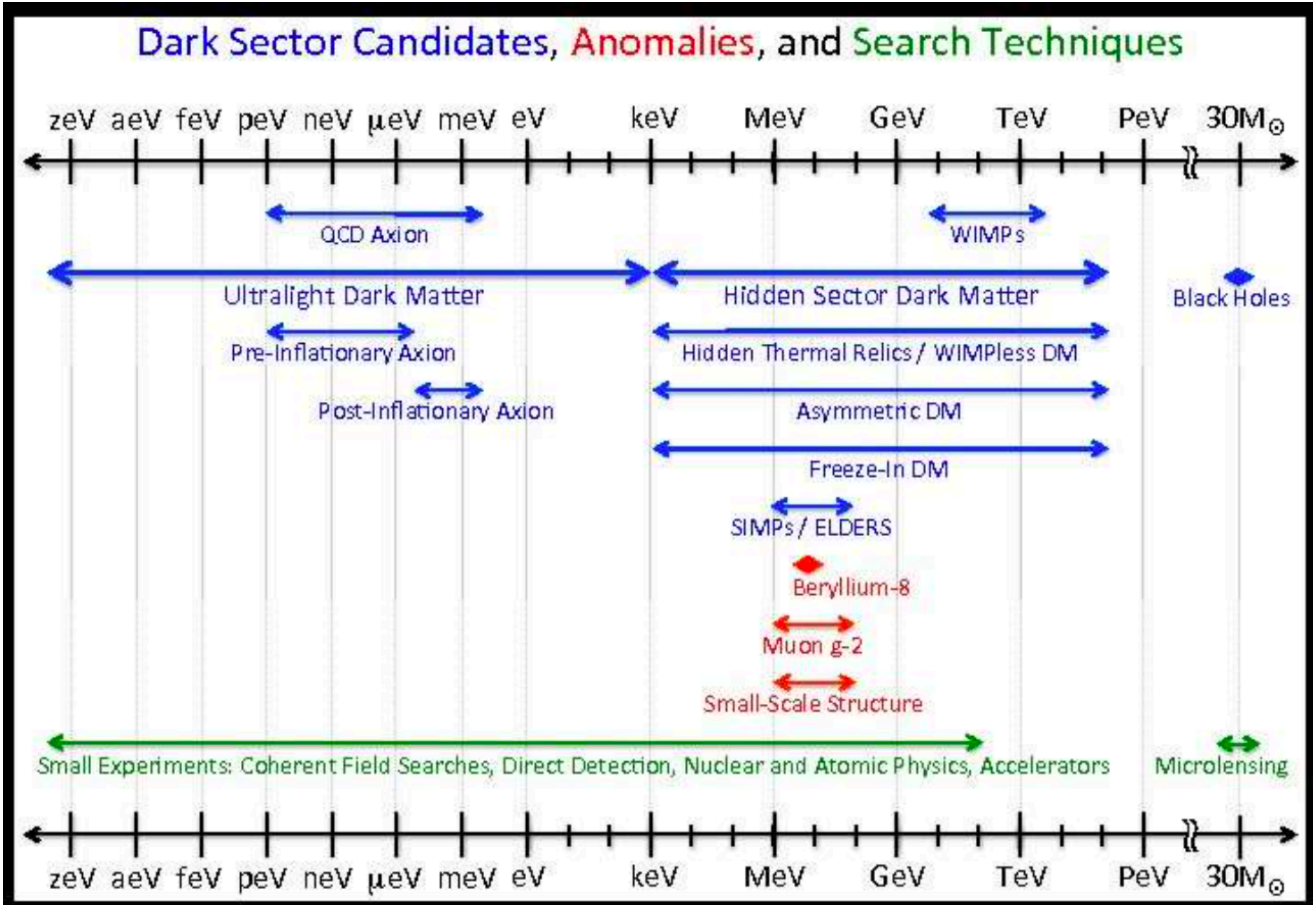
Theories of DM ?



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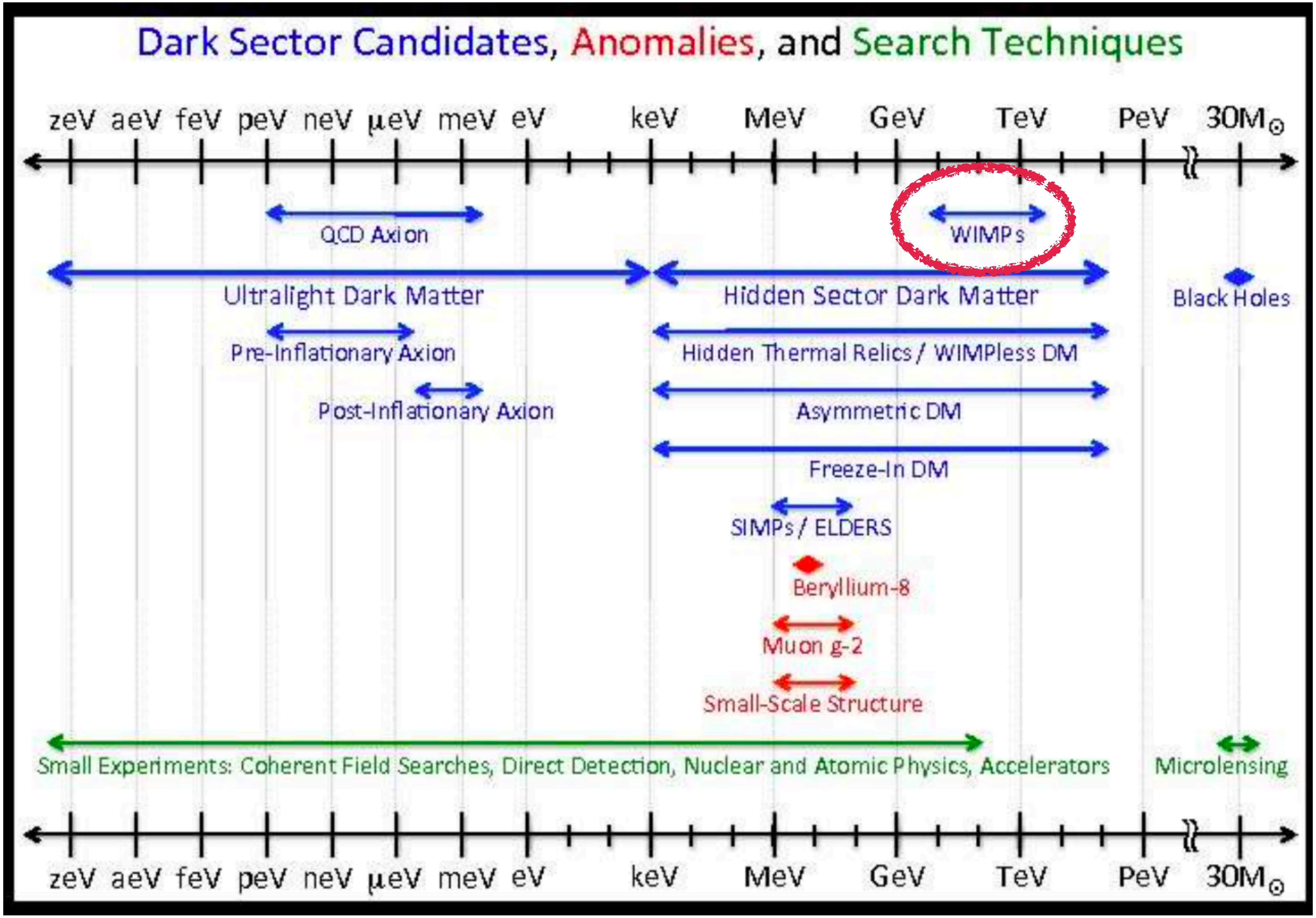

Original Idea of WIMP goes back to: Zeldovich and Gershtein (1966)

The "WIMP Miracle"

Cold thermal relic: weak scale cross section (and mass?)
($1\ \mathrm{GeV} - 1000\ \mathrm{GeV}$) WIMP (Weakly Interacting Massive Particle)

Dark matter is a complex physical phenomenon.

WIMPs are a simple, elegant, compelling explanation for a complex physical phenomenon.

Original Idea of WIMP goes back to: Zeldovich and Gershtein (1966)

Fermi National Accelerator Laboratory

FERMILAB-Pub-77/41-THY
May 1977

Cosmological Lower Bound on
Heavy Neutrino Masses

BENJAMIN W. LEE *
Fermi National Accelerator Laboratory, Batavia, Illinois 60510

AND

STEVEN WEINBERG **
Stanford University, Physics Department, Stanford, California 94305

ABSTRACT

The present cosmic mass density of possible stable neutral heavy leptons is calculated in a standard cosmological model. In order for this density not to exceed the upper limit of 2×10^{-29}g/cm^3, the lepton mass would have to be greater than a lower bound of the order of 2 GeV.

** On leave 1976-7 from Harvard University.

* Operated by Universities Research Association Inc. under contract with the Energy Research and Development Administration

Ben Lee (1935 — June 1977)

Steven Weinberg (1933— July 2021)

Original Idea of WIMP goes back to: Zeldovich and Gershtein (1966)

$$\frac{dn}{dt} = -\frac{3\dot{R}}{R}n - \langle\sigma v\rangle n^2 + \langle\sigma v\rangle n_0^{\ 2} . \qquad (2)$$

Here n is the actual number density of heavy neutrinos at time t; R is the cosmic scale factor; $\langle\sigma v\rangle$ is the average value of the $L^0\bar{L}^0$ annihilation cross-section times the relative velocity and n_0 is the number density of heavy neutrinos in thermal (and chemical) equilibrium[6]:

$$n_0(T) = \frac{2}{(2\pi)^3}\int_0^\infty 4\pi p^2 dp\left[\exp\left((m_L^{\ 2}+p^2)^{1/2}/kT\right)+1\right]^{-1} . \qquad (3)$$

(We use units with $\hbar=c=1$ throughout.)

$$\frac{dn}{dt} = -\frac{3\dot{R}}{R}n - \langle\sigma v\rangle n^2 + \langle\sigma v\rangle n_0^{\ 2}$$

where ρ is the energy density

$$\rho = N_F a T^4 = N_F \pi^2 (kT)^4/15 \qquad (5)$$

with N_F an effective number of degrees of freedom, counting ½ and 7/16 respectively for each boson or fermion species and spin state. For temperatures in the range of 10-100 MeV (which most concern us here) we must include just $\gamma, \nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, e^-$, and e^+, so $N_F = 4.5$, a value we will adopt for most purposes. However, if current ideas about the strong interactions are correct, then N_F rises steeply at a temperature of order 500 MeV to a value[7] $N_F \approx 30$.

To estimate $\langle\sigma v\rangle$, we note that the heavy neutrinos must be quite non-relativistic at the temperature T_f where they freeze

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$$\langle \sigma v \rangle = G_F^{2} m_L^{2} N_A / 2\pi$$

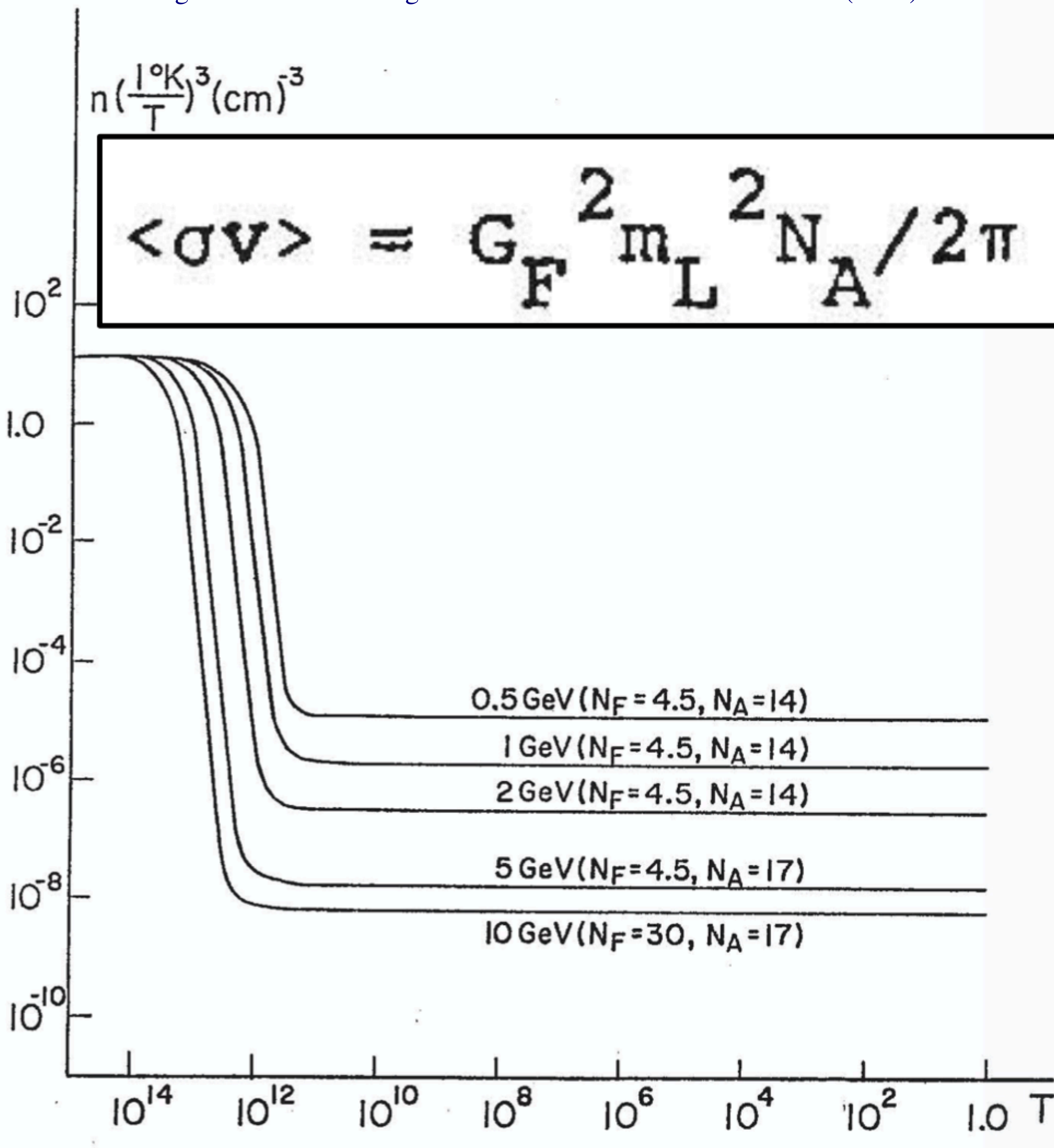

FIG. 1

en Lee (1935 — June 1977)

en Weinberg (1933— July 2021)

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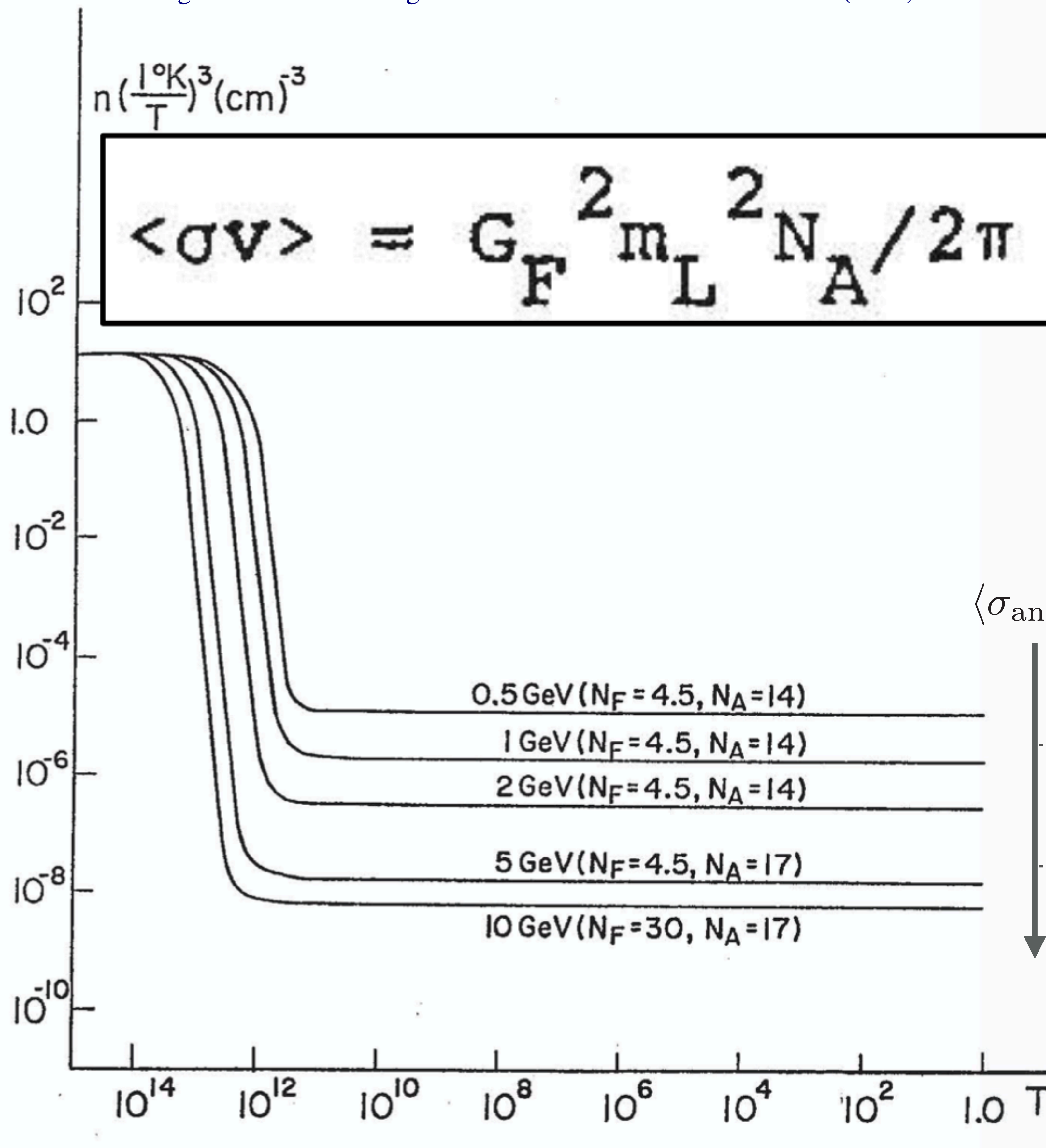


FIG. 1

...en Lee (1935 — June 1977)

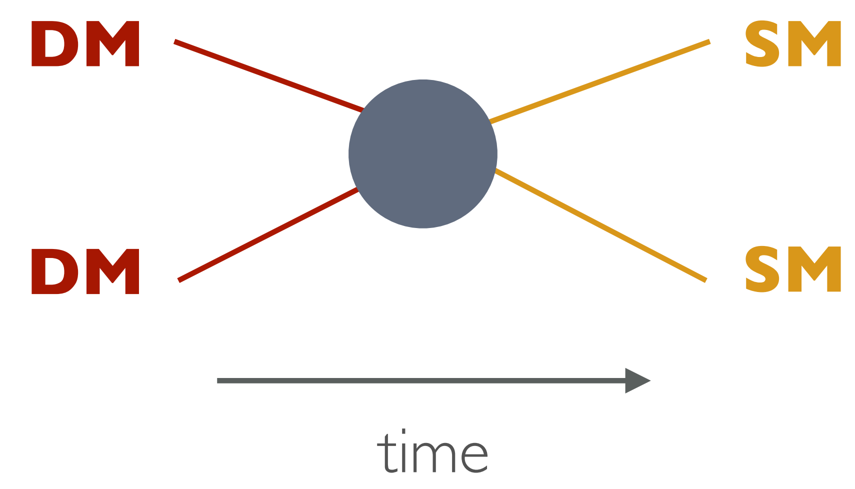
...en Weinberg (1933— July 2021)

THE WIMP MIRACLE

Insensitive to the initial conditions of the Universe:

due to the thermal equilibrium between the DM and SM gases in the early Universe

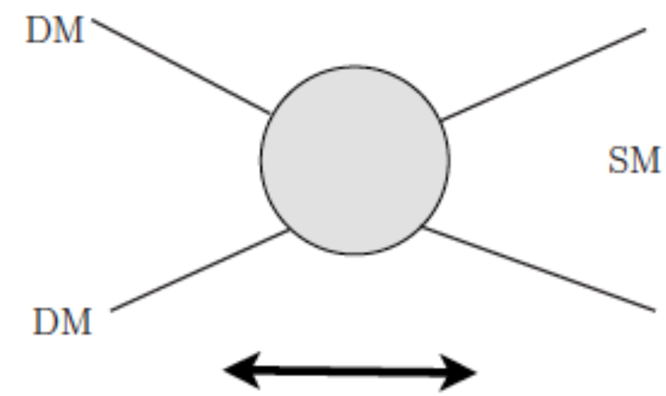
$$\text{Relic abundance} \propto \frac{1}{\text{ann. rate}}$$



Correct relic abundance for dark matter mass around the TeV scale and weak-force interactions

WIMP Dark Matter

- Original idea of WIMP Miracle

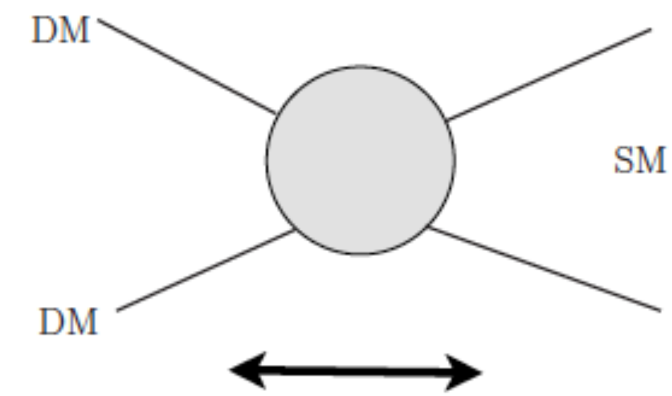


Rate in thermal eq. $\langle \sigma v \rangle \sim \frac{g_{\mathrm{D}}^4}{m_{\mathrm{DM}}^2}$

WIMP Dark Matter

- Original idea of WIMP Miracle
- => now pushed to a conner by the null results from DM direct detection experiments

Moore’s Law works in DM!



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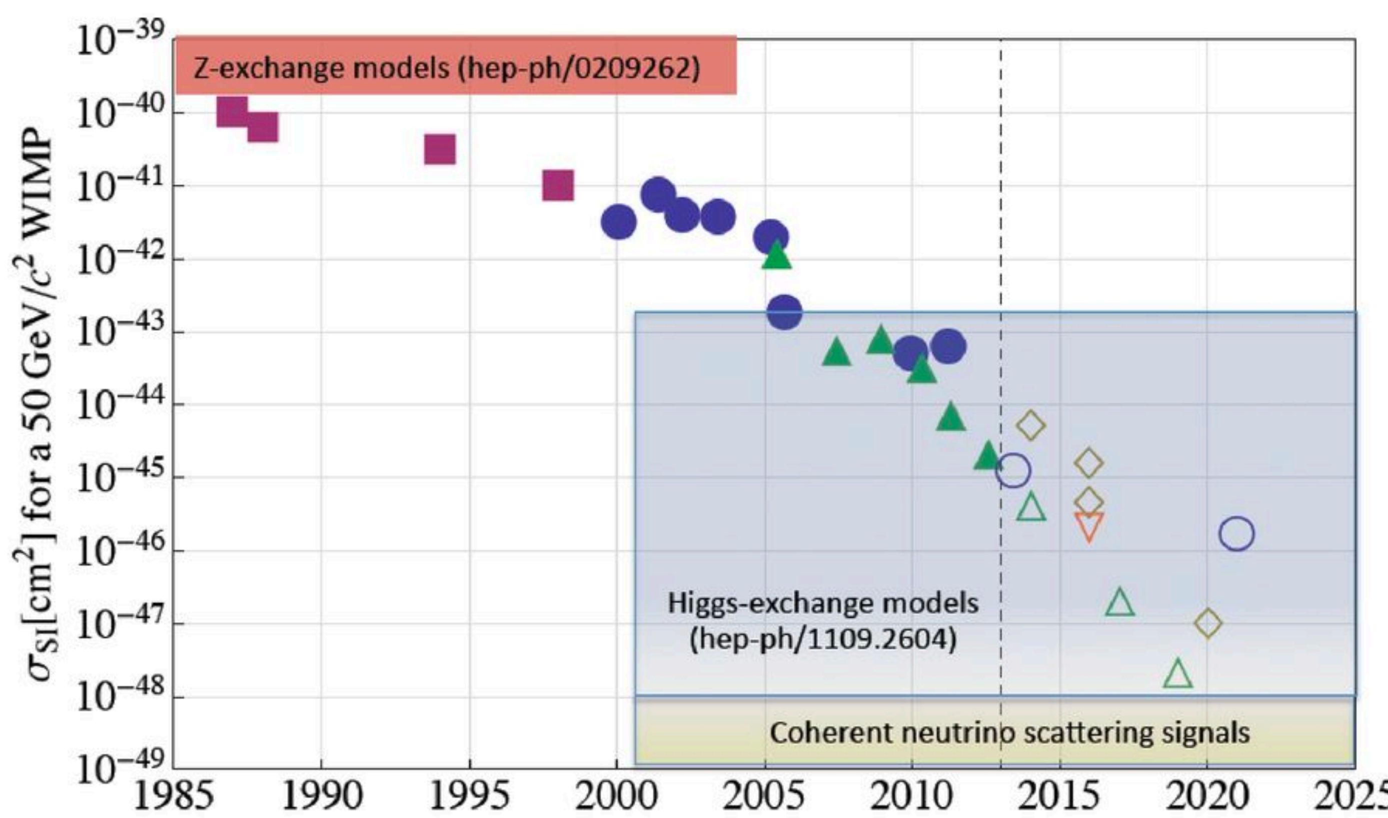

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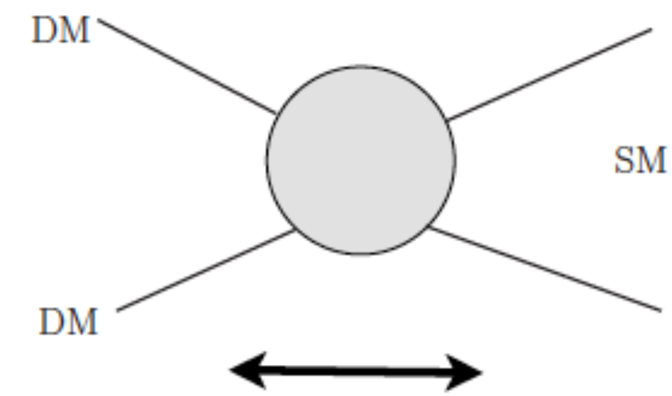

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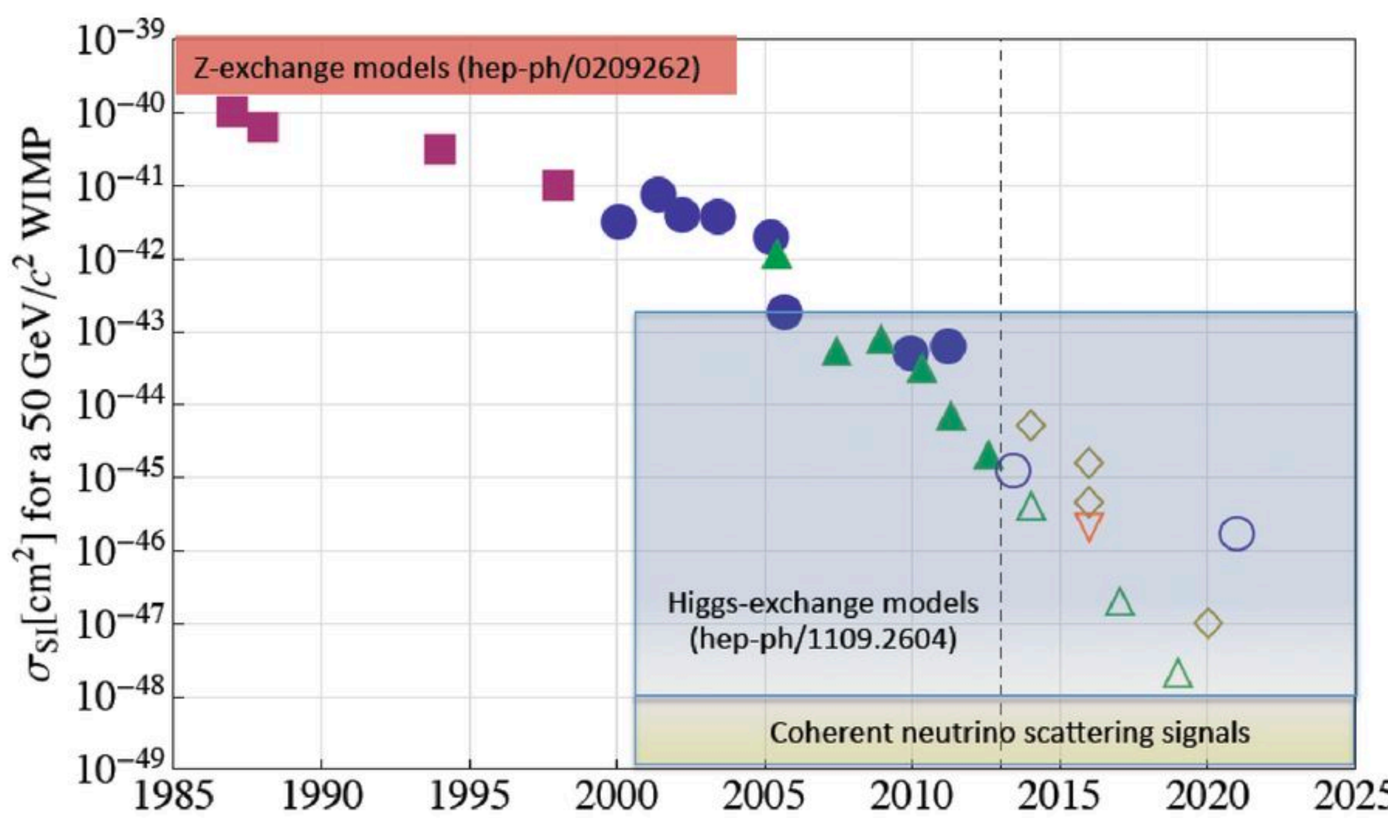
Moore's Law works in DM!

- Z boson exchange excluded except for fine-tuned corners of parameter space, and requiring tuning for Higgs mediation as well



Been searching for WIMPs…

The dominant paradigm is being challenged.

Is there another DM paradigm that gives qualitatively different signatures, but still provide the same level of simple, elegance and compelling explanation as WIMP?

DM-what we don't know

- Mass of Dark Matter (range:10^{-22} to 10^{67} eV)
- Composition of Dark Matter
- Interaction of Dark Matter

Gapped Continuum, instead of Resonances

♦ **Our Proposal:** Dark Matter is made of an ensemble of gapped continuum states

- It's not even clear whether the DM that provide successful explanations to the rotation curve of disk galaxies, CMB, and large structure formation is a localized excitation of quantum field (i.e. particle)

- continuum with a mass gap is not so uncommon in condensed matter physics: e.g. edge state in fractional quantum hall effect, topological superfluid, 2D Ising model, 2d SU(2) Thirring model, 2d SU(N) Yang-Mills theory in large-N limit ,etc

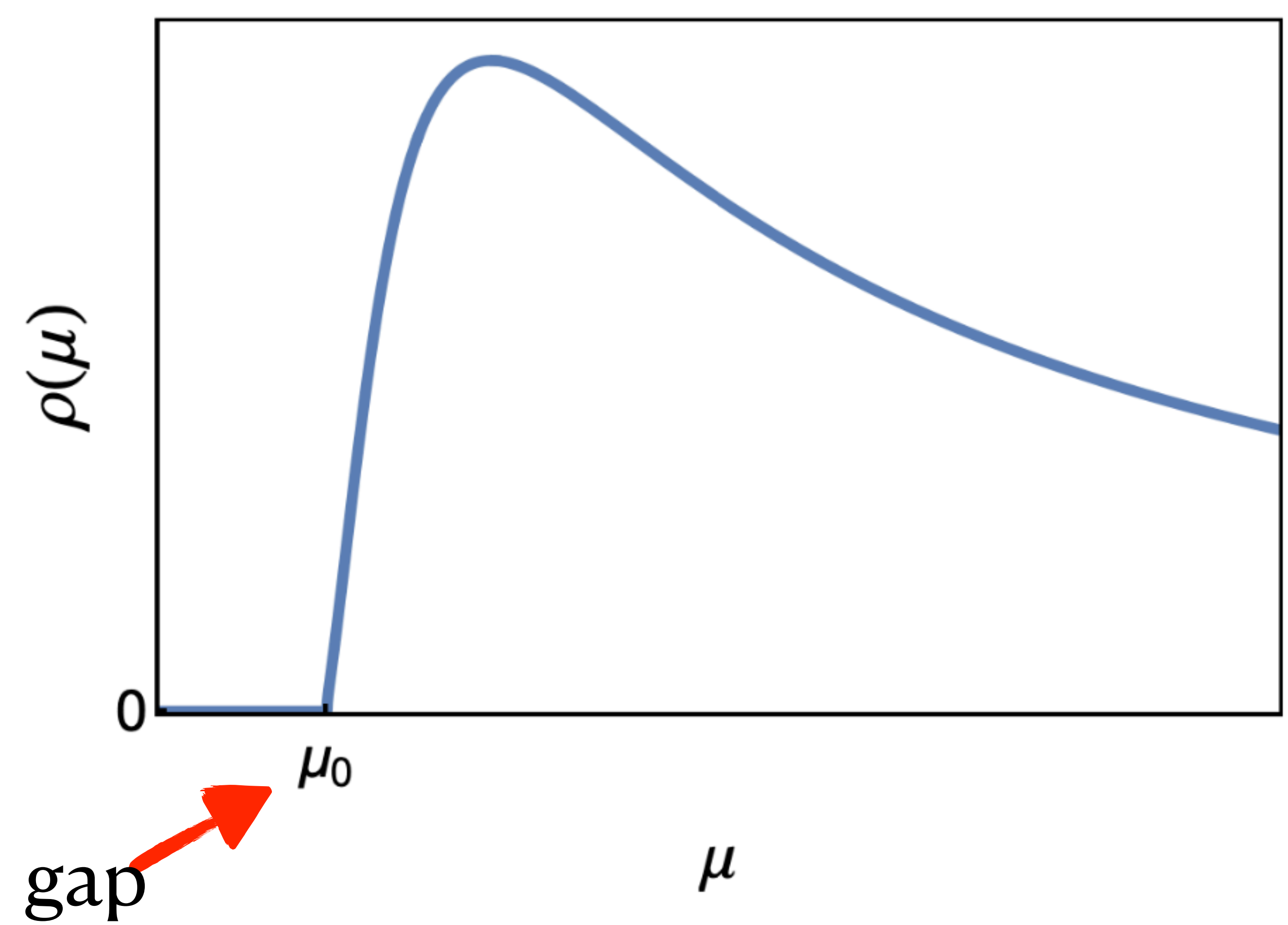
Gapped Continuum, instead of ordinary particles

♦ Continuum DM: singly-excited states are characterized by a continuous parameter μ^2, in addition to the usual 3-momentum p

The parameter μ^2 plays the role of mass in the kinematic relation $p^2 = \mu^2$ for each state. The number of states is proportional to $\int \varrho(\mu^2)\, d\mu^2$, where ϱ is the spectral density of the theory

$$\langle 0|\Phi(p)\Phi(-p)|0\rangle = \int \frac{d\mu^2}{2\pi} \frac{i\rho(\mu^2)}{p^2 - \mu^2 + i\epsilon}$$

We will construct DM models based on continuum QFT with the gap around the electroweak scale, $\mu_0 \sim 100$ GeV, and including interactions to the electroweak (EW) sector of the SM. We will call the resulting type of model the Weakly Interacting Continuum (WIC) DM model



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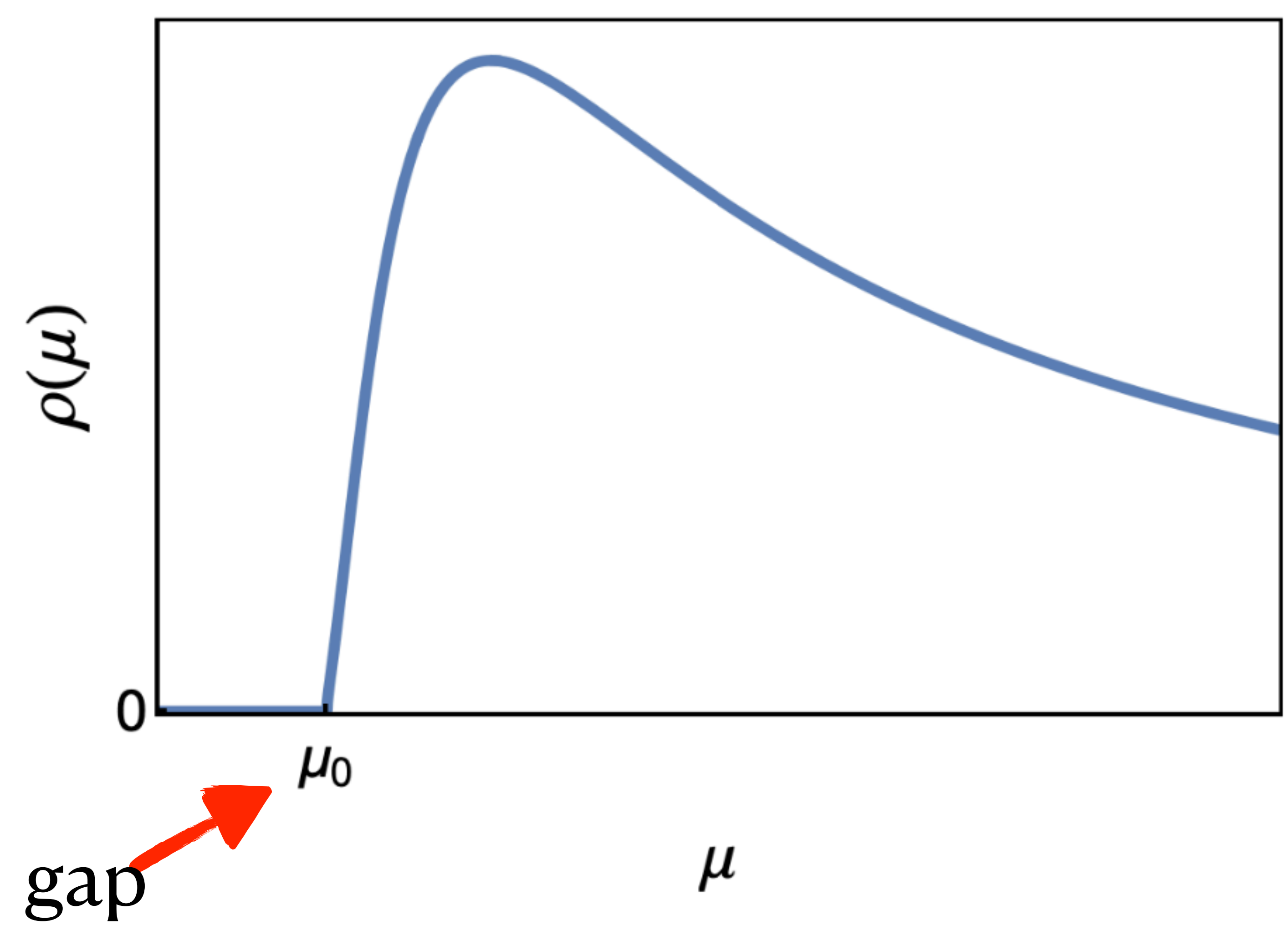

needs to be gapped: consistent with observations e.g. CMB, LSS

Gapped Continuum, instead of Resonances

♦The appearance of a continuum is very common in QFT's: e.g. spectrum of CFTs necessarily forms a continuum since the theory does not admit any mass scales (no mass gap).

♦Unparticles (Georgi): another example of gapless continuum

♦String Theory (e.g. Gubser et al, Kraus, Trivedi et al, etc): gapped continuum shows up when one has a large number of D3 branes distributed on a disc (which is dual to N = 4 SUSY broken to N = 2 via masses for two chiral adjoints)

♦Gapped Continuum in particle physics: -Softwall model (Higgs with a small mass gap (before Higgs discovery) by Terning et al, Falkowski et al

-Quantum Critical Higgs (Higgs pole + gappend continuum: after Higgs discovery) by Csaki et al (SL).

-Continuum Naturalness (for solving little hierarchy by Csaki et al (SL), and also by Quiros et al)

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So, how about DM as a gapped continuum?

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- One cannot simply plug gapped continuum into formalism developed for particle DM: need a new theoretical framework for dealing with gapped continuum in order to calculate the relic density of DM, and to deal with the finite temperature physics necessary for describing general features of cosmological history of DM

    -requires non-trivial development of theories of gapped continuum DM

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- Gapped Continuum as a DM can give striking new experimental signatures in colliders and cosmic microwave background measurements

- The strong suppression of direct detection signals reopens the possibility of a Z-mediated dark sector again (and also other continuum version of WIMP models).

Gapped Continuum Nature of DM

- Direct detection
  - quasi elastic scattering (QES): DM(μ_1) + $SM_1 \rightarrow$ DM(μ_2) + SM_2
    - even after freeze out, distribution of DM state keeps evolving: distribution is peaked at the mass gap (μ_0)at very late time (these decays are important for CMB physics), and DM states pass through the earth with non-relativistic speed ($v \sim 10^{-3}$)

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=> If incoming DM state has $\mu_1 = \mu_0 + \Delta$, accessible final continuum modes are in very narrow window $\mu_2 \in [\mu_0, \mu_0+\Delta+Q]$. For weak scale μ_0, this basically means that the integral appearing in the QES cross section is constrained to a tiny interval in μ, and leads to a significant suppression of the rate

$$\sigma \sim \int \frac{d\mu_2^2}{2\pi} \rho(\mu_2^2)\, \hat{\sigma}\,(\mu_1, \mu_2)$$

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- Direct detection
  - quasi elastic scatter
    - even after freez
      distribution is p
      decays are imp
      the earth with r

Q is the kinetic energy of the collision in the center-of-mass frame

$\Delta \ll \mu_0$ in today's universe, while $Q \ll \mu_0$ as long as ambient DM is non-relativistic.

$$\sigma_{\text{cont}} \sim \left(\frac{\Delta + Q}{\mu_0}\right)^{1+r} \sigma_{\text{particle}}$$

e.g. $\Delta \sim 100$ keV at the present time, while $Q \sim 1$ keV

μ_0 at the weak scale → ~10^9 suppression

r is a positive number that depends on the behavior of the spectral density near the gap (r=1/2 for XD)

e ... ough

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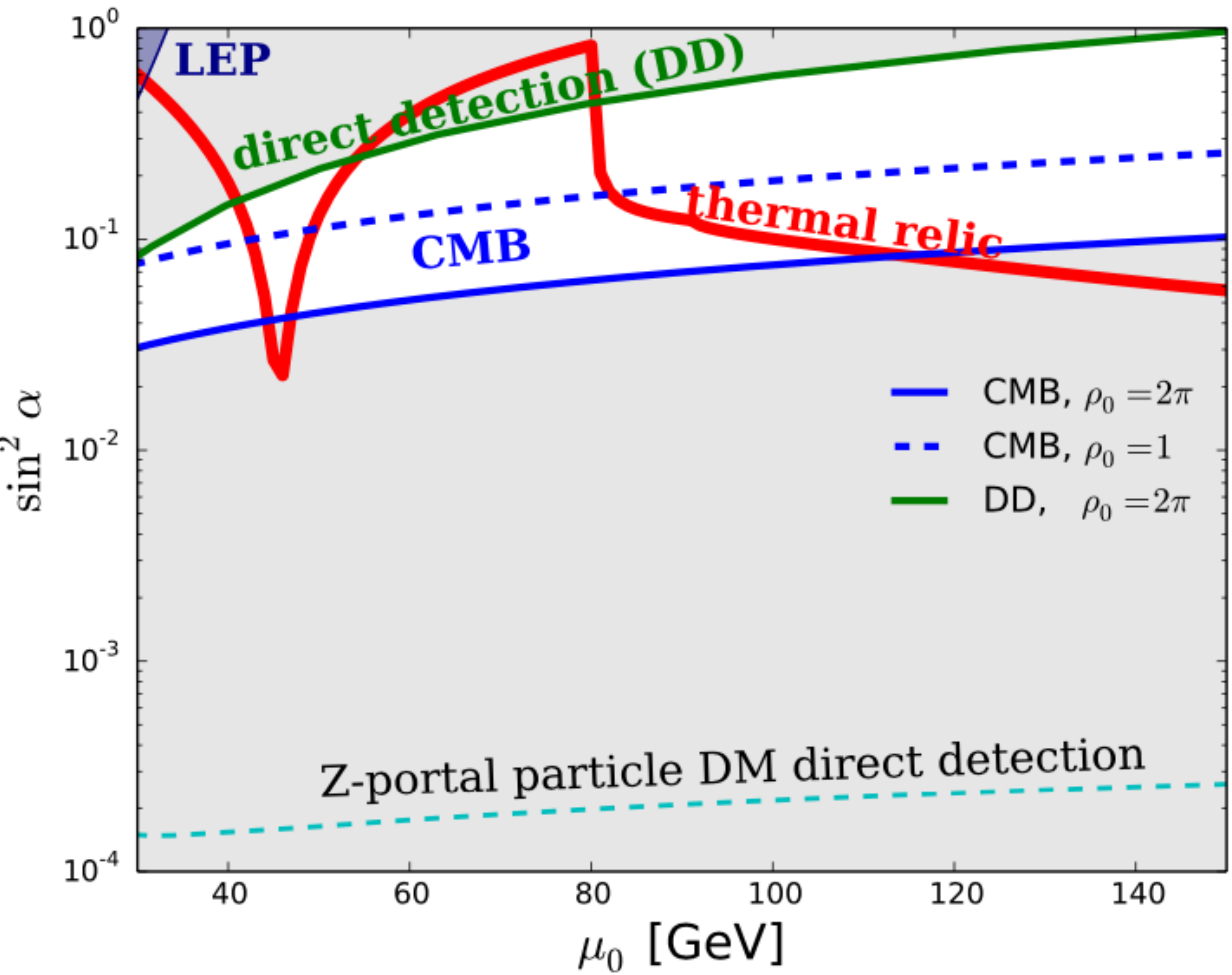

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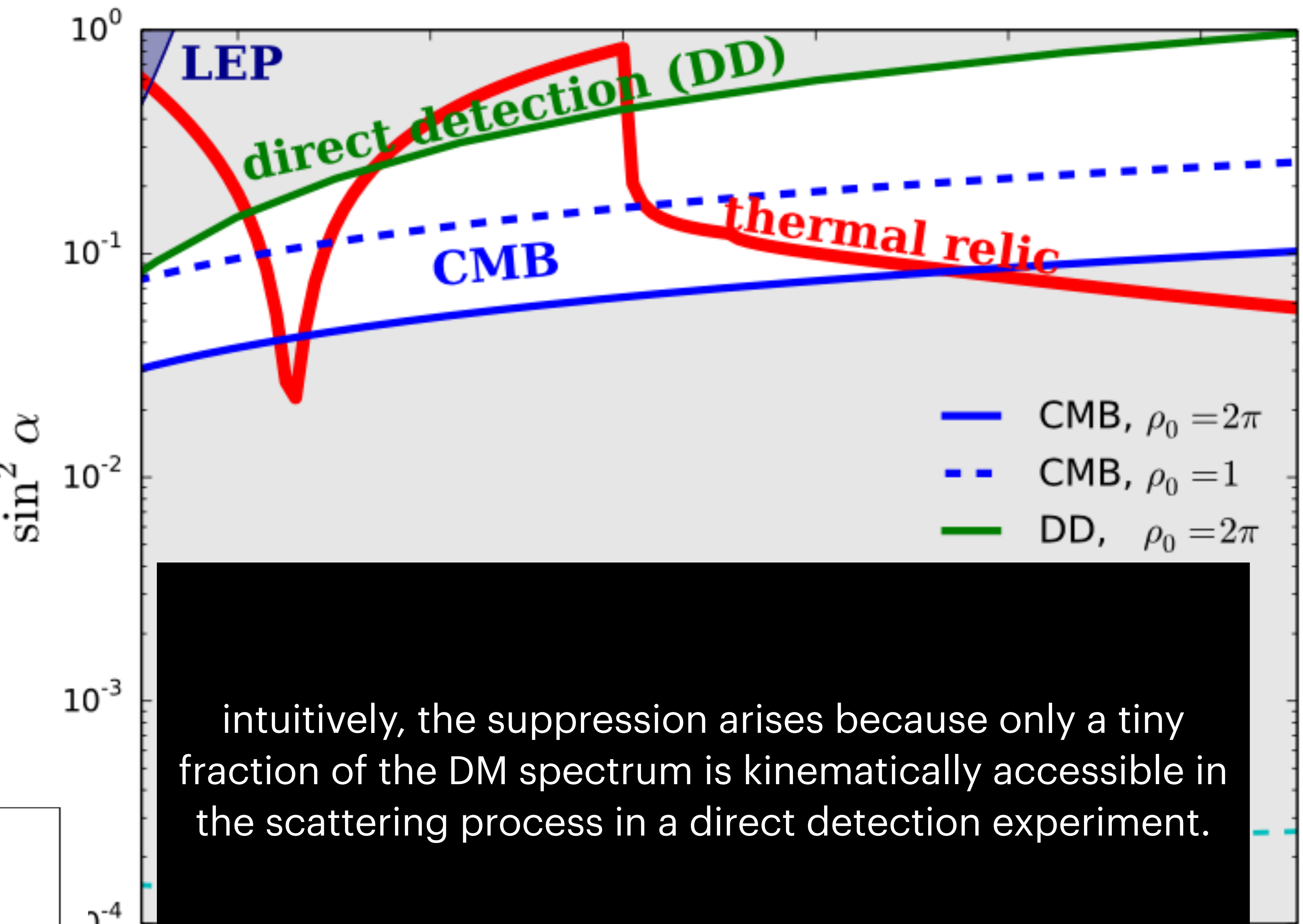
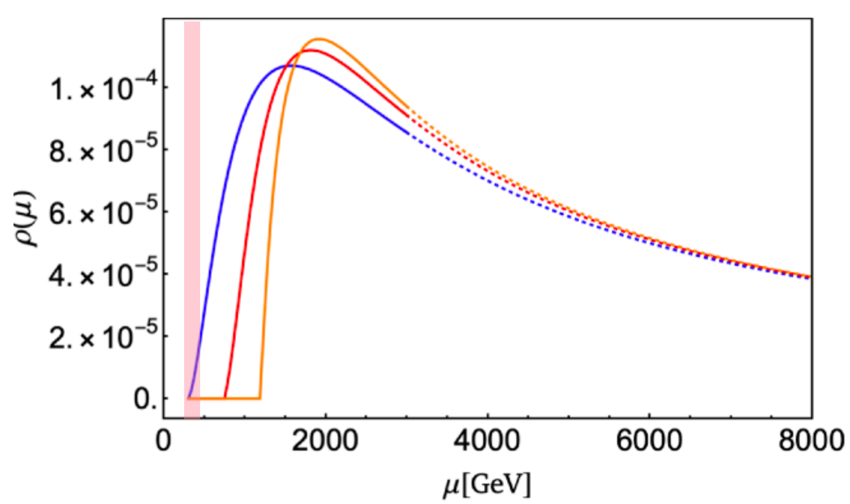


intuitively, the suppression arises because only a tiny fraction of the DM spectrum is kinematically accessible in the scattering process in a direct detection experiment.



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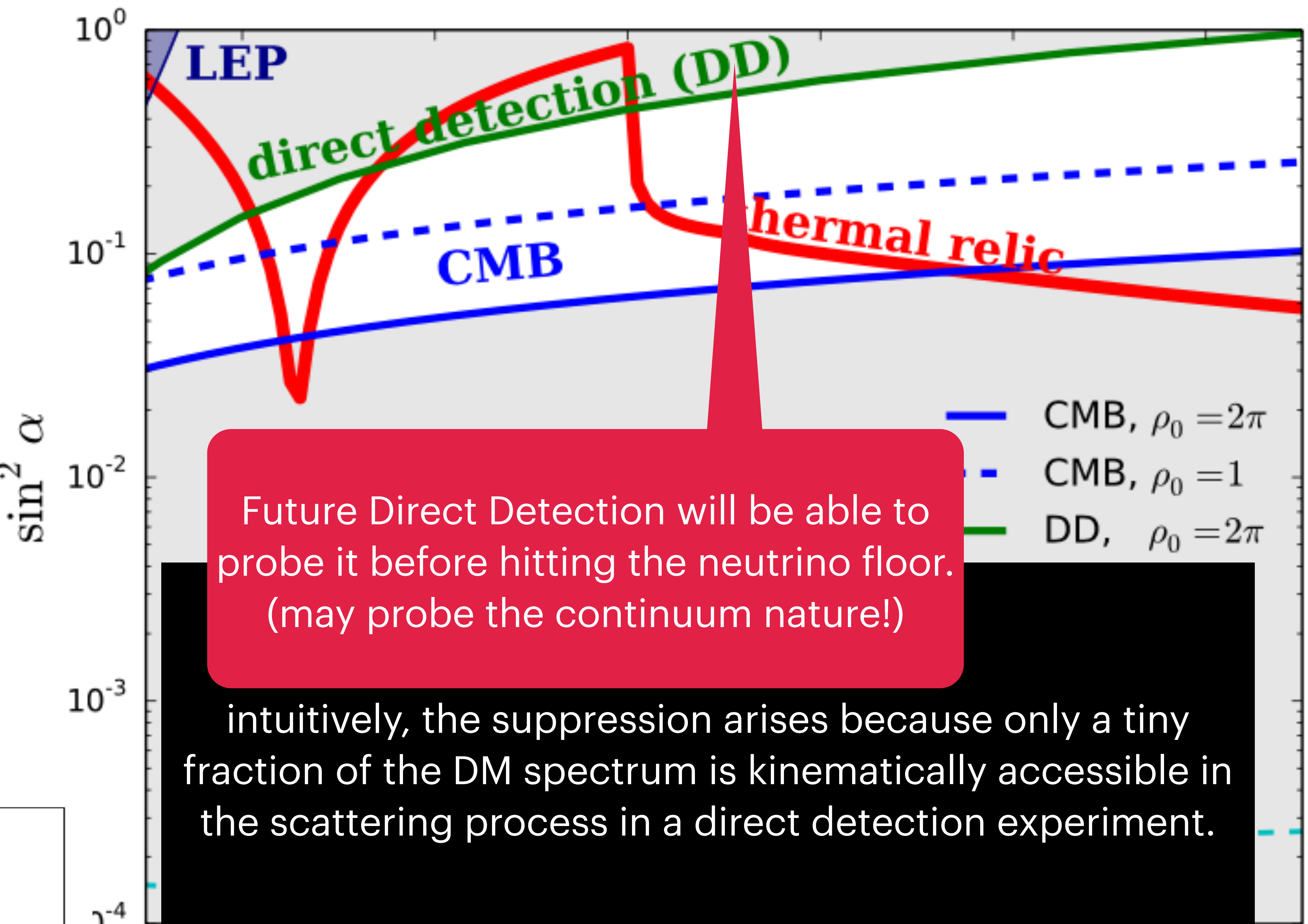

Future Direct Detection will be able to probe it before hitting the neutrino floor. (may probe the continuum nature!)

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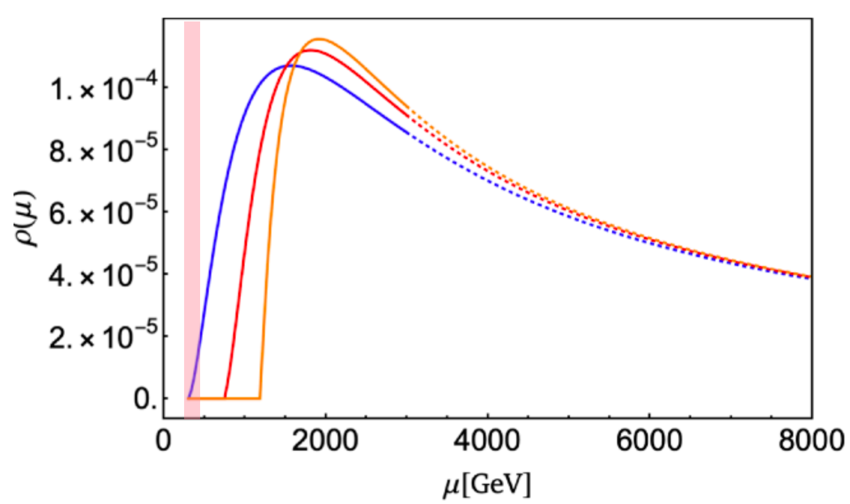

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Gapped Continuum Nature of DM

- Late decay
  - decay within the continuum state: $DM(\mu_1) \to DM(\mu_2) + SM$
    - Since all continuum states carry the same quantum number, such decays will necessarily occur continuously throughout the history of the universe.
    - In the early universe: DM in thermal and chemical equilibrium with the SM

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    - However, the mass distribution of the DM states continues to evolve, thanks to the above decays
    - The decays shift the distribution towards lower masses, closer to the gap scale.
    - Lifetime of a DM state increases with decreasing mass, due to both phase-space suppression and the fact that there are fewer states for it to decay into.
    - e.g. in our model, DM states are currently clustered within a few hundred keV above the gap scale (on average, each DM state undergoes roughly one decay per Hubble time)

Gapped Continuum Nature of DM

- Late deca
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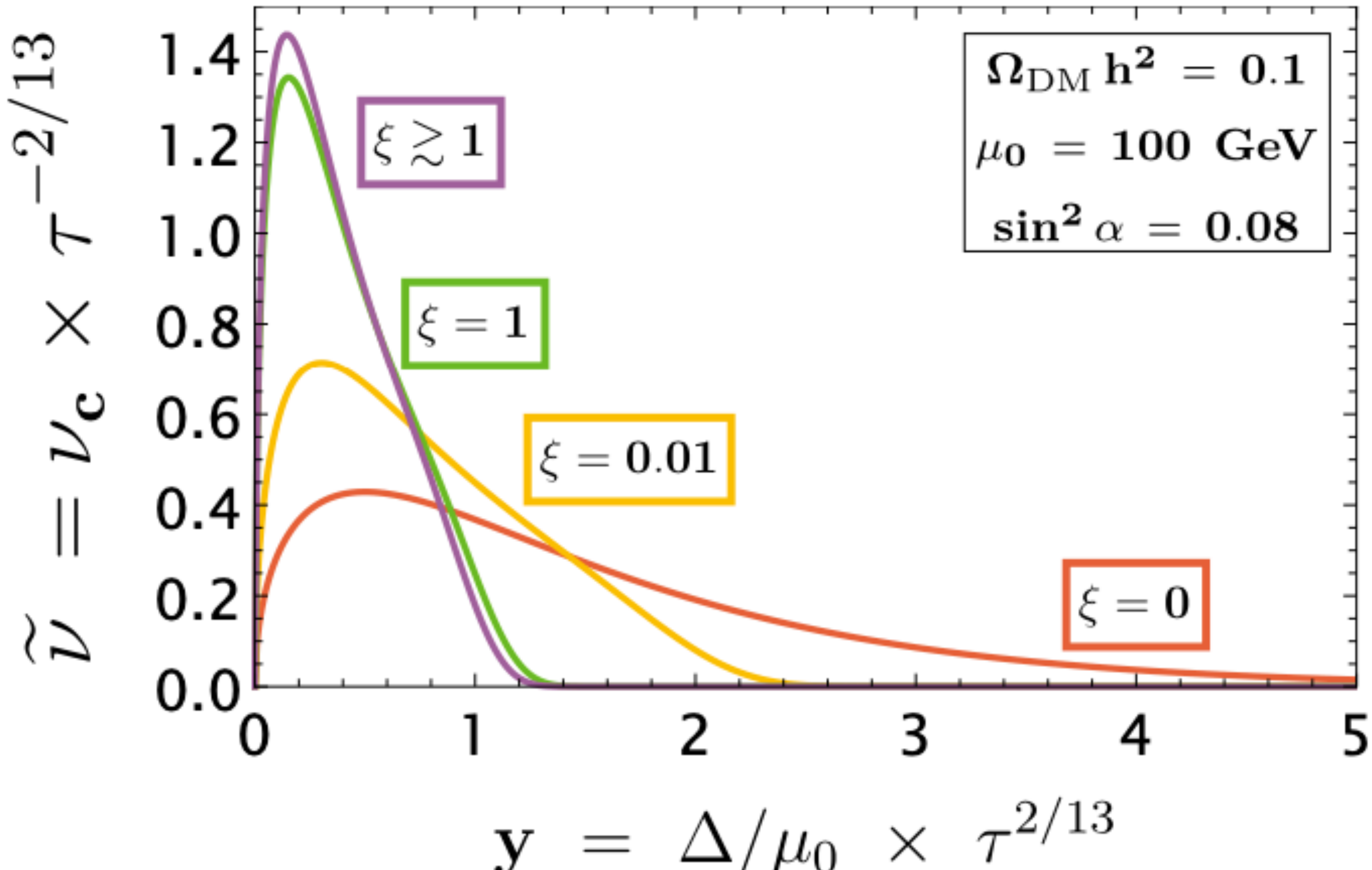

$\xi = \log(t/t_d)$, where $\tau = \Gamma_0 t$ and t_d is the time at decoupling

$\Delta = \mu - \mu_0$

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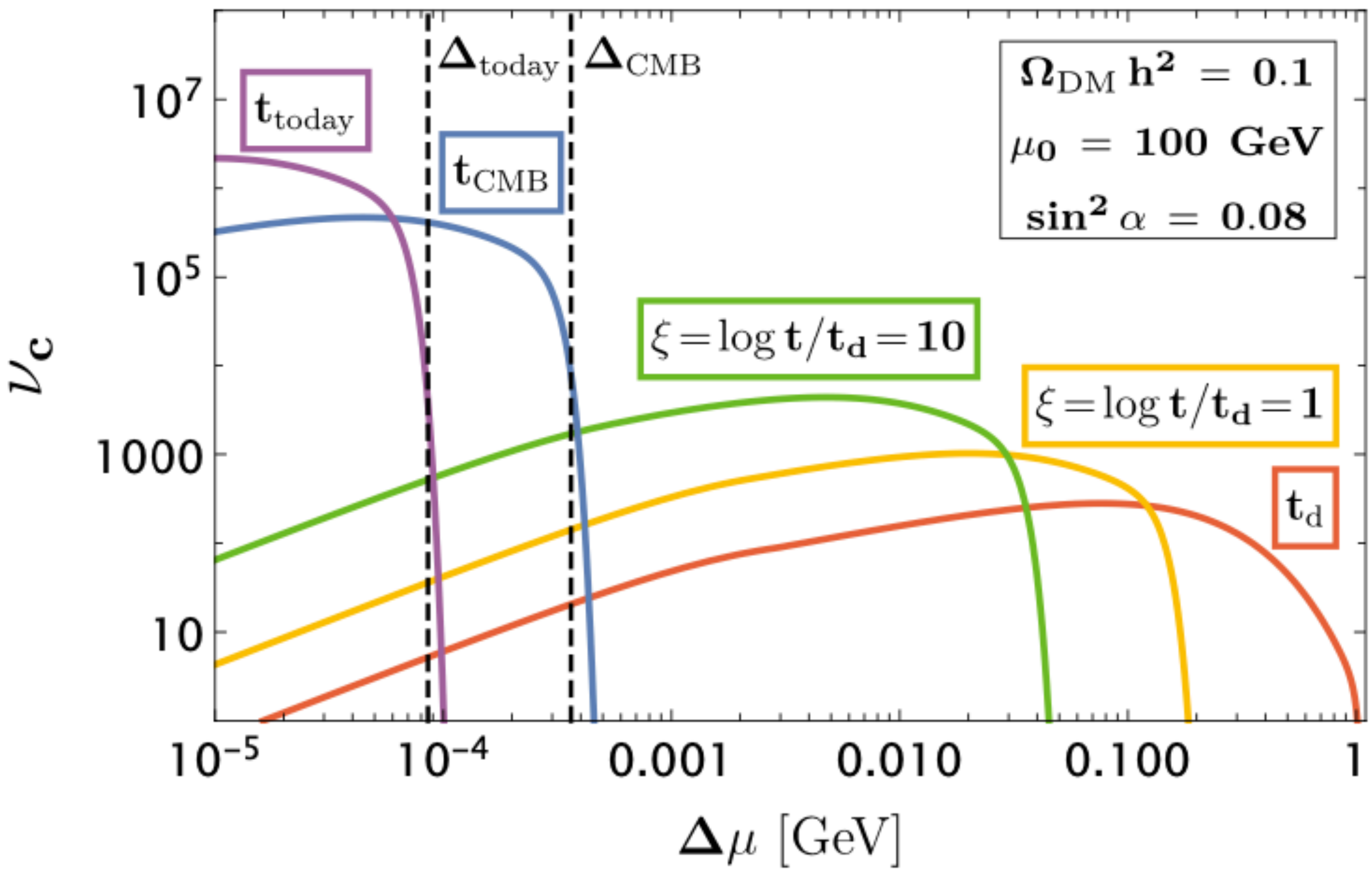

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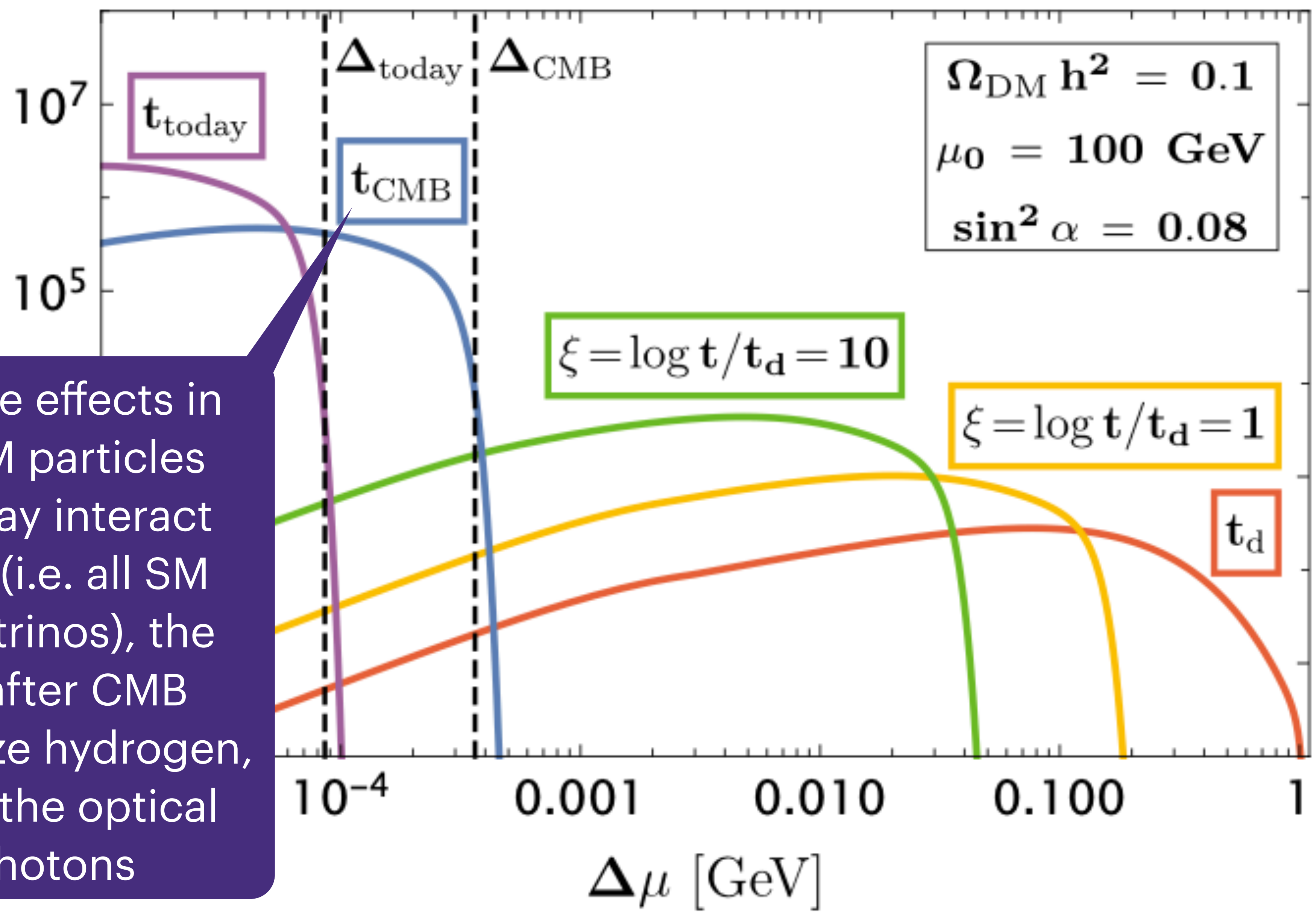


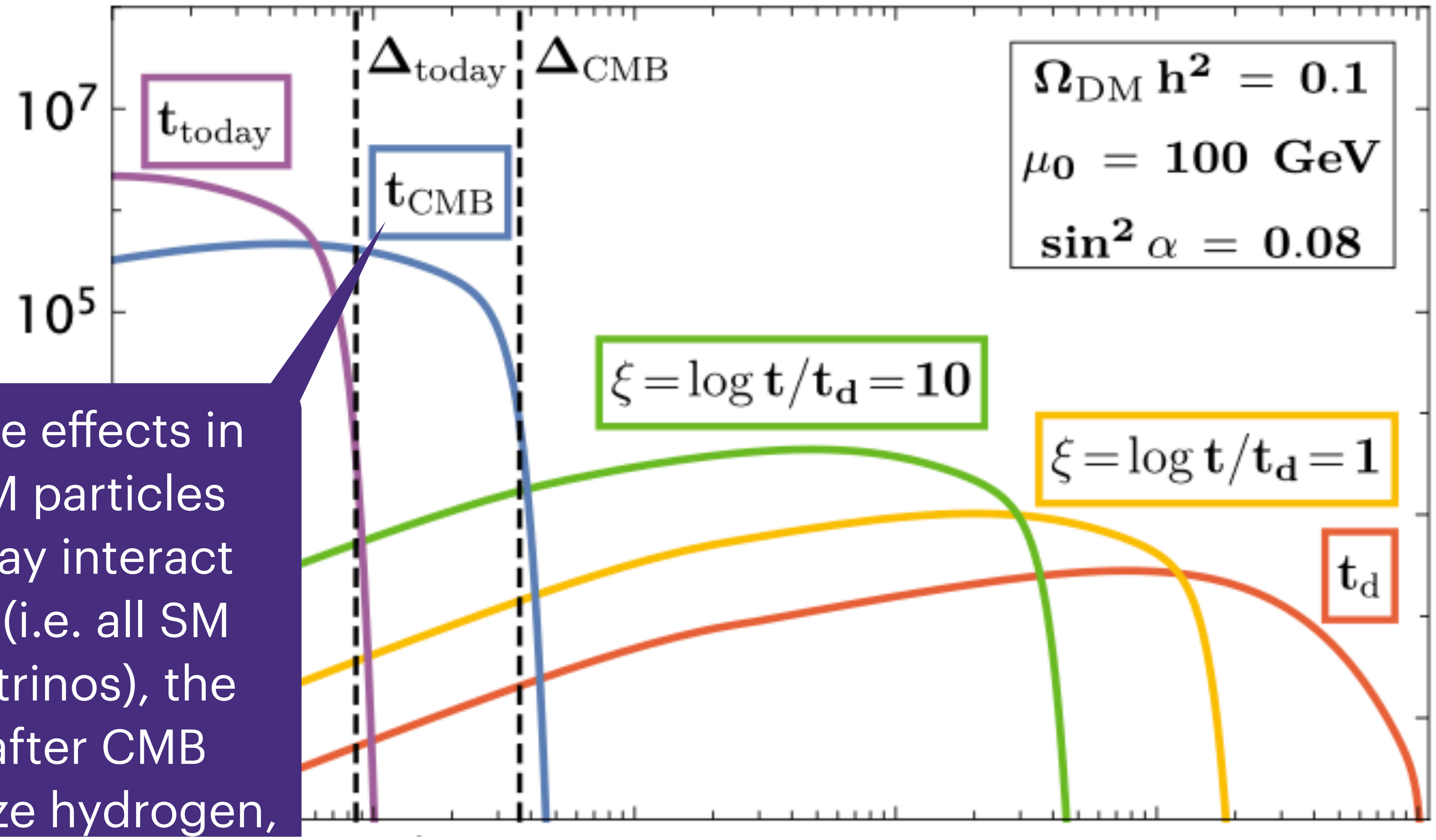
potentially observable effects in cosmology. If the SM particles produced in the decay interact electromagnetically (i.e. all SM particles except neutrinos), the decays that occur after CMB decoupling can reionize hydrogen, drastically changing the optical depth for CMB photons

$\xi = \log(t/t_d)$, where $\tau = \Gamma_0 t$ and t_d is the time at decoupling

$\Delta = \mu - \mu_0$

Gapped Continuum Nature of DM

- Late dec
  - decay w
    - Since … ssarily
      occur



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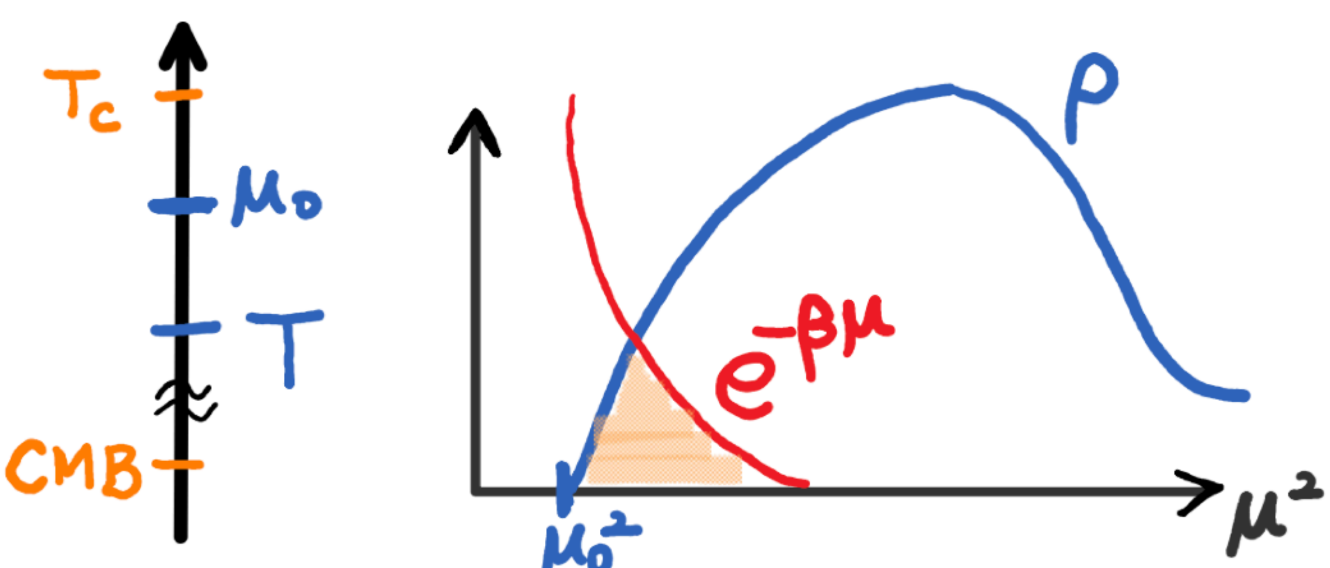

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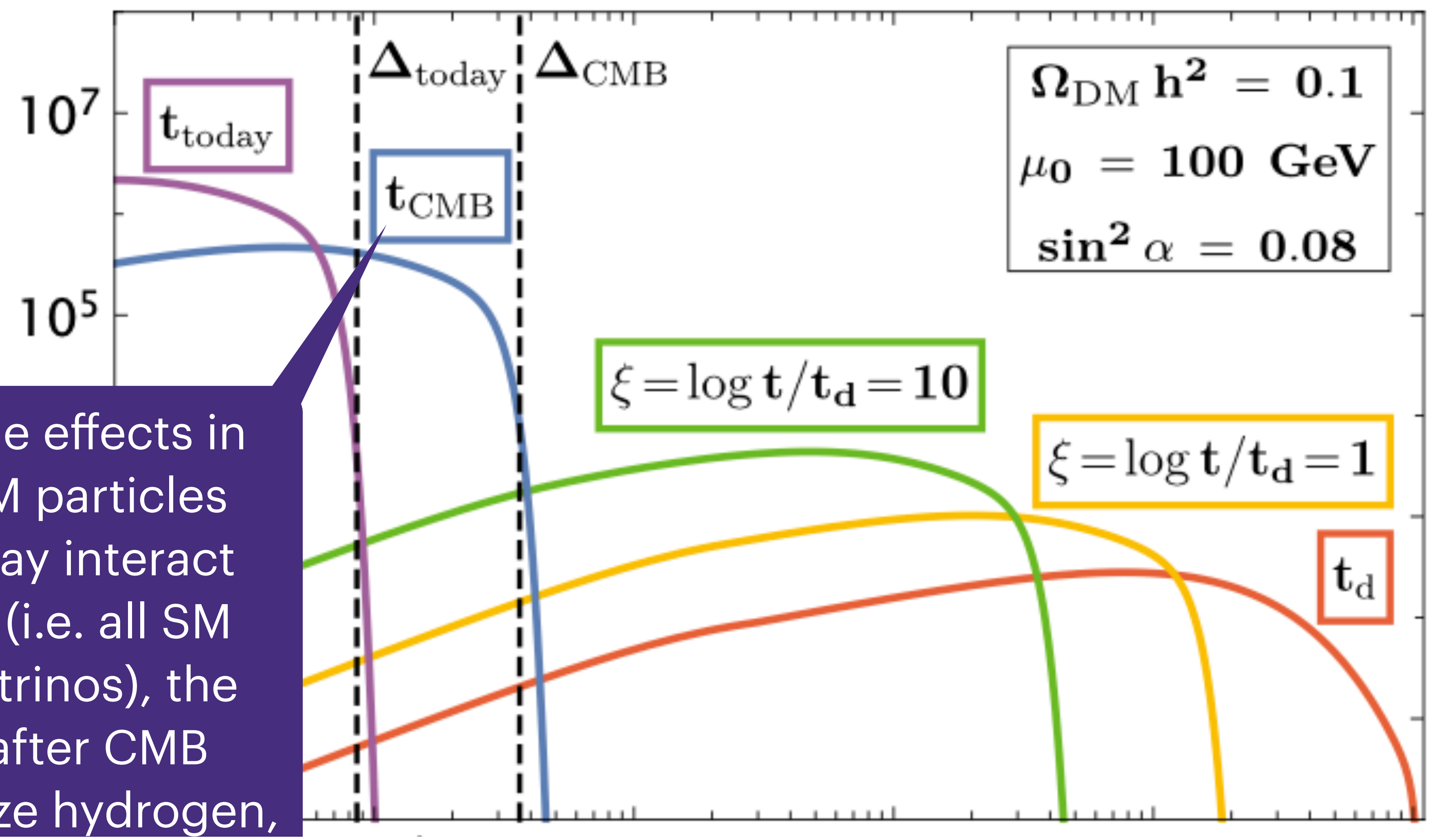

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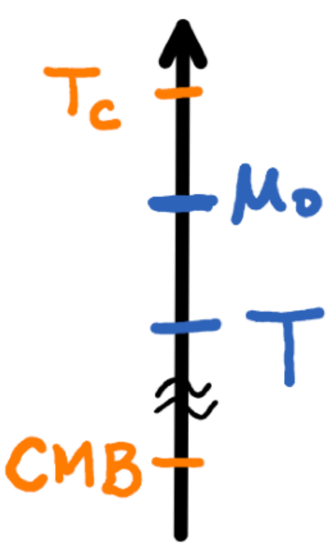

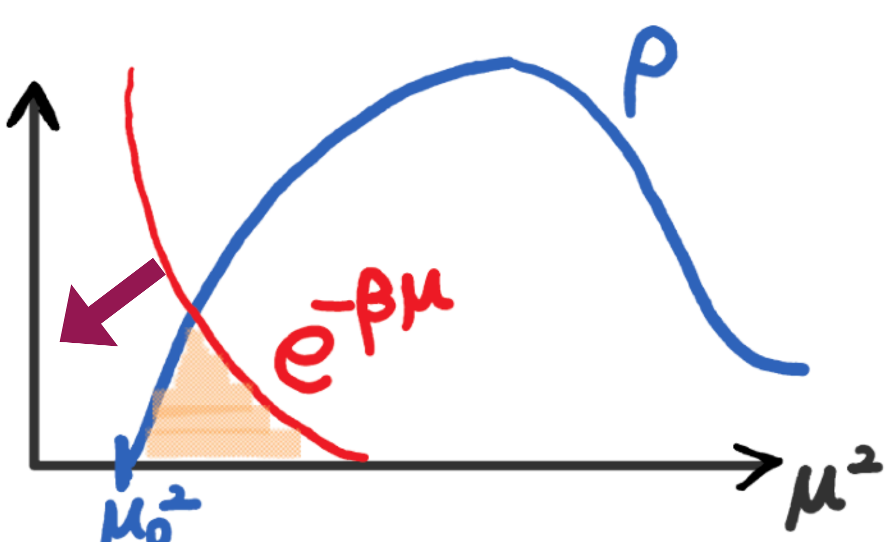

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Gapped Continuum Nature of DM

♦ Indirect Detection

$$DM(\mu_1) + DM(\mu_2) \rightarrow SM_1 + SM_2$$

- Since there is no continuum state in the final state, the rates of these processes are unsuppressed :

Gapped Continuum Nature of DM

♦ Indirect Detection

$$\mathrm{DM}(\mu_1) + \mathrm{DM}(\mu_2) \rightarrow \mathrm{SM}_1 + \mathrm{SM}_2$$

- Since there is no continuum state in the final state, the rates of these processes are unsuppressed :

  $\mu_1 \approx \mu_2 \approx \mu_0$ in the current universe $\Rightarrow$ both rates and kinematics of annihilation in the galactic halos are basically identical to those of particle DM

Gapped Continuum Nature of DM

- Colliders Phenomenology
  - for low energy experiments (low compared to gap scale): e.g. LEP bound for Z-portal WIC:

Same suppression mechanism (by continuum kinematics) as in Direct Detection applies!

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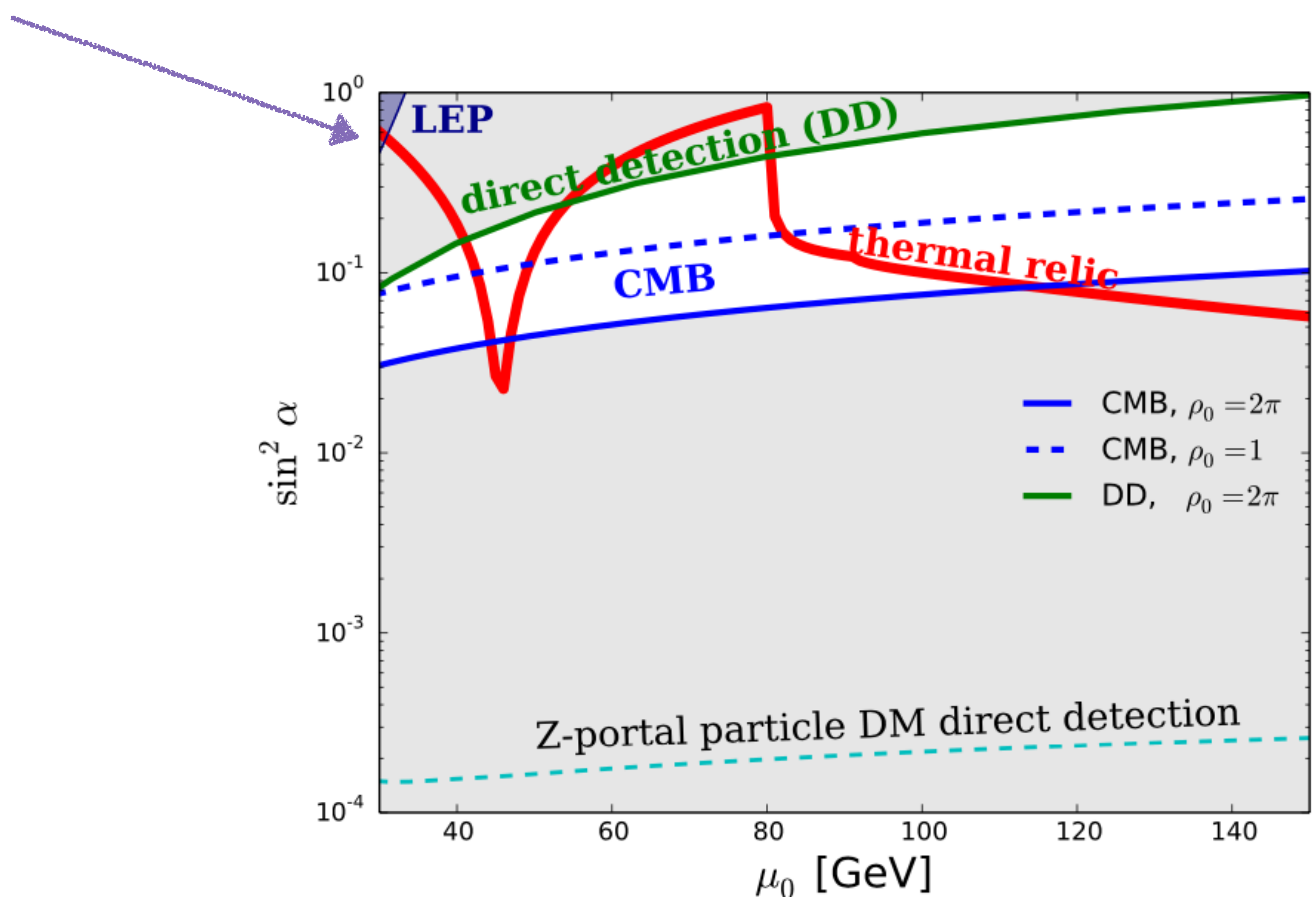
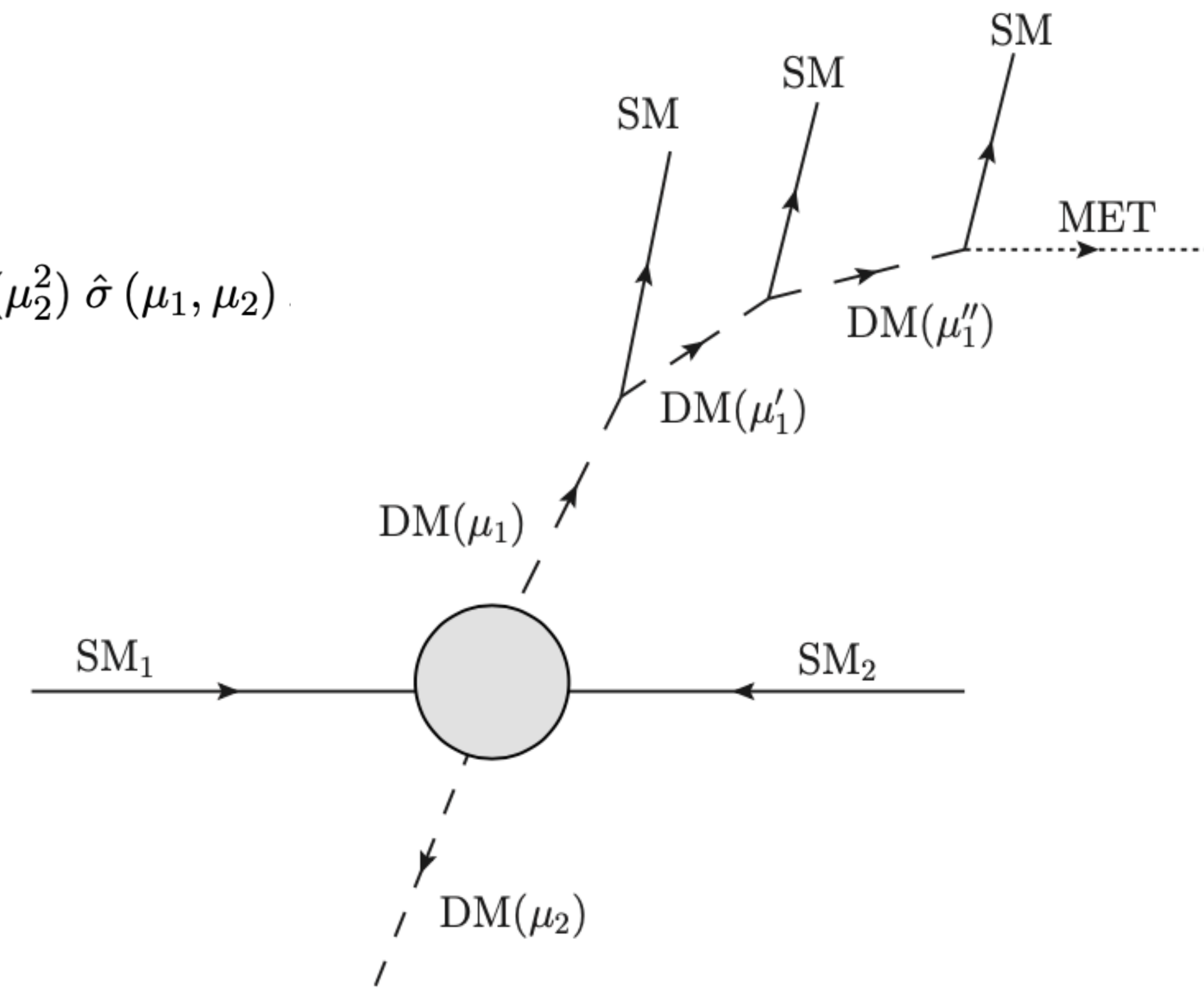


Gapped Continuum Nature of DM

♦ Colliders Phenomenology

- for high enough energy: (no suppression, an rich pheno)
  SM1+SM2 → DM(μ_1) + DM(μ_2)

$$\sigma \sim \int \frac{d\mu_1^2}{2\pi}\rho(\mu_1^2) \int \frac{d\mu_2^2}{2\pi}\rho(\mu_2^2)\, \hat{\sigma}\,(\mu_1, \mu_2)$$



Physics of Gapped Continuum DM

We develop a new formalism to deal with above questions and develop a realistic gapped continuum models to perform concrete DM phenomenology which would distinguish it from ordinary particle DM scenarios. Different areas for gapped continuum DM study for our project include:

- Gapped Continuum QFT
- Equilibrium and Non-equilibrium Thermodynamics
- Freeze-out of DM (and also Freeze-in DM)
- Gapped continuum DM from warped space model
- Realistic model building of gapped continuum DM
- Phenomenological study (both in terms of astrophysics/cosmology and collider)

Physics of Gapped Continuum DM

- CFT continuum case:
  - It's often stated that CFT's and theories with continuum spectra do not have a particle interpretation and no S-matrix can be defined: interactions leading to a non-trivial fixed point are also essential for producing the continuum spectrum of the theory
  - by turning off the interactions, the spectrum changes from continuum into that of an ordinary free particle, hence the asymptotic states defined in the usual manner would not capture the physics of the system properly
  - this means that one needs to find an alternative approach for defining scattering processes
- Our theoretic description of gapped continuum: Generalized Free Continuum (continuum analog of Generalized Free Fields: Greenberg 1961)

Also: Polyakov, early '70s- skeleton expansions

CFT completely specified by 2-point function-rest vanish

Generalized Free Continuum

♦ Generalized free continuum

-consider the case that the effects of the strong interactions can be captured by the fact that there is a non-trivial continuum (with a mass gap), and described by:

$$S = \int \frac{d^4p}{(2\pi)^4} \, \Phi^\dagger(p)\Sigma(p^2)\Phi(p)$$

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which is designed to properly reproduce the two-point function of theory

$$\int d^4x\, e^{ip(x-y)}\langle 0|T\Phi(x)\Phi^\dagger(y)|0\rangle = \langle 0|\Phi(p)\Phi^\dagger(-p)|0\rangle = \frac{i}{\Sigma(p^2)} = \int \frac{d\mu^2}{2\pi}\,\frac{i\,\rho(\mu^2)}{p^2-\mu^2+i\epsilon}$$

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- The above effective description is weakly coupled (resultin…
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This picture is supported by the concrete extra dimensional construction!

- In addition we perturb around generalized free continuum by introducing additional weak couplings to Φ and assume that the underlying structure described by the spectral density remains unchanged, resulting in a weakly interacting continuum.

Gapped Continuum QFT

Gapped Continuum Hilbert Space

- single-mode sector of the Hilbert space for continuum state consists of states $|p, \mu^2\rangle$

continuous parameter!

$$\hat{\mathbf{P}}\,|\mathbf{p},\mu^2\rangle = \mathbf{p}\,|\mathbf{p},\mu^2\rangle,$$
$$\hat{H}\,|\mathbf{p},\mu^2\rangle = \sqrt{\mathbf{p}^2+\mu^2}\,|\mathbf{p},\mu^2\rangle.$$

- Completeness relation (spectral density $\varrho(\mu^2)$ as the density of states):

$$\int \frac{d\mu^2}{2\pi}\,\rho(\mu^2)\int\frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\mathbf{p},\mu^2}}\,|\mathbf{p},\mu^2\rangle\langle\mathbf{p},\mu^2| = 1.$$

The completeness relation can also be rewritten as

$$\int\frac{d^4p}{(2\pi)^4}\rho(p^2)|\mathbf{p},\mu^2\rangle\langle\mathbf{p},\mu^2| = 1,$$

normalization (one particle state):

$$\langle\mathbf{p}',\mu'^2|\mathbf{p},\mu^2\rangle = \frac{2E_{\mathbf{p},\mu^2}}{\rho(\mu^2)}\,(2\pi)^4\delta^3(\mathbf{p}-\mathbf{p}')\,\delta(\mu^2-\mu'^2)$$

$p_0 = E_{\mathbf{p},\mu^2} = \sqrt{\mathbf{p}^2+\mu^2}$, and $p^2 = p_0^2 - \mathbf{p}^2$.

Gapped Continuum QFT

♦ Gapped Continuum Hilbert Space

- multi-mode states are built as direct products of single-mode states. e.g SM+SM→ DM+DM

$$\langle(\mathbf{p_1},\mu_1^2),(\mathbf{p_1},\mu_1^2)|\mathrm{Texp}\left(-i\int dt H_I(t)\right)|\mathbf{k}_A,\mathbf{k}_B\rangle_{\mathrm{SM}} \equiv (2\pi)^4\delta^4(k_1+k_2-p_1-p_2)\, i\mathcal{M}.$$

- Cross section:

$$\sigma = \frac{1}{2E_A}\frac{1}{2E_A}\frac{1}{|v_A-v_B|}\int\frac{d\mu_1^2}{2\pi}\rho(\mu_1^2)\int\frac{d\mu_2^2}{2\pi}\rho(\mu_2^2)\int d\Pi_1^{\mu_1^2}\, d\Pi_2^{\mu_2^2}\,(2\pi)^4\delta^4(k_1+k_2-p_1-p_2)\,|\mathcal{M}|^2$$

3D Lorentz-invariant phase space (LIPS) volume element: $d\Pi^{\mu^2} = \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\mathbf{p},\mu^2}}$

Equilibrium and Non-equilibrium Thermodynamics

♦ Equilibrium Thermodynamics

- Consider a dilute, weakly-coupled gas made out of the single-mode Gapped Continuum states

define the dimensionless phase-space density $f(p,\mu^2)$:

$$N = V \int \frac{d\mu^2}{2\pi} \rho(\mu^2) \int \frac{d^3p}{(2\pi)^3} f(\mathbf{p}, \mu^2)$$

If interactions among particles in the gas are strong enough to maintain them in thermal and chemical equilibrium with each other:

occupation number: $$f(\mathbf{p}, \mu^2) = \frac{1}{e^{\beta(E_{\mathbf{p},\mu^2} - \lambda)} \pm 1} \approx e^{-\beta(E_{\mathbf{p},\mu^2} - \lambda)}$$

- Free energy: $$F = \frac{1}{\beta} V \int \frac{d\mu^2}{2\pi} \rho(\mu^2) \int \frac{d^3p}{(2\pi)^3} \ln\left(1 - e^{-\beta E_{\mathbf{p},\mu^2}}\right)$$

$$u = \frac{1}{V}\left(\beta \frac{\partial F}{\partial \beta}|_V + F\right) = \int \frac{d\mu^2}{2\pi} \rho(\mu^2) \mathcal{U}(\mu^2)$$

$$P = -\frac{\partial F}{\partial V}|_\beta, \qquad = \int \frac{d\mu^2}{2\pi} \rho(\mu^2) \mathcal{P}(\mu^2)$$

Equilibrium and Non-equilibrium Thermodynamics

Equilibrium Thermodynamics

- Consider a dilute, weak

mode Gapped Continuum

-For T > μ_0 : energy and pressure are dominated by modes with $\mu_0 < \mu < T$, which behave as a relativistic gas

-At T < μ_0, energy and pressure are dominated by modes with $\mu \approx \mu_0$ (with details depending on behavior of spectral density in that region), which behave as a gas of non-relativistic particles. In this regime, the continuum gas can play the role of cold dark matter.

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$$N = V \int \frac{d\mu}{2\pi} \cdots$$

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♦ Non-equilibrium Thermodynamics

- Consider a dilute, weakly-coupled gas of "continuum" states, but do not assume that it is in thermal and/or chemical equilibrium

phase-space density is function of time: $f(p,\mu^2,t)$:

Boltzmann equation for toy model with 2 ↔ 2 scattering ($m_{SM} \ll \mu_0$):

$$E_\mu \frac{\partial f(\mathbf{p},\mu^2,t)}{\partial t} = -\frac{1}{2}\int \frac{d\mu'^2}{2\pi}\,\rho(\mu'^2)\int d\Pi_{\mu'}\,d\Pi_A d\Pi_B\,(2\pi)^4\delta^4(k_A+k_B-p-p')$$

$$\times|\mathcal{M}|^2\,\big(ff'(1\pm f_A)(1\pm f_B)-f_Af_B(1\pm f)(1\pm f')\big)\,,$$

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- Generalization to gas in FRW background:

$$E_\mu \frac{\partial f(E,\mu^2,t)}{\partial t} - H|\mathbf{p}|^2\frac{\partial f(E,\mu^2,t)}{\partial E} = -\frac{1}{2}\int \frac{d\mu'^2}{2\pi}\,\rho(\mu'^2)\int d\Pi_{\mu'}\,d\Pi_A d\Pi_B$$

$$\times(2\pi)^4\delta^4(k_A+k_B-p-p')\,|\mathcal{M}|^2\,\left(ff'-f_Af_B\right).$$

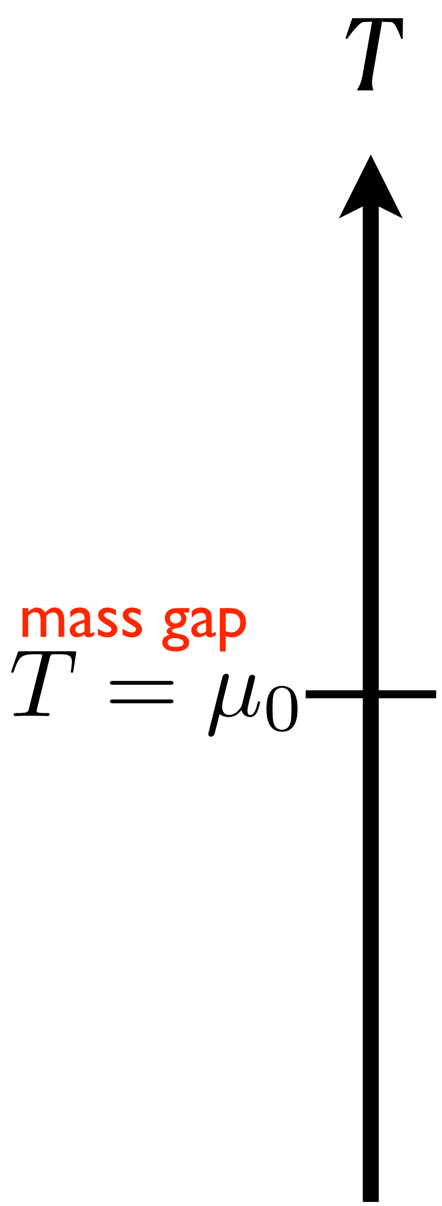
$H=\dot{a}/a$ is the Hubble, $|\mathbf{p}|^2=E^2-\mu^2$;

Freeze-Out of Gapped Continuum DM

$m_{SM} \ll \mu_0$

annihilation: DM+DM ↔ SM+SM

quasi-elastic scattering (QES): DM+SM ↔ DM+SM



$T > \mu_0$

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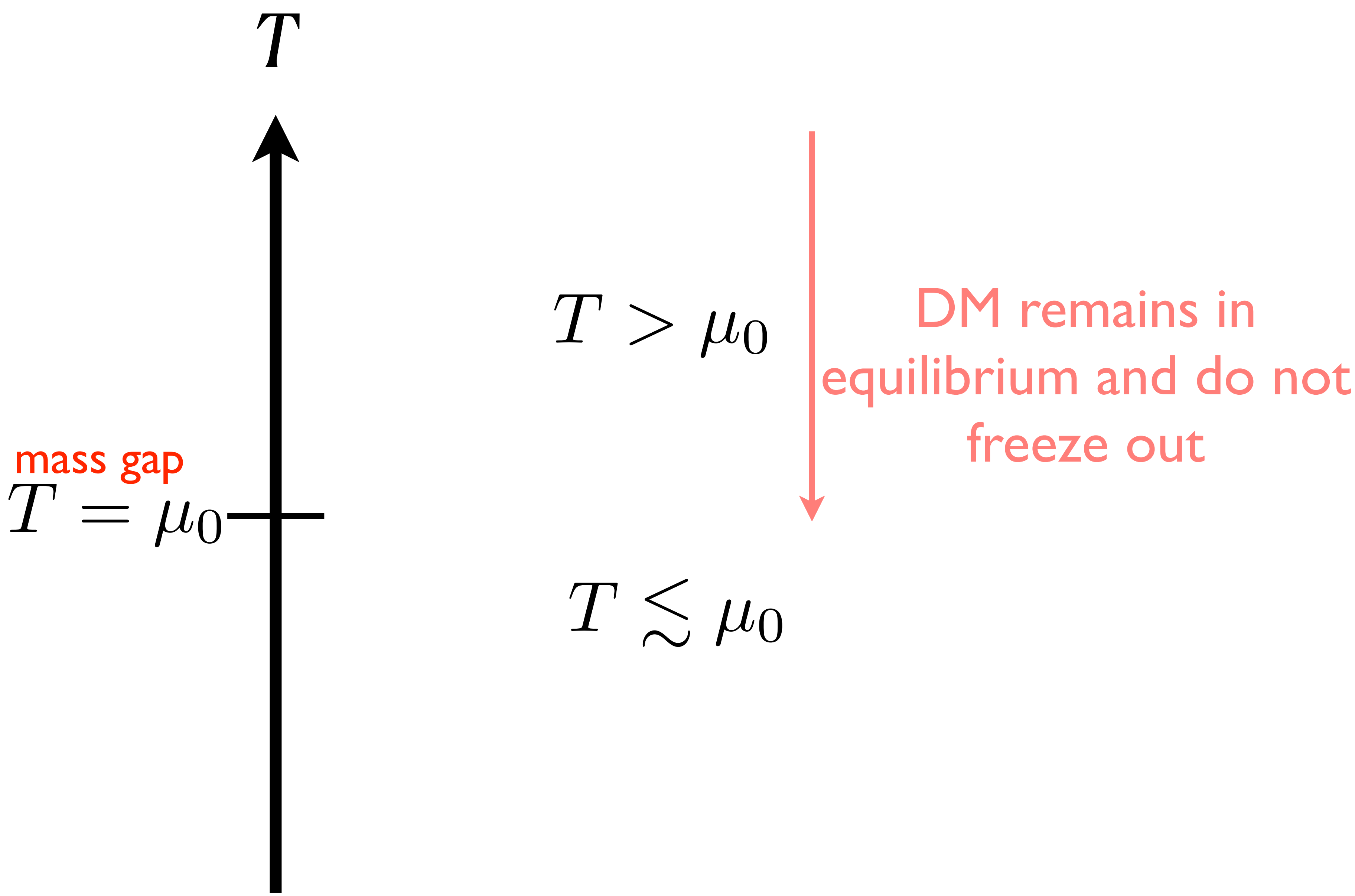

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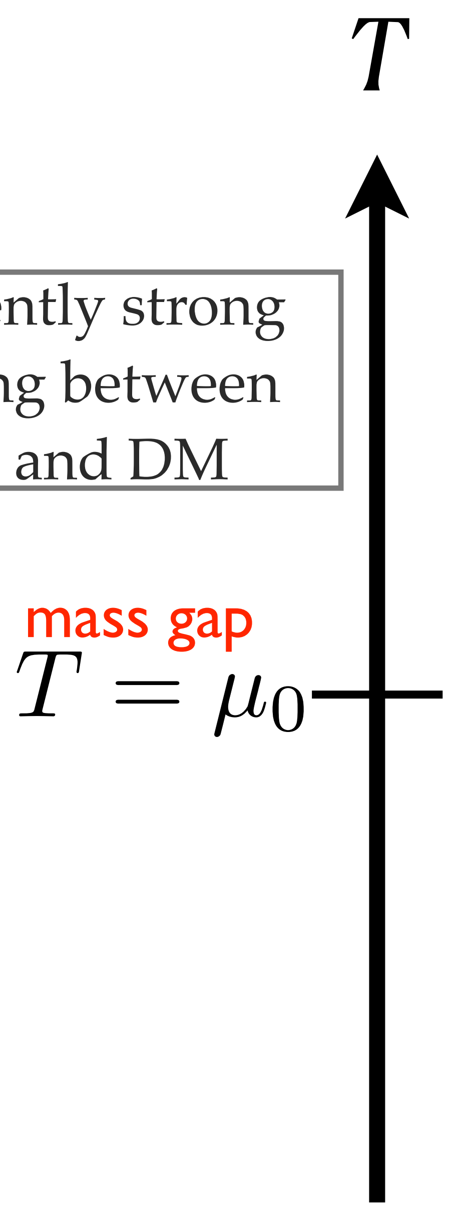

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annihilation is in equilibrium, DM particles are at the same temperature T as the SM and is at zero chemical potential

DM remains in equilibrium and do not freeze out

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annihilation rate drops exponentially, and annihilations decouple

"Freeze out"

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"Freeze out"

rate of quasi-elastic scattering of a DM particle does not experience an exponential drop : maintain thermal equilibrium between the SM and DM (same T, and chemical)

$$f_{\rm DM} = e^{-\beta\left(E_{\mathbf{p},\mu^2} - \lambda(t)\right)}, \qquad f_{\rm SM} = e^{-\beta|\mathrm{p}|}.$$

Freeze-Out of Gapped Continuum DM

♦ Boltzmann Equation for Continuum Freeze-Out

- during the freeze-out process, the DM modes are at the same temperature as the SM, T, and have a common (μ-independent) chemical potential η, which however is time-dependent and no longer vanishes. This means that in the freeze-out calculations we can assume

$$f_{\rm DM} = e^{-\beta(E_\mu - \eta(t))}, \qquad f_{\rm SM} = e^{-\beta|{\rm p}|}.$$

effective DM number density :

$$n = \int \frac{d\mu^2}{2\pi}\,\rho(\mu^2) \int \frac{d^3p}{(2\pi)^3}\, f_{\rm DM}$$

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→

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  - Integrating both sides of the Boltzmann equation eq.:

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Freeze-Out of Gapped Continuum DM

- Evolution of DM number density
  (Integrating both sides of the Boltzmann equation)

$$\frac{\partial n}{\partial t} + 3Hn = -\langle \sigma v \rangle (n^2 - n_{\rm eq}^2)$$

identical to that of the usual particle cold relic!

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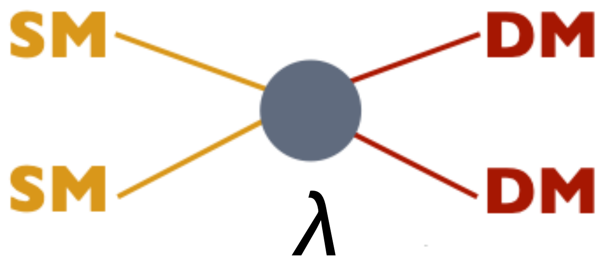
continuum physics

$$\langle \sigma v \rangle = \frac{1}{n_{\text{eq}}^2} \int \frac{d\mu^2}{2\pi} \rho(\mu^2) \int \frac{d\mu'^2}{2\pi} \rho(\mu'^2) \int d\Pi_{\mathbf{p}}^{\mu^2} \, d\Pi_{\mathbf{p}'}^{\mu'^2} \, d\Pi_A d\Pi_B \\ \times (2\pi)^4 \delta^4(k_A + k_B - p - p') \, |\mathcal{M}|^2 \, \exp\left(-\beta(E_A + E_B)\right) \, .$$

Freeze-Out of Gapped Continuum DM

- Toy Model: scalar Gapped Continuum DM coupled to SM scalars

$$\mathcal{M} = \lambda \qquad \text{@ tree level}$$



$$\langle \sigma v \rangle = \frac{\lambda^2}{32\pi} \left(\frac{I_1(\beta)}{I_2(\beta)} \right)^2 .$$

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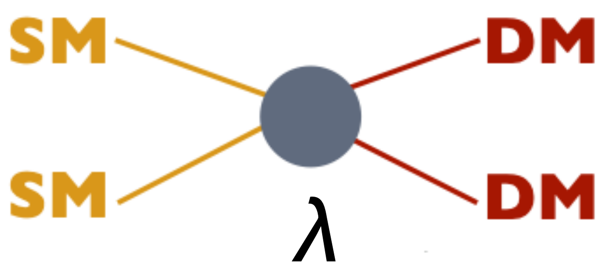

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$$I_n(\beta) = \int \frac{d\mu^2}{2\pi} \rho(\mu^2)\, \mu^n\, K_n(\beta\mu)$$

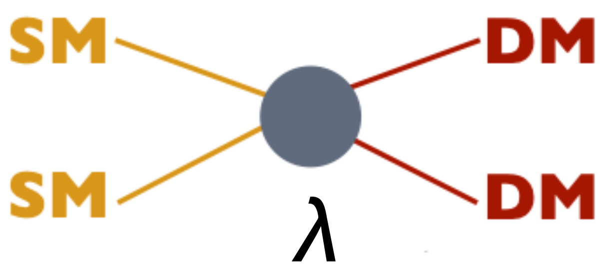
Example: 5D motivated toy spectral density

$$\rho(\mu^2) = 2\pi\rho_0 \left(\frac{\mu^2}{\mu_0^2} - 1 \right)^{1/2}$$

Freeze-Out of Gapped Continuum DM

- Toy Model: scalar Gapped Continuum DM coupled to SM scalars

$$\mathcal{M} = \lambda \qquad \text{@ tree level}$$



$$\langle \sigma v \rangle = \frac{\lambda^2}{32\pi} \left(\frac{I_1(\beta)}{I_2(\beta)} \right)^2 .$$

$$I_n(\beta) = \int \frac{d\mu^2}{2\pi} \rho(\mu^2)\, \mu^n\, K_n(\beta\mu)$$

Example: 5D motivated toy spectral density

$$\rho(\mu^2) = 2\pi\rho_0 \left(\frac{\mu^2}{\mu_0^2} - 1 \right)^{1/2} \quad \longrightarrow \quad \langle \sigma v \rangle = \frac{\lambda^2}{32\pi\mu_0^2}$$

Dark Matter Continuum Spectral Density from 5D Model

- modeling generalized free continuum by Warped 5D model

$$ds^2 = e^{-2A(y)}dx^2 + dy^2$$

- warped 5D setup we will have a 3-brane placed at the position z = R, which from the point of view of the gapped continuum field will be a UV brane cutting off the space

The 5D action of the coupled scalar-gravity system

$$S = \int d^5x\sqrt{g}\left(-M^3R + \frac{1}{2}(\partial\phi) - V(\phi)\right) - \int d^4x\sqrt{g^{ind}}V_4(\phi)$$

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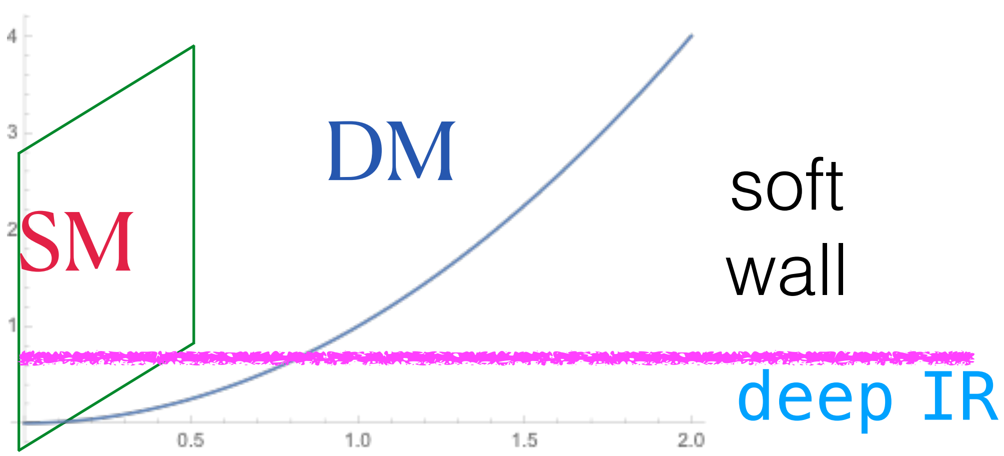
- The superpotential (w/ relation $V = 3W'^2 - 12W^2$) leading to the desired 5D background : $W = k(1 + e^{\phi})$

(fully includes the backreaction of the metric to the presence of the scalar field)



Solution

$$A(y) = -\log\left(1 - \frac{y}{y_s}\right) + ky,$$

$$\phi(y) = -\log\left(k(y_s - y)\right),$$

y_s is the finite location of the curvature singularity

Dark Matter Continuum Spectral Density from 5D Model

- Warped 5D model
  - Scalar gapped continuum: $\mathcal{L} = \sqrt{g}\left[\frac{1}{2}g^{MN}D_M\Phi^\dagger D_N\Phi - V(\Phi)\right]$

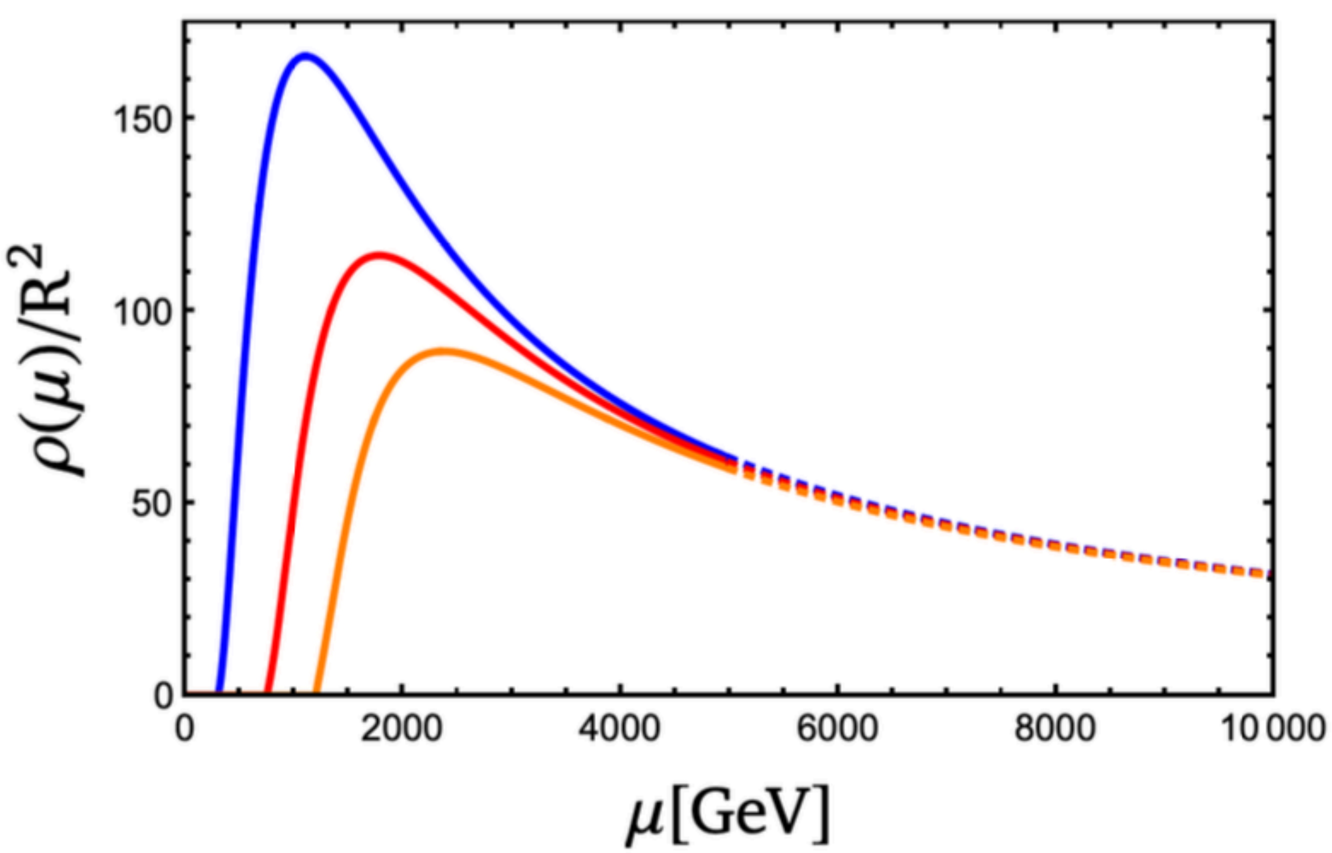
In conformally flat coordinate, Schrödinger form of eom: $\psi = e^{-\frac{3}{2}A}\Phi$

$$\left(-\partial_z^2 + \hat{V}(z)\right)\Psi(z) = p^2\Psi(z)$$

$$V(z) = \frac{3}{4y_s^2}e^{-2ky}\left(5k^2(y-y_s)^2 - 10k(y-y_s) + 3\right)$$

$$G(R,R;p) = \left(\frac{\Phi'(R,p)}{\Phi(R,p)}\right)^{-1}$$

$$\rho(p) = \frac{1}{\pi}\mathrm{Im}\, G(R,R;p).$$



Dark Matter Continuum Spectral Density from 5D Model

- Warped 5D model

  - Scalar gapped continuum near the gap:

In conformally flat coordinate, Schrödinger form of eom:

$$\left(-\partial_z^2 + \hat{V}(z)\right)\Psi(z) = p^2\Psi(z)$$

$$\lim_{z\to\infty} \hat{V}(z) = \mu_0^2 \left(1 + e^{-2z(2\mu_0/3)} + \frac{8}{3}e^{-z(2\mu_0/3)}\right)$$

$$\Psi(z,\mu) = C\, L_m^n(3e^{-2z\mu_0/3}) \exp\left(\frac{3}{2}\sqrt{1-\frac{\mu^2}{\mu_0^2}}\log\left(e^{-\frac{2\mu_0 z}{3}}\right) - \frac{3}{2}e^{-\frac{2\mu_0 z}{3}}\right)$$

can expand the arguments of the Laguerre polynomial around the mass gap

$$\rho(p) = \frac{1}{\pi}\mathrm{Im}\, G(R,R;p).$$

$$\rightarrow \quad \rho(\mu^2) \propto \left(\frac{\mu^2}{\mu_0^2} - 1\right)^{1/2}$$

Dark Matter Continuum Spectral Density from 5D Model

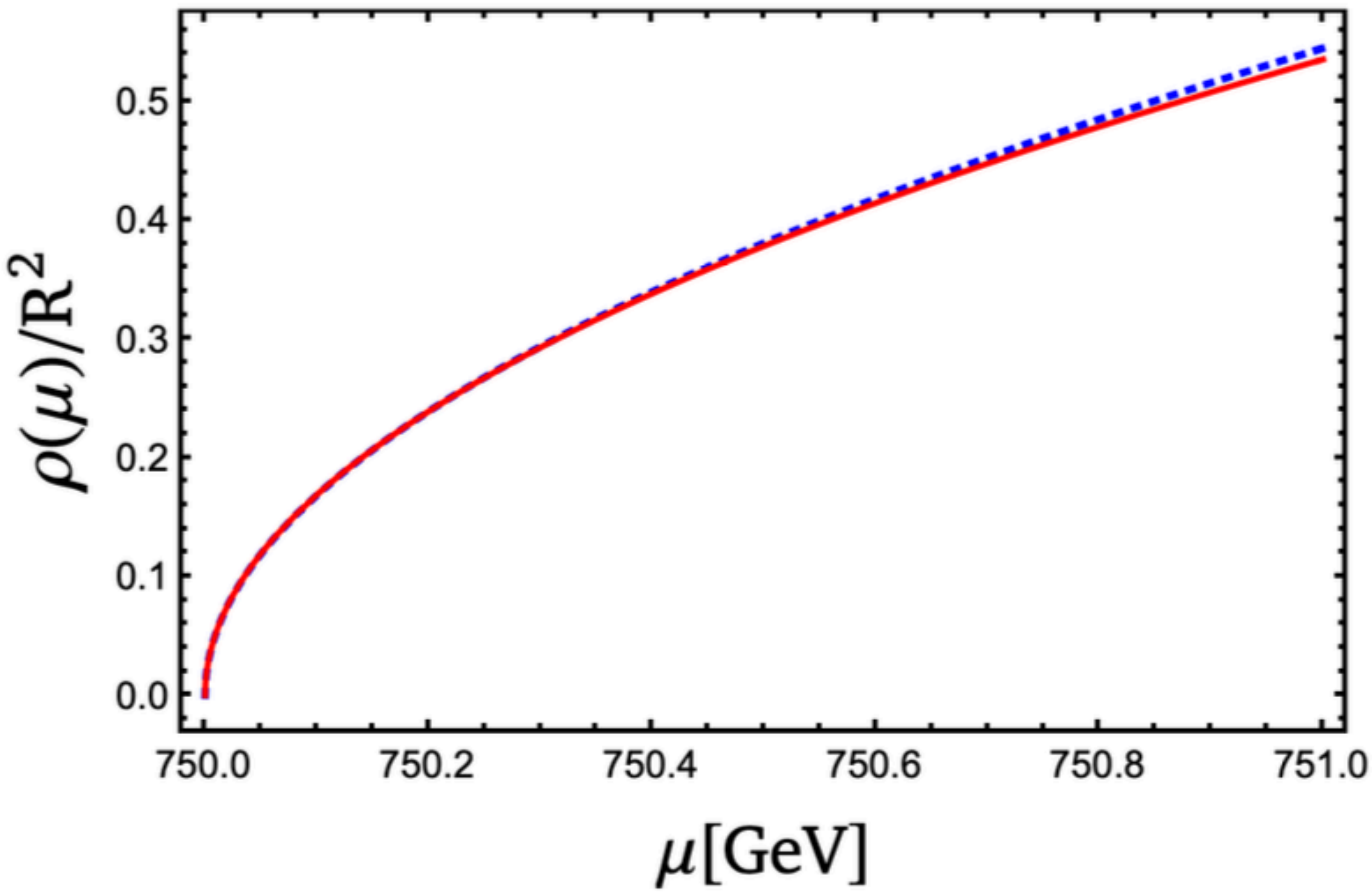
♦ Warped 5D

- Scalar gap

In conformally

$\lim_{z\to\infty} \hat{V}(z) = \mu$

$\Psi(z,\mu) = ($



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→

$$\rho(\mu^2) \propto \left(\frac{\mu^2}{\mu_0^2} - 1\right)^{1/2}$$

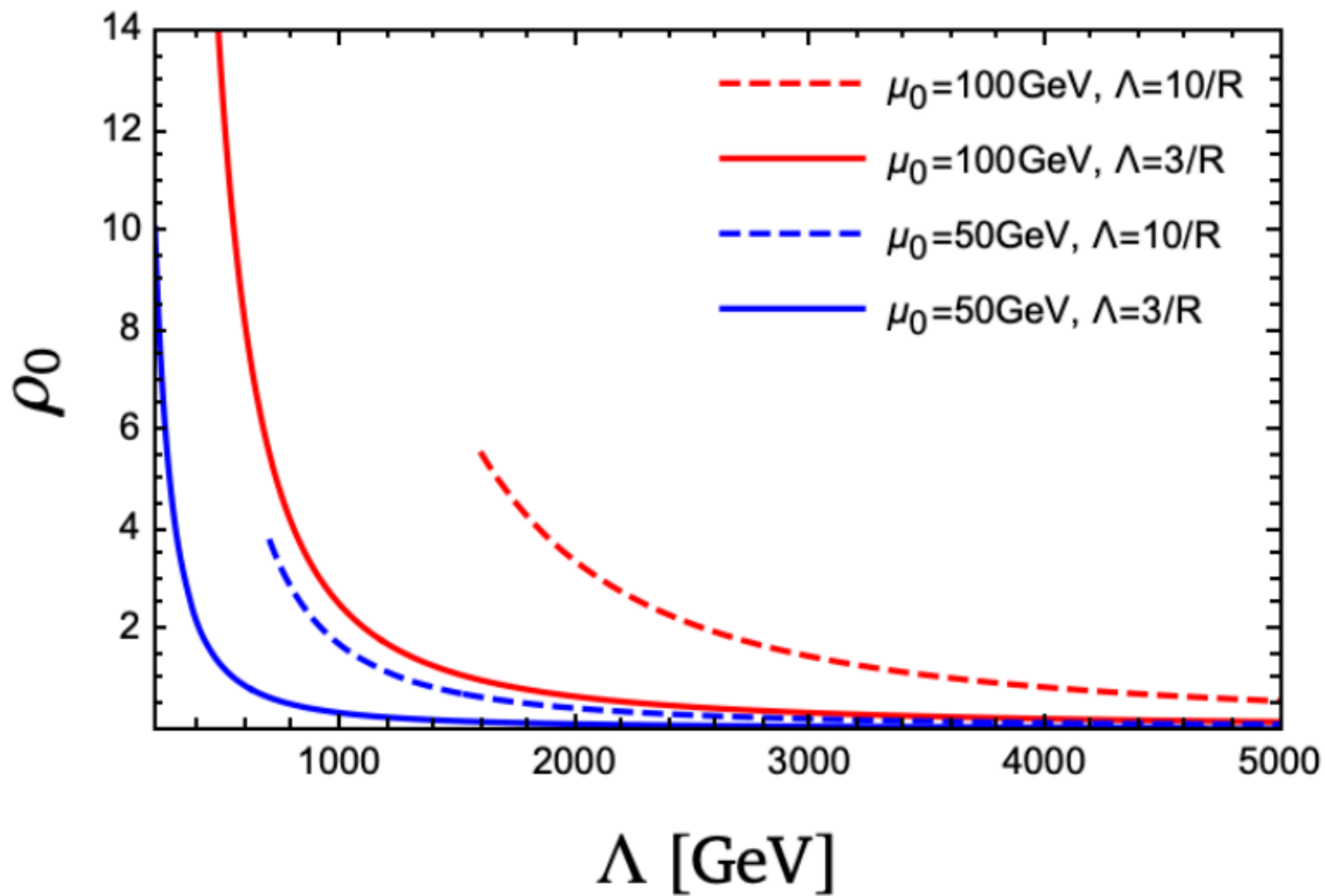
Dark Matter Continuum Spectral Density from 5D Model

- Warped 5D (generalized free) gapped continuum

  - Spectral density of the scalar gapped continuum near the mass gap:

$$\rho(\mu^2) = \frac{\rho_0}{\mu_0^2}\left(\frac{\mu^2}{\mu_0^2} - 1\right)^{1/2}$$

$$\int_{\mu_0^2}^{\Lambda^2} \frac{d\mu^2}{2\pi}\rho(\mu^2) = 1$$



Gapped Continuum Z-portal DM

Z-portal Model

- Consider a complex scalar field Φ with no SM gauge quantum numbers (this plays the role of DM field, and is lifted to 5D), and another complex scalar field χ which is a doublet under $SU(2)_L$ and carries $U(1)_Y$ charge −1/2

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\Phi} + \mathcal{L}_{\chi} + \mathcal{L}_{\rm int}$$

$$\mathcal{L}_{\Phi} = \Phi^\dagger(p)\Sigma(p^2)\Phi(p)$$

$$\mathcal{L}_{\chi} = (D_\mu\chi)^\dagger (D^\mu\chi) - m_\chi^2\chi^\dagger\chi$$

$$\mathcal{L}_{\rm int} = -\lambda\Phi\,\chi H + \text{c.c.}$$

includes couplings to the SM Z and $U(1)_Y$

spectral density: $$\rho(p^2) = \frac{1}{\pi}{\rm Im}\Sigma^{-1}(p^2)$$

- When the Higgs gets a vev, $L_{\rm int}$-term induces mass mixing between Φ and the neutral components of χ. The mass eigenstates are

$$\tilde\Phi = \cos\alpha\,\Phi + \sin\alpha\,\chi^0, \qquad \tilde\chi^0 = -\sin\alpha\,\Phi + \cos\alpha\,\chi^0.$$

Gapped Continuum Z-portal DM

Z-portal Model

$$\mathcal{L} = \sqrt{g^2 + g'^2}\,\sin^2\alpha\,\left(\tilde{\Phi}_2\partial_\mu\tilde{\Phi}_1 - \tilde{\Phi}_1\partial_\mu\tilde{\Phi}_2\right)\,Z^\mu$$

The mixing angle is given by

$$\tan 2\alpha = \frac{2\lambda v}{m_\phi^2 - m_\chi^2}$$

Gapped Continuum Z-portal DM

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The salient feature of the gapped continuum DM from our preliminary study is that there is a generic rate suppression, which makes it compatible with the current null result of direct detection experiments.

Gapped Continuum Z-portal DM

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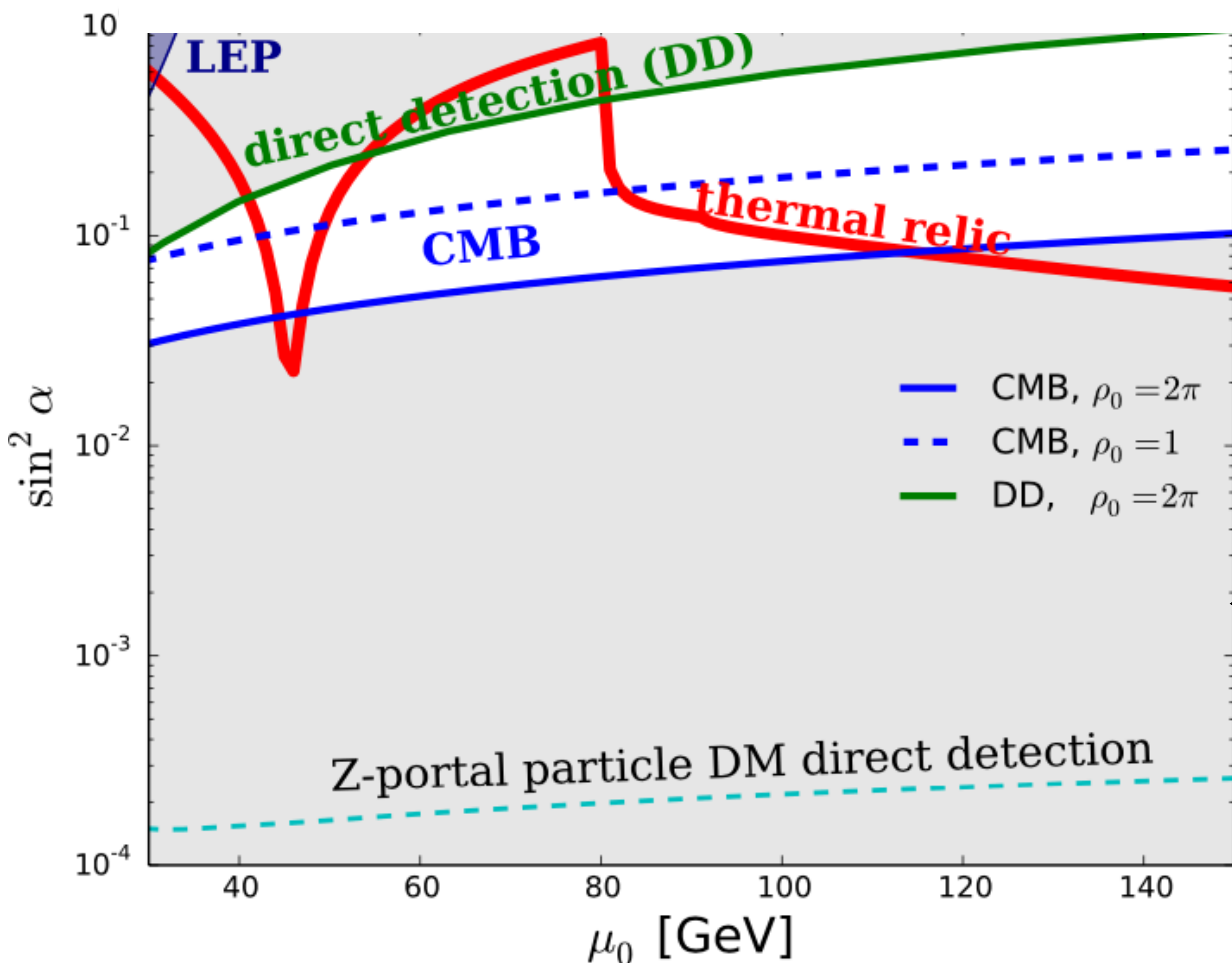

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Gapped Continuum Z-portal DM

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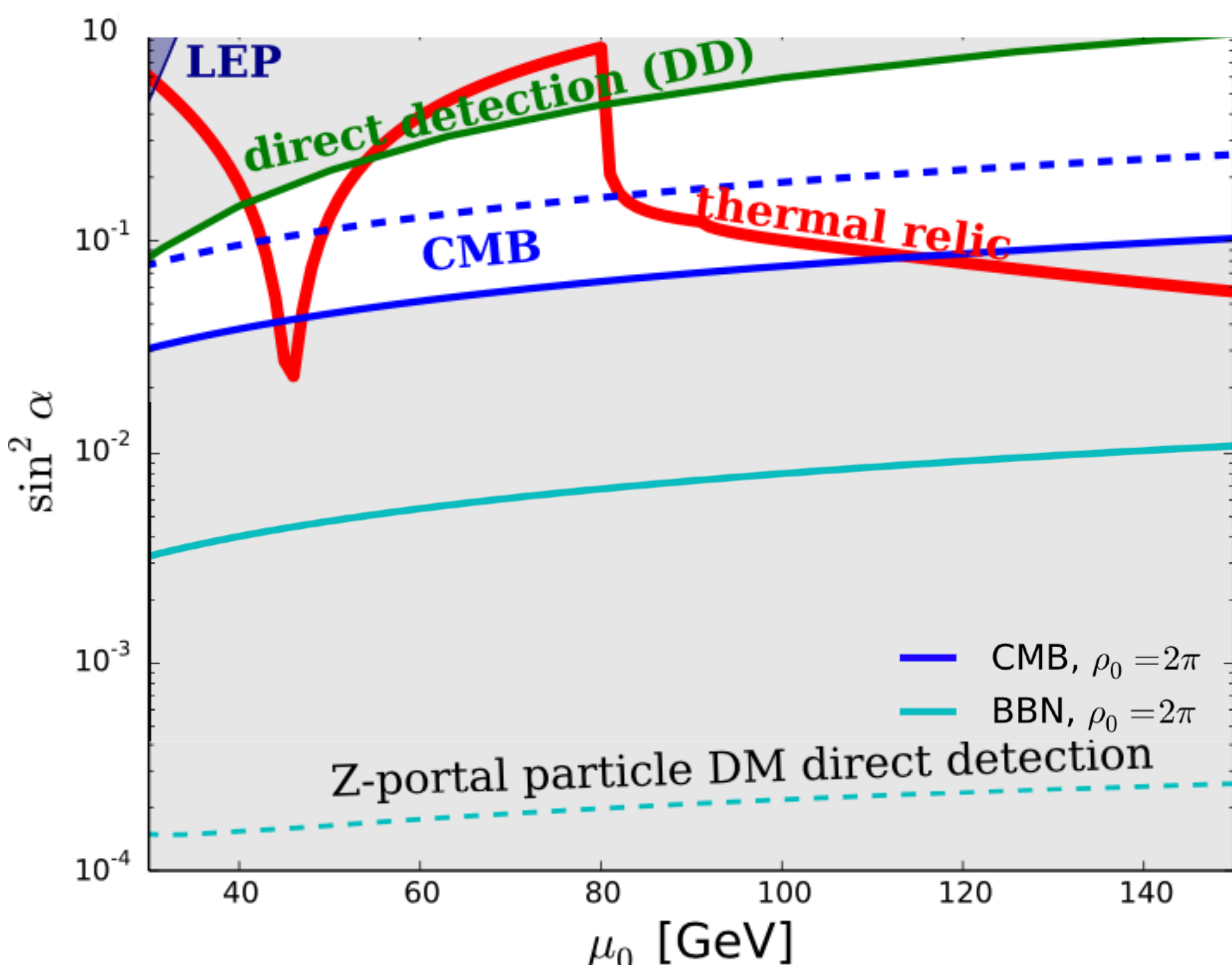

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Summary

New DM Paradigm: Weakly Interacting Massive Particle → Weakly Interacting Massive Continuum

1. Gapped Continuum DM = theoretically and phenomenologically motivating!

2. Continuum Kinematics : late decay, relaxation of direct detection bound

3. Revival of Weakly Interacting Massive Continuum (WIC) !

4. Many possible models + many detailed pheno study to be done.

5. Continuum Collider Physics = totally new → needs a systematic investigations

6. Many more...

Thank You!

Back-up

Gapped Continuum Z-portal DM

♦ 5D Z-portal Model

$$S = S_{\text{bulk}} + S_{\text{UV}}$$
$$S_{\text{bulk}} = \int d^4x dy \, \sqrt{g} \left(g^{MN} \left(\partial_M \Phi\right)^\dagger \left(\partial_N \Phi\right) - m^2 |\Phi|^2 \right)$$
$$S_{\text{UV}} = \int_{\text{UV}} d^4x \, \left(\mathcal{L}_{\text{SM}} + |D_\mu \chi|^2 - m_\chi^2 |\chi|^2 - \lambda \Phi X H + \text{h.c.}\right).$$

extra boundary term on the UV brane

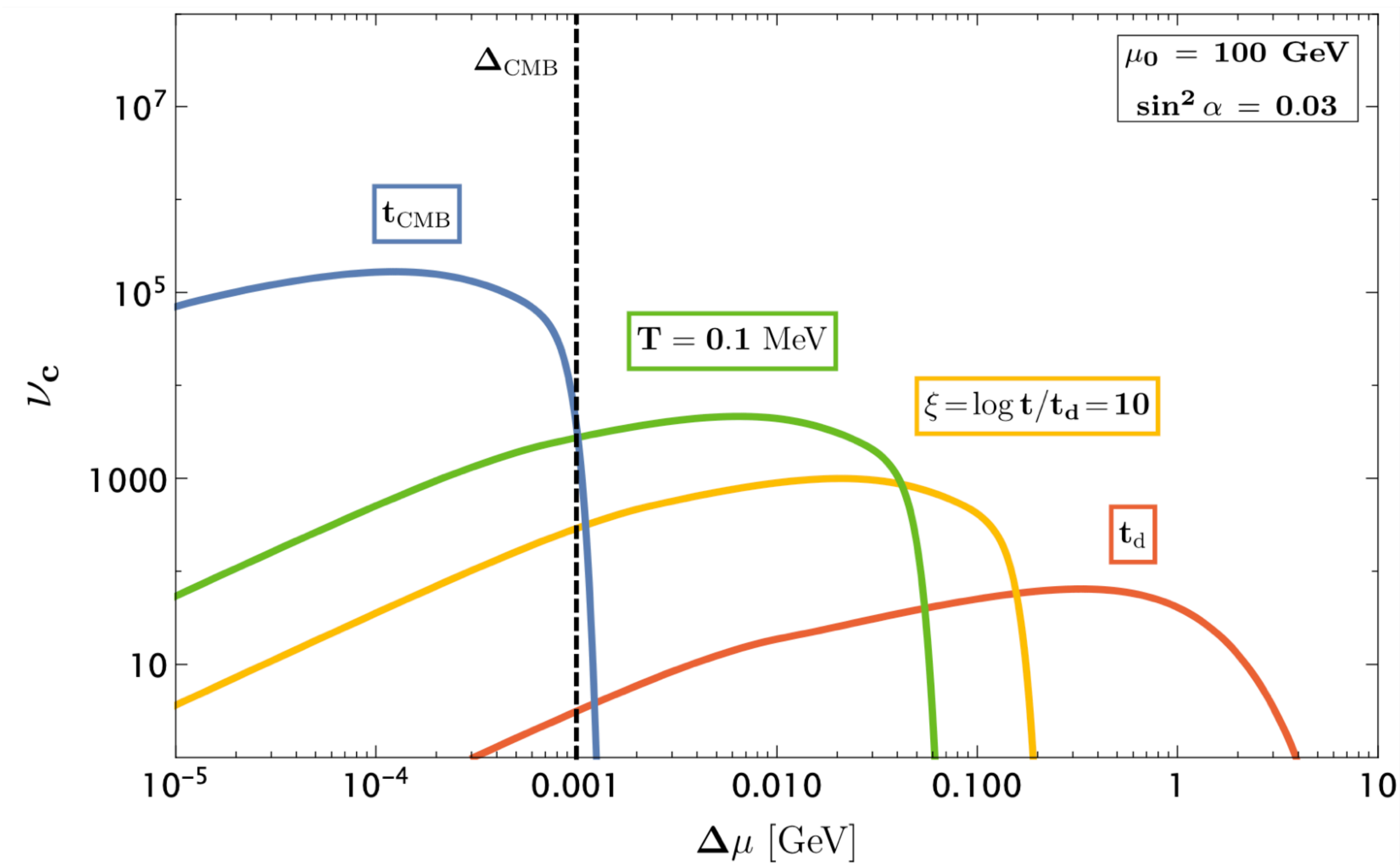
$$\Delta S_{\text{UV}} = \int_{\text{UV}} d^4x \, \Phi^\dagger \partial_y \Phi.$$

$$\Phi(p, z) = k^{1/2} \frac{f(z,p)}{f(R,p)} \hat{\phi}(p), \quad p = \sqrt{p^\mu p_\mu}$$

$$\left(-\partial_z^2 + V(z)\right) f = p^2 f$$
$$V(z) = m^2 e^{-2A} + \frac{9}{4} \left(A'\right)^2 - \frac{3}{2} A''.$$

$$V(z) \to \frac{9 e^{-2ky_s}}{4 y_s^2} = \mu_0^2 \quad \text{as} \quad z \to \infty$$

$$S_{\text{eff}} = \int \frac{d^4p}{(2\pi)^4} \, k \hat{\phi}^\dagger(p) \left(\frac{\partial_z f(z,p)}{f(z,p)} \bigg|_{z=R} \right) \hat{\phi}(p)$$
$$+ \int d^4x \, \left(\mathcal{L}_{\text{SM}} + |D_\mu \chi|^2 - m_\chi^2 |\chi|^2 - \lambda \Phi X H + \text{h.c.}\right)$$



$\mu_0 = 100$ GeV
$\sin^2\alpha = 0.03$
$\Delta_{\rm CMB}$
$t_{\rm CMB}$
$T = 0.1$ MeV
$\xi = \log t/t_d = 10$
t_d
ν_c
10^7
10^5
1000
10
10^{-5}
10^{-4}
0.001
0.010
0.100
1
10
$\Delta\mu$ [GeV]