Cosmological tensions: hints for a new concordance model?

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Cosmology Frontier in Particle Physics: Astroparticle Physics and Early Universe
NTU & NTHU

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All the models are wrong, but some are useful

The model that has now practically been selected as the "standard" cosmological model is the Lambda Cold Dark Matter (ΛCDM) model, that provides a remarkable fit to the bulk of available cosmological data.

However, despite its incredible success,
ΛCDM harbours large areas of phenomenology and ignorance.
For example, it still cannot explain key concepts in our understanding of the structure and evolution of the Universe, at the moment based on unknown quantities, that are also its largest components.
In addition, their physical evidence comes from cosmological and astrophysical observations only, without strong theoretical motivations.

The ΛCDM model

Unknown quantities:

- an early stage of accelerated expansion (Inflation) which produces the initial, tiny, density perturbations, needed for structure formation.
- a clustering matter component to facilitate structure formation (Dark Matter),
- an energy component to explain the current stage of accelerated expansion (Dark Energy).

Specific solutions for ΛCDM:

- Inflation is given by a single, minimally coupled, slow-rolling scalar field;
- Dark Matter is a pressureless fluid made of cold, i.e., with low momentum, and collisionless particles;
- Dark Energy is a cosmological constant term.

Warning!

Therefore, the 6 parameter ΛCDM model lacks the deep underpinnings a model requires to approach fundamental physics laws.
It can be rightly considered, at best, as
an effective theory of an underlying physical theory, yet to be discovered.
In this situation, we must be careful not to cling to the model too tightly or to risk missing the appearance of departures from the paradigm.

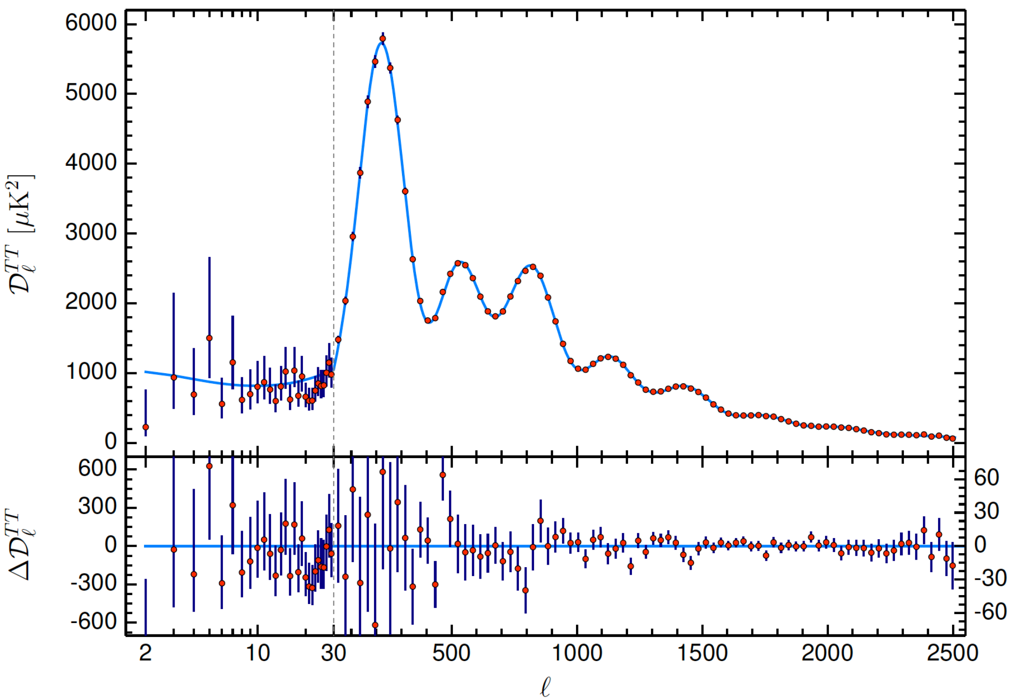
With the improvement of the number and the accuracy of the observations,
deviations from ΛCDM may be expected.
And, actually, discrepancies among key cosmological parameters of the models have emerged with different statistical significance.

While some proportion of these discrepancies may have a systematic origin, their persistence across probes should require multiple and unrelated errors, strongly hinting at cracks in the standard cosmological scenario and the necessity of new physics.

These tensions can indicate a failure of the canonical ΛCDM model.

CMB constraints

Most of the anomalies and tensions are involving the Planck data.



Parameter	TT+lowE	TE+lowE	EE+lowE	TT,TE,EE+lowE	TT,TE,EE+lowE+lensing 68% limits	TT,TE,EE+lowE+lensing+BAO 68% limits
$\Omega_b h^2$					0.02237 ± 0.00015	0.02242 ± 0.00014
$\Omega_c h^2$					0.1200 ± 0.0012	0.11933 ± 0.00091
$100\theta_{MC}$					1.04092 ± 0.00031	1.04101 ± 0.00029
τ					0.0544 ± 0.0073	0.0561 ± 0.0071
$\ln(10^{10} A_s)$					3.044 ± 0.014	3.047 ± 0.014
n_s					0.9649 ± 0.0042	0.9665 ± 0.0038
H_0 [km s^{-1} Mpc^{-1}]					67.36 ± 0.54	67.66 ± 0.42
Ω_Λ					0.6847 ± 0.0073	0.6889 ± 0.0056
Ω_m					0.3153 ± 0.0073	0.3111 ± 0.0056
$\Omega_m h^2$					0.1430 ± 0.0011	0.14240 ± 0.00087
$\Omega_m h^3$					0.09633 ± 0.00030	0.09635 ± 0.00030
σ_8					0.8111 ± 0.0060	0.8102 ± 0.0060
$S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$	0.840 ± 0.024	0.794 ± 0.024	$0.781^{+0.052}_{-0.060}$	0.834 ± 0.016	0.832 ± 0.013	0.825 ± 0.011

Planck 2018, Astron.Astrophys. 641 (2020) A6

2018 Planck results are a wonderful confirmation of the flat standard ΛCDM cosmological model, but are model dependent!

The most statistically significant and persisting anomalies and tensions of the CMB are:

- H0 with local measurements
- A_L internal anomaly
- S8 with cosmic shear data
- Ω_K different from zero

See Di Valentino et al. arXiv:2008.11283 [astro-ph.CO], arXiv:2008.11284 [astro-ph.CO], arXiv:2008.11285 [astro-ph.CO], arXiv:2008.11286 [astro-ph.CO] for an overview.

The most statistically significant and persisting anomalies and tensions of the CMB are:

- $H0$ with local measurements
- A_L internal anomaly
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See Di Valentino et al. arXiv:2008.11283 [astro-ph.CO], arXiv:2008.11284 [astro-ph.CO], arXiv:2008.11285 [astro-ph.CO], arXiv:2008.11286 [astro-ph.CO] for an overview.

The H0 tension

The most statistically significant, long-lasting and widely persisting tension is the H0 disagreement between

- The Planck estimate assuming a “vanilla" ΛCDM cosmological model:

Planck 2018, Astron.Astrophys. 641 (2020) A6

$H0 = 67.27 \pm 0.60$ km/s/Mpc in ΛCDM

- the local measurements obtained by the SH0ES collaboration.

The so called R19:

Riess et al. *Astrophys.J.* 876 (2019) 1, 85

$H0 = 74.03 \pm 1.42$ km/s/Mpc

4.4σ

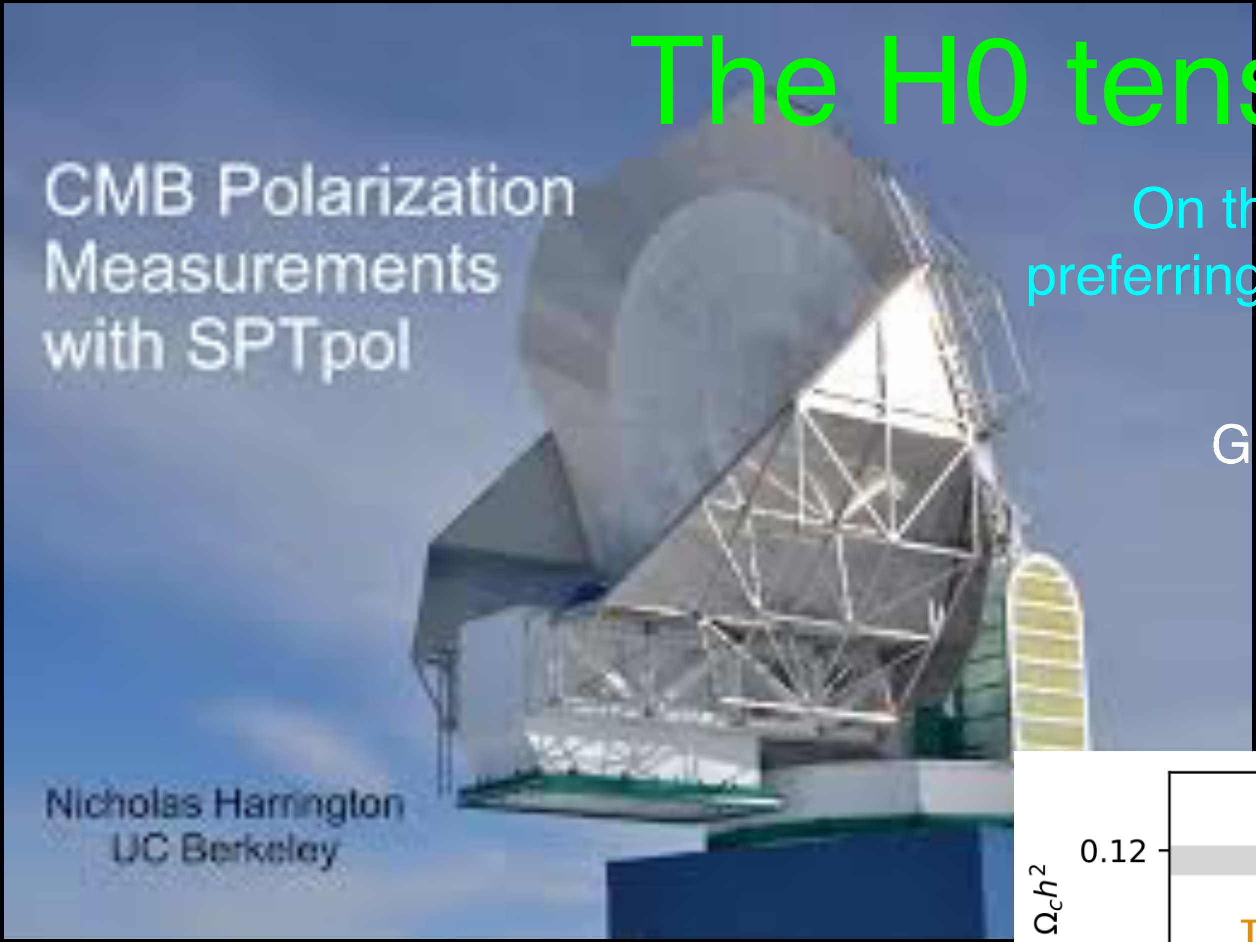
or R20 using the parallax measurements of Gaia EDR3:

Riess et al., Astrophys.J.Lett. 908 (2021) 1, L6

$H0 = 73.2 \pm 1.3$ km/s/Mpc

4.2σ

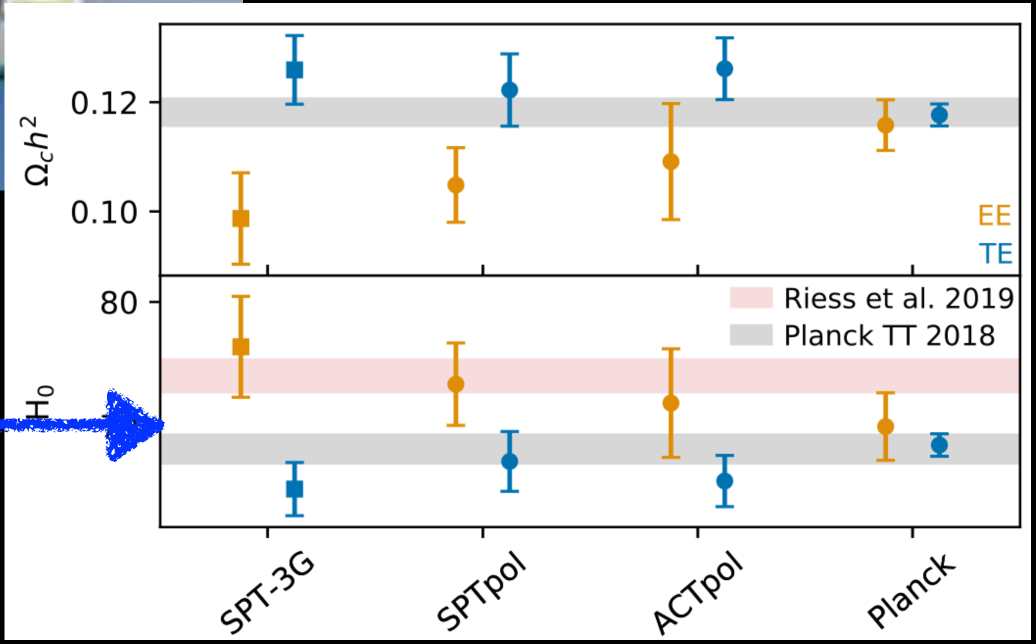
The H0 tension



On the same side of Planck, i.e. preferring smaller values of H_0 we have:

Ground based CMB telescope

SPT-3G:
H0 = 68.8 ± 1.5 km/s/Mpc in ΛCDM



SPT-3G, arXiv:2101.01684 [astro-ph.CO]

LCDM - dependent

The H0 tension

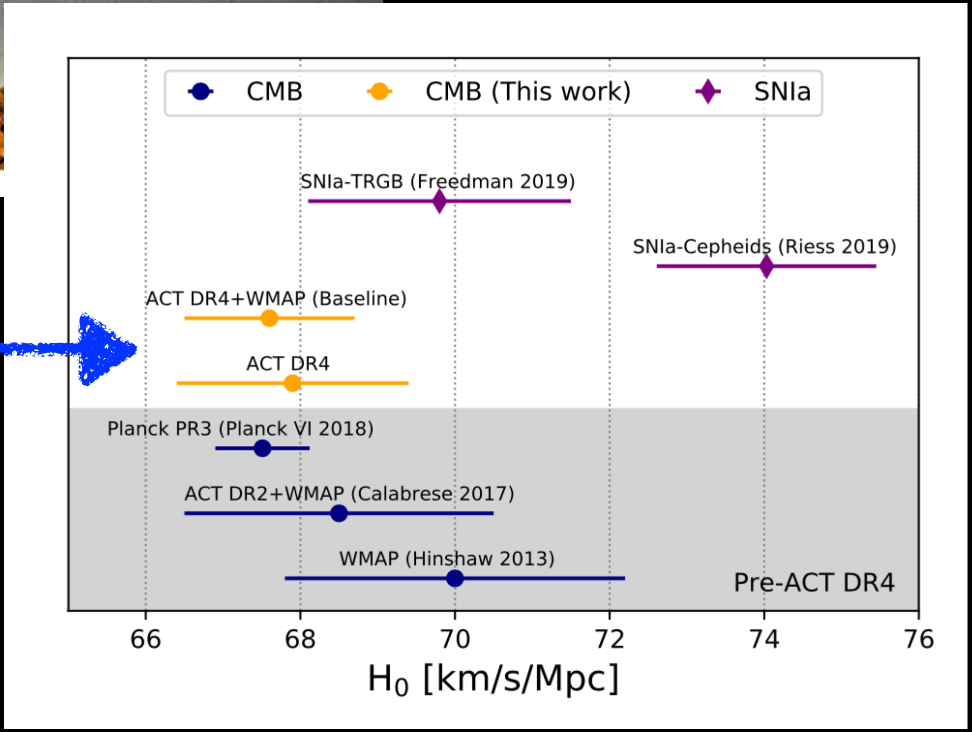
On the same side of Planck, i.e. preferring smaller values of H_0 we have:

Ground based CMB telescope

ACT-DR4:
H0 = 67.9 ± 1.5 km/s/Mpc in ΛCDM

ACT-DR4 + WMAP:
H0 = 67.6 ± 1.1 km/s/Mpc in ΛCDM

LCDM - dependent



ACT-DR4 2020, *JCAP* 12 (2020) 047

The H0 tension

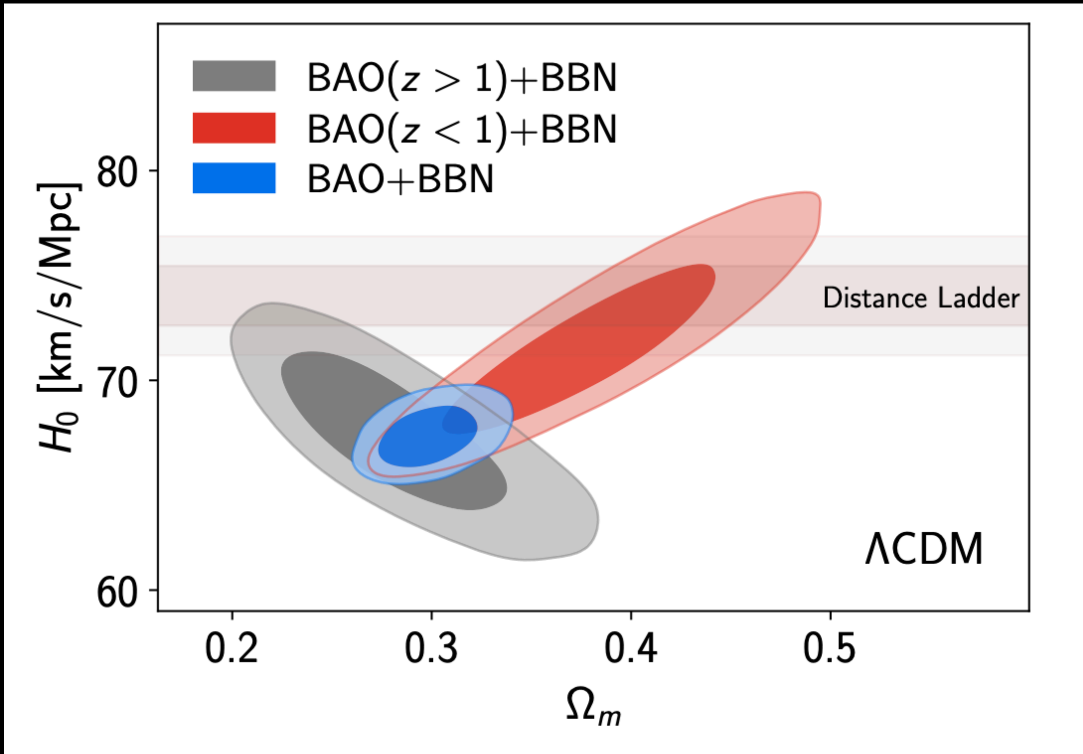
On the same side of Planck, i.e.
preferring smaller values of H_0 we have:

BAO+Pantheon+BBN+$\theta_{MC, Planck}$:
H0 = 67.9 ± 0.8 km/s/Mpc

Planck 2018, Aghanim et al., arXiv:1807.06209 [astro-ph.CO]

BAO+BBN from BOSS and eBOSS:
H_0 = 67.35 ± 0.97 km/s/Mpc

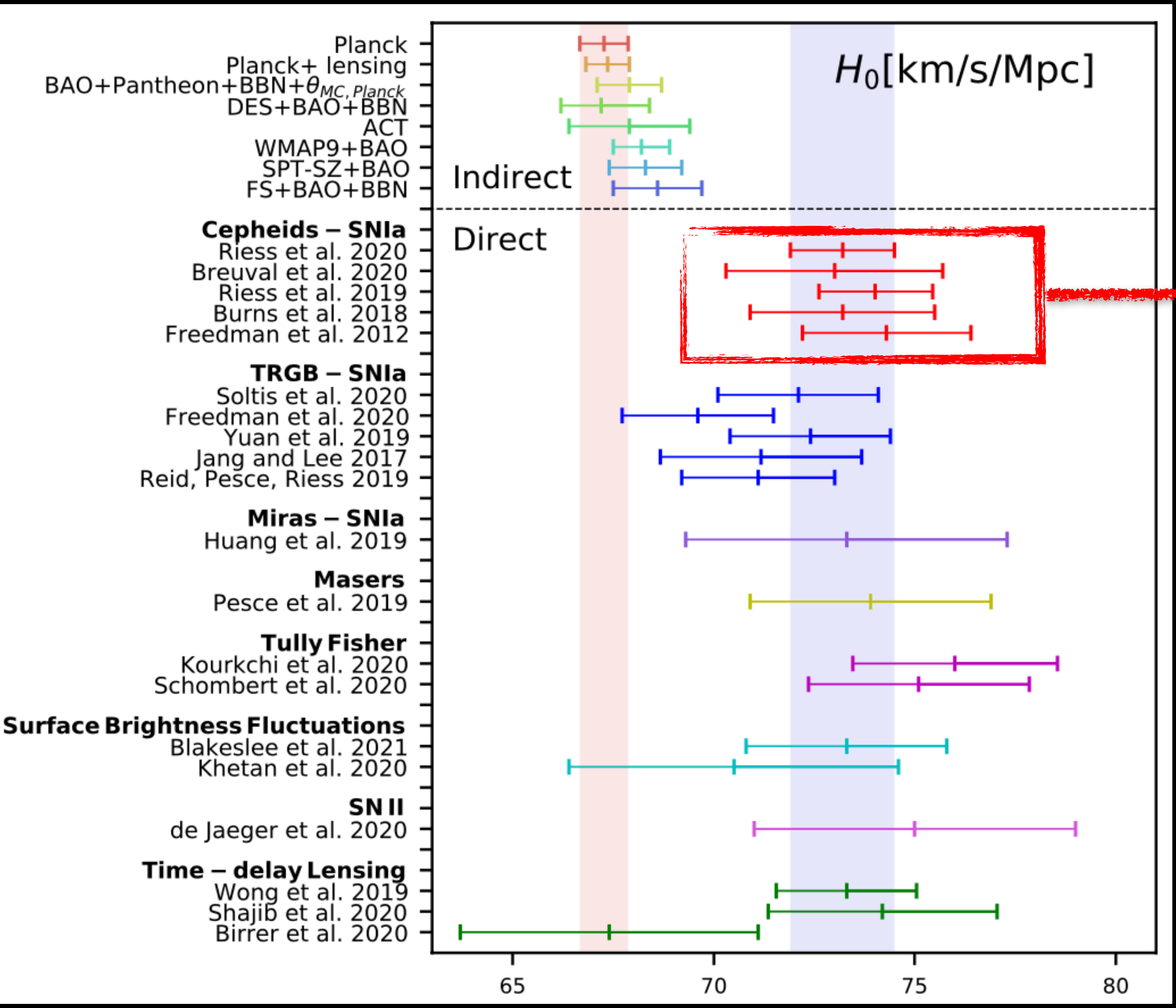
eBOSS, Alam et al., arXiv:2007.08991 [astro-ph.CO]



eBOSS, Alam et al., arXiv:2007.08991 [astro-ph.CO]

LCDM - dependent

Late universe measurements



Di Valentino, *Mon.Not.Roy.Astron.Soc.* 502 (2021) 2, 2065-2073

On the same side of SH0ES, i.e. preferring large values, we have the direct estimates of H_0.

Cepheids-SN Ia:

H0 = 73.2 ± 1.3 km/s/Mpc

Riess et al., arXiv:2012.08534 [astro-ph.CO]

H0 = 73.5 ± 1.4 km/s/Mpc

Reid et al., arXiv:1908.05625 [astro-ph.CO]

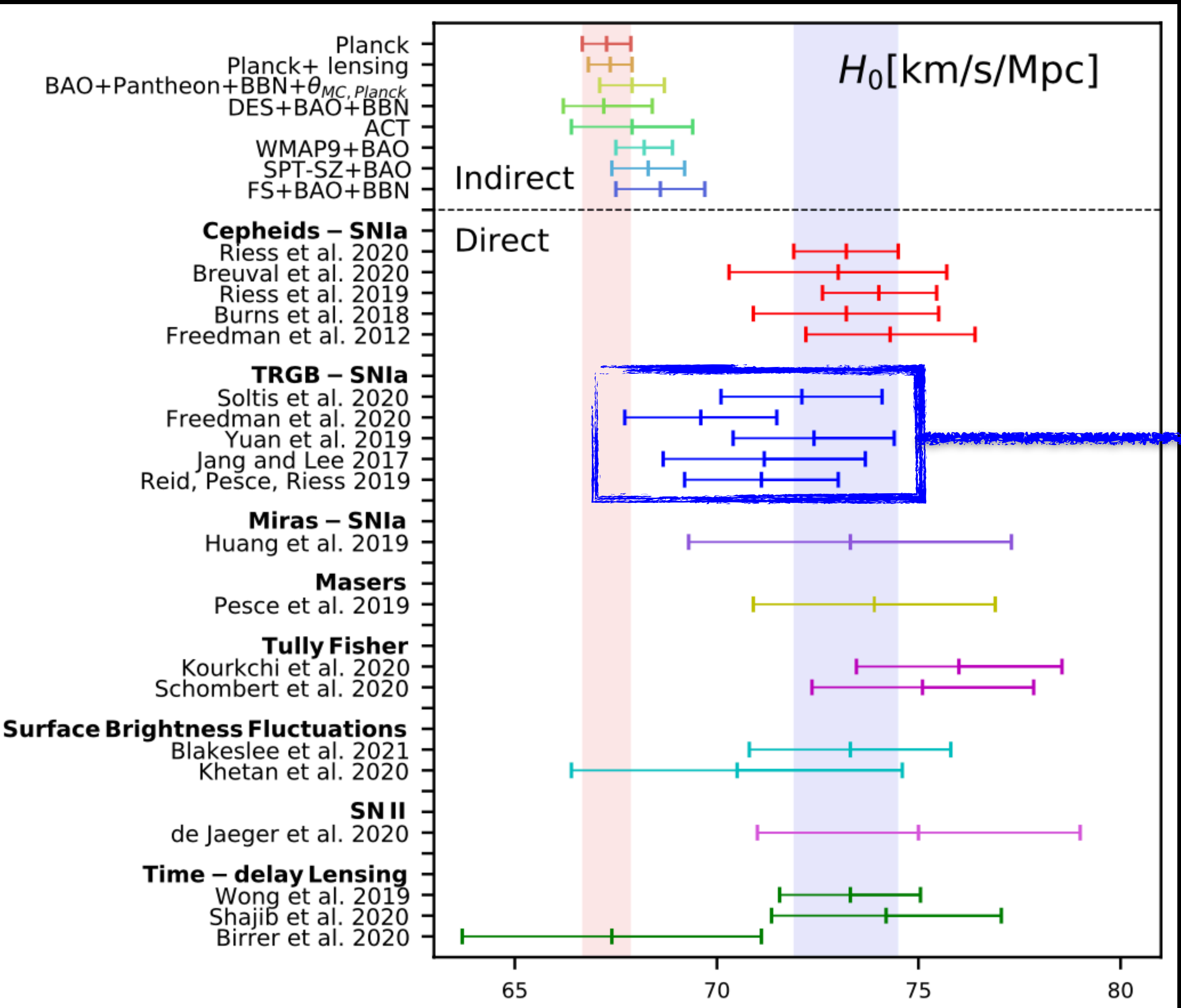
H0 = 73.0 ± 2.7 km/s/Mpc

Breuval et al., arXiv:2006.08763 [astro-ph.CO]

H0 = 73.2 ± 2.3 km/s/Mpc

Burns et al., arXiv:1809.06381 [astro-ph.CO]

Late universe measurements



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The Tip of the Red Giant Branch (TRGB) is the peak brightness reached by red giant stars after they stop using hydrogen and begin fusing helium in their core.

H0 = 72.1±1.2 km/s/Mpc

Soltis et al., arXiv:2012.09196 [astro-ph.CO]

H0 = 69.6±1.88 km/s/Mpc

Freedman et al., arXiv:2002.01550 [astro-ph.CO]

H0 = 72.4±2.0 km/s/Mpc

Yuan and Lee., arXiv:1908.00993 [astro-ph.CO]

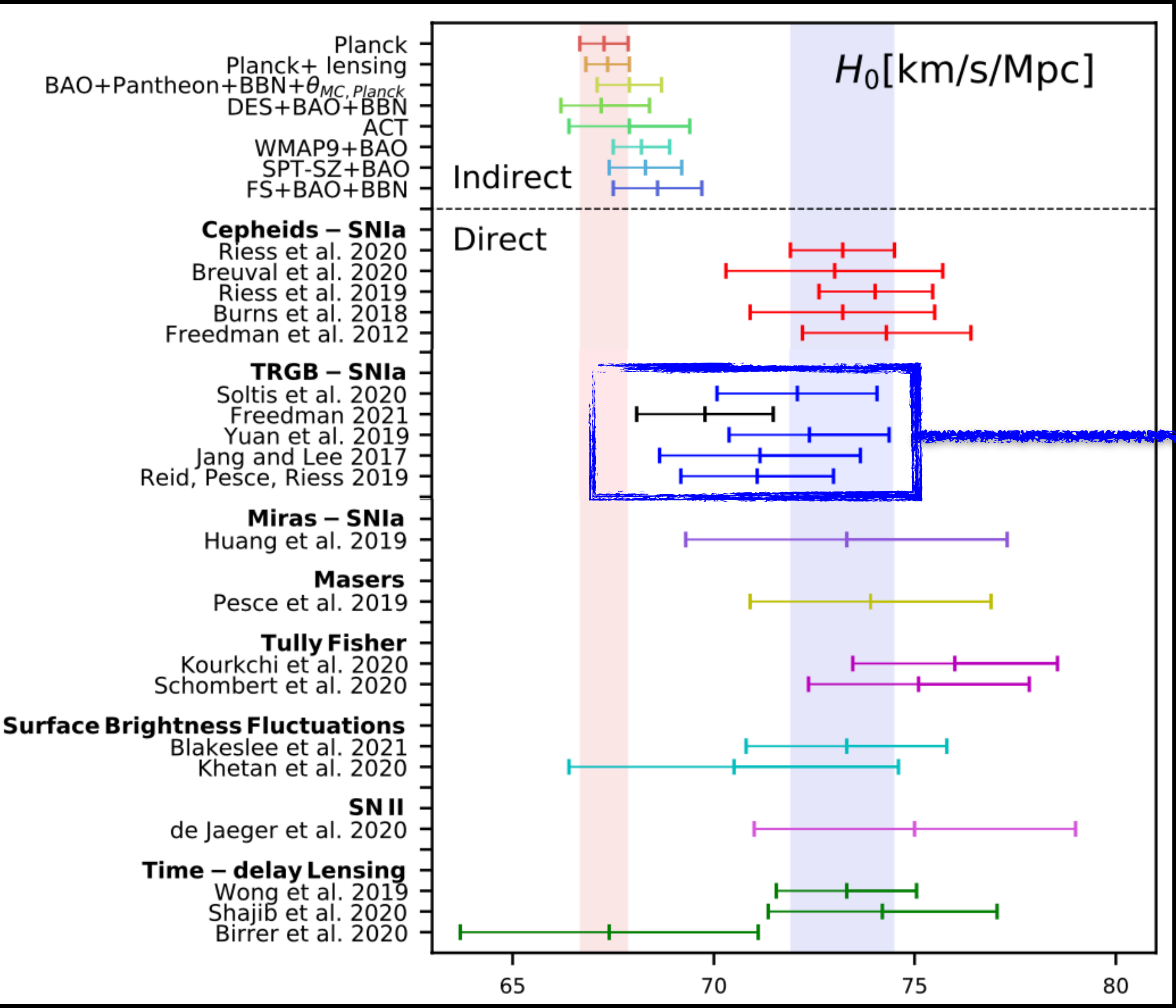
H0 = 71.17±2.50 km/s/Mpc

Jang et al., arXiv:1702.01118 [astro-ph.CO]

H0 = 71.1±1.9 km/s/Mpc

Reid et al., arXiv:1908.05625 [astro-ph.CO]

Late universe measurements



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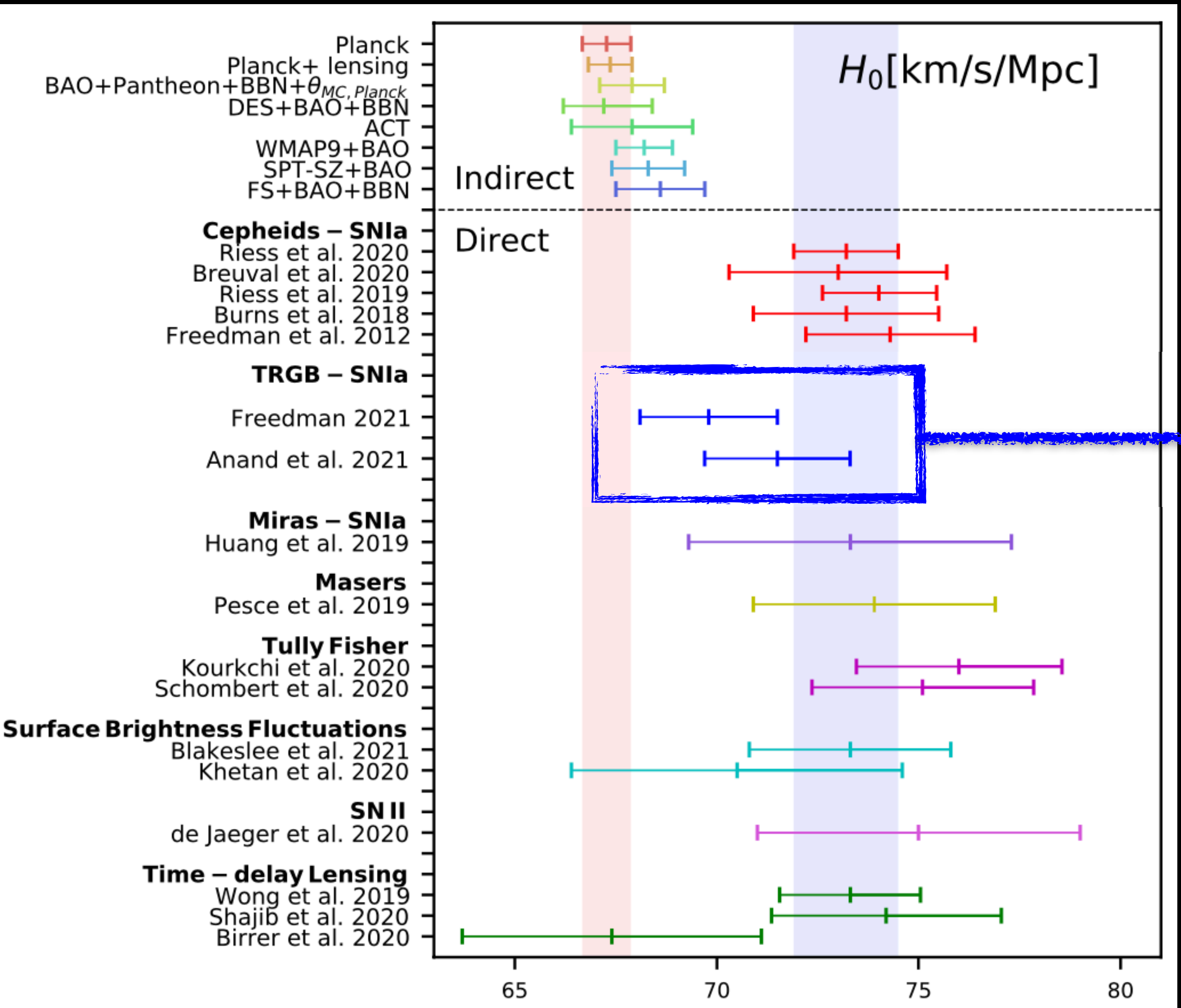

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H0 = 69.8±1.7 km/s/Mpc

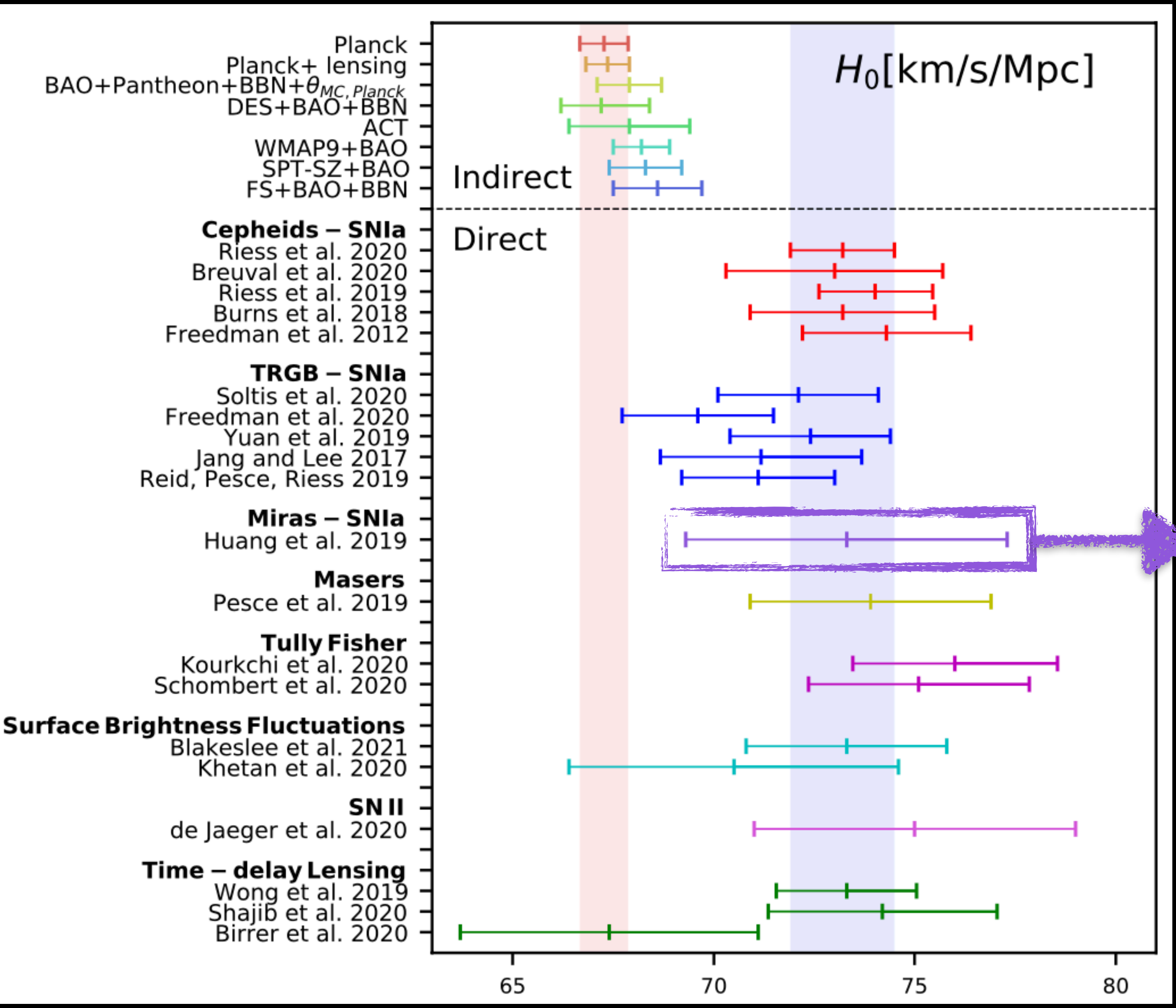
Freedman, arXiv:2106.15656 [astro-ph.CO]

New independent re-analysis of the targets presented by the Carnegie-Chicago Hubble Program (CCHP).

H0 = 71.5±1.8 km/s/Mpc

Anand et al., arXiv:2012.09196 [astro-ph.CO]

Late universe measurements



MIRAS
variable red giant stars from older stellar populations

H0 = 73.3 ± 4.0 km/s/Mpc

Huang et al., arXiv:1908.10883 [astro-ph.CO]

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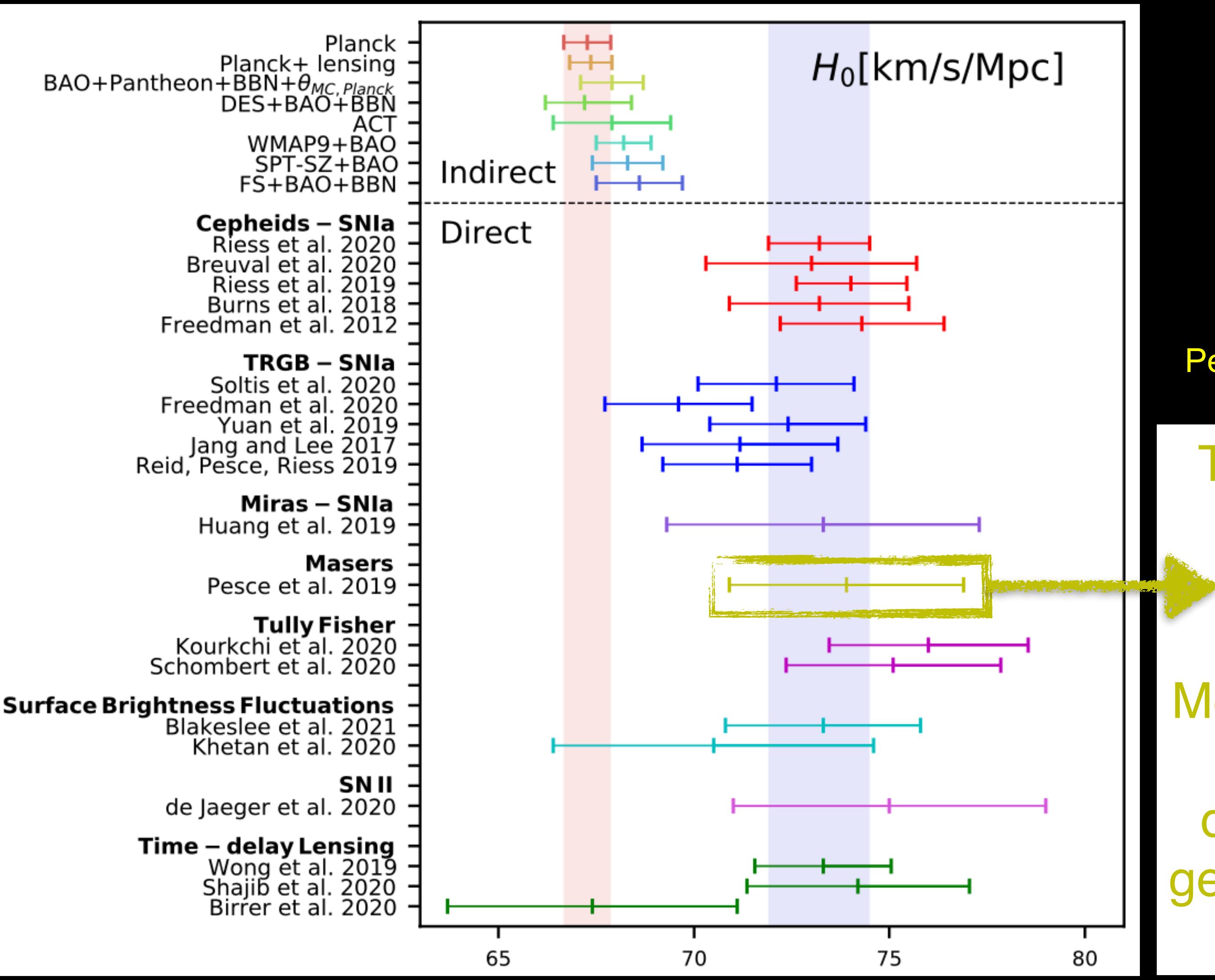
Late universe measurements



H0 = 73.9 ± 3.0 km/s/Mpc

Pesce et al. arXiv:2001.09213 [astro-ph.CO]

The Megamaser Cosmology Project measures H0 using geometric distance measurements to six Megamaser - hosting galaxies. This approach avoids any distance ladder by providing geometric distance directly into the Hubble flow.

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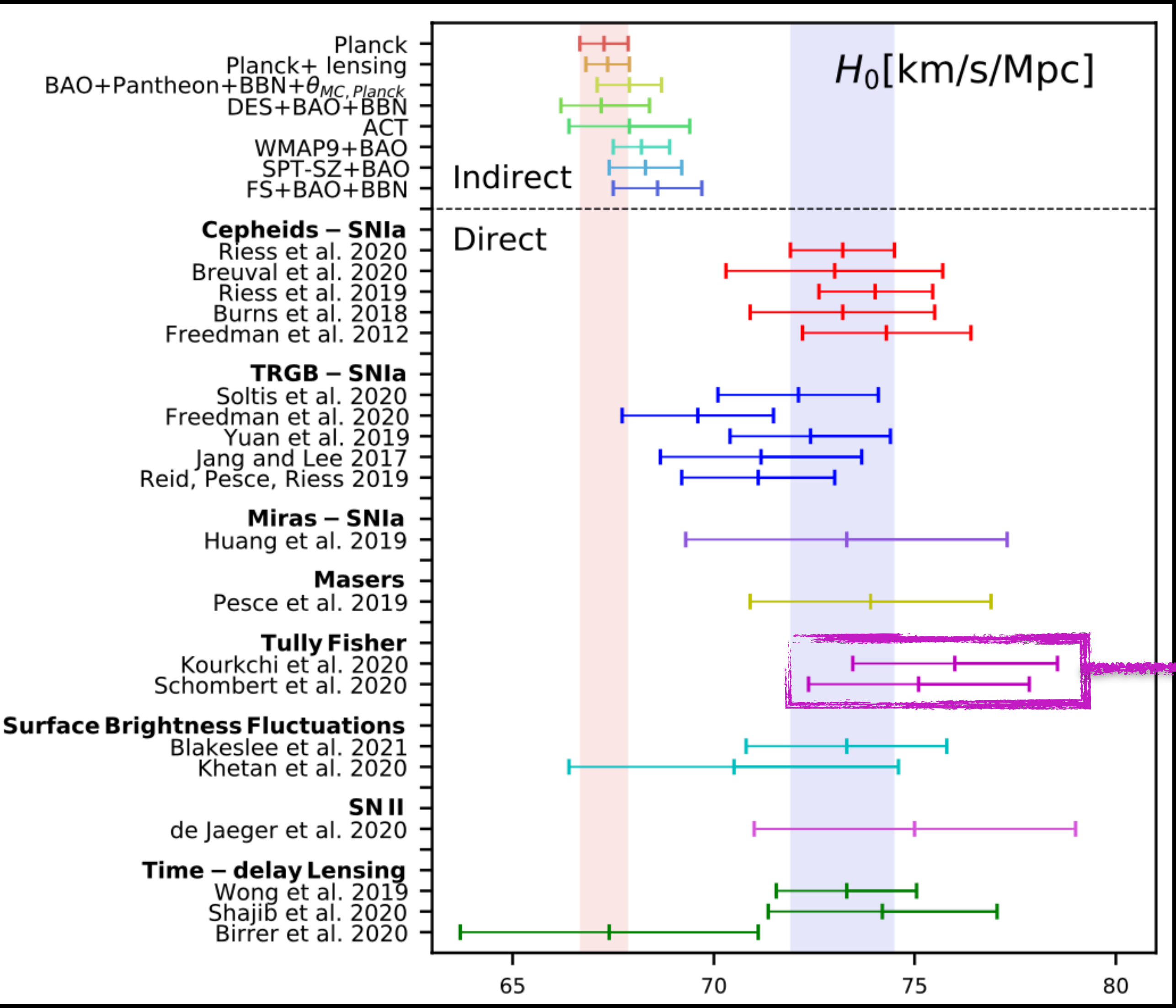
Late universe measurements



H0 = 76.00 ± 2.55 km/s/Mpc

Kourkchi et al. arXiv:2004.14499 [astro-ph.CO]

H0 = 75.10 ± 2.75 km/s/Mpc

Schombert et al. arXiv:2006.08615 [astro-ph.CO]

Tully-Fisher Relation (based on the correlation between the rotation rate of spiral galaxies and their absolute luminosity, and using as calibrators Cepheids and TRGB)

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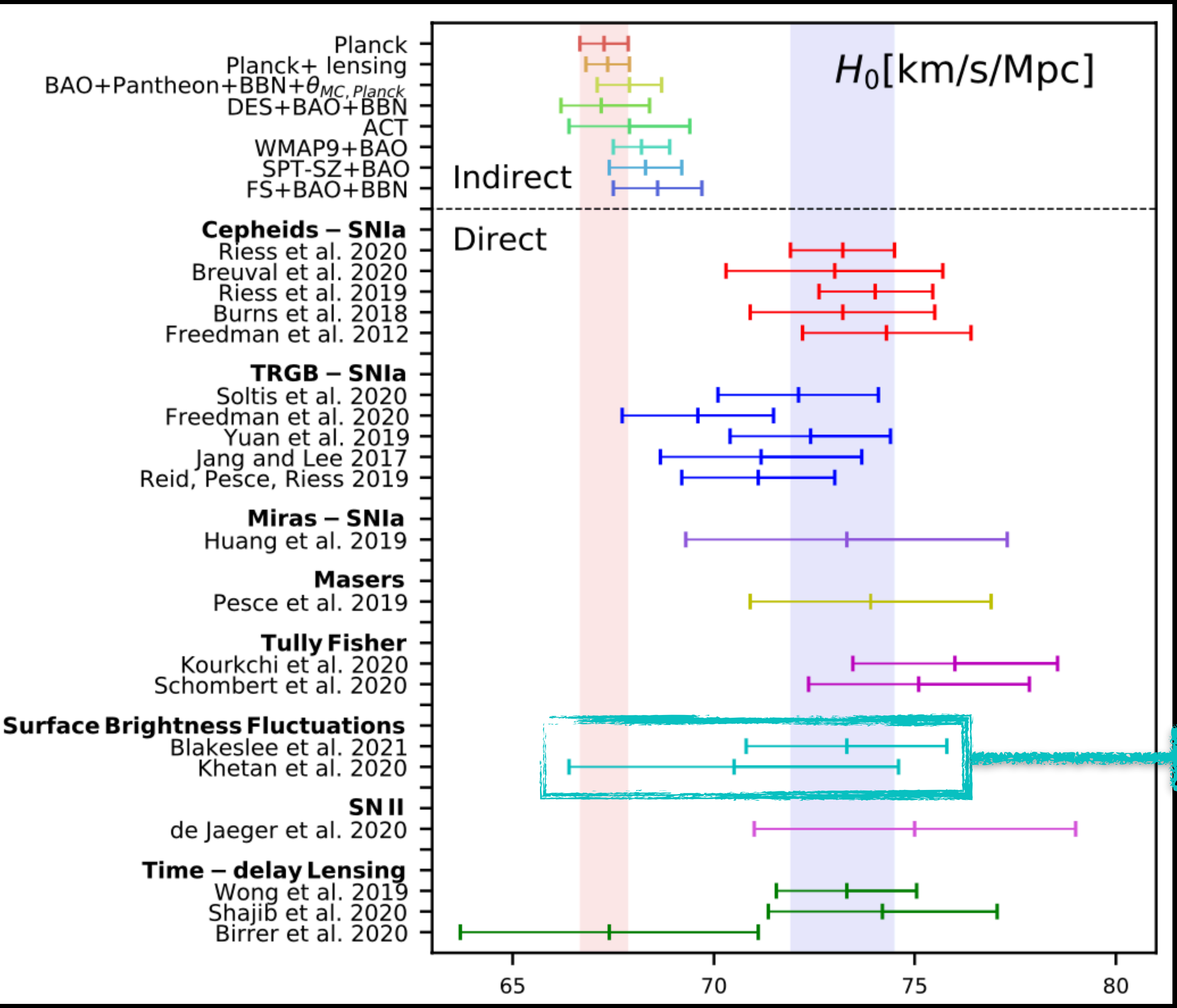
Late universe measurements



H0 = 73.3 ± 2.5 km/s/Mpc

Blakeslee et al., arXiv:2101.02221 [astro-ph.CO]

H0 = 70.5 ± 4.1 km/s/Mpc

Khetan et al. arXiv:2008.07754 [astro-ph.CO]

Surface Brightness Fluctuations (substitutive distance ladder for long range indicator, calibrated by both Cepheids and TRGB)

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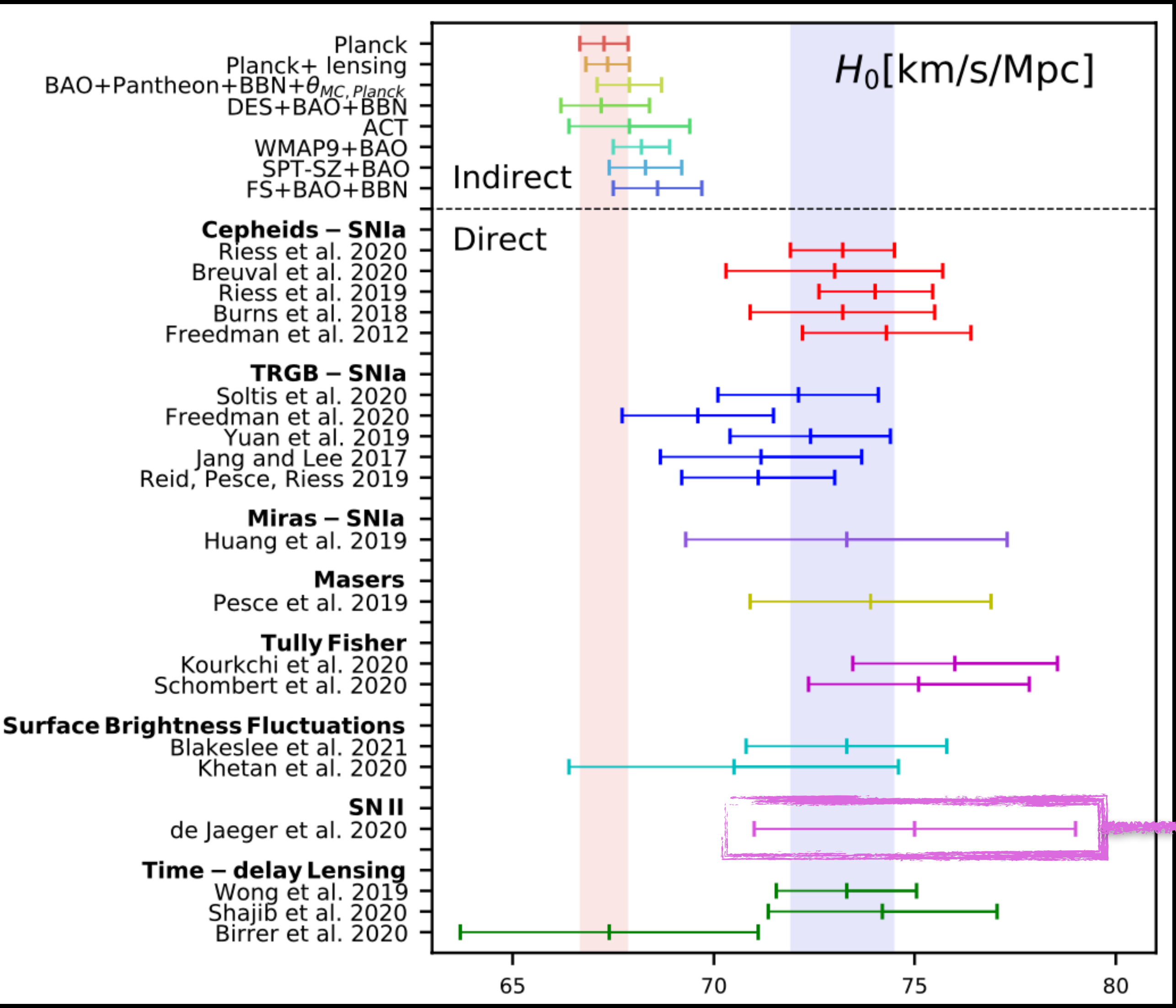
Late universe measurements



$H0 = 75.8^{+5.2}_{-4.9}$ km/s/Mpc

de Jaeger et al., arXiv:2006.03412 [astro-ph.CO]

Type II supernovae used as standardisable candles and calibrated by both Cepheids and TRGB

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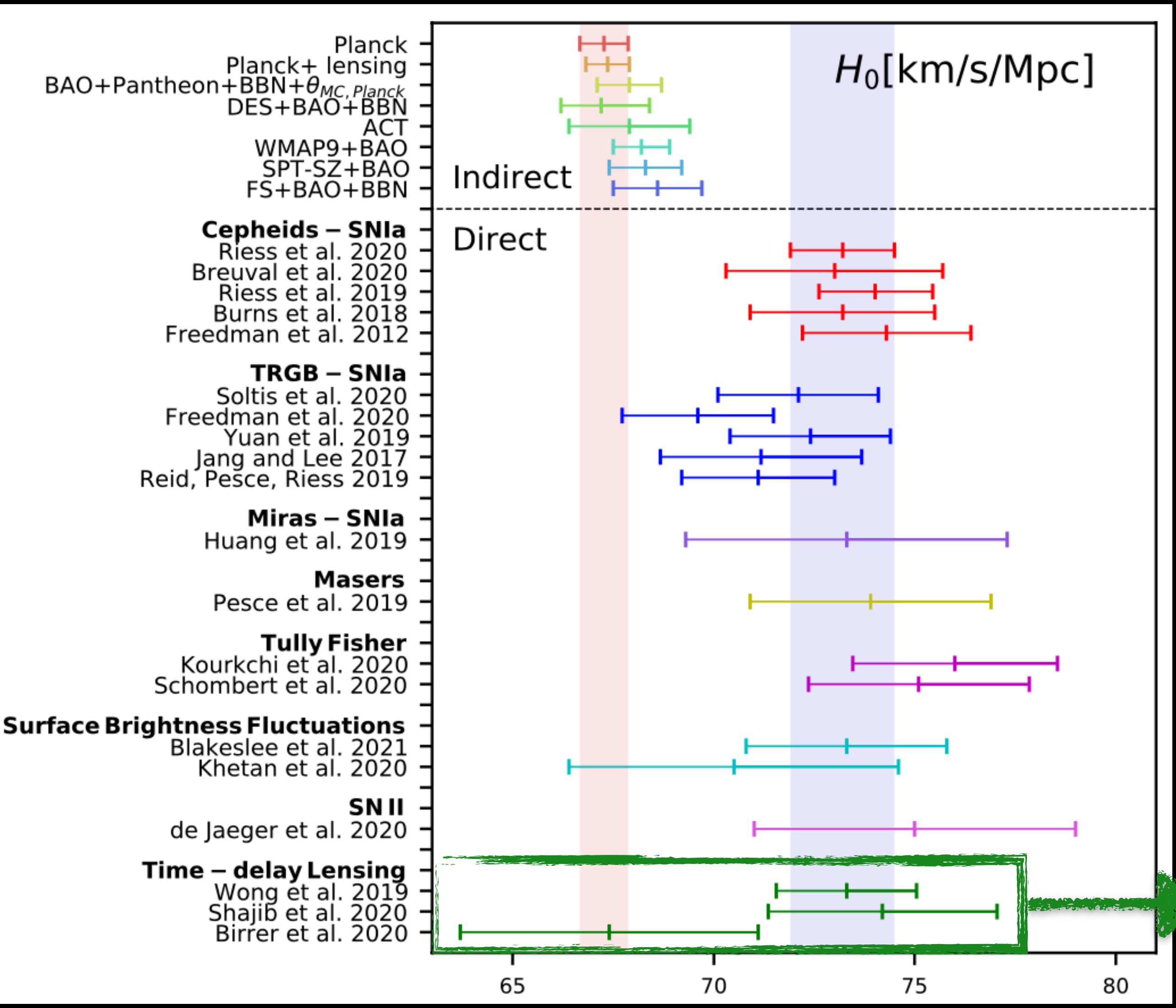
Late universe measurements



HOLiCOW:

$H0 = 73.3^{+1.7}_{-1.8}$ km/s/Mpc

Wong et al. arXiv:1907.04869 [astro-ph.CO]

STRIDES:

$H0 = 74.2^{+2.7}_{-3.0}$ km/s/Mpc

Shajib et al. arXiv:1910.06306 [astro-ph.CO]

TDCOSMO+SLAC:

$H0 = 67.4^{+4.1}_{-3.2}$ km/s/Mpc

Birrer et al. arXiv:2007.02941 [astro-ph.CO]

Strong Lensing measurements of the time delays of multiple images of quasar systems caused by the strong gravitational lensing from a foreground galaxy. Uncertainties coming from the lens mass profile.

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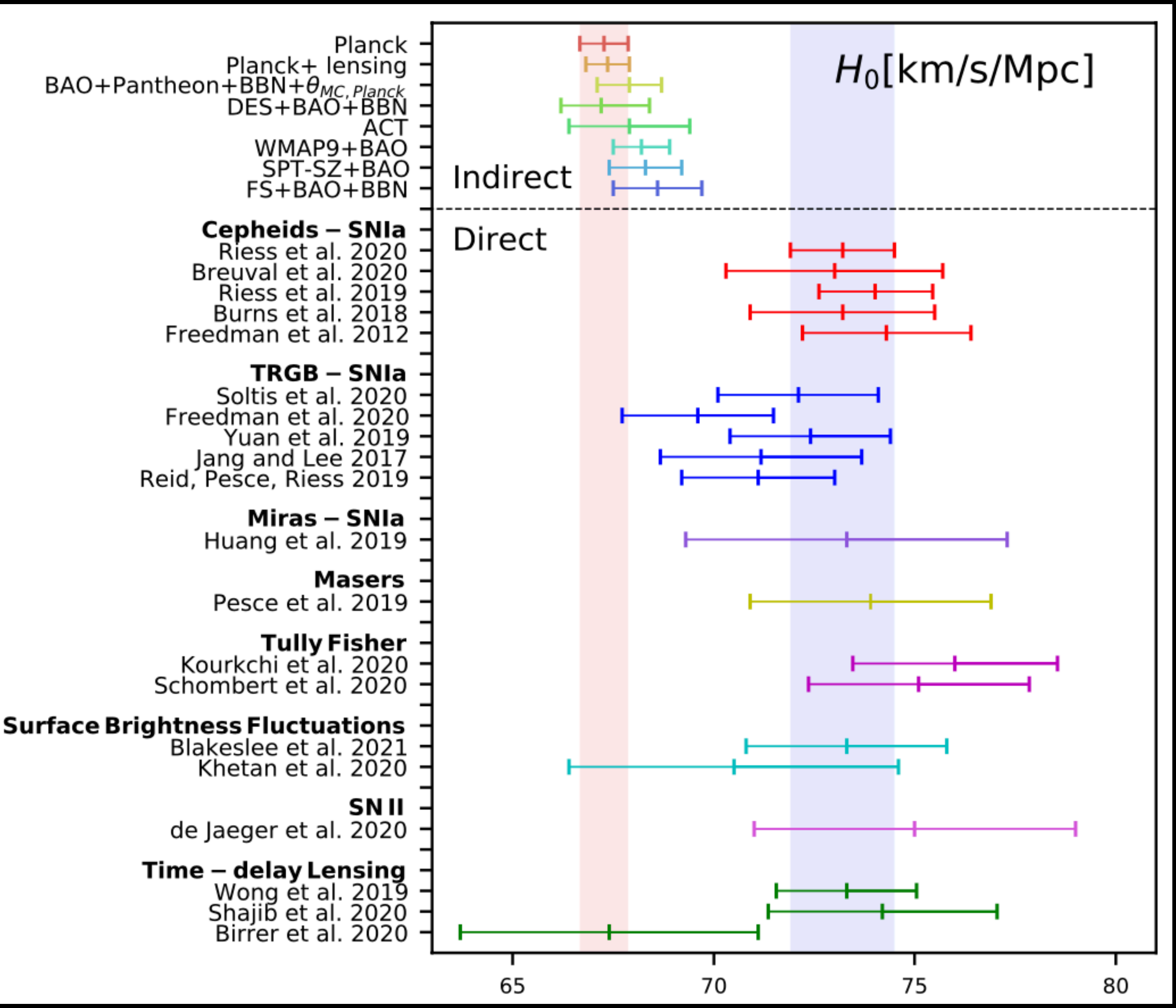
Astrophysical model dependent

Late universe measurements



Combining all of them together (+Standard Sirens and + γ-ray Attenuation) we obtain our

Optimistic estimate (5.9σ tension with Planck)

H0 = 72.94 ± 0.75 km/s/Mpc

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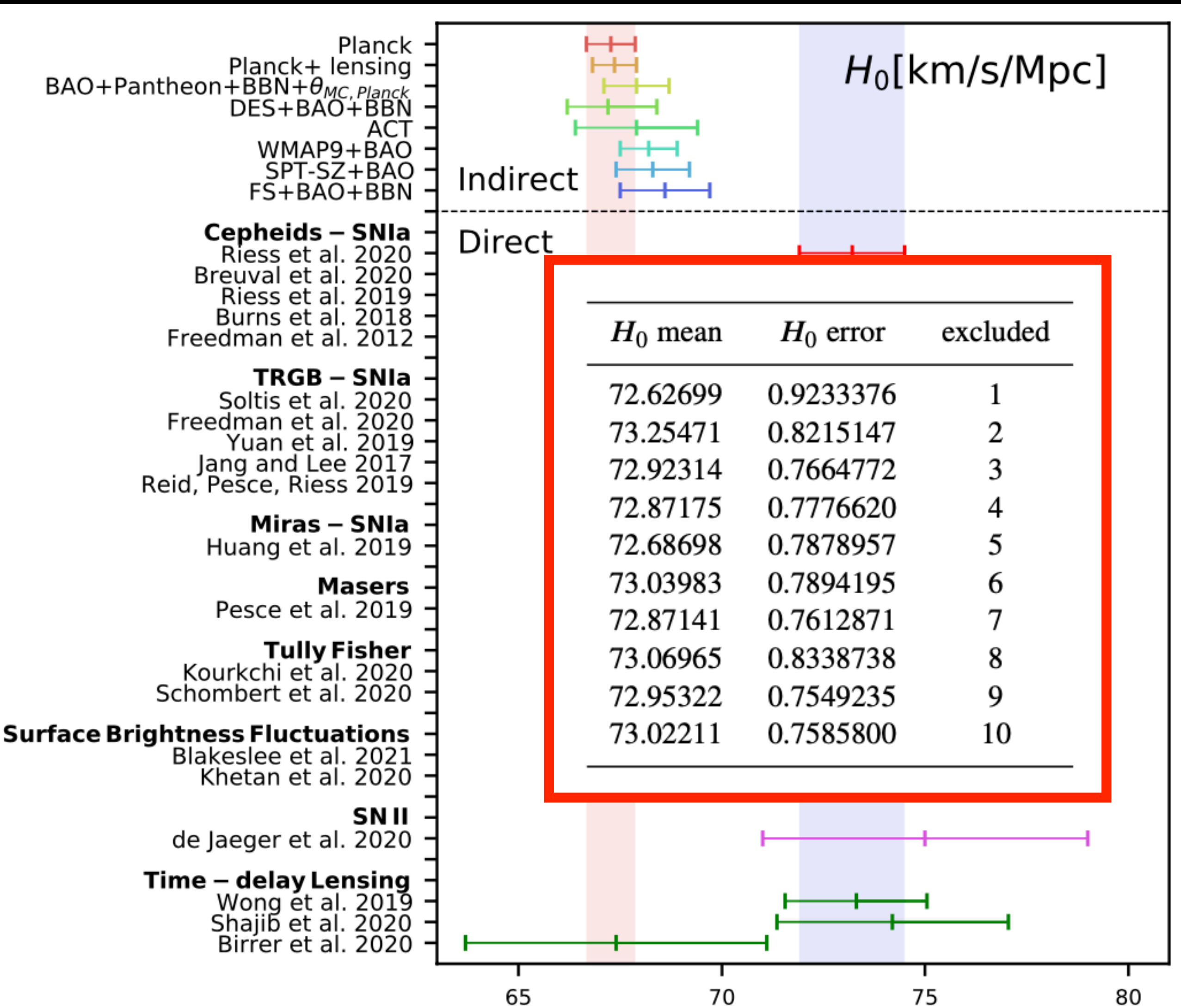
Late universe measurements



H_0 mean	H_0 error	excluded
72.62699	0.9233376	1
73.25471	0.8215147	2
72.92314	0.7664772	3
72.87175	0.7776620	4
72.68698	0.7878957	5
73.03983	0.7894195	6
72.87141	0.7612871	7
73.06965	0.8338738	8
72.95322	0.7549235	9
73.02211	0.7585800	10

Excluding one group of data and taking the result with the largest error bar, i.e. excluding the most precise measurements based on Cepheids-SN Ia, we obtain our

Conservative estimate (4.8σ tension with Planck)

H0 = 72.63 ± 0.92 km/s/Mpc

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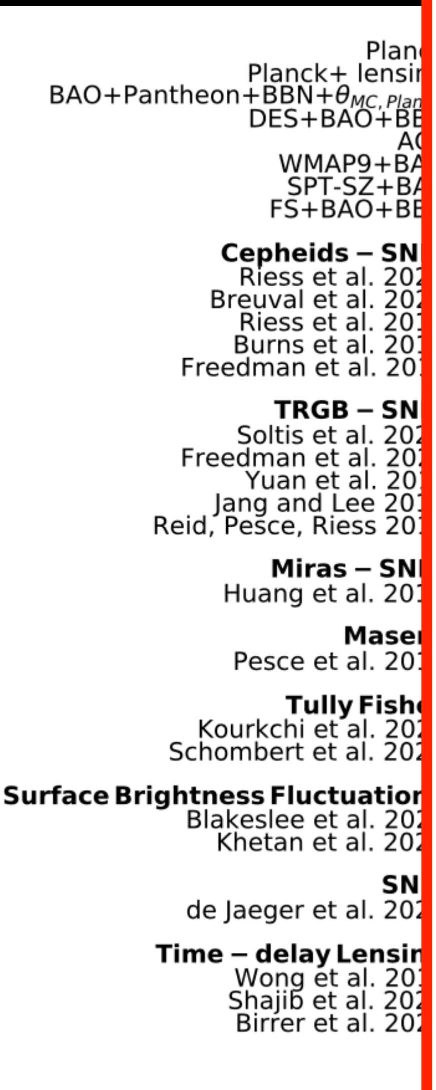
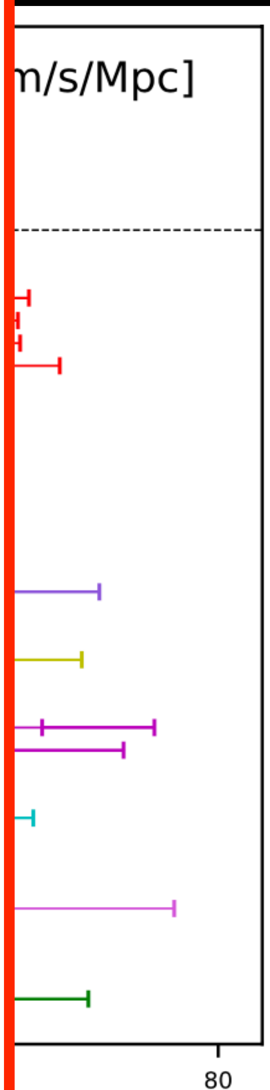
measurements



H_0 mean	H_0 error	excluded	excluded
73.05838	1.059989	1	2
72.58911	0.9489663	1	3
72.49379	0.9704452	1	4
72.18593	0.9905548	1	5
72.74182	0.9935880	1	6
72.51725	0.9391693	1	7
72.73386	1.086945	1	8
72.64958	0.9272990	1	9
72.74957	0.9341007	1	10
73.25271	0.8394086	2	3
73.20239	0.8541644	2	4
72.98889	0.8677809	2	5
73.41869	0.8698181	2	6
73.18552	0.8326054	2	7
73.51043	0.9304035	2	8
73.27682	0.8243009	2	9
73.36285	0.8290675	2	10
72.85492	0.7927890	3	4
72.66224	0.8036401	3	5
73.02929	0.8052573	3	6
72.85530	0.7754612	3	7
73.05918	0.8526064	3	8
72.94041	0.7687386	3	9
73.01174	0.7726005	3	10
72.59712	0.8165602	4	5
72.97585	0.8182568	4	6
72.80062	0.7870499	4	7
73.00013	0.7800242	4	9
72.96215	0.7840596	4	10
72.77377	0.8302036	5	6
72.60931	0.7976639	5	7
72.77266	0.8823864	5	8
72.70377	0.7903527	5	9
72.77669	0.7945514	5	10
72.97070	0.7992452	6	7
73.21607	0.8845285	6	8
73.05890	0.7918909	6	9
73.13590	0.7961143	6	10
72.99312	0.8454798	7	8
72.88815	0.7635028	7	9
72.95798	0.7672859	7	10
73.09115	0.8367882	8	9
73.17762	0.8417761	8	10
73.03960	0.7607720	9	10



Excluding two groups of data and taking the result with the largest error bar, i.e. excluding the most precise measurements based on Cepheids-SN Ia and Time-delay Lensing, we obtain our

Ultra-conservative estimate (3σ tension with Planck)

H0 = 72.7 ± 1.1 km/s/Mpc

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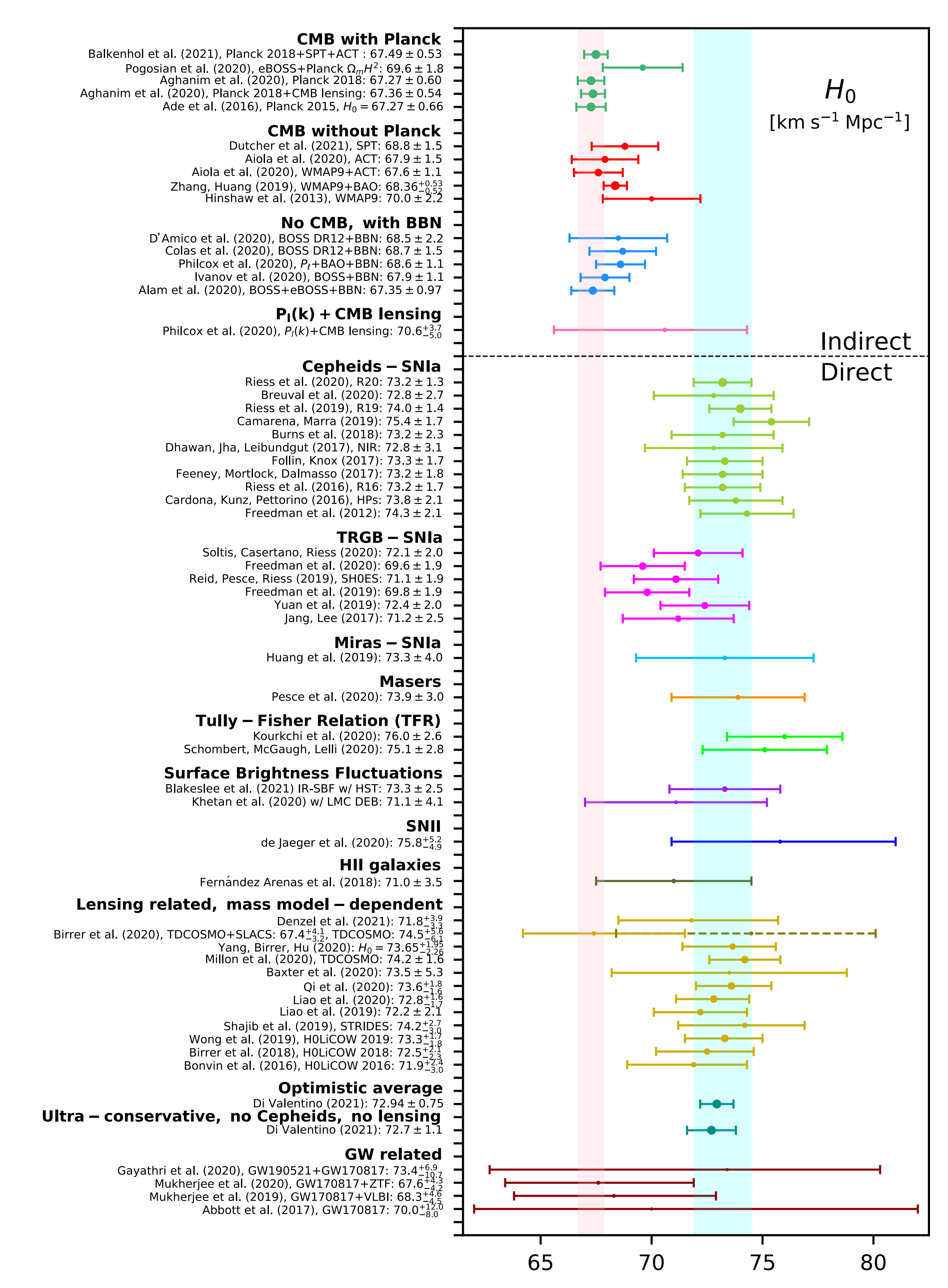


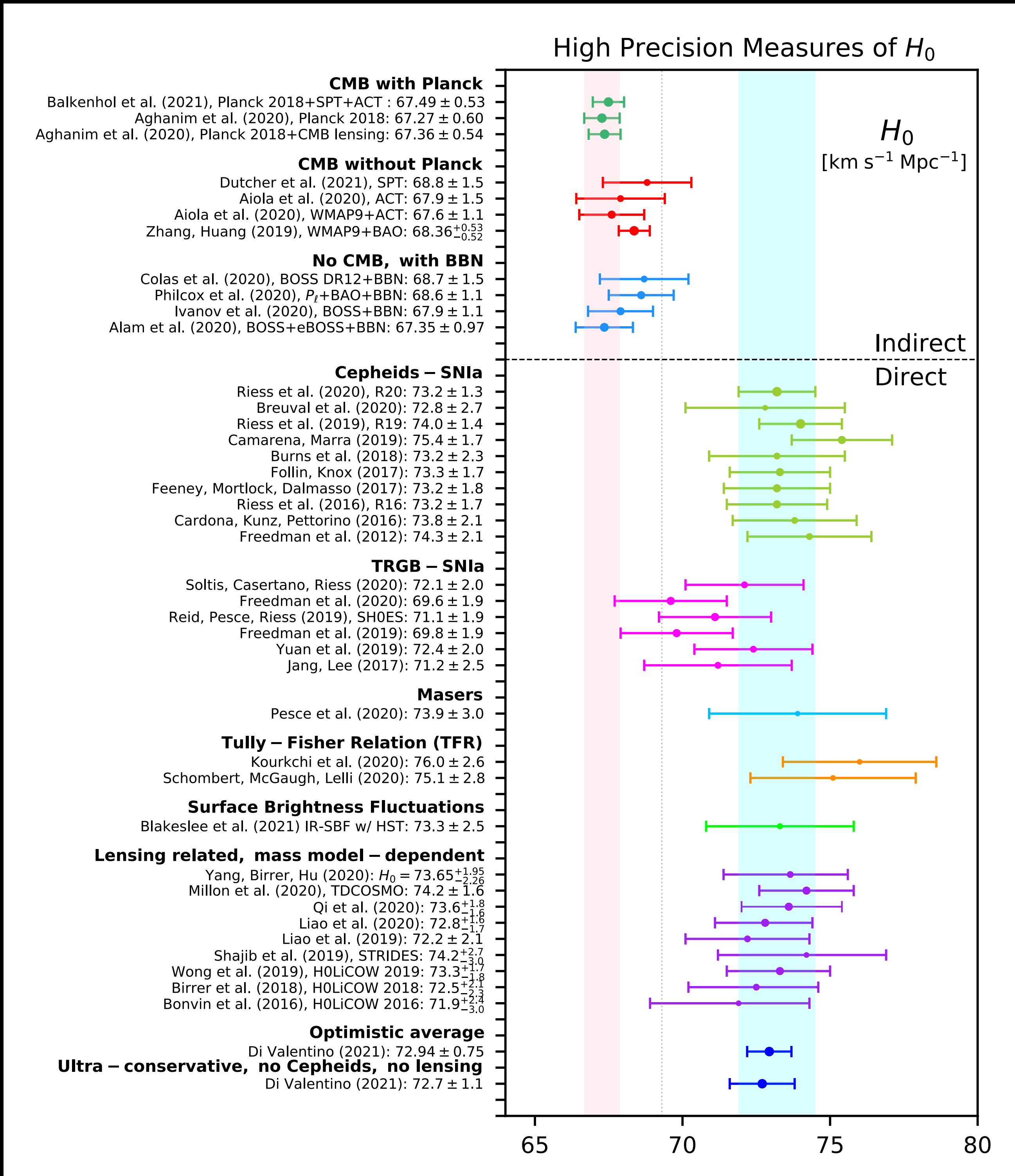
Hubble constant direct and indirect measurements made by different astronomical missions and groups over the years.

The cyan vertical band corresponds to the H0 value from SH0ES Team and the light pink vertical band corresponds to the H0 value as reported by Planck 2018 team within a ΛCDM scenario.

A sample code for producing similar figures with any choice of the data is made publicly available online at github.com/lucavisinelli/H0TensionRealm.

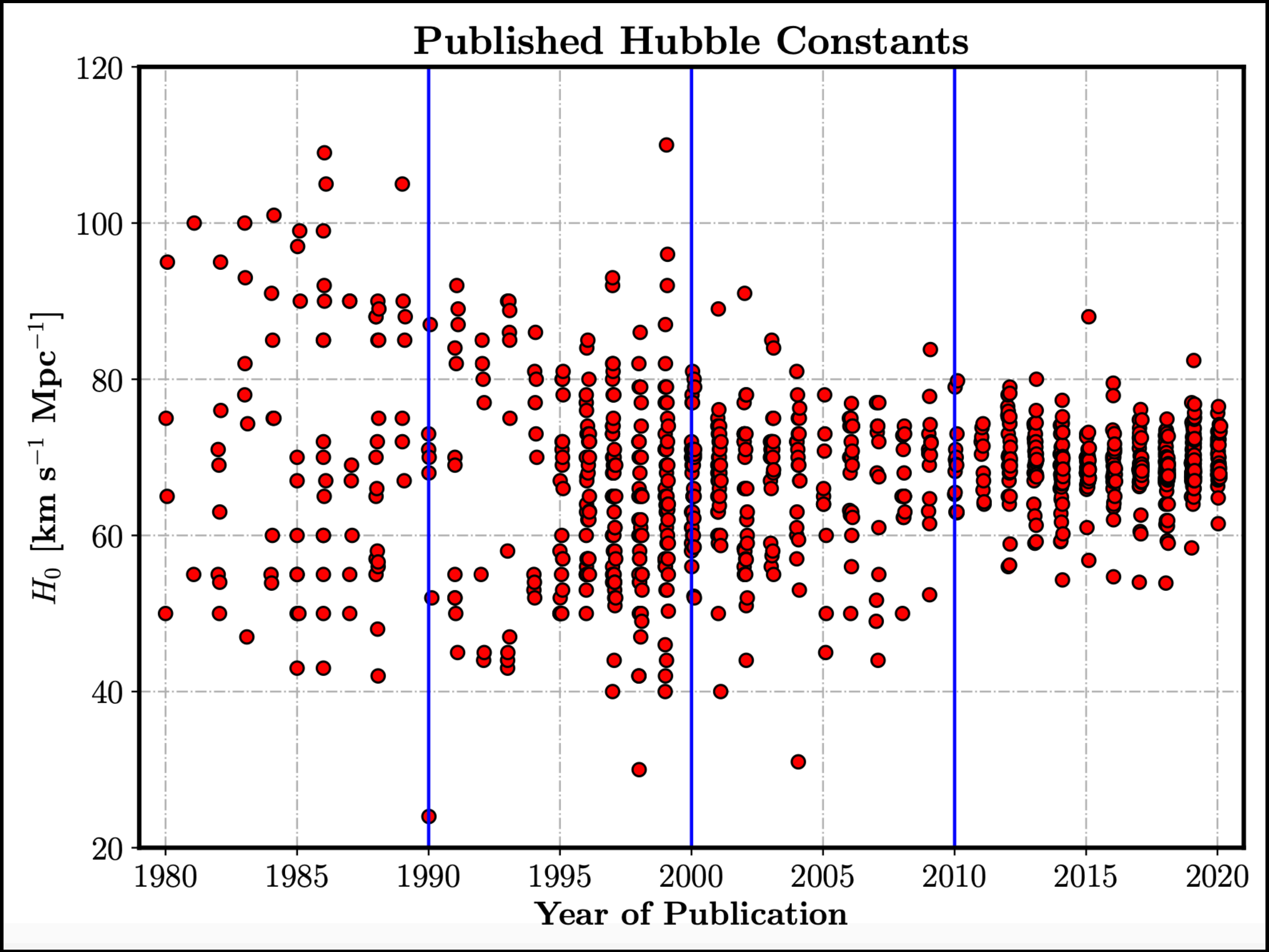
Make your plot!

Di Valentino et al., Class.Quant.Grav. (2021), arXiv:2103.01183 [astro-ph.CO]



The high precision and consistency of the data at both ends present strong challenges to the possible solution space and demands a hypothesis with enough rigor to explain multiple observations – whether these invoke new physics, unexpected large-scale structures or multiple, unrelated errors.

Di Valentino et al., Class.Quant.Grav. (2021), arXiv:2103.01183 [astro-ph.CO]



Freedman, arXiv:2106.15656 [astro-ph.CO]

In the past the tension was within the same types of measurements and at the same redshifts and thus pointing directly to systematics.

Now there are no late universe measurements below early ones and vice versa.

It is hard to conceive of a single type of systematic error that would apply to the measurements of the disparate phenomena we saw before as to effectively resolve the Hubble constant tension.
Because the tension remains with the removal of the measurements of any single type of object, mode or calibration, it is challenging to devise a single error that would suffice.
While multiple, unrelated systematic errors have a great deal more flexibility to resolve the tension but become less likely by their inherent independence.

Since the indirect constraints are model dependent, we can try to expand the cosmological scenario and see which extensions work in solving the tensions between the cosmological probes.

The Neutrino effective number

We can consider modifications in the dark matter sector.

A classical extension is the effective number of relativistic degrees of freedom, i.e. additional relativistic matter at recombination, corresponding to a modification of the expansion history of the universe at early times.

The Neutrino effective number

The expected value is Neff = 3.046, if we assume standard electroweak interactions and three active massless neutrinos. If we measure a Neff > 3.046, we are in presence of extra radiation.

If we compare the Planck 2015 constraint on Neff at 68% cl

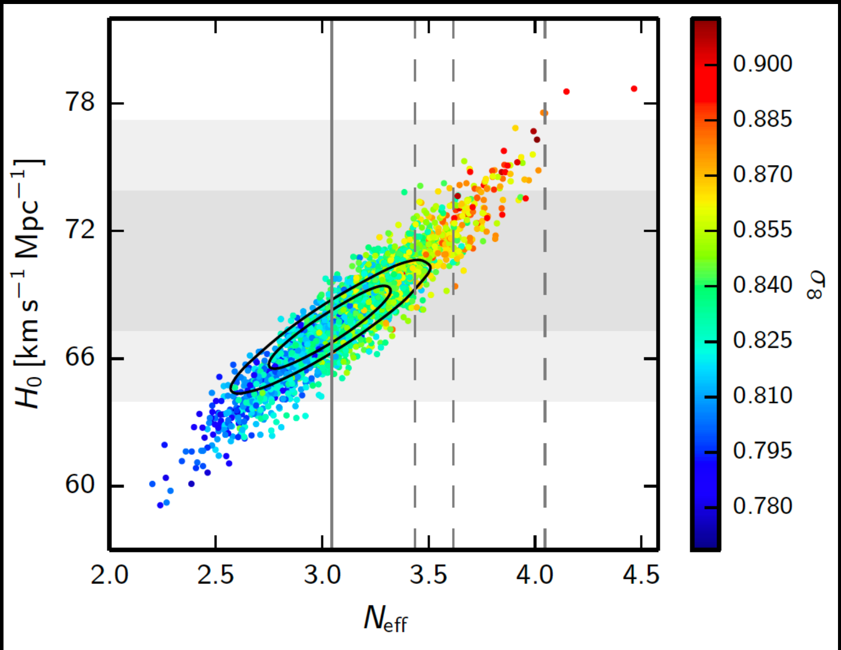
$$N_{\rm eff} = 3.13 \pm 0.32 \quad \textit{Planck} \text{ TT+lowP},$$
$$N_{\rm eff} = 3.15 \pm 0.23 \quad \textit{Planck} \text{ TT+lowP+BAO},$$

with the new Planck 2018 bound,

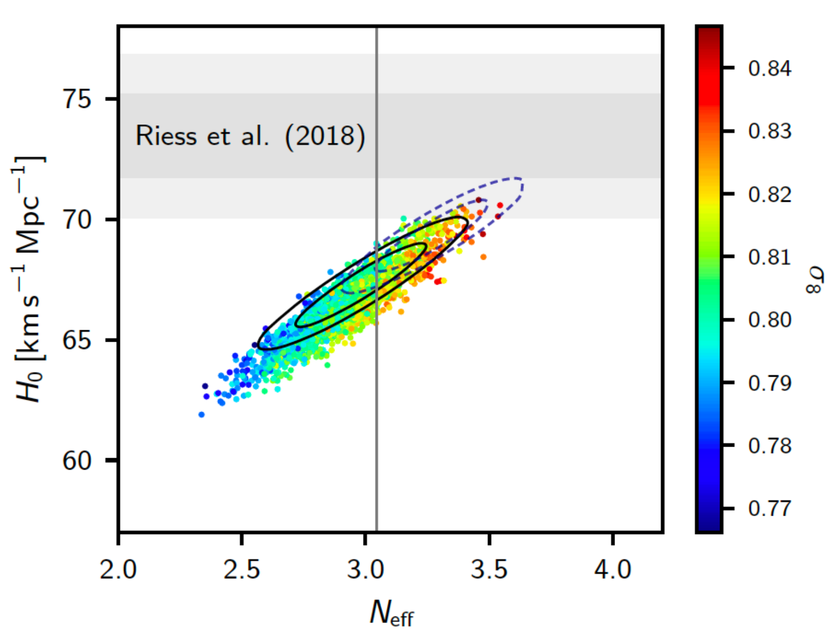
$$N_{\rm eff} = 2.92^{+0.36}_{-0.37} \quad (95\,\%, \textit{Planck} \text{ TT,TE,EE+lowE}),$$

we see that the neutrino effective number is now very well constrained.

H0 passes from 68.0 ± 2.8 km/s/Mpc (2015) to 66.4 ± 1.4 km/s/Mpc (2018), and the tension with R20 increases from 1.7σ to 3.6σ also varying Neff.



Planck collaboration, 2015



Planck collaboration, 2018

The Neutrino effective number

The expected value is Neff = 3.046, if we assume standard electroweak interactions and three active massless neutrinos. If we measure a Neff > 3.046, we are in presence of extra radiation.

If we compare the Planck 2015 constraint on Neff at 68% cl

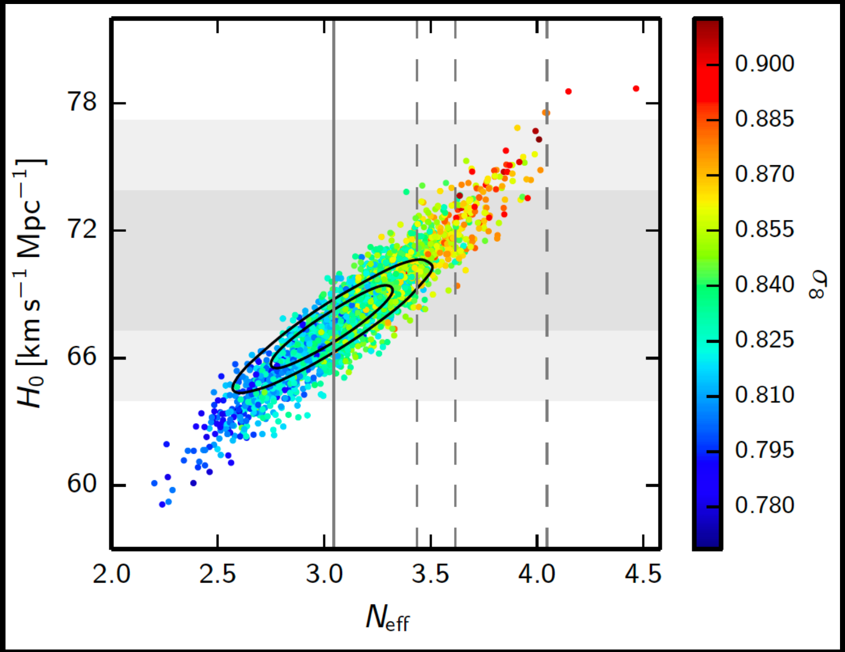
$$N_{\rm eff} = 3.13 \pm 0.32 \quad \textit{Planck}\ \text{TT+lowP},$$
$$N_{\rm eff} = 3.15 \pm 0.23 \quad \textit{Planck}\ \text{TT+lowP+BAO},$$

with the new Planck 2018 bound,

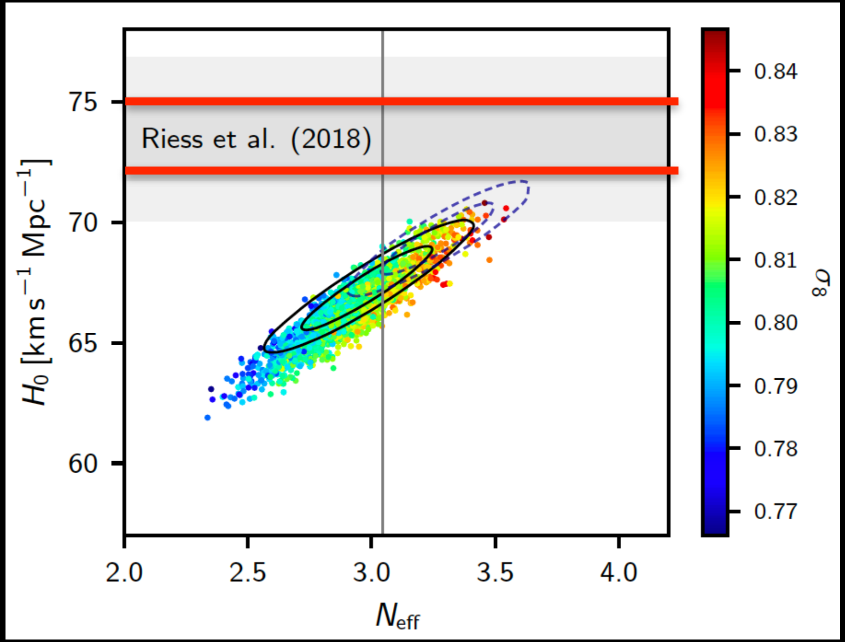
$$N_{\rm eff} = 2.92^{+0.36}_{-0.37} \quad (95\,\%,\ \textit{Planck}\ \text{TT,TE,EE+lowE}),$$

we see that the neutrino effective number is now very well constrained.

H0 passes from 68.0 ± 2.8 km/s/Mpc (2015) to 66.4 ± 1.4 km/s/Mpc (2018), and the tension with R20 increases from 1.7σ to 3.6σ also varying Neff.



Planck collaboration, 2015



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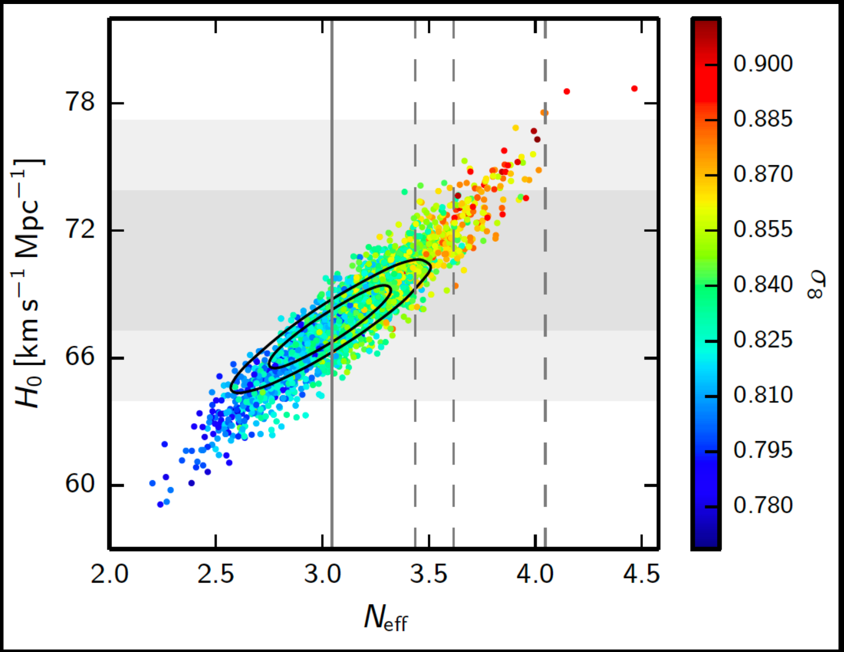
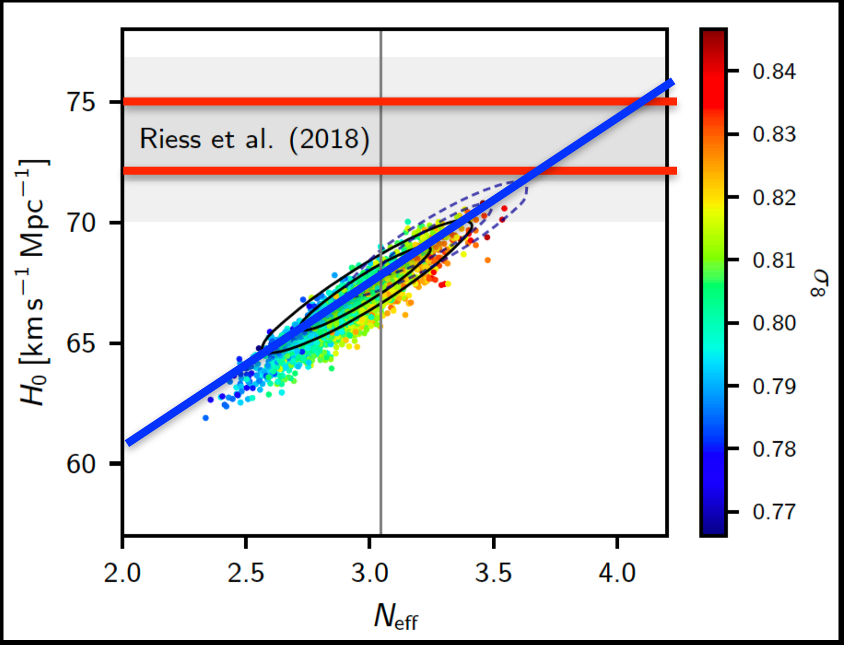


Planck collaboration, 2015



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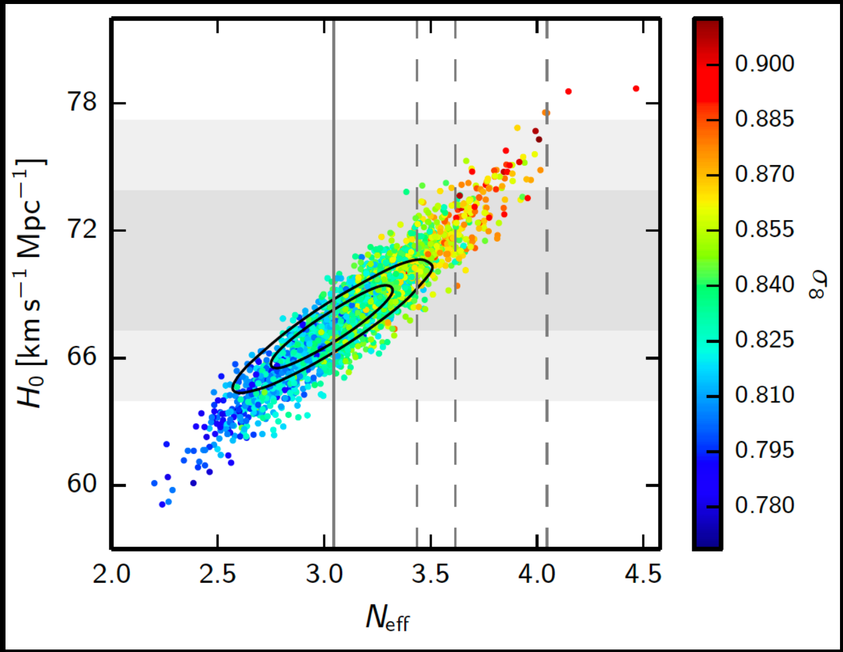
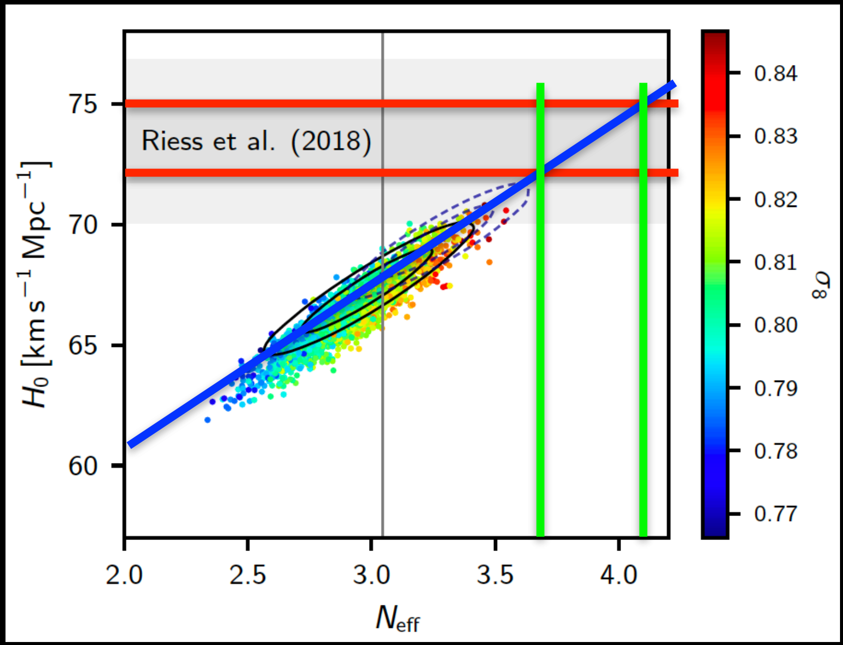


Planck collaboration, 2015



Planck collaboration, 2018

The Dark energy equation of state

For example, we can consider modifications in the dark energy sector.

A classical extension is a varying dark energy equation of state, that is a modification of the expansion history of the universe at late times.

The Dark energy equation of state

Changing the dark energy equation of state w, we are changing the expansion rate of the Universe:

$$H^2 = H_0^2 \left[\Omega_m (1+z)^3 + \Omega_r (1+z)^4 + \Omega_{\rm de} (1+z)^{3(1+w)} + \Omega_k (1+z)^2 \right]$$

w introduces a geometrical degeneracy with the Hubble constant that will be unconstrained using the CMB data only, resulting in agreement with R20.

We have in 2018 $w = -1.58^{+0.52}_{-0.41}$ with $H0 > 69.9$ km/s/Mpc at 95% c.l.

Planck data prefer a phantom dark energy, with an energy component with $w < -1$, for which the density increases with time in an expanding universe that will end in a Big Rip. A phantom dark energy violates the energy condition $\rho \geq |p|$, that means that the matter could move faster than light and a comoving observer measure a negative energy density, and the Hamiltonian could have vacuum instabilities due to a negative kinetic energy.

Formally successful models in solving H0

tension $\leq 1\sigma$ *"Excellent models"*	tension $\leq 2\sigma$ *"Good models"*	tension $\leq 3\sigma$ *"Promising models"*
Dark energy in extended parameter spaces [289]	Early Dark Energy [235]	Early Dark Energy [229]
Dynamical Dark Energy [309]	Phantom Dark Energy [11]	Decaying Warm DM [474]
Metastable Dark Energy [314]	Dynamical Dark Energy [11, 281, 309]	Neutrino-DM Interaction [506]
PEDE [392, 394]	GEDE [397]	Interacting dark radiation [517]
Elaborated Vacuum Metamorphosis [400–402]	Vacuum Metamorphosis [402]	Self-Interacting Neutrinos [700, 701]
IDE [314, 636, 637, 639, 652, 657, 661–663]	IDE [314, 653, 656, 661, 663, 670]	IDE [656]
Self-interacting sterile neutrinos [711]	Critically Emergent Dark Energy [997]	Unified Cosmologies [747]
Generalized Chaplygin gas model [744]	$f(\mathcal{T})$ gravity [814]	Scalar-tensor gravity [856]
Galileon gravity [876, 882]	Über-gravity [59]	Modified recombination [986]
Power Law Inflation [966]	Reconstructed PPS [978]	Super ΛCDM [1007]
$f(\mathcal{T})$ [818]		Coupled Dark Energy [650]

Table B1. Models solving the H_0 tension with R20 within the 1σ, 2σ and 3σ confidence levels considering the *Planck* dataset only.

Planck only

Di Valentino et al., Class.Quant.Grav. (2021), arXiv:2103.01183 [astro-ph.CO]

Let’s see an example…

Parker Vacuum Metamorphosis

There is a model considered in the early days of dark energy investigations that possesses the phenomenological properties needed to solve the H0 tension, but is based on a sound theoretical foundation: the vacuum metamorphosis model of Parker and Raval, Phys. Rev. D 62, 083503 (2000), Parker and Vanzella, Phys. Rev. D 69, 104009 (2004), Caldwell, Komp, Parker and Vanzella,Phys. Rev. D 73, 023513 (2006), which has a phase transition in the nature of the vacuum.

Vacuum metamorphosis arises from a nonperturbative summation of quantum gravity loop corrections due to a massive scalar field.

We found that the Parker vacuum metamorphosis model, physically motivated by quantum gravitational effects, with the same number of parameters as LCDM, but not nested with it, can remove the H_0 tension, because can mimic a phantom DE behaviour at low redshifts.

First principles theory

Parker Vacuum Metamorphosis

When the Ricci scalar evolves during cosmic history to reach the scalar field mass squared, then a phase transition occurs and R freezes with

$R = 6(\dot{H} + 2H^2 + ka^{-2}) = m^2$ and defining $M = m^2/(12H_0^2)$

The expansion behaviour above and below the phase transition is

$$H^2/H_0^2 = \Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_k(1+z)^2 + M\left\{1 - \left[3\left(\frac{4}{3\Omega_m}\right)^4 M(1-M-\Omega_k-\Omega_r)^3\right]^{-1}\right\}, \quad z > z_t$$

$$H^2/H_0^2 = (1-M-\Omega_k)(1+z)^4 + \Omega_k(1+z)^2 + M\,, \quad z \le z_t$$

with

$$z_t = -1 + \frac{3\Omega_m}{4(1-M-\Omega_k-\Omega_r)}$$

We see that above the phase transition, the universe behaves as one with matter (plus radiation plus spatial curvature) plus a constant, and after the phase transition it effectively has a dark radiation component that rapidly redshifts away leaving a de Sitter phase.

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$$H^2/H_0^2 = (1-M-\Omega_k)(1+z)^4 + \Omega_k(1+z)^2 + M\,,\quad z \le z_t$$

with

$$z_t = -1 + \frac{3\Omega_m}{4(1-M-\Omega_k-\Omega_r)}$$

The original model did not include an explicit high redshift cosmological constant; we see that this implies that

$$\Omega_m = \frac{4}{3}\left[3M(1-M-\Omega_k-\Omega_r)^3\right]^{1/4}$$

i.e. the parameter M is fixed and depends on the matter density, and this model has the same number of degrees of freedom as ΛCDM.

Parker Vacuum Metamorphosis

When the Ricci scalar evolves during cosmic history to reach the scalar field mass squared, then a phase transition occurs and R freezes with

$R = 6(\dot{H} + 2H^2 + ka^{-2}) = m^2$ and defining $M = m^2/(12H_0^2)$

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with

$$z_t = -1 + \frac{3\Omega_m}{4(1-M-\Omega_k-\Omega_r)}$$

However, we can also consider an extended VM where M is an independent parameter. In this case, the massive scalar field has a vacuum expectation value that manifests as a cosmological constant, and these conditions are assumed:

$$\frac{4}{3}(1-M-\Omega_k-\Omega_r) \le \Omega_m \le \frac{4}{3}\left[3M(1-M-\Omega_k-\Omega_r)^3\right]^{1/4}$$

corresponding to

$z_t \ge 0$ (left inequality) $\qquad \Omega_{\rm de}(z > z_t) \ge 0$ (right inequality)

A Vacuum Phase Transition Solves the H_0 Tension

The effective dark energy equation of state below the phase transition is

$$w(z) = -1 - \frac{1}{3}\frac{3\Omega_m(1+z)^3 - 4(1-M-\Omega_k-\Omega_r)(1+z)^4}{M+(1-M-\Omega_k-\Omega_r)(1+z)^4 - \Omega_m(1+z)^3}$$

The equation of state behaviour is phantom today, and more deeply phantom in the past.

In the case without the cosmological constant there is no DE above the transition.

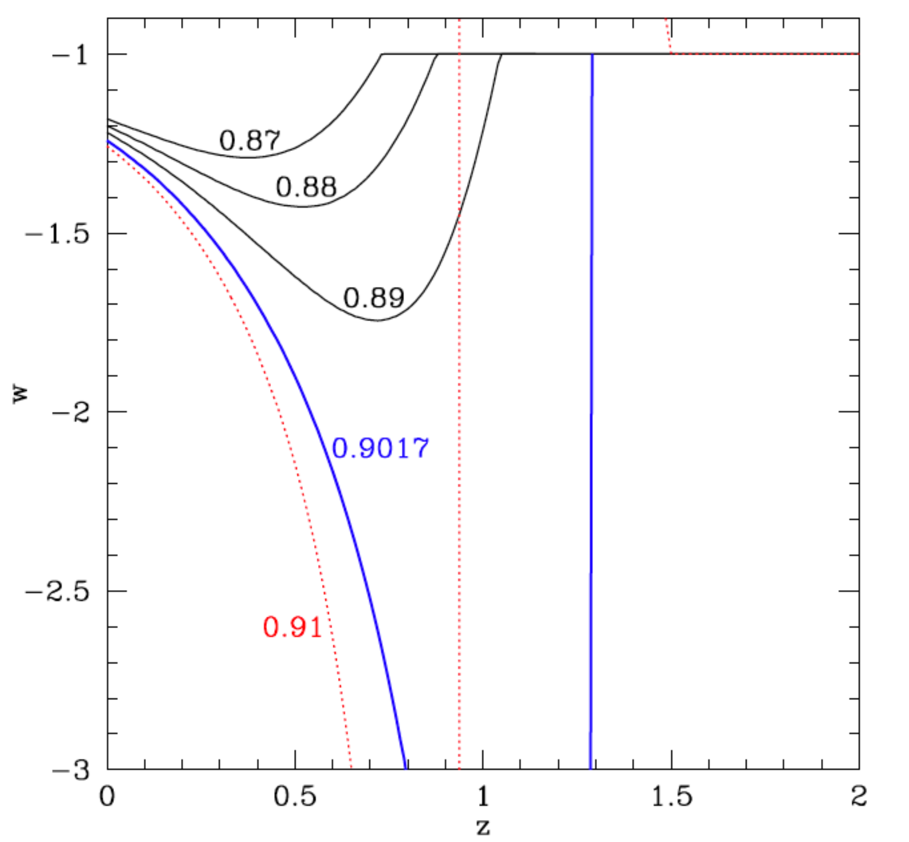


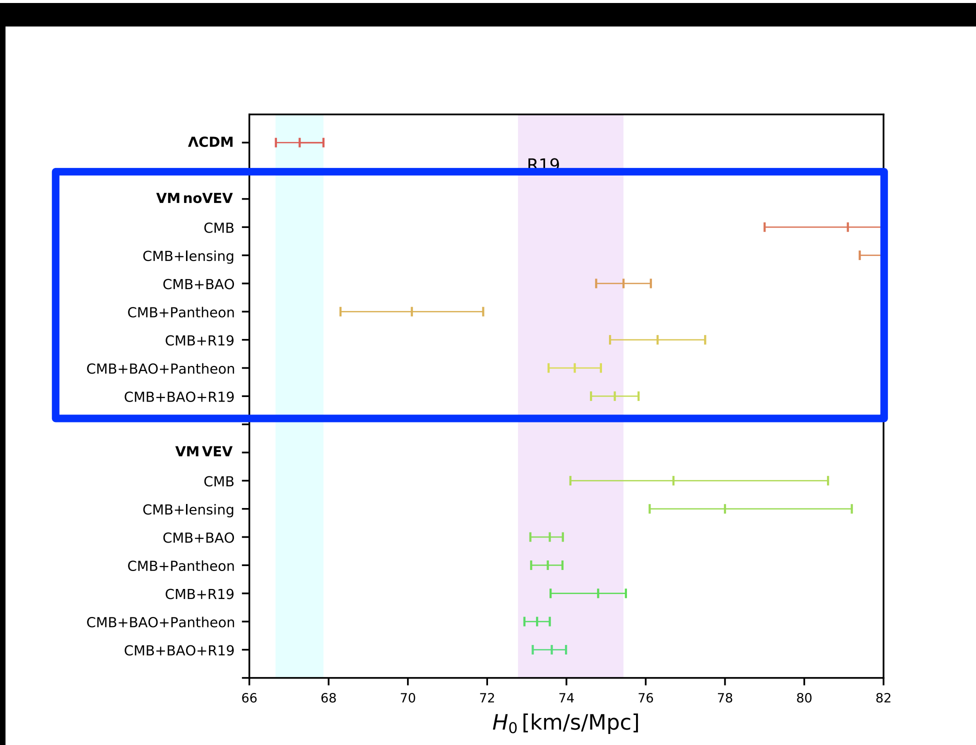
Figure 1. The effective dark energy equation of state evolution is plotted vs redshift for several values of the mass parameter M, for $\Omega_m = 0.3$. The bold blue curve shows the original case (our preferred model) where there is no cosmological constant, while the medium black curves show the elaborated case with an added cosmological constant, and the dotted red curve shows one with a negative cosmological constant (causing w to first shoot up to large positive values before it plummets to highly negative values).

Di Valentino et al., Phys.Rev. D97 (2018) no.4, 043528

Constraints at 68% cl.

A Vacuum Phase Transition Solves the H_0 Tension

Parameters	CMB	CMB+lensing	CMB+BAO	CMB+Pantheon	CMB+R19	CMB+BAO+Pantheon	CMB+BAO+R19
$\Omega_b h^2$	0.02238 ± 0.00014	0.02242 ± 0.00013	0.02218 ± 0.00012	0.02201 ± 0.00013	0.02221 ± 0.00012	0.02213 ± 0.00012	0.02217 ± 0.00012
$100\theta_{MC}$	1.04091 ± 0.00030	1.04097 ± 0.00029	1.04060 ± 0.00029	1.04033 ± 0.00031	1.04063 ± 0.00029	1.04053 ± 0.00029	1.04060 ± 0.00029
τ	0.0524 ± 0.0078	0.0510 ± 0.0078	$0.0458^{+0.0083}_{-0.0067}$	$0.039^{+0.010}_{-0.007}$	0.0469 ± 0.0075	$0.0449^{+0.0079}_{-0.0065}$	$0.0456^{+0.0083}_{-0.0068}$
M	$0.9363^{+0.0055}_{-0.0044}$	0.9406 ± 0.0034	0.9205 ± 0.0023	$0.8996^{+0.0081}_{-0.0073}$	$0.9230^{+0.0042}_{-0.0036}$	0.9163 ± 0.0023	0.9198 ± 0.0020
$\ln(10^{10}A_s)$	3.041 ± 0.016	3.036 ± 0.015	$3.035^{+0.017}_{-0.014}$	$3.027^{+0.020}_{-0.014}$	3.036 ± 0.016	$3.035^{+0.017}_{-0.014}$	$3.035^{+0.017}_{-0.015}$
n_s	0.9643 ± 0.0039	0.9663 ± 0.0036	0.9572 ± 0.0031	0.9511 ± 0.0036	0.9585 ± 0.0033	0.9560 ± 0.0031	0.9571 ± 0.0031
H_0[km/s/Mpc]	81.1 ± 2.1	82.9 ± 1.5	75.44 ± 0.69	70.1 ± 1.8	76.3 ± 1.2	74.21 ± 0.66	75.22 ± 0.60
σ_8	0.9440 ± 0.0077	0.9392 ± 0.0067	$0.9456^{+0.0083}_{-0.0070}$	$0.9419^{+0.0098}_{-0.0069}$	0.9457 ± 0.0075	$0.9461^{+0.0080}_{-0.0068}$	$0.9457^{+0.0082}_{-0.0073}$
S_8	0.805 ± 0.022	0.783 ± 0.014	0.865 ± 0.010	0.927 ± 0.023	0.856 ± 0.015	0.880 ± 0.010	0.8675 ± 0.0098
Ω_m	$0.218^{+0.010}_{-0.012}$	0.2085 ± 0.0076	0.2510 ± 0.0046	0.291 ± 0.015	$0.2458^{+0.0074}_{-0.0084}$	0.2593 ± 0.0046	0.2525 ± 0.0040
$\chi^2_{\rm bf}$	2767.74	2776.23	2806.22	3874.13	2777.04	3910.01	2808.34
$\Delta\chi^2_{\rm bf}$	-4.91	-5.81	$+26.51$	$+66.63$	-14.80	$+95.83$	$+11.29$



We dont solve the tension, we do obtain H0~73-74 km/s/Mpc !!

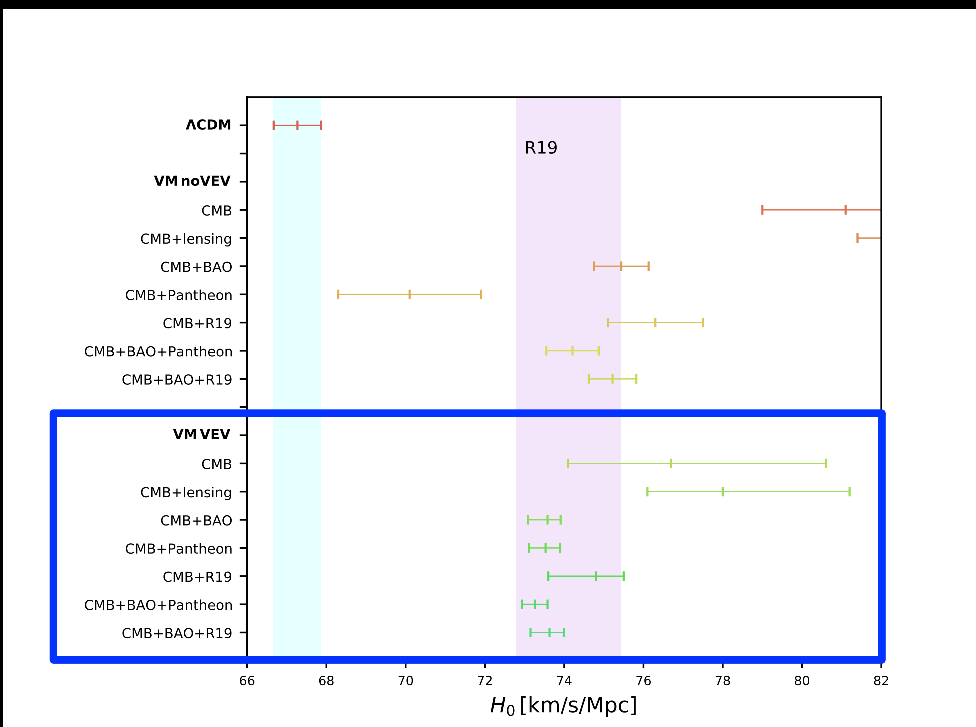
H0 is exactly in agreement with R20 even if BAO and Pantheon are included. However, this worsen considerably the fit of the data because the model fails in recover the shape of H(z) at low redshifts.

Di Valentino et al., Phys.Dark Univ. 30 (2020) 100733

Constraints at 68% cl.

A Vacuum Phase Transition Solves the H_0 Tension

Parameters	CMB	CMB+lensing	CMB+BAO	CMB+Pantheon	CMB+R19	CMB+BAO+Pantheon	CMB+BAO+R19
$\Omega_b h^2$	0.02238 ± 0.00015	0.02242 ± 0.00015	0.02229 ± 0.00014	0.02233 ± 0.00015	0.02236 ± 0.00015	0.02228 ± 0.00014	0.02230 ± 0.00014
$\Omega_c h^2$	0.1200 ± 0.0013	0.1194 ± 0.0012	0.1213 ± 0.0012	0.1208 ± 0.0014	0.1203 ± 0.0014	0.1217 ± 0.0012	0.1212 ± 0.0011
$100\theta_{MC}$	1.04092 ± 0.00031	1.04098 ± 0.00030	1.04079 ± 0.00030	1.04086 ± 0.00031	1.04090 ± 0.00032	1.04077 ± 0.00030	1.04080 ± 0.00031
τ	0.0541 ± 0.0078	0.0529 ± 0.0076	0.0527 ± 0.0077	0.0529 ± 0.0077	0.0537 ± 0.0079	0.0524 ± 0.0078	0.0530 ± 0.0077
M	$0.914^{+0.021}_{-0.009}$	$0.920^{+0.017}_{-0.007}$	$0.8950^{+0.0013}_{-0.0033}$	$0.8940^{+0.0012}_{-0.0022}$	$0.9028^{+0.0046}_{-0.0085}$	$0.8929^{+0.0010}_{-0.0016}$	$0.8953^{+0.0014}_{-0.0034}$
$\ln(10^{10}A_s)$	3.044 ± 0.016	3.039 ± 0.015	3.044 ± 0.016	3.043 ± 0.016	3.044 ± 0.016	3.044 ± 0.016	3.045 ± 0.016
n_s	0.9653 ± 0.0044	0.9666 ± 0.0040	0.9620 ± 0.0041	0.9632 ± 0.0025	0.9644 ± 0.0044	0.9612 ± 0.0040	0.9623 ± 0.0038
H_0[km/s/Mpc]	$76.7^{+3.9}_{-2.6}$	$78.0^{+3.2}_{-1.9}$	$73.58^{+0.33}_{-0.49}$	$73.53^{+0.37}_{-0.42}$	$74.8^{+0.7}_{-1.2}$	73.26 ± 0.32	$73.63^{+0.33}_{-0.48}$
σ_8	$0.895^{+0.016}_{-0.026}$	$0.900^{+0.024}_{-0.019}$	0.876 ± 0.010	0.872 ± 0.010	$0.880^{+0.012}_{-0.016}$	0.8756 ± 0.0091	$0.8760^{+0.0093}_{-0.0099}$
S_8	0.805 ± 0.016	$0.796^{+0.013}_{-0.015}$	0.825 ± 0.014	0.821 ± 0.015	0.813 ± 0.015	0.830 ± 0.013	0.825 ± 0.013
Ω_m	$0.243^{+0.017}_{-0.025}$	$0.235^{+0.011}_{-0.020}$	$0.2664^{+0.0048}_{-0.0043}$	0.2661 ± 0.0050	$0.2561^{+0.0081}_{-0.0068}$	0.2695 ± 0.0041	0.2660 ± 0.0044
$\chi^2_{\rm bf}$	2769.74	2778.93	2790.75	3840.55	2772.09	3857.21	2789.76
$\Delta\chi^2_{\rm bf}$	−2.91	−3.11	+11.04	+33.05	−19.75	+43.03	−7.29



And a more ad hoc VM model that includes a cosmological constant, i.e. allowing the vacuum criticality parameter M to float, is even better.

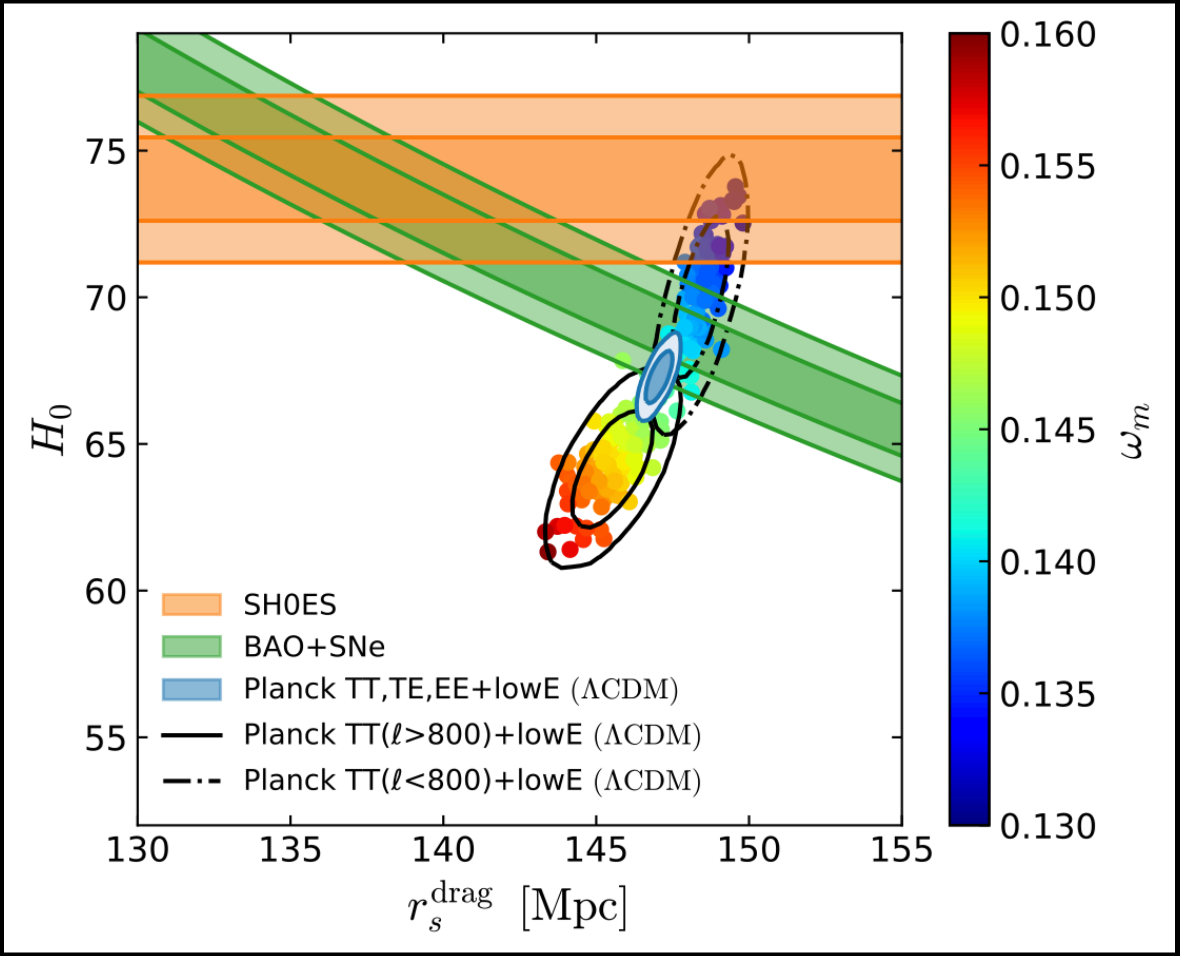
For all the dataset combinations H0~73-74 km/s/Mpc !!

Di Valentino et al., Phys.Dark Univ. 30 (2020) 100733

What about BAO+Pantheon?

BAO+Pantheon measurements constrain the product of H0 and the sound horizon r_s .

In order to have a higher H0 value in agreement with R20, we need r_s near 137 Mpc. However, Planck by assuming ΛCDM, prefers r_s near 147 Mpc. Therefore, a cosmological solution that can increase H0 and at the same time can lower the sound horizon inferred from CMB data is promising to put in agreement all the measurements.

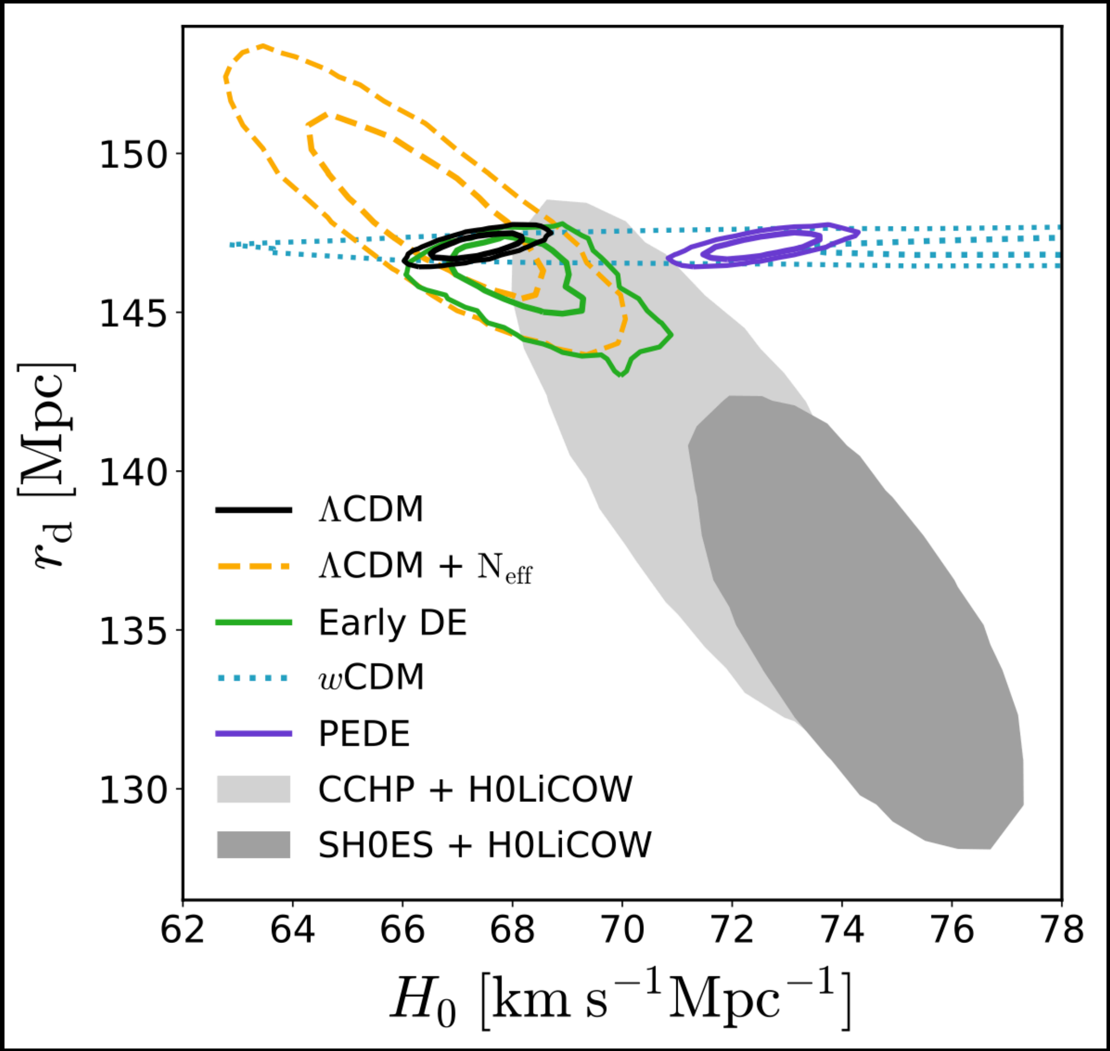


Knox and Millea, *Phys.Rev.D* 101 (2020) 4, 043533

Early vs late time solutions

Here we can see the comparison of the 2σ credibility regions of the CMB constraints and the measurements from late-time observations (SN + BAO + H0LiCOW + SH0ES).

We see that the late time solutions, as wCDM, increase H0 but leave r_s unaltered.
However, the early time solutions, as Neff or Early Dark Energy, move in the right direction both the parameters, but can't solve completely the H0 tension with R20.



Arendse et al., Astron.Astrophys. 639 (2020) A57

Formally successful models in solving H0

tension $\leq 1\sigma$ *"Excellent models"*	tension $\leq 2\sigma$ *"Good models"*	tension $\leq 3\sigma$ *"Promising models"*
Early Dark Energy [228, 235, 240, 250]	Early Dark Energy [212, 229, 236, 263]	DE in extended parameter spaces [289]
Exponential Acoustic Dark Energy [259]	Rock 'n' Roll [242]	Dynamical Dark Energy [281, 309]
Phantom Crossing [315]	New Early Dark Energy [247]	Holographic Dark Energy [350]
Late Dark Energy Transition [317]	Acoustic Dark Energy [257]	Swampland Conjectures [370]
Metastable Dark Energy [314]	Dynamical Dark Energy [309]	MEDE [399]
PEDE [394]	Running vacuum model [332]	Coupled DM - Dark radiation [534]
Vacuum Metamorphosis [402]	Bulk viscous models [340, 341]	Decaying Ultralight Scalar [538]
Elaborated Vacuum Metamorphosis [401, 402]	Holographic Dark Energy [350]	BD-ΛCDM [852]
Sterile Neutrinos [433]	Phantom Braneworld DE [378]	Metastable Dark Energy [314]
Decaying Dark Matter [481]	PEDE [391, 392]	Self-Interacting Neutrinos [700]
Neutrino-Majoron Interactions [509]	Elaborated Vacuum Metamorphosis [401]	Dark Neutrino Interactions [716]
IDE [637, 639, 657, 661]	IDE [659, 670]	IDE [634–636, 653, 656, 663, 669]
DM - Photon Coupling [685]	Interacting Dark Radiation [517]	Scalar-tensor gravity [855, 856]
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Über-Gravity [59]	Self-interacting sterile neutrinos [711]	Modified recombination [986]
Galileon Gravity [875]	$f(\mathcal{T})$ gravity theory [817]	Effective Electron Rest Mass [989]
Unimodular Gravity [890]	Über-Gravity [871]	Super ΛCDM [1007]
Time Varying Electron Mass [990]	VCDM [893]	Axi-Higgs [991]
ΛCDM [995]	Primordial magnetic fields [992]	Self-Interacting Dark Matter [479]
Ginzburg-Landau theory [996]	Early modified gravity [859]	Primordial Black Holes [545]
Lorentzian Quintessential Inflation [979]	Bianchi type I spacetime [999]	
Holographic Dark Energy [351]	$f(\mathcal{T})$ [818]	

Table B2. Models solving the H_0 tension with R20 within 1σ, 2σ and 3
Planck in combination with additional cosmological probes. D
datasets are discussed in the main text.

Combination of datasets

Di Valentino et al., Class.Quant.Grav. (2021), arXiv:2103.01183 [astro-ph.CO]

Formally successful models in solving H0

tension $\leq 1\sigma$ *"Excellent models"*	tension $\leq 2\sigma$ *"Good models"*	tension $\leq 3\sigma$ *"Promising models"*
Early Dark Energy [228, 235, [illegible]]	Early Dark Energy [212, 229, 236, 263]	DE in extended parameter spaces [289]
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Phantom Crossing [315]	New Early Dark Energy [247]	Holographic Dark Energy [350]
Late Dark Energy Transition [317]	Acoustic Dark Energy [257]	Swampland Conjectures [370]
Metastable Dark Energy [314]	Dynamical Dark Energy [309]	MEDE [399]
PEDE [394]	Running vacuum model [332]	Coupled DM - Dark radiation [534]
Vacuum Metamorphosis [402]	Bulk viscous models [340, 341]	Decaying Ultralight Scalar [538]
Elaborated Vacuum Metamorphosis [[illegible], 402]	Holographic Dark Energy [350]	BD-ΛCDM [852]
Sterile Neutrinos [433]	Phantom Braneworld DE [378]	Metastable Dark Energy [314]
Decaying Dark Matter [481]	PEDE [391, 392]	Self-Interacting Neutrinos [700]
Neutrino-Majoron Interactions [509]	Elaborated Vacuum Metamorphosis [401]	Dark Neutrino Interactions [716]
IDE [637, 639, 657, 661]	IDE [659, 670]	IDE [634–636, 653, 656, 663, 669]
DM - Photon Coupling [685]	Interacting Dark Radiation [517]	Scalar-tensor gravity [855, 856]
$f(\mathcal{T})$ gravity theory [812]	Decaying Dark Matter [471, 474]	Galileon gravity [877, 881]
BD-ΛCDM [851]	DM - Photon Coupling [686]	Nonlocal gravity [886]
Über-Gravity [59]	Self-interacting sterile neutrinos [711]	Modified recombination [986]
Galileon Gravity [875]	$f(\mathcal{T})$ gravity theory [817]	Effective Electron Rest Mass [989]
Unimodular Gravity [890]	Über-Gravity [871]	Super ΛCDM [1007]
Time Varying Electron Mass [990]	VCDM [893]	Axi-Higgs [991]
ΛCDM [995]	Primordial magnetic fields [992]	Self-Interacting Dark Matter [479]
Ginzburg-Landau theory [996]	Early modified gravity [859]	Primordial Black Holes [545]
Lorentzian Quintessential Inflation [979]	Bianchi type I spacetime [999]	
Holographic Dark Energy [351]	$f(\mathcal{T})$ [818]	

Table B2. Models solving the H_0 tension with R20 within 1σ, 2σ and [illegible] *Planck* in combination with additional cosmological probes. D[illegible] datasets are discussed in the main text.

Combination of datasets

Di Valentino et al., Class.Quant.Grav. (2021), arXiv:2103.01183 [astro-ph.CO]

How the H0 determination works?

The SH0ES Collaboration, combining together anchors, cepheids and calibrators, produces a constraint on the SN absolute magnitude MB, that depends on the astrophysical properties of the sources and is independent of cosmology.

At this point, in order to obtain the H0 measurement, it uses the Hubble-flow supernovae in the redshift range $0.023 \leq z \leq 0.15$ to probe the luminosity distance-redshift relation, and it adopts a cosmography with $q0 = -0.55$ and $j0 = 1$.

Camarena & Marra, arXiv:1906.11814

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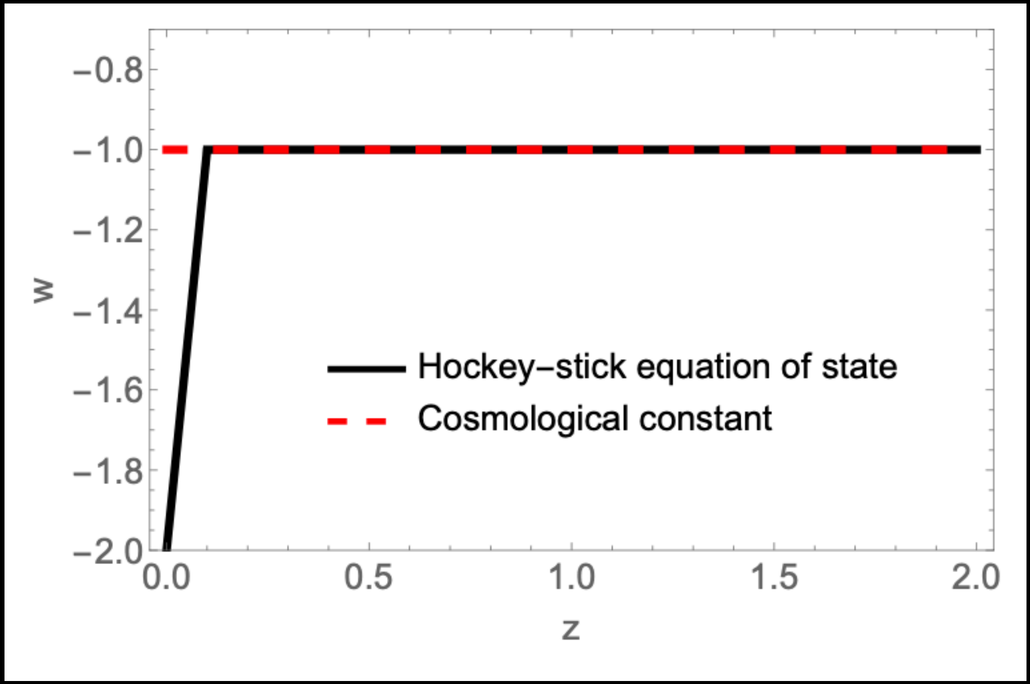
Camarena & Marra, arXiv:1906.11814



Camarena & Marra, arXiv:2101.08641

If we use a H0 prior as obtained by SH0ES and we claim to solve the H0 tension, we have to be careful with hockey-stick dark energy scenarios, i.e. models that have a transition at very-low redshift ($z < 0.1$).
For these classes of models it is better to consider a MB prior!

The MB tension at 3.4σ

Therefore, rather than explaining the H0 tension, one should instead focus on the supernova absolute magnitude tension, because the SH0ES team does not directly measure H0.
The MB approach allows to skip the cosmography part, and to avoid the double counting of SN in the redshift range $0.023 \leq z \leq 0.15$.
Actually, imposing a H0 prior, all information on the shape of the SN magnitude-redshift relation is lost.

The Planck constraint on the SN absolute magnitude MB using the parametric-free inverse distance ladder predicts

$$M_B^{\mathrm{P18}} = -19.401 \pm 0.027 \text{ mag}$$

while the the SN measurements from SH0ES corresponds to

$$M_B^{\mathrm{R20}} = -19.244 \pm 0.037 \text{ mag}$$

corresponding to a 3.4σ tension.

Camarena & Marra, arXiv:2101.08641

Successful models in solving MB

Model	$\Delta N_{\rm param}$	M_B	Gaussian Tension
ΛCDM	0	-19.416 ± 0.012	4.4σ
$\Delta N_{\rm ur}$	1	-19.395 ± 0.019	3.6σ
SIDR	1	-19.385 ± 0.024	3.2σ
DR-DM	2	-19.413 ± 0.036	3.3σ
mixed DR	2	-19.388 ± 0.026	3.2σ
SIν+DR	3	-19.440 ± 0.038	3.7σ
Majoron	3	-19.380 ± 0.027	3.0σ
primordial B	1	-19.390 ± 0.018	3.5σ
varying m_e	1	-19.391 ± 0.034	2.9σ
varying m_e+Ω_k	2	-19.368 ± 0.048	2.0σ
EDE	3	-19.390 ± 0.016	3.6σ
NEDE	3	-19.380 ± 0.021	3.2σ
CPL	2	-19.400 ± 0.016	3.9σ
PEDE	0	-19.349 ± 0.013	2.7σ
MPEDE	1	-19.400 ± 0.022	3.6σ
DM $\to$ DR+WDM	2	-19.410 ± 0.013	4.2σ
DM $\to$ DR	2	-19.410 ± 0.011	4.3σ

Schöneberg et al., arXiv:2107.10291 [astro-ph.CO]

Only 3 models alleviate the MB tension below 3σ, and none of them below 2σ.

Planck + BAO + Pantheon

Successful models in solving MB

Model	$\Delta N_{\rm param}$	M_B	Gaussian Tension	$\Delta\chi^2$	ΔAIC		Finalist	
ΛCDM	0	-19.416 ± 0.012	4.4σ	0.00	0.00	X	X	
$\Delta N_{\rm ur}$	1	-19.395 ± 0.019	3.6σ	-4.60	-2.60	X	X	
SIDR	1	-19.385 ± 0.024	3.2σ	-3.77	-1.77	X	X	
DR-DM	2	-19.413 ± 0.036	3.3σ	-7.82	-3.82	X	X	
mixed DR	2	-19.388 ± 0.026	3.2σ	-6.40	-2.40	X	X	
SIν+DR	3	-19.440 ± 0.038	3.7σ	-3.56	2.44	X	X	
Majoron	3	-19.380 ± 0.027	3.0σ	-13.74	-7.74	✓	✓	2
primordial B	1	-19.390 ± 0.018	3.5σ	-10.83	-8.83	✓	✓	3
varying m_e	1	-19.391 ± 0.034	2.9σ	-9.87	-7.87	✓	✓	3
varying m_e+Ω_k	2	-19.368 ± 0.048	2.0σ	-16.11	-12.11	✓	✓	1
EDE	3	-19.390 ± 0.016	3.6σ	-20.80	-14.80	✓	✓	2
NEDE	3	-19.380 ± 0.021	3.2σ	-17.70	-11.70	✓	✓	2
CPL	2	-19.400 ± 0.016	3.9σ	-4.23	-0.23	X	X	
PEDE	0	-19.349 ± 0.013	2.7σ	4.76	4.76	X	X	
MPEDE	1	-19.400 ± 0.022	3.6σ	-2.21	-0.21	X	X	
DM $\rightarrow$ DR+WDM	2	-19.410 ± 0.013	4.2σ	-4.18	-0.18	X	X	
DM $\rightarrow$ DR	2	-19.410 ± 0.011	4.3σ	0.11	4.11	X	X	

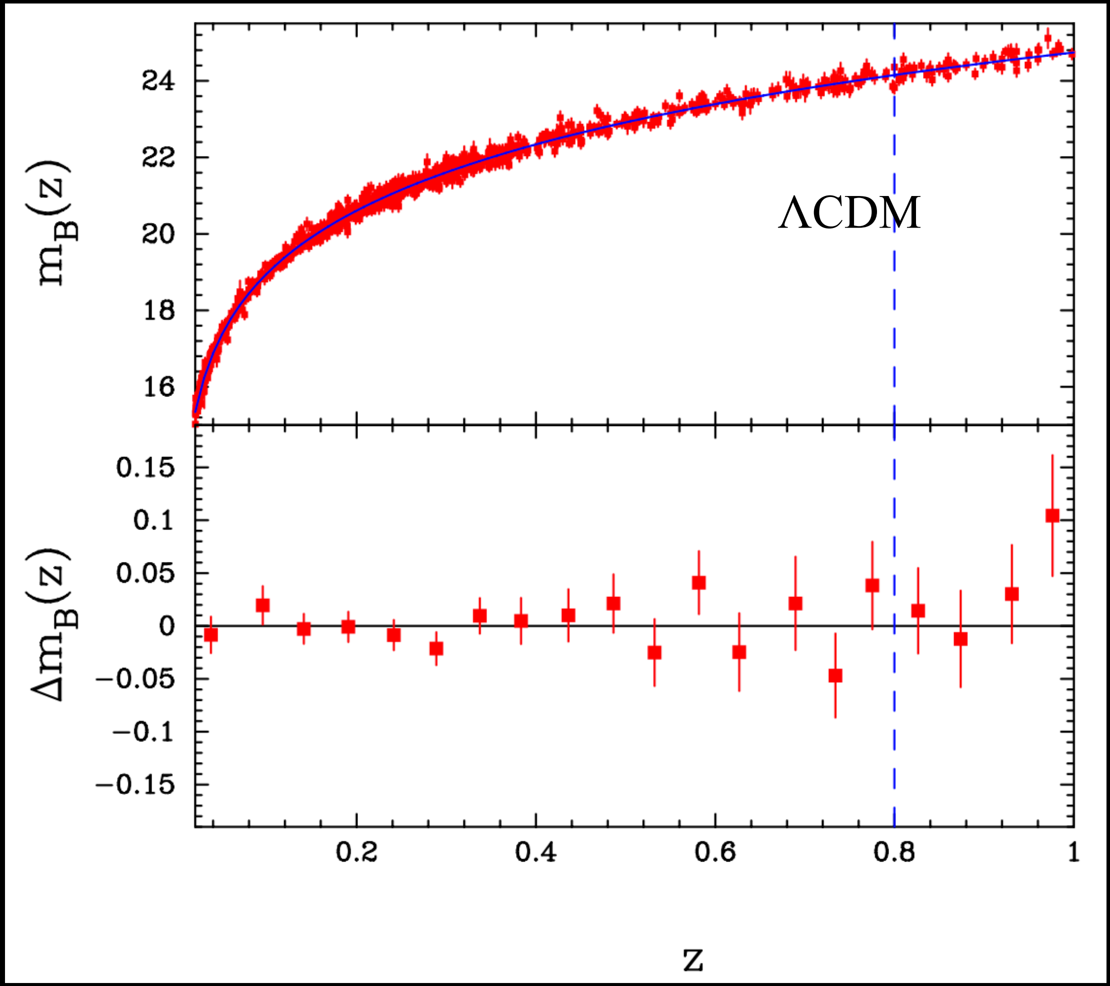
Schöneberg et al., arXiv:2107.10291 [astro-ph.CO]

Planck + BAO + Pantheon

However, between these models, only a closed universe with a varying electron mass is also favoured by the χ^2 and a model comparison with respect to the ΛCDM scenario.

Sekiguchi & Takahashi, arXiv:2007.03381 [astro-ph.CO]

The MB tension



It has been argued that the MB tension is another reason to exclude modifications of the expansion history at late times. Even if they can give a higher H0 value, these are strongly disfavoured by the Pantheon magnitude-redshift relation.

Efstathiou, arXiv:2103.08723

Let’s see another example…

IDE can solve the H0 tension

In the standard cosmological framework, the dark matter is assumed to be collisionless. In practice this means that one arbitrarily sets the dark matter interactions to zero when predicting the angular power spectrum of the CMB.

In particular, dark matter and dark energy are described as separate fluids not sharing interactions beyond gravitational ones. However, from a microphysical perspective it is hard to imagine how non-gravitational DM-DE interactions can be avoided, unless forbidden by a fundamental symmetry. This has motivated a large number of studies based on models where DM and DE share interactions other than gravitational.

IDE can solve the H0 tension

At the background level, the conservation equations for the pressureless DM and DE components can be decoupled into two separate equations with an inclusion of an arbitrary function, Q, known as the coupling or interacting function:

$$\begin{aligned} \dot{\rho}_c + 3\mathcal{H}\rho_c &= Q\,, \\ \dot{\rho}_x + 3\mathcal{H}(1+w)\rho_x &= -Q\,, \end{aligned}$$

and we assume the phenomenological form for the interaction rate:

$$Q = \xi\mathcal{H}\rho_x$$

proportional to the dark energy density ρ_x and the conformal Hubble rate $\mathcal{H}$, via a negative dimensionless parameter ξ quantifying the strength of the coupling, to avoid early-time instabilities.

Gavela et al. J. Cosmol. Astropart. Phys. 07 (2009) 034

IDE can solve the H0 tension

In this scenario of IDE the tension on H0 between the Planck satellite and R19 is completely solved. The coupling could affect the value of the present matter energy density Ω_m. Therefore, if within an interacting model Ω_m is smaller (because for negative ξ the dark matter density will decay into the dark energy one), a larger value of H0 would be required in order to satisfy the peaks structure of CMB observations, which accurately determine the value of $\Omega_m h^2$.

Parameter	*Planck*	*Planck+R19*
$\Omega_b h^2$	0.02239 ± 0.00015	0.02239 ± 0.00015
$\Omega_c h^2$	< 0.105	< 0.0615
n_s	0.9655 ± 0.0043	0.9656 ± 0.0044
$100\theta_s$	$1.0458^{+0.0033}_{-0.0021}$	1.0470 ± 0.0015
τ	0.0541 ± 0.0076	0.0534 ± 0.0080
ξ	$-0.54^{+0.12}_{-0.28}$	$-0.66^{+0.09}_{-0.13}$
$H_0\,[\mathrm{km\,s^{-1}\,Mpc^{-1}}]$	$72.8^{+3.0}_{-1.5}$	$74.0^{+1.2}_{-1.0}$

TABLE I. Mean values with their 68% C.L. errors on selected cosmological parameters within the $\xi\Lambda$CDM model, considering either the *Planck* 2018 legacy dataset alone, or the same dataset in combination with the *R19* Gaussian prior on H_0 based on the latest local distance measurement from *HST*. The quantity quoted in the case of $\Omega_c h^2$ is the 95% C.L. upper limit.

Di Valentino et al., Phys.Dark Univ. 30 (2020) 100666

IDE is in agreement with the near universe

Within interacting cosmologies the growth of dark matter perturbations will be larger than in uncoupled models.

This feature will be general for models with negative coupling and in which the energy exchange among the dark sectors is proportional to ρ_X, due to a suppression of the friction term and an enhancement of the source term in the differential growth equation.

arXiv.org > astro-ph > arXiv:0905.0492

Astrophysics > Cosmology and Nongalactic Astrophysics

[Submitted on 5 May 2009 (v1), last revised 24 Jun 2009 (this version, v2)]

The Growth of Structure in Interacting Dark Energy Models

Gabriela Caldera-Cabral, Roy Maartens, Bjoern Malte Schaefer

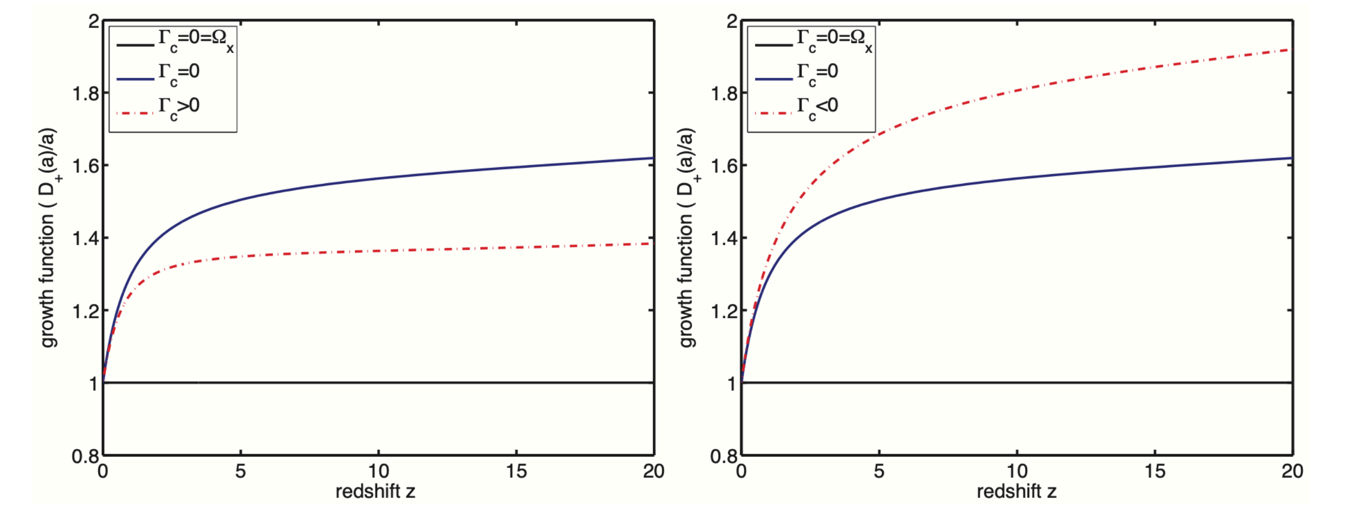
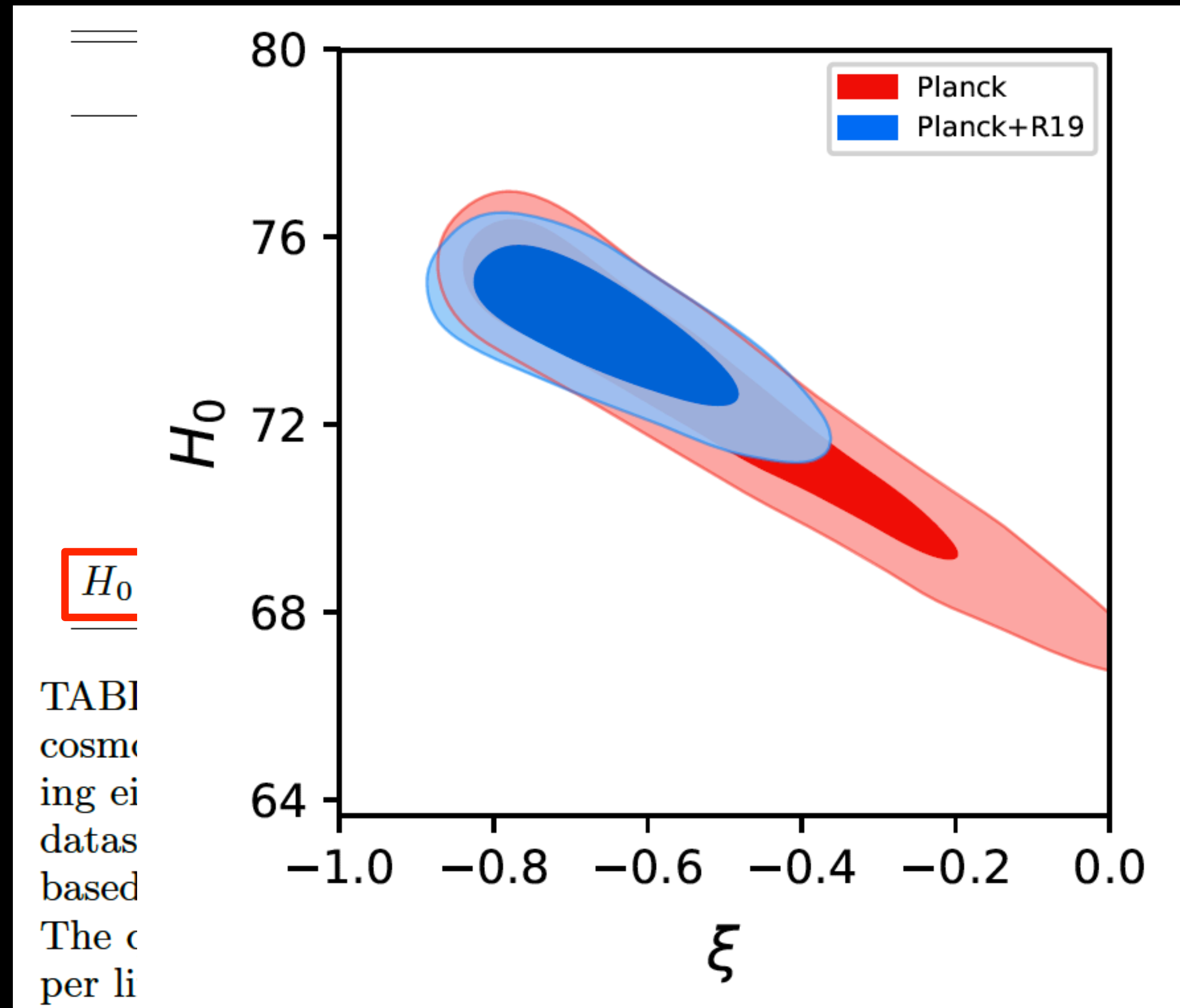


FIG. 2: Linear growth function $D_+ = \delta_c/\delta_{c0}$, normalized to today's value, relative to its value in a pure-matter model ($D_+ = a$). The interacting models (dashed-dotted lines), with $\Gamma_c = \pm 0.3 H_0$, are shown in comparison to non-interacting models (solid lines).

IDE can solve the H0 tension

Therefore we can safely combine the two datasets together, and we obtain a non-zero dark matter-dark energy coupling ξ at more than FIVE standard deviations.



Di Valentino et al., Phys.Dark Univ. 30 (2020) 100666

Bayes factor

Anyway it is clearly interesting to quantify the better accordance of a model with the data respect to another by using the marginal likelihood also known as the Bayesian evidence.

The Bayesian evidence weights the simplicity of the model with the improvement of the fit of the data. In other words, because of the Occam's razor principle, models with additional parameters are penalised, if don't improve significantly the fit.

Given two competing models M_0 and M_1 it is useful to consider the ratio of the likelihood probability (the Bayes factor):

$$ln\mathcal{B} = p(\boldsymbol{x}|M_0)/p(\boldsymbol{x}|M_1)$$

According to the revised Jeffrey's scale by Kass and Raftery 1995, the evidence for M_0 (against M_1) is considered as "weak" if $| \ln B | > 1.0$, "moderate" if $| \ln B | > 2.5$, and "strong" if $| \ln B | > 5.0$.

IDE can solve the H0 tension

Computing the Bayes factor for the IDE model with respect to LCDM for the Planck dataset we find lnB = 1.2, i.e. a weak evidence for the IDE model.
If we consider Planck + R19 we find the extremely high value lnB=10.0, indicating a strong evidence for the IDE model.

Parameter	*Planck*	*Planck+R19*
$\Omega_b h^2$	0.02239 ± 0.00015	0.02239 ± 0.00015
$\Omega_c h^2$	< 0.105	< 0.0615
n_s	0.9655 ± 0.0043	0.9656 ± 0.0044
$100\theta_s$	$1.0458^{+0.0033}_{-0.0021}$	1.0470 ± 0.0015
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ξ	$-0.54^{+0.12}_{-0.28}$	$-0.66^{+0.09}_{-0.13}$
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TABLE I. Mean values with their 68% C.L. errors on selected cosmological parameters within the $\xi\Lambda$CDM model, considering either the *Planck* 2018 legacy dataset alone, or the same dataset in combination with the *R19* Gaussian prior on H_0 based on the latest local distance measurement from *HST*. The quantity quoted in the case of $\Omega_c h^2$ is the 95% C.L. upper limit.

Di Valentino et al., Phys.Dark Univ. 30 (2020) 100666

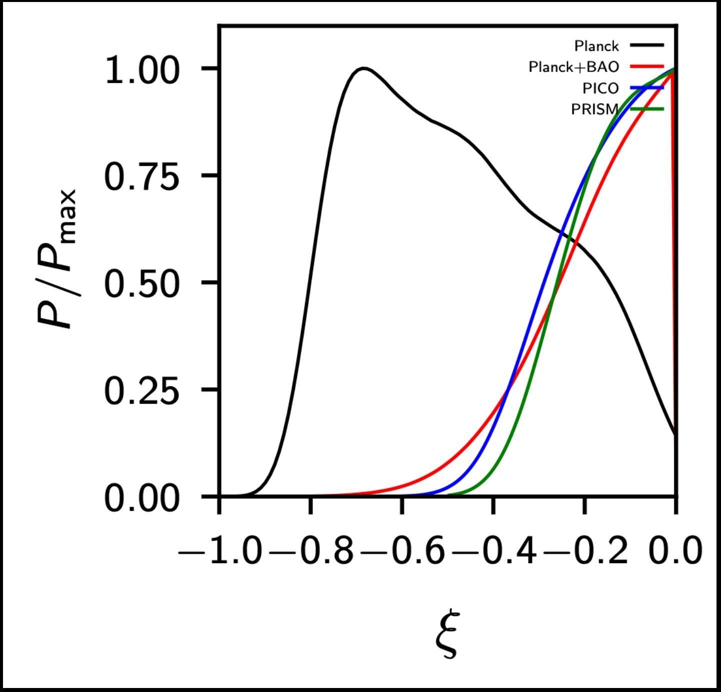
fake IDE detection

Parameters	Fiducial model	Planck	Planck+BAO	PICO	PRISM
$\Omega_b h^2$	0.02236	0.02238 ± 0.00015	0.02230 ± 0.00014	0.022364 ± 0.000029	0.022361 ± 0.000019
$\Omega_c h^2$	0.1202	$0.056^{+0.025}_{-0.047}$	$0.101^{+0.019}_{-0.006}$	$0.100^{+0.019}_{-0.008}$	$0.103^{+0.016}_{-0.007}$
$100\theta_{MC}$	1.04090	$1.0451^{+0.0021}_{-0.0032}$	$1.0419^{+0.0005}_{-0.0011}$	$1.04206^{+0.0005}_{-0.0011}$	$1.04191^{+0.00042}_{-0.00094}$
τ	0.0544	$0.0528^{+0.010}_{-0.009}$	0.0517 ± 0.0098	$0.0543^{+0.0016}_{-0.0019}$	$0.0542^{+0.0017}_{-0.0019}$
n_s	0.9649	0.9652 ± 0.0041	0.9624 ± 0.0036	0.9571 ± 0.0014	0.9657 ± 0.0012
$\ln(10^{10} A_s)$	3.045	$3.041^{+0.020}_{-0.018}$	3.042 ± 0.019	$3.0436^{+0.0030}_{-0.0034}$	3.0435 ± 0.0032
ξ	0	$-0.48^{+0.16}_{-0.30}$	> -0.223	> -0.220	> -0.195

Di Valentino & Mena, Mon.Not.Roy.Astron.Soc. 500 (2020) 1, L22-L26, arXiv:2009.12620

For a simulated Planck-like experiment, due to the strong correlation present between the standard and the exotic physics parameters, there is a dangerous detection at more than 3σ for a coupling between dark matter and dark energy different from zero, even if the fiducial model has ξ =0:

$-0.85 < \xi < -0.02$ at 99% CL



Simulated experiments

fake IDE detection

Parameters	Fiducial model	Planck	Planck+BAO	PICO	PRISM
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$100\theta_{MC}$	1.04090	$1.0451^{+0.0021}_{-0.0032}$	$1.0419^{+0.0005}_{-0.0011}$	$1.04206^{+0.0005}_{-0.0011}$	$1.04191^{+0.00042}_{-0.00094}$
τ	0.0544	$0.0528^{+0.010}_{-0.009}$	0.0517 ± 0.0098	$0.0543^{+0.0016}_{-0.0019}$	$0.0542^{+0.0017}_{-0.0019}$
n_s	0.9649	0.9652 ± 0.0041	0.9624 ± 0.0036	0.9571 ± 0.0014	0.9657 ± 0.0012
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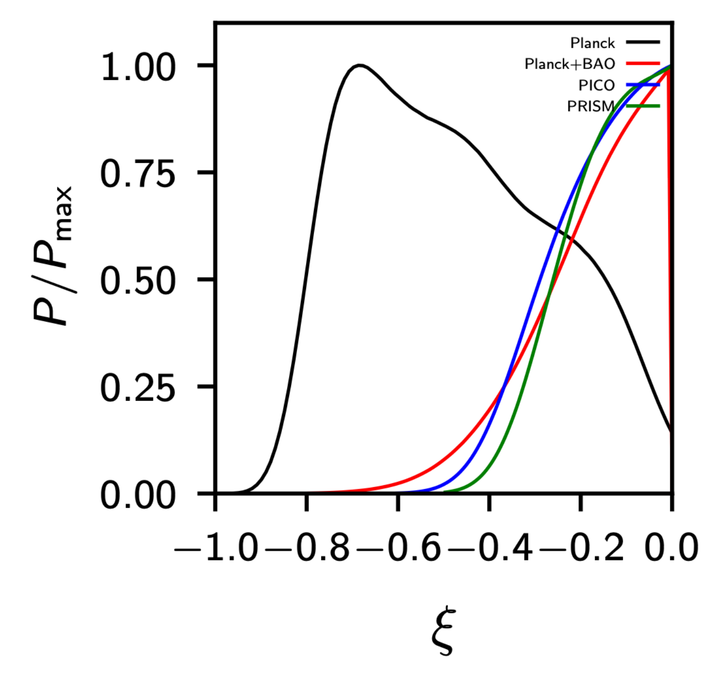
ξ	0	$-0.48^{+0.16}_{-0.30}$	> -0.223	> -0.220	> -0.195

Di Valentino & Mena, Mon.Not.Roy.Astron.Soc. 500 (2020) 1, L22-L26, arXiv:2009.12620

The inclusion of simulated BAO data, a mock dataset built using the same fiducial cosmological model than that of the CMB, helps in breaking the degeneracy, providing a lower limit for the coupling ξ in perfect agreement with zero.



Simulated experiments

Constraints at 68% cl

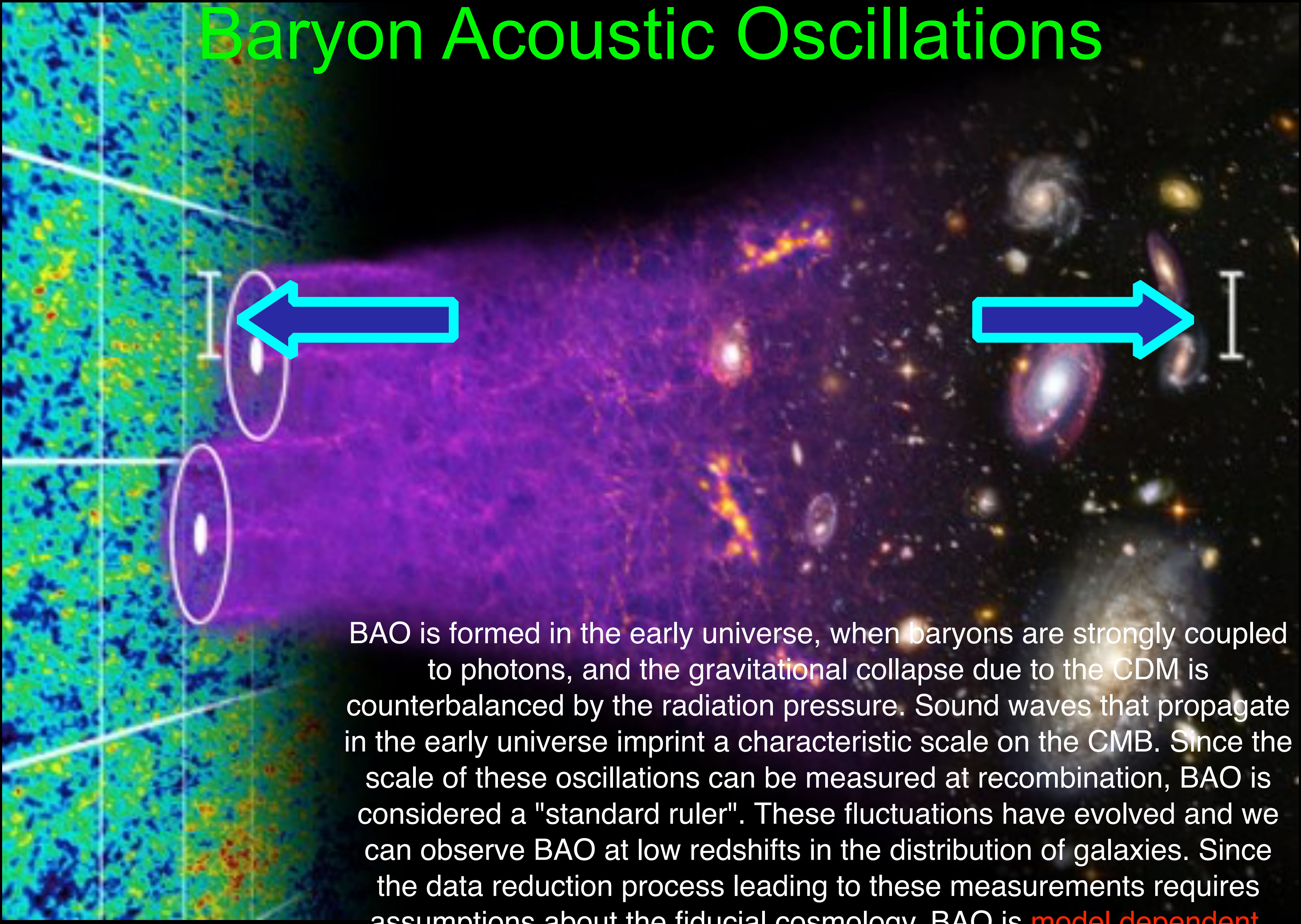
IDE can solve the H0 tension

Parameters	Planck	Planck +R19	Planck +lensing	Planck +BAO	Planck + Pantheon
$\Omega_b h^2$	0.02239 ± 0.00015	0.02239 ± 0.00015	0.02241 ± 0.00014	0.02236 ± 0.00014	0.02235 ± 0.00015
$\Omega_c h^2$	< 0.0634	$0.031^{+0.013}_{-0.023}$	< 0.0675	$0.095^{+0.022}_{-0.008}$	$0.103^{+0.013}_{-0.007}$
$100\theta_{\rm MC}$	$1.0458^{+0.0033}_{-0.0021}$	1.0470 ± 0.0015	$1.0456^{+0.0031}_{-0.0024}$	$1.0424^{+0.0006}_{-0.0013}$	$1.04185^{+0.00049}_{-0.00078}$
τ	0.0541 ± 0.0076	0.0534 ± 0.0080	0.0526 ± 0.0074	0.0540 ± 0.0076	0.0540 ± 0.0076
n_s	0.9655 ± 0.0043	0.9656 ± 0.0044	0.9663 ± 0.0040	0.9647 ± 0.0040	0.9643 ± 0.0042
$ln(10^{10} A_s)$	3.044 ± 0.016	3.042 ± 0.017	$3.039^{+0.013}_{-0.015}$	3.044 ± 0.016	3.044 ± 0.016
ξ	$-0.54^{+0.12}_{-0.28}$	$-0.66^{+0.09}_{-0.13}$	$-0.51^{+0.12}_{-0.29}$	$-0.22^{+0.21}_{-0.05}$	$-0.15^{+0.12}_{-0.06}$
H_0[km/s/Mpc]	$72.8^{+3.0}_{+1.5}$	$74.0^{+1.2}_{-1.0}$	$72.8^{+3.0}_{+1.6}$	$69.4^{+0.9}_{-1.5}$	$68.6^{+0.8}_{-1.0}$
σ_8	$2.3^{+0.4}_{-1.4}$	$2.71^{+0.05}_{-1.3}$	$2.2^{+0.4}_{-1.4}$	$1.05^{+0.03}_{-0.24}$	$0.95^{+0.04}_{-0.12}$
S_8	$1.30^{+0.17}_{-0.44}$	$1.44^{+0.17}_{-0.34}$	$1.30^{+0.15}_{-0.42}$	$0.93^{+0.03}_{-0.10}$	$0.892^{+0.028}_{-0.054}$

The addition of low-redshift measurements, as BAO data, still hints to the presence of a coupling, albeit at a lower statistical significance. Also for this data sets the Hubble constant values is larger than that obtained in the case of a pure LCDM scenario, enough to bring the H0 tension at 2.4σ.

Di Valentino et al., *Phys.Rev.D* 101 (2020) 6, 063502

Baryon Acoustic Oscillations



BAO is formed in the early universe, when baryons are strongly coupled to photons, and the gravitational collapse due to the CDM is counterbalanced by the radiation pressure. Sound waves that propagate in the early universe imprint a characteristic scale on the CMB. Since the scale of these oscillations can be measured at recombination, BAO is considered a "standard ruler". These fluctuations have evolved and we can observe BAO at low redshifts in the distribution of galaxies. Since the data reduction process leading to these measurements requires assumptions about the fiducial cosmology, BAO is model dependent.

IDE can solve the H0 tension

In other words, the tension between Planck+BAO and R19 could be due to a statistical fluctuation in this case.

Moreover, BAO data is extracted under the assumption of LCDM, and the modified scenario of interacting dark energy could affect the result.
In fact, the full procedure which leads to the BAO constraints carried out by the different collaborations might be not necessarily valid in extended DE models.
For instance, the BOSS collaboration advises caution when using their BAO measurements (both the pre- and post- reconstruction measurements) in more exotic dark energy cosmologies.

BAO constraints themselves might need to be revised in a non-trivial manner when applied to constrain extended dark energy cosmologies.

Di Valentino et al., *Phys.Rev.D* 101 (2020) 6, 063502

IDE can solve the MB tension

Constraints at 95% cl.

Model	M_B	H_0	Ω_m	w	ξ	Tension
ΛCDM	$-19.393^{+0.042}_{-0.040}$	$68.5^{+1.2}_{-1.1}$	$0.302^{+0.025}_{-0.025}$	-1	0	3.4σ
wCDM	$-19.376^{+0.095}_{-0.11}$	$69.2^{+3.3}_{-3.7}$	$0.306^{+0.032}_{-0.034}$	$-1.02^{+0.11}_{-0.11}$	0	2.1σ
IDE	$-19.385^{+0.094}_{-0.089}$	$69.1^{+3.1}_{-2.8}$	$0.274^{+0.044}_{-0.050}$	-0.999	> -0.35	2.4σ

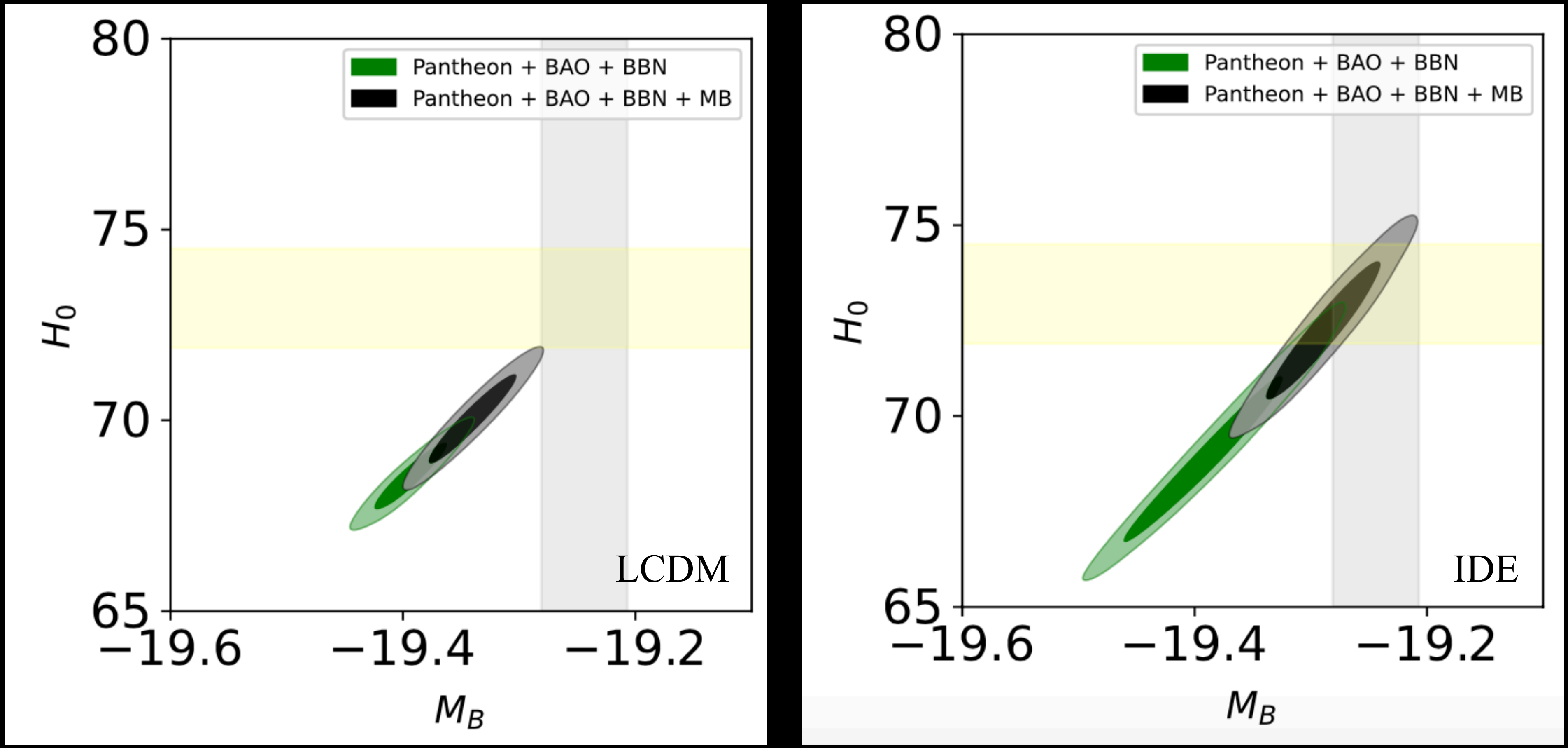
An IDE model reduces also the MB tension at 2.4σ considering a combination of Pantheon + BAO + BBN,

Model	M_B	H_0	Ω_m	w	ξ	Tension
ΛCDM	$-19.340^{+0.047}_{-0.046}$	$70.0^{+1.4}_{-1.4}$	$0.314^{+0.024}_{-0.025}$	-1	0	2.1σ
wCDM	$-19.285^{+0.049}_{-0.060}$	$72.4^{+1.9}_{-2.2}$	$0.323^{+0.028}_{-0.025}$	$-1.10^{+0.09}_{-0.10}$	0	1.0σ
IDE	$-19.288^{+0.064}_{-0.063}$	$72.3^{+2.4}_{-2.3}$	$0.256^{+0.061}_{-0.065}$	-0.999	$-0.31^{+0.27}_{-0.28}$	0.9σ

while it solves the MB tension below 1σ when a gaussian prior on MB is included, with an indication for a coupling different from zero at more than 95% CL.

Nunes & Di Valentino, *Phys.Rev.D* (2021), arXiv:2107.09151

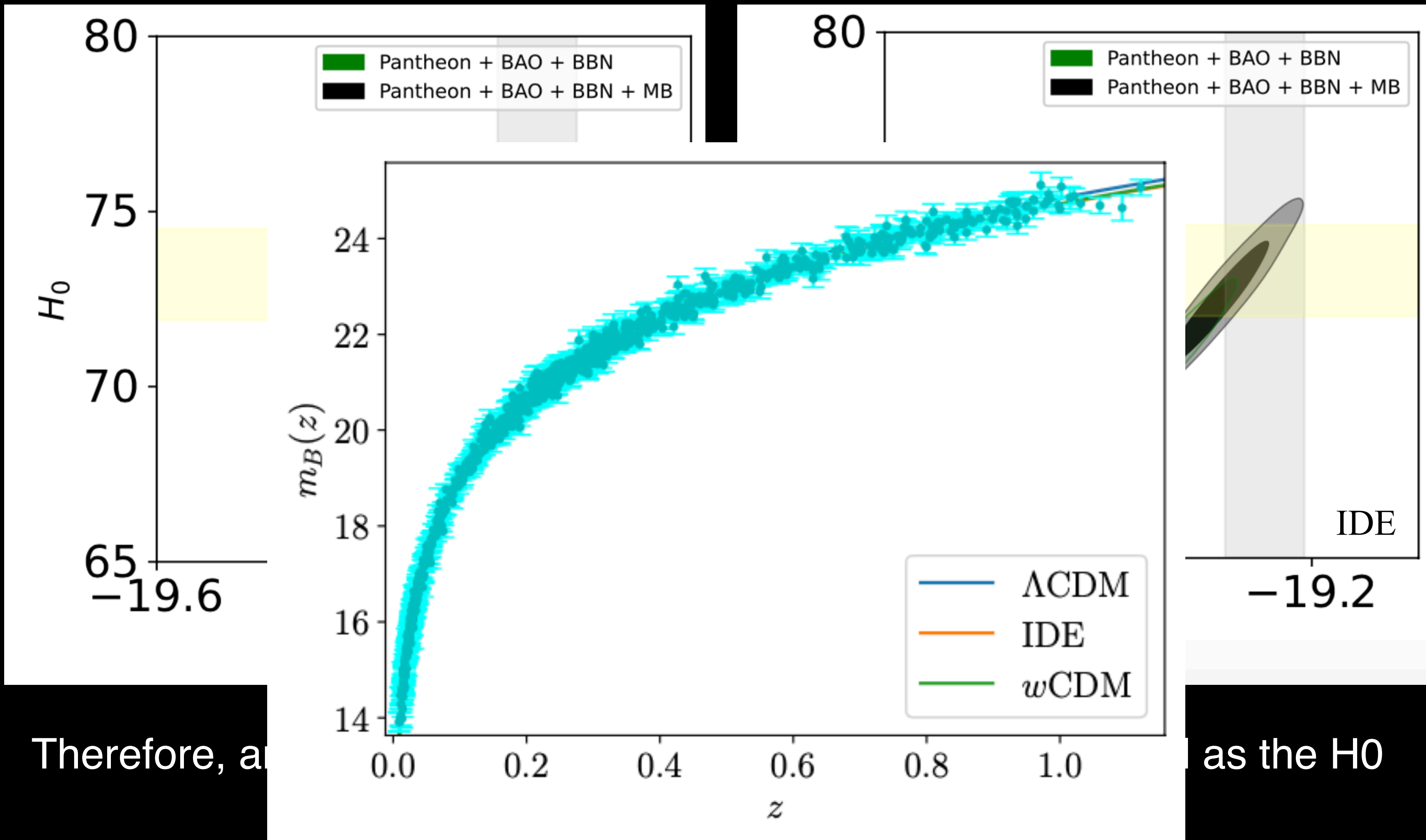
IDE can solve the MB tension



Therefore, an IDE model can alleviate the MB tension, as well as the H0 disagreement.

Nunes & Di Valentino, *Phys.Rev.D* (2021), arXiv:2107.09151

IDE can solve the MB tension



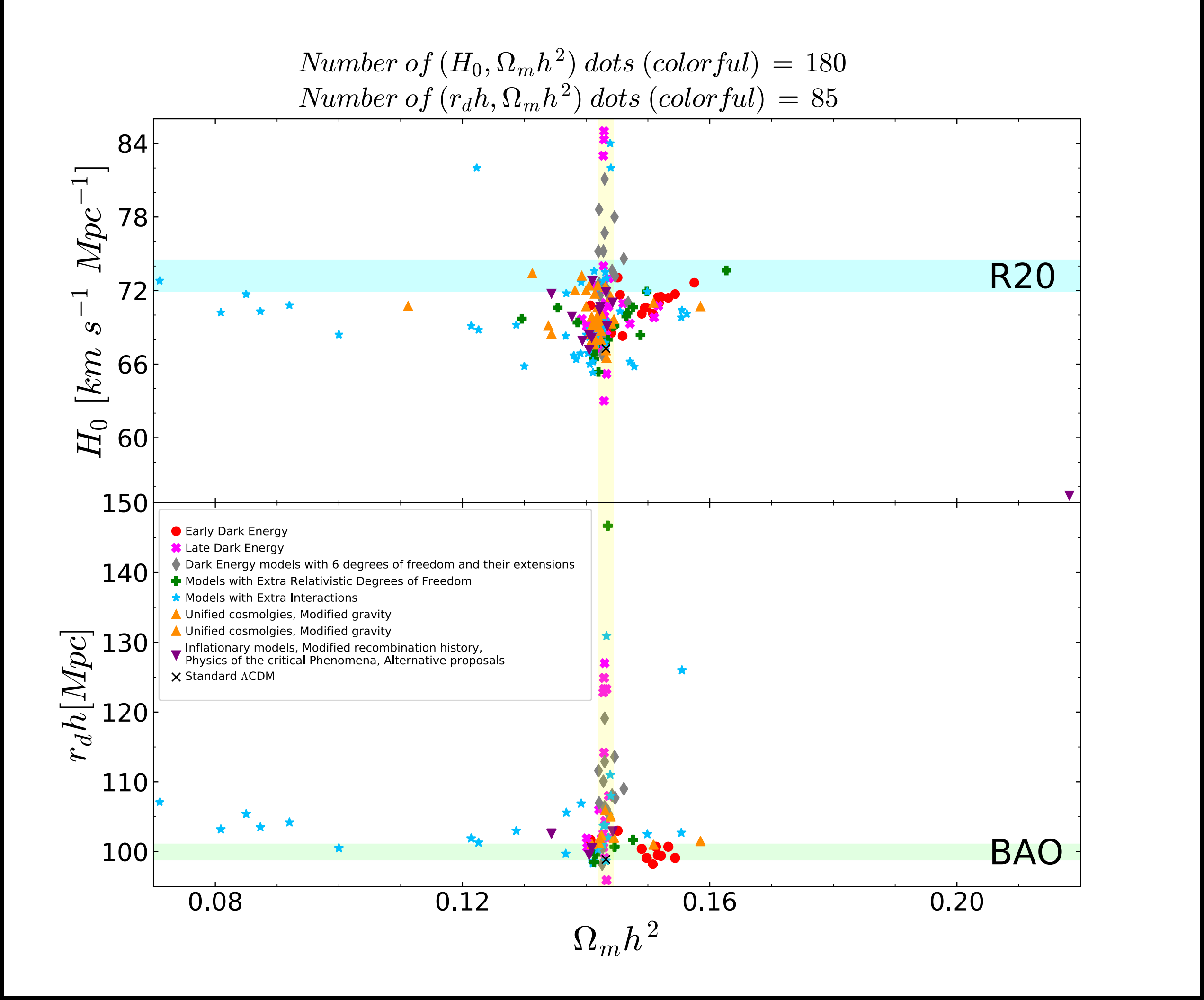
Therefore, a… as the H0

Moreover, it is not disfavoured by the Pantheon magnitude-redshift relation.

Nunes & Di Valentino, *Phys.Rev.D* (2021), arXiv:2107.09151

Successful models?

This is the density of the proposed cosmological models:



At the moment no specific proposal makes a strong case for being highly likely or far better than all others !!!

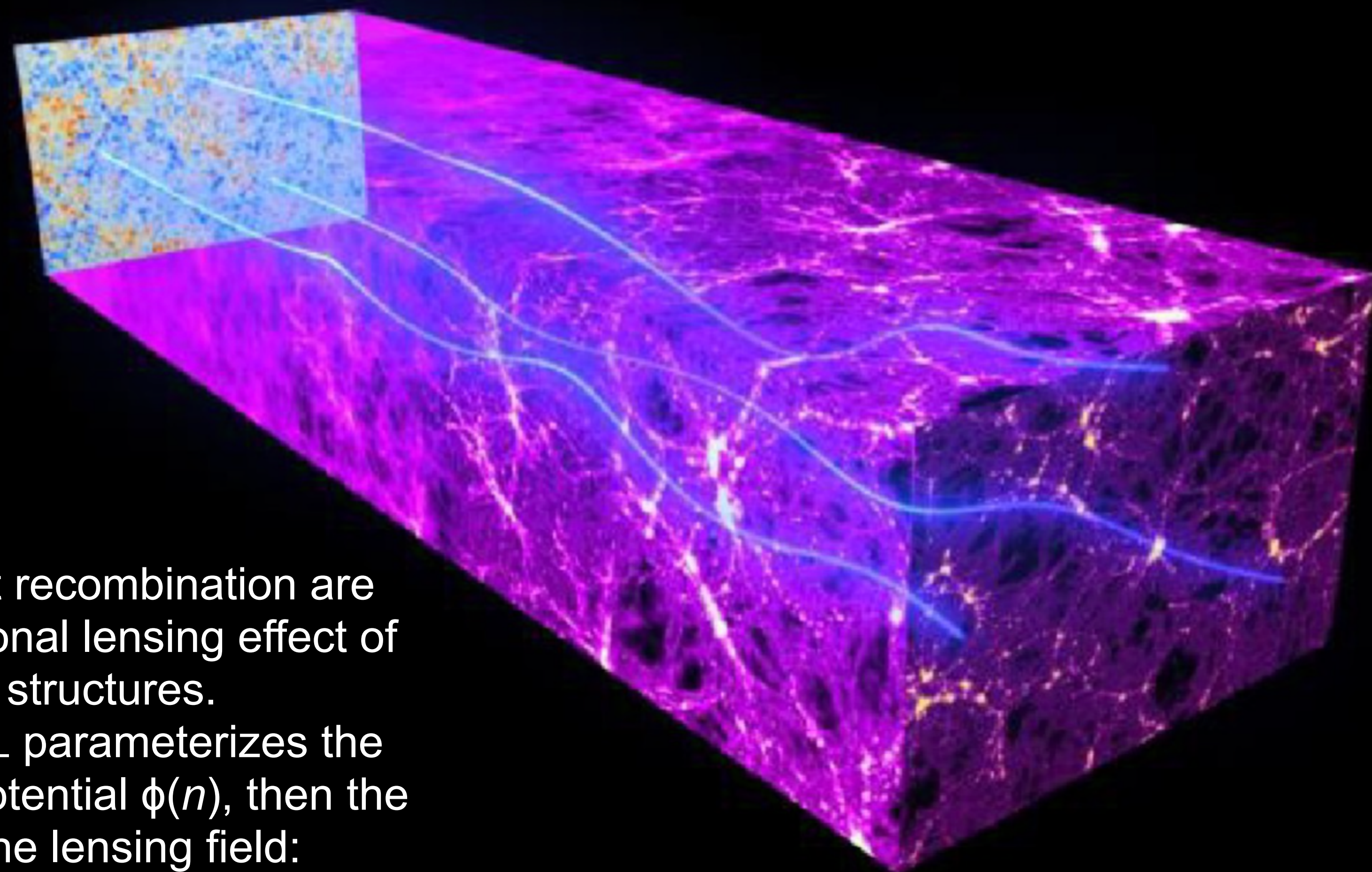
Di Valentino et al., Class.Quant.Grav. (2021), arXiv:2103.01183 [astro-ph.CO]

The most statistically significant and persisting anomalies and tensions of the CMB are:

- H0 with local measurements
- A_L internal anomaly
- S8 with cosmic shear data
- Ω_K different from zero

See Di Valentino et al. arXiv:2008.11283 [astro-ph.CO], arXiv:2008.11284 [astro-ph.CO], arXiv:2008.11285 [astro-ph.CO], arXiv:2008.11286 [astro-ph.CO] for an overview.

A_L internal anomaly



CMB photons emitted at recombination are deflected by the gravitational lensing effect of massive cosmic structures.
The lensing amplitude A_L parameterizes the rescaling of the lensing potential $\phi(n)$, then the power spectrum of the lensing field:

$$C_\ell^{\phi\phi} \rightarrow A_\mathrm{L} C_\ell^{\phi\phi}$$

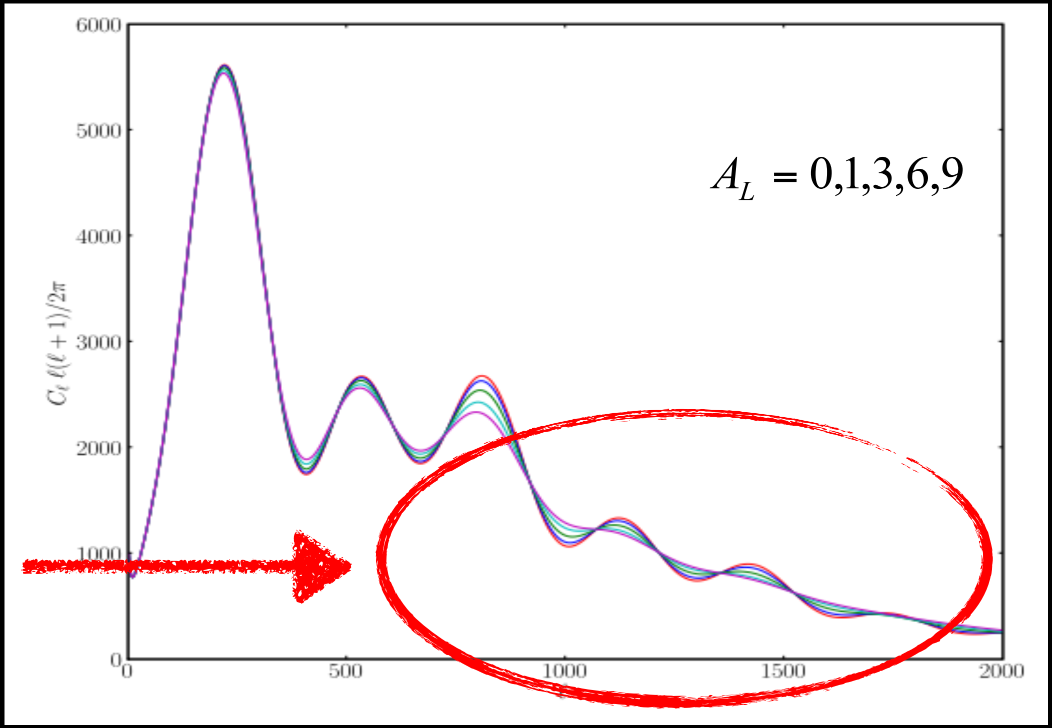
The gravitational lensing deflects the photon path by a quantity defined by the gradient of the lensing potential $\phi(n)$, integrated along the line of sight n, remapping the temperature field.

A_L internal anomaly

Its effect on the power spectrum is the smoothing of the acoustic peaks, increasing AL.

Interesting consistency checks is if the amplitude of the smoothing effect in the CMB power spectra matches the theoretical expectation AL = 1 and whether the amplitude of the smoothing is consistent with that measured by the lensing reconstruction.

If AL =1 then the theory is correct, otherwise we have a new physics or systematics.



Calabrese et al., Phys. Rev. D, 77, 123531

A_L : a failed consistency check

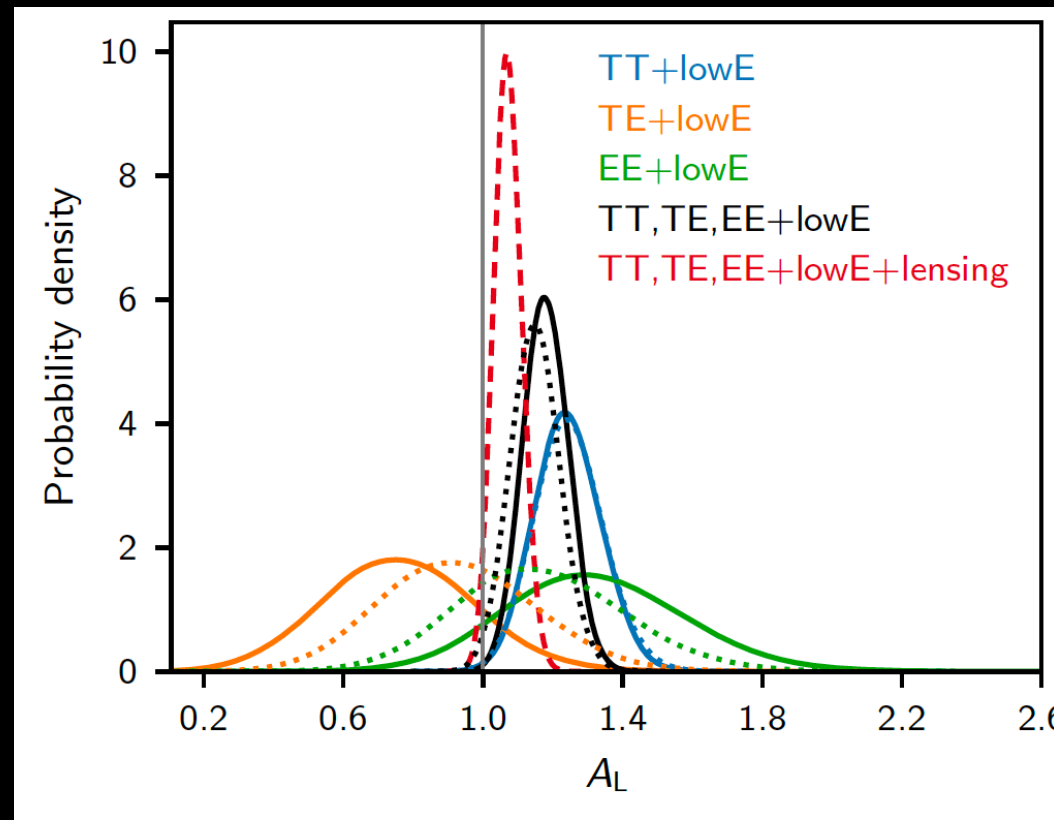
The Planck lensing-reconstruction power spectrum is consistent with the amplitude expected for LCDM models that fit the CMB spectra, so the Planck lensing measurement is compatible with AL = 1.

However, the distributions of AL inferred from the CMB power spectra alone indicate a preference for AL > 1.

The joint combined likelihood shifts the value preferred by the TT data downwards towards AL = 1, but the error also shrinks, increasing the significance of AL > 1 to 2.8σ.

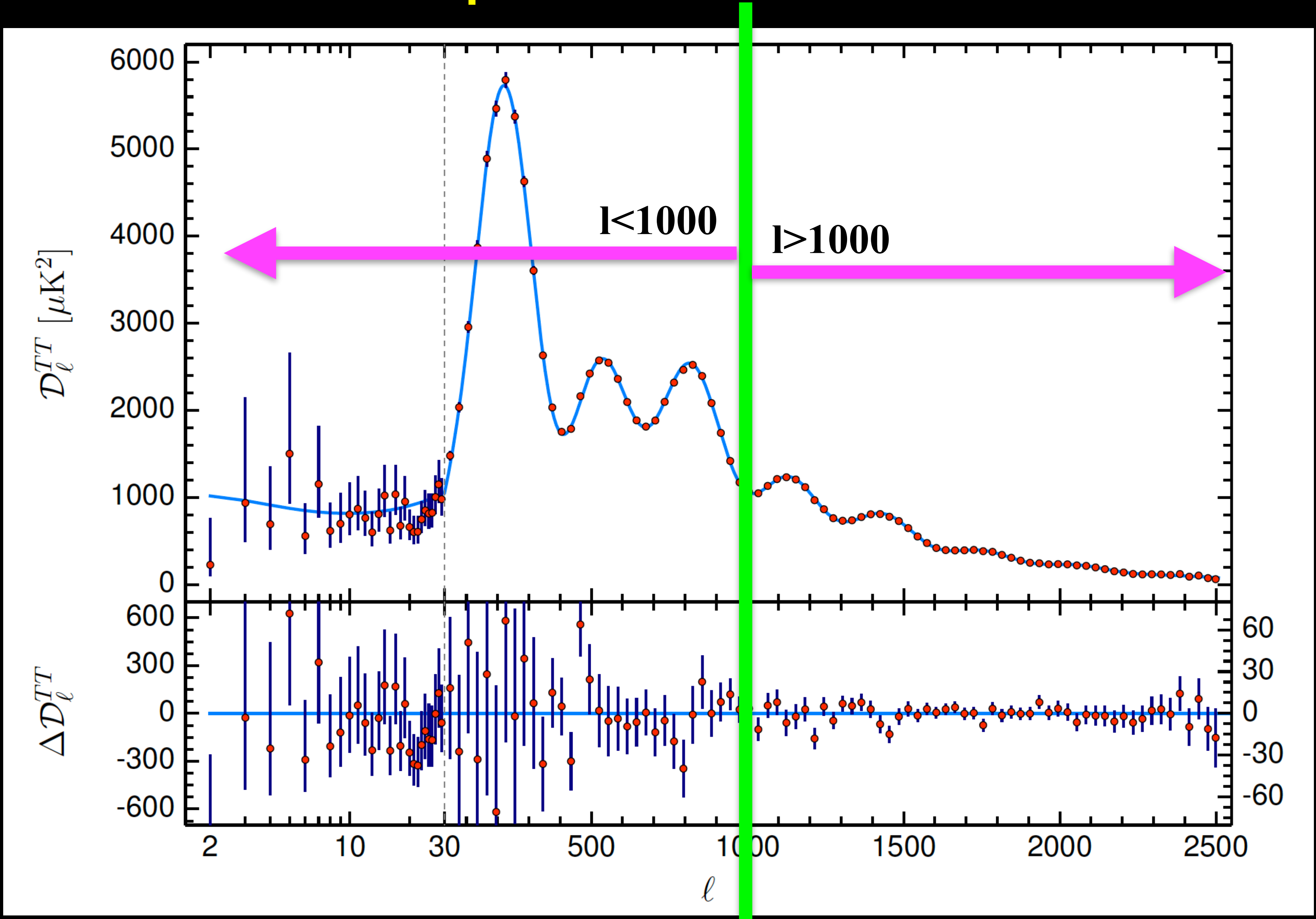
The preference for high AL is not just a volume effect in the full parameter space, with the best fit improved by $\Delta\chi^2 \sim 9$ when adding AL for TT+lowE and 10 for TTTEEE+lowE.

Planck 2018, Astron.Astrophys. 641 (2020) A6



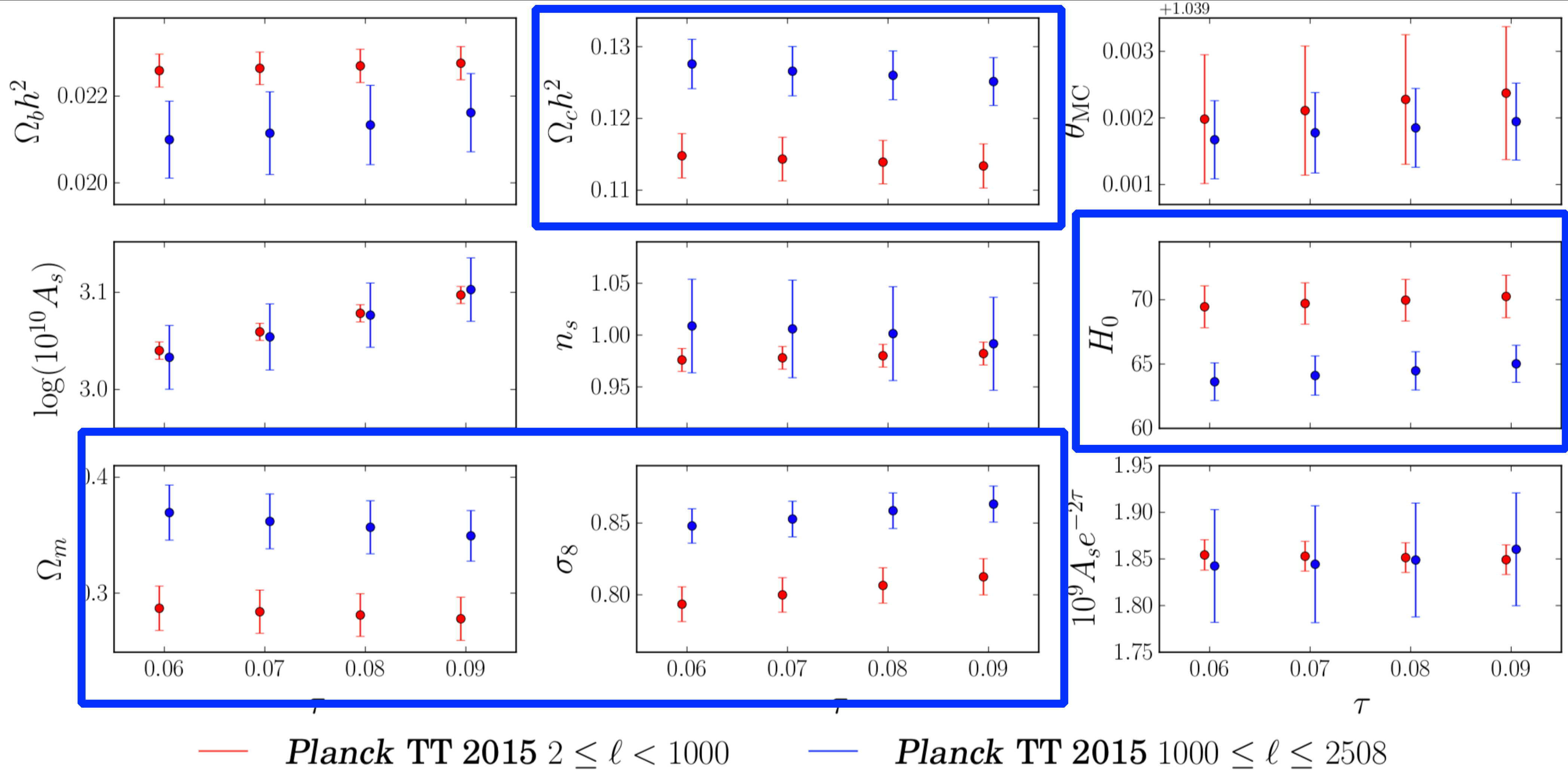
$$A_L = 1.243 \pm 0.096 \quad (68\,\%,\ \textit{Planck}\ \mathrm{TT{+}lowE}),$$
$$A_L = 1.180 \pm 0.065 \quad (68\,\%,\ \textit{Planck}\ \mathrm{TT,TE,EE{+}lowE}),$$

A_L can explain internal tension



Planck 2018, Astron.Astrophys. 641 (2020) A6

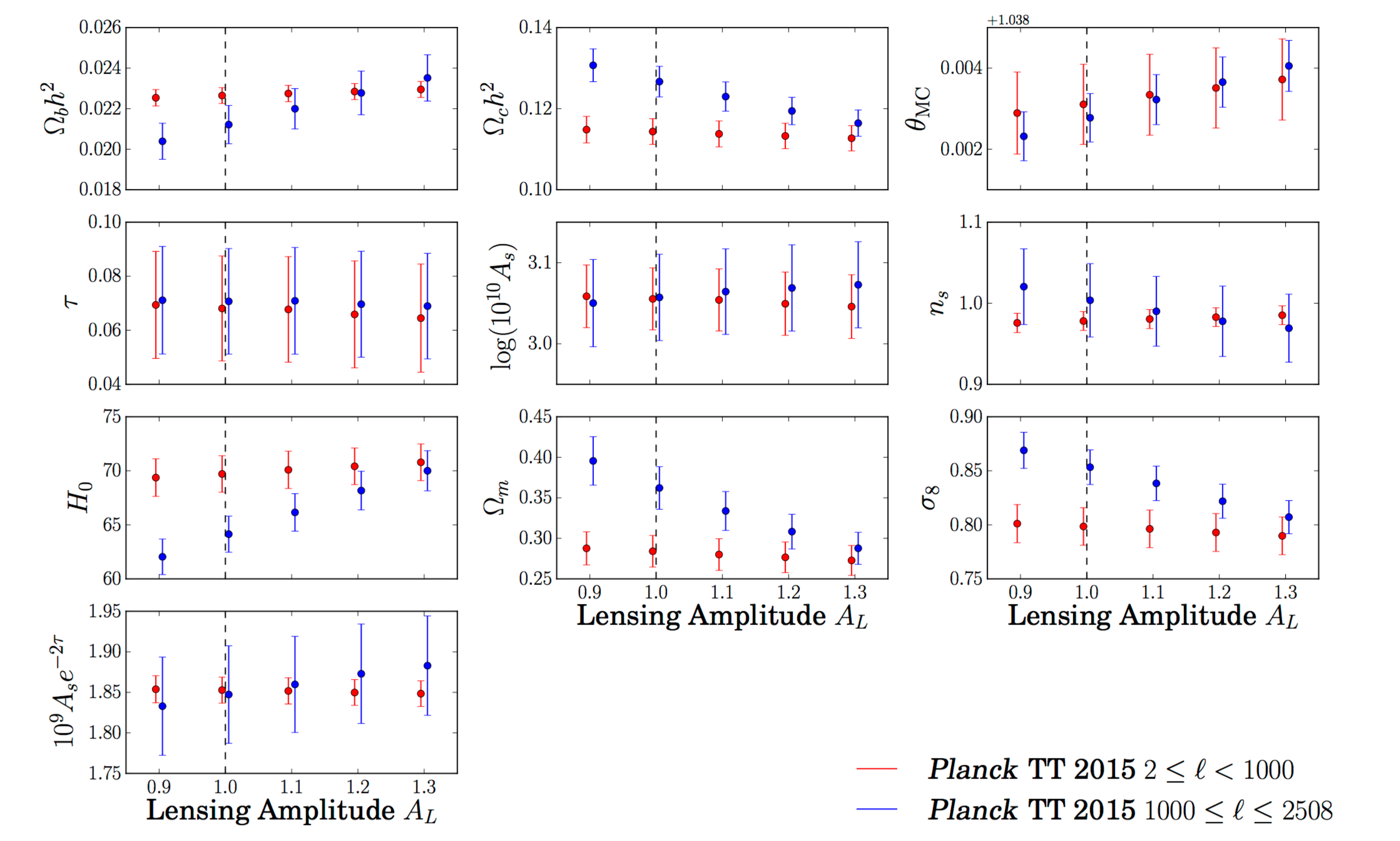
A_L can explain internal tension



Marginalized 68.3% confidence ΛCDM parameter constraints from fits to the l < 1000 and l ≥ 1000 Planck TT 2015 spectra. Tension at more than 2σ level appears in $\Omega_c h^2$ and derived parameters, including H0, Ωm, and σ8.

Addison et al., Astrophys.J. 818 (2016) no.2, 132

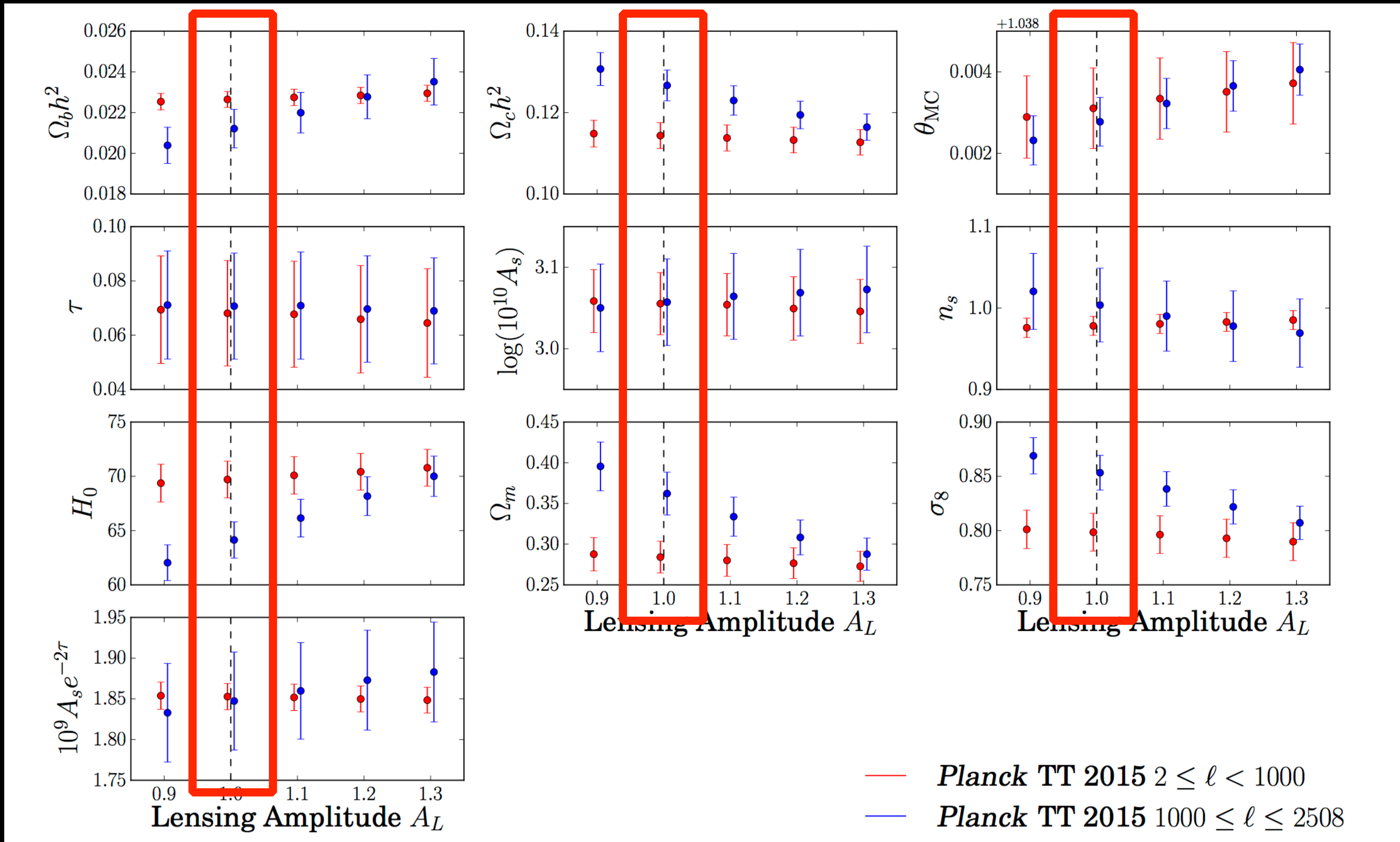
A_L can explain internal tension



Increasing AL smooths out the high order acoustic peaks, improving the agreement between the two multipole ranges.

Addison et al., Astrophys.J. 818 (2016) no.2, 132

A_L can explain internal tension



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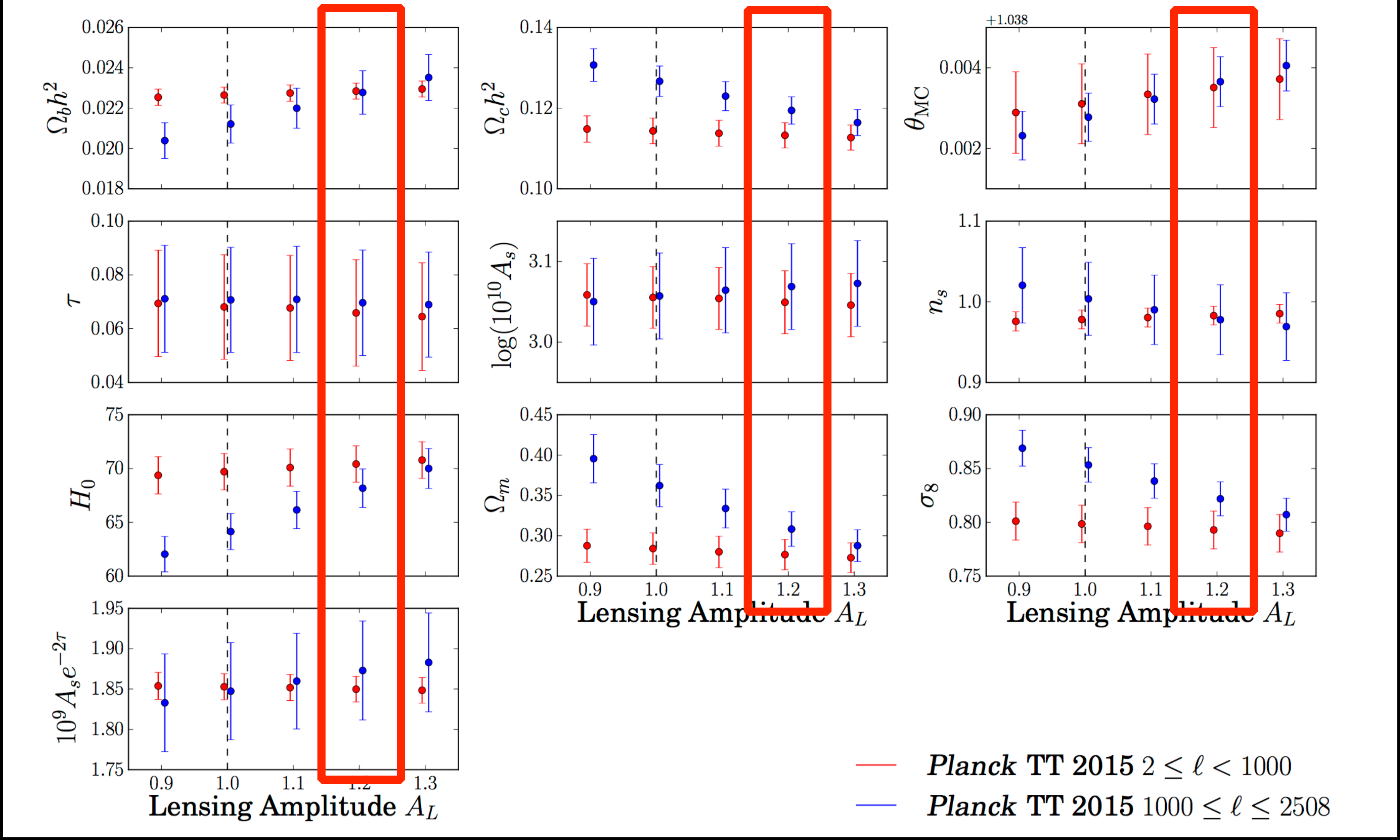
Addison et al., Astrophys.J. 818 (2016) no.2, 132

A_L can explain internal tension

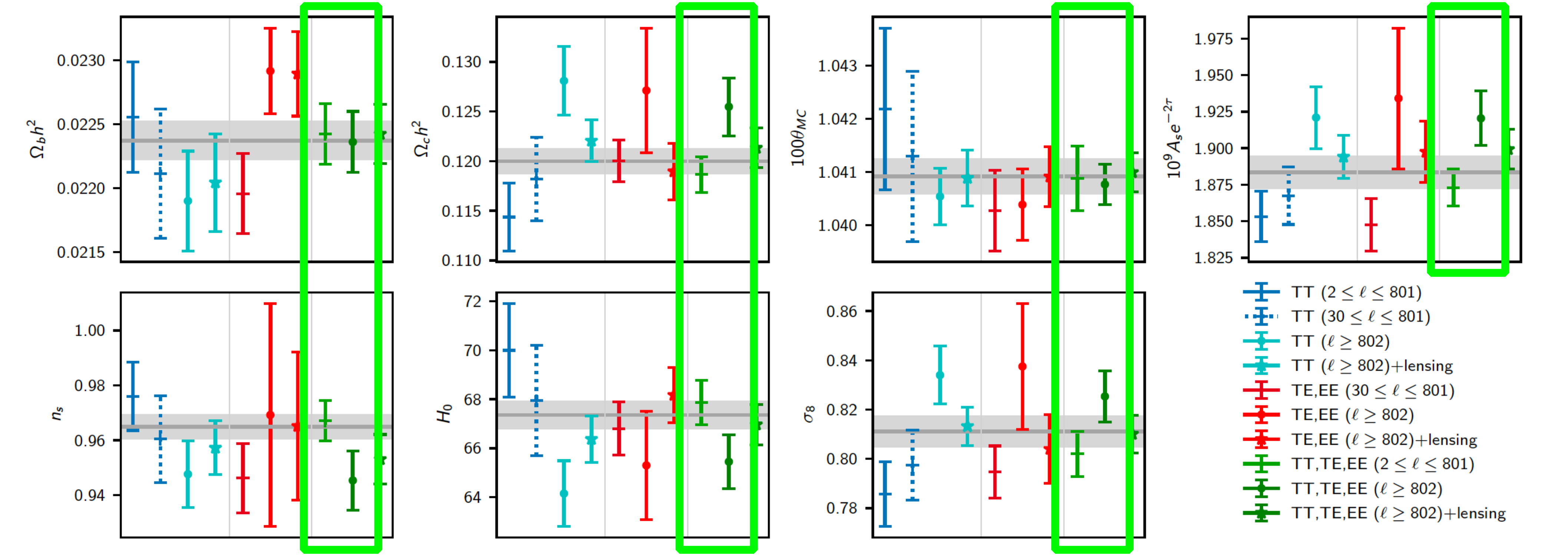


Increasing AL smooths out the high order acoustic peaks, improving the agreement between the two multipole ranges.

Addison et al., Astrophys.J. 818 (2016) no.2, 132

A_L can explain internal tension

Planck 2018, Aghanim et al., arXiv:1807.06209 [astro-ph.CO]



LCDM 68% marginalized parameter constraints for l=[2-801] (points marked with a cross), l>802 (points marked with a circle), and l>802 + lensing (points marked with a star). Correcting for the lensing, all the results from high multipoles are in better consistency with the results from lower multipoles.

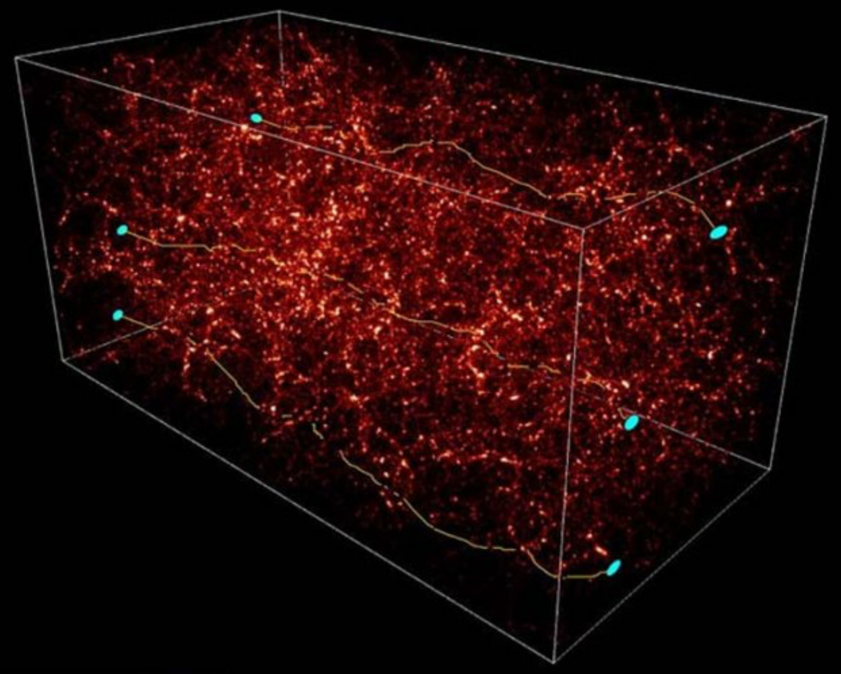
Dotted error bars are the results from l=[30-801], without the large-scaleTT likelihood, showing that l< 30 pulls the low-multipole parameters further from the joint result.

The most statistically significant and persisting anomalies and tensions of the CMB are:

- H0 with local measurements
- A_L internal anomaly
- **S8 with cosmic shear data**
- Ω_K different from zero

See Di Valentino et al. arXiv:2008.11283 [astro-ph.CO], arXiv:2008.11284 [astro-ph.CO], arXiv:2008.11285 [astro-ph.CO], arXiv:2008.11286 [astro-ph.CO] for an overview.

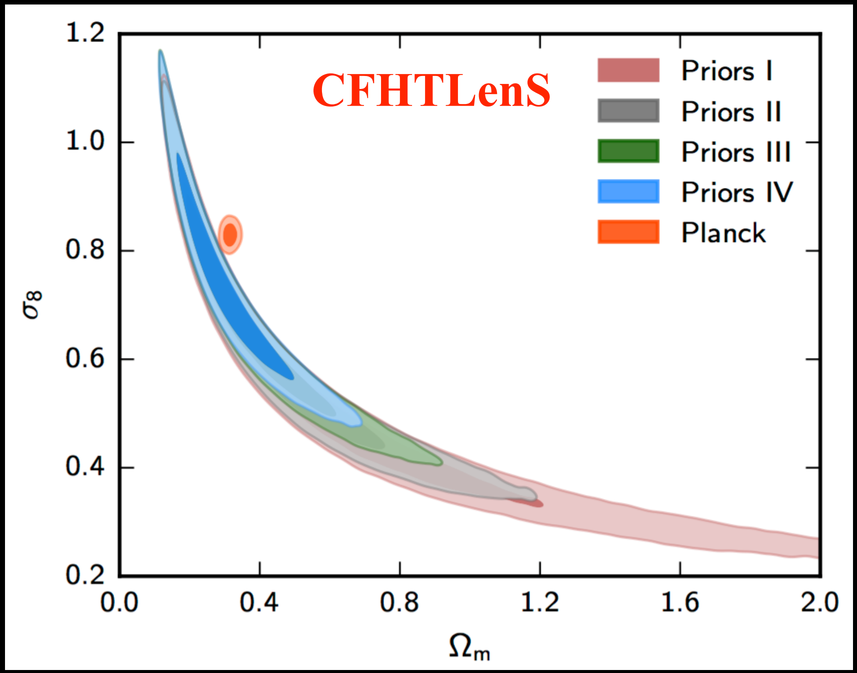
The S8 tension



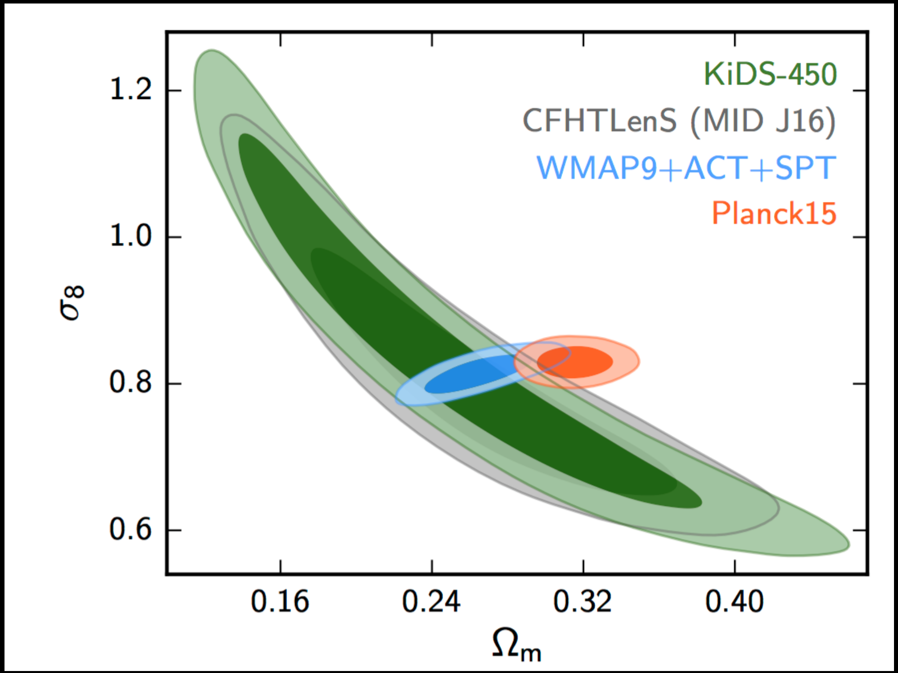
$$S_8 \equiv \sigma_8 \sqrt{\Omega_m / 0.3}$$

A tension on S8 is present between the Planck data in the ΛCDM scenario and the cosmic shear data.

The S8 tension



Joudaki et al, arXiv:1601.05786



KiDS-450, Hildebrandt et al., arXiv:1606.05338.

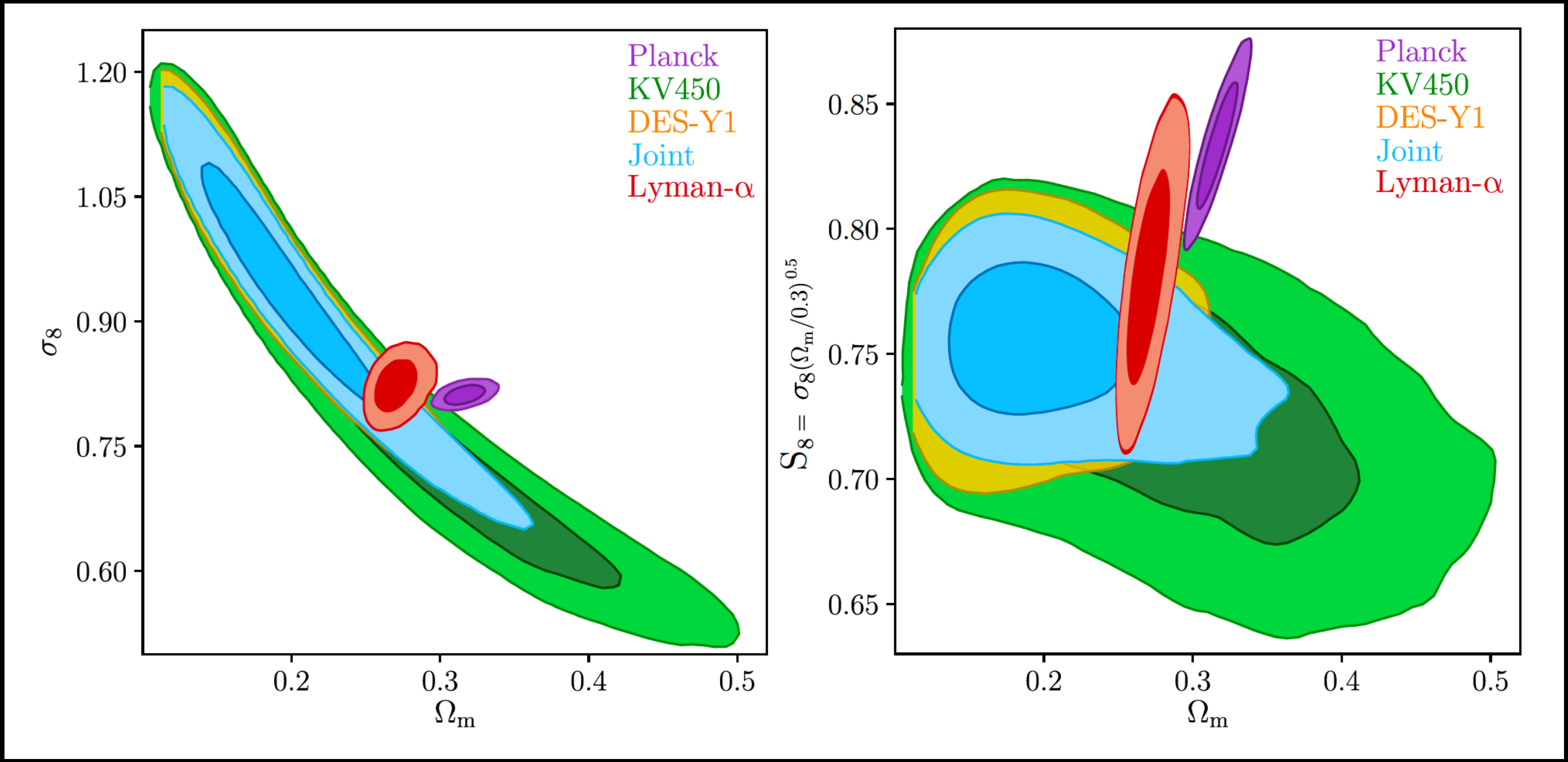
The S8 tension is at about 2.6σ level between Planck assuming ΛCDM and CFHTLenS survey and KiDS-450.

$S_8 = 0.834 \pm 0.016$
Planck 2018, Aghanim et al., arXiv:1807.06209 [astro-ph.CO]

$S_8 = 0.745 \pm 0.035$
KiDS-450, Hildebrandt et al., arXiv:1606.05338 [astro-ph.CO]

The S8 tension

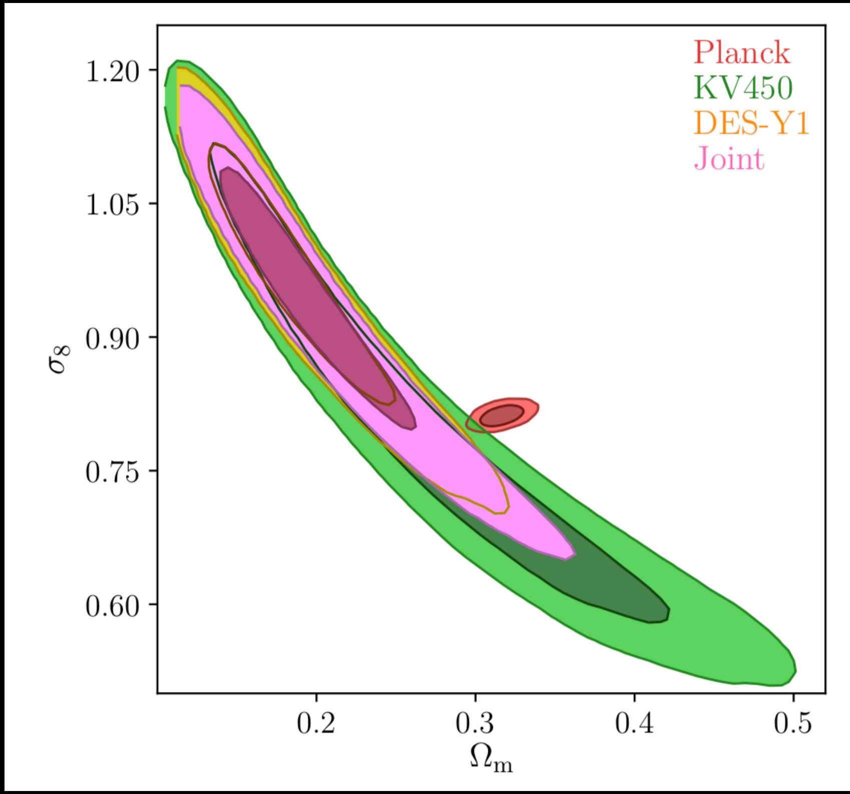
Palanque-Delabrouille et al., arXiv:1911.09073 [astro-ph.CO]



A tension on S8 at more than 2.5σ is present between Planck assuming ΛCDM and DES-Y1 results including galaxy clustering, and Planck and Ly-α (sharing a similar range of scales).

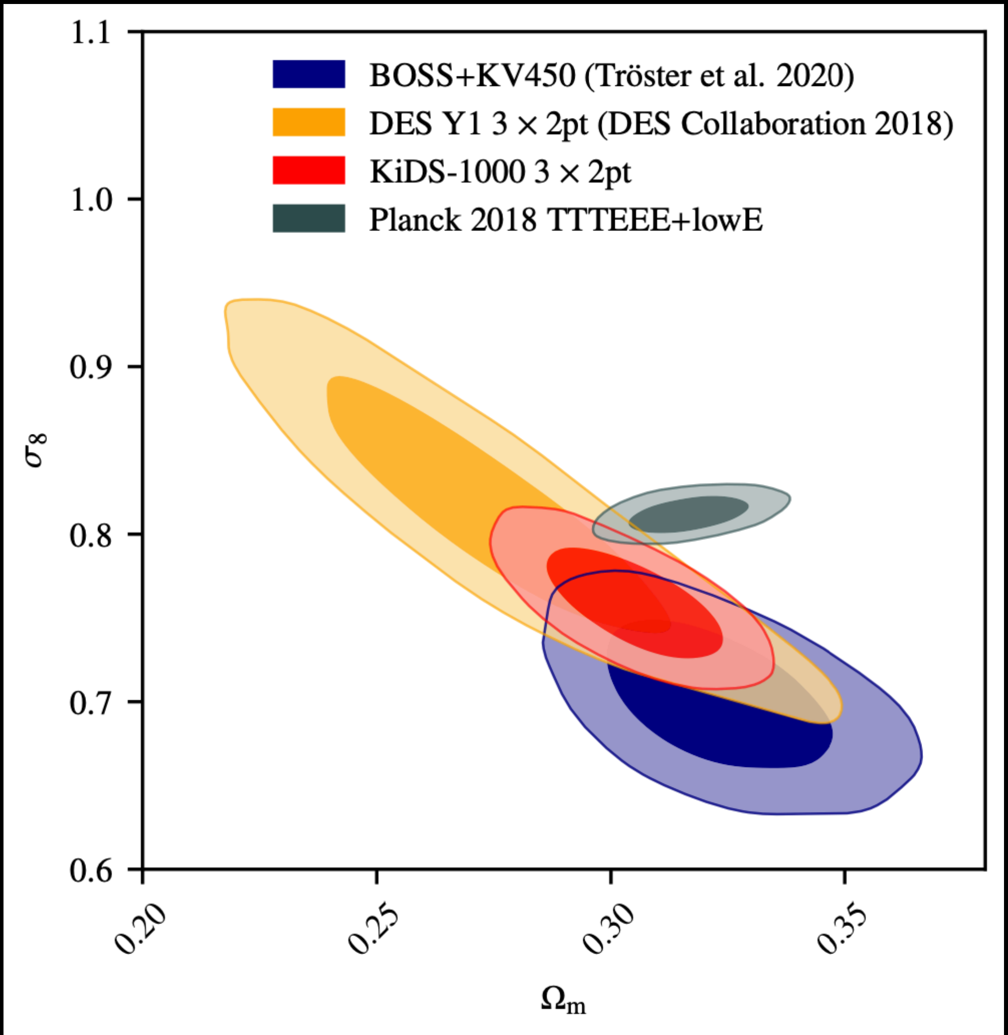
The S8 tension

Asgari et al., arXiv:1910.05336 [astro-ph.CO]



A tension on S8 at 3.2σ is present between Planck assuming ΛCDM and KiDS+VIKING-450 and DES-Y1 combined together.

The S8 tension



KiDS-1000, Heymans et al., arXiv:2007.15632 [astro-ph.CO]

The S8 tension is present at 3.4σ between Planck assuming ΛCDM and KiDS+VIKING-450 and BOSS combined together, or 3.1σ with KiDS-1000.

$S_8 = 0.834 \pm 0.016$

Planck 2018, Aghanim et al., arXiv:1807.06209 [astro-ph.CO]

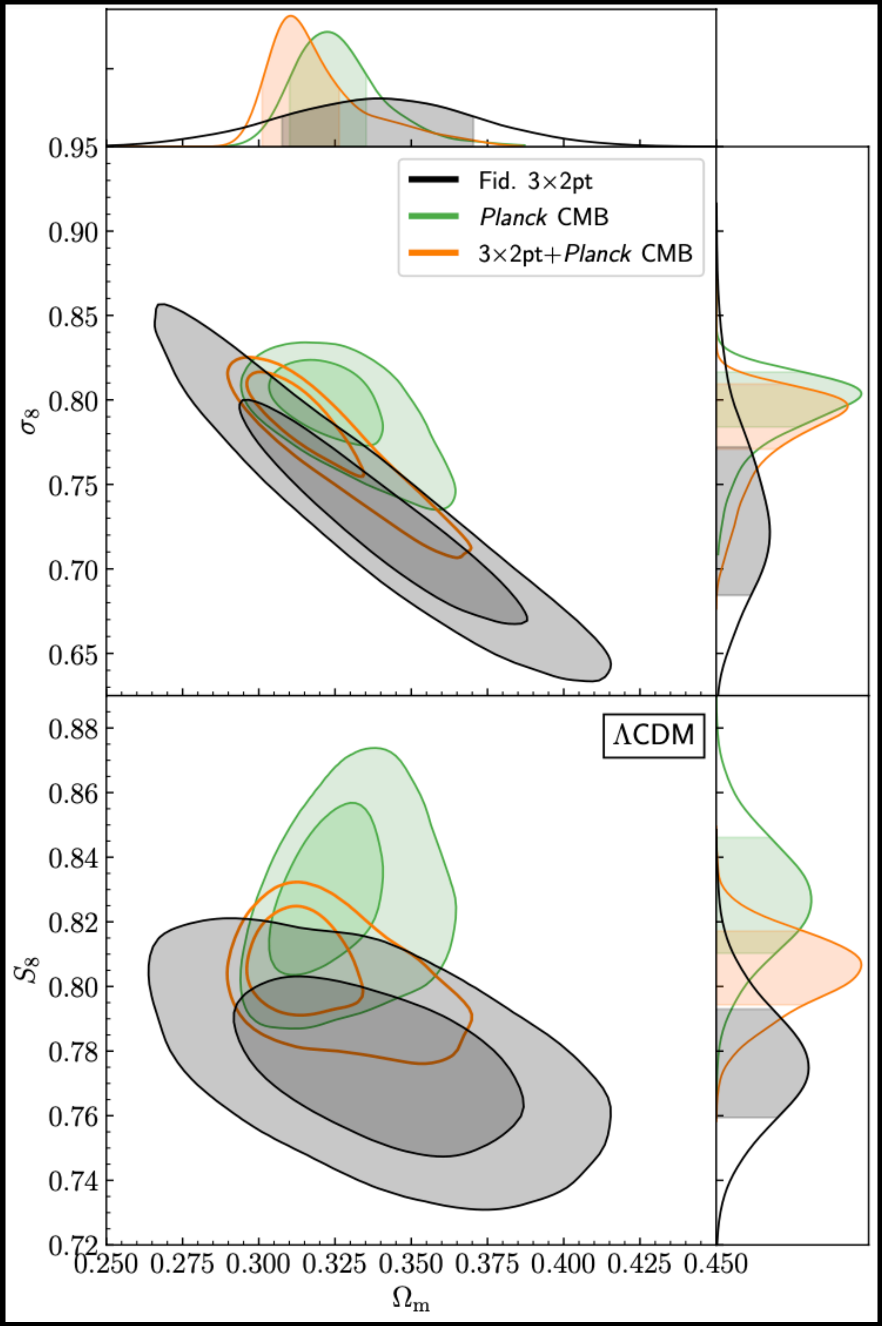
$S_8 = 0.728 \pm 0.045$

Troster et al., arXiv:1909.11006 [astro-ph.CO]

$S_8 = 0.766^{+0.020}_{-0.014}$

KiDS-1000, Heymans et al., arXiv:2007.15632 [astro-ph.CO]

The S8 tension



DES-Y3, Abbott et al., arXiv:2105.13549 [astro-ph.CO]

The S8 tension is present at 2.5σ between Planck assuming ΛCDM and DES-Y3.

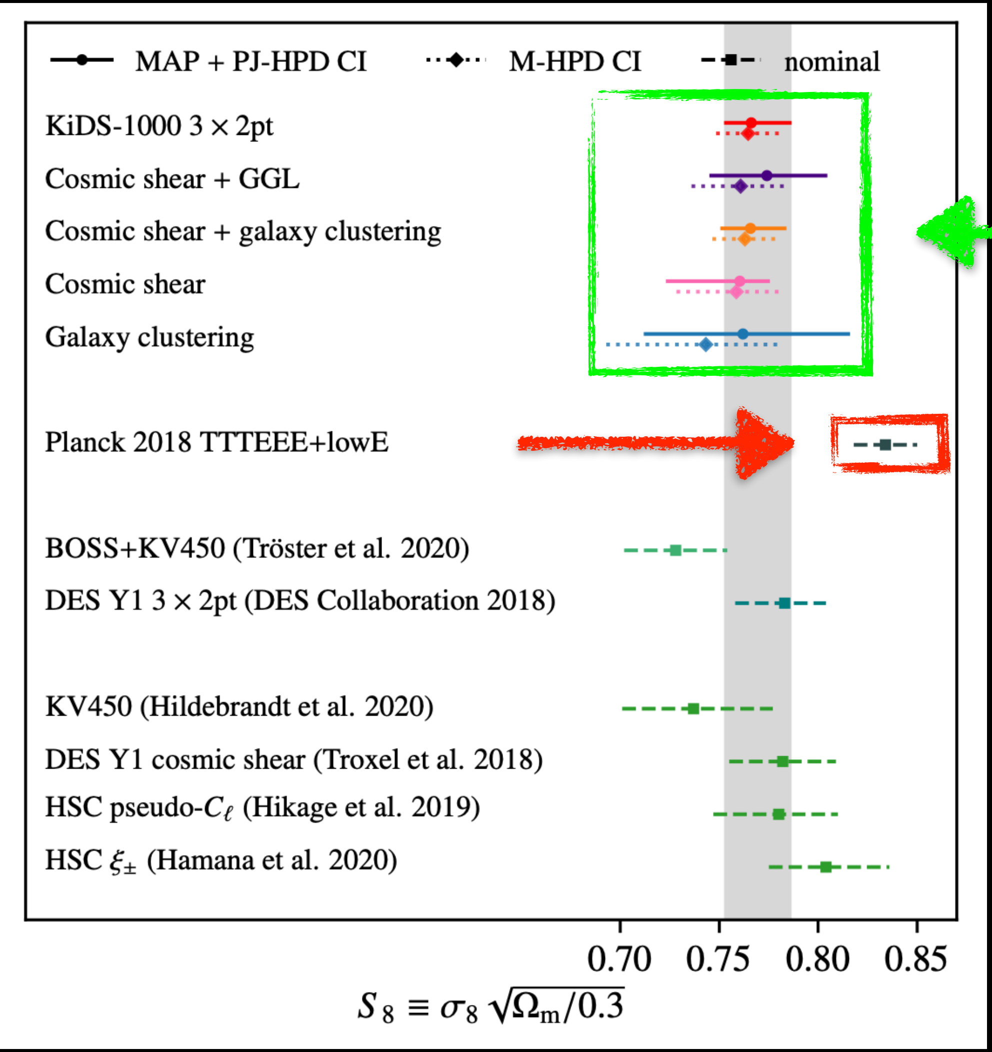
$S_8 = 0.834 \pm 0.016$

Planck 2018, Aghanim et al., arXiv:1807.06209 [astro-ph.CO]

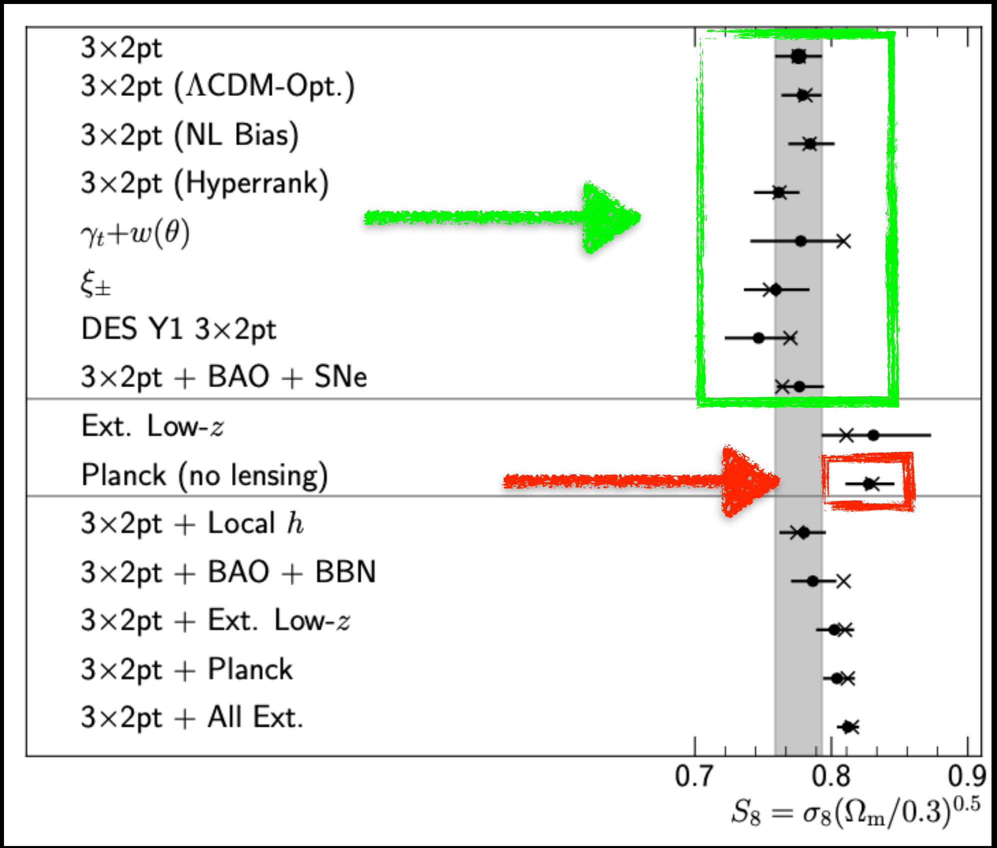
$S_8 = 0.776^{+0.017}_{-0.017}$

DES-Y3, Abbott et al., arXiv:2105.13549 [astro-ph.CO]

The S8 tension



KiDS-1000, Heymans et al., arXiv:2007.15632 [astro-ph.CO]



DES-Y3, Abbott et al., arXiv:2105.13549 [astro-ph.CO]

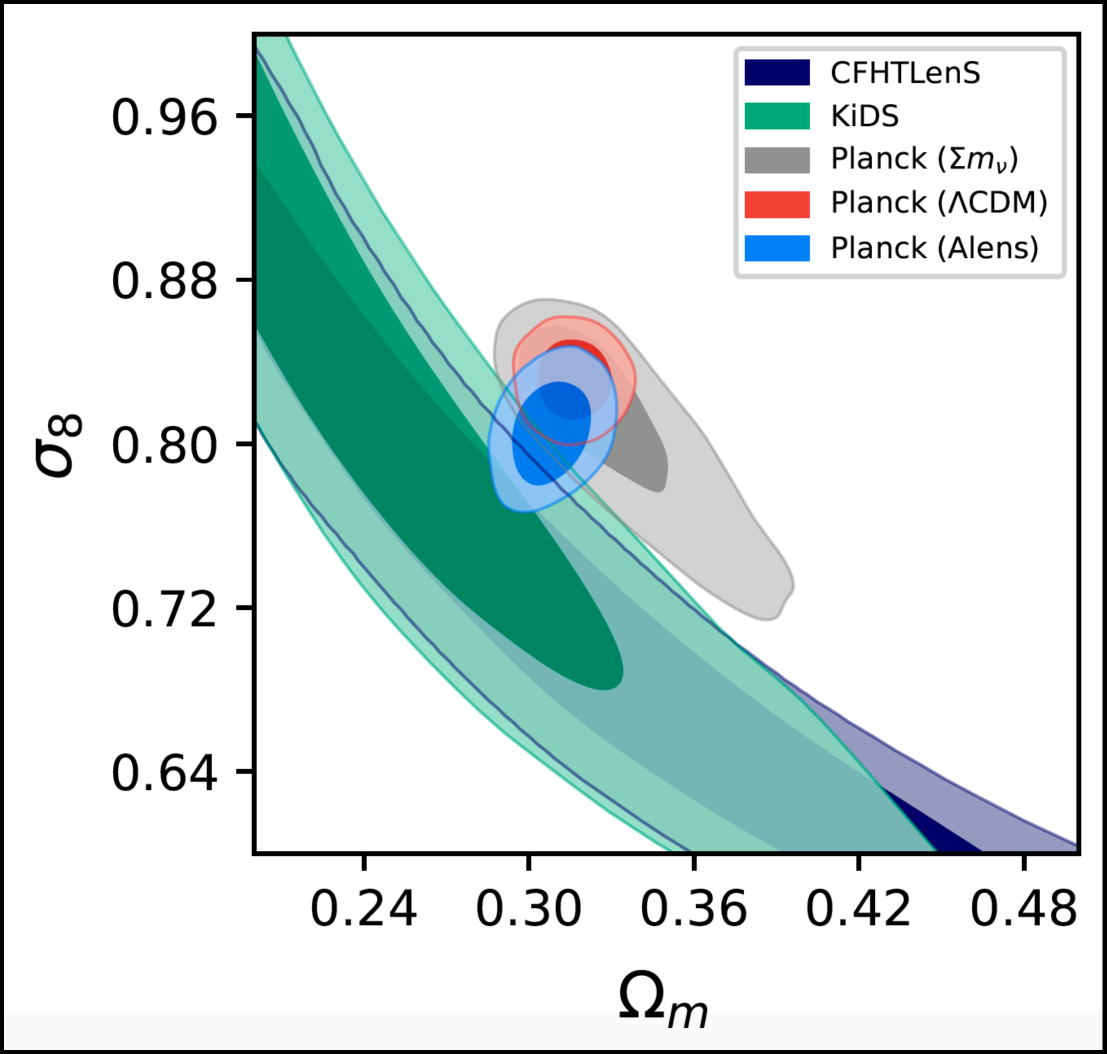
Proposals for solving the S8 tension are:

- Axion monodromy inflation (Meerburg arXiv:1406.3243, etc…)
- Extended parameter spaces involving Alens>1 (Di Valentino et al. arXiv:1507.06646, Di Valentino et al. arXiv:1606.00634, Di Valentino et al. arXiv:1704.00762, Di Valentino et al. arXiv:1908.01391, etc…)
- Active and Sterile Neutrinos (Battye & Moss arXiv:1308.5870 , Bohringer & Chon arXiv:1610.02855, etc…)
- Interacting Dark Energy models (Di Valentino et al. arXiv:1908.04281, Di Valentino et al. arXiv: 1910.09853, etc.)
- Decaying dark matter (Chudaykin et al. arXiv:1711.06738, Abellan et al. arXiv:2008.09615, Berezhiani et al. arXiv:1505.03644, Anchordoqui et al. arXiv:1506.08788, Abellan et al. arXiv:2102.12498 [astro-ph.CO], etc..)
- Cannibal dark matter (Heimersheim et al. arXiv:2008.08486, etc…)
- Minimally and non-minimally coupled scalar field models (Davari et al. arXiv:1911.00209, etc…)
- Modified Gravity models (Di Valentino et al. arXiv:1509.07501, Sola Peracaula et al. arXiv:1909.02554, Sola et al. arXiv: 2006.04273, etc…)
- Running Vacuum models (Gomez-Valent & Sola arXiv:1711.00692, Lambiase et al. arXiv:1804.07154, Sola et al. arXiv:1506.05793, Sola et al. arXiv:1709.07451, Sola et al. arXiv:1602.02103, etc…)
- Quartessence (Camera et al. arXiv:1704.06277, etc…)

See Di Valentino et al. arXiv:2008.11285 [astro-ph.CO] for a summary of other possible candidates.

A_L can explain the S8 tension

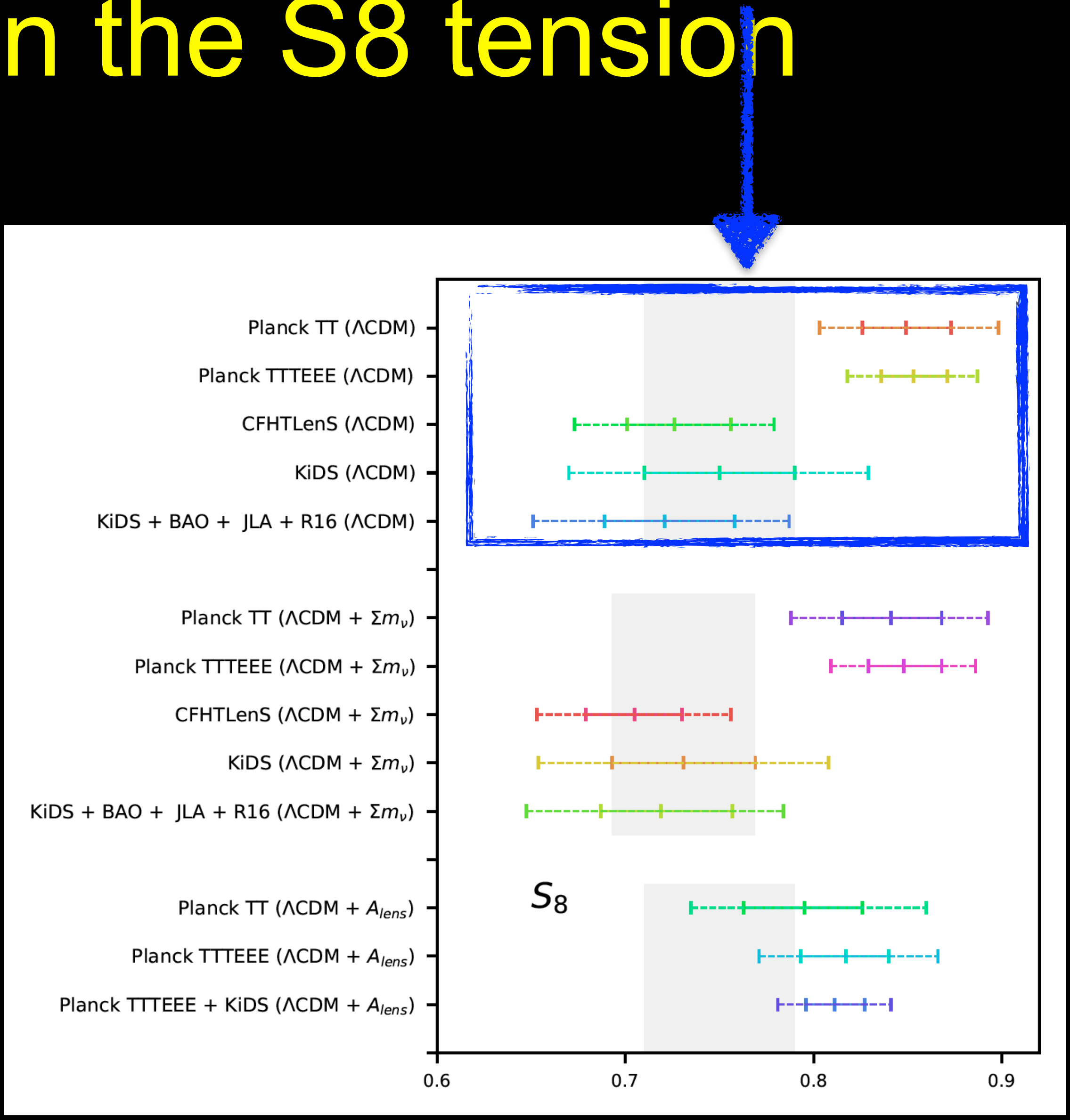
If we include the additional scaling parameter on the CMB lensing amplitude A_L, we find that this can put in agreement Planck 2015 with the cosmic shear data.



Di Valentino and Bridle, Symmetry 10 (2018) no.11, 585

A_L can explain the S8 tension

If we include the additional scaling parameter on the CMB lensing amplitude A_L, we find that this can put in agreement Planck 2015 with the cosmic shear data.



Di Valentino and Bridle, Symmetry 10 (2018) no.11, 585

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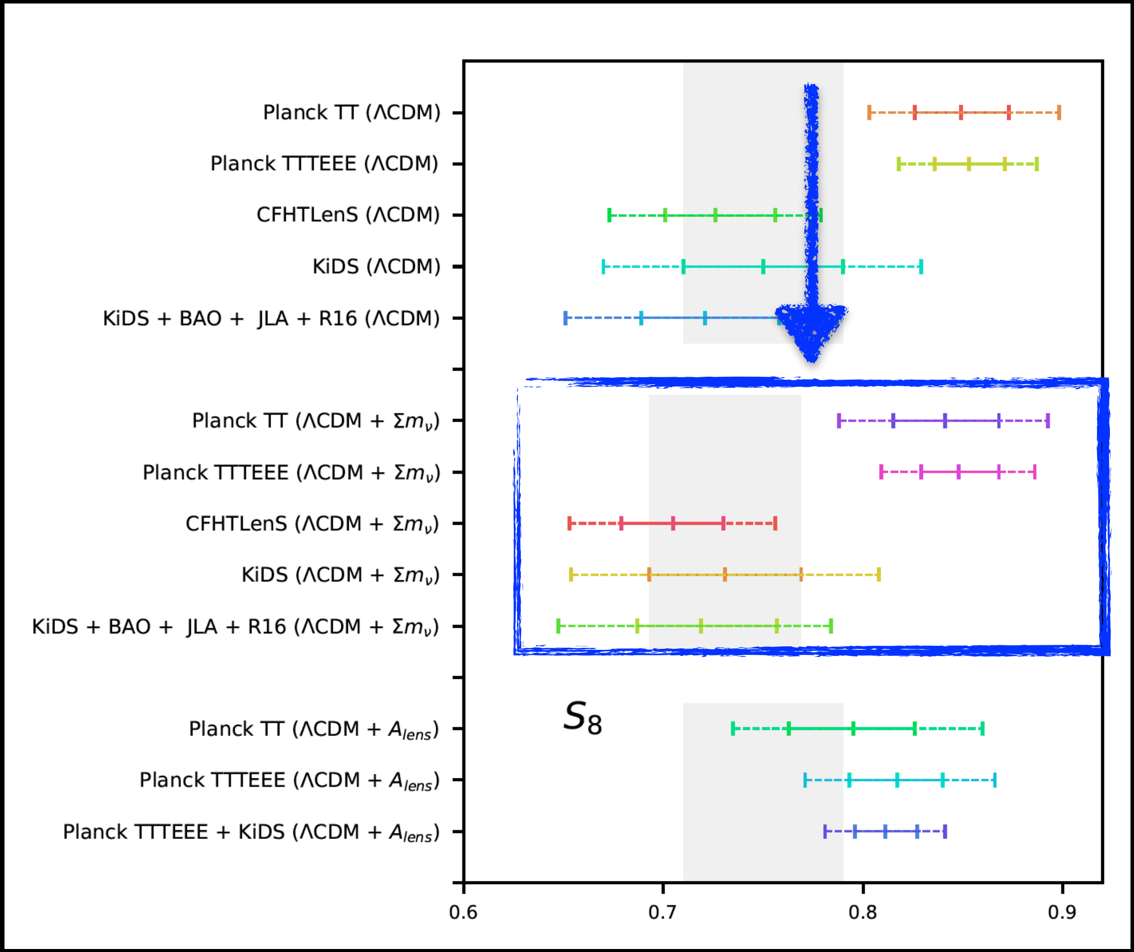

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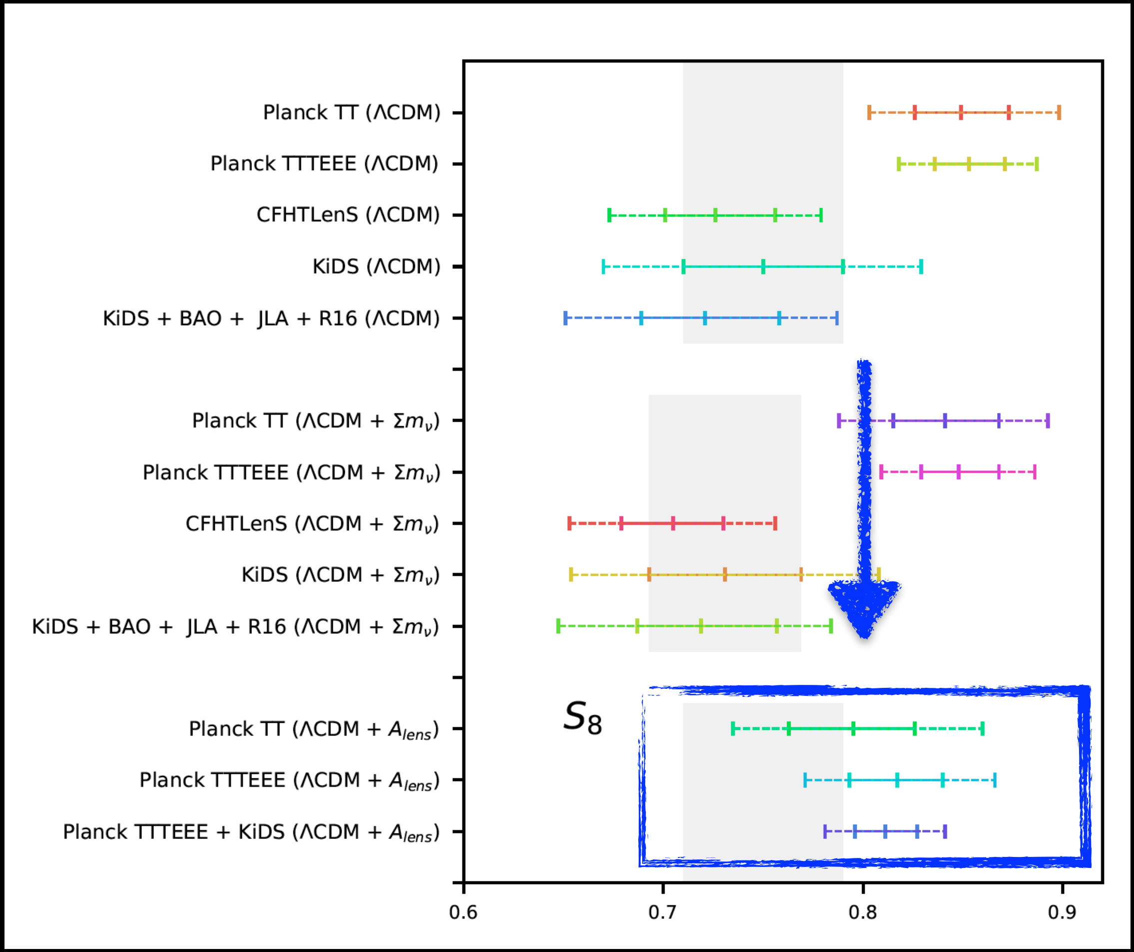

Di Valentino and Bridle, Symmetry 10 (2018) no.11, 585

Beyond six parameters: extending ΛCDM

What happens if we vary all the parameters together?

In practice we do not try to solve any single tension with a specific theoretical mechanism, but we allow for a significant number of motivated extensions of ΛCDM, looking for a possible combination of parameters that could solve or at least ameliorate, the current discordances.

While this ”minimal” 6 parameter approach is justified by the good fit to the data, some of the assumptions or simplifications made are indeed not anymore fully justified and risk an oversimplification of the physics that drives the evolution of the Universe.

Beyond six parameters: extending ΛCDM

- The total neutrino mass is fixed arbitrary to 0.06eV. However, we know that neutrinos are massive and that current cosmological datasets are sensitive to variations in the absolute neutrino mass scale of order ~ 100 meV.
- The cosmological constant offers difficulties in any theoretical interpretation: fixing the dark energy equation of state to –1 is not favoured by any theoretical argument. Moreover, while both matter and radiation evolve rapidly, Λ is assumed not to change with time, so its recent appearance in the standard cosmological model implies an extreme fine-tuning of initial conditions. This fine-tuning is known as the coincidence problem. Therefore it seems reasonable to incorporate in the analysis a possible dynamical dark energy component, constant with redshift w, or redshift dependent w(z)=w0+(1-a)wa (CPL).
- Any inflationary model, because it is a dynamical process, predicts a running of the scalar spectral index, expected for slow rolling inflation at the level of $(1\text{-}ns)^2 \sim 10^{-3}$.
- The effective number of relativistic degrees of freedom Neff could be easily different from the standard expected value of 3.046, for example for the presence of sterile neutrinos or thermal axions.
- We need to take into account the anomalous value for the lensing amplitude A_L. While this parameter is purely phenomenological, one should clearly consider it and check if the cosmology obtained is consistent with other datasets.

Constraints at 68% cl.

Beyond six parameters: extending ΛCDM

Parameters	Planck	Planck +R19	Planck +lensing	Planck +BAO	Planck + Pantheon
$\Omega_b h^2$	0.02246 ± 0.00028	$0.02248^{+0.00028}_{-0.00032}$	0.02228 ± 0.00026	0.02264 ± 0.00026	0.02250 ± 0.00028
$\Omega_c h^2$	0.1172 ± 0.0033	0.1174 ± 0.0035	0.1164 ± 0.0033	0.1175 ± 0.0033	$0.1174^{+0.0031}_{-0.0035}$
$100\theta_{\rm MC}$	1.04112 ± 0.00051	1.04111 ± 0.00052	1.04119 ± 0.00050	1.04120 ± 0.00049	1.04111 ± 0.00050
τ	0.0496 ± 0.0086	0.0508 ± 0.0091	$0.0494^{+0.0086}_{-0.0076}$	0.0502 ± 0.0087	$0.0499^{+0.0086}_{-0.0078}$
Σm_ν [eV]	< 0.863	< 0.821	< 0.714	< 0.352	< 0.822
w	-1.27 ± 0.53	$-1.33^{+0.17}_{-0.11}$	-1.33 ± 0.52	$-1.009^{+0.092}_{-0.070}$	$-1.071^{+0.073}_{-0.050}$
$N_{\rm eff}$	2.95 ± 0.24	2.97 ± 0.26	2.85 ± 0.23	3.04 ± 0.23	$2.98^{+0.23}_{-0.25}$
A_L	$1.25^{+0.09}_{-0.14}$	$1.21^{+0.09}_{-0.10}$	$1.116^{+0.061}_{-0.096}$	$1.213^{+0.076}_{-0.088}$	1.232 ± 0.090
$\ln(10^{10} A_s)$	3.027 ± 0.020	3.030 ± 0.022	3.024 ± 0.020	3.030 ± 0.020	$3.028^{+0.020}_{-0.018}$
n_s	0.964 ± 0.012	0.965 ± 0.013	0.958 ± 0.012	0.971 ± 0.012	0.965 ± 0.012
α_S	-0.0053 ± 0.0085	-0.0047 ± 0.0082	-0.0066 ± 0.0082	-0.0041 ± 0.0081	-0.0049 ± 0.0086
H_0[km/s/Mpc]	73^{+10}_{-20}	74.0 ± 1.4	74^{+10}_{-20}	67.9 ± 1.7	66.9 ± 2.0
σ_8	$0.79^{+0.15}_{-0.13}$	$0.811^{+0.051}_{-0.035}$	$0.80^{+0.15}_{-0.13}$	0.782 ± 0.025	$0.750^{+0.055}_{-0.034}$
S_8	$0.754^{+0.053}_{-0.041}$	$0.758^{+0.039}_{-0.027}$	$0.757^{+0.047}_{-0.038}$	$0.791^{+0.025}_{-0.019}$	$0.775^{+0.036}_{-0.026}$

In this Table we show the constraints obtained assuming our extended 11 parameters space, assuming a constant dark energy equation of state w.

Di Valentino, Melchiorri and Silk, JCAP 2001 (2020) no.01, 013

Constraints at 68% cl.

Beyond six parameters: extending ΛCDM

Parameters	Planck	Planck +R19	Planck +lensing	Planck +BAO	Planck + Pantheon
$\Omega_b h^2$	0.02246 ± 0.00028	$0.02248^{+0.00028}_{-0.00032}$	0.02228 ± 0.00026	0.02264 ± 0.00026	0.02250 ± 0.00028
$\Omega_c h^2$	0.1172 ± 0.0033	0.1174 ± 0.0035	0.1164 ± 0.0033	0.1175 ± 0.0033	$0.1174^{+0.0031}_{-0.0035}$
$100\theta_{\rm MC}$	1.04112 ± 0.00051	1.04111 ± 0.00052	1.04119 ± 0.00050	1.04120 ± 0.00049	1.04111 ± 0.00050
τ	0.0496 ± 0.0086	0.0508 ± 0.0091	$0.0494^{+0.0086}_{-0.0076}$	0.0502 ± 0.0087	$0.0499^{+0.0086}_{-0.0078}$
Σm_ν [eV]	< 0.863	< 0.821	< 0.714	< 0.352	< 0.822
w	-1.27 ± 0.53	$-1.33^{+0.17}_{-0.11}$	-1.33 ± 0.52	$-1.009^{+0.092}_{-0.070}$	$-1.071^{+0.073}_{-0.050}$
$N_{\rm eff}$	2.95 ± 0.24	2.97 ± 0.26	2.85 ± 0.23	3.04 ± 0.23	$2.98^{+0.23}_{-0.25}$
A_L	$1.25^{+0.09}_{-0.14}$	$1.21^{+0.09}_{-0.10}$	$1.116^{+0.061}_{-0.096}$	$1.213^{+0.076}_{-0.088}$	1.232 ± 0.090
$\ln(10^{10} A_s)$	3.027 ± 0.020	3.030 ± 0.022	3.024 ± 0.020	3.030 ± 0.020	$3.028^{+0.020}_{-0.018}$
n_s	0.964 ± 0.012	0.965 ± 0.013	0.958 ± 0.012	0.971 ± 0.012	0.965 ± 0.012
α_S	-0.0053 ± 0.0085	-0.0047 ± 0.0082	-0.0066 ± 0.0082	-0.0041 ± 0.0081	-0.0049 ± 0.0086
H_0[km/s/Mpc]	73^{+10}_{-20}	74.0 ± 1.4	74^{+10}_{-20}	67.9 ± 1.7	66.9 ± 2.0
σ_8	$0.79^{+0.15}_{-0.13}$	$0.811^{+0.051}_{-0.035}$	$0.80^{+0.15}_{-0.13}$	0.782 ± 0.025	$0.750^{+0.033}_{-0.034}$
S_8	$0.754^{+0.053}_{-0.041}$	$0.758^{+0.039}_{-0.027}$	$0.757^{+0.047}_{-0.038}$	$0.791^{+0.025}_{-0.019}$	$0.775^{+0.036}_{-0.026}$

We find a relaxed value for the Hubble constant, with respect to the one derived under the assumption of ΛCDM.

Di Valentino, Melchiorri and Silk, JCAP 2001 (2020) no.01, 013

Constraints at 68% cl.

Beyond six parameters: extending ΛCDM

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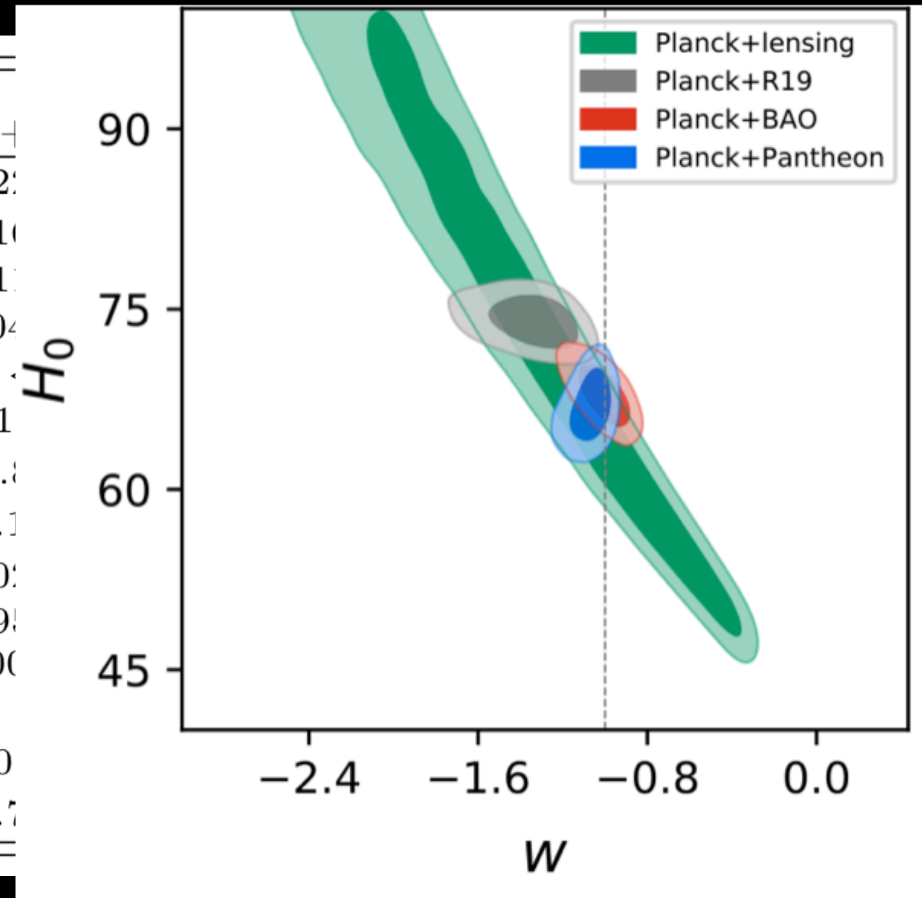
n_s	0.964 ± 0.012	0.965 ± 0.013
α_S	-0.0053 ± 0.0085	-0.0047 ± 0.0082
H_0[km/s/Mpc]	73^{+10}_{-20}	74.0 ± 1.4
σ_8	$0.79^{+0.15}_{-0.13}$	$0.811^{+0.051}_{-0.035}$
S_8	$0.754^{+0.053}_{-0.041}$	$0.758^{+0.039}_{-0.027}$



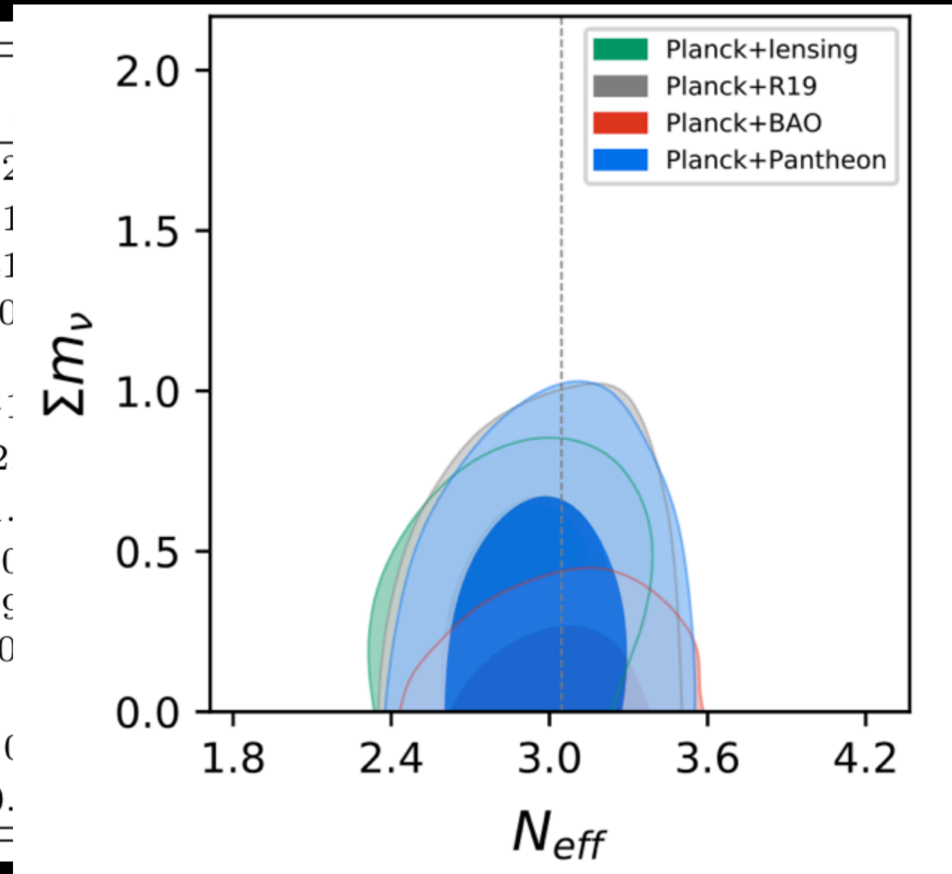
Since now datasets are fully compatible, we combine Planck 2018 with R19 (H_0=74.03 +/- 1.42 km/s/Mpc), in order to see which parameter is preferred by the data to solve the tension. We find a phantom-like dark energy component with an equation of state $w<-1$ at more than three standard deviations, while the neutrino effective number is fully compatible with standard expectations.

Di Valentino, Melchiorri and Silk, JCAP 2001 (2020) no.01, 013

Constraints at 68% cl.

Beyond six parameters: extending ΛCDM

Parameters	Planck	Planck +R19	
$\Omega_b h^2$	0.02246 ± 0.00028	$0.02248^{+0.00028}_{-0.00032}$	0.022
$\Omega_c h^2$	0.1172 ± 0.0033	0.1174 ± 0.0035	0.11
$100\theta_{\rm MC}$	1.04112 ± 0.00051	1.04111 ± 0.00052	1.041
τ	0.0496 ± 0.0086	0.0508 ± 0.0091	0.0
Σm_ν [eV]	< 0.863	< 0.821	
w	-1.27 ± 0.53	$-1.33^{+0.17}_{-0.11}$	−
$N_{\rm eff}$	2.95 ± 0.24	2.97 ± 0.26	2
A_L	$1.25^{+0.09}_{-0.14}$	$1.21^{+0.09}_{-0.10}$	1.
$\ln(10^{10} A_s)$	3.027 ± 0.020	3.030 ± 0.022	3.0
n_s	0.964 ± 0.012	0.965 ± 0.013	0.9
α_S	-0.0053 ± 0.0085	-0.0047 ± 0.0082	−0.0
H_0[km/s/Mpc]	73^{+10}_{-20}	74.0 ± 1.4	
σ_8	$0.79^{+0.15}_{-0.13}$	$0.811^{+0.051}_{-0.035}$	(
S_8	$0.754^{+0.053}_{-0.041}$	$0.758^{+0.039}_{-0.027}$	0.



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Di Valentino, Melchiorri and Silk, JCAP 2001 (2020) no.01, 013

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Beyond six parameters: extending ΛCDM

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$100\theta_{MC}$	1.04112 ± 0.00051	1.04111 ± 0.00052
τ	0.0496 ± 0.0086	0.0508 ± 0.0091
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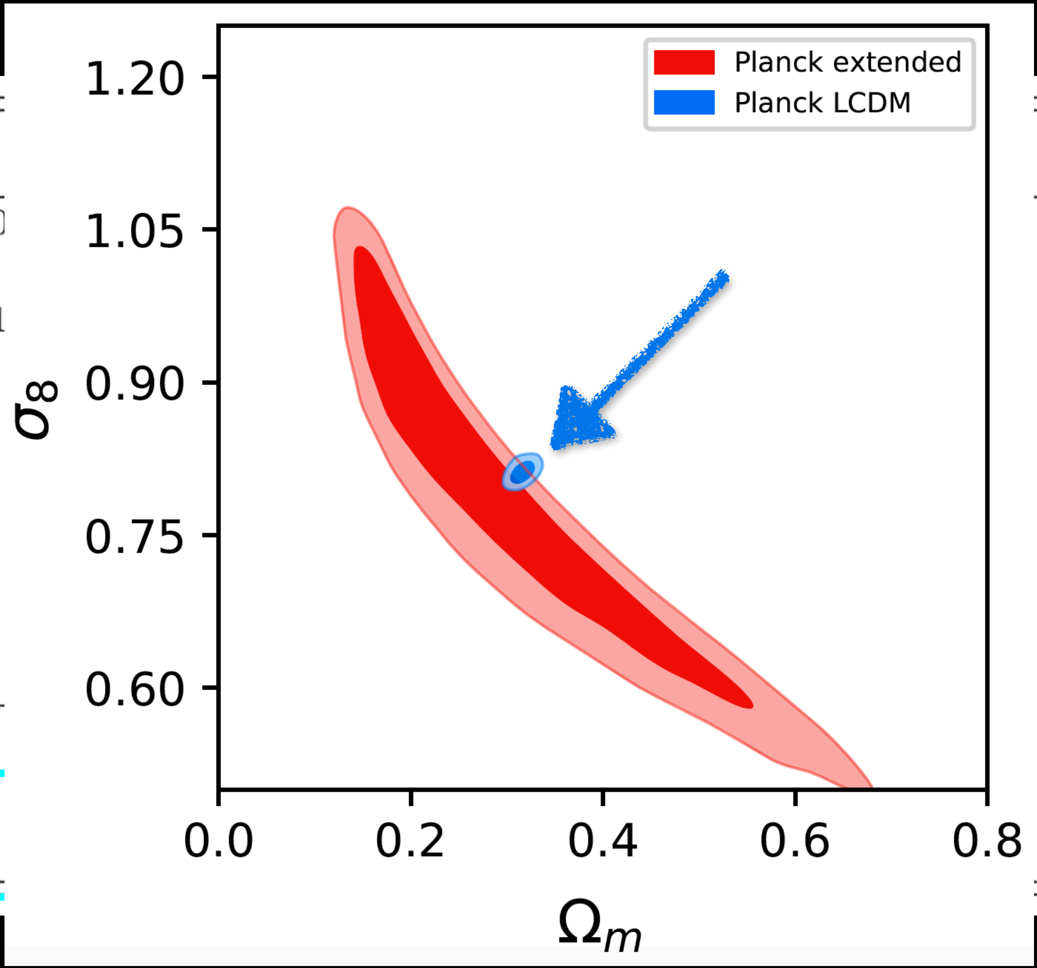
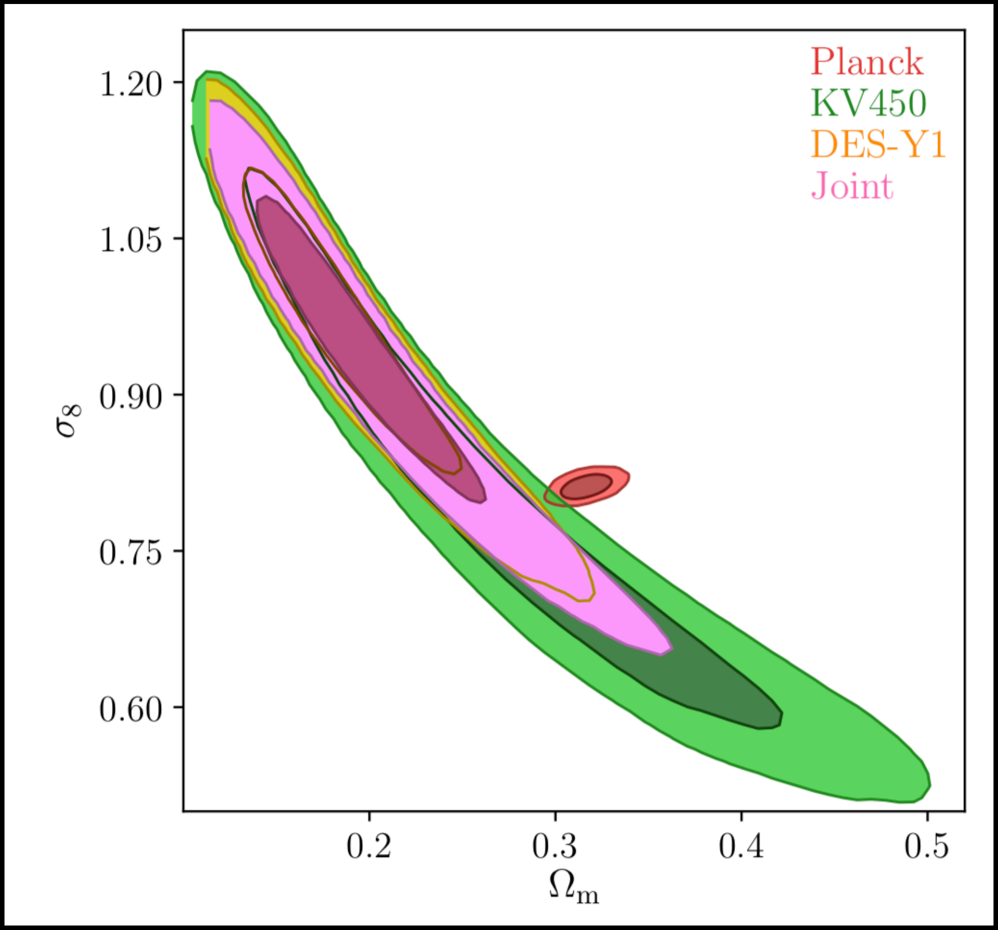
S_8	$0.754^{+0.053}_{-0.041}$	$0.758^{+0.039}_{-0.027}$



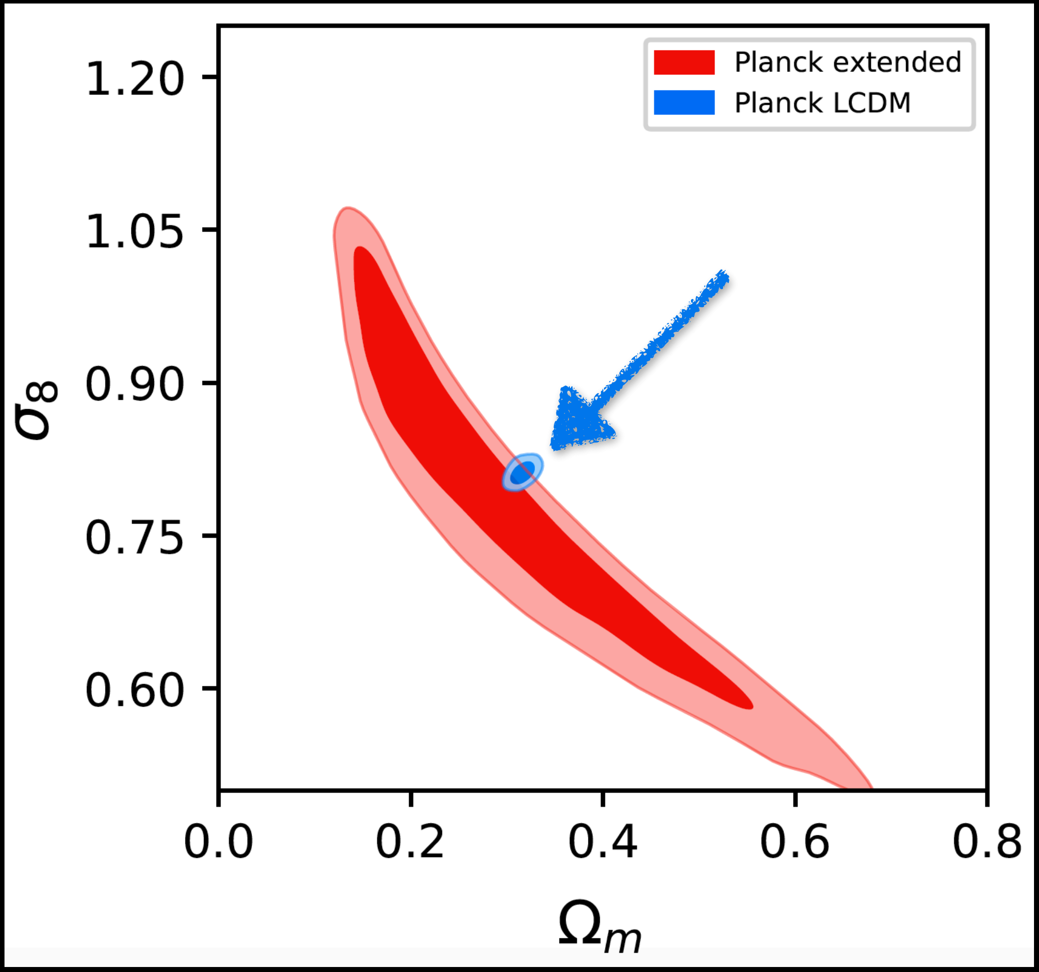
We find relaxed and lower values for the clustering parameter σ8 and S8, with respect to those derived under the assumption of ΛCDM.

Di Valentino, Melchiorri and Silk, JCAP 2001 (2020) no.01, 013

Beyond six parameters: extending ΛCDM



Asgari et al., arXiv:1910.05336 [astro-ph.CO]



Di Valentino, Melchiorri and Silk, JCAP 2001 (2020) no.01, 013

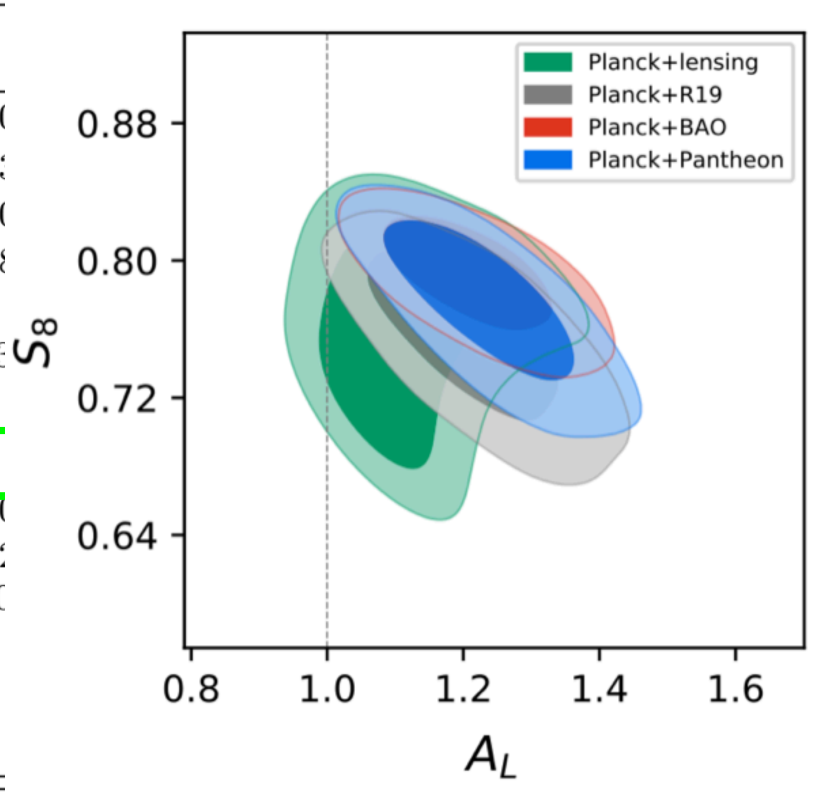
$$S_8 \equiv \sigma_8 \sqrt{\Omega_m / 0.3}$$

In this way, we can solve the existing S8 tensions with the CFHTlenS and KiDS-450 cosmic shear surveys.

Constraints at 68% cl.

Beyond six parameters: extending ΛCDM

Parameters	Planck	Planck +BAO	Planck + Pantheon
$\Omega_b h^2$	0.02246 ± 0.000	02264 ± 0.00026	0.02250 ± 0.00028
$\Omega_c h^2$	0.1172 ± 0.00	$.1175 \pm 0.0033$	$0.1174^{+0.0031}_{-0.0035}$
$100\theta_{MC}$	1.04112 ± 0.000	04120 ± 0.00049	1.04111 ± 0.00050
τ	0.0496 ± 0.00	$.0502 \pm 0.0087$	$0.0499^{+0.0086}_{-0.0078}$
Σm_ν [eV]	< 0.863	< 0.352	< 0.822
w	-1.27 ± 0.5	$-1.009^{+0.092}_{-0.070}$	$-1.071^{+0.073}_{-0.050}$
$N_{\rm eff}$	2.95 ± 0.24	3.04 ± 0.23	$2.98^{+0.23}_{-0.25}$
A_L	$1.25^{+0.09}_{-0.14}$	$1.213^{+0.076}_{-0.088}$	1.232 ± 0.090
$\ln(10^{10} A_s)$	3.027 ± 0.02	3.030 ± 0.020	$3.028^{+0.020}_{-0.018}$
n_s	0.964 ± 0.01	0.971 ± 0.012	0.965 ± 0.012
α_S	-0.0053 ± 0.00	0.0041 ± 0.0081	-0.0049 ± 0.0086
H_0[km/s/Mpc]	73^{+10}_{-20}	67.9 ± 1.7	66.9 ± 2.0
σ_8	$0.79^{+0.15}_{-0.13}$	0.782 ± 0.025	$0.750^{+0.055}_{-0.034}$
S_8	$0.754^{+0.053}_{-0.041}$	$0.791^{+0.025}_{-0.019}$	$0.775^{+0.036}_{-0.026}$



And in fact, the only notable exception is the angular power spectrum lensing amplitude, A_L that is larger than the expected value at about 3 standard deviations even when combining the Planck data with BAO and supernovae type Ia external datasets.

Di Valentino, Melchiorri and Silk, JCAP 2001 (2020) no.01, 013

But…
assuming General Relativity,
is there a physical explanation
for A_L?

The most statistically significant and persisting anomalies and tensions of the CMB are:

- H0 with local measurements
- A_L internal anomaly
- S8 with cosmic shear data
- Ω_K different from zero

See Di Valentino et al. arXiv:2008.11283 [astro-ph.CO], arXiv:2008.11284 [astro-ph.CO], arXiv:2008.11285 [astro-ph.CO], arXiv:2008.11286 [astro-ph.CO] for an overview.

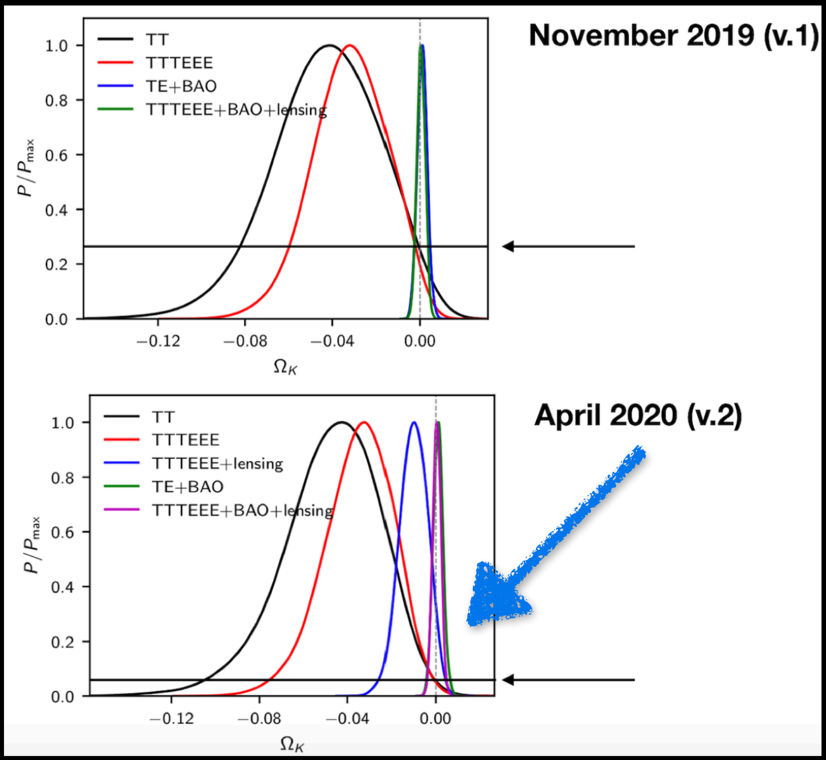
Curvature of the universe

The ΛCDM model assumes that the universe is specially flat.
The combination of the Planck temperature and polarization power spectra gives:

$$\Omega_K = -0.044^{+0.018}_{-0.015} \quad (68\,\%, \textit{Planck}\ \mathrm{TT,TE,EE+lowE}),$$

Planck 2018, Astron.Astrophys. 641 (2020) A6

a detection of curvature at about 3.4σ,
with a 99% probability region of $-0.095 \leq \Omega_K \leq -0.007$.



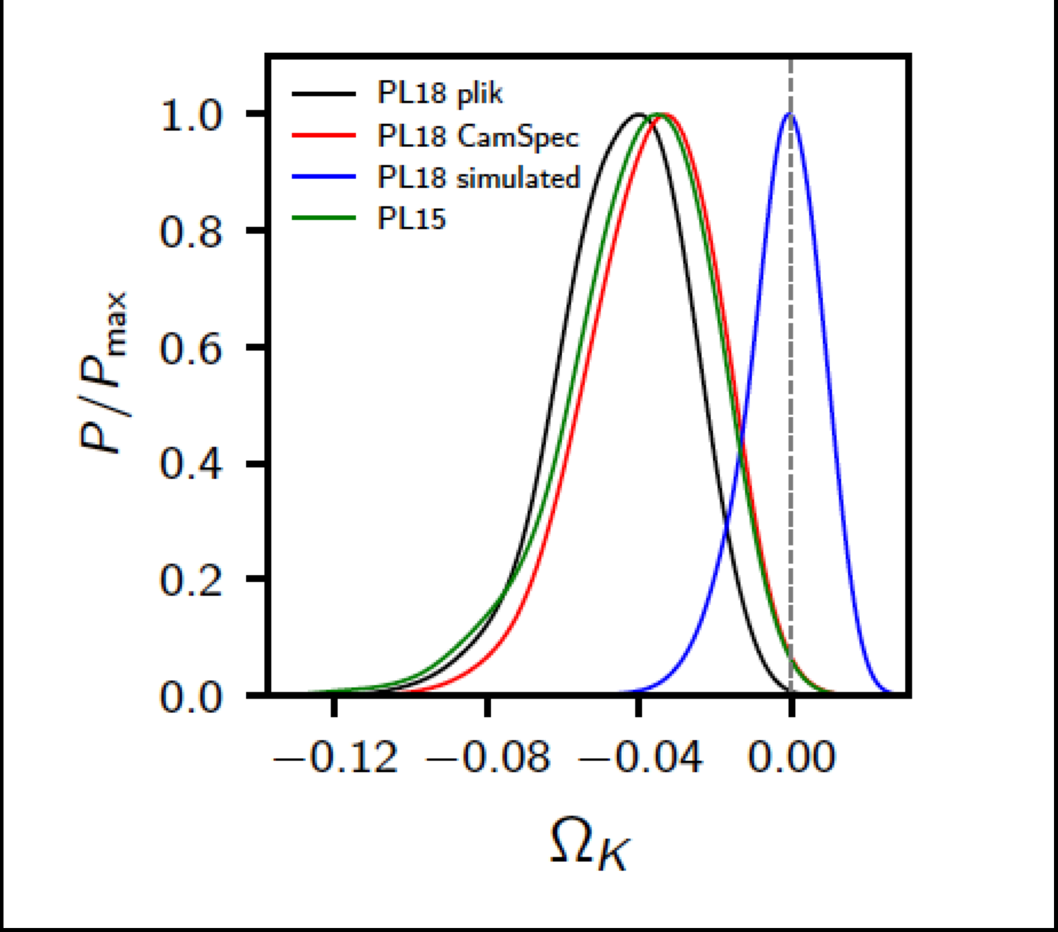
Efstathiou and Gratton, arXiv:1910.00483

This result has been obtained by using Plik, i.e. the baseline likelihood of Planck, but is NOW confirmed by CamSpec

$-0.083 \leq \Omega_K \leq -0.001$ at 99% CL.

Curvature of the universe

Can Planck provide an unbiased and reliable estimate of the curvature of the Universe?
This may not be the case since a "geometrical degeneracy" is present with Ωm.
When precise CMB measurements at arc-minute angular scales are included, since gravitational lensing depends on the matter density, its detection breaks the geometrical degeneracy. The Planck experiment with its improved angular resolution offers the unique opportunity of a precise measurement of curvature from a single CMB experiment.
We simulated Planck, finding that such experiment could constrain curvature with a 2% uncertainty, without any significant bias towards closed models.

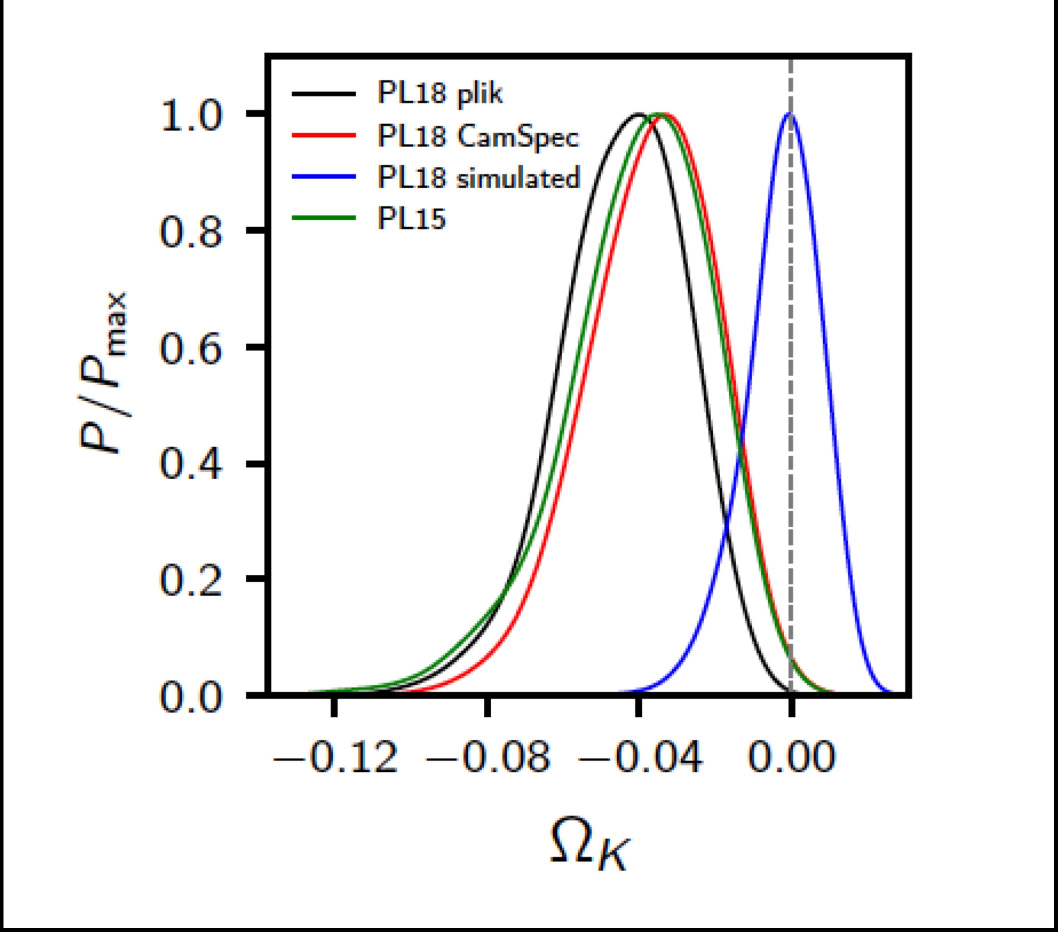


Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

Curvature of the universe

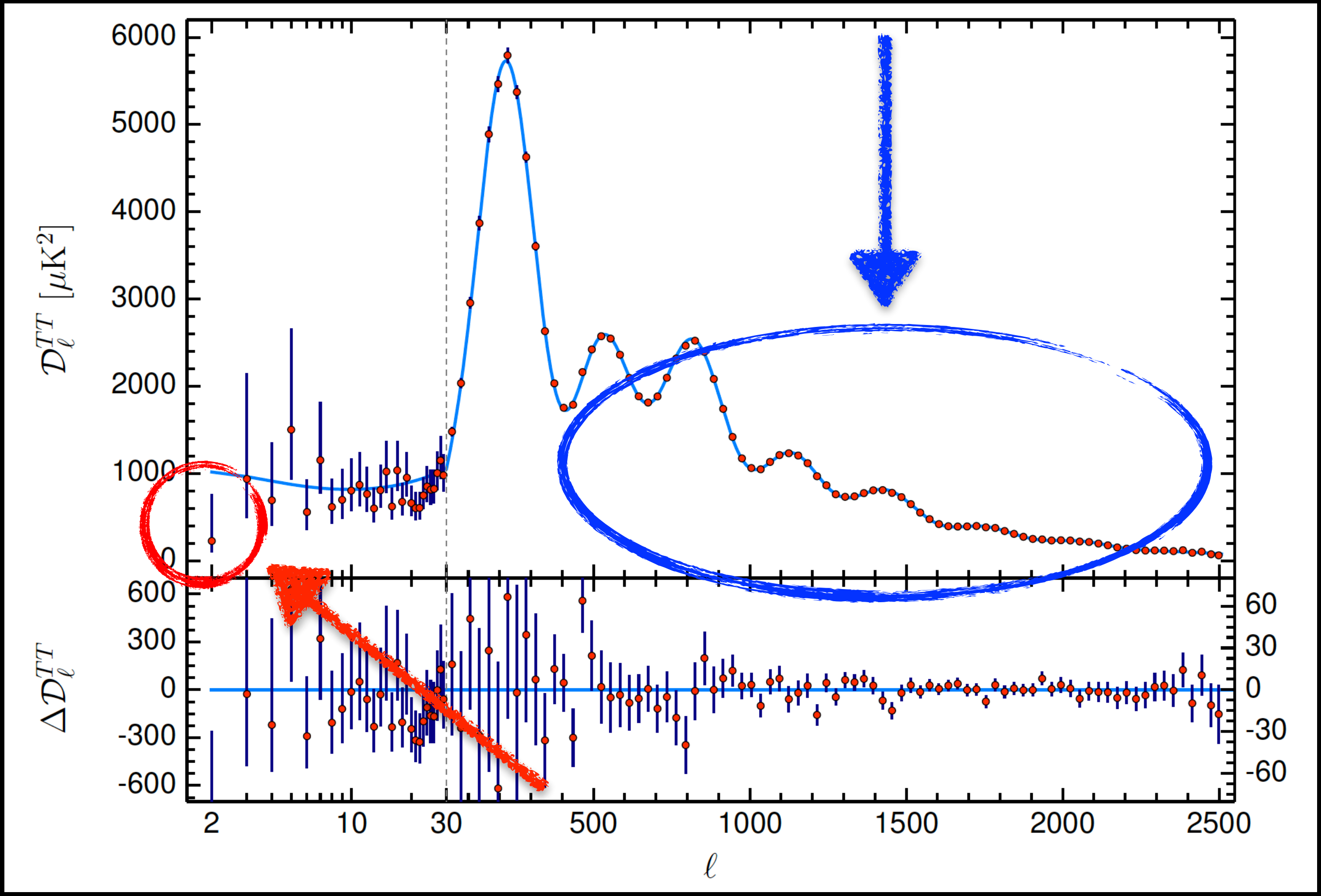
Planck favours a closed Universe ($\Omega k<0$) with 99.985% probability.
A closed Universe with $\Omega K = -0.0438$ provides a better fit to PL18 with respect to a flat model.

This is not entirely a volume effect, since the best-fit $\Delta\chi^2$ changes by -11 compared to base ΛCDM when adding the one additional curvature parameter.
The improvement is due also to the fact that closed models could also lead to a large-scale cut-off in the primordial density fluctuations in agreement with the observed low CMB anisotropy quadrupole.



Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

A closed universe fits Planck better than A_L



Planck 2018, Astron.Astrophys. 641 (2020) A6

A model with $\Omega_K < 0$ is slightly preferred with respect to a flat model with $A_L > 1$, because closed models better fit not only the damping tail, but also the low-multipole data, especially the quadrupole.

arXiv.org > astro-ph > arXiv:astro-ph/0303127

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Astrophysics

[Submitted on 5 Mar 2003 (v1), last revised 30 Jul 2003 (this version, v2)]

Is the Low CMB Quadrupole a Signature of Spatial Curvature?

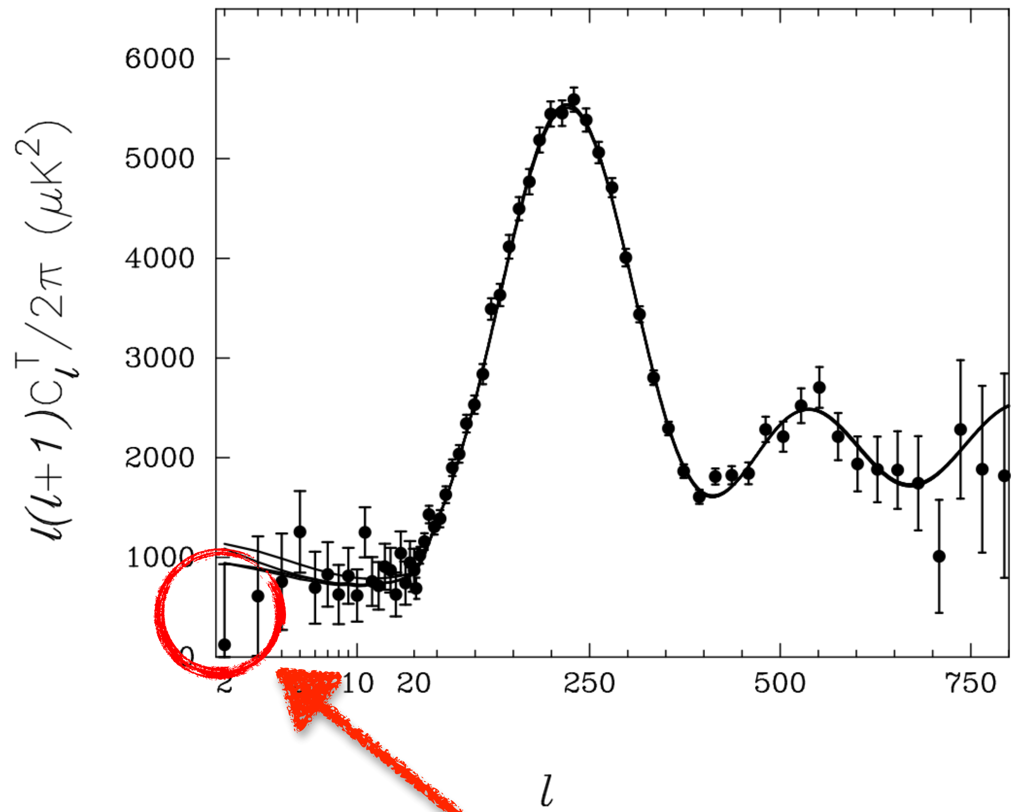
G. Efstathiou (University of Cambridge)

The temperature anisotropy power spectrum measured with the Wilkinson Microwave Anisotropy Probe (WMAP) at high multipoles is in spectacular agreement with an inflationary Lambda-dominated cold dark matter cosmology. However, the low order multipoles (especially the quadrupole) have lower amplitudes than expected from this cosmology, indicating a need for new physics. Here we speculate that the low quadrupole amplitude is associated with spatial curvature. We show that positively curved models are consistent with the WMAP data and that the quadrupole amplitude can be reproduced if the primordial spectrum truncates on scales comparable to the curvature scale.

Comments: 4 pages, Latex, 2 figs, revised version accepted by MNRAS
Subjects: **Astrophysics (astro-ph)**
Journal reference: Mon.Not.Roy.Astron.Soc. 343 (2003) L95
DOI: 10.1046/j.1365-8711.2003.06940.x
Cite as: arXiv:astro-ph/0303127
(or arXiv:astro-ph/0303127v2 for this version)

Submission history

From: George Efstathiou [view email]
[v1] Wed, 5 Mar 2003 23:30:33 UTC (21 KB)
[v2] Wed, 30 Jul 2003 10:16:45 UTC (22 KB)



A lower quadrupole than predicted by the ΛCDM was already present in WMAP, and a closed universe to explain this effect was already taken into account.

Curvature of the universe

We can compute the Bayesian evidence ratio by making use of the Savage–Dickey density ratio. In this case the Bayes factor can be written as

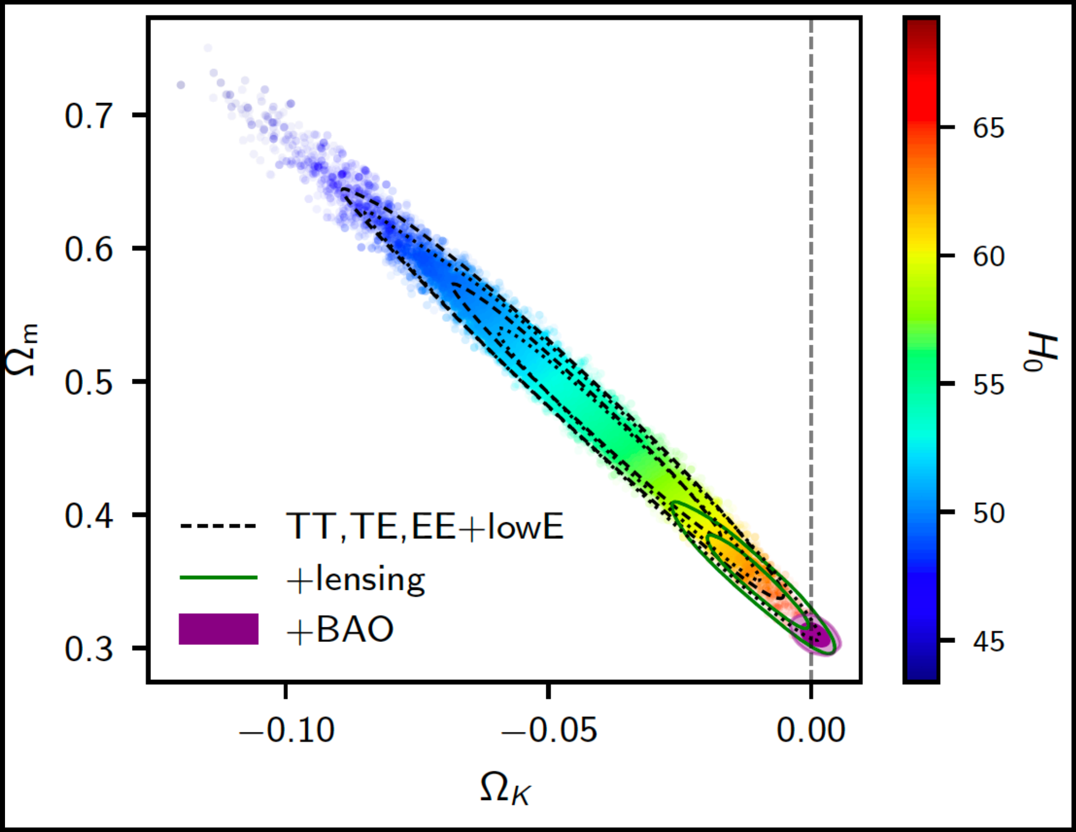
$$B_{01} = \left. \frac{p(\Omega_K | d, M_1)}{\pi(\Omega_K | M_1)} \right|_{\Omega_K = 0}$$

where M1 denotes the model with curvature, $p(\Omega_K|d, M1)$ is the posterior for Ω_K in this theoretical framework, computed from a specific dataset d, and $\pi(\Omega_K|M1)$ is the prior on Ω_K that we assume to be flat in the range $-0.2 \leq \Omega_K \leq 0$.

For Planck we obtain a Bayes ratio of $|\ln B_{01}| = 3.3$, i.e. a strong evidence for a closed universe with respect to a flat one.

Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

What about Planck+BAO?



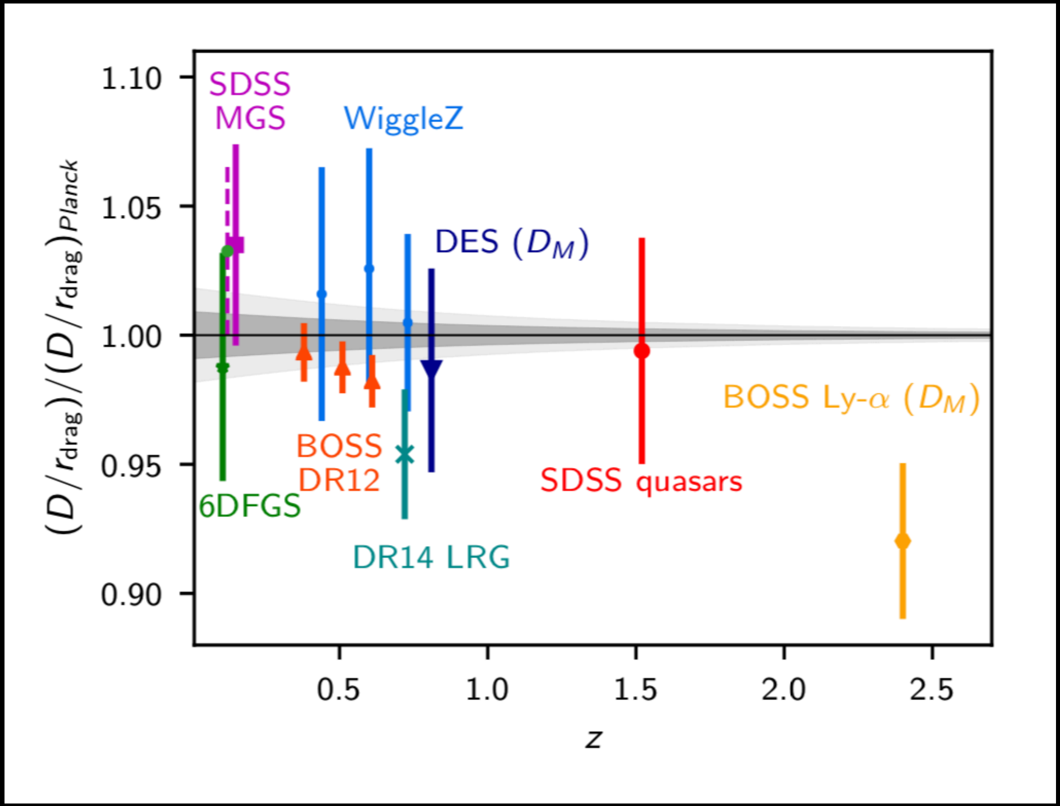
Planck 2018, Astron.Astrophys. 641 (2020) A6

Adding BAO data, a joint constraint is very consistent with a flat universe.

$$\Omega_K = 0.0007 \pm 0.0019 \quad \text{(68 \%, TT,TE,EE+lowE +lensing+BAO).}$$

Given the significant change in the conclusions from Planck alone, it is reasonable to investigate whether they are actually consistent. In fact, a basic assumption for combining complementary datasets is that these ones must be consistent, i.e. they must plausibly arise from the same cosmological model.

BAO tension

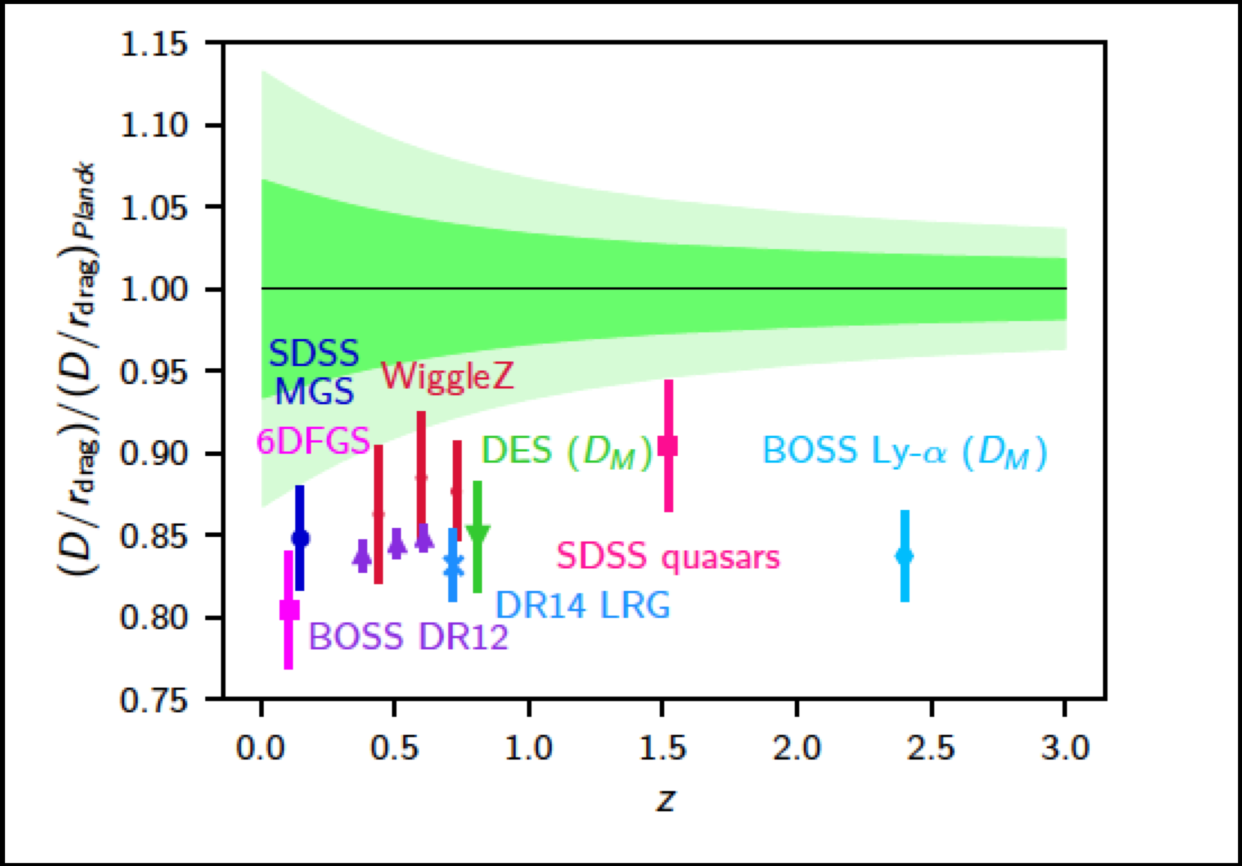


Planck 2018, Astron.Astrophys. 641 (2020) A6

This is a plot of the acoustic-scale distance ratio, DV(z)/rdrag, as a function of redshift, taken from several recent BAO surveys, and divided by the mean acoustic-scale ratio obtained by Planck adopting a model. rdrag is the comoving size of the sound horizon at the baryon drag epoch, and DV, the dilation scale, is a combination of the Hubble parameter H(z) and the comoving angular diameter distance DM(z).

In a ΛCDM model the BAO data agree really well with the Planck measurements…

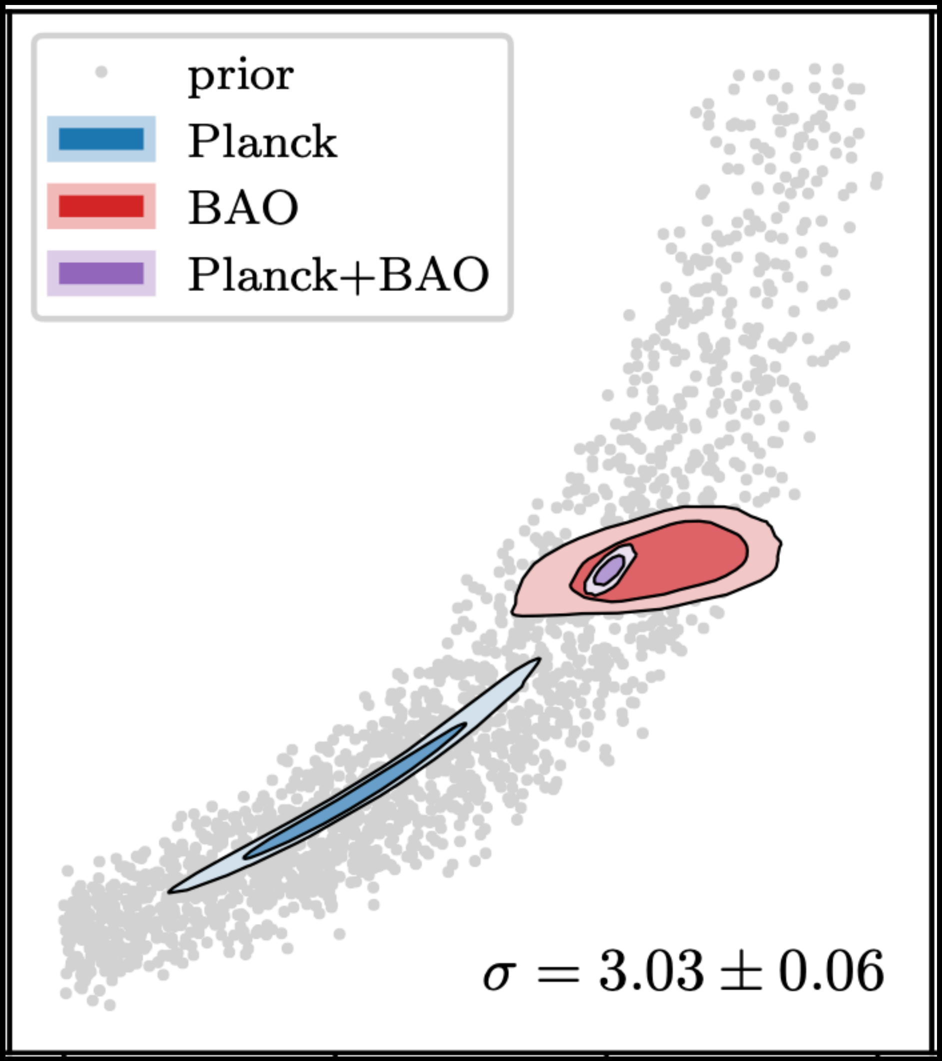
BAO tension



Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

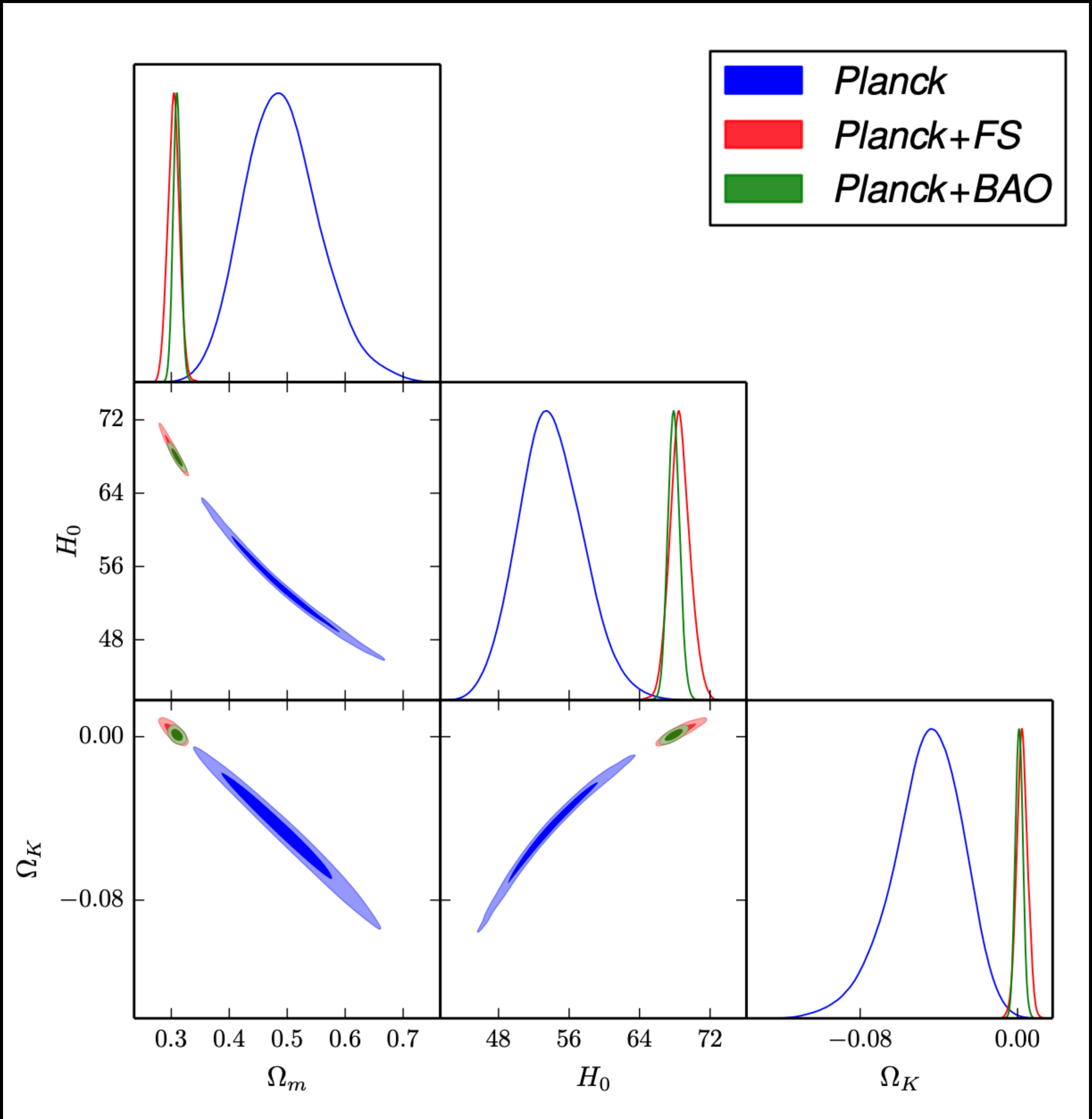
… but when we let curvature to vary
there is a striking disagreement between Planck spectra and BAO measurements!

BAO tension



In agreement with
Handley, Phys.Rev.D 103 (2021) 4, L041301

What about Planck+FS?

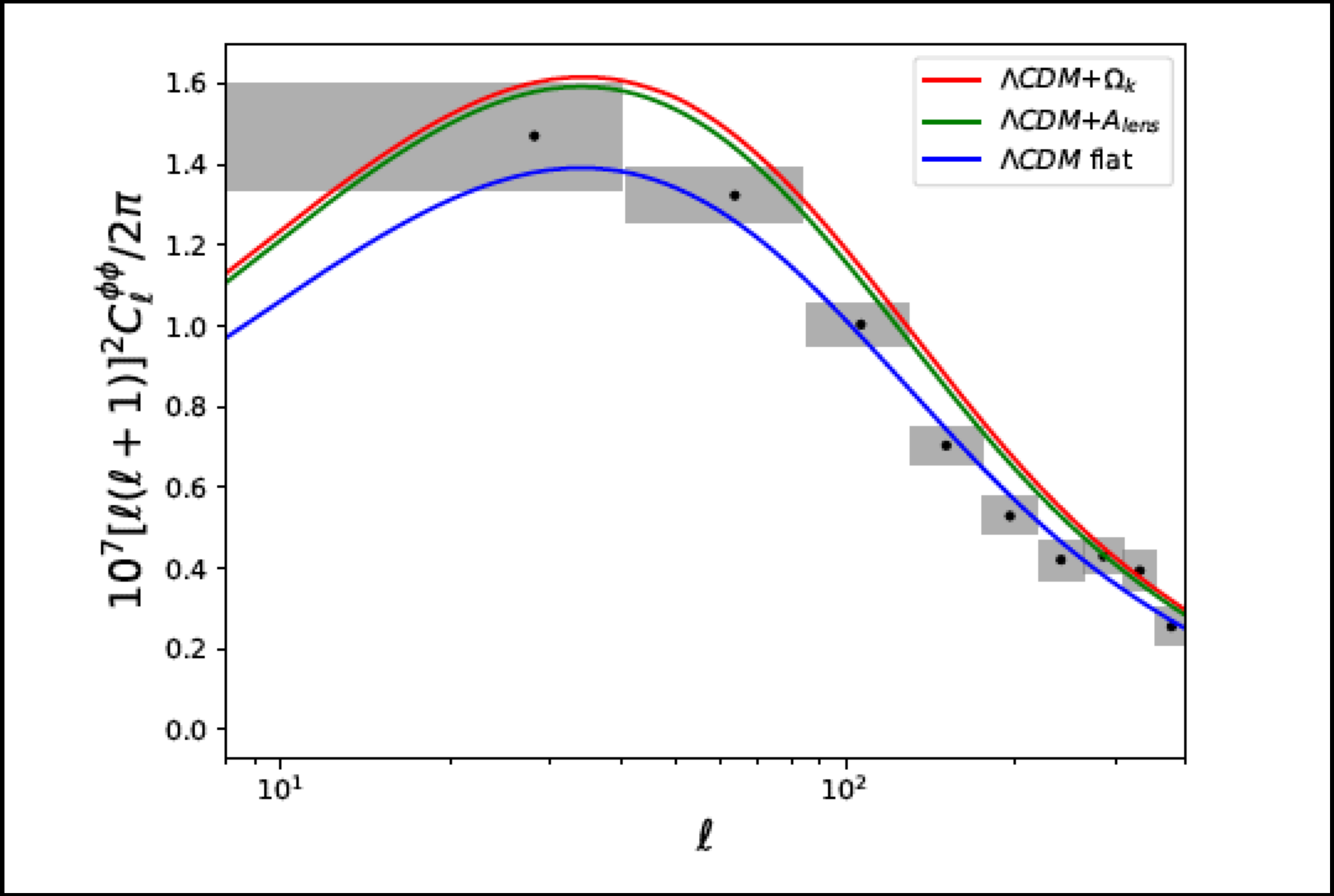


The strong disagreement between Planck and BAO it is evident in this triangular plot, as well as that with the full-shape (FS) galaxy power spectrum measurements from the BOSS DR12 CMASS sample, at an effective redshift $z_{eff} = 0.57$.

Vagnozzi, Di Valentino, et al., *Phys.Dark Univ.* 33 (2021) 100851

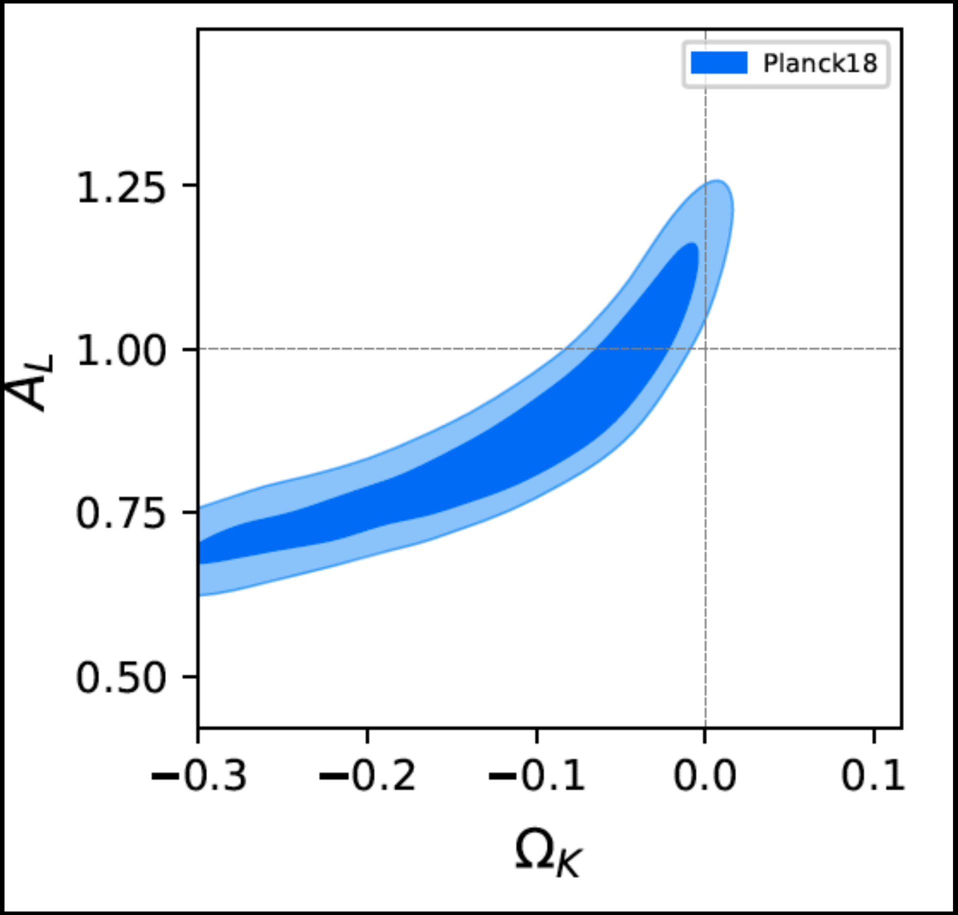
What about CMB lensing?

Closed models predict substantially higher lensing amplitudes than in ΛCDM, because the dark matter content can be greater, leading to a larger lensing signal. The reasons for the pull towards negative values of Ω_K are essentially the same as those that lead to the preference for AL > 1.



Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

A closed universe (Friedmann 1922) can explain A_L!



Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

A degeneracy between curvature and the AL parameter is clearly present. A closed universe can provide a robust physical explanation to the enhancement of the lensing amplitude. In fact, the curvature of the Universe is not new physics beyond the standard model, but it is predicted by the General Relativity, and depends on the energy content of the Universe.

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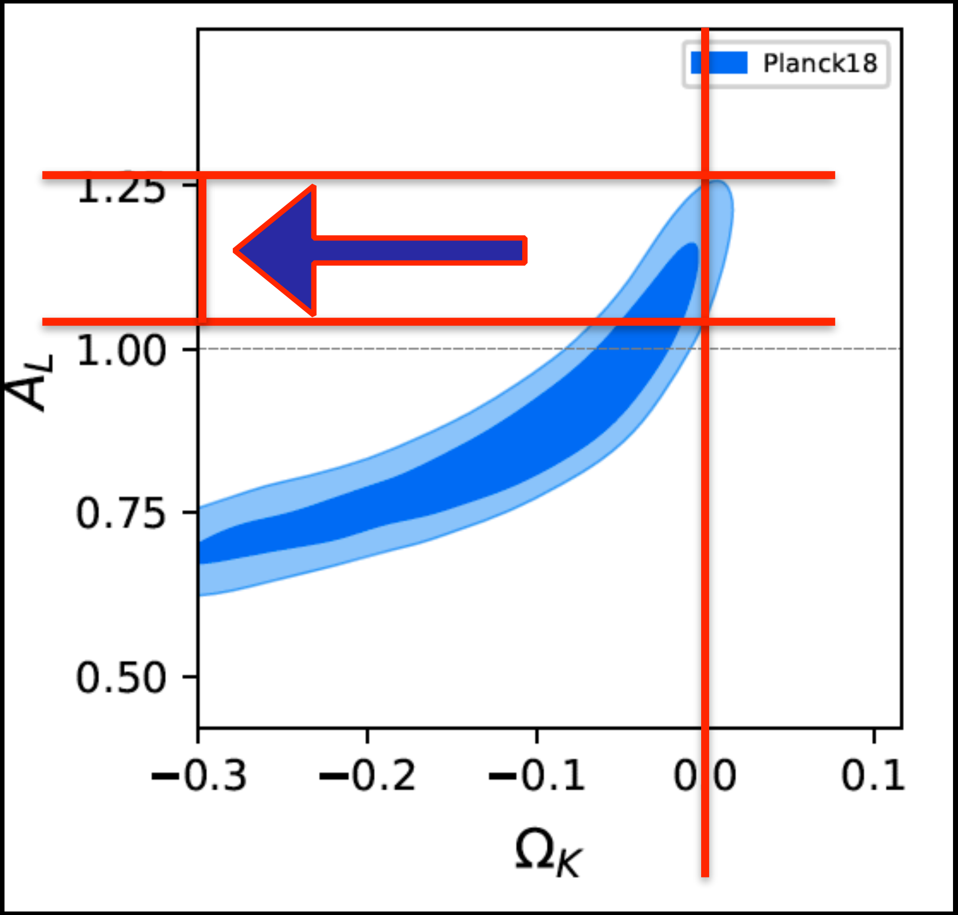

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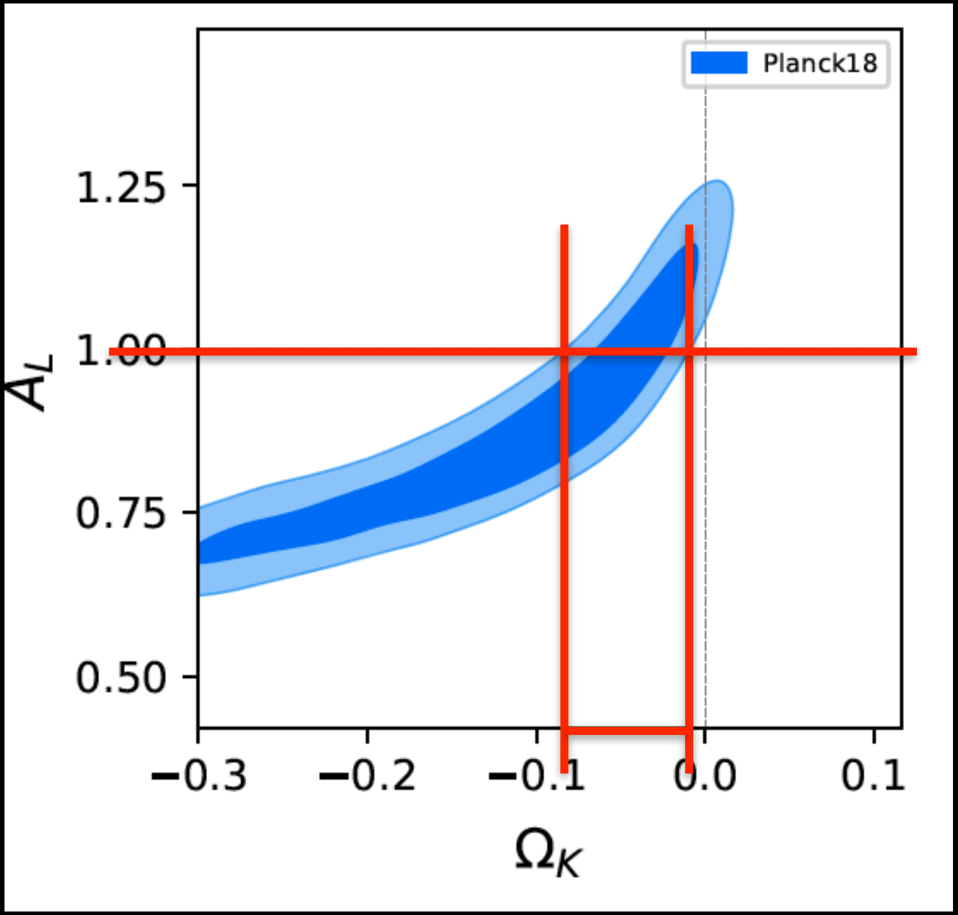

Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

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The evolution over time of the geometry of the universe is described by Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^2} T_{\mu\nu} + \Lambda g_{\mu\nu}$$

which relate the purely geometric properties of space-time, with the distribution of energy of the universe. For this it is sufficient to know the energy content of the Universe to determine its geometry and vice-versa.

Adopting a 4-dimensional coordinate system for the space-time and the Cosmological Principle, i.e. a universe homogeneous and isotropic at large scales, the resulting metric is the Friedmann-Lemaitre-Robertson-Walker (FLRW), that describes the distance between two events in space-time.

$$ds^2 = c^2dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + sin^2\theta d\varphi^2\right)\right]$$

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The curvature parameter k can be positive, null or negative, depending on the value of the curvature of the universe: positive, flat or negative.

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The evolution over time of the geometry of the universe is described by Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^2} T_{\mu\nu} + \Lambda g_{\mu\nu}$$

which relate the purely geometric properties of space-time, with the distribution of energy of the universe. For this it is sufficient to know the energy content of the Universe to determine its geometry and vice-versa.

Combining together the FLRW metric and Einstein's equations we obtain the Friedmann equations that describe the expansion history of the universe:

1st
$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

2n
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3P) + \frac{\Lambda}{3}$$

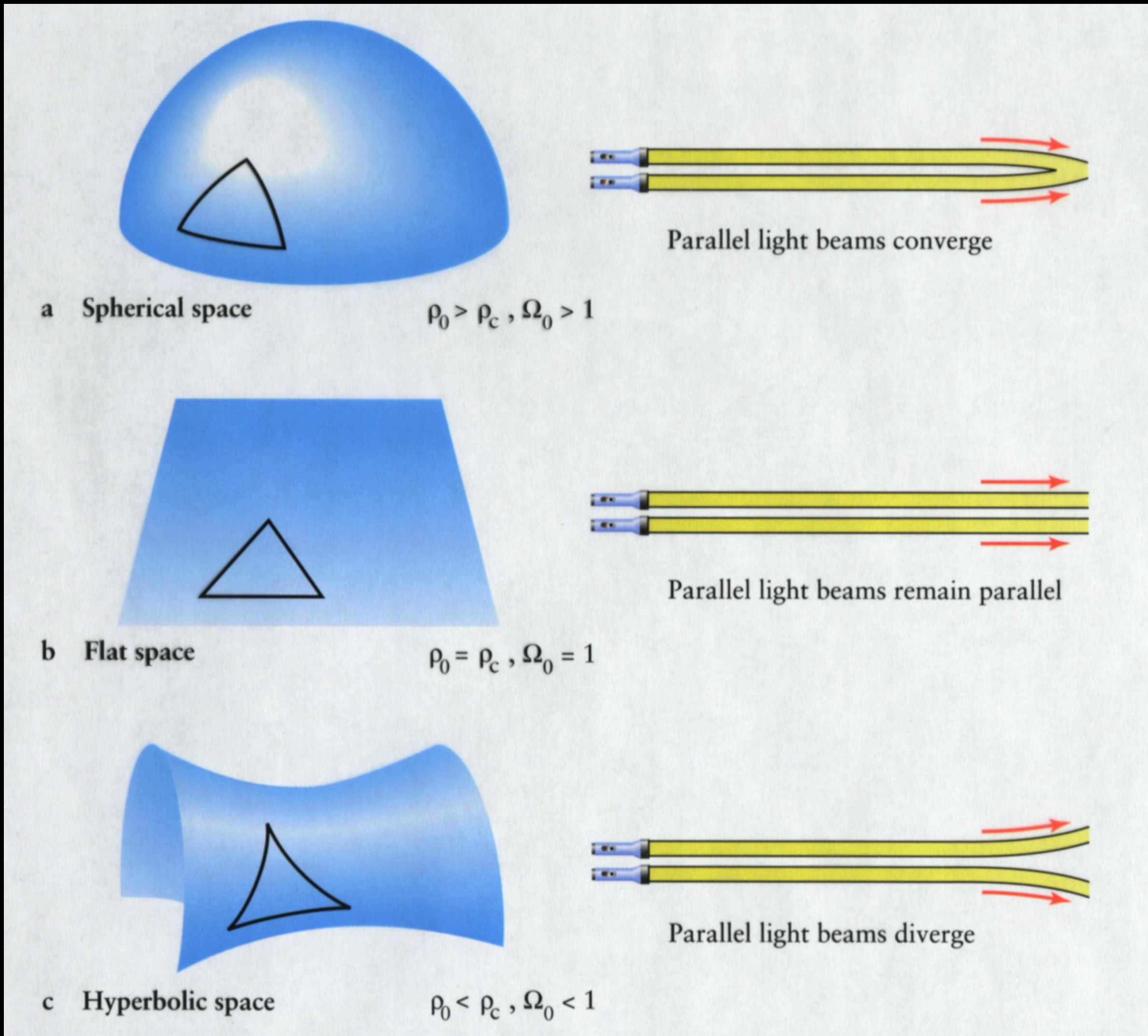


Figure: http://w3.phys.nthu.edu.tw

If we divide the 1[st] Friedmann equation, for the critical density (density of a flat universe), we obtain today:

$$\Omega = \sum_i \Omega_i = \Omega_m + \Omega_\Lambda + \Omega_r = 1 - \Omega_k$$

From this equation it is possible to estimate the curvature of the universe, independently measuring the various contributions to the total density parameter Ω.

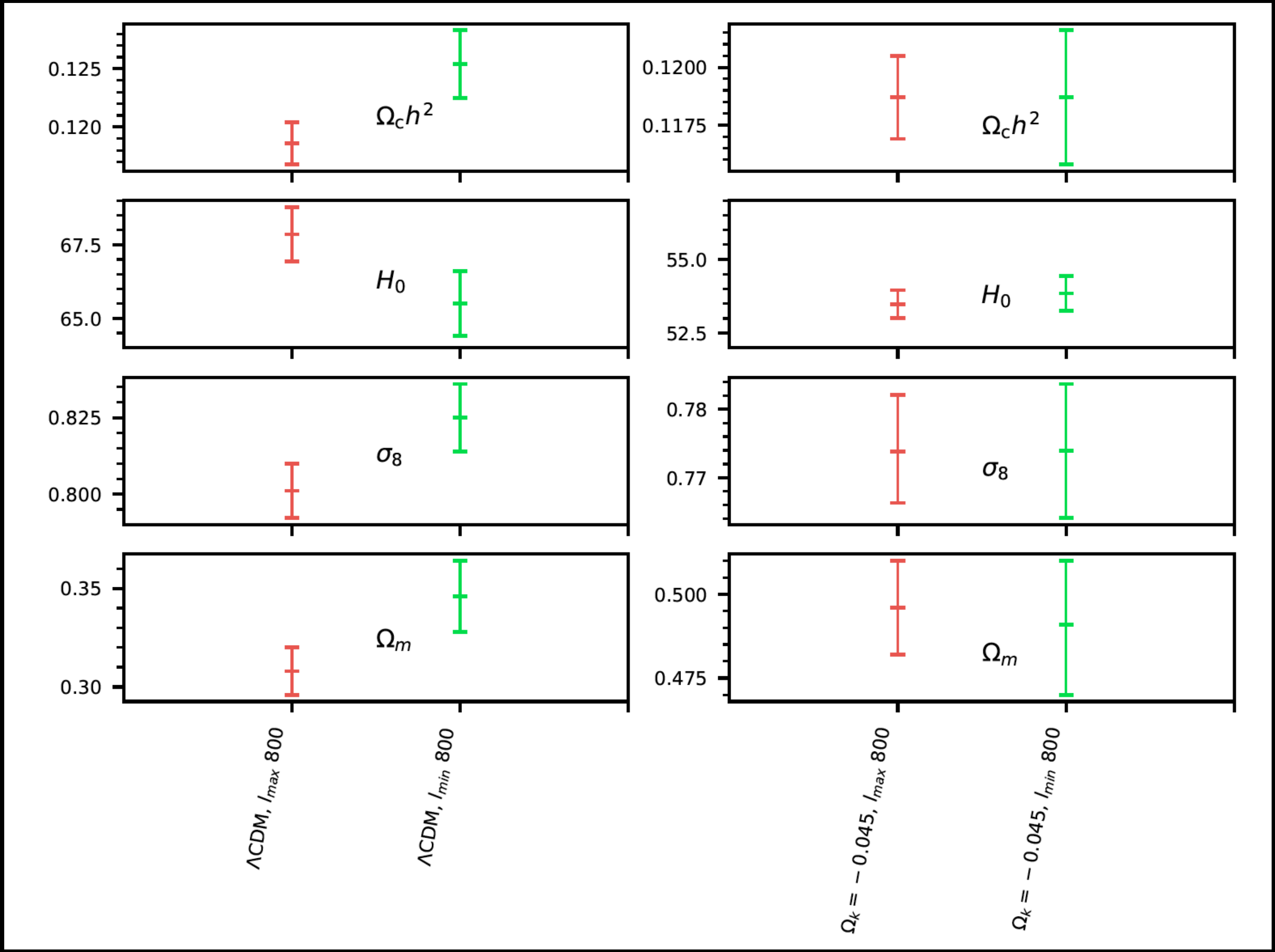
$$\begin{cases} \Omega > 1 & \Omega_k < 0 \\ \Omega = 1 & \Omega_k = 0 \\ \Omega < 1 & \Omega_k > 0 \end{cases}$$

→ k>0 : closed Universe

→ k=0 : flat Universe

→ k<0 : open Universe

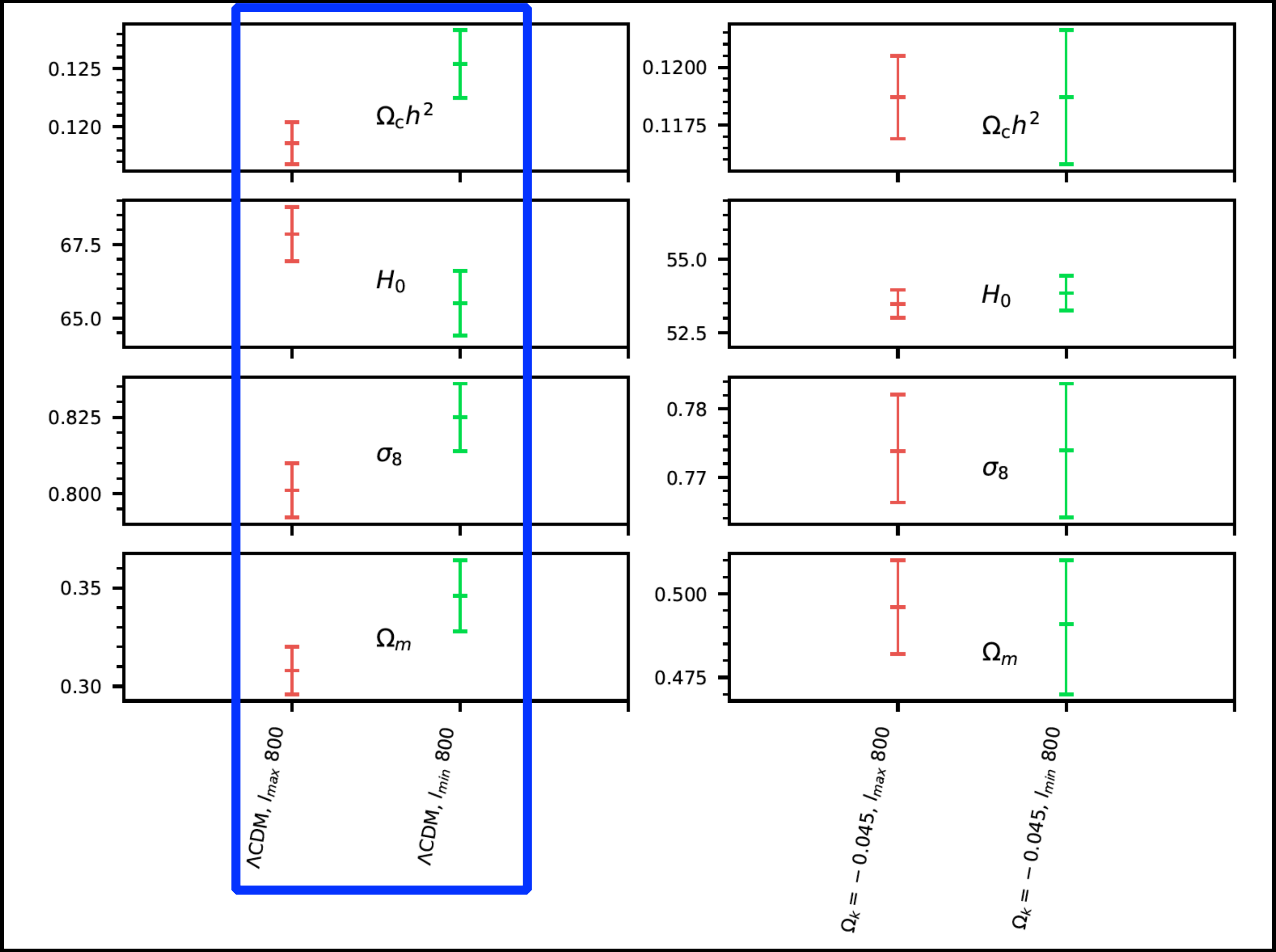
Curvature can explain internal tension



Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

In a closed Universe with ΩK = –0.045, the cosmological parameters derived in the two different multipole ranges are now fully compatible.

Curvature can explain internal tension



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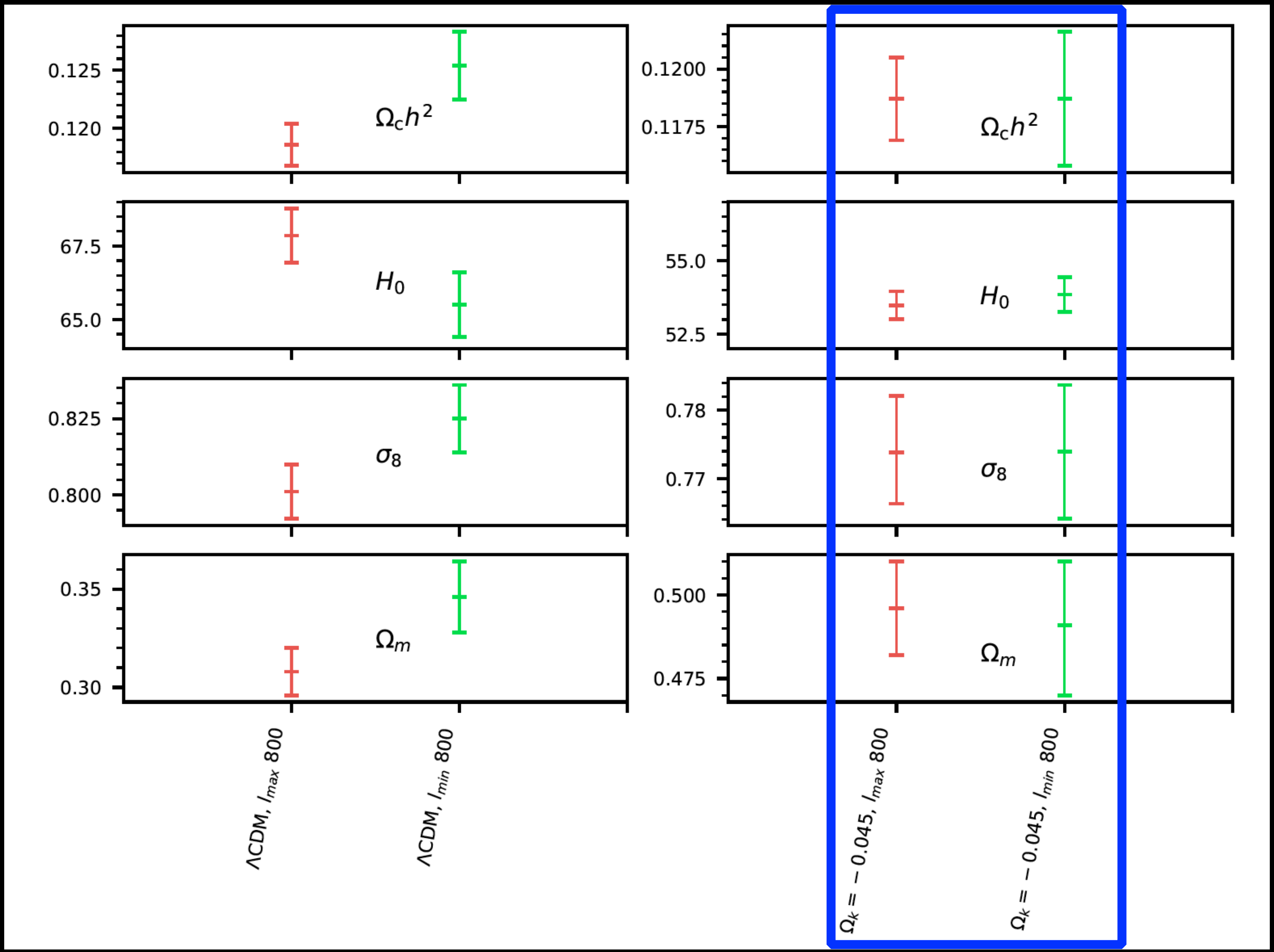
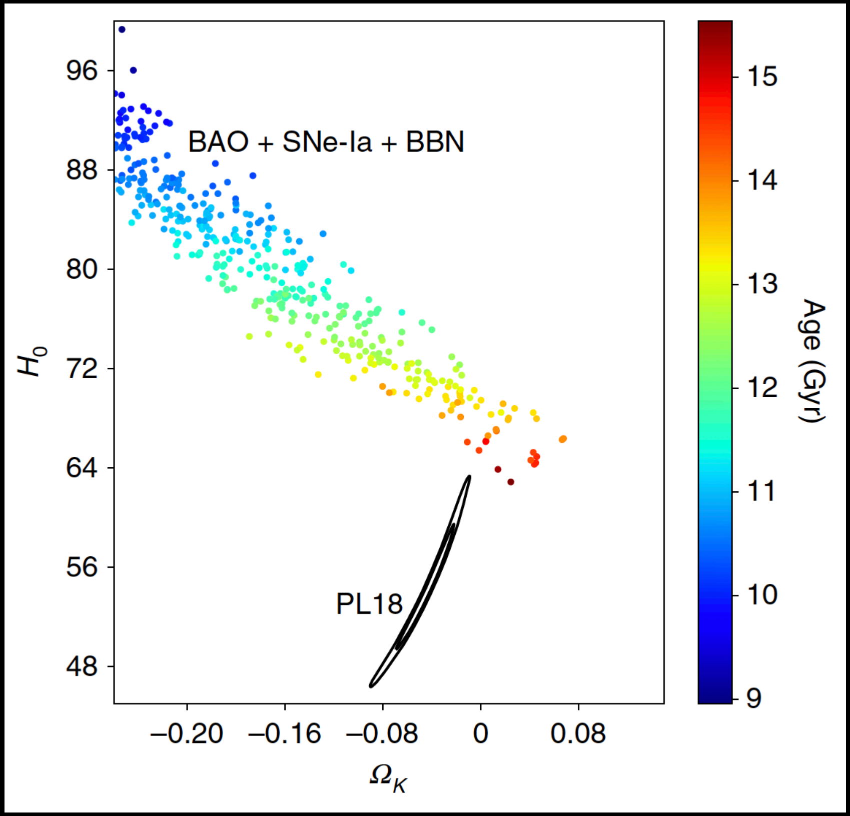


Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

In a closed Universe with ΩK = –0.045, the cosmological parameters derived in the two different multipole ranges are now fully compatible.

What about non-CMB data?



Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

It is now interesting to address the compatibility of Planck with combined datasets, like BAO + type-Ia supernovae + big bang nucleosynthesis data.
In principle, each dataset prefers a closed universe,
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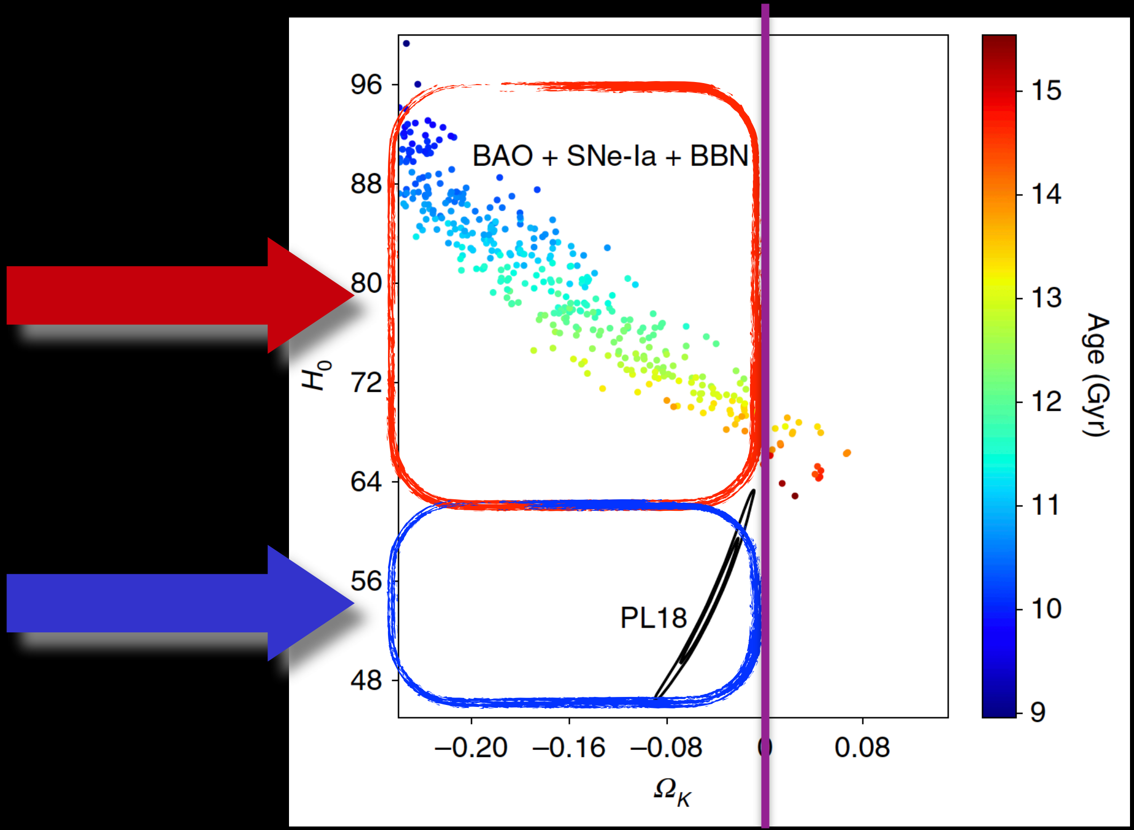

Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

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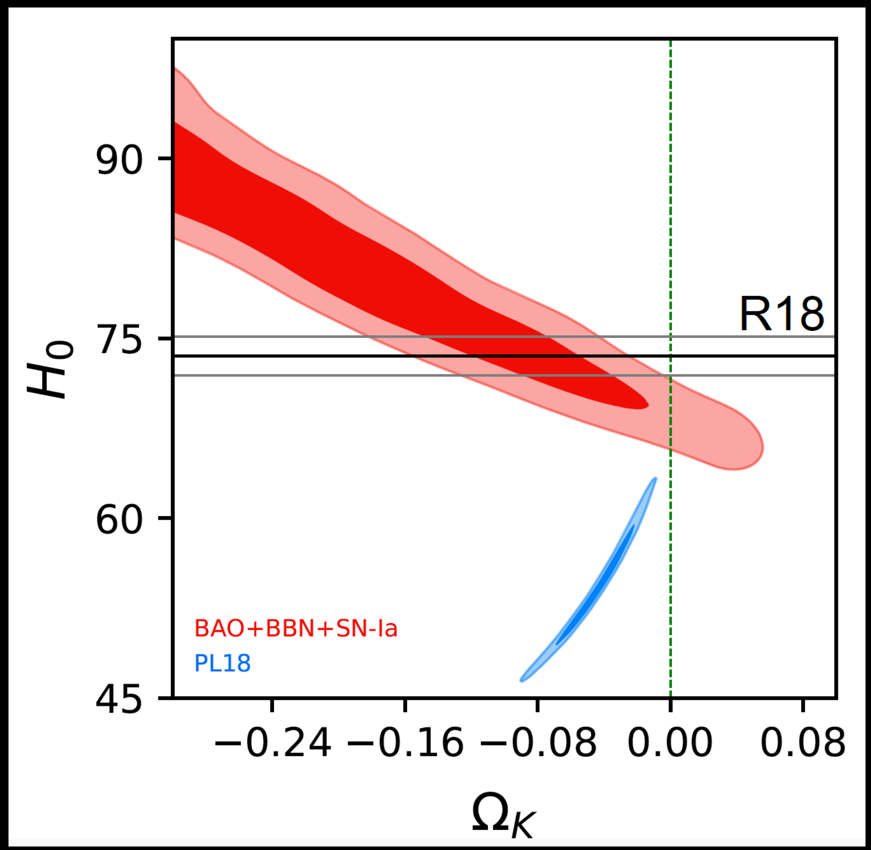
BAO+SNIa+BBN+R18 gives Ωk = −0.091 ± 0.037 at 68%cl.

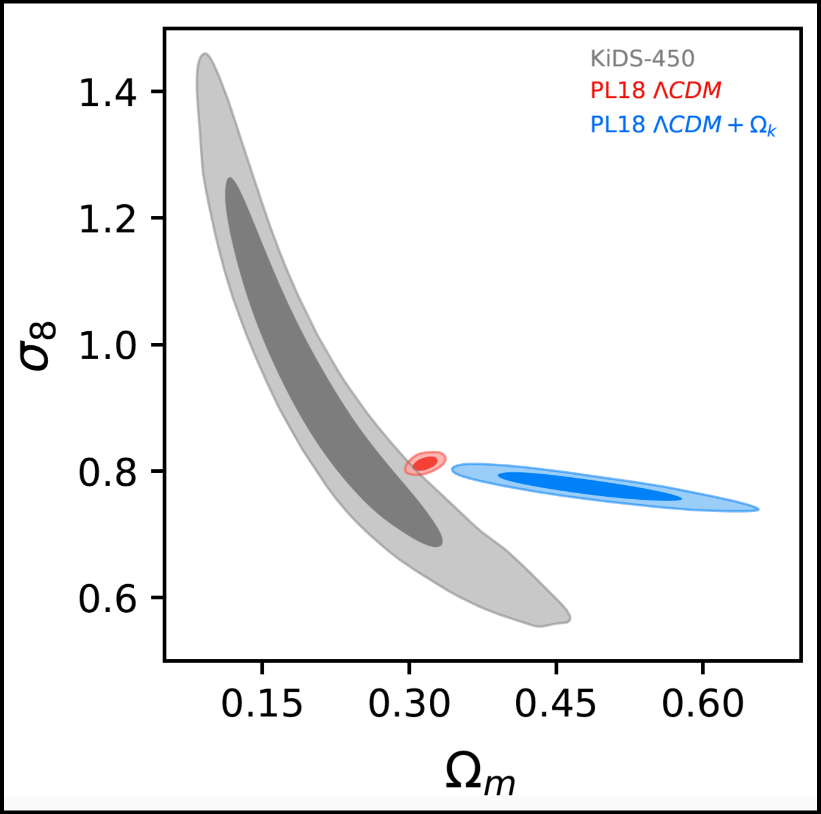
Curvature can't explain external tensions



Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

Varying Ωκ, both the well know tensions on H0 and S8 are exacerbates.
In a ΛCDM + ΩK model, Planck gives $H0 = 54.4^{+3.3}_{-4.0}$ km/s/Mpc at 68% cl., increasing the tension with R20 at 5.4σ.

Curvature can’t explain external tensions



Di Valentino, Melchiorri and Silk, *Nature Astron.* 4 (2019) 2, 196-203

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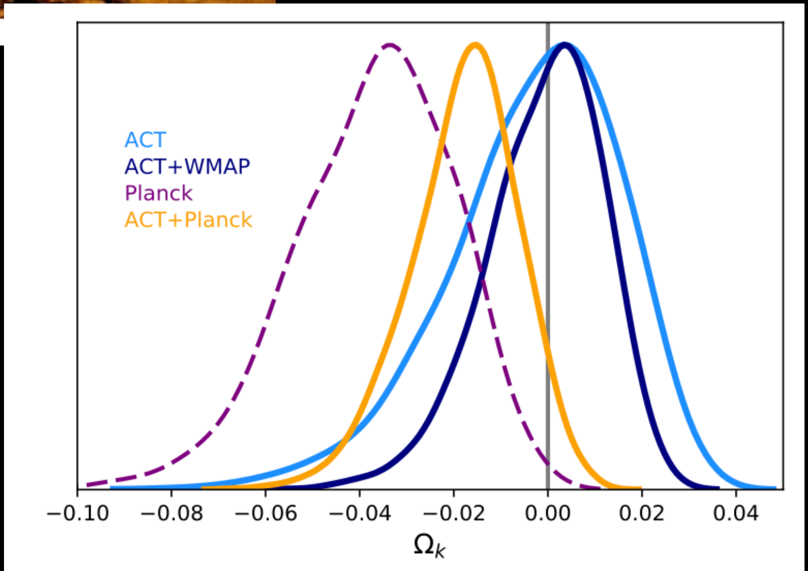
In a ΛCDM + ΩK model, Planck gives S8 in disagreement at about 3.8σ with KiDS-450, and more than 3.5σ with DES.

What about different CMB experiments?

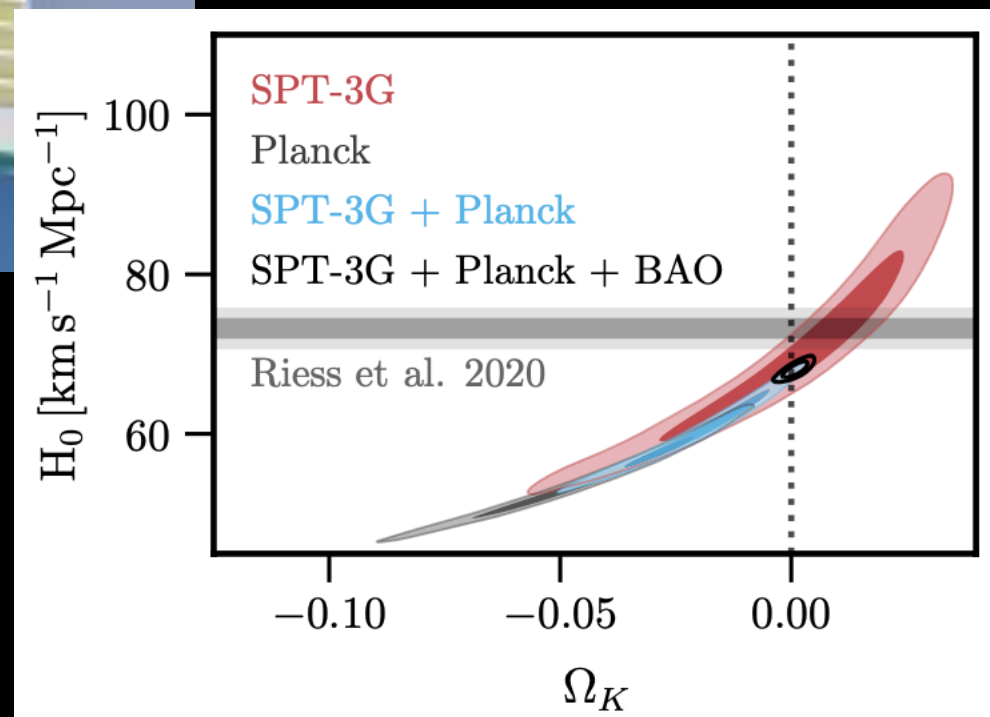


ACT-DR4 + WMAP gives at 68% CL

$$\Omega_k = -0.001 \pm 0.012$$

ACT-DR4 2020, Aiola et al., arXiv:2007.07288 [astro-ph.CO]

What about different CMB experiments?

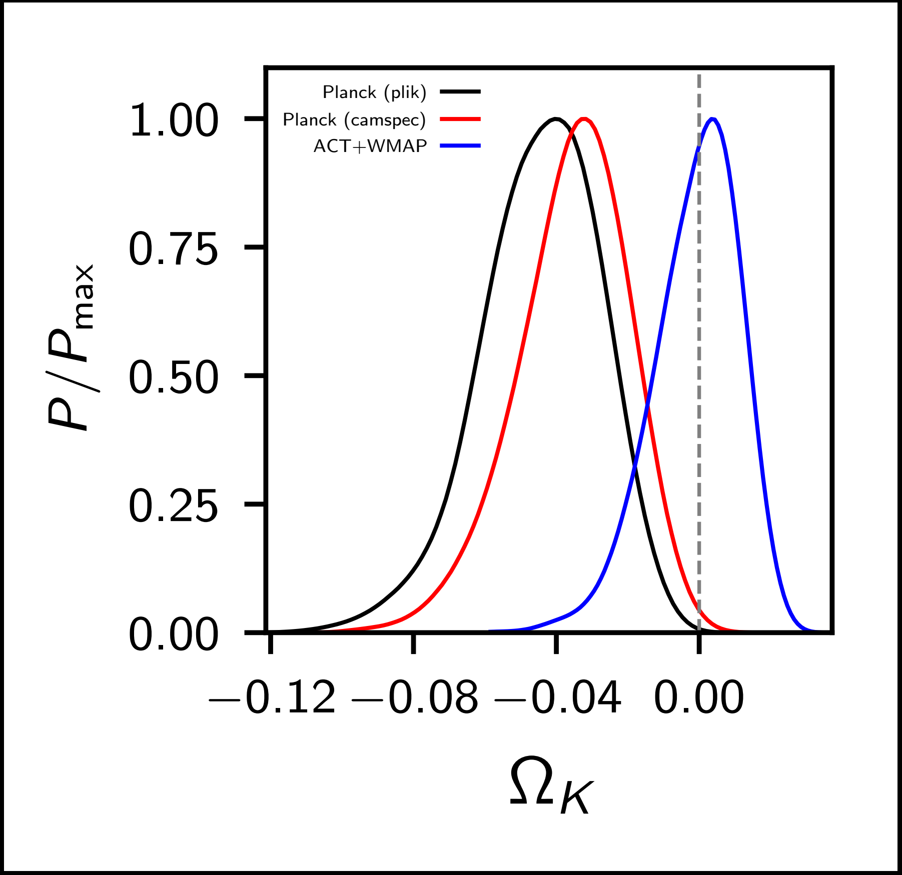


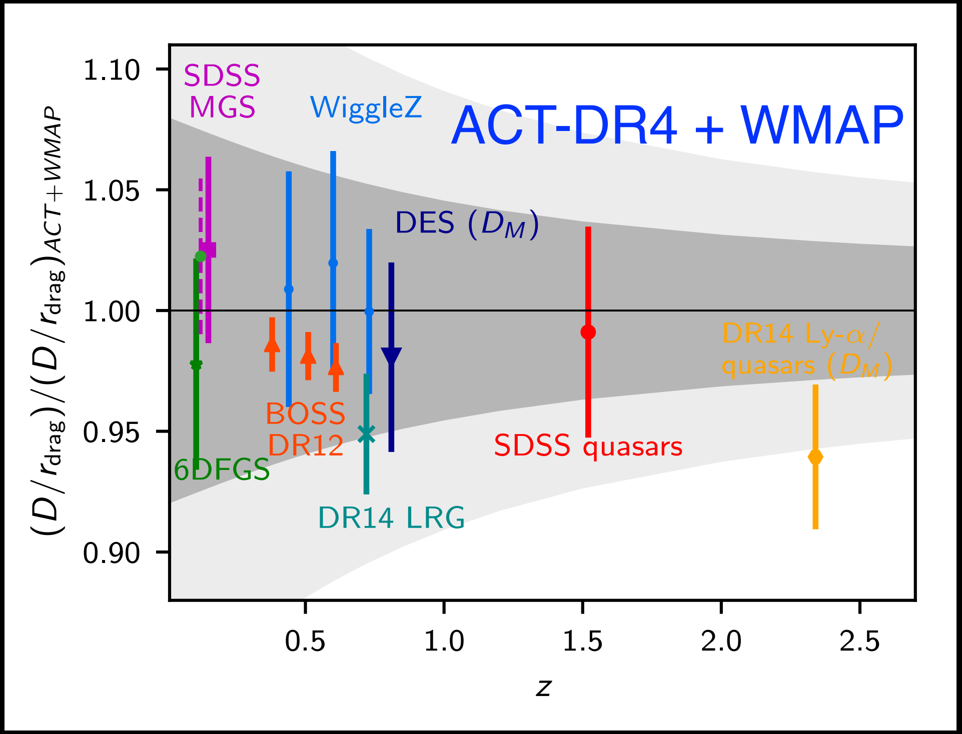
SPT-3G gives at 68% CL:

$$\Omega_K = 0.001^{+0.018}_{-0.019}$$

SPT-3G, arXiv:2103.13618 [astro-ph.CO]

ACT-DR4



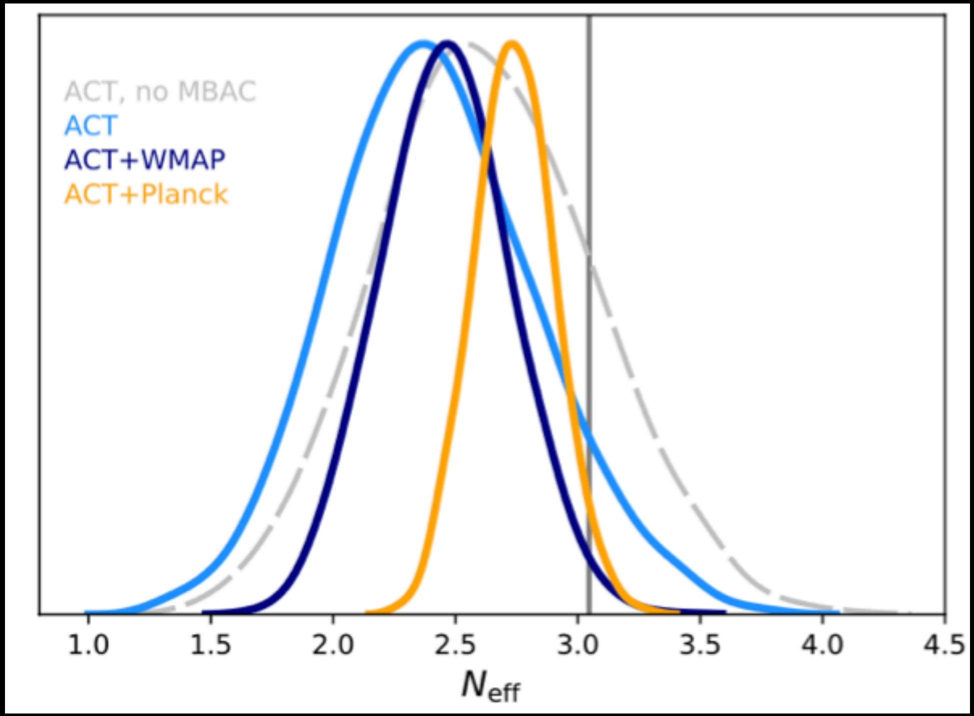


Di Valentino et al. in preparation

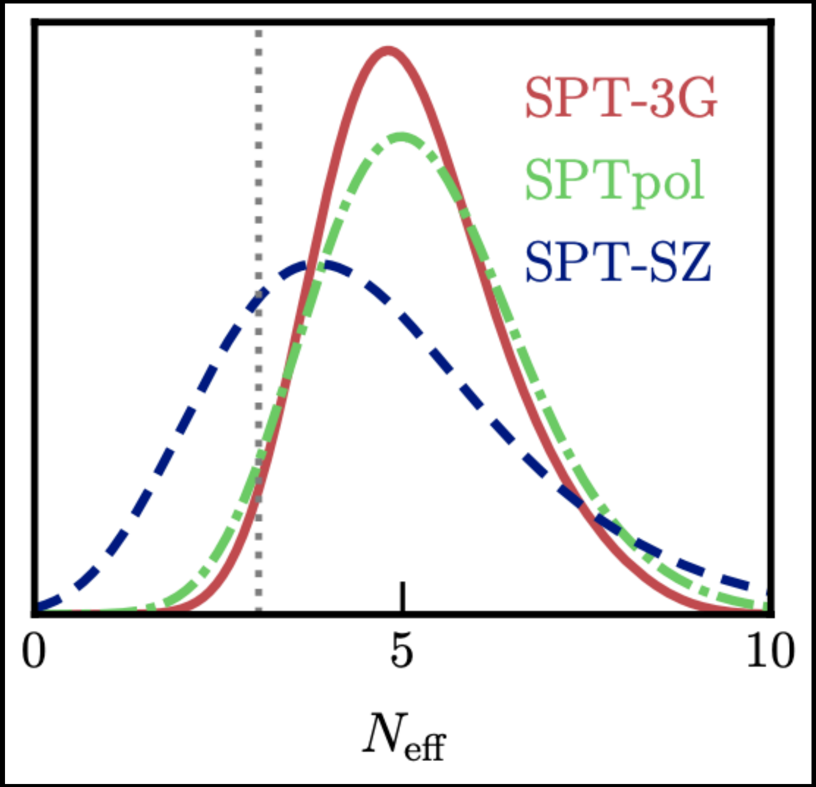
Confirmation that from a CMB experiment you can obtain Omegak!

When precise CMB measurements at arc-minute angular scales are included, since gravitational lensing depends on the matter density, its detection breaks the geometrical degeneracy.

ACT-DR4 vs SPT-3G



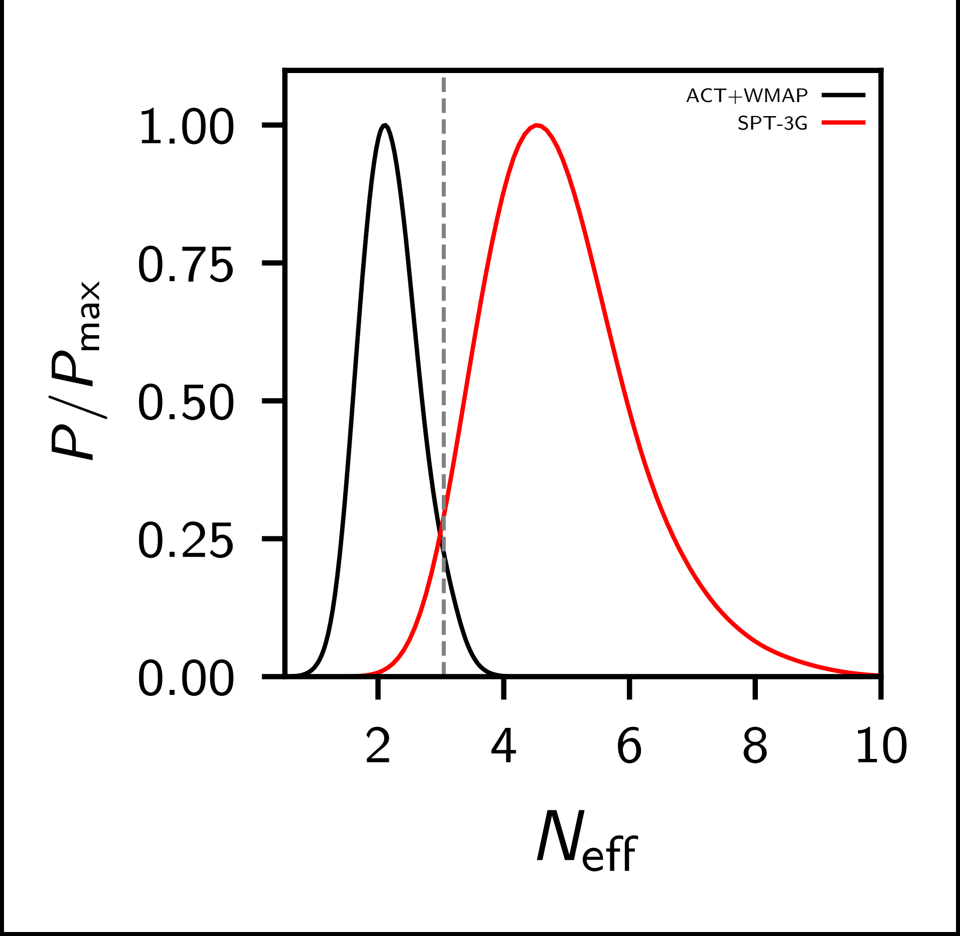
ACT-DR4 2020, Aiola et al., arXiv:2007.07288 [astro-ph.CO]



SPT-3G, arXiv:2103.13618 [astro-ph.CO]

N_{eff} tension?

ACT-DR4 vs SPT-3G



Di Valentino et al. in preparation

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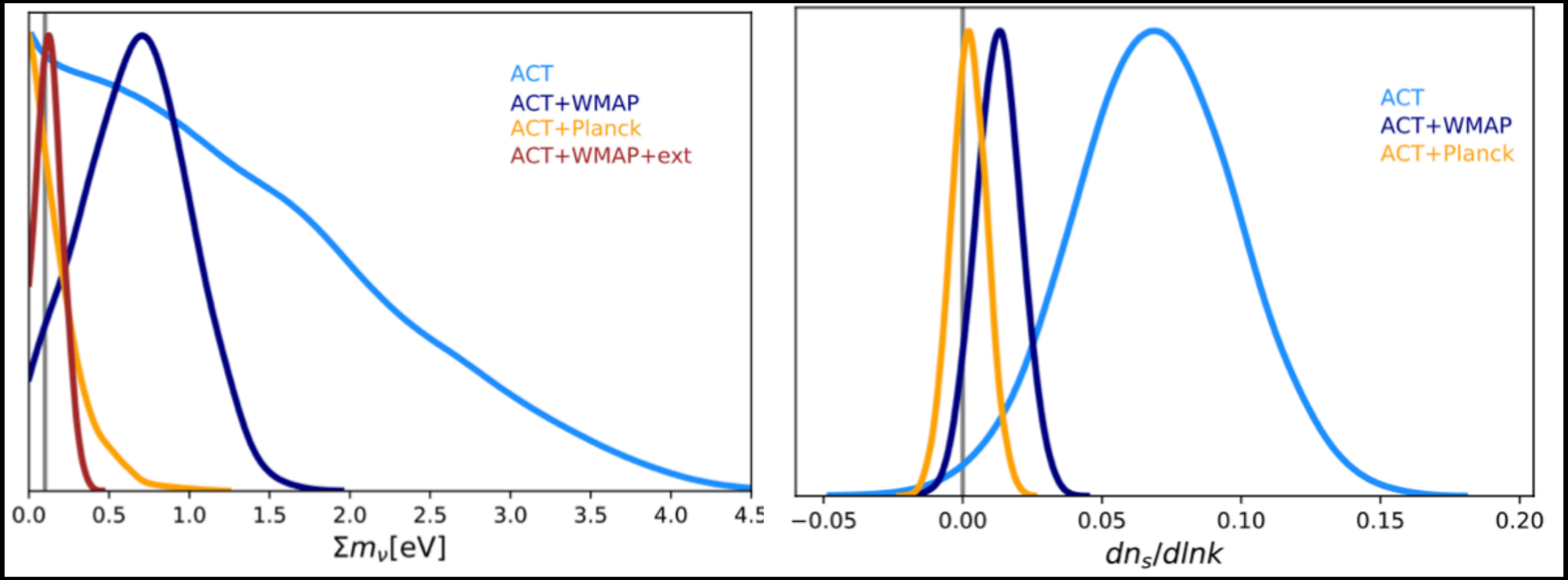
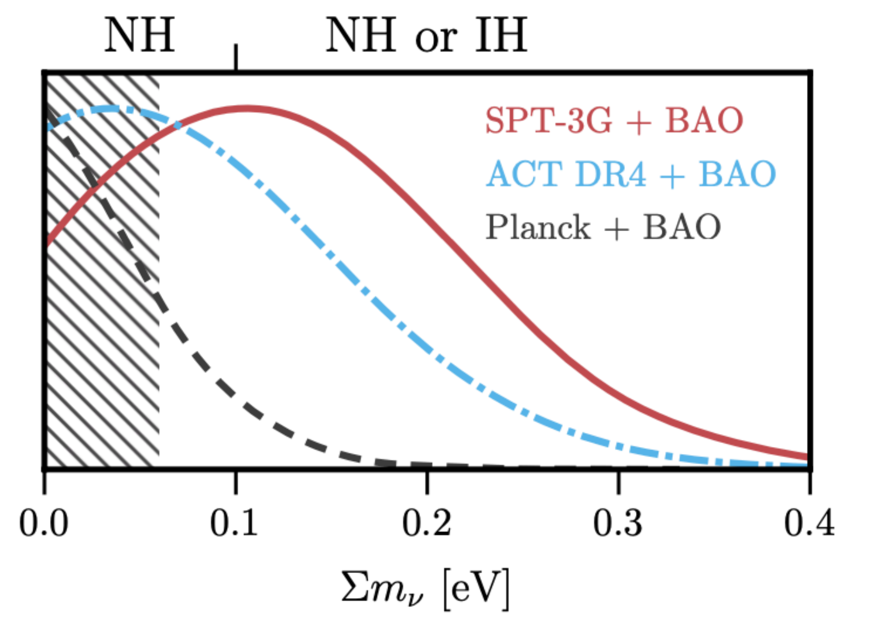


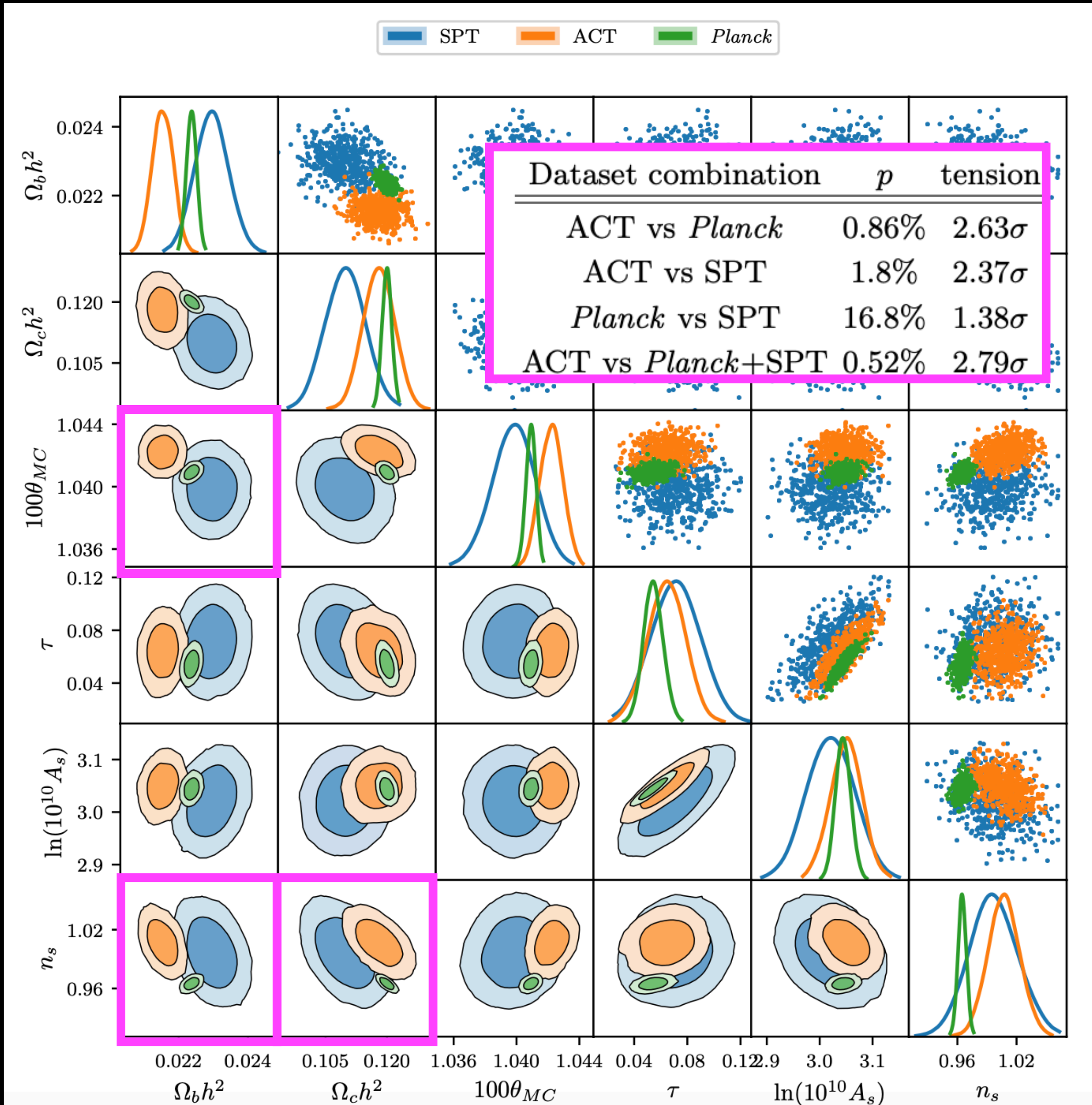
ACT-DR4 2020, Aiola et al., arXiv:2007.07288 [astro-ph.CO]



SPT-3G, arXiv:2103.13618 [astro-ph.CO]

ACT-DR4

Handley and Lemos, arXiv:2007.08496 [astro-ph.CO]



Dataset combination	p	tension
ACT vs *Planck*	0.86%	2.63σ
ACT vs SPT	1.8%	2.37σ
Planck vs SPT	16.8%	1.38σ
ACT vs *Planck*+SPT	0.52%	2.79σ

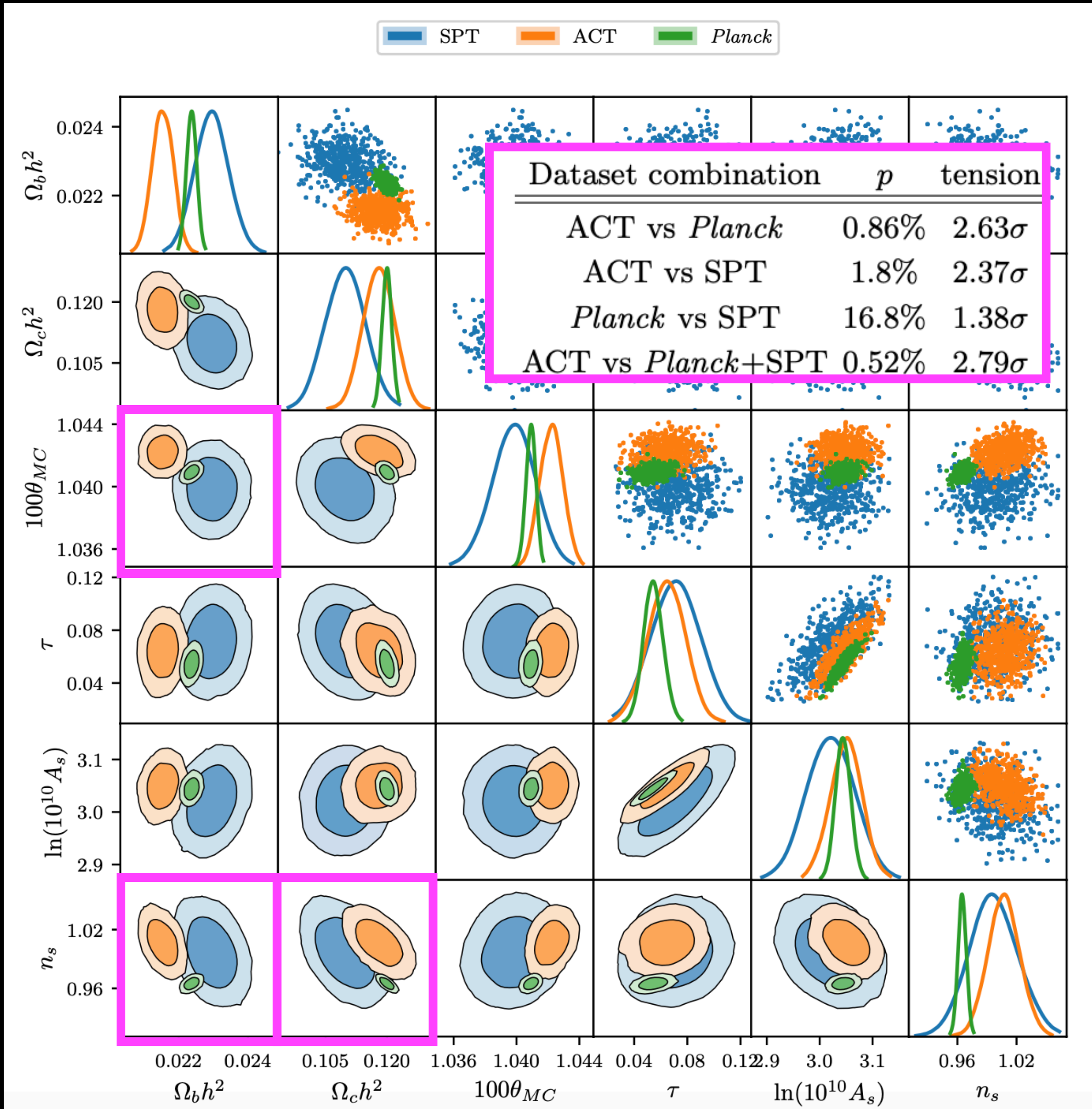
Global tensions between CMB datasets.

For each pairing of datasets this is the tension probability p that such datasets would be this discordant by (Bayesian) chance, as well as a conversion into a Gaussian-equivalent tension.

Between Planck and ACT there is a 2.6σ tension.

ACT-DR4

Handley and Lemos, arXiv:2007.08496 [astro-ph.CO]



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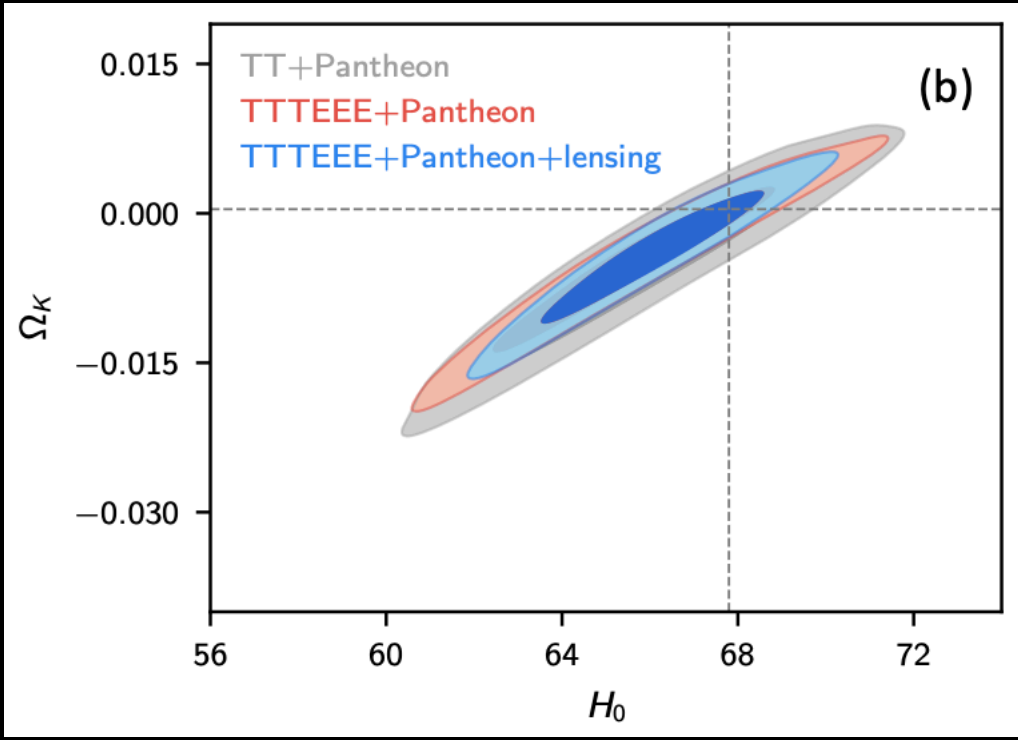
At this point, given the quality of all the analyses, it is more likely that these discrepancies are indicating a problem with the underlying cosmology and our understanding of the Universe, rather than the presence of systematic effects.

And this suspect is corroborated by the many other tensions we saw emerging between the other cosmological probes.

What about Planck + Pantheon?



Efstathiou and Gratton, Mon.Not.Roy.Astron.Soc. 496 (2020) 1, L91-L95

Adding Pantheon data, a joint constraint is very consistent with a flat universe.

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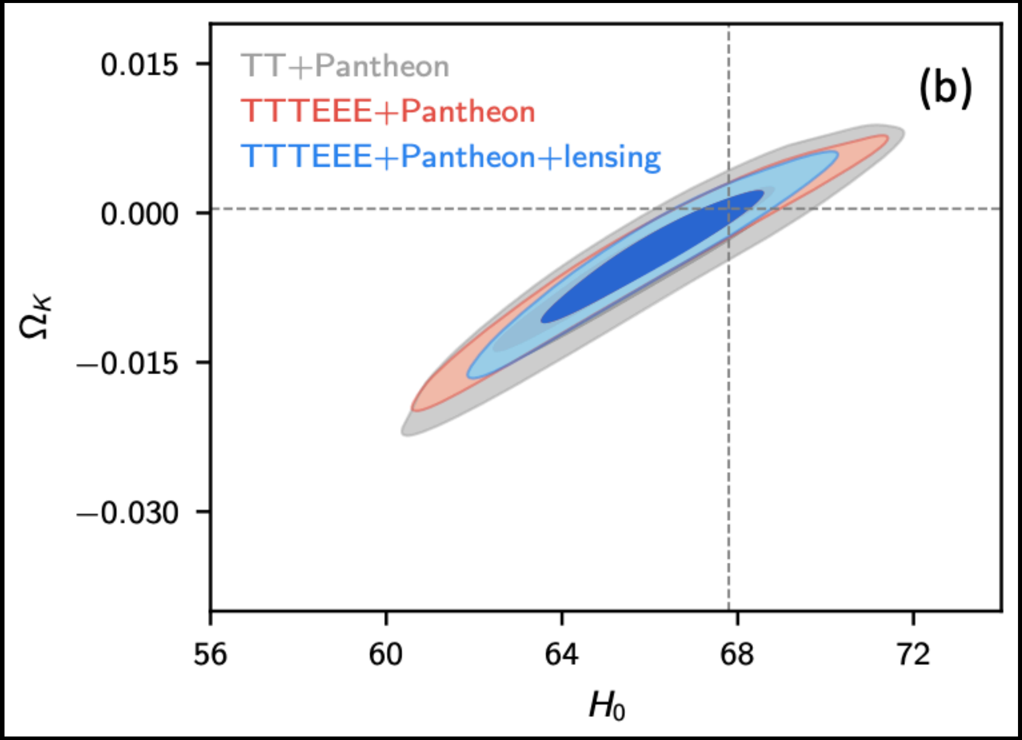


Efstathiou and Gratton, Mon.Not.Roy.Astron.Soc. 496 (2020) 1, L91-L95

Adding Pantheon data, a joint constraint is very consistent with a flat universe.

Again, what happens if we vary all the parameters together?

Constraints at 68% cl.

10 parameters: replacing Alens with curvature

Parameters	Planck	Planck +R19	Planck +F20	Planck +BAO	Planck + Pantheon
$\Omega_b h^2$	0.02253 ± 0.00019	$0.02253^{+0.00020}_{-0.00016}$	$0.02255^{+0.00019}_{-0.00017}$	0.02243 ± 0.00016	0.02255 ± 0.00018
$\Omega_c h^2$	0.1183 ± 0.0016	$0.1187^{+0.0015}_{-0.0018}$	0.1184 ± 0.0015	0.1198 ± 0.0014	0.1186 ± 0.0015
$100\theta_{\rm MC}$	1.04099 ± 0.00035	$1.04103^{+0.00034}_{-0.00031}$	1.04105 ± 0.00034	1.04095 ± 0.00032	1.04107 ± 0.00034
τ	0.0473 ± 0.0083	$0.052^{+0.009}_{-0.011}$	0.0491 ± 0.0079	0.0563 ± 0.0081	0.0506 ± 0.0082
Σm_ν [eV]	$0.43^{+0.16}_{-0.37}$	< 0.513	$0.28^{+0.11}_{-0.23}$	< 0.194	< 0.420
w	$-1.6^{+1.0}_{-0.8}$	$-2.11^{+0.35}_{-0.77}$	-2.14 ± 0.46	$-1.038^{+0.098}_{-0.088}$	$-1.27^{+0.14}_{-0.09}$
Ω_k	$-0.074^{+0.058}_{-0.025}$	$-0.0192^{+0.0036}_{-0.0099}$	$-0.0263^{+0.0060}_{-0.0077}$	$0.0003^{+0.0027}_{-0.0037}$	$-0.029^{+0.011}_{-0.010}$
$\ln(10^{10}A_s)$	3.025 ± 0.018	$3.037^{+0.016}_{-0.026}$	3.030 ± 0.017	3.049 ± 0.017	3.034 ± 0.017
n_s	0.9689 ± 0.0054	$0.9686^{+0.0056}_{-0.0050}$	0.9693 ± 0.0051	0.9648 ± 0.0048	0.9685 ± 0.0051
α_S	-0.0005 ± 0.0067	-0.0012 ± 0.0066	-0.0010 ± 0.0068	-0.0054 ± 0.0068	-0.0023 ± 0.0065
H_0[km/s/Mpc]	53^{+6}_{-16}	73.8 ± 1.4	69.3 ± 2.0	$68.6^{+1.5}_{-1.8}$	60.5 ± 2.5
σ_8	$0.74^{+0.08}_{-0.16}$	0.932 ± 0.040	0.900 ± 0.039	0.821 ± 0.027	$0.812^{+0.031}_{-0.018}$
S_8	$0.989^{+0.095}_{-0.063}$	0.874 ± 0.032	$0.900^{+0.034}_{-0.031}$	0.826 ± 0.016	0.927 ± 0.037
Age[Gyr]	$16.10^{+0.92}_{-0.80}$	$14.90^{+0.72}_{-0.32}$	$15.22^{+0.054}_{-0.038}$	13.77 ± 0.10	14.98 ± 0.39
Ω_m	$0.61^{+0.21}_{-0.34}$	$0.264^{+0.010}_{-0.013}$	$0.300^{+0.017}_{-0.020}$	0.305 ± 0.016	$0.393^{+0.030}_{-0.036}$
$\Delta\chi^2_{bestfit}$	0.0	0.62	0.88	14.77	1037.82

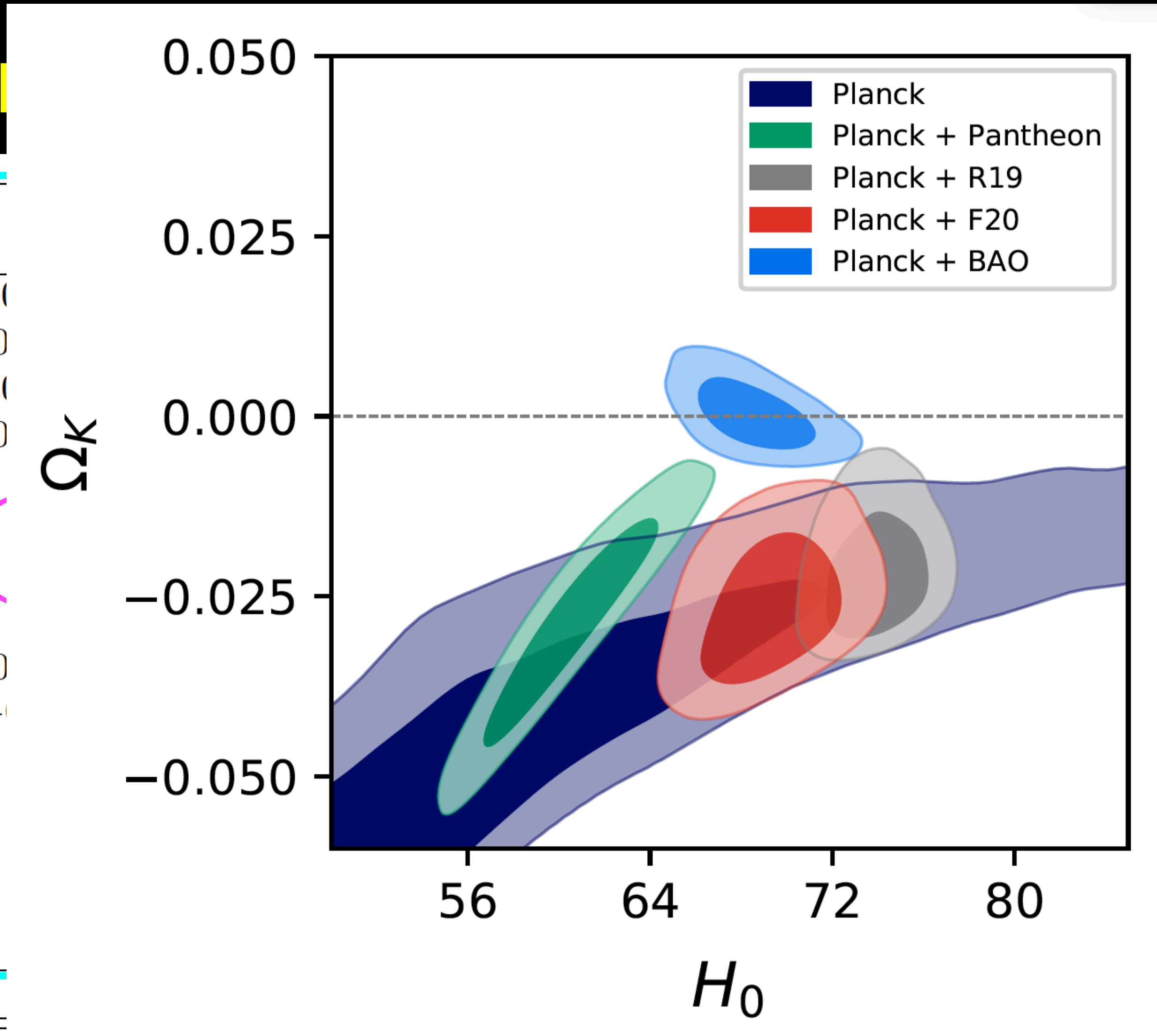
Therefore, now we want to check the robustness of these results further increasing the number of parameters, in addition to curvature.

Di Valentino, Melchiorri and Silk, ApJ Letters, 908, L9 (2021), arXiv:2003.04935

Constraints at 68% cl.

10 para curvature

Parameters			Planck + Pantheon
$\Omega_b h^2$	0.(	)016	0.02255 ± 0.00018
$\Omega_c h^2$	0	)14	0.1186 ± 0.0015
$100\theta_{MC}$	1.(	)032	1.04107 ± 0.00034
τ	0	)81	0.0506 ± 0.0082
Σm_ν [eV]			< 0.420
w		8 8	$-1.27^{+0.14}_{-0.09}$
Ω_k		7 7	$-0.029^{+0.011}_{-0.010}$
$\ln(10^{10} A_s)$		7	3.034 ± 0.017
n_s	0	48	0.9685 ± 0.0051
α_S	−	068	-0.0023 ± 0.0065
H_0[km/s/Mpc]			60.5 ± 2.5
σ_8		7	$0.812^{+0.031}_{-0.018}$
S_8		6	0.927 ± 0.037
Age[Gyr]		)	14.98 ± 0.39
Ω_m		6	$0.393^{+0.030}_{-0.036}$
$\Delta\chi^2_{bestfit}$			1037.82

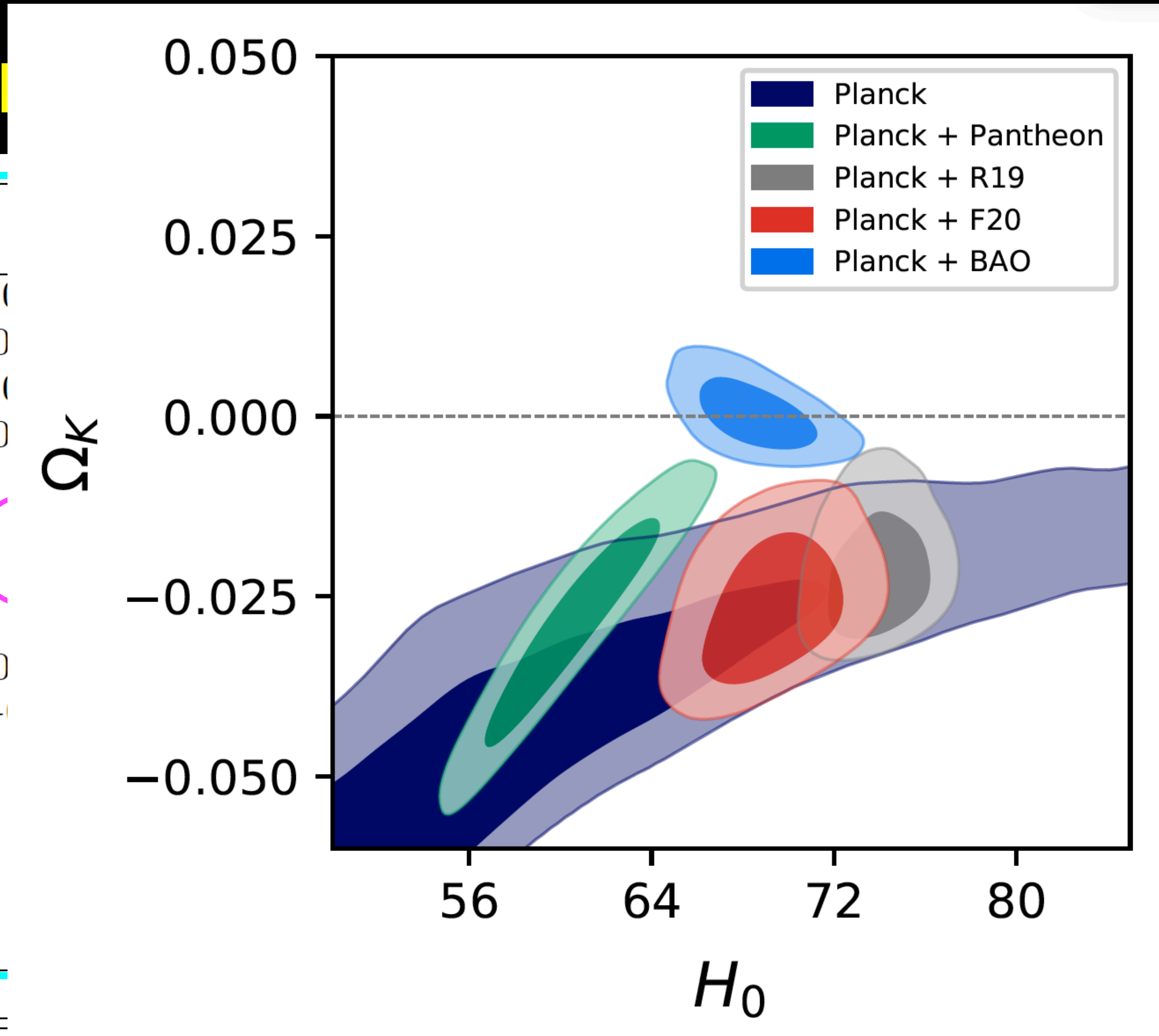


The confidence levels from Planck are clearly below the $\Omega_k = 0$ line that describes a flat universe. On the other hand, the Planck data are now in perfect agreement with the Pantheon, R19, and F20 (Freedman et al. arXiv:2002.01550) measurements, while they are still in strong tension with the BAO measurements, so their combination should be considered with some caution.

Di Valentino, Melchiorri and Silk, ApJ Letters, 908, L9 (2021), arXiv:2003.04935

Constraints at 68% cl.

10 paran... curvature



Parameters	Planck + Pantheon
$\Omega_b h^2$	0.02255 ± 0.00018
$\Omega_c h^2$	0.1186 ± 0.0015
$100\theta_{MC}$	1.04107 ± 0.00034
τ	0.0506 ± 0.0082
Σm_ν [eV]	< 0.420
w	$-1.27^{+0.14}_{-0.09}$
Ω_k	$-0.029^{+0.011}_{-0.010}$
$\ln(10^{10} A_s)$	3.034 ± 0.017
n_s	0.9685 ± 0.0051
α_S	-0.0023 ± 0.0065
H_0[km/s/Mpc]	60.5 ± 2.5
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S_8	0.927 ± 0.037
Age[Gyr]	14.98 ± 0.39
Ω_m	$0.393^{+0.030}_{-0.036}$
$\Delta\chi^2_{bestfit}$	1037.82

Moreover, all the 95% confidence regions from the Planck+Pantheon, Planck+F20, and Planck+R19 datasets are well below the $\Omega_k = 0$ line. This clearly shows that the recent claims of a closed universe as being incompatible with luminosity distance measurements are simply due to the assumption of a cosmological constant.

Di Valentino, Melchiorri and Silk, ApJ Letters, 908, L9 (2021), arXiv:2003.04935

Constraints at 68% cl.

10 parameters: replacing Alens with curvature

Parameters	Planck	Planck +	Planck	Planck	Planck + Pantheon
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$100\theta_{MC}$	1.04099 ± 0.00035	1.0410		32	1.04107 ± 0.00034
τ	0.0473 ± 0.0083	0.05		1	0.0506 ± 0.0082
Σm_ν [eV]	$0.43^{+0.16}_{-0.37}$	$<$			< 0.420
w	$-1.6^{+1.0}_{-0.8}$	$-2.$			$-1.27^{+0.14}_{-0.09}$
Ω_k	$-0.074^{+0.058}_{-0.025}$	-0.01			$-0.029^{+0.011}_{-0.010}$
$\ln(10^{10} A_s)$	3.025 ± 0.018	3.03			3.034 ± 0.017
n_s	0.9689 ± 0.0054	0.968		8	0.9685 ± 0.0051
α_S	-0.0005 ± 0.0067	-0.001		68	-0.0023 ± 0.0065
H_0[km/s/Mpc]	53^{+6}_{-16}	73.			60.5 ± 2.5
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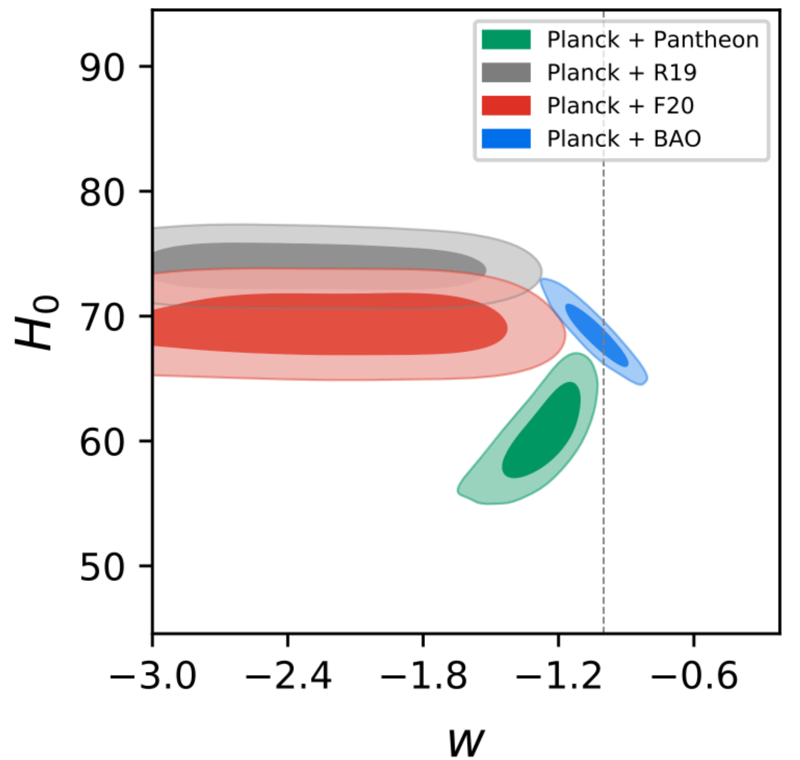
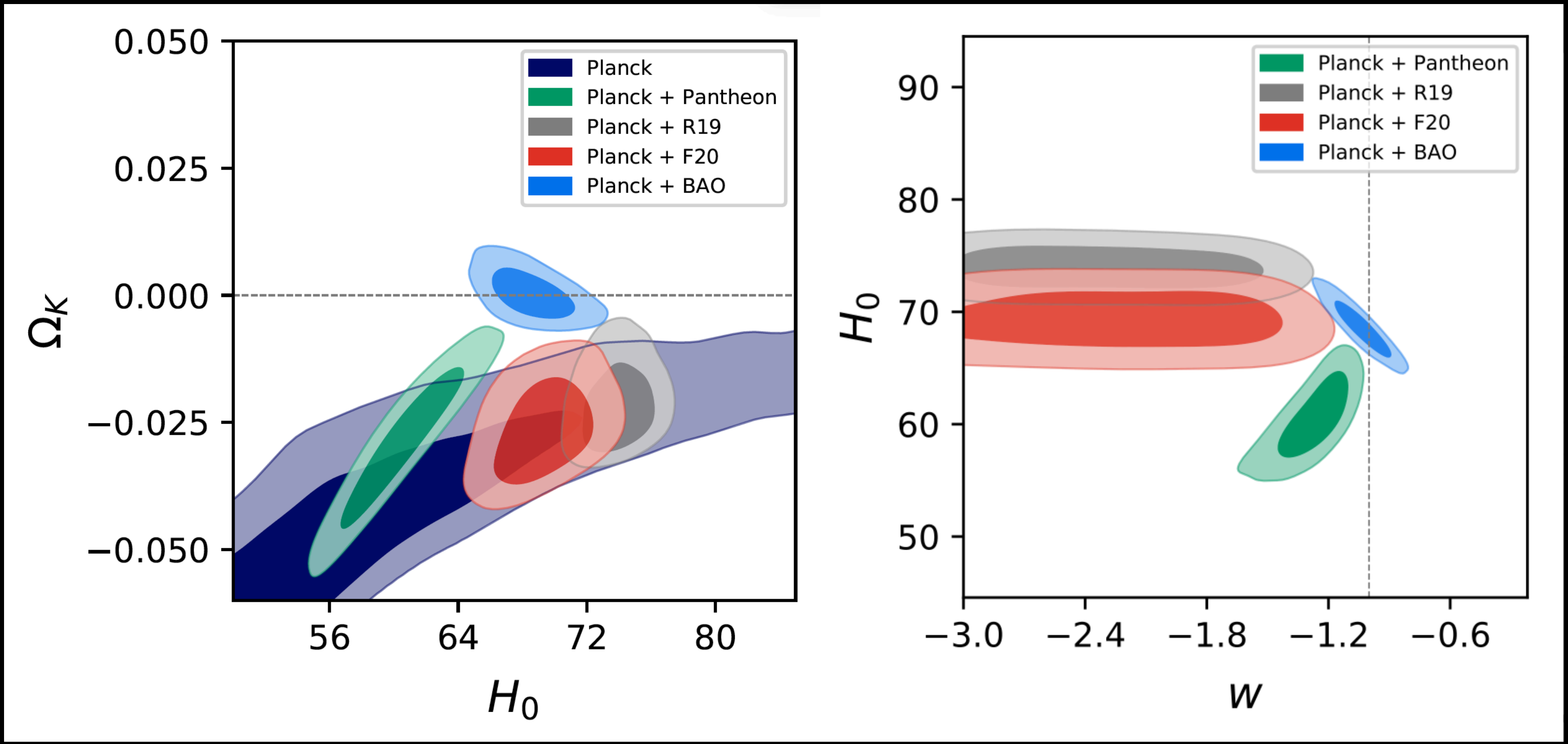
S_8	$0.989^{+0.095}_{-0.063}$	0.874			0.927 ± 0.037
Age[Gyr]	$16.10^{+0.92}_{-0.80}$	14.			14.98 ± 0.39
Ω_m	$0.61^{+0.21}_{-0.34}$	0.26			$0.393^{+0.030}_{-0.036}$
$\Delta\chi^2_{bestfit}$	0.0	0.62	0.88	14.77	1037.82



Indeed, all the three datasets, combined with Planck, exclude a cosmological constant, clearly preferring a value of w < −1.

Di Valentino, Melchiorri and Silk, ApJ Letters, 908, L9 (2021), arXiv:2003.04935

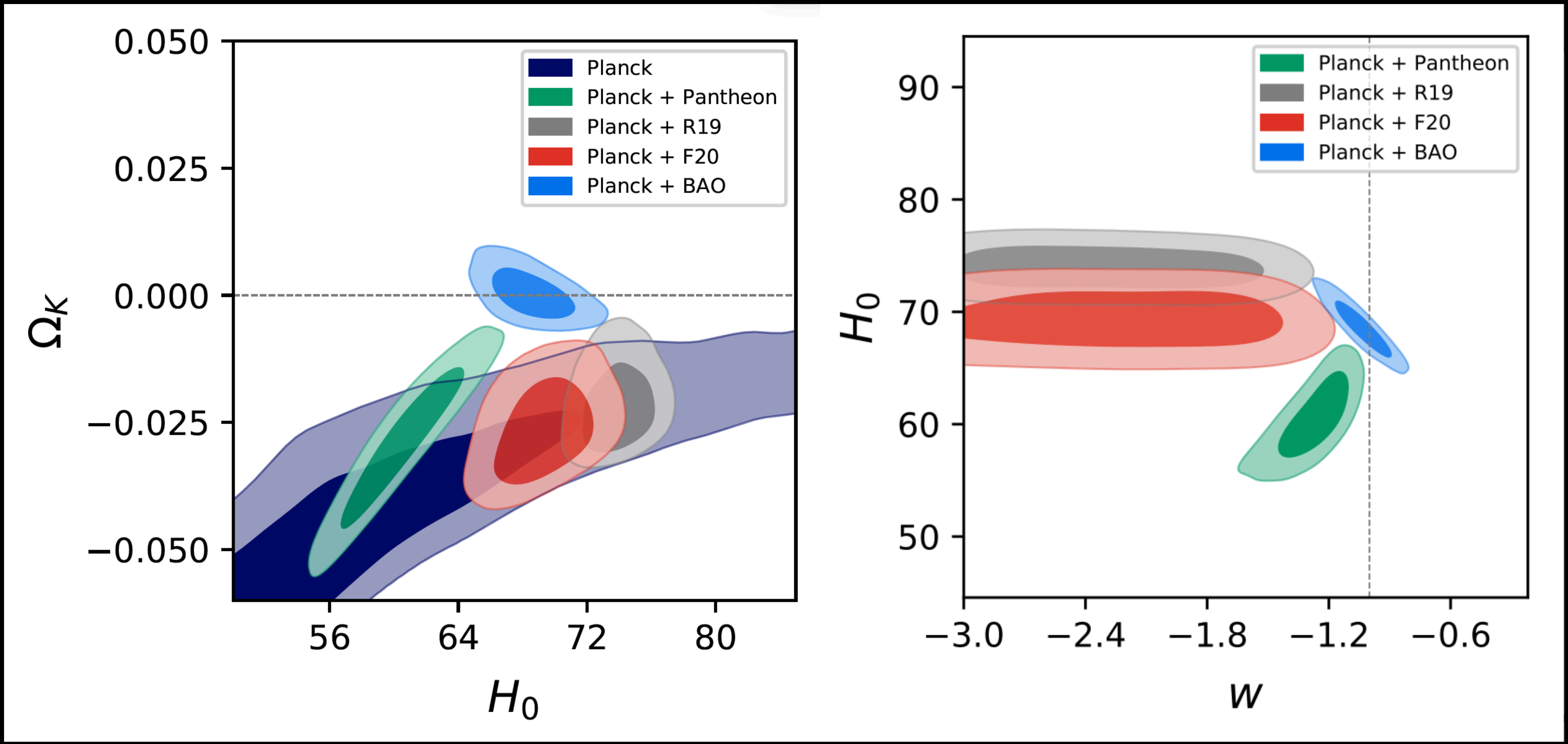
Cosmic Discordance



In practice, Planck+Pantheon, Planck+R19, and Planck+F20
all exclude both
a cosmological constant and a flat universe at more than 99% C.L.

Di Valentino, Melchiorri and Silk, ApJ Letters, 908, L9 (2021), arXiv:2003.04935

Cosmic Discordance



Evidence for a phantom closed Universe at more than 99% CL!!

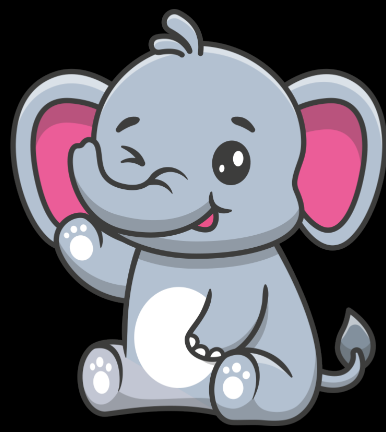
It is interesting to note that if a closed universe increases the fine-tuning of the theory, the removal of a cosmological constant reduces it. It is, therefore, difficult to decide whether a phantom closed model is less or more theoretically convoluted than ΛCDM.

Di Valentino, Melchiorri and Silk, ApJ Letters, 908, L9 (2021), arXiv:2003.04935

Concluding…

Most of the anomalies and tensions are involving the Planck data:

- H0 tension
- S8 tension
- $A_L > 1$ or $\Omega_K < 0$



presenting a serious limitation to the precision cosmology.

Are we sure that
the 2018 Planck results are still a confirmation of
the flat standard ΛCDM cosmological model?

Watch out for the elephant in the room!

These cosmic discordances
call for new observations and stimulate the investigation of
alternative theoretical models and solutions.

Thank you!

e.divalentino@sheffield.ac.uk