Vector dark matter in Kaluza-Klein gravity

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Outline

1. Introduction and motivations
2. Action and bulk profile of 4D metric
3. Stability mechanism of dark matter (DM)
4. DM production
5. Conclusion

Introduction and motivations

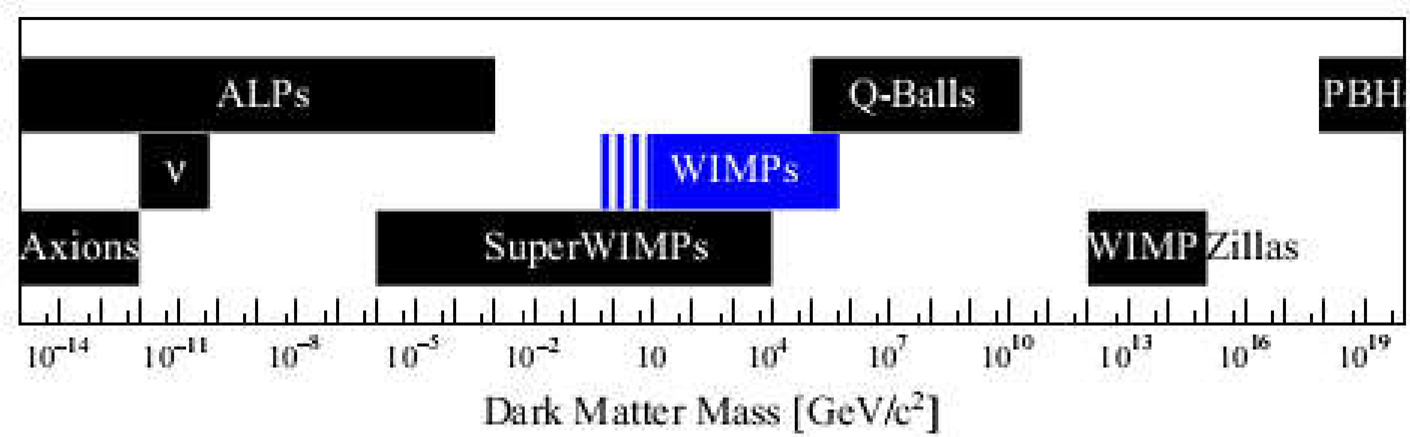
- DM accounts for about 26% of the energy-mass density of universe

$$\Omega_{DM}h^2 \approx 0.12$$ Planck Collaboration, 2016

- DM properties: non-baryonic, electrically neutral, stable (lifetime larger than Universe's age)

- The predicted mass range

M. Schumann, 2019



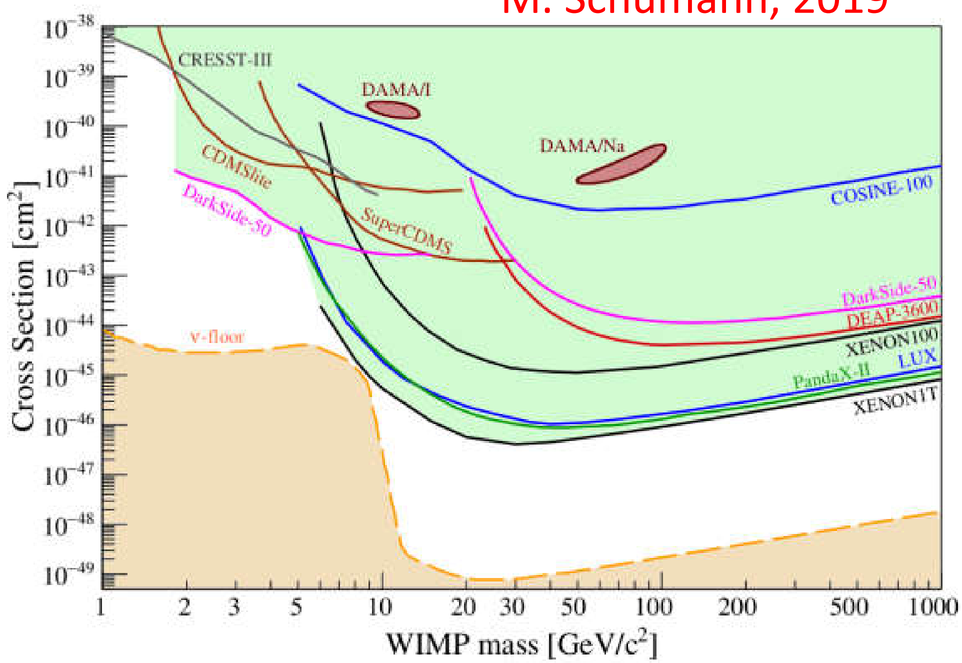
- Production mechanisms: thermal freeze-out, thermal freeze-in, gravitational production, decay of "mother" particles , asymmetric DM,….

➢ The most studied DM candidates are weakly interacting massive particles (WIMPs) which are produced via the thermal freeze-out and found in many particle physics models beyond the standard model:

- Lightest supersymmetric particle (stabilized by R-parity)
- Lightest KK excitation in the theories with extra dimensions (stabilized by KK-parity)
- ....

➢ This paradigm is currently challenged by no WIMP signal observed at the LHC and in the direct detection experiments

M. Schumann, 2019



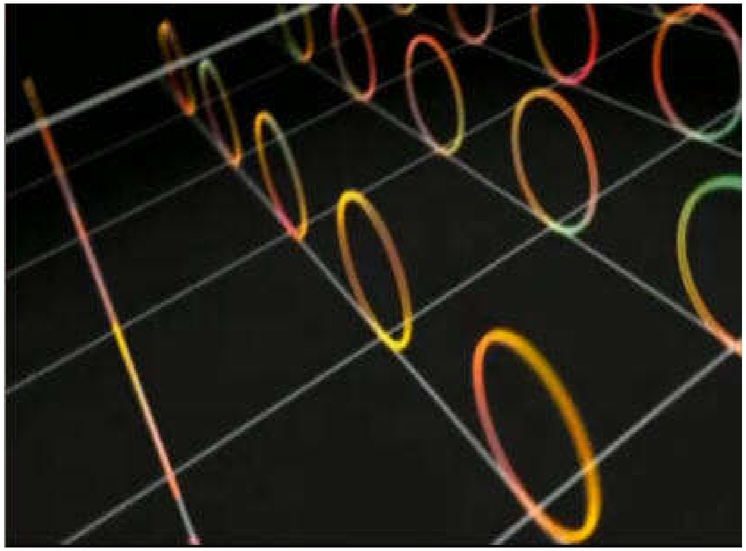
- Other well-motivated DM candidates have been proposed with various production mechanisms.

- Indeed, what we known about the DM only comes from the gravitational interaction which is described by a metric field of spacetime in general relativity (GR).

- Since, it is reasonable to believe that the DM is possibly described by new gravitational degrees of freedom corresponding to a hidden topological/geometrical structure of spacetime.

- Kaluza-Klein (KK) theory (Kaluza, 1921 & Klein, 1926) offers one of the most beautiful and attractive way to unify gauge interactions and gravitation based on the geometry of higher dimensional spacetime or matter and geometry: the $\mu5(5\mu)$ −component of the bulk metric emerges in the 4D effective theory as a mediator of the interaction → the original KK idea failed in describing the fact → finding other possible suggestions.

Action and bulk profile of 4D metric



Source: physociety.wordpress.com

- A most general setting of 5D KK spacetime is a principal bundle with the typical fiber to be $U(1)$ or S^1.

- The local coordinate: (x^μ, θ) with $\theta \in [-\pi, \pi]$ to be the fundamental domain of S^1.

The boundary condition

$$\Phi(x, -\pi) = \Phi(x, \pi)$$

- General coordinate transformation

$$x^\mu \longrightarrow x'^\mu = x'^\mu(x)$$
$$\theta \longrightarrow \theta' = \theta + \alpha(x)$$

$$X_\mu \longrightarrow X'_\mu = \frac{\partial x^\nu}{\partial x'^\mu}\left(X_\nu - \frac{1}{g_X}\partial_\nu \alpha(x)\right)$$

- Bulk metric

$$\left\{\partial_\mu - g_X X_\mu \partial_\theta\right\} \equiv \left\{\hat{\partial}_\mu\right\} \qquad \partial_\theta$$

Dual

Dual

$$ds_5^2 = g_{\mu\nu}(x,\theta)dx^\mu dx^\nu - T^2(x,\theta)\left[d\theta + g_X X_\mu(x)dx^\mu\right]^2$$

In this talk, I will point out this vector field is a natural candidate for superheavy DM.

- Action of Einstein-Hilbert (EB) gravity + cosmological constant + matter fields

$$S = \int d^4x d\theta \sqrt{-G} \left[\frac{M_*^3}{2} \left(\mathcal{R}^{(5)} - 2\Lambda \right) + \mathcal{L}_{\rm SM}\delta(\theta) + \mathcal{L}_{\rm exo} \right]$$

- Expanding the first two terms

$$S \supset \int d^4x d\theta \sqrt{-g} \frac{M_*^3}{2} \; T \left[\hat{\mathcal{R}} - 2\Lambda + \boxed{\frac{1}{4T^2} \left(\partial_\theta g^{\mu\nu} \partial_\theta g_{\mu\nu} + g^{\mu\nu} g^{\rho\lambda} \partial_\theta g_{\mu\nu} \partial_\theta g_{\rho\lambda} \right)} - \frac{g_X^2 T^2}{4} g^{\mu\rho} g^{\nu\lambda} X_{\mu\nu} X_{\rho\lambda} + \frac{6}{T^2} g^{\mu\nu} \left(\hat{\partial}_\mu T \right) \left(\hat{\partial}_\nu T \right) \right],$$

$$\hat{\mathcal{R}} \equiv g^{\mu\nu} (\hat{\partial}_\nu \Gamma^\lambda_{\lambda\mu} - \hat{\partial}_\lambda \Gamma^\lambda_{\nu\mu} + \Gamma^\rho_{\lambda\mu} \Gamma^\lambda_{\nu\rho} - \Gamma^\rho_{\nu\mu} \Gamma^\lambda_{\lambda\rho})$$

$$X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$$

$$\Gamma^\rho_{\mu\nu} \equiv \frac{g^{\rho\lambda}}{2} (\hat{\partial}_\mu g_{\lambda\nu} + \hat{\partial}_\nu g_{\lambda\mu} - \hat{\partial}_\lambda g_{\mu\nu})$$

- In the following, we consider system in the vacuum: $\langle T \rangle \equiv R.$

The stabilizing potential for the size of the extra dimension: magnetic flux compactification (Douglas and Kachru, 2007); coupling to the matter fields (Goldberger and Wise, 1999); quantum corrections (Fukazawa, Inami, and Koyama, 2013); classical potential (CHN, 2019 & 2021).

- We solve the equations of motion of the metric $g_{\mu\nu}(x,\theta)$ in the presence of cosmological constant

$$g_{\mu\nu}(x,\theta) = \chi(\theta)\bar{g}_{\mu\nu}(x)$$

$$\mathcal{R}^{(4)}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\mathcal{R}^{(4)} + \frac{k}{4R^2}\bar{g}_{\mu\nu} = 0,$$

$$6\chi''(\theta) + 4\frac{\chi'(\theta)^2}{\chi(\theta)} + \frac{4\Lambda}{R^{-2}}\chi(\theta) = k,$$

The boundary condition

$$\chi(-\pi) = \chi(\pi)$$

$C_2 = 0$

- Analytical solution with $k \approx 0$ and $\Lambda < 0$:

It is essential to find the bulk profile of the fields and thus the stability of DM

$$\chi(\theta) = C_1 \cosh^{3/5}\left[\frac{\sqrt{10}}{3}\kappa(\theta + C_2)\right] \qquad \kappa \equiv \sqrt{\frac{-\Lambda}{R^{-2}}}$$

C_1 can be set to be unity by absorbing it into the 4D effective metric

- Solutions for $\Lambda \geq 0$ lead to the negative mass squared for X_μ in the 4D effective theory and thus there cases are in general excluded.

- The 4D effective EH action

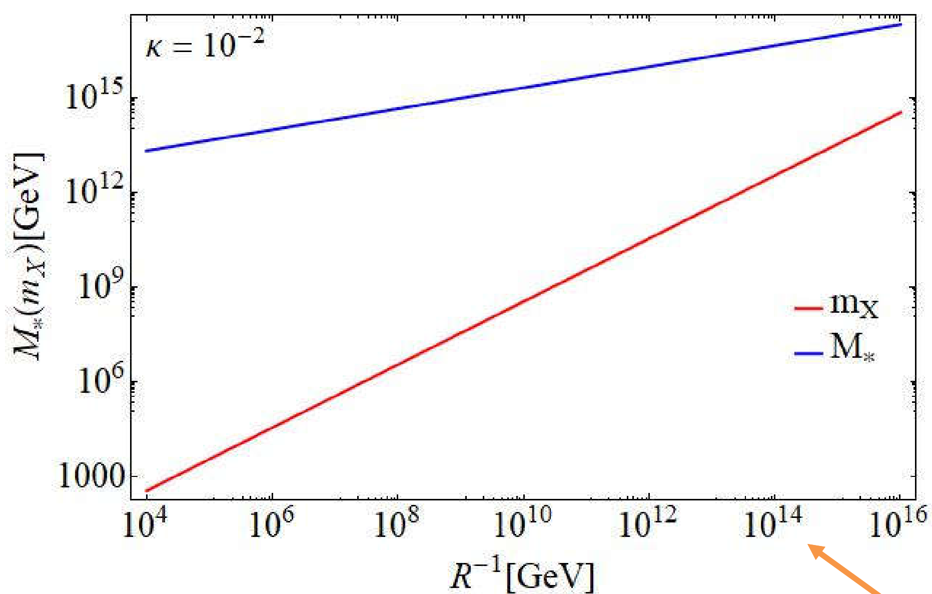
$$S_{EH}^{\rm eff} = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}\mathcal{R}^{(4)} - \frac{1}{4}X^{\mu\nu}X_{\mu\nu} + \frac{m_X^2}{2}X^{\mu}X_{\mu}\right]$$

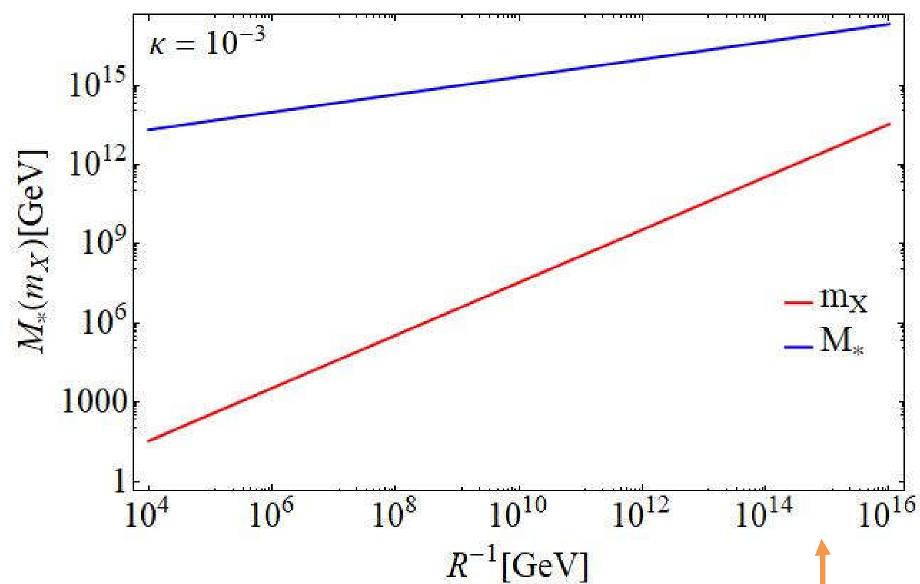
$$M_{\rm Pl}^2 = \frac{3}{4\kappa}\sqrt{\frac{5}{2}}\cosh^{\frac{8}{5}}\left(\frac{\sqrt{10}}{3}\kappa\pi\right)\Re\left[i\,{}_2F_1\left(\frac{1}{2},\frac{4}{5};\frac{9}{5};\cosh^2\left(\frac{\sqrt{10}}{3}\kappa\pi\right)\right)\right]\frac{M_*^3}{R^{-1}}$$

$$m_X^2 = \frac{27\bar{\kappa}}{80\pi}\left\{\frac{56\sinh(\bar{\kappa}\pi)}{\cosh^{\frac{2}{5}}(\bar{\kappa}\pi)} - 15\cosh^{\frac{8}{5}}(\bar{\kappa}\pi)\Re\left[i\,{}_2F_1\left(\frac{1}{2},\frac{4}{5};\frac{9}{5};\cosh^2(\bar{\kappa}\pi)\right)\right]\right\}\frac{1}{R^2}$$

real part

$$\bar{\kappa} \equiv \frac{\sqrt{10}}{3}\kappa$$





No large hierarchy among M_*, R^{-1}, Λ

Stability mechanism of DM

- The couplings of X_μ to the fields are found from the invariant terms in the action related to the covariant derivative $\partial_\mu - g_X X_\mu \partial_\theta$.

- A general expression for the coupling of X_μ to any field $\Psi(x,\theta)$ with the KK expansion given as

$$\Psi(x,\theta) = \sum_n \psi_n(x) Y_n(\theta)$$

$$Y(-\pi) = Y(\pi)$$

From Christoffel and spin connections

From direct action of $\partial_\mu - g_X X_\mu \partial_\theta$ on the field

As seen later on

$$\mathcal{L}_{X\psi_n} = \sum_{\{k\}} \int_{-\pi}^{\pi} d\theta \mathcal{F}(\chi, Y_n) \left(\partial_\theta \chi\right)^m \left(\partial_\theta Y_{\{k\}}\right)^n \times \left[X_{\mu_1} X_{\mu_2} \cdots X_{\mu_{m+n}}\right] J^{\mu_1 \mu_2 \cdots \mu_{m+n}}(x).$$

- χ and Y_n are even functions on the fundamental domain → $\partial_\theta \chi$ and $\partial_\theta Y_n$ all are odd on this domain → the integral would vanish for the odd number of X_μ → the stability mechanism preventing X_μ from its decay.

- Complex scalar fields

Three-point vertex $\sim \left(\int_{-\pi}^{\pi} d\theta \chi Y_n \partial_\theta Y_m\right)\left(\phi_m^\dagger \partial_\mu \phi_n + \phi_m \partial_\mu \phi_n^\dagger\right) X^\mu$

vanishing in our model

Non-zero in original KK theory except $n = 0$, because $Y_n = e^{in\theta}$ → it is proportional to the KK mode number n

Four-point vertex $\sim \left(\int_{-\pi}^{\pi} d\theta \chi \partial_\theta Y_n \partial_\theta Y_m\right) \phi_m^\dagger \phi_n X_\mu X^\mu$

Nonzero in general

- Fermions

From direct action of $\partial_\mu - g_X X_\mu \partial_\theta$ on the field

From spin connection

Three-point vertex $\sim \int_{-\pi}^{\pi} d\theta \left[\mathcal{F}_1(\chi, Y_m)\partial_\theta Y_n + \mathcal{F}_2(\chi, Y_n, Y_m)\partial_\theta \chi\right] \bar{\psi}_m \gamma^\mu \psi_n X_\mu$

vanishing in our model

- The profile of the bulk fields along the fifth dimension

Bulk scalar fields
$$Y_n'' + \frac{2\chi'}{\chi} Y_n' + R^2 \left(\frac{m_n^2}{\chi} - m_\Phi^2 \right) Y_n = 0.$$

Bulk fermions
$$\begin{aligned} Y_{Ln}' + \left(\frac{m_\Psi}{R^{-1}} + \frac{\chi'}{\chi} \right) Y_{Ln}' - \frac{m_n}{R^{-1}\chi} Y_{Rn} &= 0, \\ -Y_{Rn}' + \left(\frac{m_\Psi}{R^{-1}} - \frac{\chi'}{\chi} \right) Y_{Rn}' - \frac{m_n}{R^{-1}\chi} Y_{Ln} &= 0, \end{aligned}$$

Bulk gauge bosons
$$Y_n'' + \frac{\chi'}{\chi} Y_n' + R^2 \frac{m_n^2}{\chi} Y_n = 0,$$

- In general, it is not easily to obtain the analytical solutions of these equations. However, for the small κ, we can find the analytical solutions for these equations by expanding χ in κ. The solutions are given up to the second order of κ as

$$Y_n(\theta) = e^{-a_n\theta^2} \left[N_n H_{2n} \left(\sqrt{\frac{b_n}{3}}\theta \right) + {}_1F_1 \left(-n; \frac{1}{2}; \frac{b_n}{3}\theta^2 \right) \right]$$

$n = 0,\ 1,\ 2,\ \cdots$

N_n is the normalization constant

H_{2n} and ${}_1F_1$ are Hermite polynomial and confluent hypergeometric function, respectively

DM production

- X_μ DM candidate would be superheavy in nature → it could not be produced by the thermal freeze-out because their relic abundance would dramatically be overproduction due to the unitarity limits on the cross-section (Griest and Kamionkowski, 1990).

- On the other hand, the coupling of X_μ DM particles to the particles in the plasma bath must be very weak → they must not have been in the thermal equilibrium → the X_μ DM particles presently observed would be the non-thermal relics.

- Several non-thermal production mechanisms of superheavy DM:

  - Gravitational DM production: due to the non-adiabatic expansion of the background spacetime acting on the vacuum quantum fluctuations at the end of inflation → in general depending on the inflation model.

  - DM production after the inflationary stage:

Reheating efficiency $\gamma \equiv \sqrt{\frac{\Gamma_\phi}{H_i}}$

- $\gamma = 1$: reheating is instantaneous → DM is produced via freeze-in.
- $\gamma < 1$: reheating is mon-instantaneous → DM production during the reheating.

- The Boltzmann equations for the reheating dynamics are generically given by

$$\frac{d\rho_\phi}{dt} = -3H\rho_\phi - \Gamma_\phi\rho_\phi,$$
$$\frac{d\rho_R}{dt} = -4H\rho_R + (1-B_X)\Gamma_\phi\rho_\phi + 2\langle\sigma v\rangle\langle E_X\rangle\left[n_X^2 - (n_X^{\rm eq})^2\right]$$
$$\frac{dn_X}{dt} = -3Hn_X + B_X\Gamma_\phi\rho_\phi - \langle\sigma v\rangle\left[n_X^2 - (n_X^{\rm eq})^2\right],$$

Chung, *et. al.*,1998 & Giudice, *et. al.*, 2001

$$n_X^{\rm eq} = \frac{g}{2\pi^2}m_X^2 T K_2\left(\frac{m_X}{T}\right)$$

- With $B_X \approx 0$ and $n_X \ll n_X^{eq}$

$$a_f \to \infty$$

$$X_f = \frac{1}{T_{\rm rh}^3}\int_{a_i}^{a_f} da\frac{a^2}{H(a)}\langle\sigma v\rangle\left(n_X^{\rm eq}\right)^2$$

$$a_i = 1$$

$$T(a) \simeq \frac{0.20\sqrt{\gamma M_{\rm Pl}H_i}}{(1+3w/5)^{1/4}}\left[a^{-3(1-w)/2} - a^{-4}\right]^{1/4}$$
$$H(a) = H_i\left(\frac{a}{a_i}\right)^{-3(1+w)/2},$$

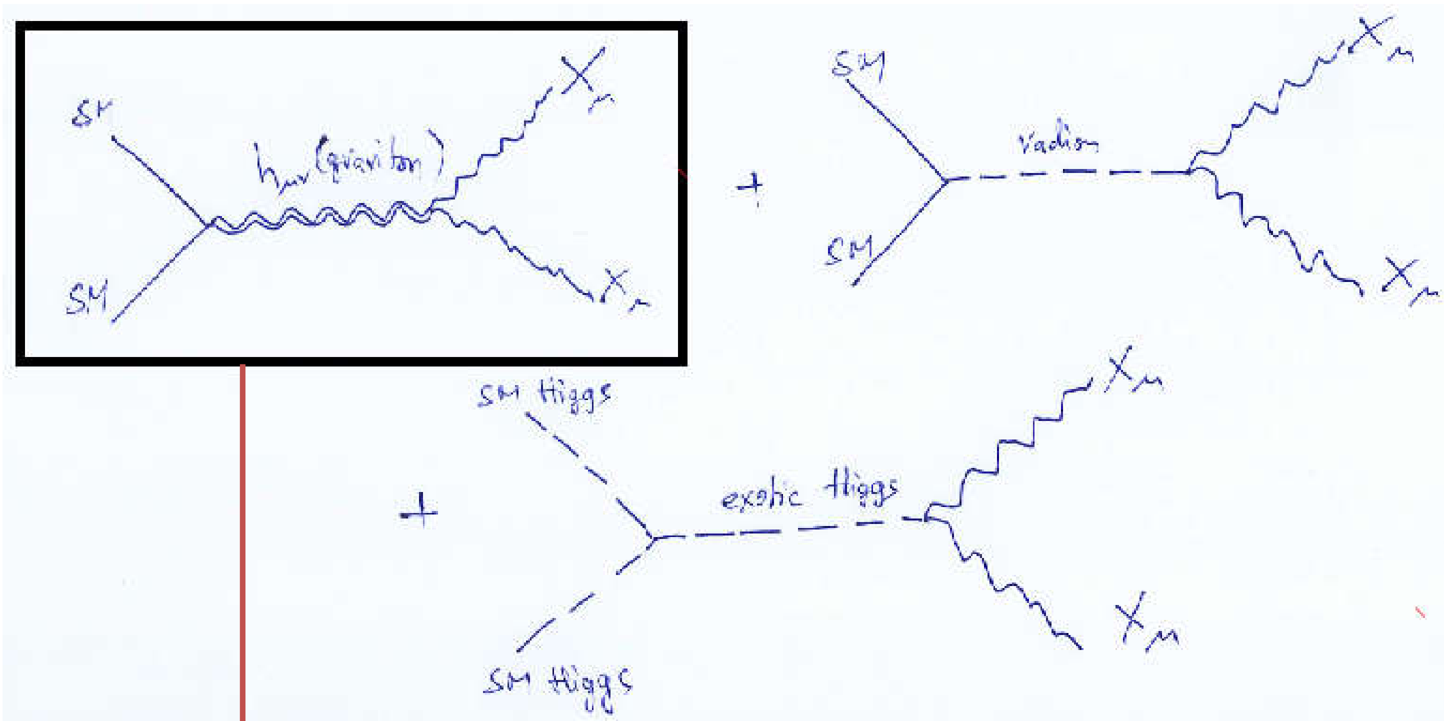
$$H_i = \gamma^{-2}\sqrt{\frac{4\pi^3 g_{\rm rh}}{45}\frac{T_{\rm rh}^2}{M_{\rm Pl}}}$$

$$\Omega_X h^2 \simeq 9.2\times10^{24}\times\gamma^{4/(1+w)}\frac{m_X}{M_{\rm Pl}}X_f$$

$$\langle\sigma v\rangle = 4\langle\sigma v\rangle_0 + 45\langle\sigma v\rangle_{1/2} + 12\langle\sigma v\rangle_1$$

- Communication between DM and the SM particles

Garny, et. al., 2016



$$\langle \sigma v \rangle_0 = \frac{3\pi m_X^2}{8 M_{\rm Pl}^4}\left[\frac{3}{5}\frac{K_1^2}{K_2^2} + \frac{2}{5} + \frac{4}{5}\frac{T}{m_X}\frac{K_1}{K_2} + \frac{8}{5}\frac{T^2}{m_X^2}\right]$$

$$\langle \sigma v \rangle_{1/2} = \langle \sigma v \rangle_1 = \frac{\pi m_X^2}{M_{\rm Pl}^4}\left[\frac{11}{20}\frac{K_1^2}{K_2^2} + \frac{9}{20} + \frac{13}{20}\frac{T}{m_X}\frac{K_1}{K_2} + \frac{13}{10}\frac{T^2}{m_X^2}\right]$$

Garny, et. al., 2018

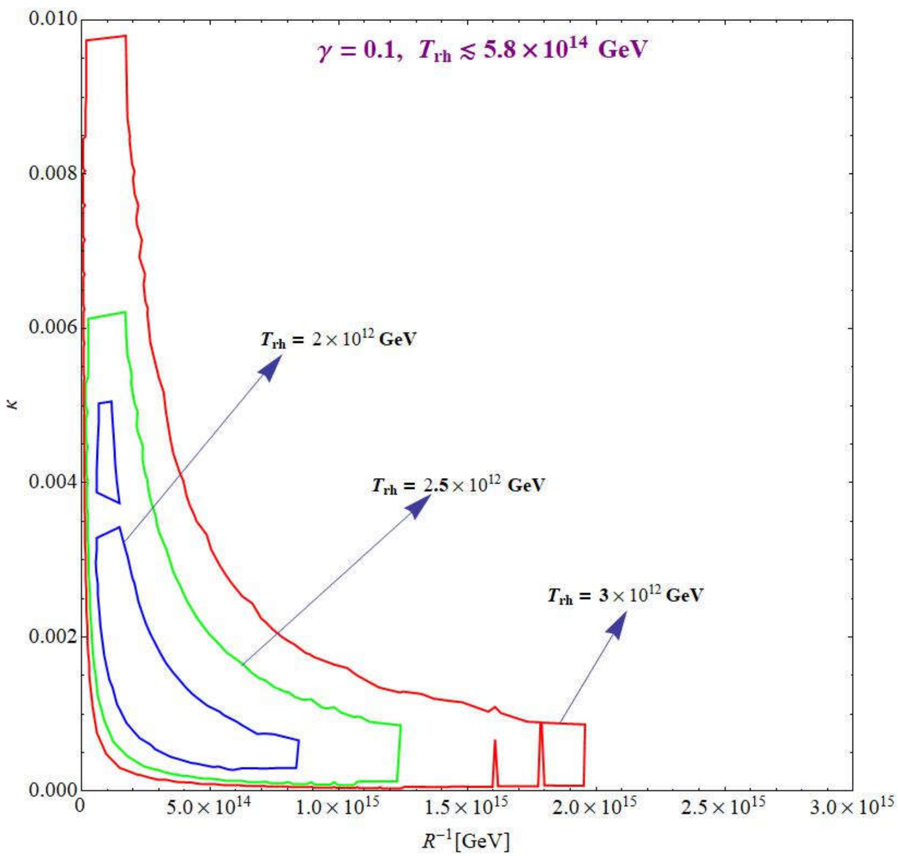
- The region of the parameter space with the correct DM relic abundance for various of the reheating temperature and the efficiency of reheating

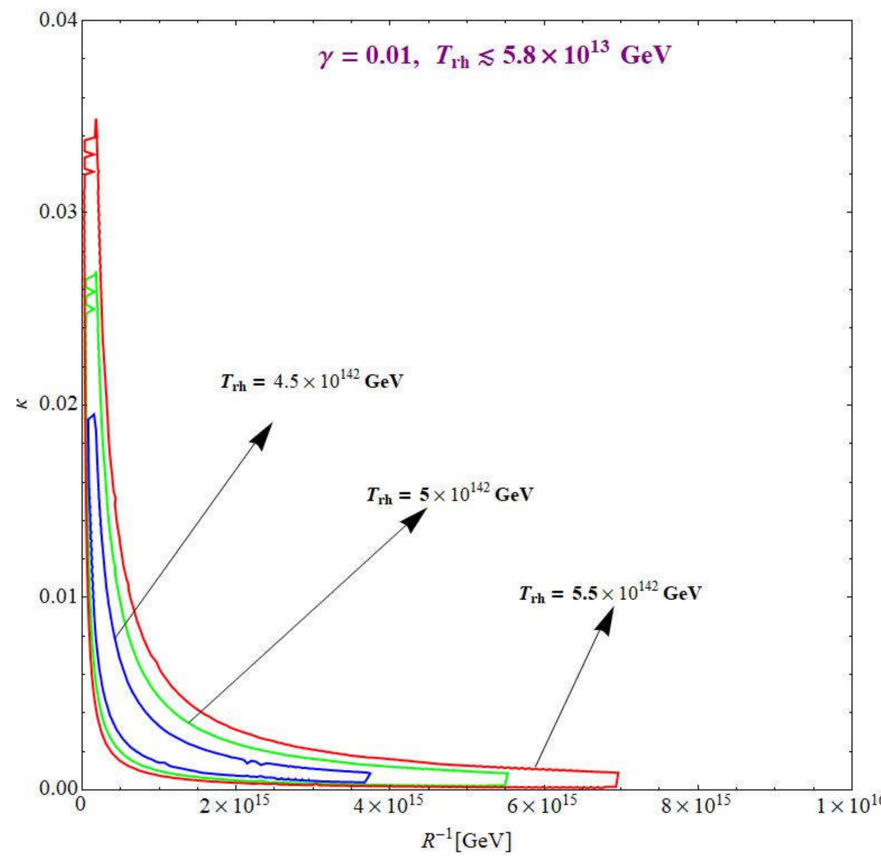
Upper bound on tensor-to-scalar ratio →

$$H_i < 6.6 \times 10^{-6} \times M_{\rm Pl} \left(\frac{r}{0.1}\right)^{1/2}$$

→ Upper bound on the reheating temperature

Planck Collaboration, 2018





Conclusion

- We have revisited Kaluza-Klein gravity from which the excitation of spacetime from the ground state $M_4 \times S^1$ emerging in the 4D effective theory is a natural superheavy candidate for DM.

- A stability mechanism preventing DM from its decay is found as a result of its coupling form to other fields in the 5D theory (which are mathematically fixed), the nontrivial dynamics of the 4D gravitational field along the extra dimension, and the boundary condition on the extra dimension.

- We have presented DM production during the heating and pointed out the region of the parameter space with the correct DM relic abundance.

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