Stochastic spatial models in ecology

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Overview

Tutorial (morning):

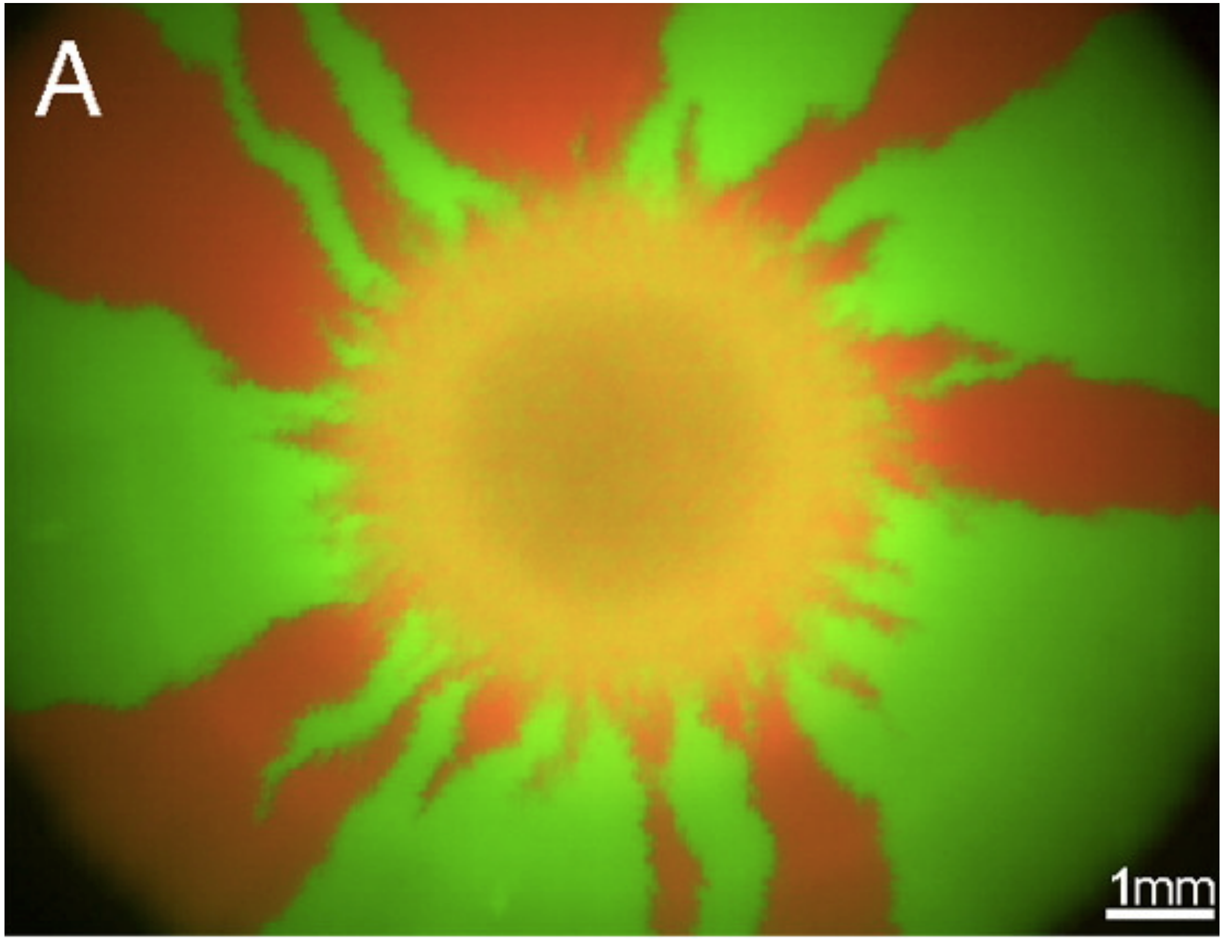
- Stochastic competition
- Biodiversity and neutral theory
- Spatial competition and scaling phenomena

Seminar (afternoon):

- Bet-hedging in jellyfish
- Population waves
- Bet-hedging in expanding populations

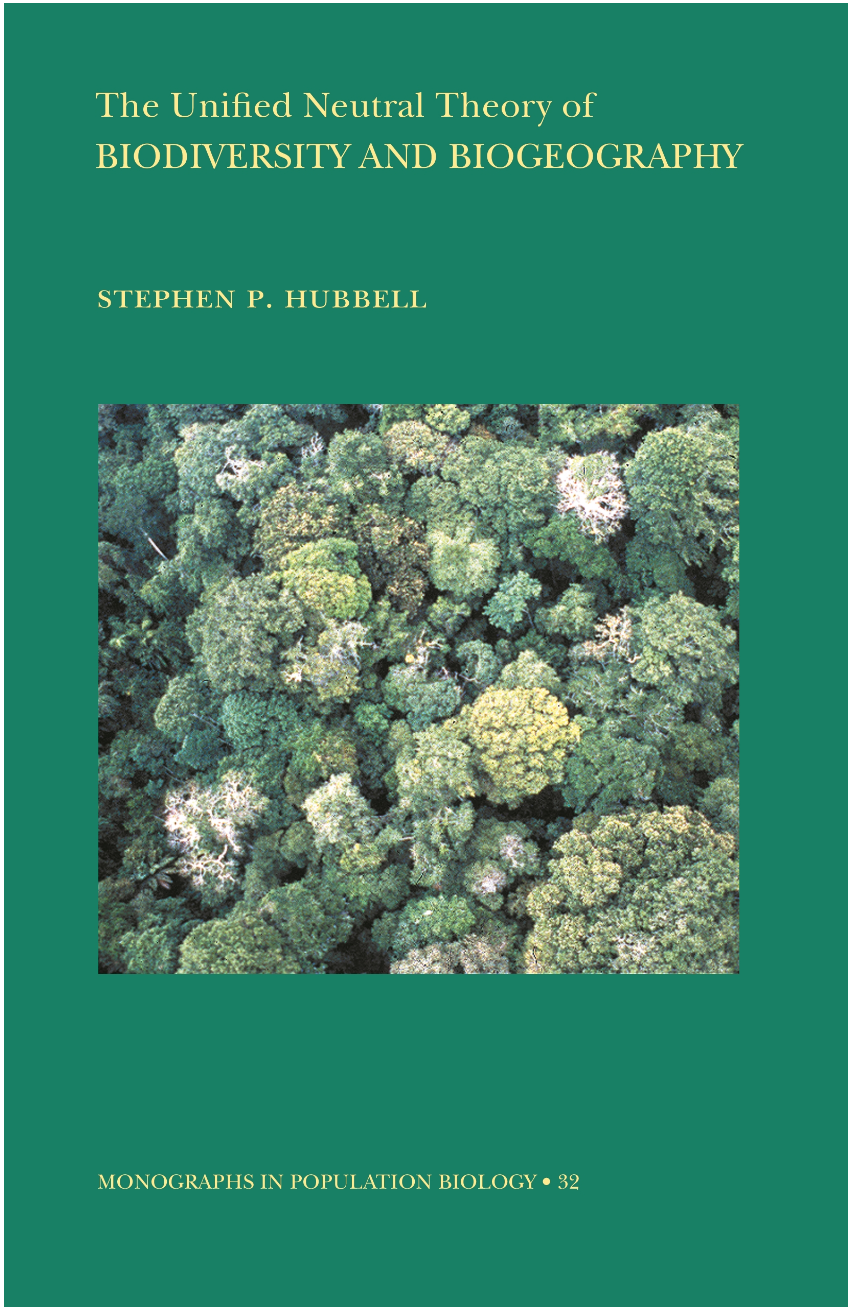
Tutorial: SP et al. JSP 2018
Seminar: Azaña et al. Am. Nat (2018), Villa-Martin et al. Plos. Comp. Biol. (2019)

Neutral competition



growth of a colony of two neutral
E.Coli strains

Hallatscheck and Nelson (2007)



S. Hubbell (2001)

A simple neutral model

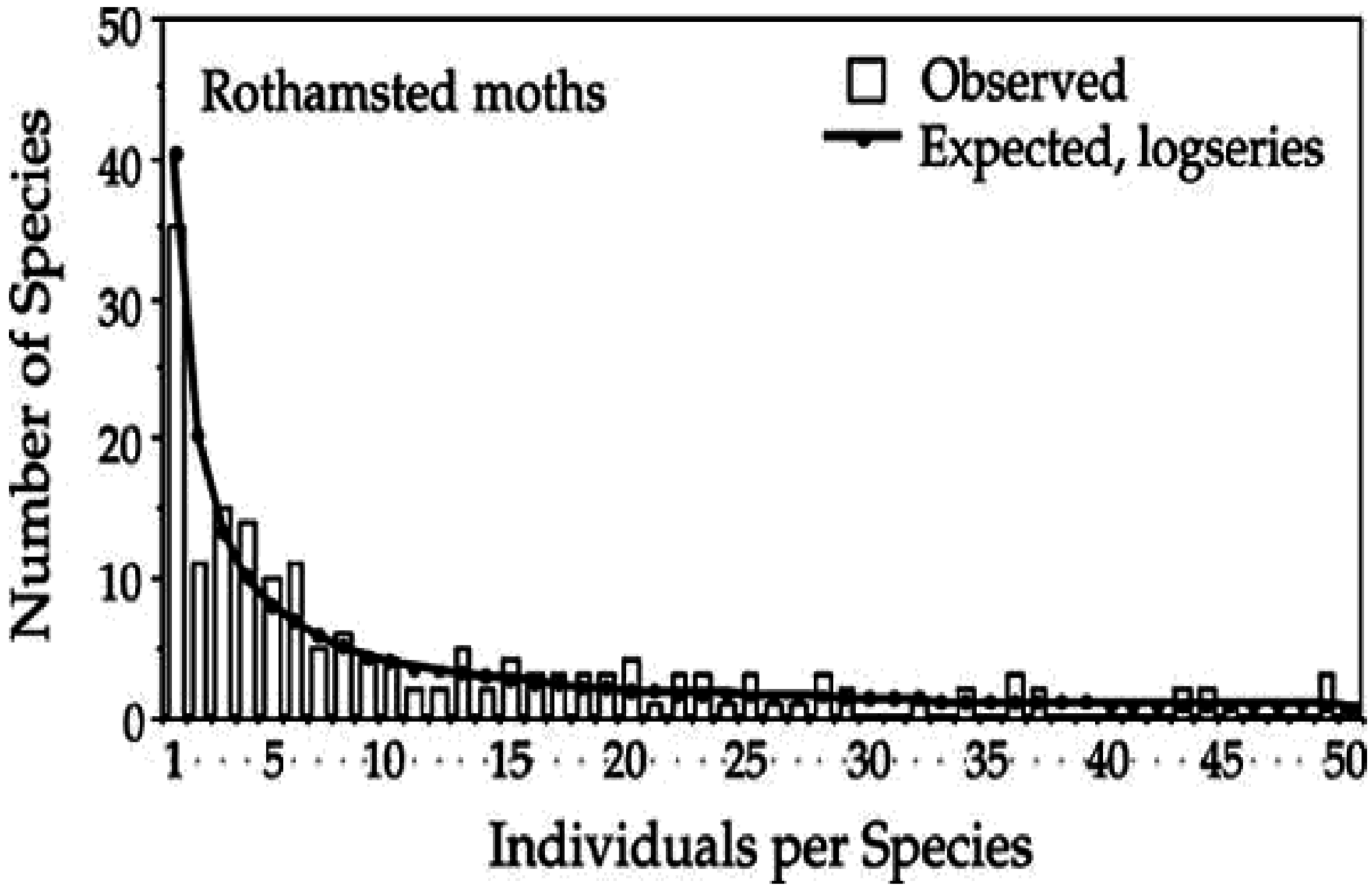
- Birth rate b
- Death rate d
- Extinct species are reintroduced at a small rate

Master equation:

$$\frac{d}{dt}p_n(t) = b(n-1)p_{n-1}(t) + d(n+1)p_{n-1}(t) - (bn+dn)p_n(t)$$

At equilibrium:

$$b(n-1)p_{n-1} = dnp_n \longrightarrow p_n \propto \frac{\prod_{j=1}^{n-1} b\,j}{\prod_{j=1}^{n} d\,j} = \frac{(b/d)^n}{n}$$



$$p_n \propto \frac{(b/d)^n}{n}$$

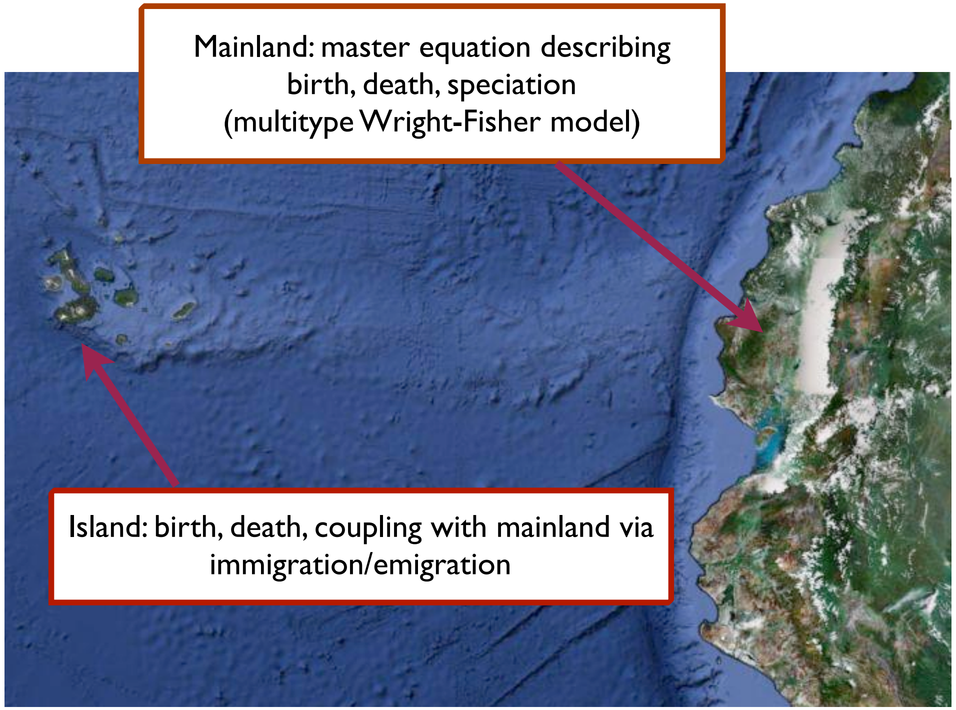
S. Hubbell (2001)

Neutral theory

Neutrality as a working hypothesis for biodiversity

Classical case study: tropical forests. Recently applied to plankton and bacterial communities

property	population genetics	community ecology
unit	gene	individual
diversity	alleles	species
source of diversity	mutation	speciation



Hubbell (2001), Alonso et al. (2006)

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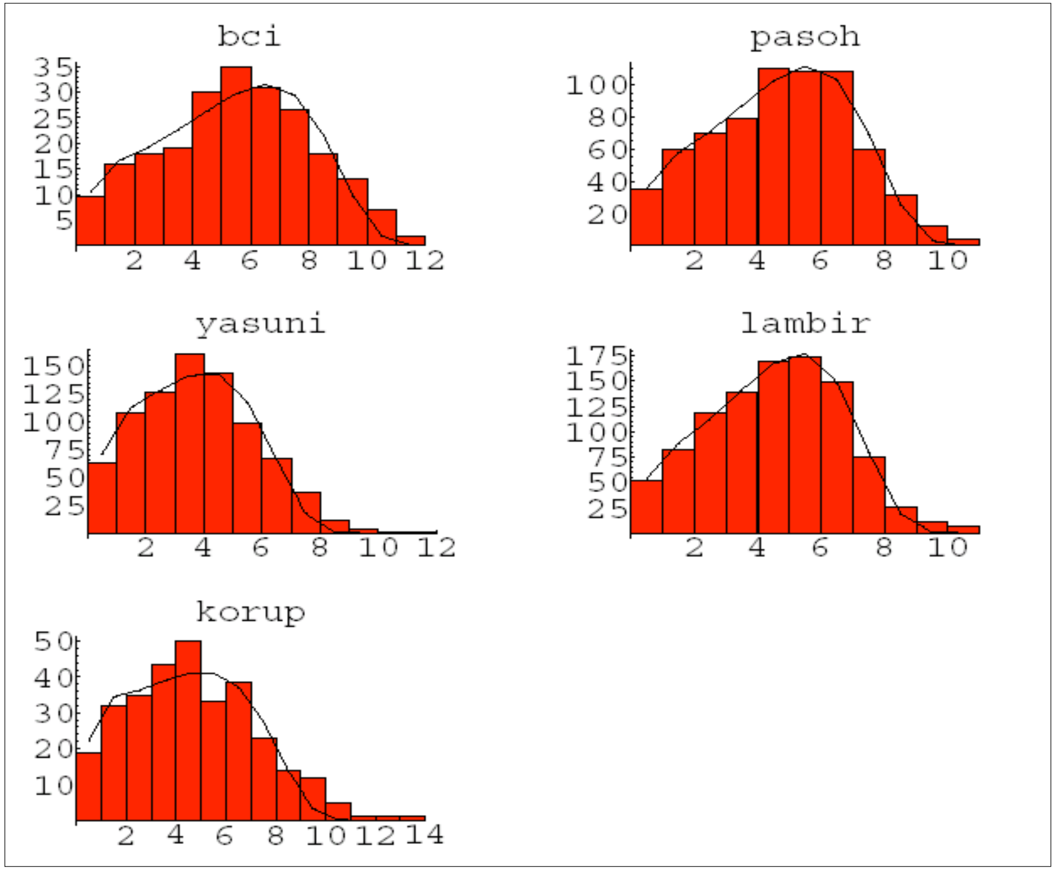
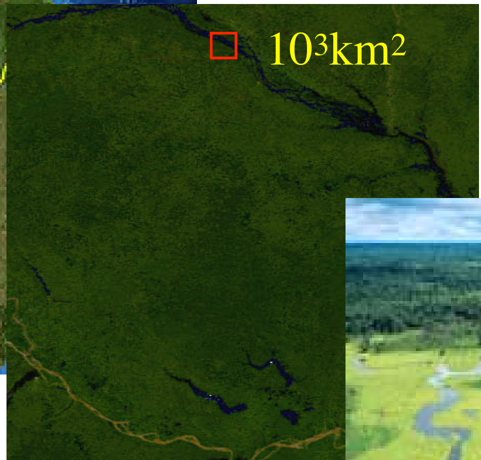


Hubbell (2001), SP et al. (2004), Alonso et al. (2006)

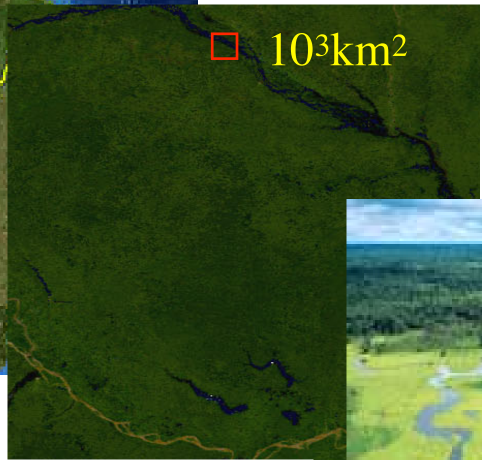
Biodiversity across spatial scales



Species Area Laws =

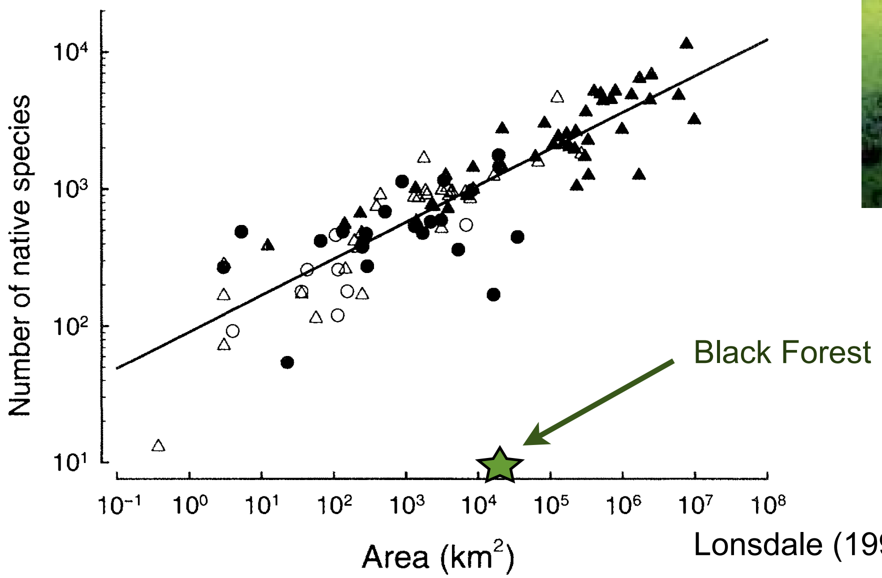
S vs A?

Biodiversity across spatial scales



Species Area Laws =

S vs A?



Lonsdale (1999)

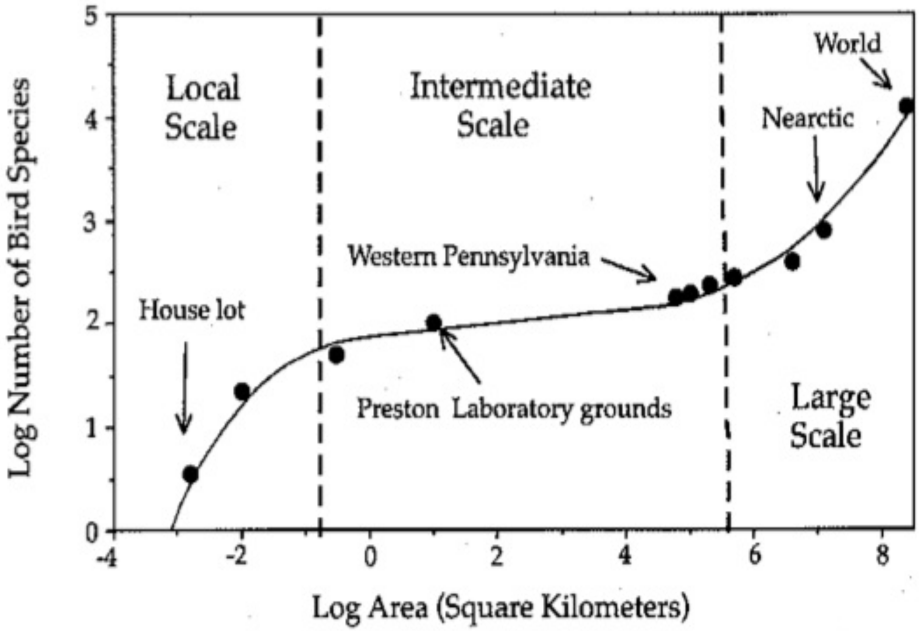
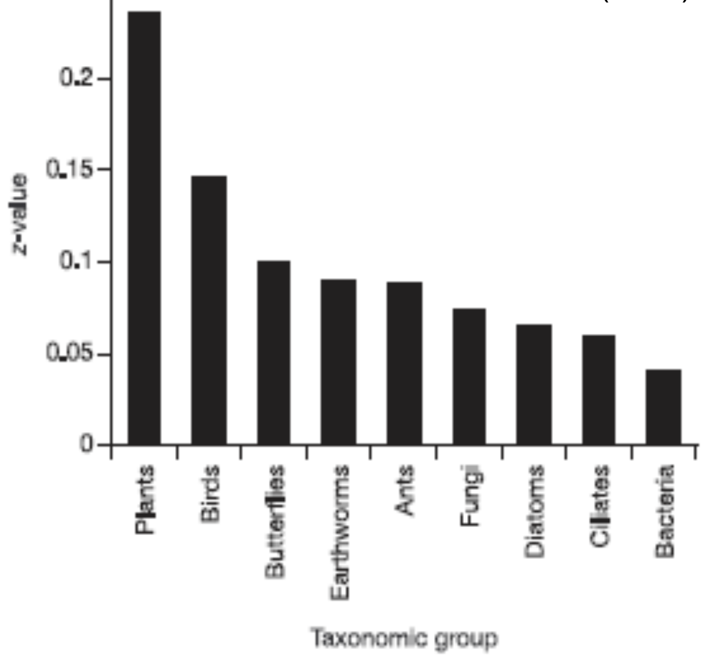


FIG. 6.2. Species-area curve for the world's avifauna, spanning spatial scales from less than one acre to the entire surface of the Earth. The S-shaped curve suggests that the sampling units change as area is increased, from individuals, to species ranges, and finally to different biogeographic realms at local, regional to subcontinental, and finally to intercontinental spatial scales. Data from Preston (1960).

from Horner-Devine et al (2004)



$S \propto cA^z$ z is scale-dependent

z≈1 for small & large scales

z<1 non-trivial (intermediate scales)

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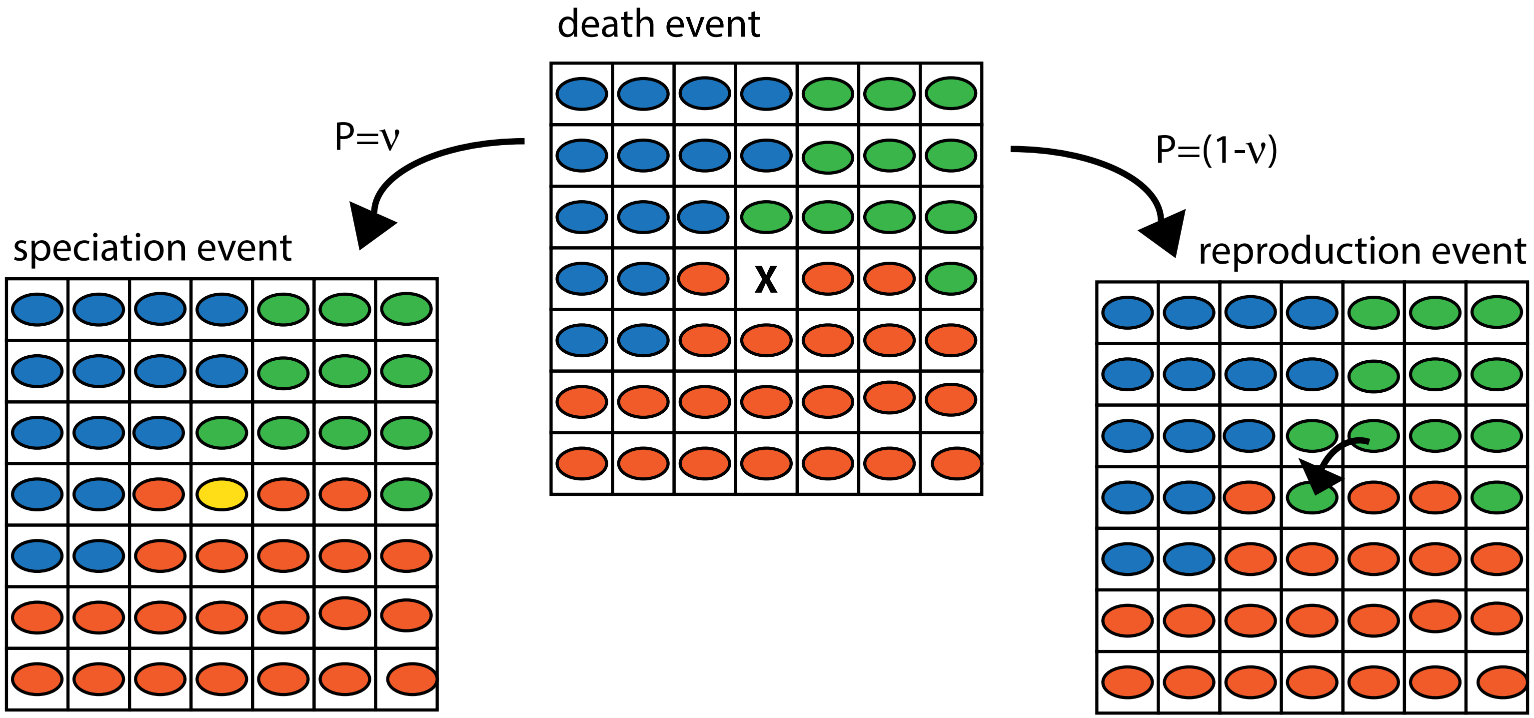
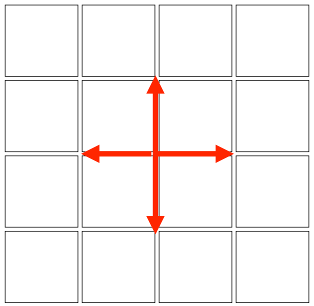
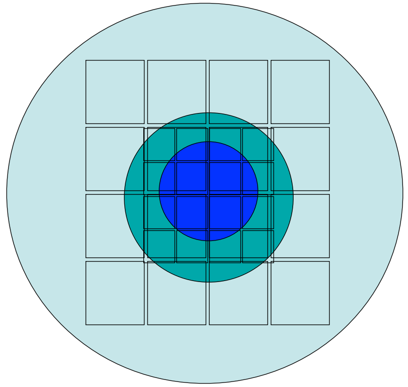
Spatial neutral model



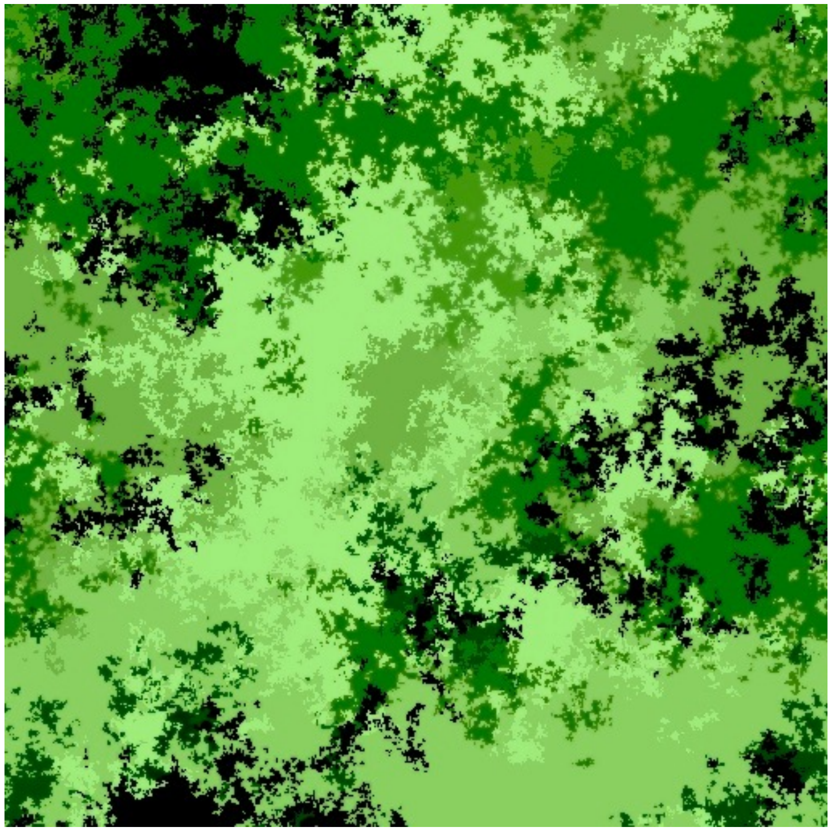
Choice of dispersal kernel

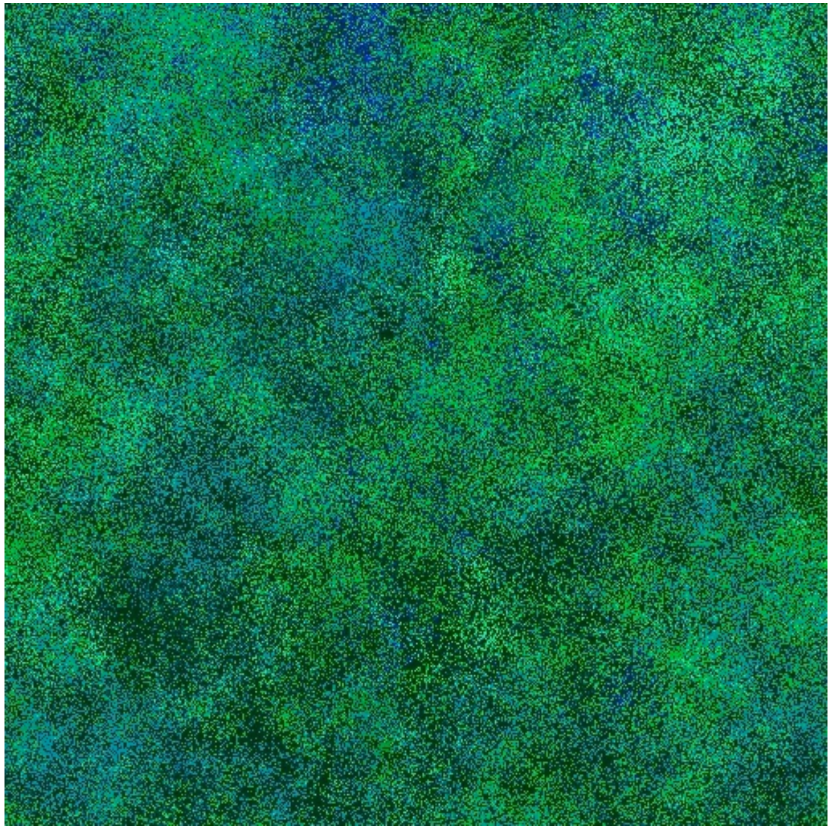


Nearest-Neighbor

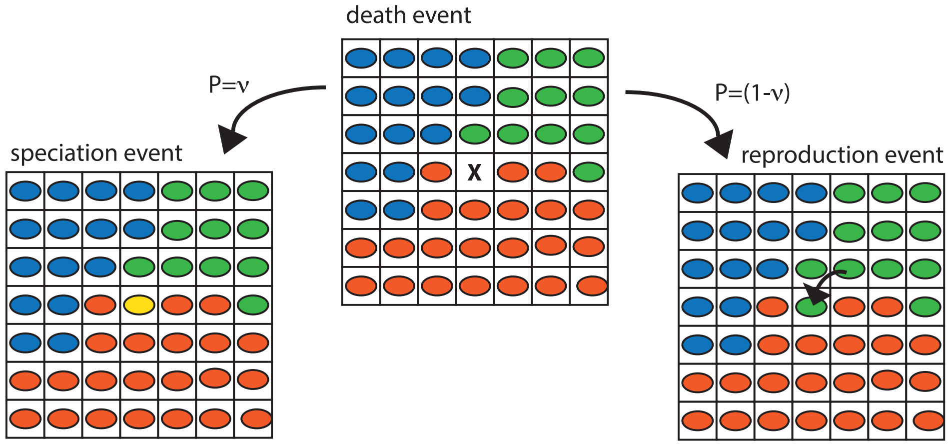


General Kernel
of range K
(e.g. Gaussian, Square)

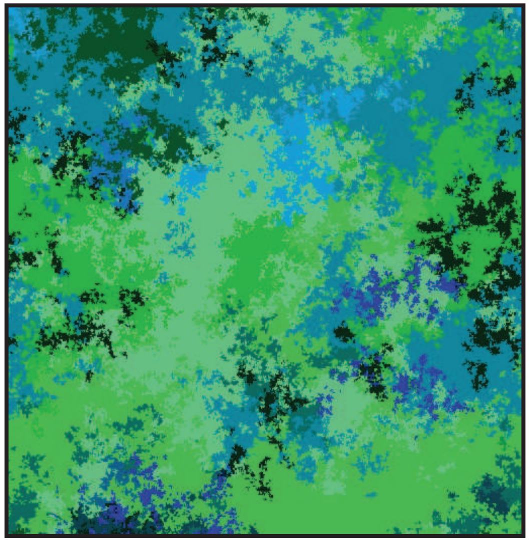




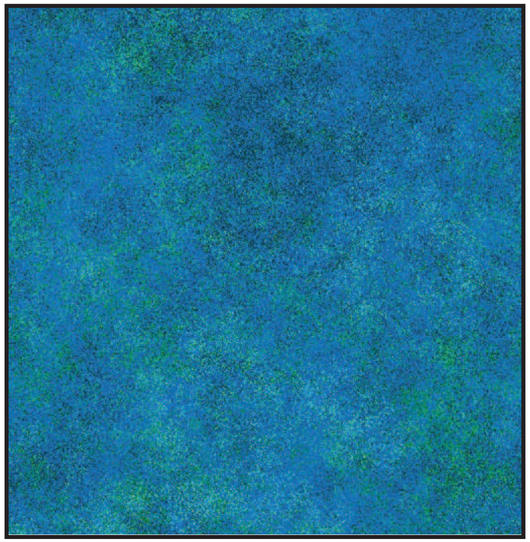
Spatial neutral models



Nearest-neighbor dispersal

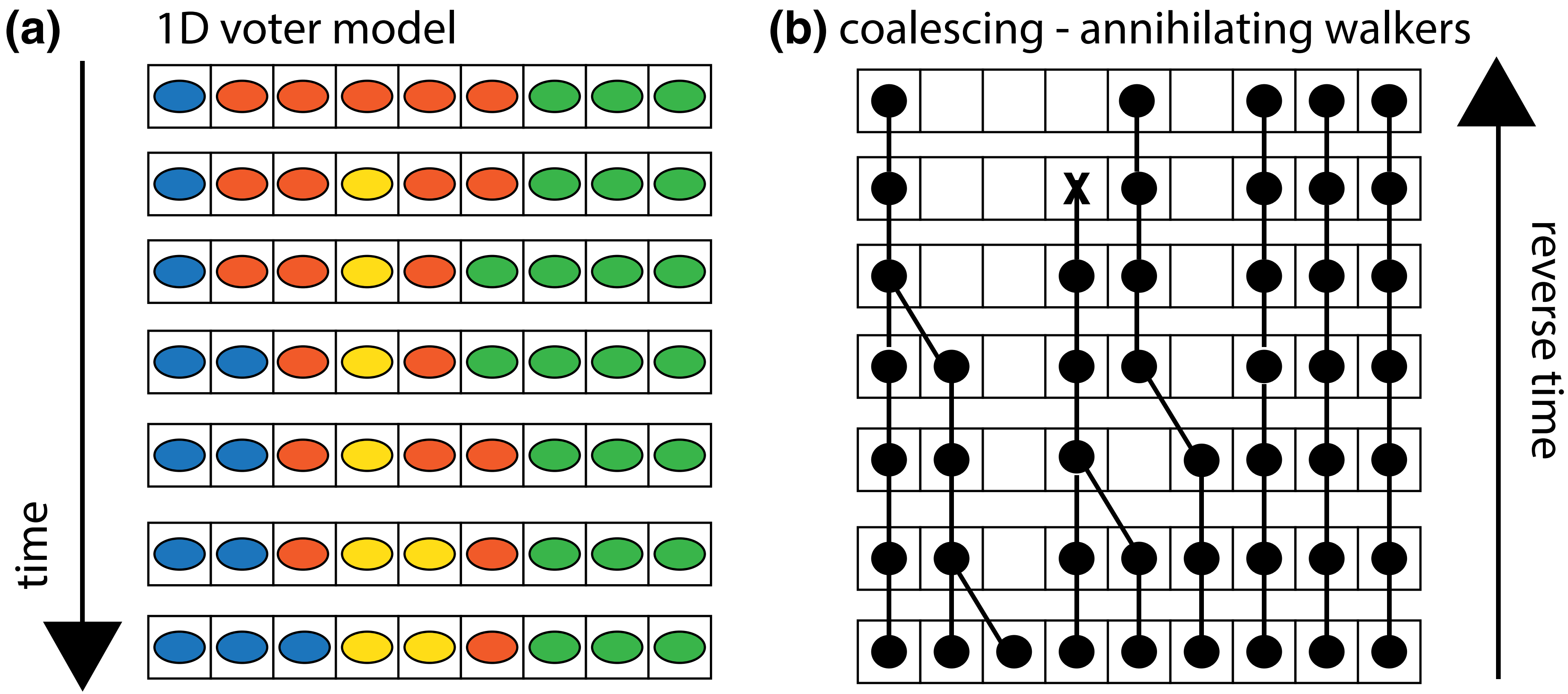


Long-distance dispersal

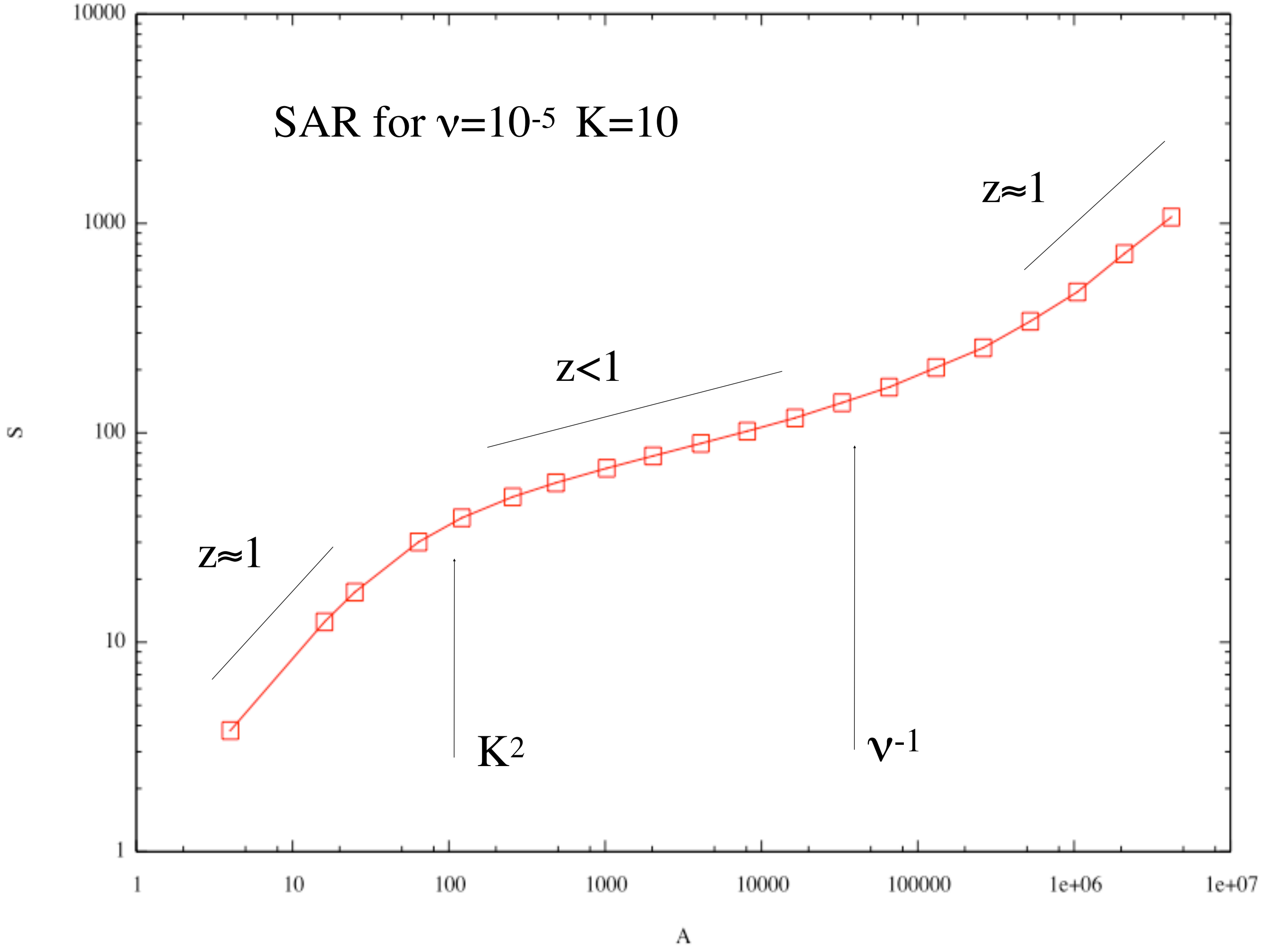


SP and M. Cencini, JTB 2009; SP et al. JSP 2018

Duality


(a)
1D voter model
time
(b) coalescing - annihilating walkers
x
reverse time

Results



Species-area laws are qualitatively reproduced by the model

(Chave et al. 2002, Rosindell et al 2007)

1. How does z depend on the dispersal kernel?
2. How does z depend on the speciation rate?

SP and M. Cencini, JTB (2009)

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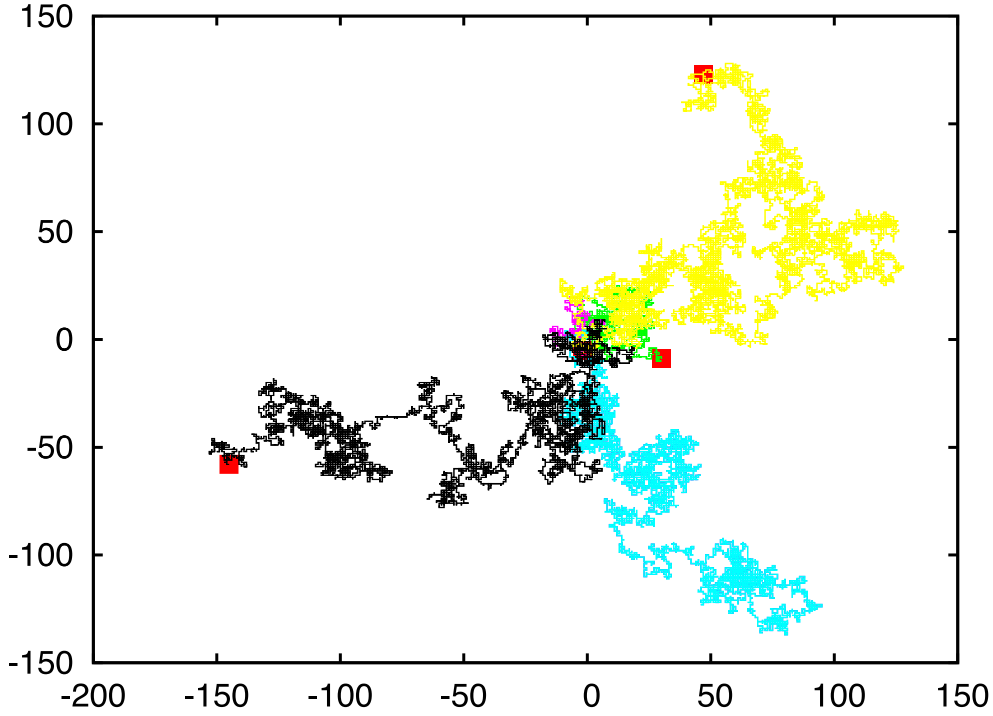
Coalescing random walkers

A + A -> A



Dimensional analysis suggests:

$$\frac{d}{dt}\rho(t) \propto -\rho^2 \qquad \rightarrow \qquad \rho(t) \propto \frac{1}{t}$$

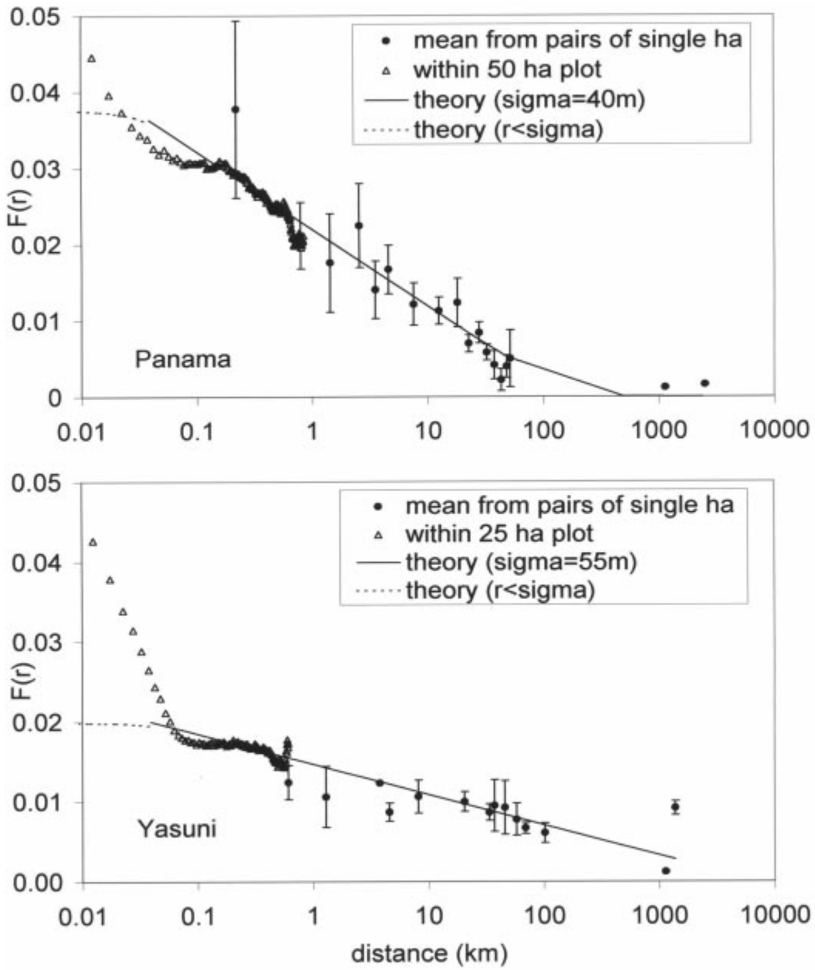
Asymptotic behavior of the density of walkers (Bramson and Lebowitz 1991):

$$\rho(t) = \begin{cases} \frac{C}{\sqrt{t}} & d = 1 \\ \frac{C}{\ln t} & d = 2 \\ C_0 + \frac{C}{\sqrt{t}} & d = 3 \end{cases}$$

2D is the critical dimension (logarithmic decay, finite asymptotic density above 2D)

Outcome determined by the interplay between S(t) and the killing rate -> power laws only in 2D

Beta diversity (correlation function)

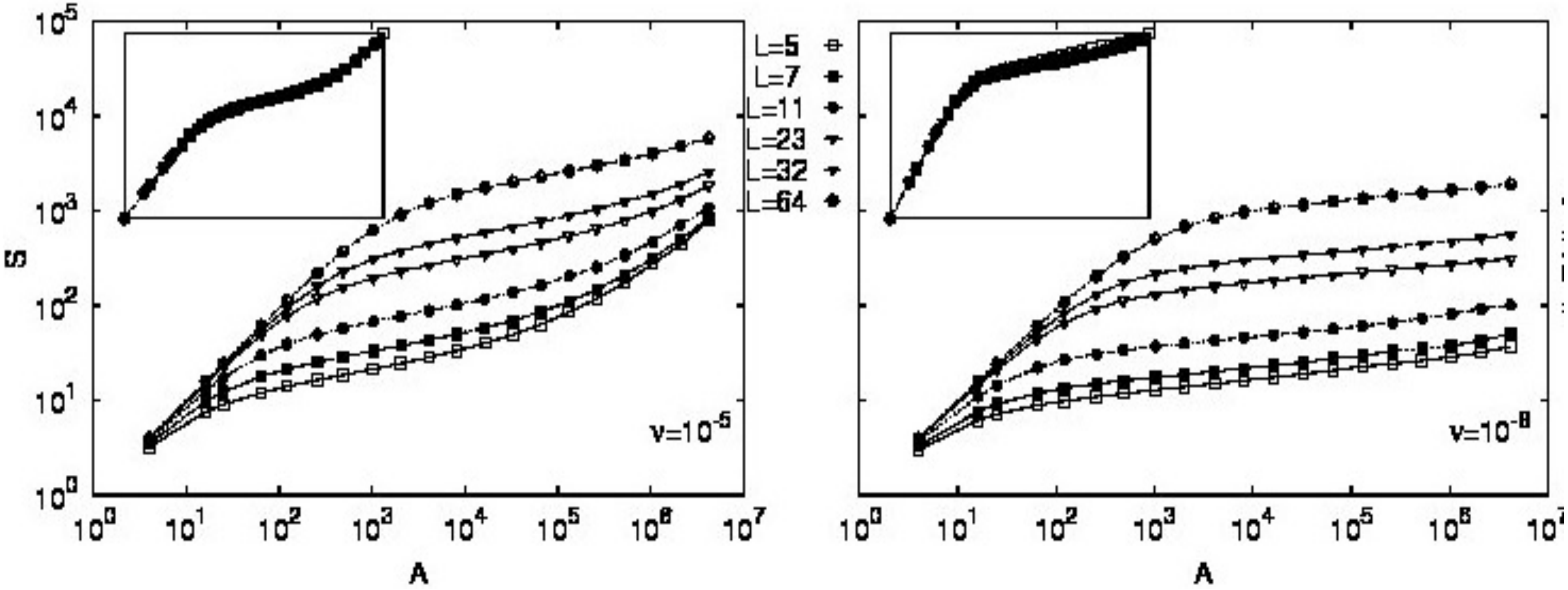


Condit et al. (2005)

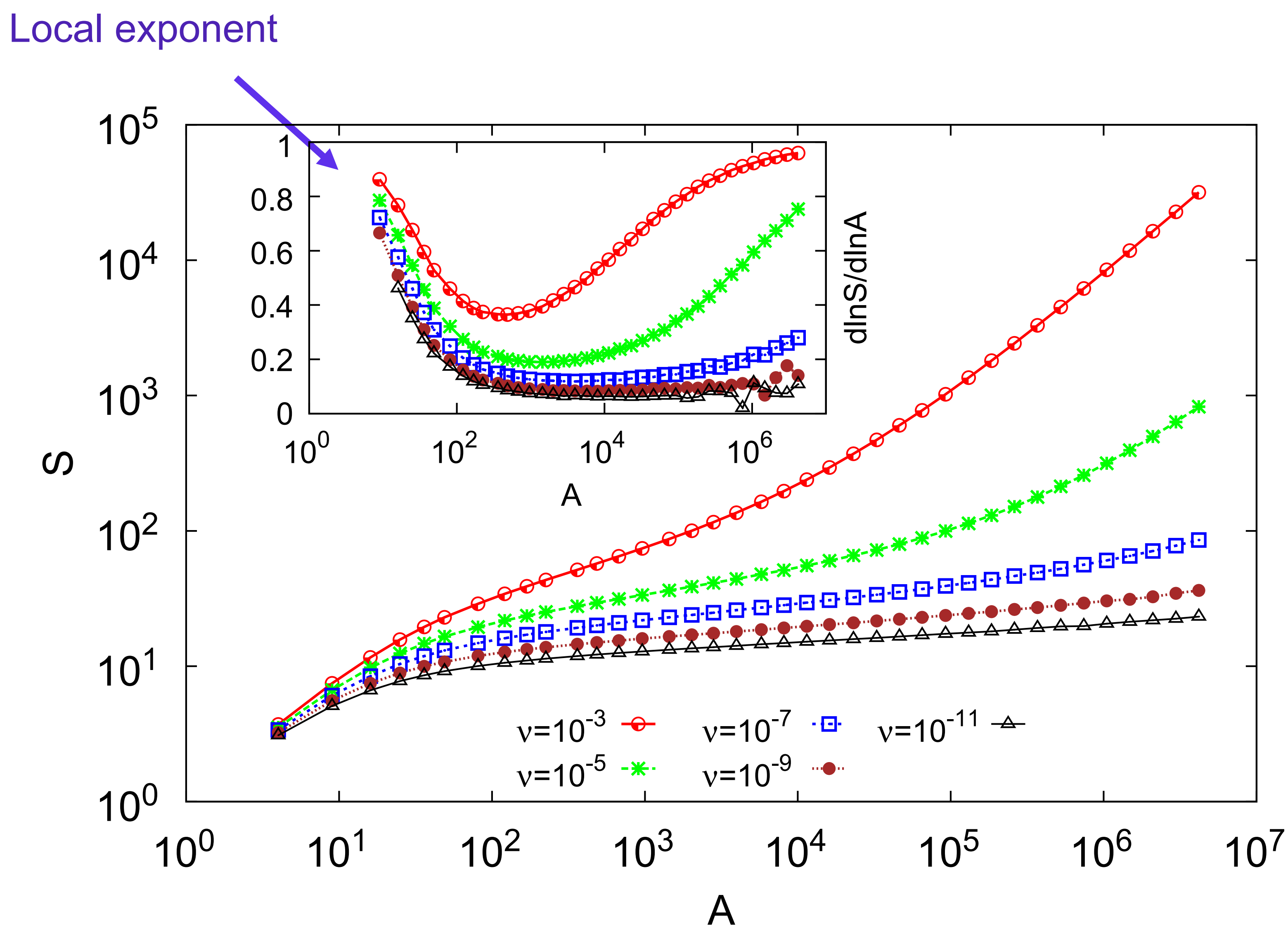
Probability that two trees at a distance r belong to the same species

$$F(r) \sim \frac{1}{\ln r}$$

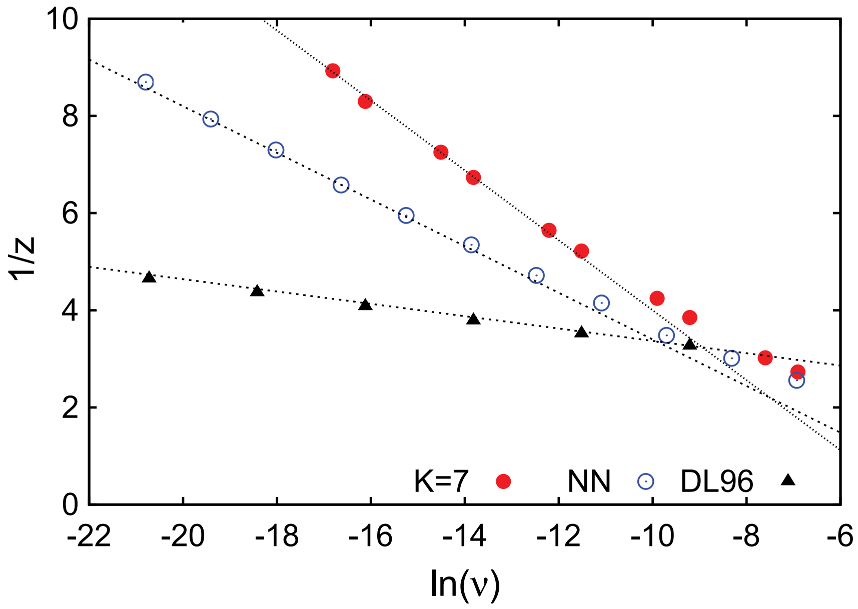
The exponent depends weakly on dispersal

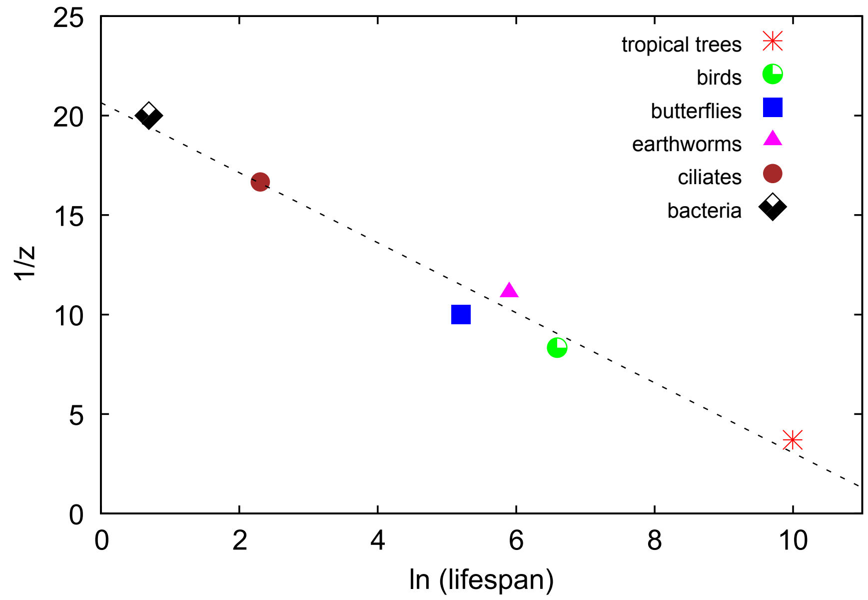


Dependence on the speciation rate



Dependence on the speciation rate





$$1/z \propto \ln(\nu)$$

$$1/z \propto \ \ln(\text{lifespan})?$$

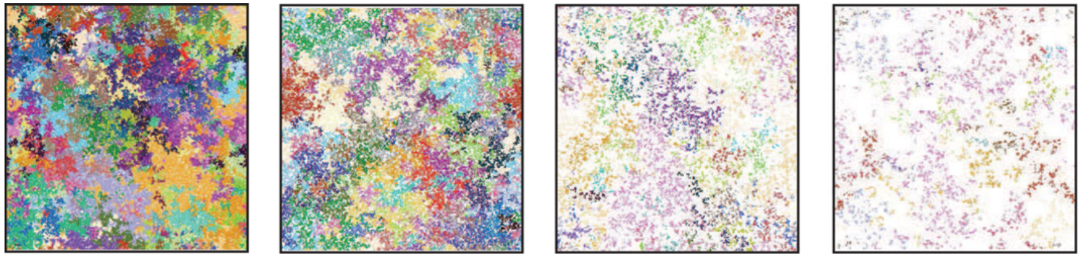
Ecosystems and critical phenomena

- Ecosystems show power-law behaviors consistent with the voter model universality class

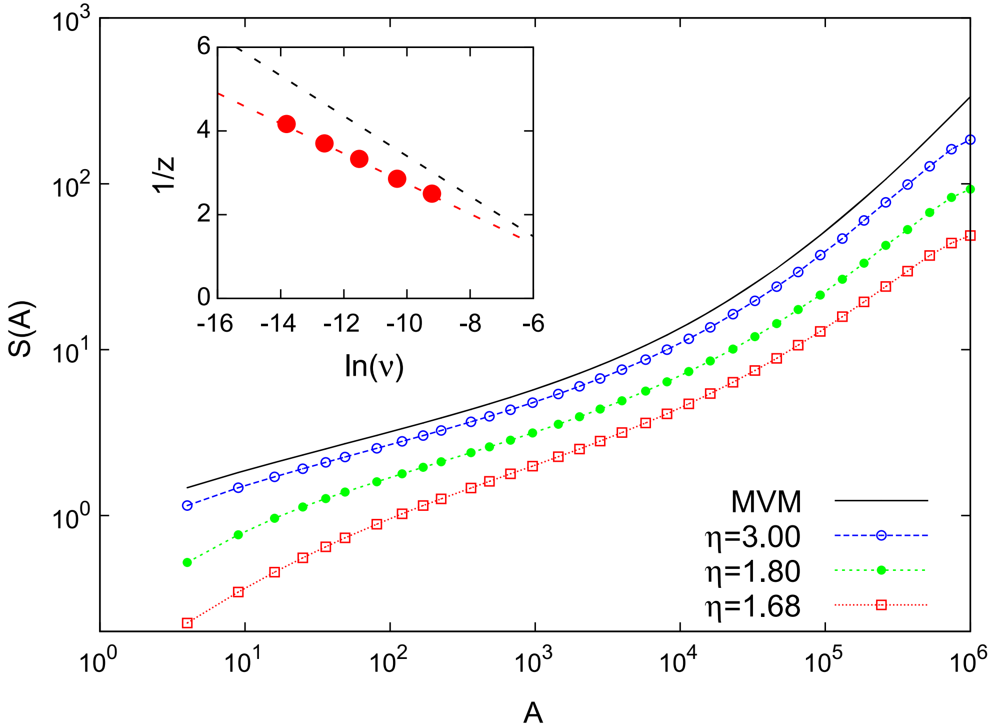
- 2D is the critical dimension, logarithmic correction

- Scale invariance is broken at the critical dimension (logarithmic corrections)

Non-saturated environments

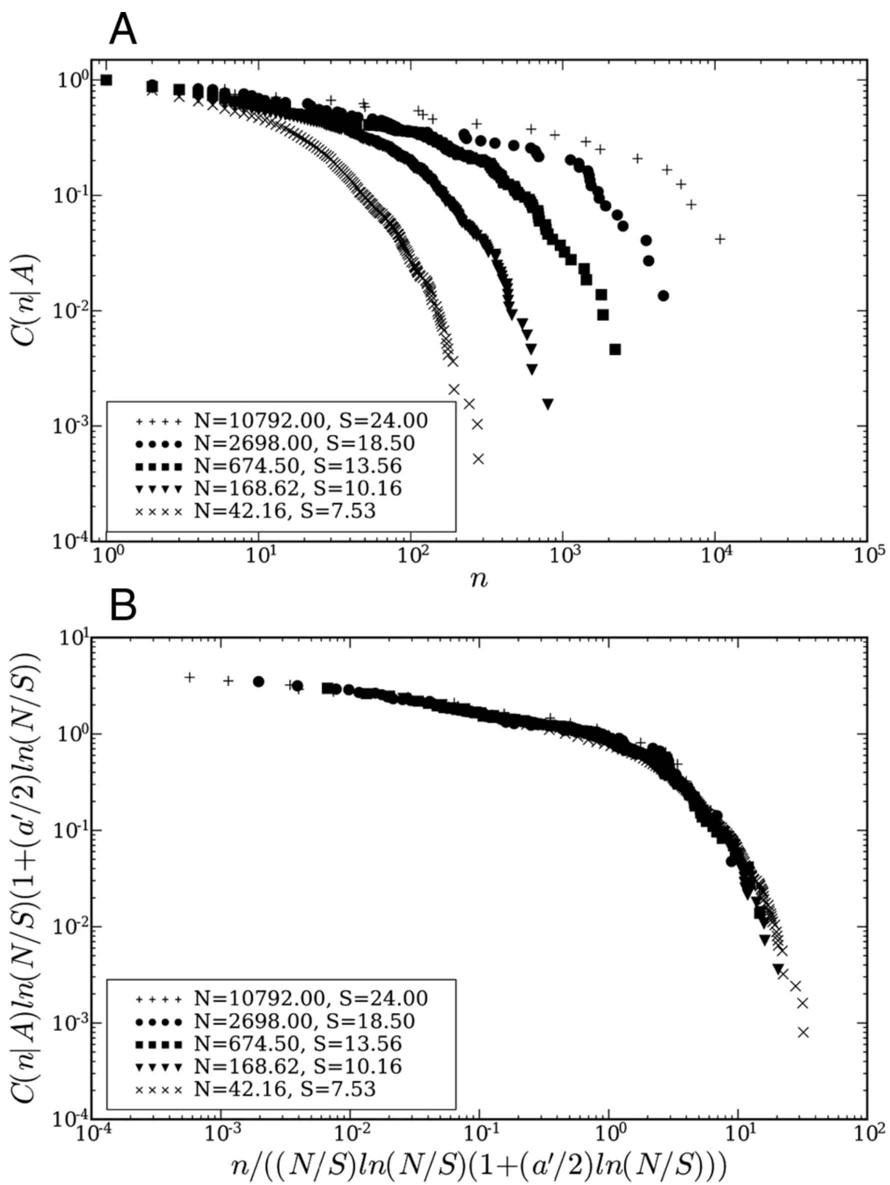


η = 10 η = 3 η= 1.8 η = 1.68



Cencini et al. Plos One (2012), Pigolotti et al. JSP (2018)

Species-Abundance distributions



Zillio et al. PNAS 2008

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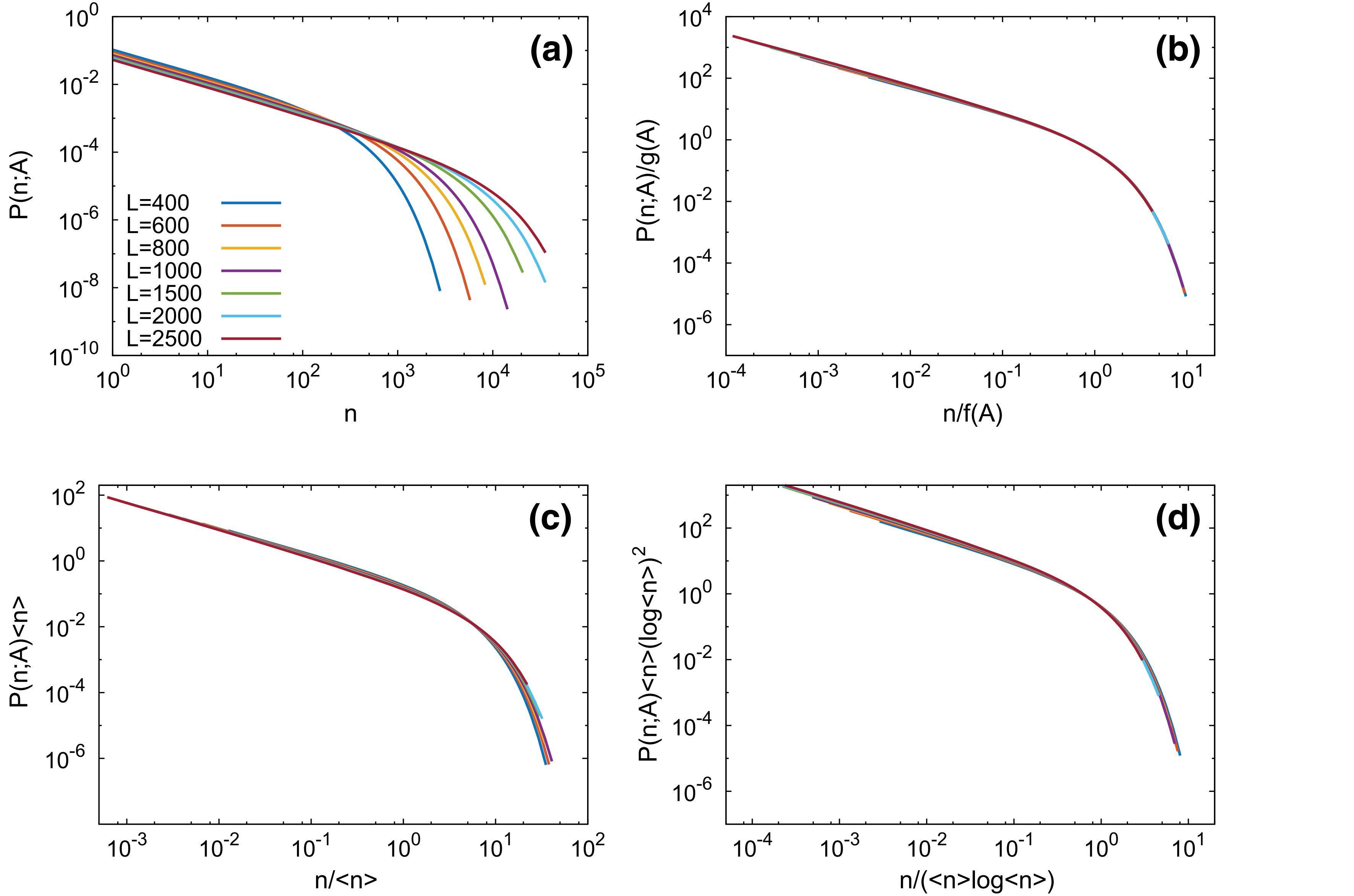

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Conclusion

- Ecosystems are non-equilibrium complex systems characterized by scaling behavior
- 2D is the critical dimension for simple stochastic competition models
- Consequences: non-trivial behavior of biodiversity, corrections to scaling