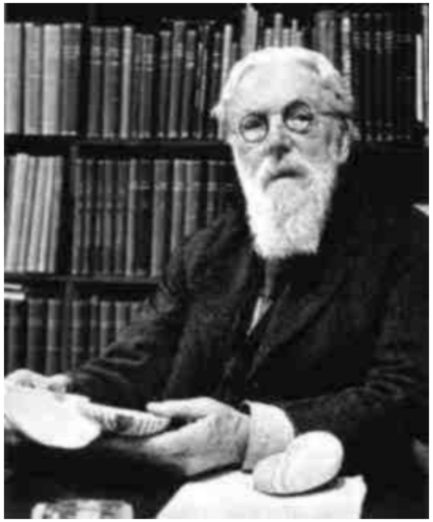
First lecture:

Precision measurement in atomic physics
---selected topics

Second lecture:

Laser spectroscopy of simple atoms
---work performed at NTHU

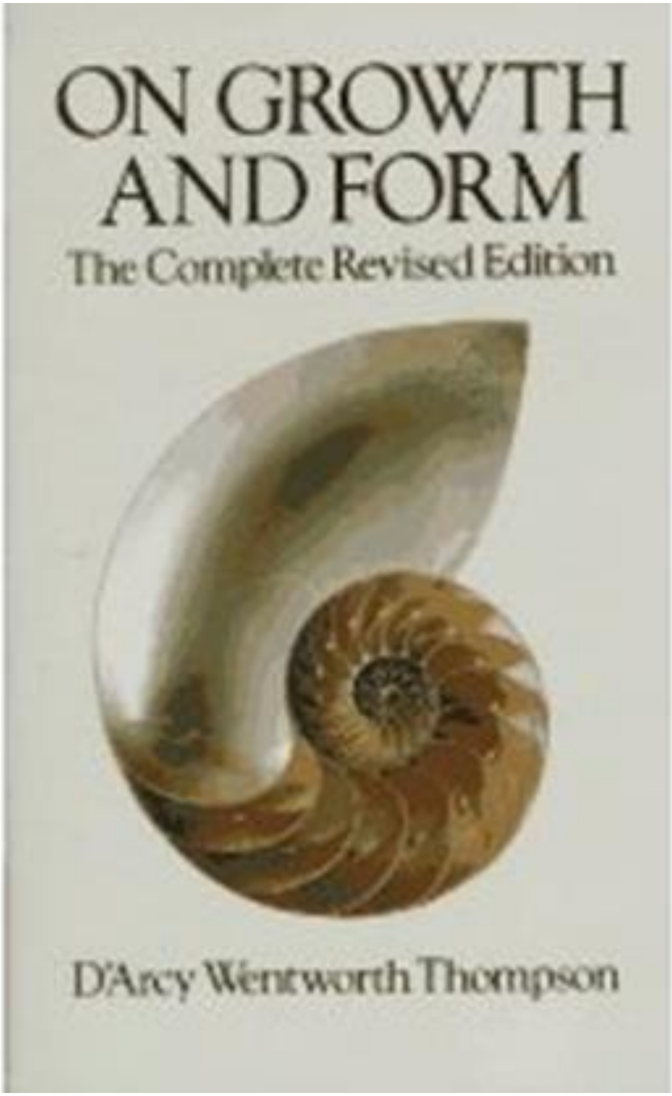
王立邦 Li-Bang Wang
清華大學物理系NTHU Physics



Thompson, D'Arcy Wentworth

On Growth and Form, 1917

...numerical precision is the
very soul of science...



ON GROWTH
AND FORM
The Complete Revised Edition

D'Arcy Wentworth Thompson

How precise?

- Magnetic moment of electron,

  g_e (exp) = 2.00231930436146(56)

- Rydberg constant = $\dfrac{2\pi^2 m_e e^4}{h^3 c}$

  = 109,737.31568160(21)

- Electric dipole moment of electron

  $|d_e| < 1.1\times 10^{-29}$ e·cm

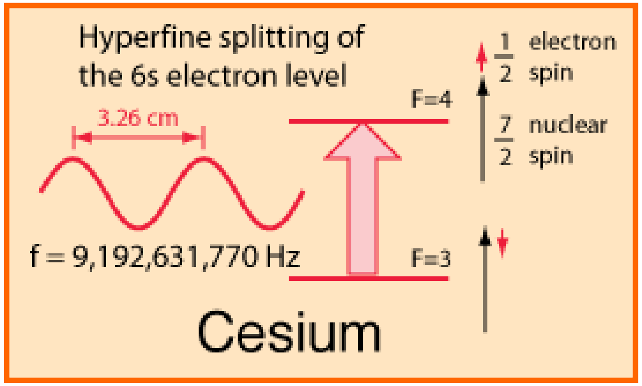
- Time variation of fine structure constant

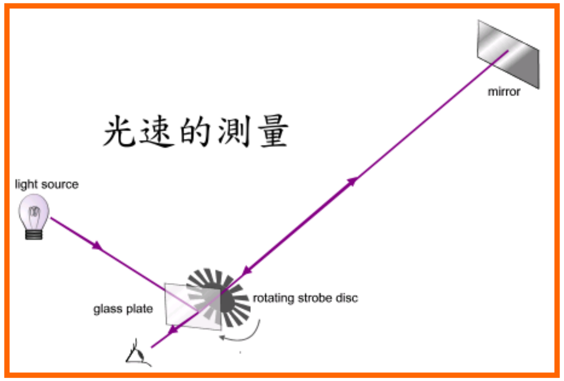
  $\delta\alpha/\alpha = (-1.6 \pm 2.3)\mathrm{x}10^{-17}$/year $\qquad \alpha = e^2/\hbar c \approx 1/137$

[1] D. Hanneke *et al*., *Phys. Rev. Lett.* **100,** 120801 (2008)
[2] 2018 CODATA value (2019)
[3] The ACME Collaboration, Nature **562,** 355 (2018)
[4] T. Rosenband *et al.*, *Science* **319**, 1808 (2008)

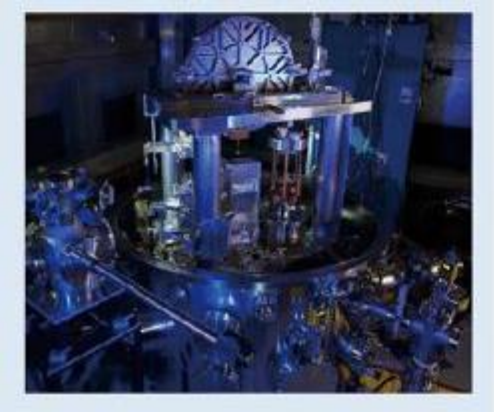
Classical measurements

- Time, Length, Mass, Charge
- Definition of Unit
- Good Tools





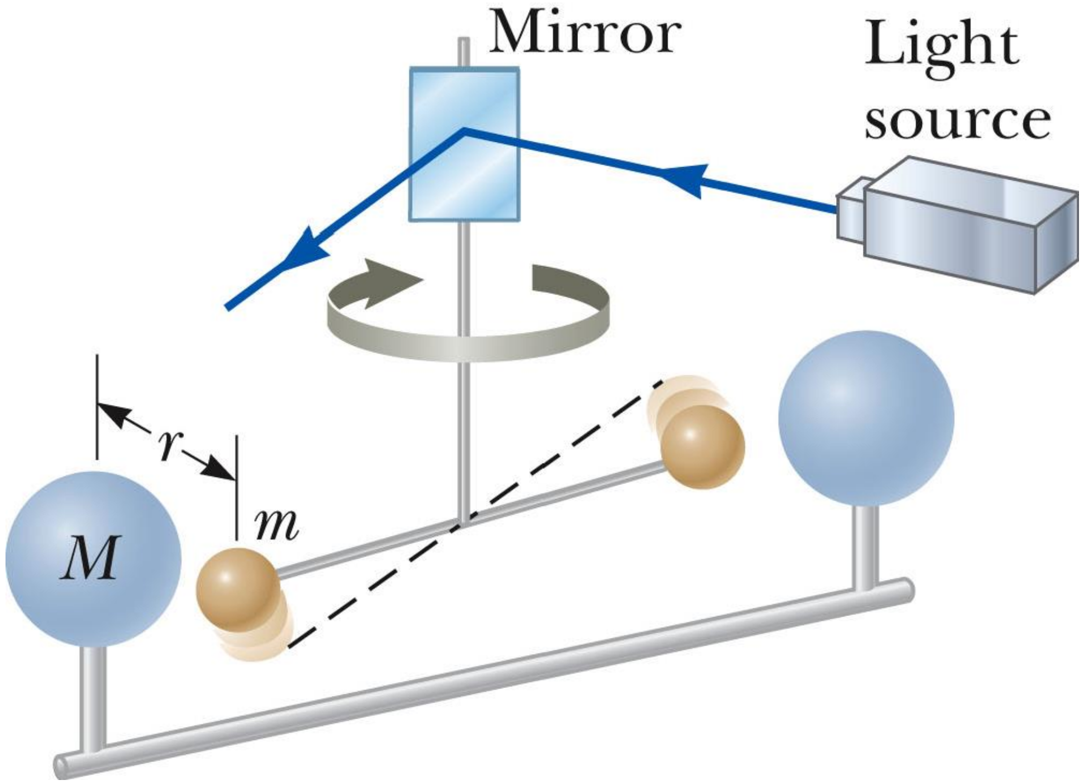
Kibble balance



Silicon sphere

Example

- Cavendish 1798, G = 6.74 x 10^{-11}
- G = (6.674 $\pm$ 0.001) x 10^{-11}, $\delta G/G \sim 100$ ppm



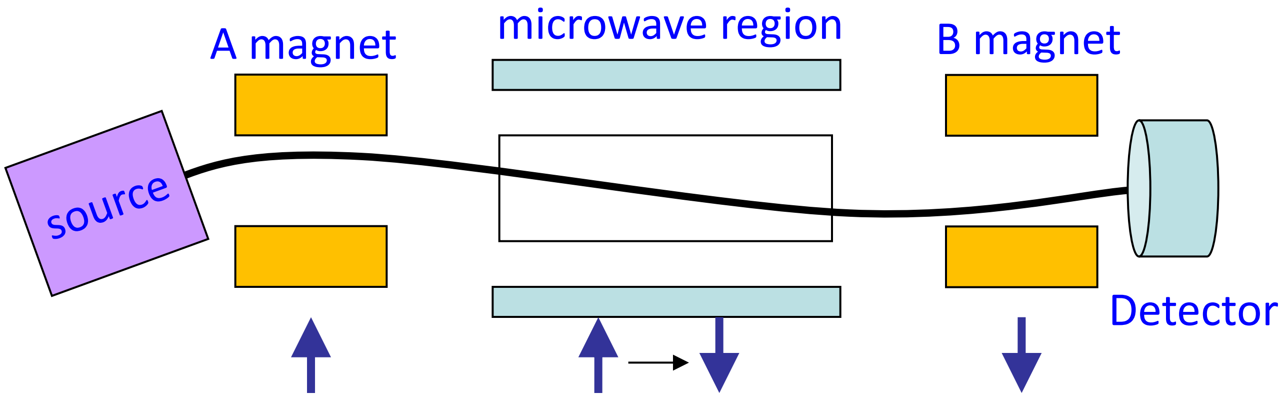
Outline

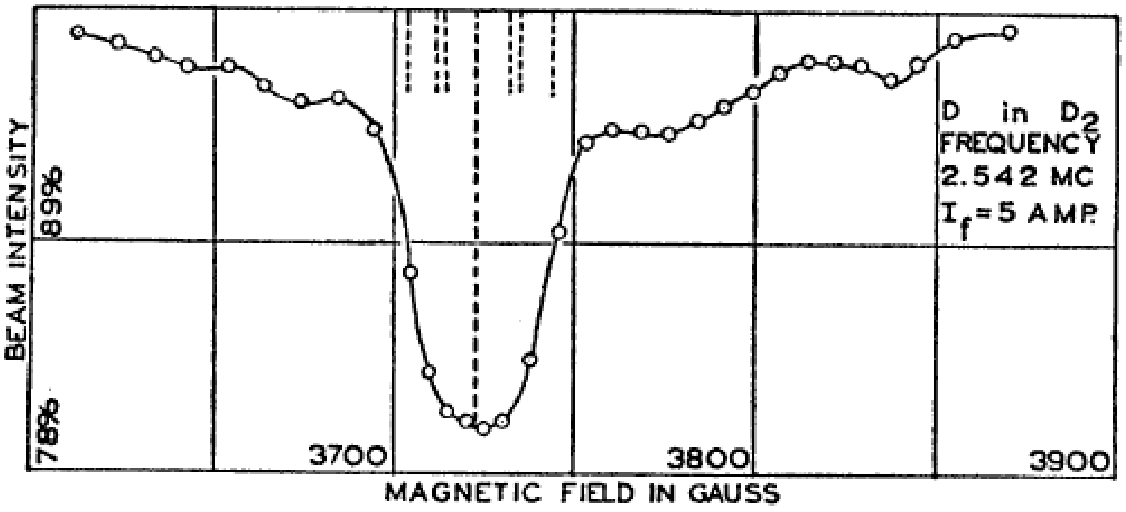
- Historical Review:
  Precision measurement lead to new physics
- Modern approach: resonance and frequency measurement
- Selected topics

Resonance Method

- First Motivated by Stern-Gerlach experiment, 1922
- First atomic spectroscopy by resonance method

NMR, Rabi and Ramsey, 1934-1939





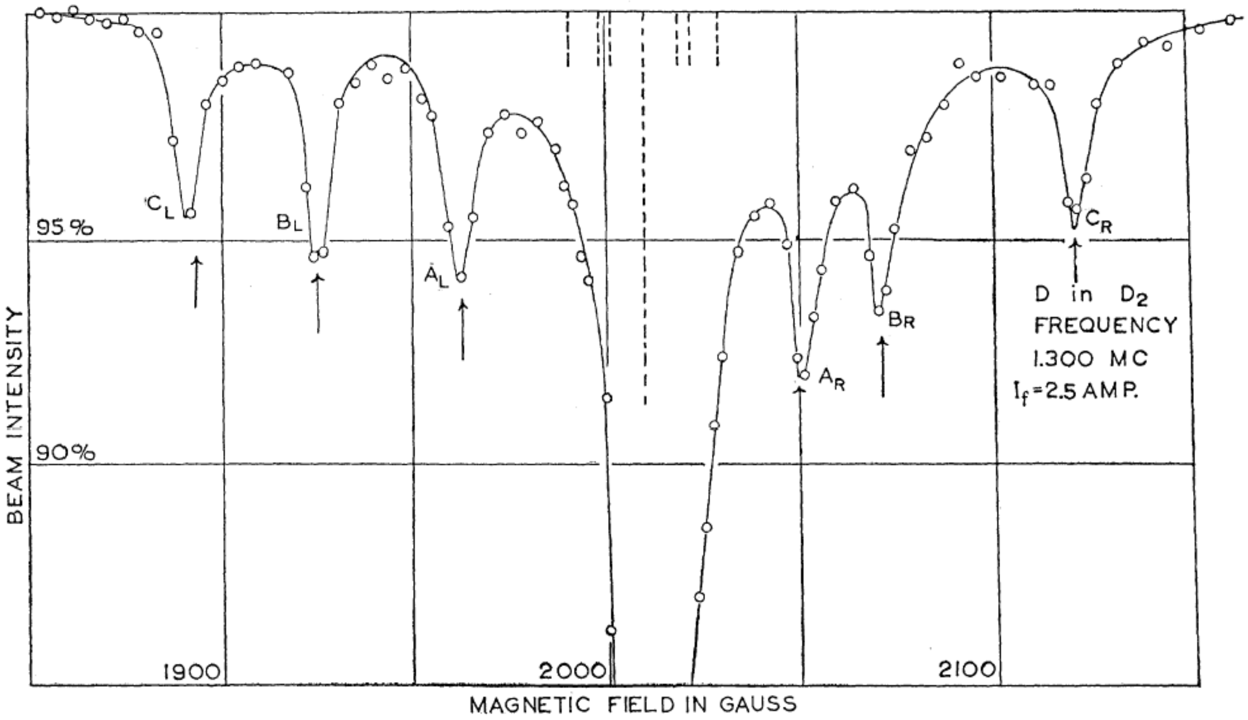
NMR for Deuteron

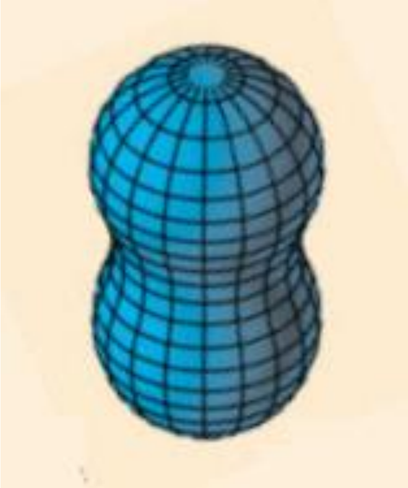
Isidor Isaac Rabi

Norman F. Ramsey

Electric Quadrupole Moment

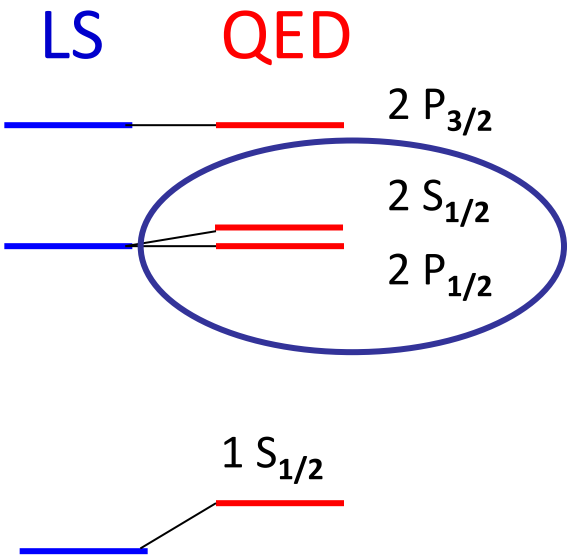
- Electric Quadrupole moment of Deuteron discovered
- Nuclear force is Non-central! Tensor force

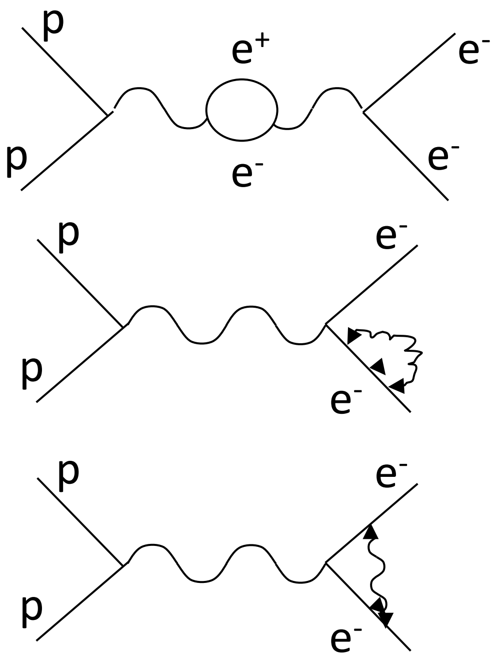




J. M. Kellogg, I. I. Rabi, N. F. Ramsey, and J. R. Zacharias Phys. Rev. 57, 677 (1940)

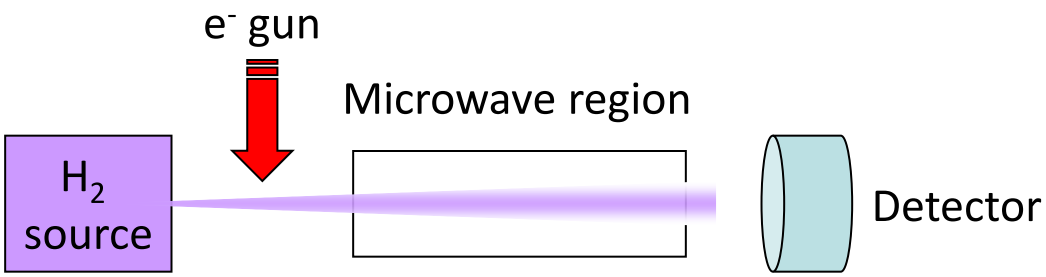
The Lamb Shift





Willis Eugene Lamb
Nobel Prize in 1955
"for his discoveries concerning the fine structure of the hydrogen spectrum"

1947 by Lamb ~1060 MHz
now: 1057.846(4) MHz



QED

- Quantum Electrodynamics
- The most precisely tested theory

Sin-Ichiro Tomonaga (朝永振一郎), Julian Schwinger, Richard P. Feynman
Nobel Prize in Physics 1965

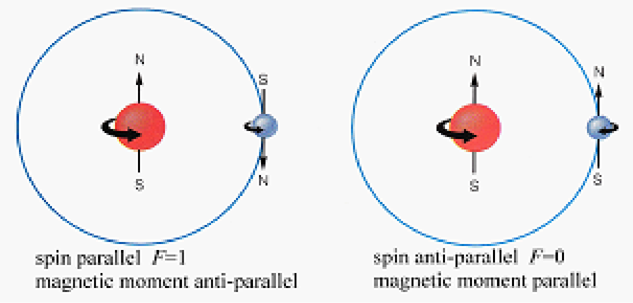
"for their fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles"

Selected topics in precision measurement

- Hyperfine structure: probing nuclear moment
- Test QED: the g-2 of electron and muon
- Strong interaction: size of proton
- Time and frequency standard: making a better clock
- Constancy of fundamental constant: can speed of light change?
- New source of CP violation: electric dipole moment of electron and nucleon

Hyperfine structure

- Nuclear magnetic moment
- F = I + J
- a and b constant

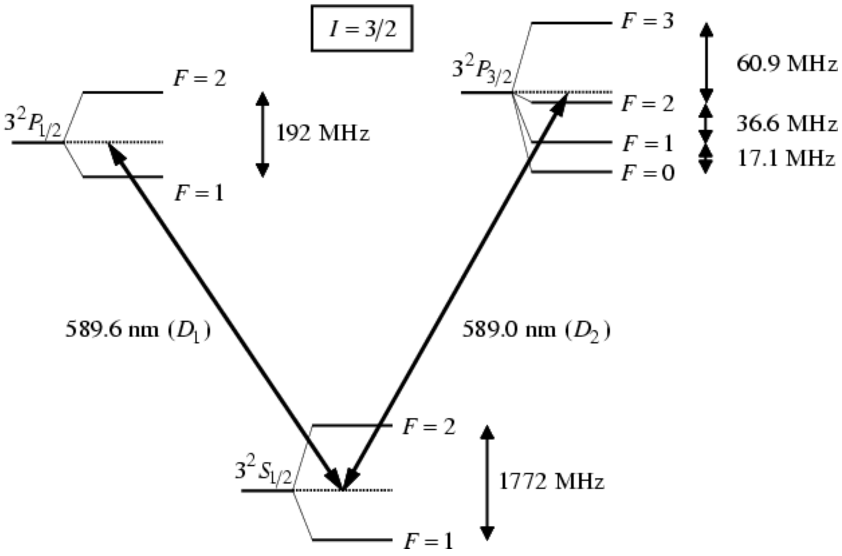


spin parallel F=1
magnetic moment anti-parallel

spin anti-parallel F=0
magnetic moment parallel

$$\Delta E_{HFS} = \tfrac{1}{2} aK + \tfrac{1}{4} b \frac{\tfrac{3}{2} K(K+1) - 2I(I+1)J(J+1)}{I(2I-1)J(2J-1)}$$

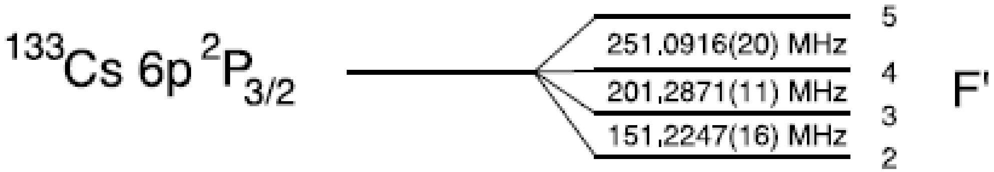
$$K = F(F+1) - J(J+1) - I(I+1)$$

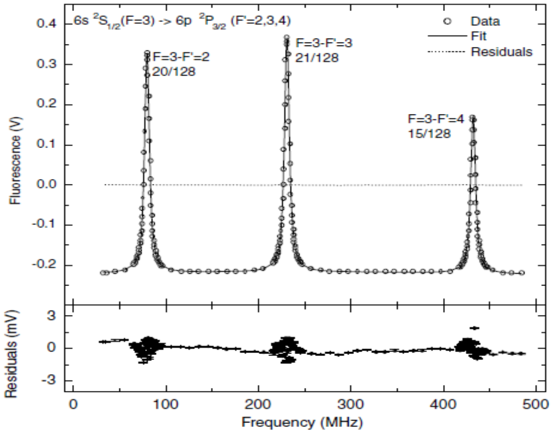


Magnetic octupole moment

- Observe magnetic octupole moment of ^{133}Cs

V. Gerginov, A. Derevianko, and C. E. Tanner, PRL 91, 072501 (2003)





$$H_{\text{dipole}} = aI \cdot J, \qquad H_{\text{quadrupole}} = b\frac{3(I \cdot J)^2 + \frac{3}{2}(I \cdot J) - I(I+1)J(J+1)}{2I(2I-1)J(2J-1)},$$

$$H_{\text{octupole}} = c\frac{\{10(I \cdot J)^3 + 20(I \cdot J)^2 + 2(I \cdot J)[-3I(I+1)J(J+1) + I(I+1) + J(J+1) + 3] - 5I(I+1)J(J+1)\}}{I(I-1)(2I-1)J(J-1)(2J-1)}$$

$$\Delta\nu_{54} = 5a + \frac{5}{7}b + \frac{40}{7}c \ + \ (-0.000520 \text{ MHz}),$$

$$\Delta\nu_{43} = 4a - \frac{2}{7}b - \frac{88}{7}c \ + \ (+0.000119 \text{ MHz}),$$

$$\Delta\nu_{32} = 3a - \frac{5}{7}b + \frac{88}{7}c \ + \ (+0.000401 \text{ MHz}),$$

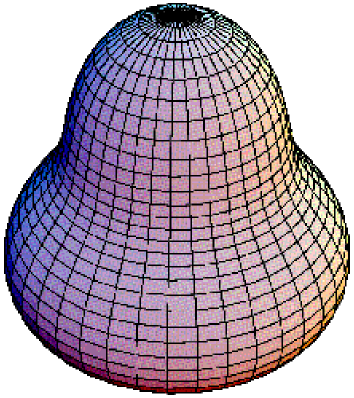
a (MHz)	b (MHz)	c (kHz)
50.288 27(23)	−0.4934(17)	0.56(7)

Magnetic octupole moment

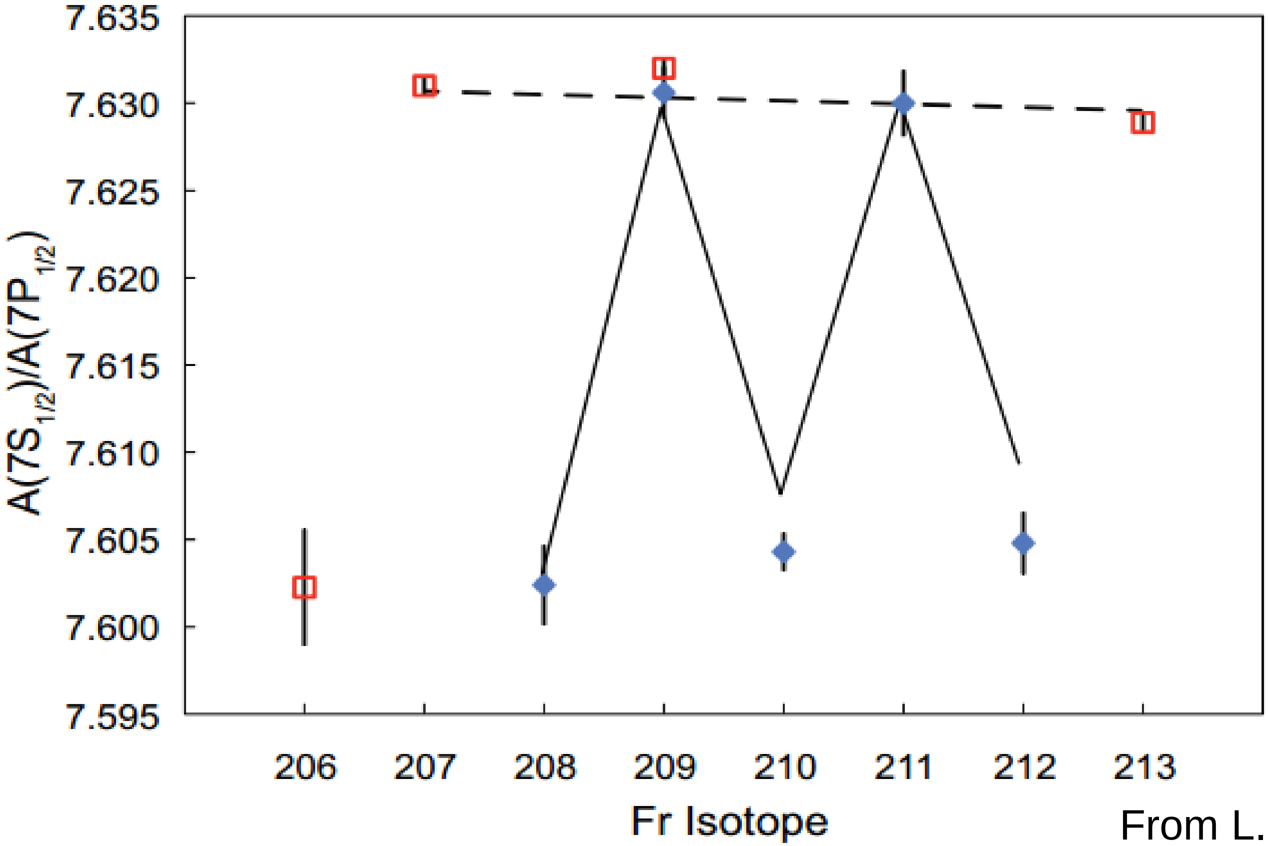
- Observe magnetic octupole moment of ^{87}Rb from spectroscopy of hyperfine intervals

  V Gerginov, C E Tanner, W R Johnson, Can. J. Phys., 87(1): 101-104 (2009)

- Large deformation for some heavy nuclei
- Comparison to nuclear shell model



Hyperfine anomaly

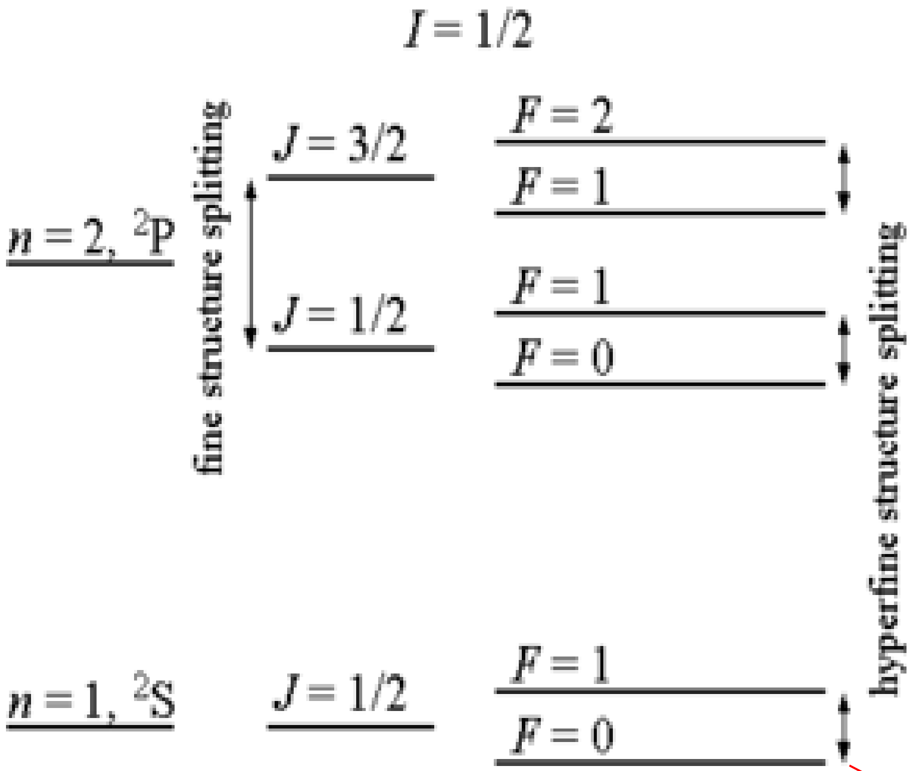


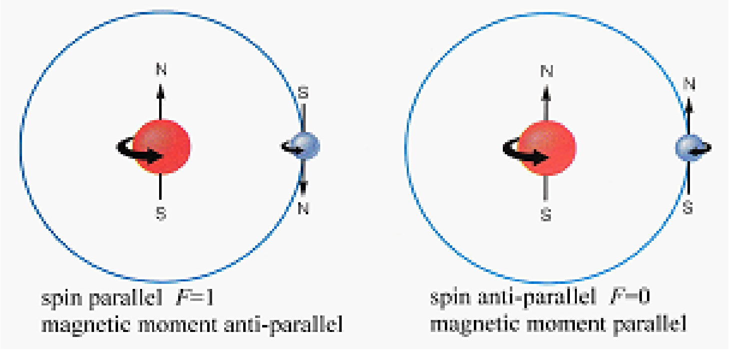
From L. Orozco, UMD

Blue points: Grossman et al., PRL **83**, 935–938 (1999)
Red points: new measurements at TRIUMF (error as of May 30, 2013)
Dashed line: magnetization radius = charge radius, normalized to ^{211}Fr
^{213}Fr ground state: Duong, et al. Europhys. Lett., **3**(2), 175-182 (1987)
^{206}Fr ground state: Voss, et al. To be published.

- Measure hyperfine constant of isotopes
- Magnetic radius ≠electric radius

Hydrogen HFS





E_{HFS} = 1420.4057517667(9) MHz = 21 cm

$$E_F^N = \frac{8}{3\pi}\alpha^3\mu_B\mu_N\frac{m_e^3 m_N^3}{(m_N + m_e)^3}$$

HFS and Zemach moment

calculable

unknown

$$E_{HFS} = (1 + \Delta_{QED} + \Delta_R + \Delta_S)E_F$$

where $$E_F = \frac{8}{3\pi}\alpha^3 \mu_B \mu_p \frac{m_e^3 m_p^3}{(m_e + m_p)^3}$$ = known

$$\Delta_S = \Delta_Z + \Delta_{pol}$$

calculable

$$= -2\alpha m_e \boxed{< r >_Z}(1 + \delta_Z^{rad}) + \boxed{\Delta_{pol}}$$

$$< r >_Z = -\frac{4}{\pi}\int_0^\infty \frac{dQ}{Q^2}\left[\frac{2G_E G_M}{g} - 1\right]$$

Zemach moment,
to be precisely determined

g-2 of electron

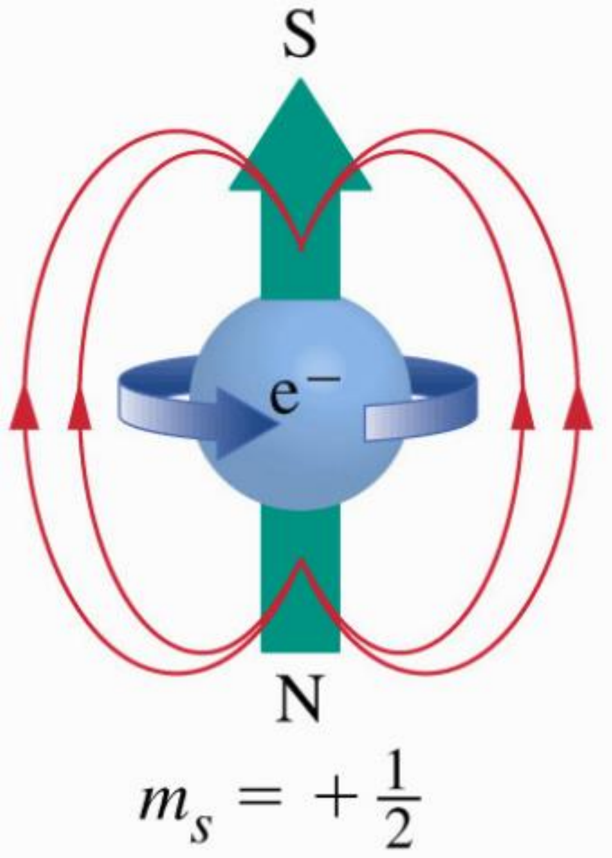
What is g-factor?

The magnetic moment of the electron is proportional to the electron spin

$$\frac{|\bar{\mu}|}{\mu_B} = g \cdot \frac{|\bar{s}|}{\hbar}$$

- Classical non-relativistic g=1
- In QM, Dirac Theory predict g=2

In reality, g= 2.002319304

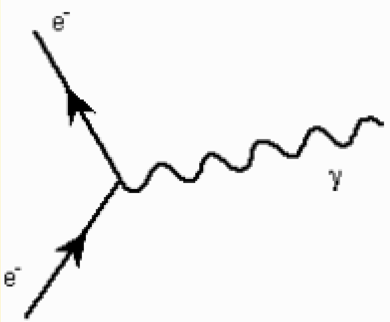


$$m_s = +\frac{1}{2}$$

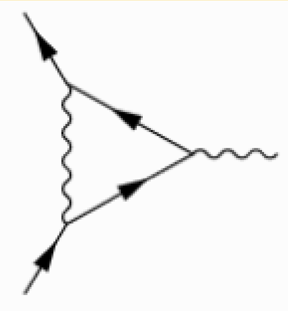
QED and g factor

- Vacuum is not real vacuum

0th order:

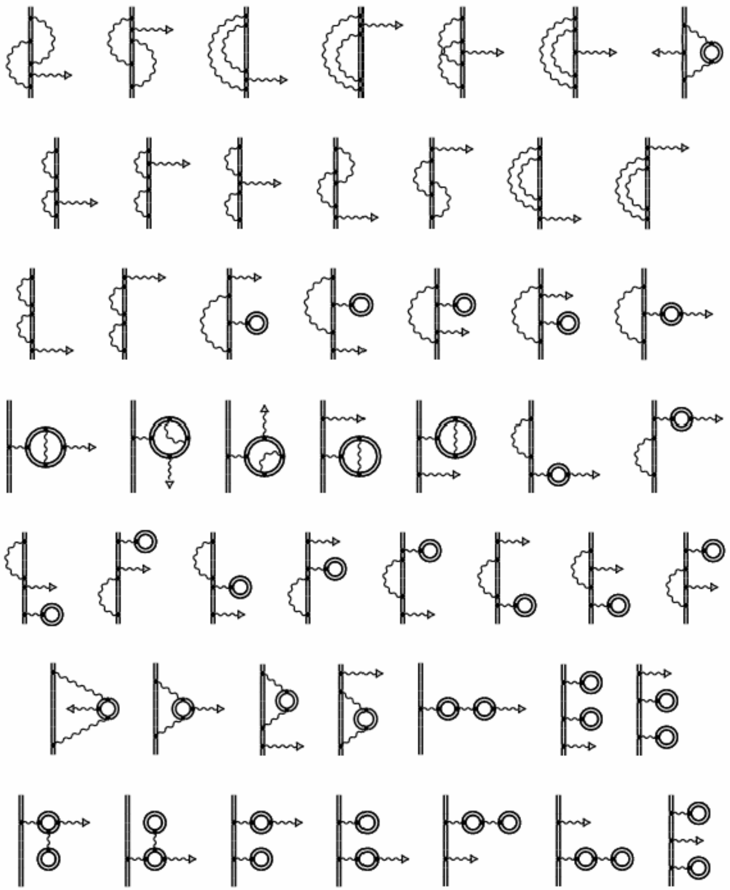


1st order:



+ …

and a lot more....



Why measure g factor

- Test QED
- Search for structure of electrons
- g can be expanded as a function of α

$$\frac{g}{2} = 1 + C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + \ldots\ldots$$

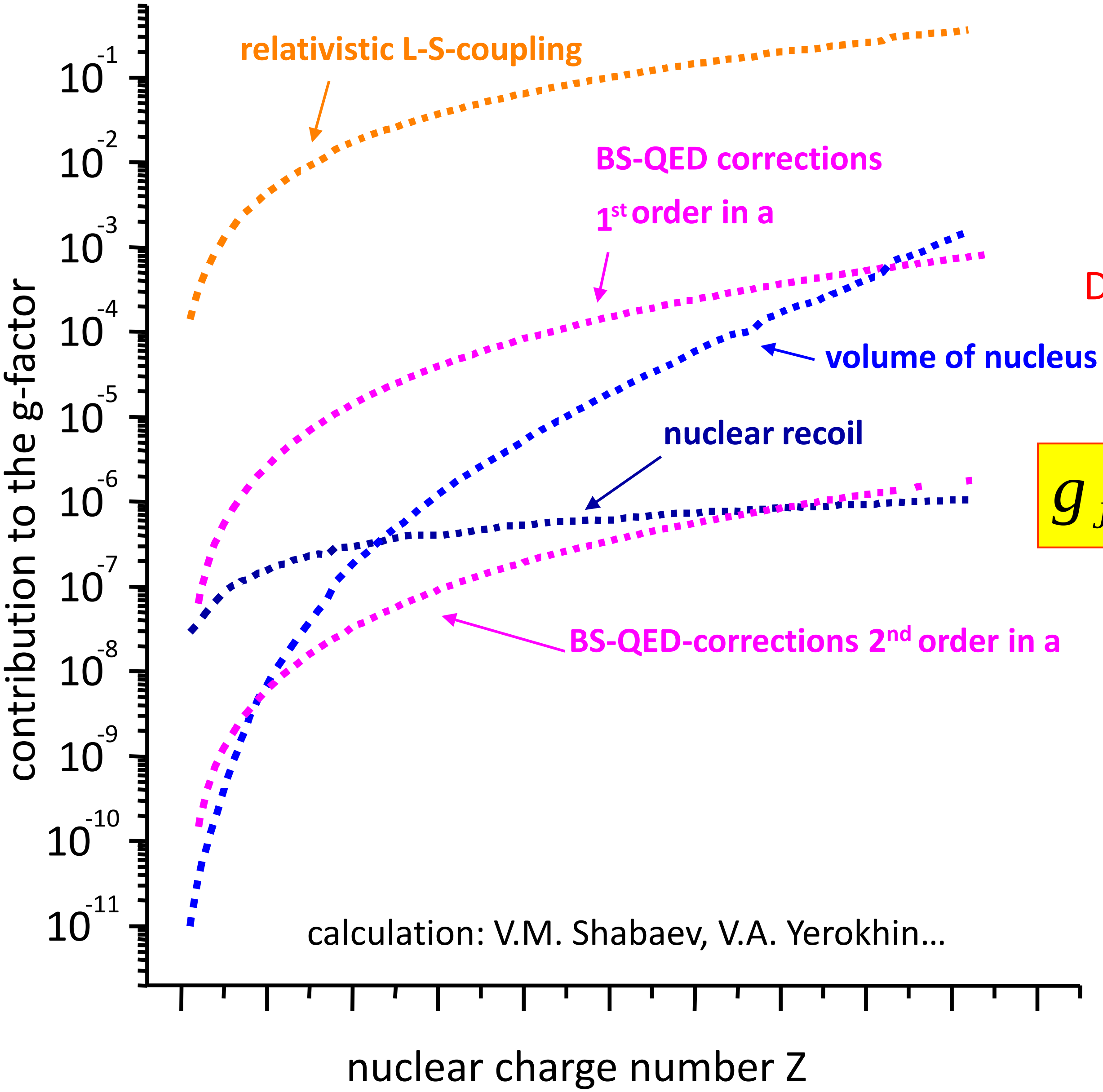
$$+ a_{\mu\tau} + a_{hadronic} + a_{weak}$$

$$\alpha = e^2/\hbar c \approx 1/137$$

α^{-1} = 137.035 999 074 (44)

D. Hanneke, S. Fogwell, and G. Gabrielse, *PRL* **100,** 120801 (2008)

Measure electron in bound system



calculation: V.M. Shabaev, V.A. Yerokhin…

QED of free electron
(exp. Dehmelt et al., Gabrielse
et al. theo. Kinoshita)

Dirac value

Nuclear
correction

$$g_J = 2 + a_e + a_b + b + c$$

Relativistic
correction

QED of bound
electron

Measure g in free space

Spin precession (Larmor) frequency

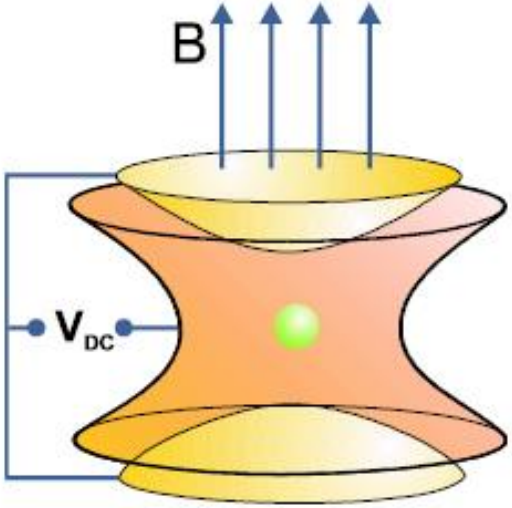
$$\hbar\omega_L = m_s g \cdot \mu_B \cdot B$$

$$\mu_B = \frac{e\hbar}{2m}$$

Calibration of magnetic field by cyclotron frequency:

$$\hbar\omega_C = \frac{q}{M} B$$

$$\boxed{g = 2\frac{\omega_L}{\omega_C}\frac{q}{e}\frac{m}{M} = 2\frac{\omega_L}{\omega_C}}$$



Measurement performed on a sing electron in Penning trap

Current status

g-2 experiment of electron and muon:

g_e (exp) = 2.00231930436146(56)

g_e (th) to determine fundamental constant

$$\left.\begin{array}{l} g_\mu \text{ (exp)} = 2.0023318\underline{416}(12)^* \\ g_\mu \text{ (th)} = 2.0023318\underline{367}(13)^* \end{array}\right\} 5\,\sigma \text{ deviation}$$

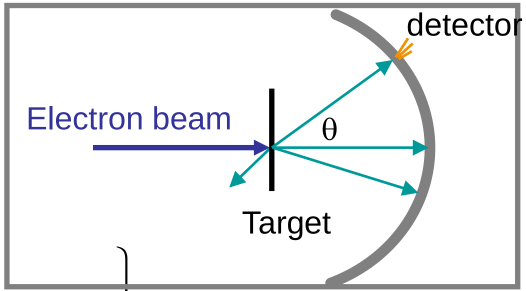
G.W. Bennett et al., *Phys Rev Lett* 92, 161802 (2004)

Muon *g*–2 at Fermilab data taken

Size of proton

- Fundamental quantity bothering for decades
- Very important for nuclear calculation
- Lattice QCD not able to calculate precisely

Charge radius of proton by electron scattering

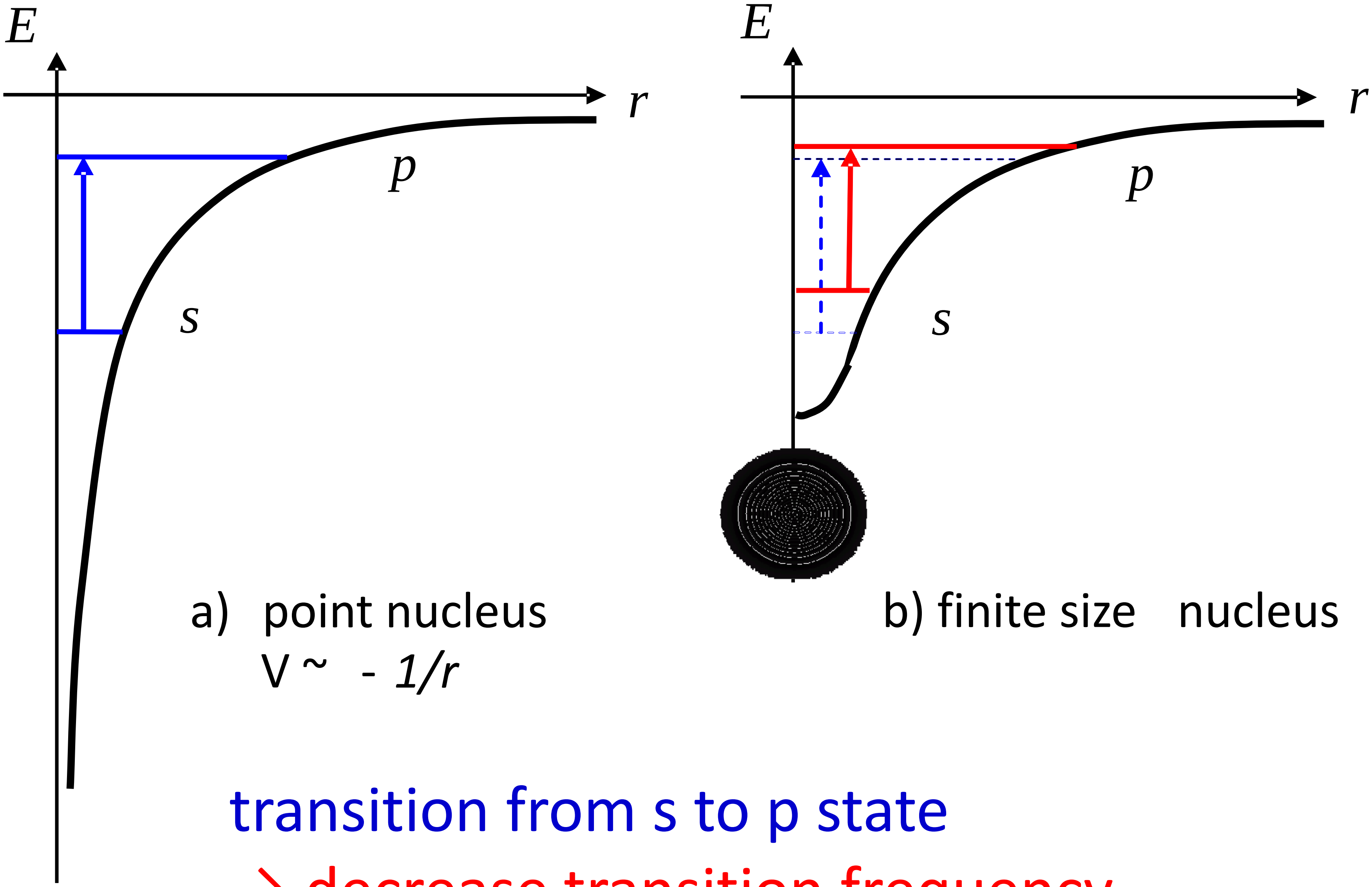


$$\left(\frac{d\sigma}{d\Omega}\right)_{Rosenbluth} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott}\left\{\left(\frac{G_E^2(Q^2)+\frac{Q^2}{4M^2}G_M^2(Q^2)}{1+\frac{Q^2}{4M^2}}\right)+\frac{Q^2}{2M^2}\cdot G_M^2(Q^2)\cdot\tan^2(\theta/2)\right\}$$

$$\Rightarrow \quad < r_c^{\;2} > = -6\frac{dG_E(Q^2)}{dQ^2}\Big|_{Q^2=0}$$

- Simon et al, (1980) R_{rms} = 0.862(12) fm
- I. Sick, (2003). R_{rms} = 0.895(18) fm

Nuclear size effect



a) point nucleus
V ~ - *1/r*

b) finite size nucleus

transition from s to p state

→ decrease transition frequency

Laser spectroscopy method

- Measure H transition frequency 1s →2s at 121 nm, uncertainty: (th) 16 kHz, (exp) 22 kHz

→ charge radius of proton R_{rms} = 0.883(14) fm

Lack of precision (only1~2%) very annoying!

Kirill Melnikov and Timo van Ritbergen, Phys. Rev. Lett. **84**, 1673(2000)

Nuclear size effect

$$E(nS) \approx \frac{-R_\infty}{n^2} + \frac{L_{1S}}{n^3} \qquad L_{1S} = (8172 + 1.56 \cdot r_p{}^2)MHz$$

- Two unknowns: R_∞ and r_p

0.883(14) fm, hydrogen spectroscopy, ENS Paris

C. Schwob et al., Phys. Rev. Lett. **82**, 4960 (1999).

K. Melnikov and T. van Ritbergen, Phys. Rev. Lett. **84**, 1673(2000).

0.890(14) fm, hydrogen spectroscopy, MPI Garching

T. Udem et al. Phys. Rev. Lett. **79**, 2646, (1997).

0.862(12) fm, original electron scattering result

G.G. Simon et.al. Nucl. Phys. A **333**, 381 (1980)

0.895(18) fm, re-analysis of world data

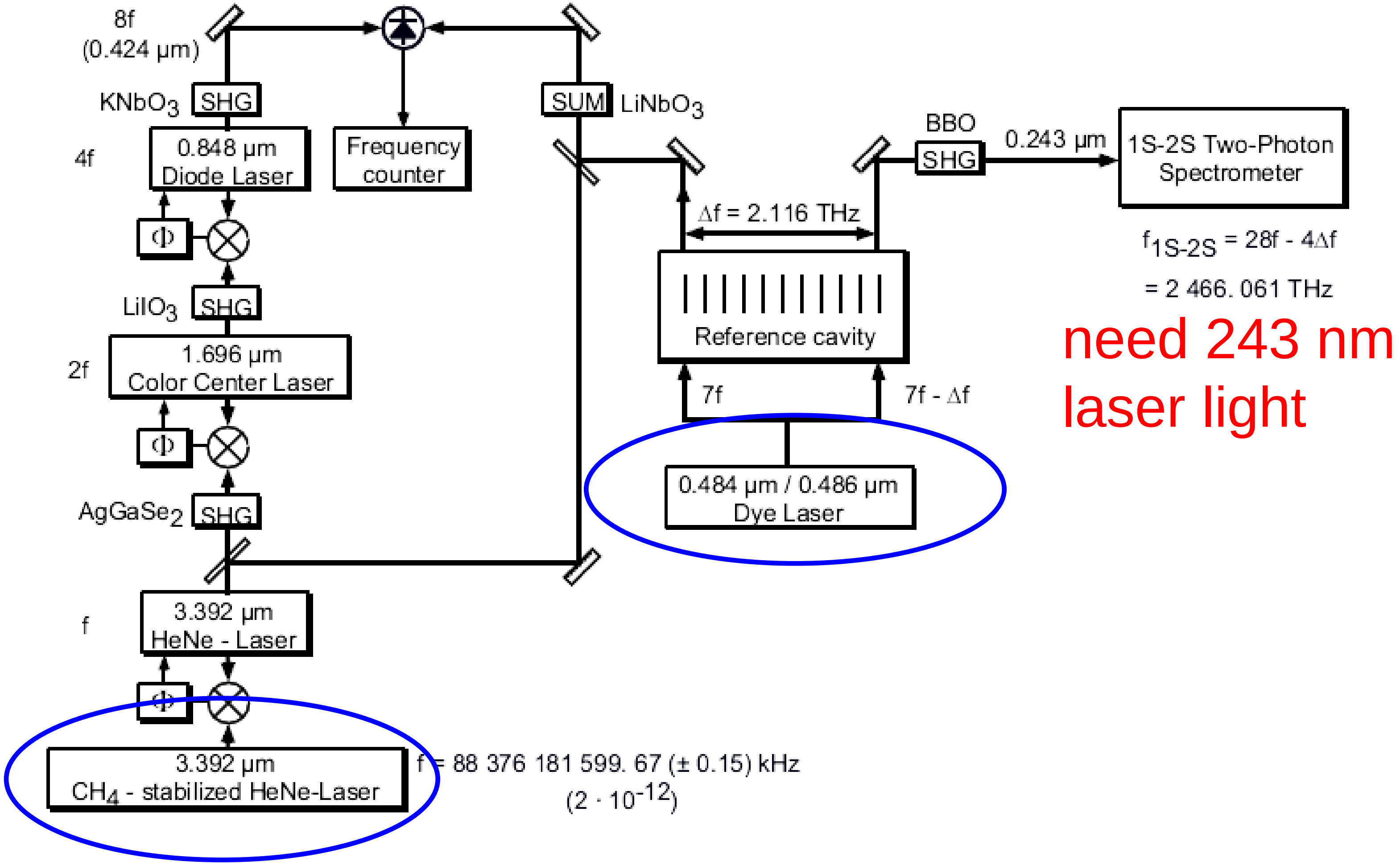
I. Sick, Phys. Lett. **B 576**, 62-67 (2003)

0.879(8) fm, new experiment by GSI

J. C. Bernauer et al., Phys. Rev. Lett. **105**, 242001 (2010)

Optical frequency measurement

Precision Spectroscopy of Atomic Hydrogen 23



need 243 nm laser light

Fig. 4. The first 1992 Garching frequency chain for the measurement of the $1S - 2S$ transition in atomic hydrogen (Φ: phase-locked loop, SHG: second harmonic generation)

Frequency chain

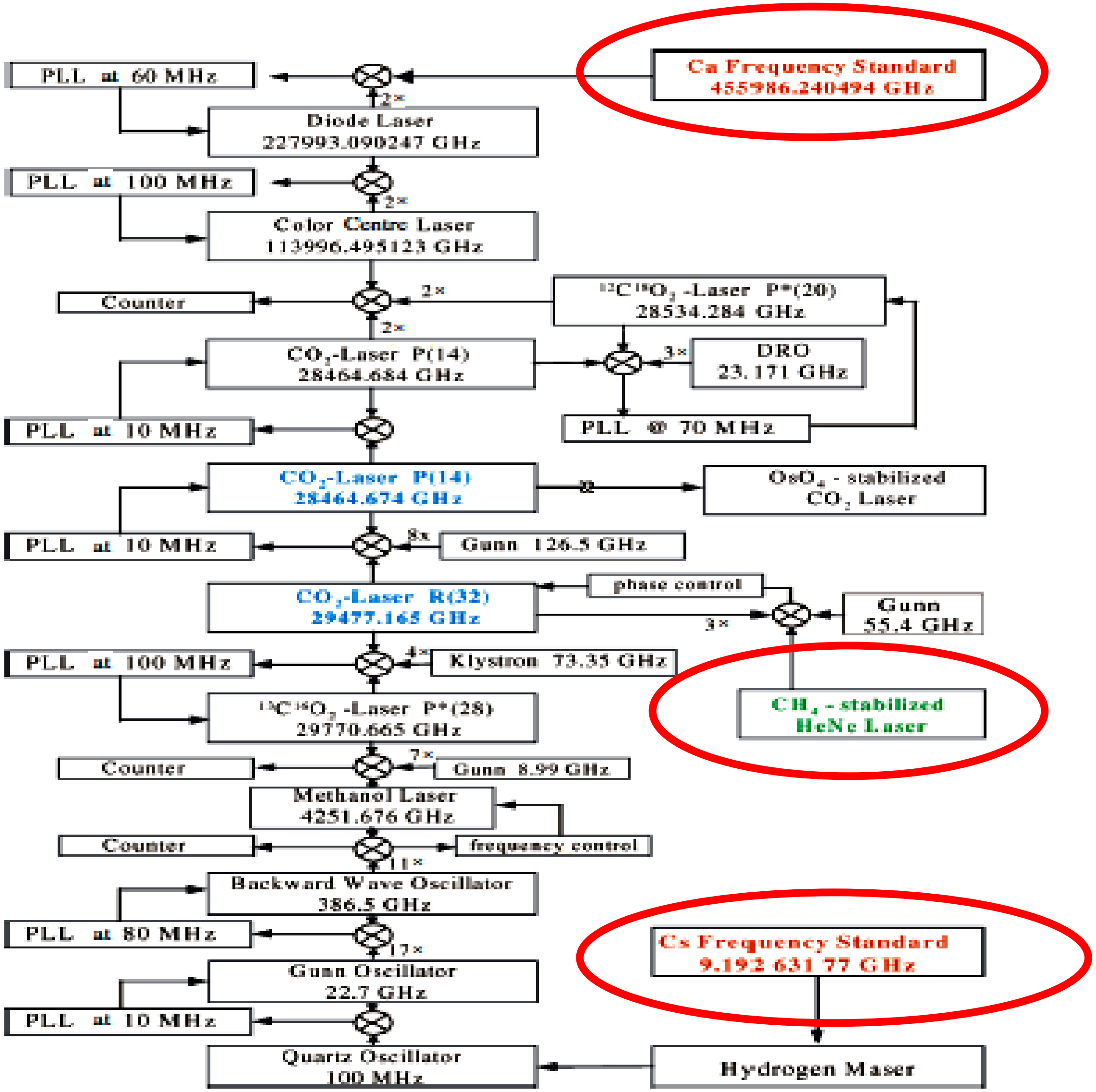
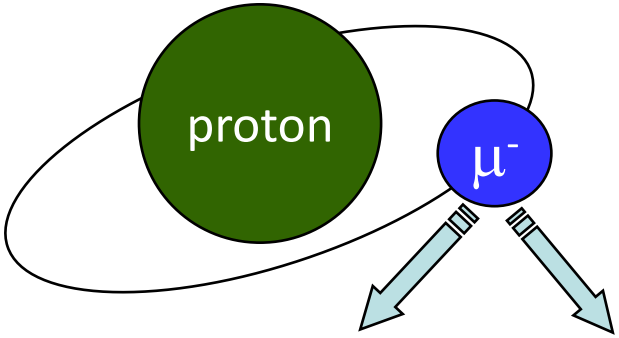


Figure 5. The PTB harmonic frequency chain was the first to achieve a phase-coherent connection between the caesium primary standard and an optical-frequency reference in the visible range. In this case, the target was the calcium optical frequency standard at 457 THz (657 nm). (Adapted with permission from [96].)

Muonic atom result



muon decay $\mu^- \rightarrow e^- \nu \bar{\nu}$

- m_μ/m_e ~ 200
- Bohr radius $$a_0 = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2}$$
- Energy level $$E_n = -\frac{e^4}{2(4\pi\varepsilon_0\hbar)^2}\frac{m_e}{n^2}$$
- Wave function $$\Psi(r) \sim a_0^{-3/2} e^{-r/a_0}$$
- Energy shift due to nuclear size~ $|\Psi(0)|^2 \langle r^2 \rangle$
- Sensitivity ~ $(m_\mu/m_e)^2$

r_p = 0.84184(67) fm

Pohl et al., Nature **466**, 213, 2010

r_p = 0.84087(39) fm

Antognini et al., Science **339**, 417, 2013

Problem in old hydrogen experiments?

Thomas et. al, arXiv:1903.04252 (2019)

r_p = 0.877(13) fm by 1S-3S spectroscopy

nature

OIL SPILLS

PLAGIARISM

CHIMPANZEES

SHRINKING THE PROTON

Frequency standard

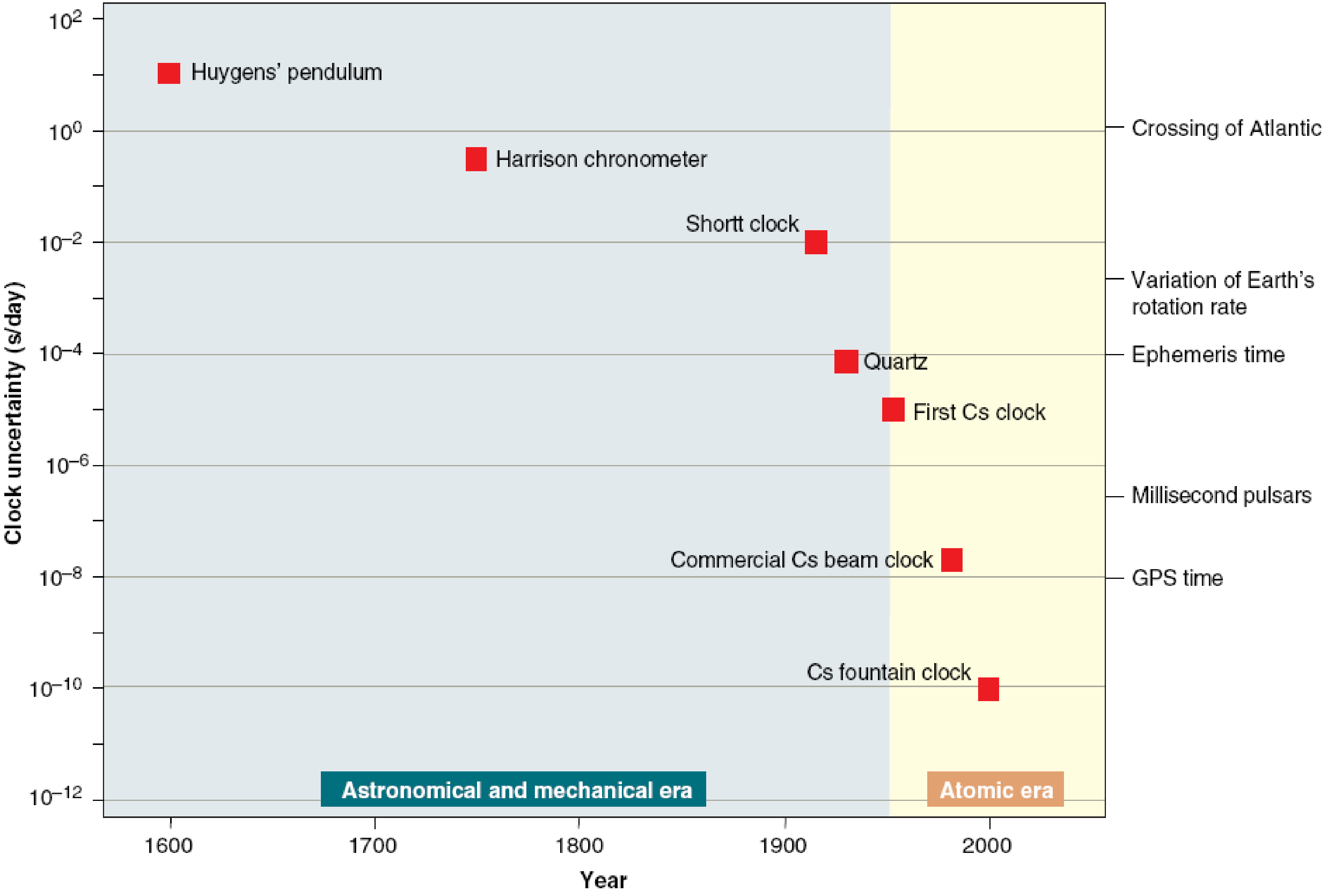
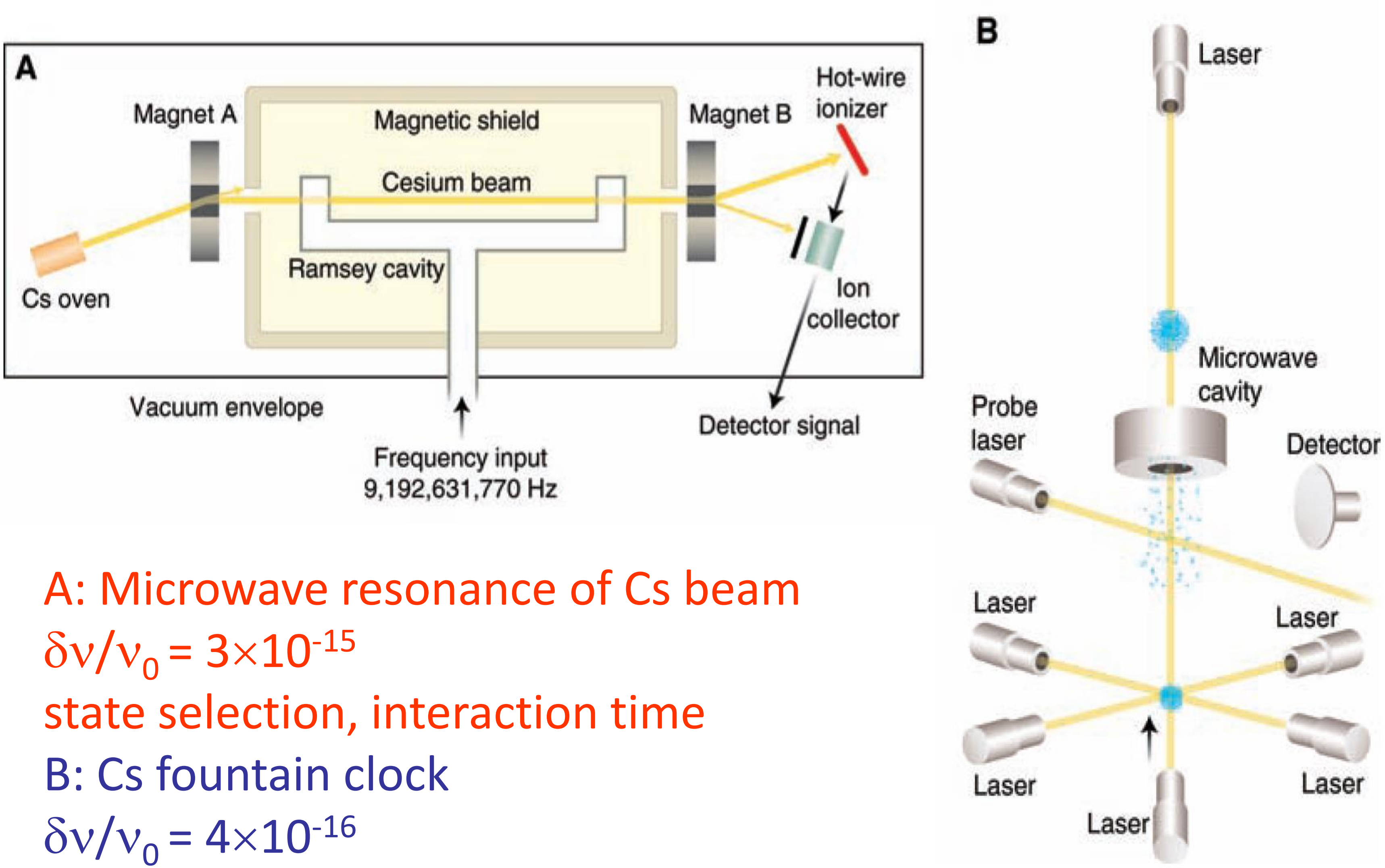


Figure: S.A. Diddams et al.,Science 306, 1318, 2004

Why better clock?

- When you measure something very precisely, new physics will come out by itself!
- Applications:
  - GPS
  - Test special relativity, gravitational red-shift
  - Gravitational wave radiation in binary pulsar
  - Test linearality of quantum mechanics
  - Variation of fundamental constant

Microwave Cs clock



A: Microwave resonance of Cs beam

$\delta\nu/\nu_0 = 3\times10^{-15}$

state selection, interaction time

B: Cs fountain clock

$\delta\nu/\nu_0 = 4\times10^{-16}$

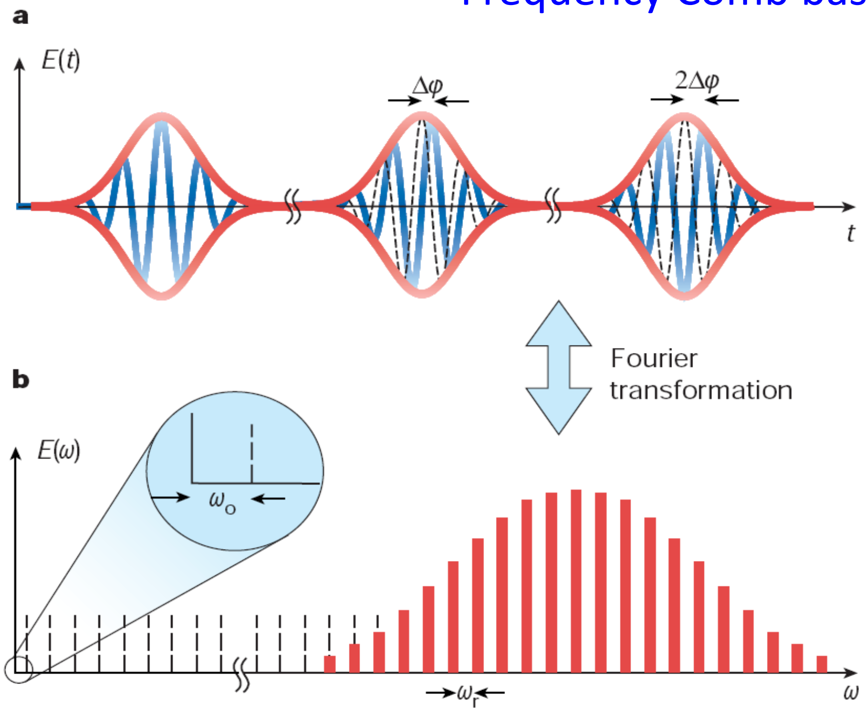
> To improve $\delta\nu/\nu_0$?

NIST Cs fountain

Optical Atomic Clock

- Frequency comb: pioneering work by T.W. Hänsch and J. Hall made frequency measurement possible

Frequency Comb basics



Picture from Th. Udem, R. Holzwarth, T.W. Hänsch, Nature 416, 233, 2002

ω_n= n ω_r + ω_{offset}

ω_r : 50MHz ~ 1GHz

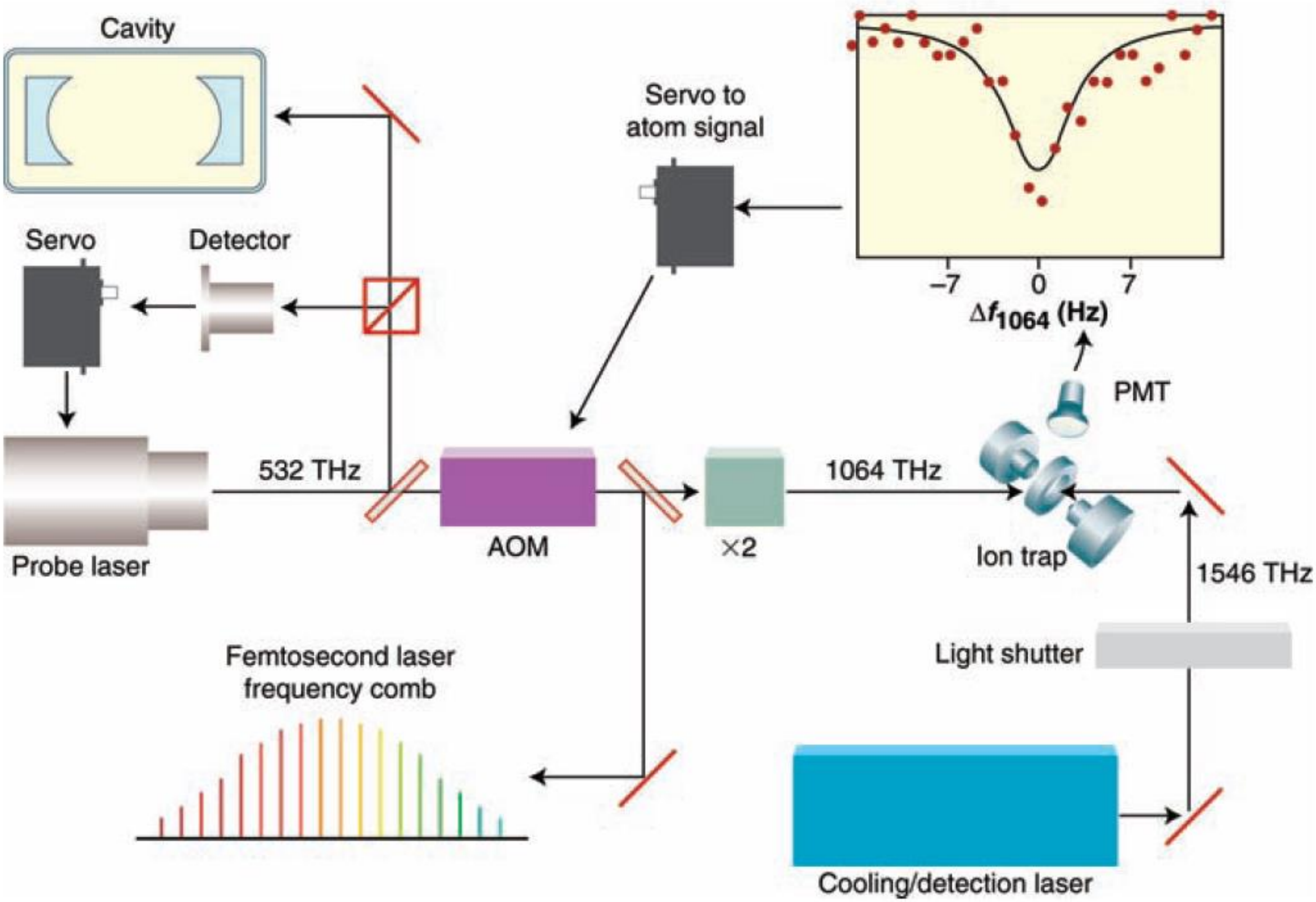
ω_r:
Determined by cavity length

ω_{offset}:
measured by f-2f technique

Optical Atomic Clock

- Trapped ions: Hg^+, Ca^+, Sr^+, Yb^+, Ba^+, In^+, Al^+.....
  - Long interaction time, can be laser cooled
  - Single ion detection, poor S/N
- Trapped neutral atoms: Ca, Sr, Yb, Mg, H.....
  - Large number of atoms, very high S/N
  - Complicated systematics
- Precision ~ 10^{-18} $(Hz^{1/2})$

How optical clock work



S.A. Diddams, J.C. Bergquist, S.R. Jefferts, C.W. Oats, Science 306, 1318, 2004

Constancy of Fine Structure Constant

$$\alpha = e^2/\hbar c \approx 1/137$$

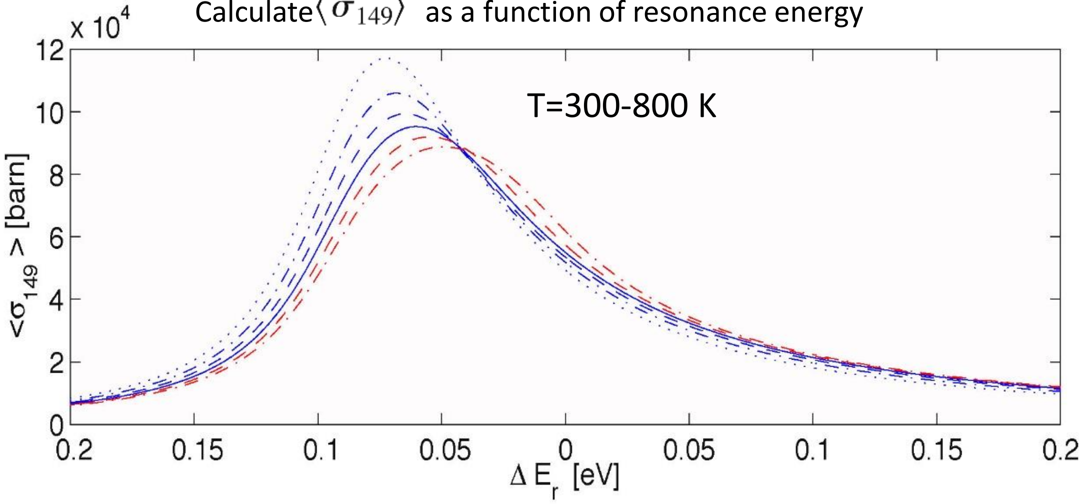
α^{-1} = 137.035 999 074 (44)

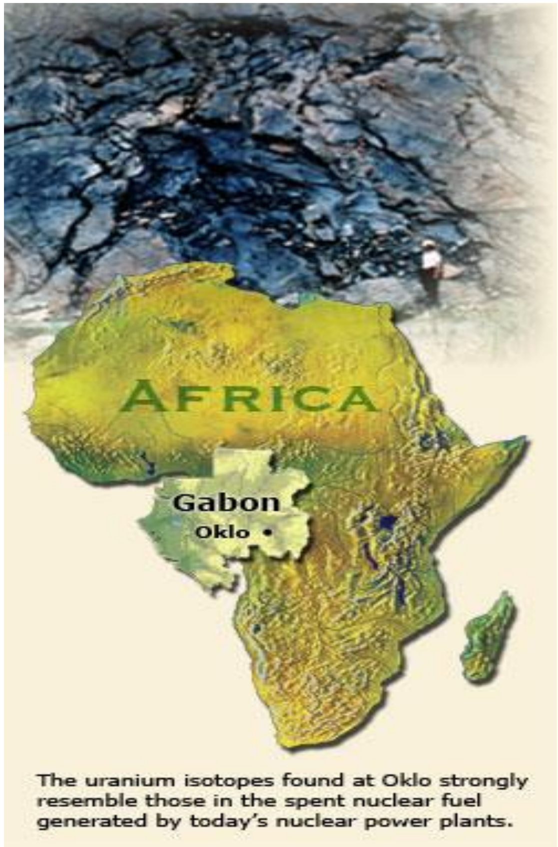
- How constant is the constant?
- Two reports show the constant may change in time
- Oklo phenomenon
- Quasar spectra

The Oklo Natural Nuclear Reactor

- Natural nuclear reactor 1.9 billion years ago
- ^{235}U abundance very low
- Fission of ^{235}U, neutron slowed by water
- $n + {}^{149}Sm \rightarrow {}^{150}Sm + \gamma$, $E_r = 97.3$ meV

Calculate $\langle \sigma_{149} \rangle$ as a function of resonance energy





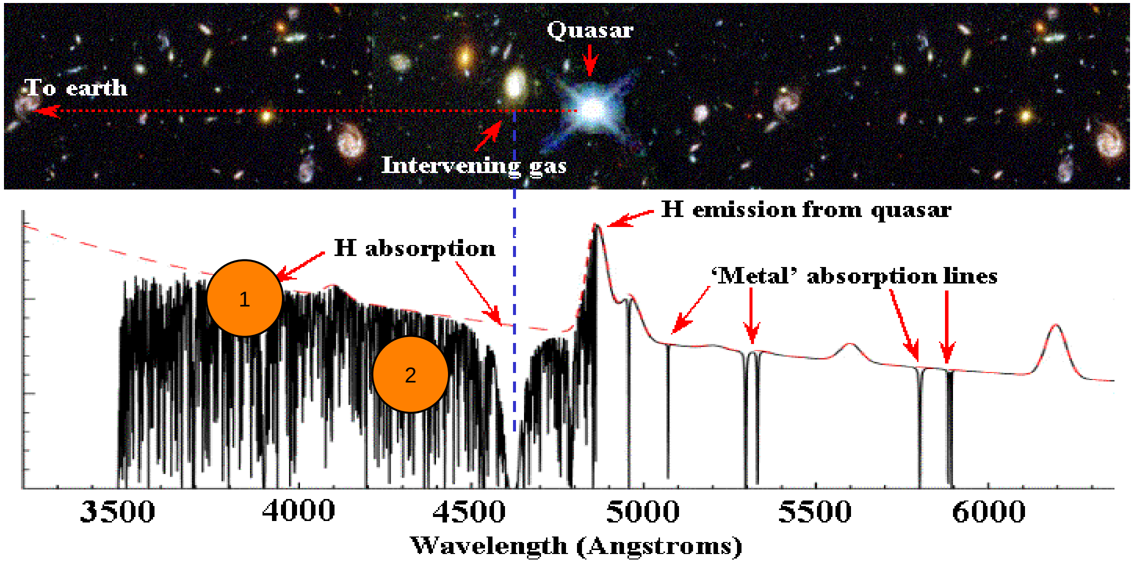
The uranium isotopes found at Oklo strongly resemble those in the spent nuclear fuel generated by today's nuclear power plants.

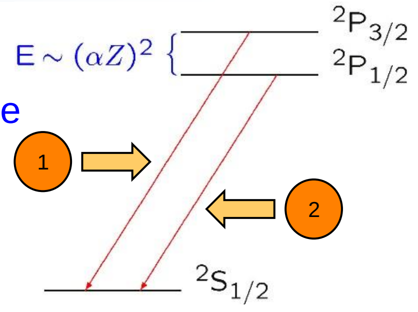
- ^{149}Sm isotope abundance, → neutron absorption cross section σ_{149}
- From $\sigma_{149} \rightarrow \Delta E_r$, assume reasonable T

$$\frac{\Delta E_r}{1.1MeV} = \frac{\Delta\alpha}{\alpha}$$

Δ Er < 0.1 eV, $\Delta\alpha/\alpha < 10^{-7}$

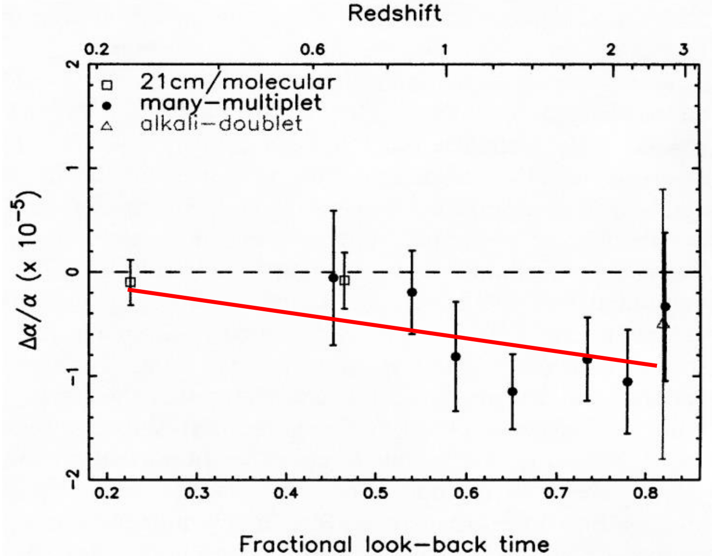
Quasar Absorption Spectra





fine structure doublet

$\Delta\alpha/\alpha$	z	t
$(7.2 \pm 1.8)\cdot 10^{-6}$	0.5–2.6	0.38–0.82
$\longrightarrow (6.4 \pm 1.4)\cdot 10^{-16}/\mathrm{yr}$		



(1) Atoms located in different regions with different velocities

→ different Doppler red shift

(2) Isotopic abundance in the past may change

→ shift the position of absorption line

(3) Magnetic field shift the energy levels by Zeeman effect

Results From Searches for dα/dt

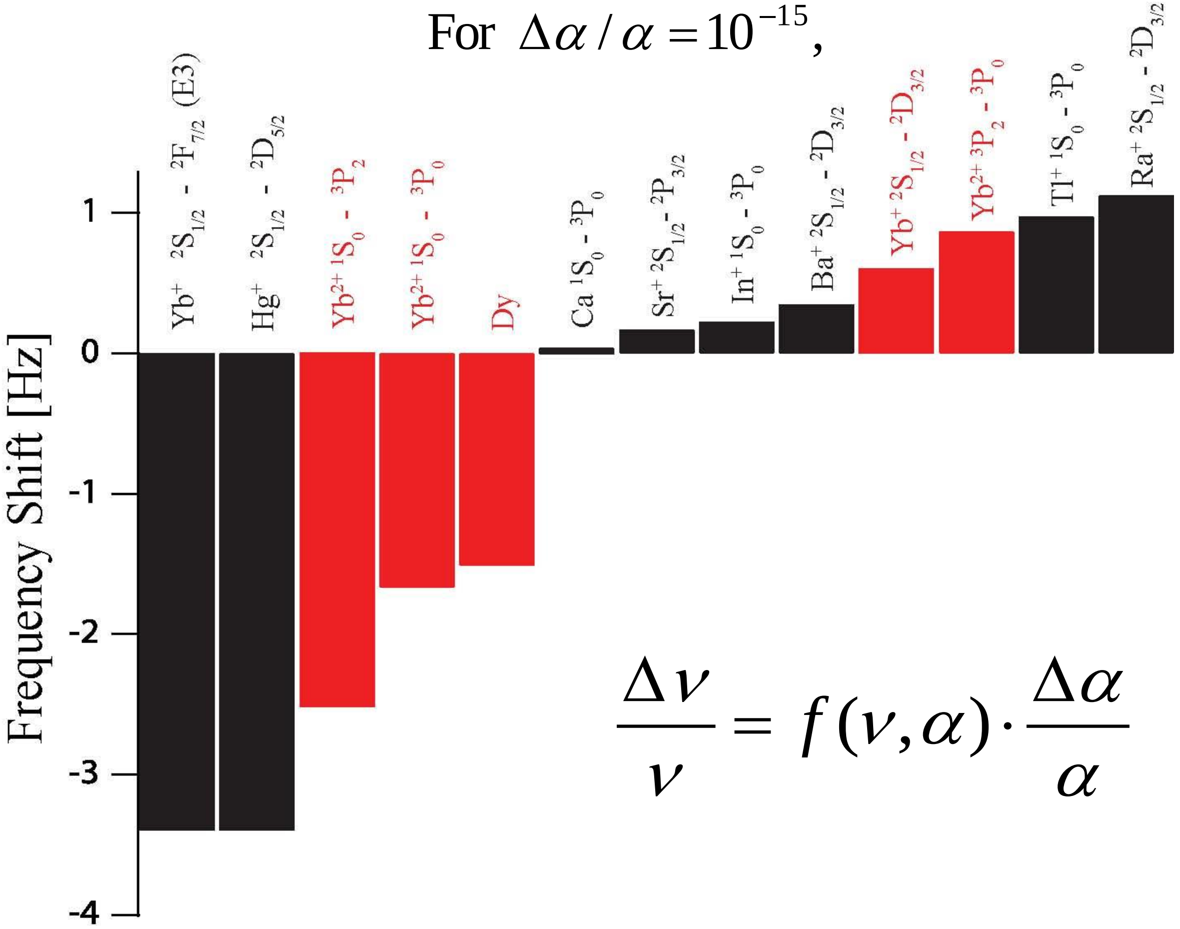
Δα/α per year	method	quantity	ref
$-(2.3\ +0.7/-0.3)\ 10^{-17}$	Oklo nuclear reactor	$\alpha(m_\pi/\Lambda_{qcd})^{0.5}$	1
$<\ (0.8)\ 10^{-17}$	Oklo nuclear reactor	$\alpha(m_\pi/\Lambda_{qcd})^{0.5}$	2
$(7.2 \pm 1.8)\ 10^{-16}$	Quasar spectra	α	3
$(6.4 \pm 1.4)\ 10^{-16}$	Quasar spectra	α	4
$(0.6 \pm 0.6)\ 10^{-16}$	Quasar spectra	α	5
$(4.4 \pm 1.6)\ 10^{-16}$	Quasar spectra	α	6
$(-2.7 \pm 2.6)\ 10^{-15}$	Dy atomic beam	α	7
$(-0.9 \pm 2.9)\ 10^{-15}$	Opt-H ↔ Cs	$g_{Cs} m_e/m_p \alpha^{2.8}$	8
$(0.3 \pm 2.0)\ 10^{-15}$	Opt-Yb$^+$ ↔ Cs	$g_{Cs} m_e/m_p \alpha^{1.9}$	9
$(0.02 \pm 0.7)\ 10^{-15}$	Rb ↔ Cs	$g_{Cs}/g_{Rb} \alpha^{0.44}$	10
$(-6.2 \pm 6.5)\ 10^{-17}$	Opt-Hg$^+$ ↔ Cs	$g_{Cs} m_e/m_p \alpha^{6.0}$	11
$(-1.6 \pm 2.3) 10^{-17}$	Opt-Hg$^+$ ↔ Opt-Al$^+$	α	12

[1] S. K. Lamoreaux and J. R. Torgerson, Phys. Rev. D **69**, 121701R (2004).
[2] Y. Fujii *et al.*, Nuclear Physics B **573, 377** (2000).
[3] J. K. Webb *et al.*, Phys. Rev. Lett. **87**, 091301 (2001).
[4] M. T. Murphy, J. K. Webb, and V. V. Flambaum, Mon. Not. R. Astron. Soc. **345**,609 (2003).
[5] R. Srianand *et al.*, Phys. Rev. Lett. **92**, 121302 (2004).
[6] M. T. Murphy, J. K. Webb, and V. V. Flambaum, astro-ph/0612407 (2006).
[7] A. Cingöz *et al.*, Phys. Rev. Lett. **98**, 040801 (2007).
[8] M. Fischer *et al.*, Phys. Rev. Lett. **92**, 230802 (2004).
[9] E. Peik *et al.*, Phys. Rev. Lett. **93**, 170801 (2004).
[10] H. Marion *et al.*, Phys. Rev. Lett. **90**, 150801 (2003).
[11] T. M. Fortier *et al.*, Phys. Rev. Lett. **98**, 070801 (2007).
[12] T. Rosenband et al., Science **319**, 1808 (2008)

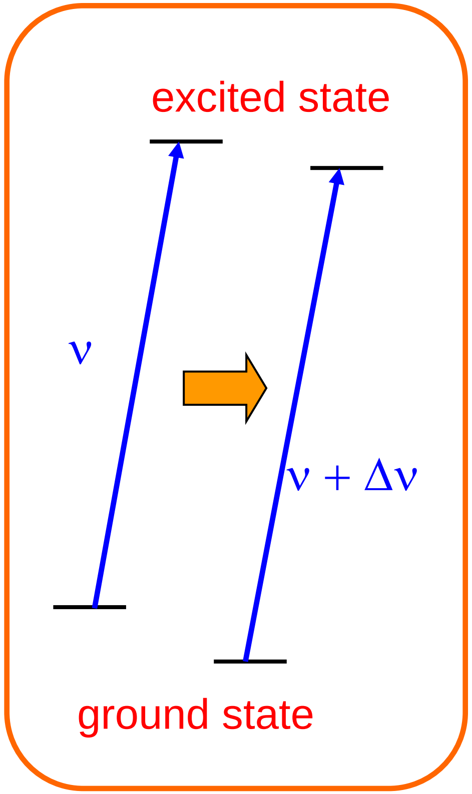
Laboratory Test

- Laboratory and controllable test strongly desired
- Ultra-cold trapped ions and atoms best testing ground

For $\Delta\alpha/\alpha = 10^{-15}$,



$$\frac{\Delta\nu}{\nu} = f(\nu,\alpha)\cdot\frac{\Delta\alpha}{\alpha}$$



Results From Searches for dα/dt

Δα/α per year	method	quantity	ref
-(2.3 +0.7/-0.3) 10^{-17}	Oklo nuclear reactor	$\alpha(m_\pi/\Lambda_{qcd})^{0.5}$	1
< 0.8 10^{-17}	Oklo nuclear reactor	$\alpha(m_\pi/\Lambda_{qcd})^{0.5}$	2
(7.2 ± 1.8) 10^{-16}	Quasar spectra	α	3
(6.4 ± 1.4) 10^{-16}	Quasar spectra	α	4
(0.6 ± 0.6) 10^{-16}	Quasar spectra	α	5
(4.4 ± 1.6) 10^{-16}	Quasar spectra	α	6
(-2.7 ± 2.6) 10^{-15}	Dy atomic beam	α	7
(-0.9 ± 2.9) 10^{-15}	Opt-H ↔ Cs	$g_{Cs} m_e/m_p \alpha^{2.8}$	8
(0.3 ± 2.0) 10^{-15}	Opt-Yb⁺ ↔ Cs	$g_{Cs} m_e/m_p \alpha^{1.9}$	9
(0.02 ± 0.7) 10^{-15}	Rb ↔ Cs	$g_{Cs}/g_{Rb} \alpha^{0.44}$	10
(-6.2 ± 6.5) 10^{-17}	Opt-Hg⁺ ↔ Cs	$g_{Cs} m_e/m_p \alpha^{6.0}$	11
(-1.6 ± 2.3)10^{-17}	Opt-Hg⁺ ↔ Opt-Al⁺	α	12

[1] S. K. Lamoreaux and J. R. Torgerson, Phys. Rev. D **69**, 121701R (2004).
[2] Y. Fujii *et al*., Nuclear Physics B **573, 377** (2000).
[3] J. K. Webb *et al.*, Phys. Rev. Lett. **87**, 091301 (2001).
[4] M. T. Murphy, J. K. Webb, and V. V. Flambaum, Mon. Not. R. Astron. Soc. **345**,609 (2003).
[5] R. Srianand *et al.*, Phys. Rev. Lett. **92**, 121302 (2004).
[6] M. T. Murphy, J. K. Webb, and V. V. Flambaum, astro-ph/0612407 (2006).
[7] A. Cingöz *et al.*, Phys. Rev. Lett. **98**, 040801 (2007).
[8] M. Fischer *et al.*, Phys. Rev. Lett. **92**, 230802 (2004).
[9] E. Peik *et al.*, Phys. Rev. Lett. **93**, 170801 (2004).
[10] H. Marion *et al.*, Phys. Rev. Lett. **90**, 150801 (2003).
[11] T. M. Fortier *et al.*, Phys. Rev. Lett. **98**, 070801 (2007).
[12] T. Rosenband et al., Science **319**, 1808 (2008)

Permanent electric dipole moment

- 1957 Landau first pointed out the electric dipole moment (EDM) of a fundamental particle would suggest P and T violation

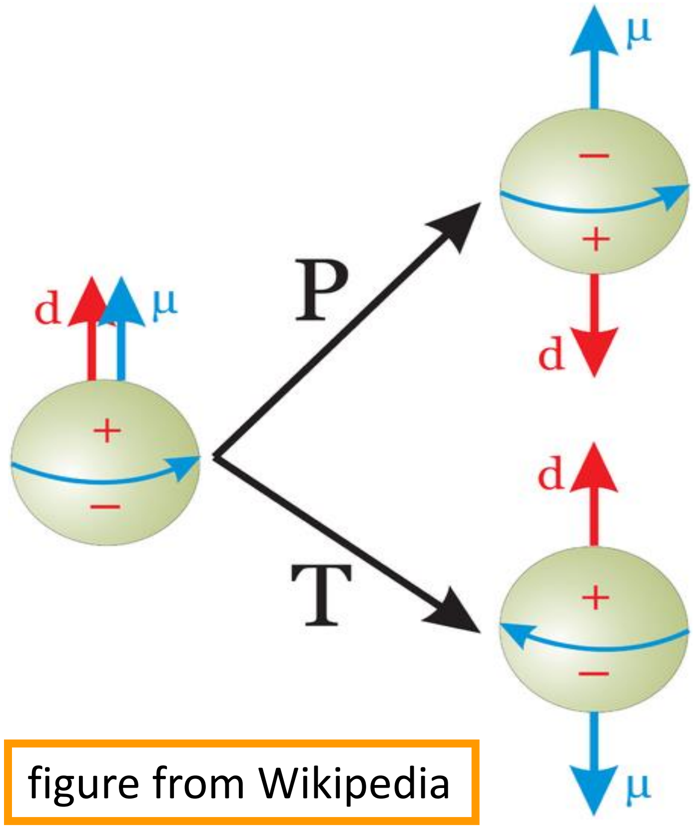


figure from Wikipedia

d: electric dipole moment
= vector
u: spin or magnetic moment
= pseudo-vector, (like **r** x **p**)

Neutron EDM: history

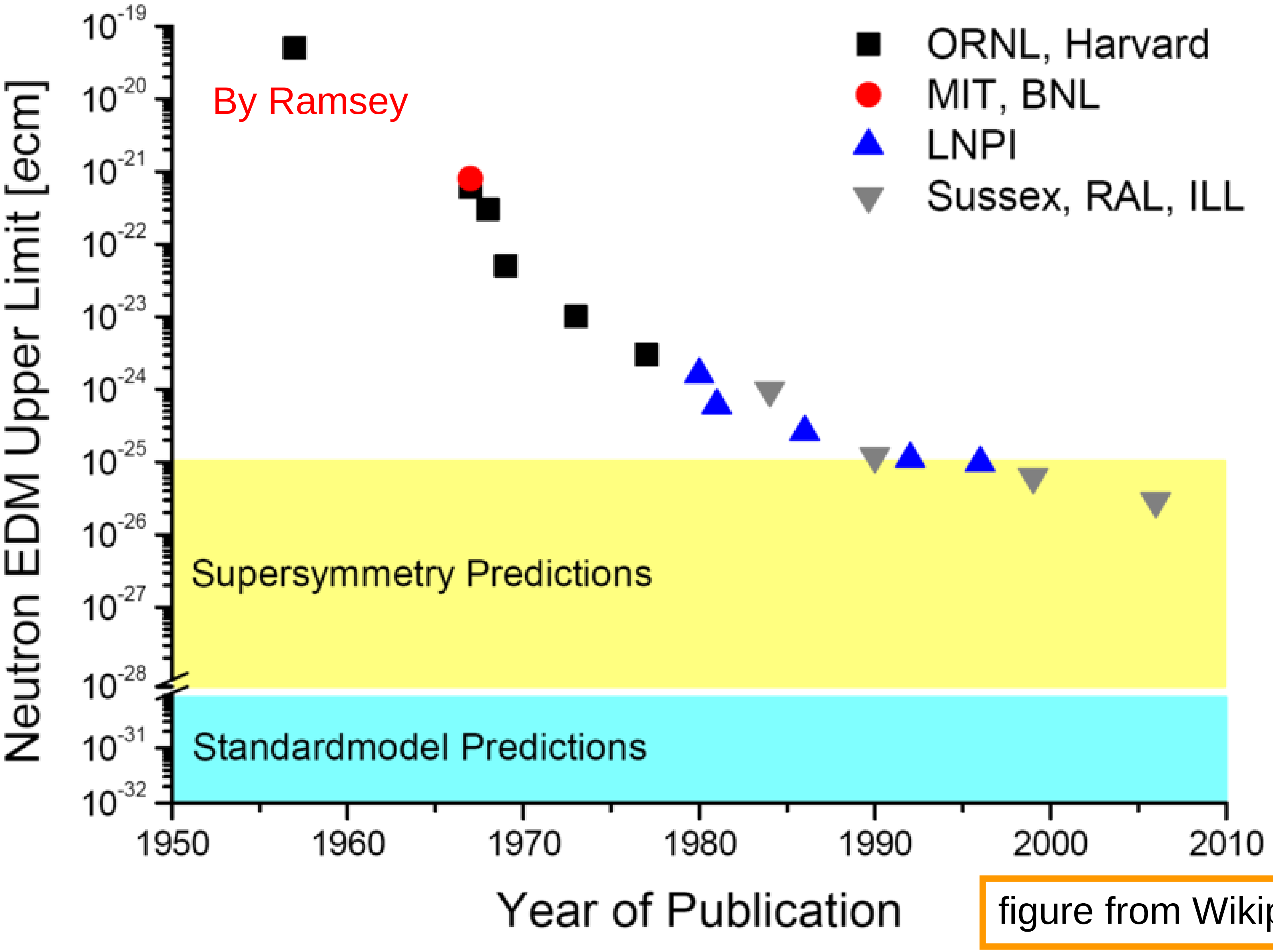


figure from Wikipedia

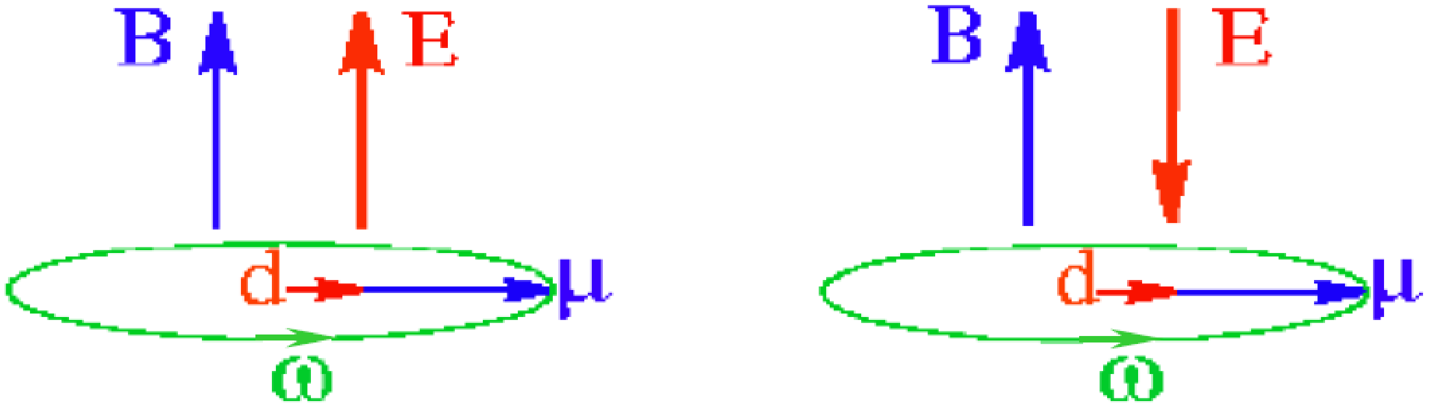
CP violation without strangeness

- Matter–antimatter asymmetry suggest another source of CP violation
  - Electron: intrinsic ?
  - Quark: intrinsic ?
  - Neutron/proton:
    from quark EDM ? New interaction?
  - Atoms, molecules:
    large enhancement factors

Atoms: Tl, Cs, Hg, Xe, Rn, Ra, Fr
Molecule: PbO, YbF, ThO
Ions, solid state systems, etc…

How to measure EDM

$$H = -\vec{\mu}\cdot\vec{B} - \vec{d}\cdot\vec{E}$$



$$\omega_1 = \frac{2\mu B + 2dE}{\hbar}$$

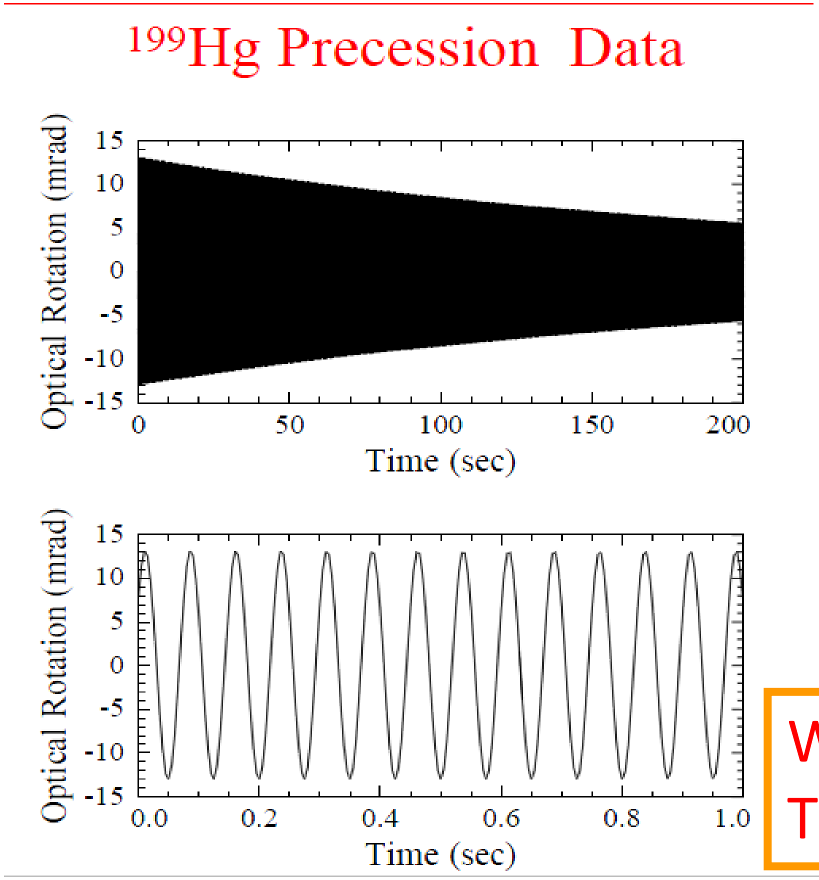
$$\omega_2 = \frac{2\mu B - 2dE}{\hbar}$$

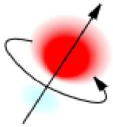
$$\omega_1 - \omega_2 = \frac{4dE}{\hbar}$$

figure from M. Romalis, Princeton

Problem:
reverse E field cause leakage current,
huge noise from B field

Current status





Typical value

E ~ 10 kV/cm

ω~ 10Hz, $\delta\omega$ ~ nHz

Sensitivity approaching
some Standard Model extension

What if someone observes EDM?
Theories ready with many parameters

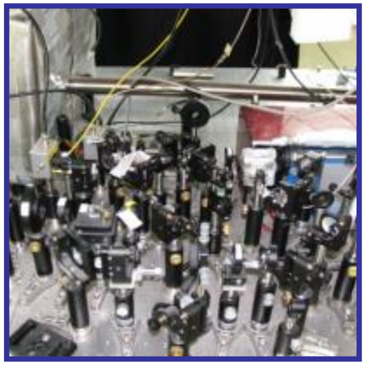
figure from M. Romalis, Princeton

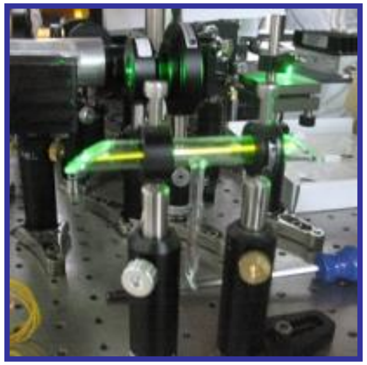
Laser spectroscopy

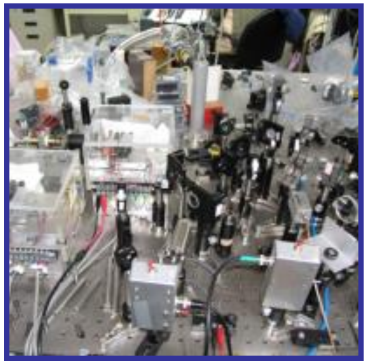
Second lecture:

Laser spectroscopy of simple atoms

work performed at NTHU Physics







Atomic spectroscopy

Precision spectroscopy of simple atoms

- Hydrogen: exact solution exist
- Helium: numerical method
- Lithium: numerical method

Remain as input for atomic calculation

- Magnetic moment of nucleus
- Charge radius of nucleus

Motivation

- Calculable atomic structure: precision measurement test fundamental physics
- Advances in atomic theories, e.g. in H, He, Li, He^+, Li^+, Be^+ etc…
- New experimental techniques, e.g.: cold atoms by atom and ion trap, frequency comb, exotic atoms, etc…
- What fundamental physics to study?

Atomic structure (Hydrogen)

- non-relativistic

$$E_n = -\alpha^2 mc^2\left(\frac{1}{2n^2}\right) = \frac{-13.6eV}{n^2}$$

- relativistic correction

$$-\alpha^4 mc^2 \frac{1}{4n^2}\left[\frac{2n}{(l+1/2)} - \frac{3}{2}\right]$$

- spin-orbit interaction (L·S, fine structure)

$$-\alpha^4 mc^2 \frac{1}{4n^2}\left[\frac{2n}{(j+1/2)} - \frac{3}{2}\right]$$

- nuclear moment (HFS)

$$\left(\frac{m}{m_p}\right)\alpha^4 mc^2 \frac{4\gamma_p}{3n^2}[f(f+1)-3/2]$$

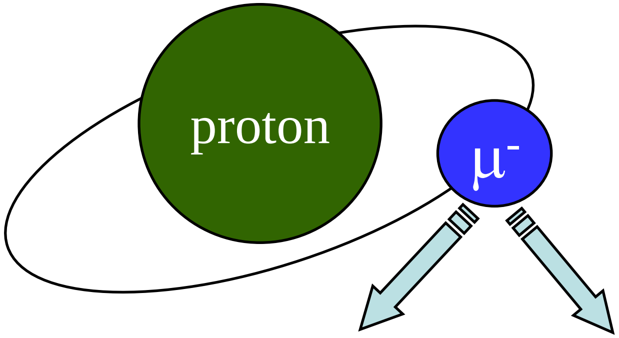
- QED effect (Lamb shift)

$$\alpha^5 mc^2 \frac{1}{4n^3}\left\{k(n,l) \pm \frac{1}{\pi(j+1/2)(l+1/2)}\right\}$$

- nuclear size effect

$$\frac{2\pi}{3} Ze^2 |\psi(0)|^2 \langle r^2 \rangle_{proton}$$

Muonic atom result



muon decay $\mu^- \rightarrow e^- \nu \bar{\nu}$

- $m_\mu/m_e \sim 200$
- Bohr radius $a_0 = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2}$
- Energy level $E_n = -\frac{e^4}{2(4\pi\varepsilon_0\hbar)^2}\frac{m_e}{n^2}$
- Wave function $\Psi(r) \sim a_0^{-3/2} e^{-r/a_0}$
- Energy shift due to nuclear size~ $|\Psi(0)|^2 \langle r^2 \rangle$
- Sensitivity ~ $(m_\mu/m_e)^2$

$r_p = 0.84184(67)$ fm

Pohl et al., Nature **466**, 213, 2010

$r_p = 0.84087(39)$ fm

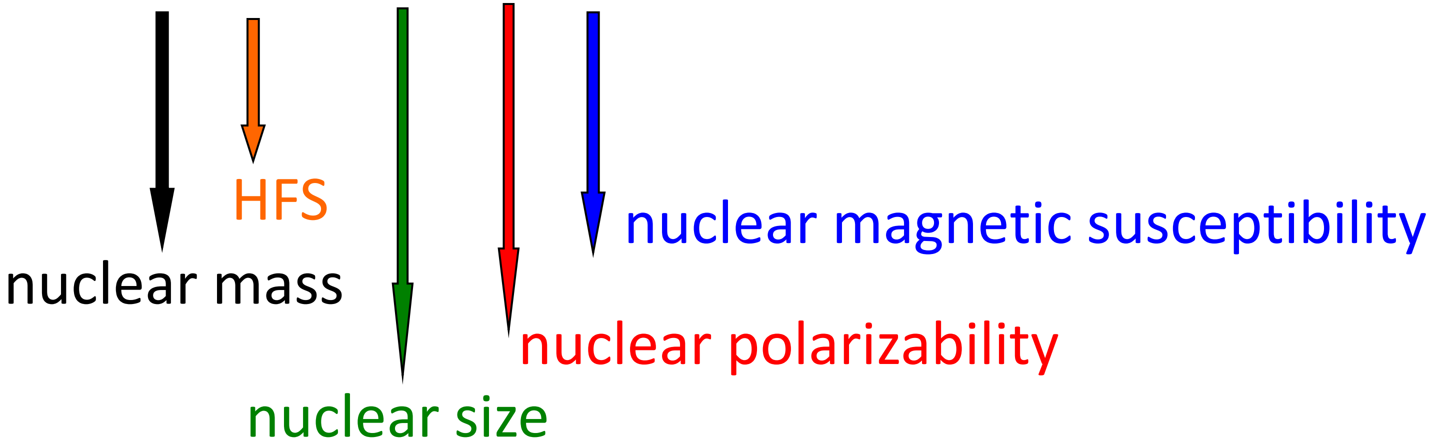
Antognini et al., Science **339**, 417, 2013

Need more experimental verification of atomic QED calculations

Hydrogen isotope shift

- Measure isotope shift (H-D), 1s - 2s at 121 nm

$\Delta\nu$ = 670 994.33464(15) MHz



HFS

nuclear mass

nuclear magnetic susceptibility

nuclear polarizability

nuclear size

Reference:

[1] Kirill Melnikov and Timo van Ritbergen, Phys. Rev. Lett. **84**, 1673(2000)

[2] A. Huber, Th. Udem, B. Gross, J. Reichert, M. Kourogi, K. Pachucki, M. Weitz, and T.W. Hänsch. Phys. Rev. Lett. **80**, 468 (1998)

Precision of Theory

Total transition frequency (optical):

- QED effects
- H: ~10 kHz, He: ~1 MHz, Li: > 10 MHz

Isotope shift:

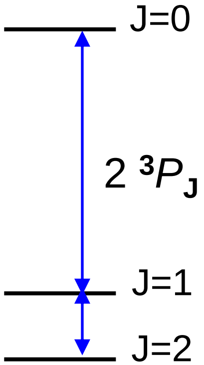
- Nuclear charge radius
- H: ~1 kHz, He: ~1 kHz, Li: ~10 kHz

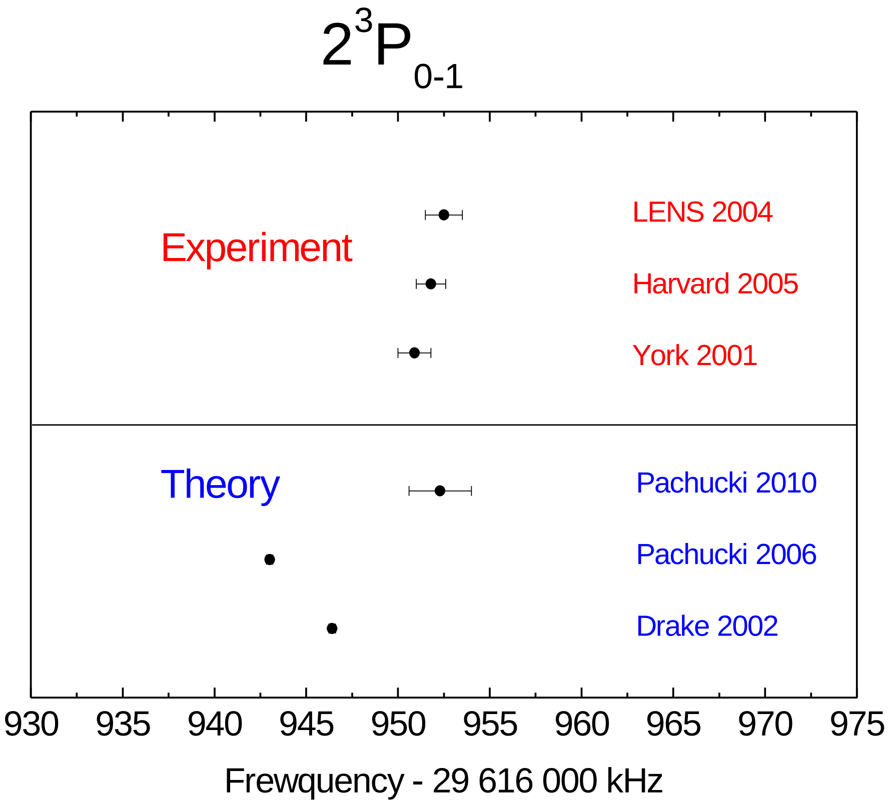
Fine and hyperfine structure :

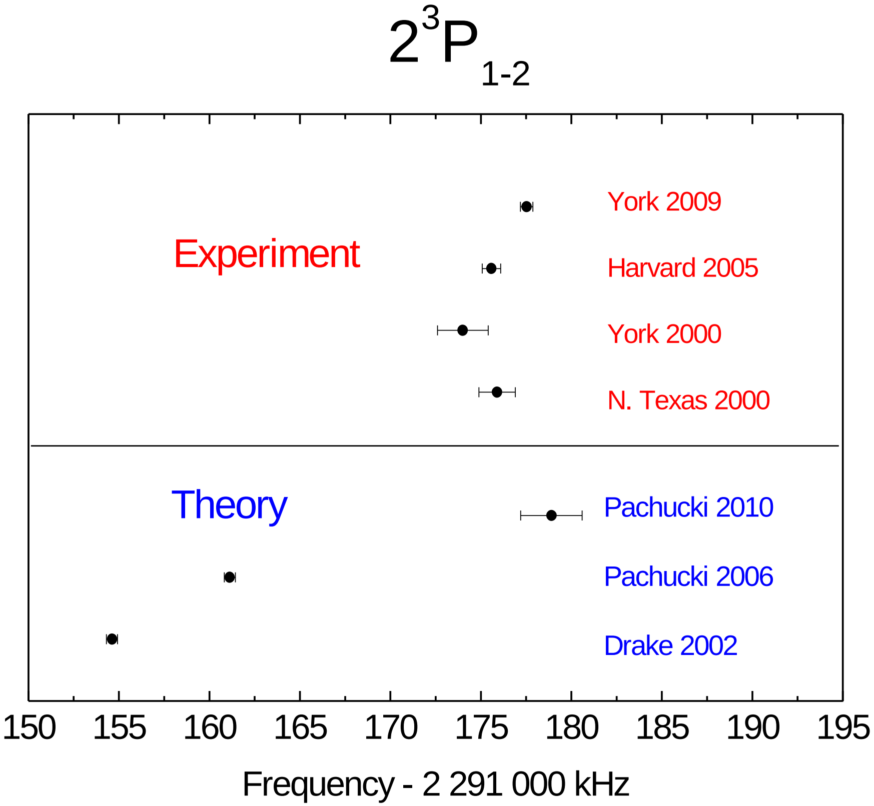
- State mixing, find hyperfine constant
- H: <1 kHz, He: <1 kHz, Li: several kHz

Previous attempt for helium FS

- uncertainty of theory and exp ~ 1 kHz
- Discrepancy = 10 kHz and 20 kHz for 10 years
- Include high-order terms ($\alpha^5 Ry$)



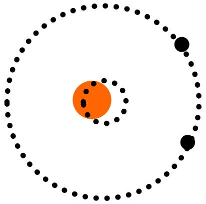




Isotope shift

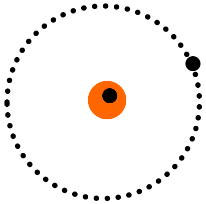
Isotope Shift $\delta\nu = \delta\nu_{MS} + \delta\nu_{FS}$

Mass shift: due to nucleus recoil



$$\delta\nu_{MS} \propto \frac{A-A'}{AA'}$$

Field shift: due to nuclear size



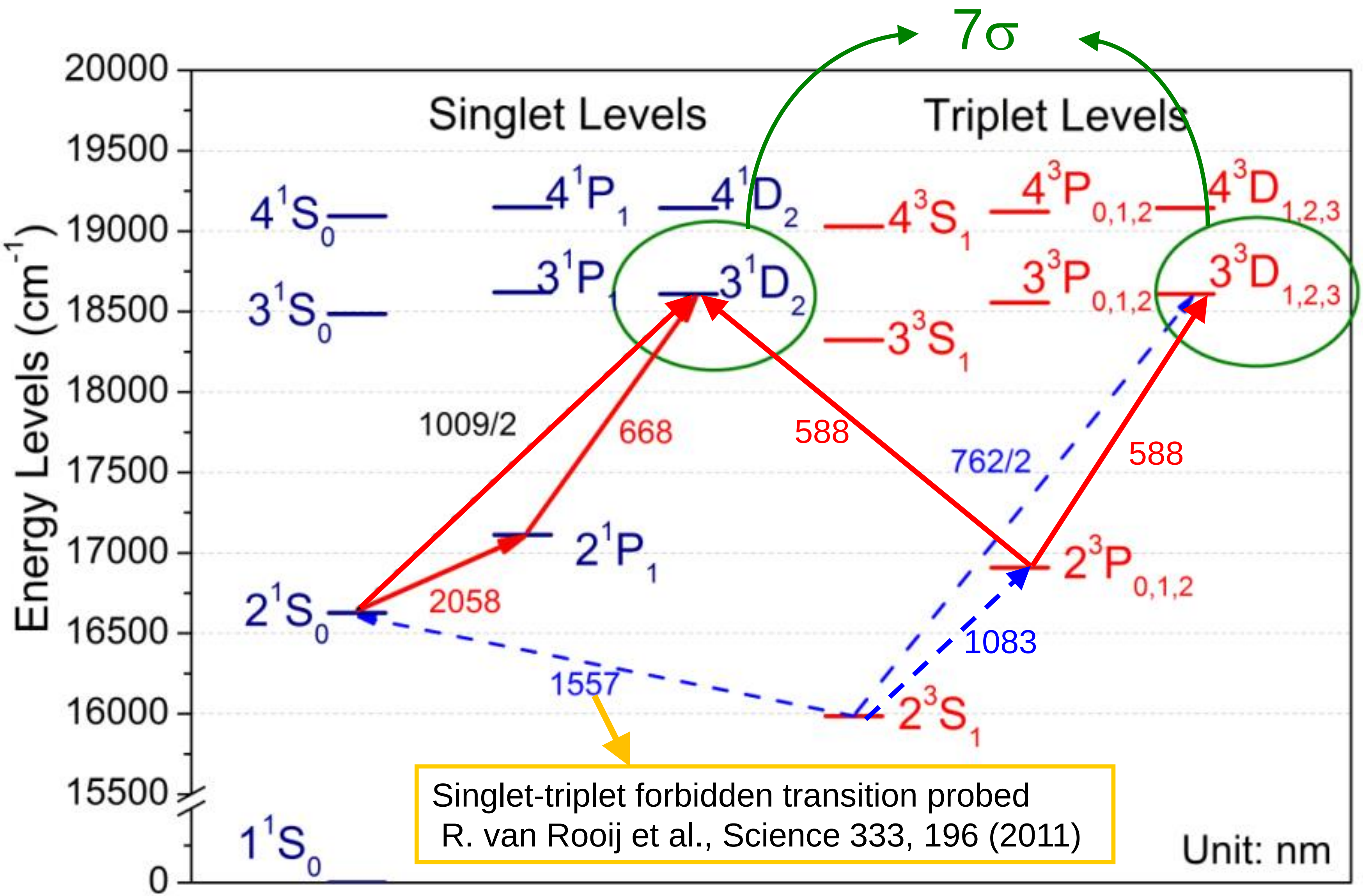
$$\delta\nu_{FS} \propto [\Psi(0)]^2 \times \delta < r_c^2 >$$

Nuclear mass precisely known

Atomic structure $\Psi(0)$ calculable, H, He, Li.

This method widely used in H-D, ^{4}He-^{3}He, and ^{6}Li-^{7}Li and determine the difference in the nuclear size

Helium energy levels

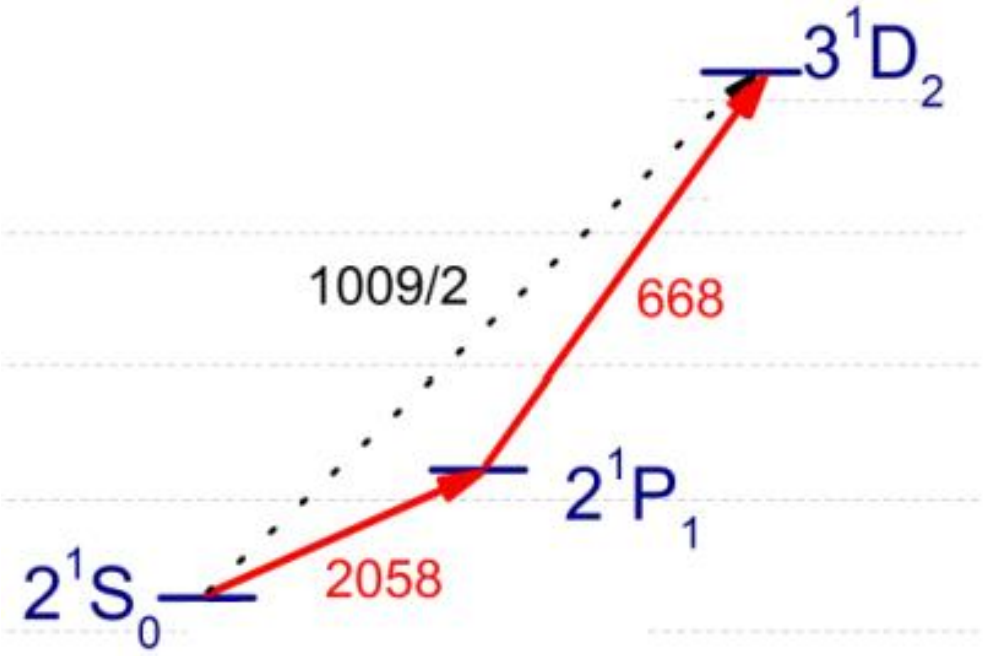


Singlet-triplet forbidden transition probed
R. van Rooij et al., Science 333, 196 (2011)

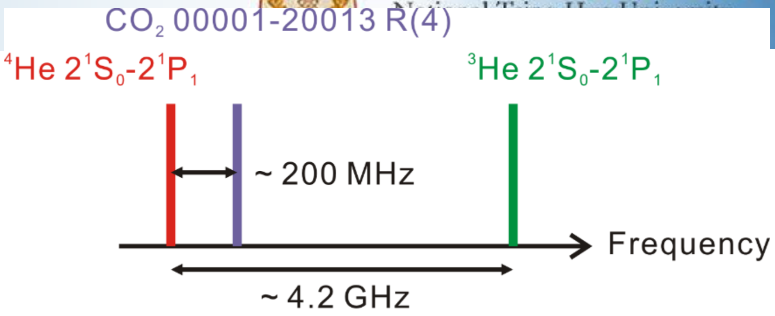
Motivation

- Better theory
- Experiments mostly performed in triplet state
- Singlet-triplet forbidden transition probed

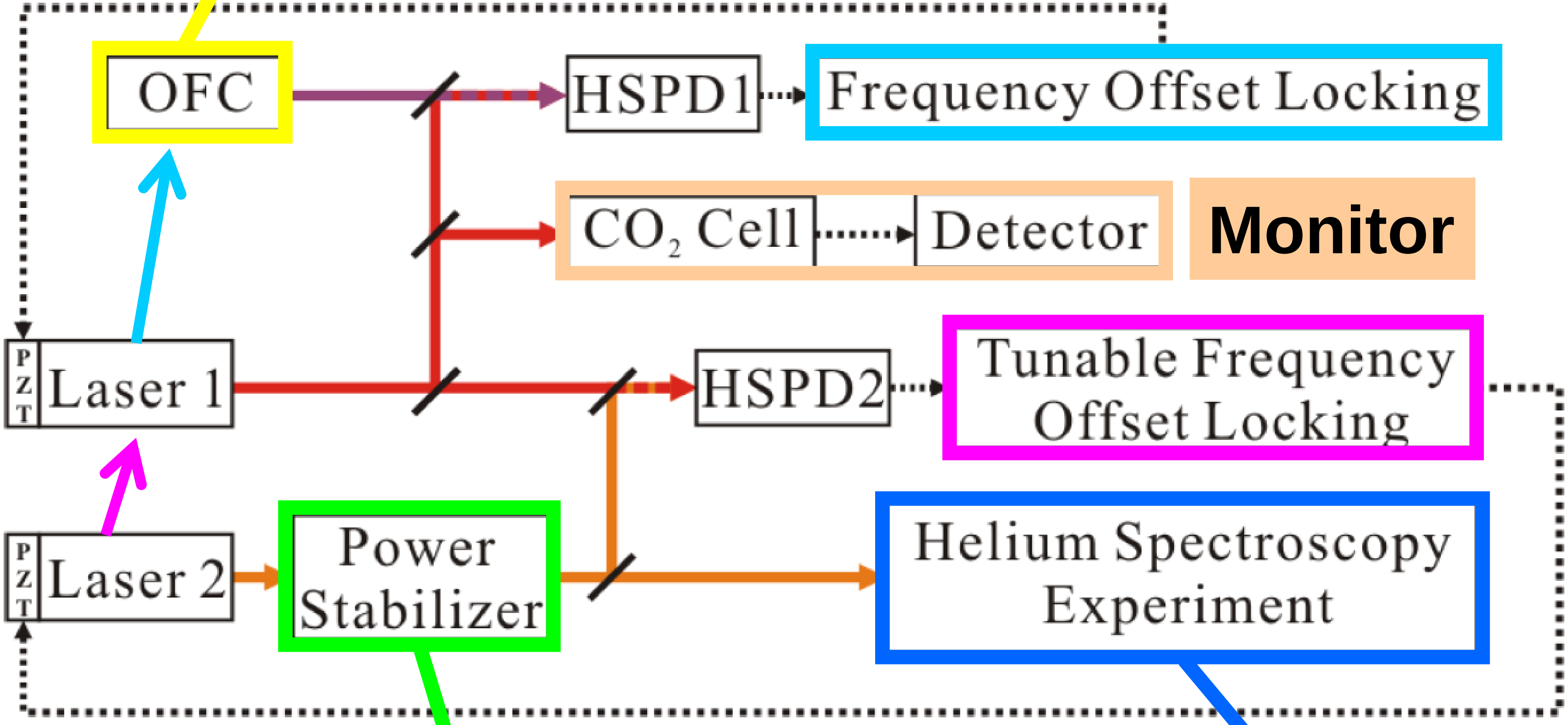
R. van Rooij et al., Science **333**, 196 (2011)



2058 nm transition



Accuracy = 10^{-12} @ 1000-sec

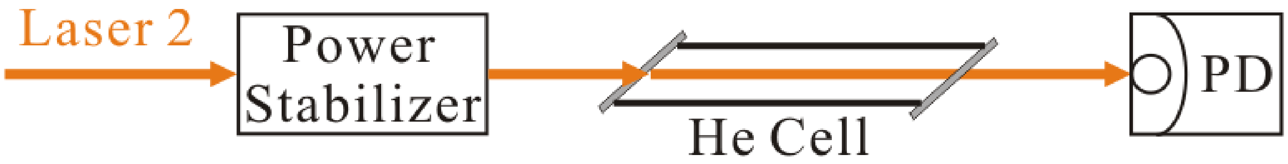


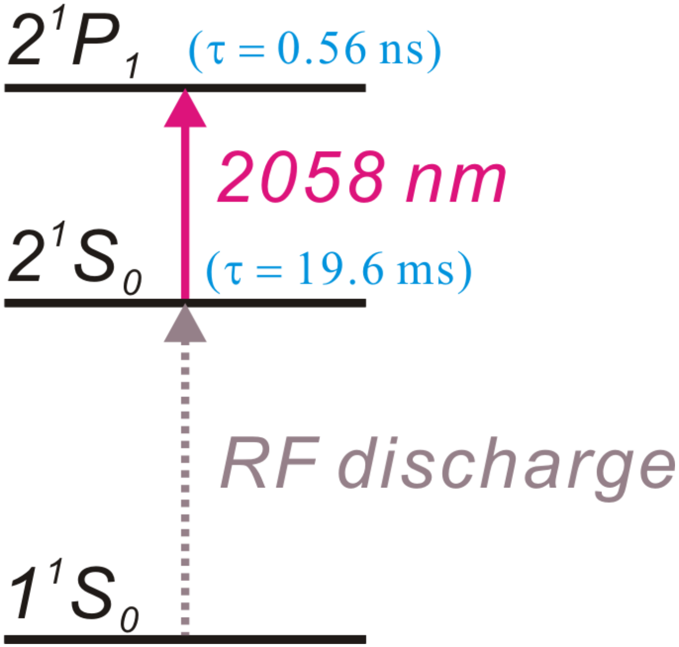
Monitor

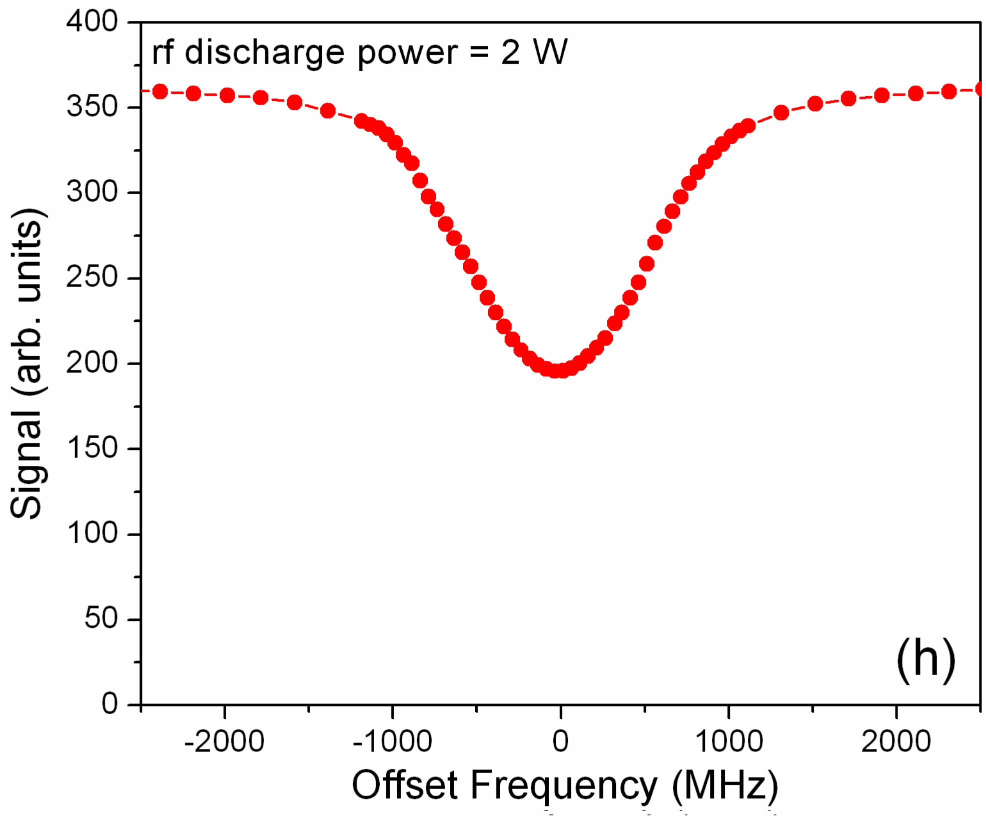
Stability < 1%

RF Discharge He Cell

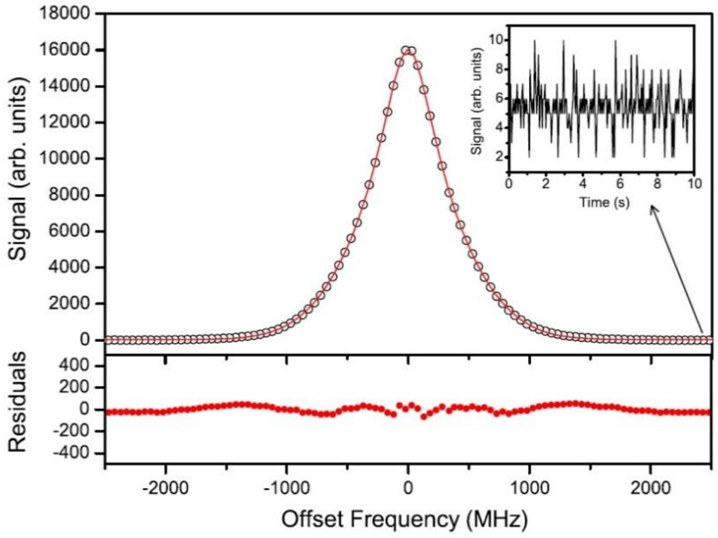
Direct Absorption Spectrum

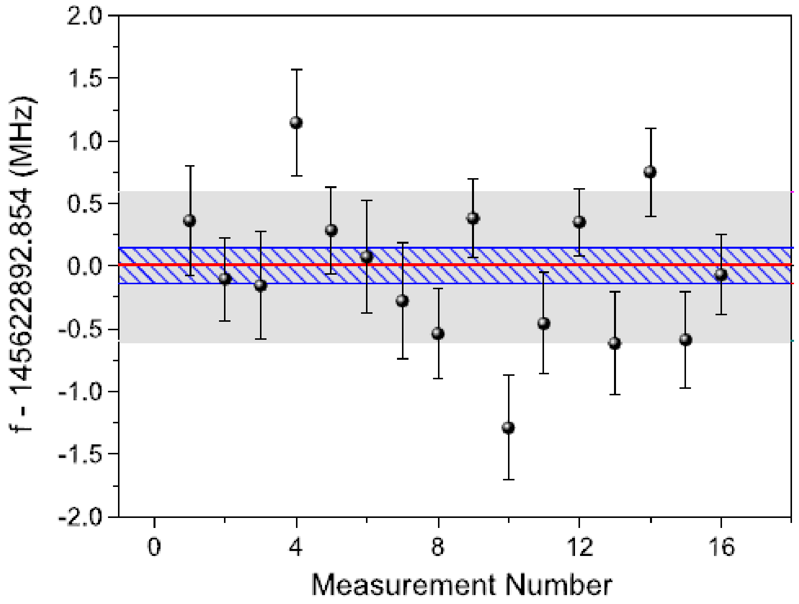




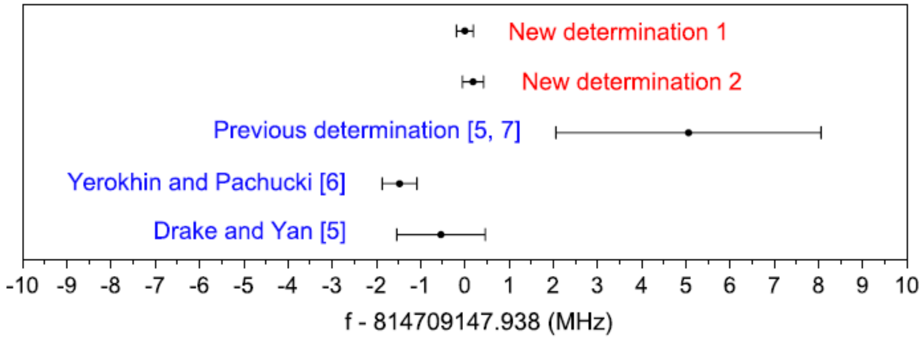


Spectroscopic results





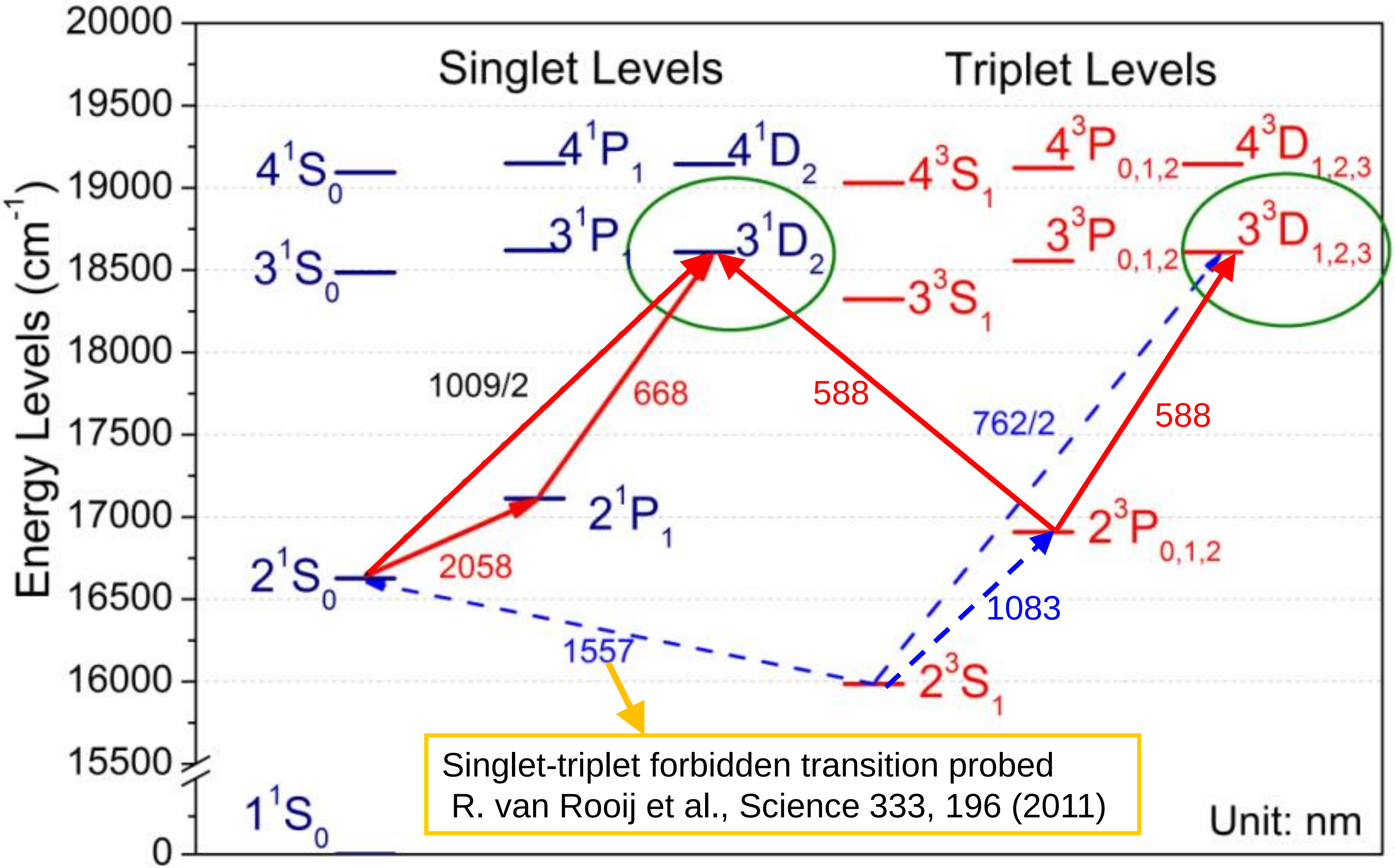
Source	kHz
Statistical error	141
OFC accuracy	<5
Frequency locking stability of laser 1 and OFC	<5
Frequency offset locking stability of two lasers	<2
Pressure shift	9.3
Light intensity	66
rf discharge power	82
Zeeman effect	<7
Overall uncertainty	183



P.-L. Luo et al., *Phys. Rev. Lett.* **111**, 013002 (2013)

- First absolute frequency measurement on this transition
- 10 time more precise in isotope shift

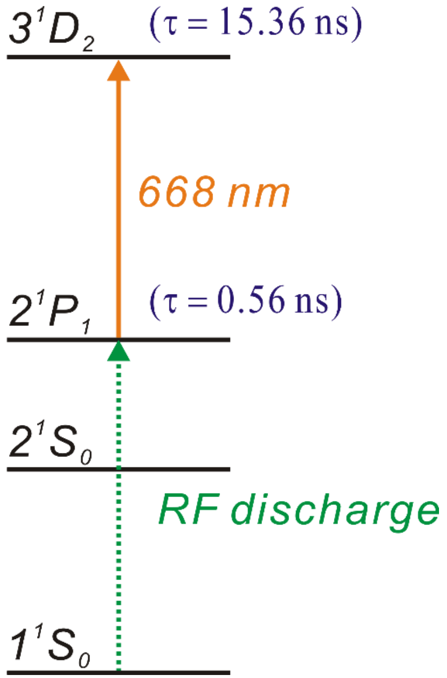
Helium energy levels

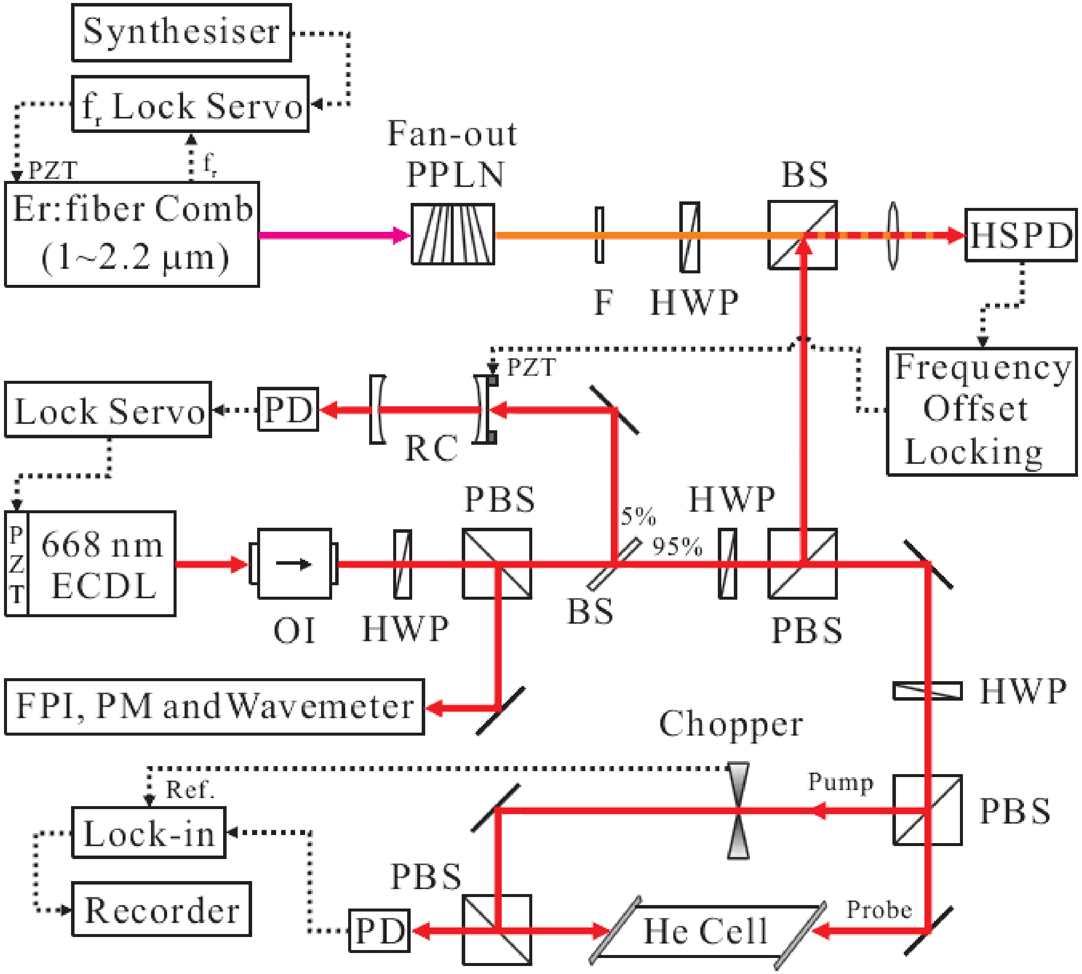


Singlet-triplet forbidden transition probed
R. van Rooij et al., Science 333, 196 (2011)

668 nm transition

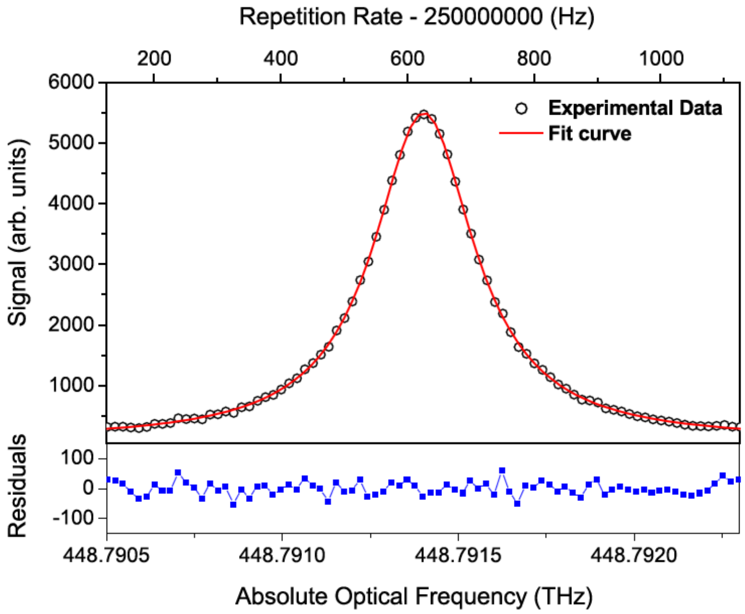
- ECDL at 668 nm locked to frequency comb
- Comb → PPLN, change rep rate

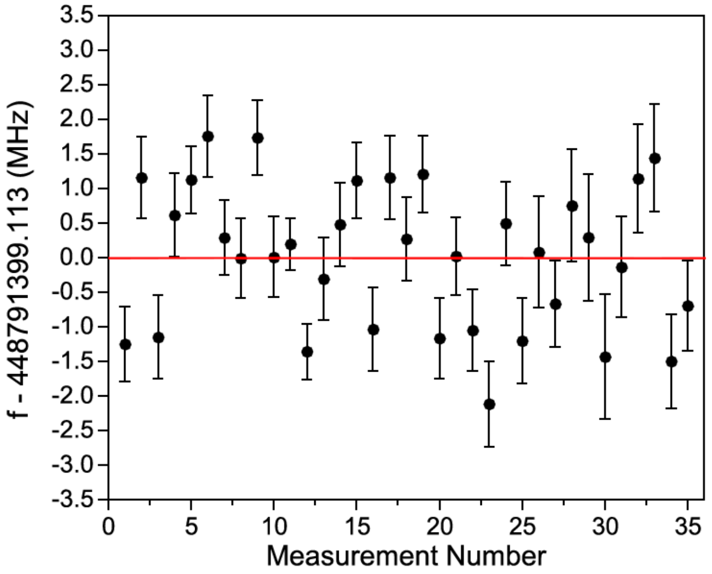




$$f_L = n \times f_r + 2 \times f_o \pm f_b$$

Results



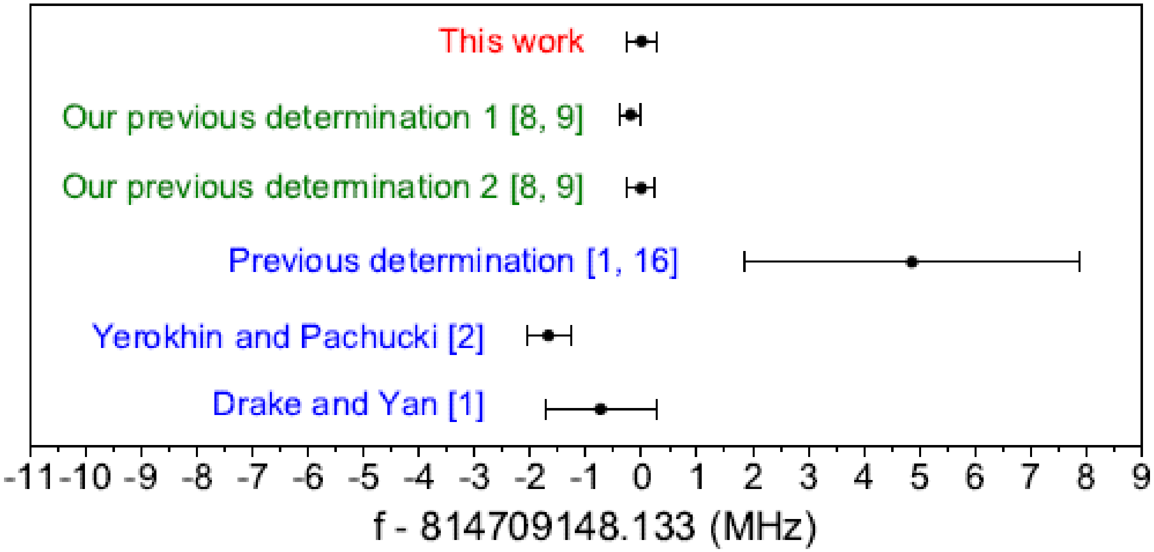


Source	shift	uncertainty
Statistical error	0	177
OFC accuracy	0	<5
Frequency locking of ECDL and OFC	0	<5
Pressure shift	0	192
Zeeman effect	0	<7
Overall	0	268

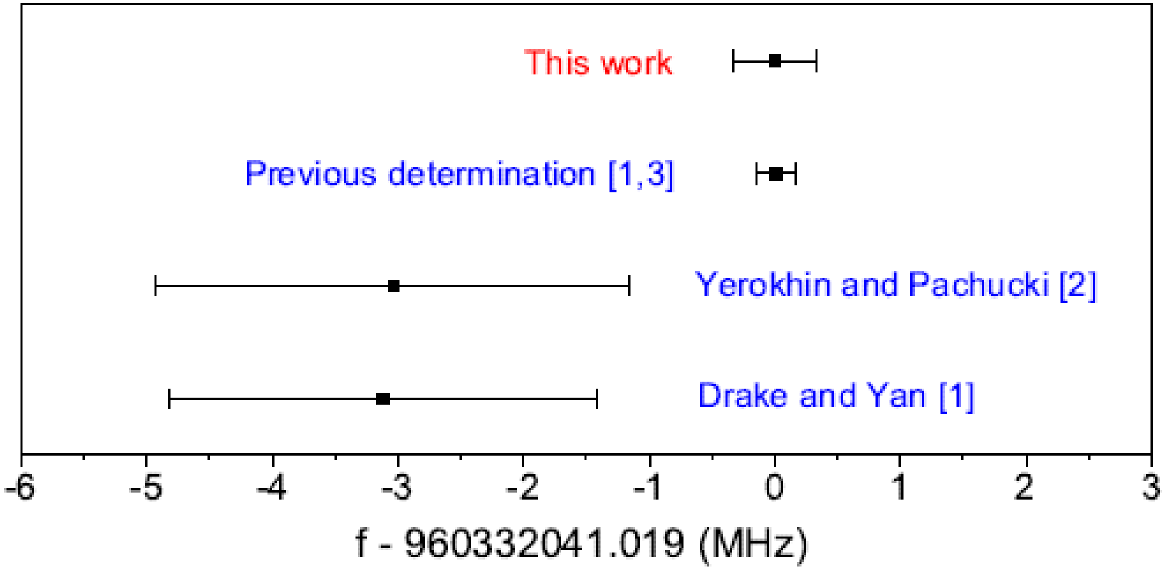
P.-L. Luo et al., *Phys. Rev. A* **88**, 054501 (2013)

10 time more precise than previous measurement

Ionization energy



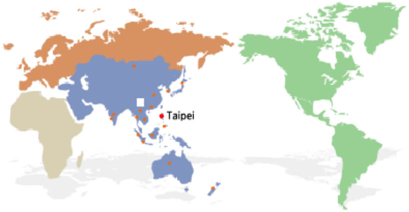
2P



2S

P.-L. Luo et al., *Phys. Rev. A* **88**, 054501 (2013)

AAPPS Bulletin
Association of Asia Pacific Physical Societies



Reported in Physics Focus of AAPPS Bulletin

AAPPS Bulletin

OCTOBER 2013 VOL. 23 NO. 5

PHYSICS FOCUS

Precision Laser Spectroscopy of Helium Testing QED Atomic Calculations

Helium is the simplest multi-electron atom and has been the best testing ground for many-body QED atomic calculations. Unlike the hydrogen atom, for which analytical solutions exists, similar studies of helium requires extensive numerical calculations in order to determine its electronic structures. Precision laser spectroscopy of He can improve the theoretical value of Lamb shift and be used to determine the nuclear charge radii of helium.

Recently, researchers from a multi- had to control precisely many experimental parameters, e.g., laser power stability, magnetic field shielding, discharge condition, and helium gas pressure. Otherwise, any systematic error will limit the final precision of the measurement. A typical spectrum is shown in Fig. 2. This represents the first Doppler-free measurement of the $2^1S_0 \rightarrow 2^1P_1$ transition.

For the ionization energy of the 2^1P_1 state, a discrepancy of 3.5 σ with the most precise theoretical value is found. This is shown in Fig. 3. This

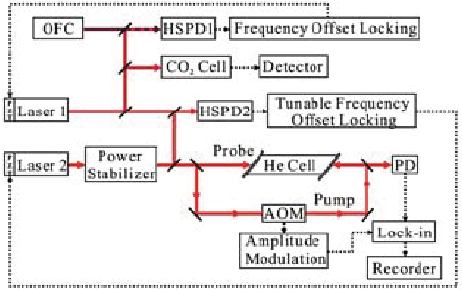
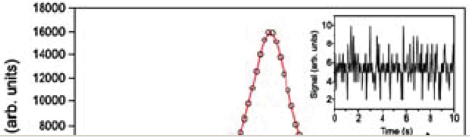
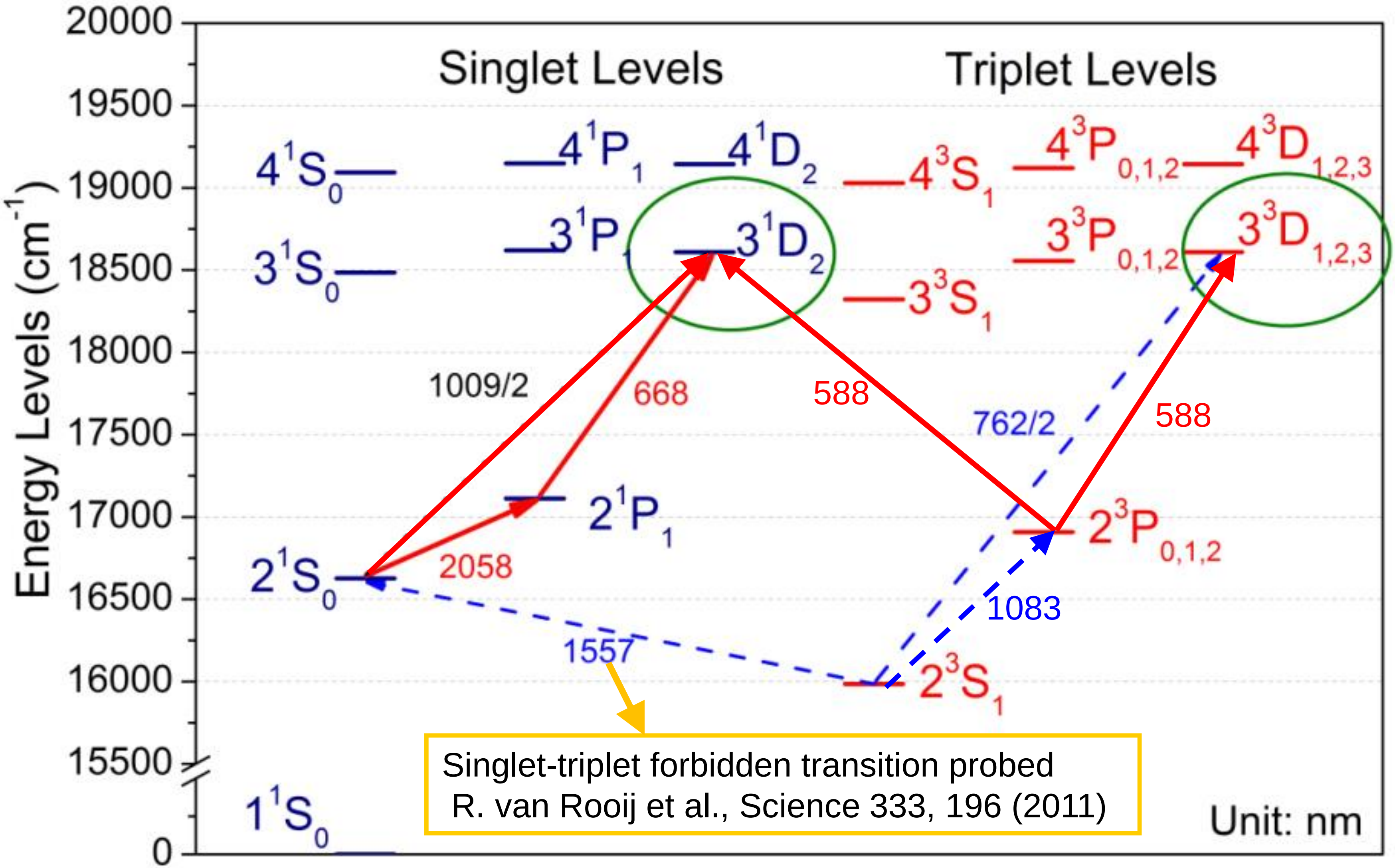


Fig. 1: Schematic of the experimental setup. OFC: optical frequency comb; PD: photodetector; HSPD: high-speed photodetector; AOM: acousto-optical modulator.

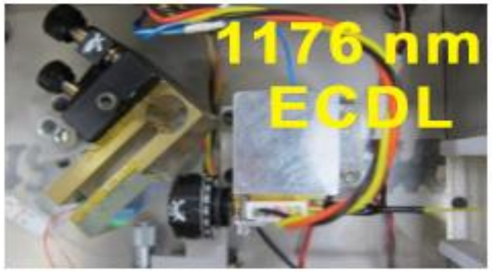


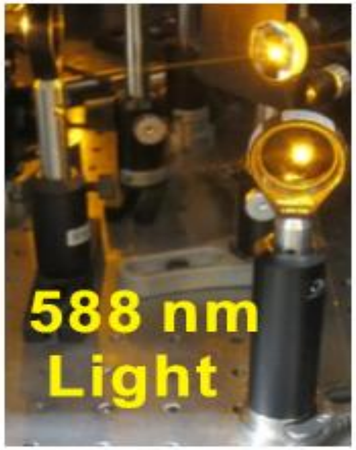
Helium energy levels



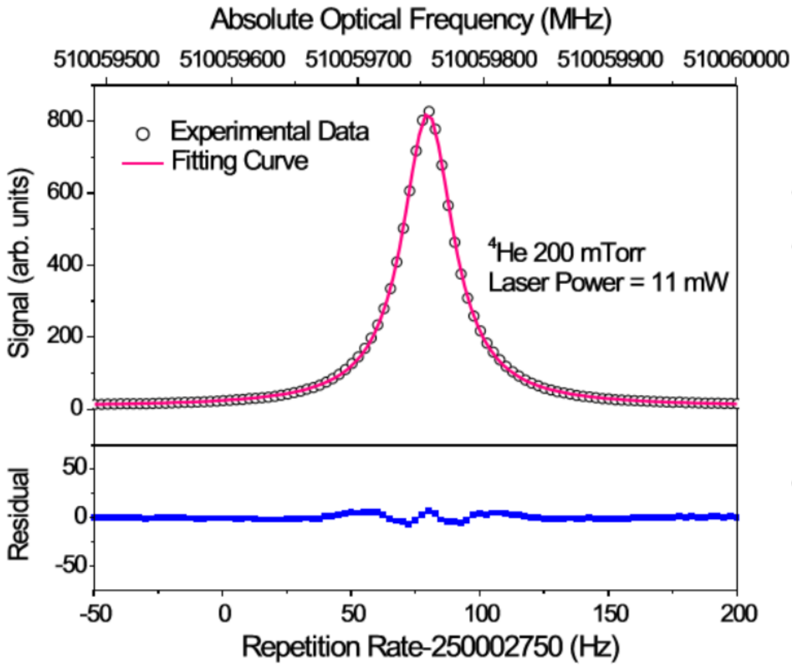
Singlet-triplet forbidden transition probed
R. van Rooij et al., Science 333, 196 (2011)

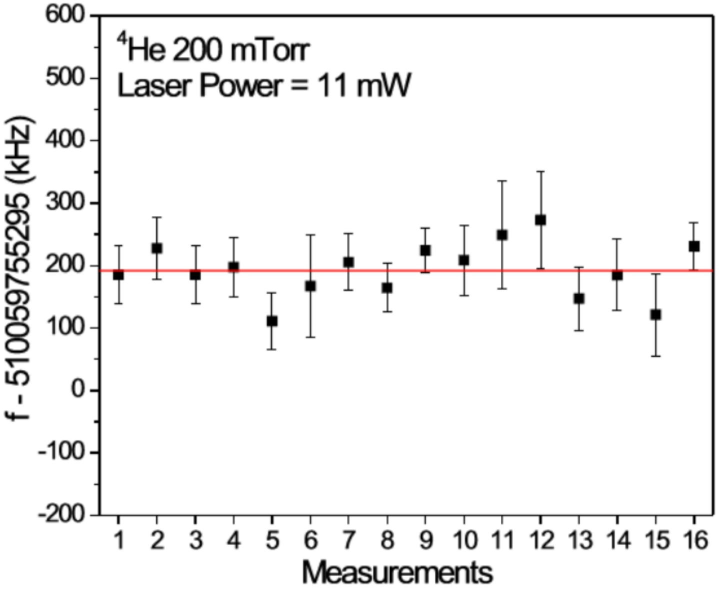
588 nm light source



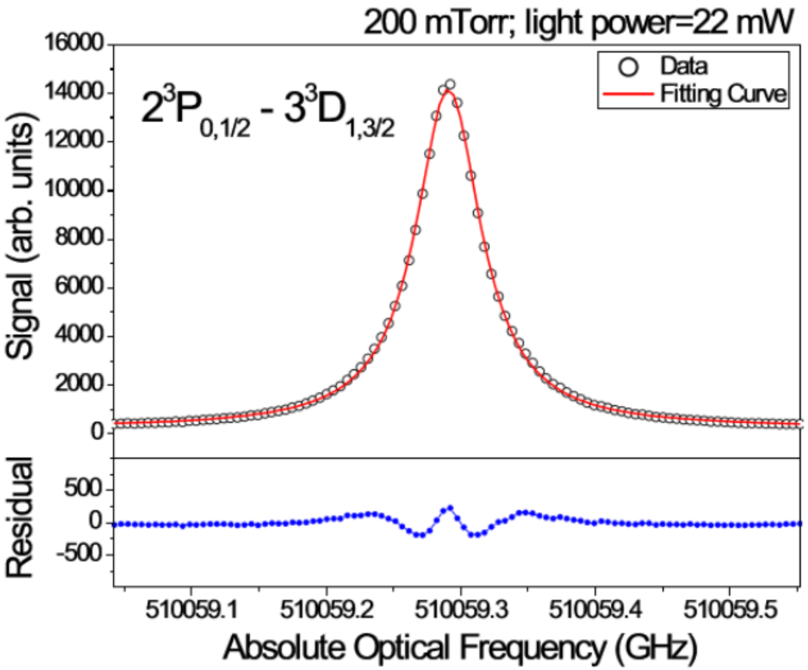


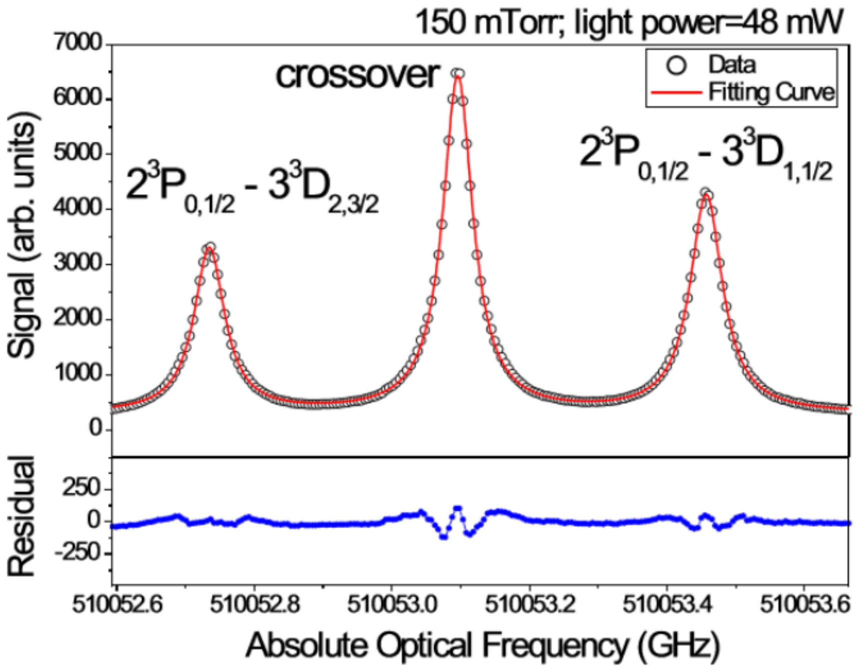
P.-L. Luo et al., Applied Phys. B **120**, 279 (2015)





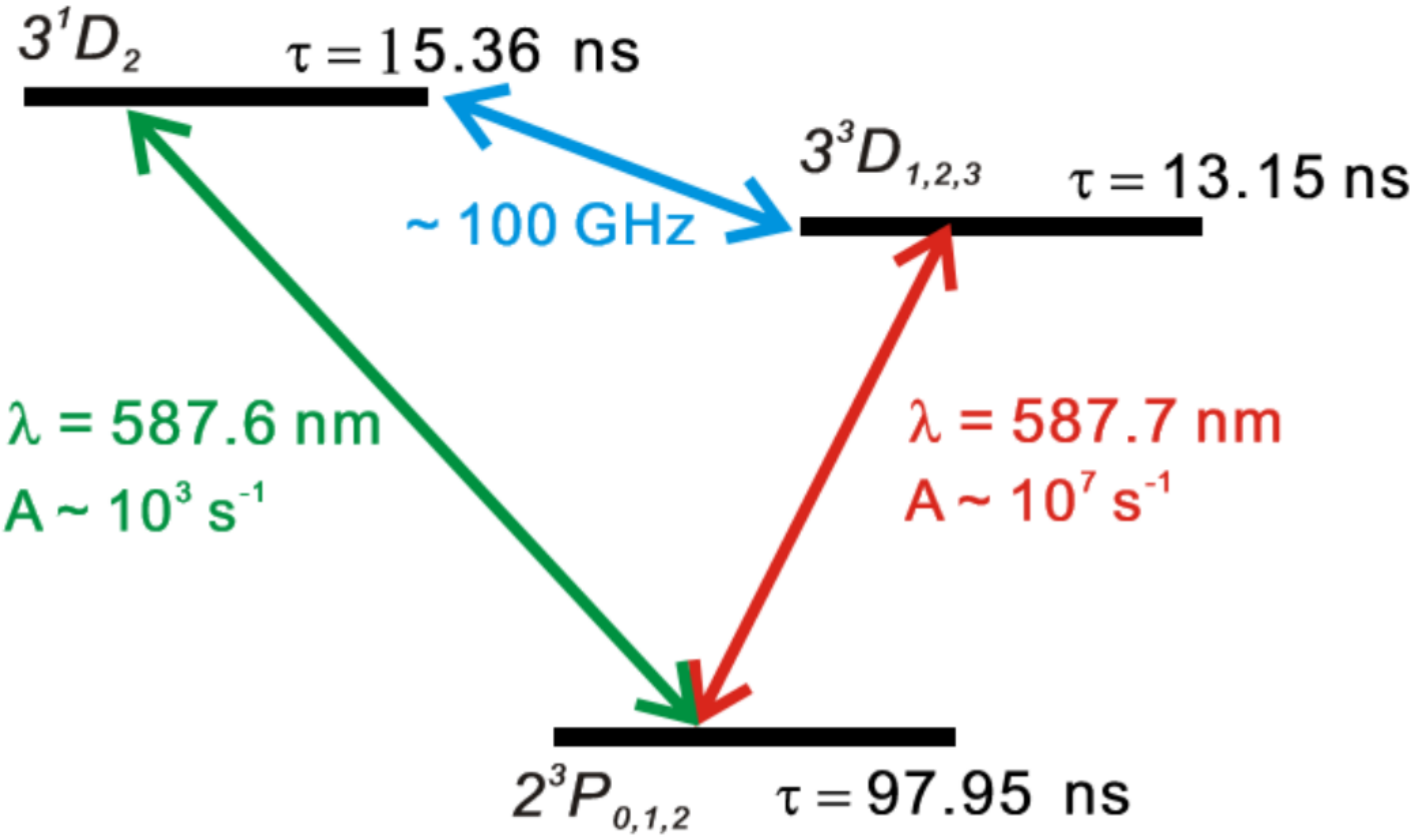
Helium-3





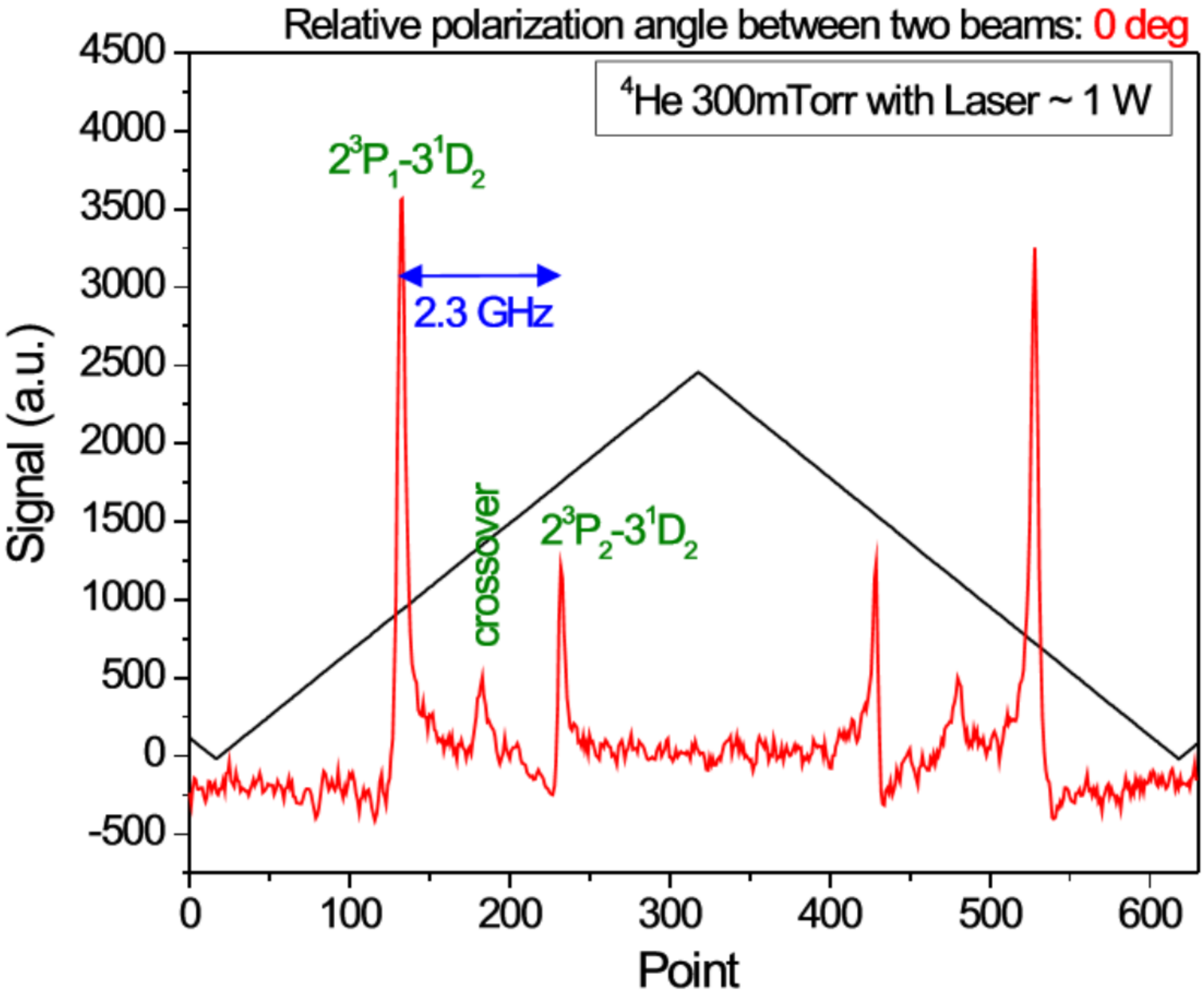
Separation	Our work	Theory [5]
$^3He\ 3^3D_{1,3/2} - 3^3D_{1,1/2}$	5834.073 (0.134)	5834.03 (0.04)
$^3He\ 3^3D_{1,3/2} - 3^3D_{2,3/2}$	6556.458 (0.149)	6556.42 (0.03)
$^3He\ 3^3D_{1,1/2} - 3^3D_{2,3/2}$	722.385 (0.183)	722.39 (0.04)

Spin-forbidden transition



- Only Doppler-broadened spectrum observed before

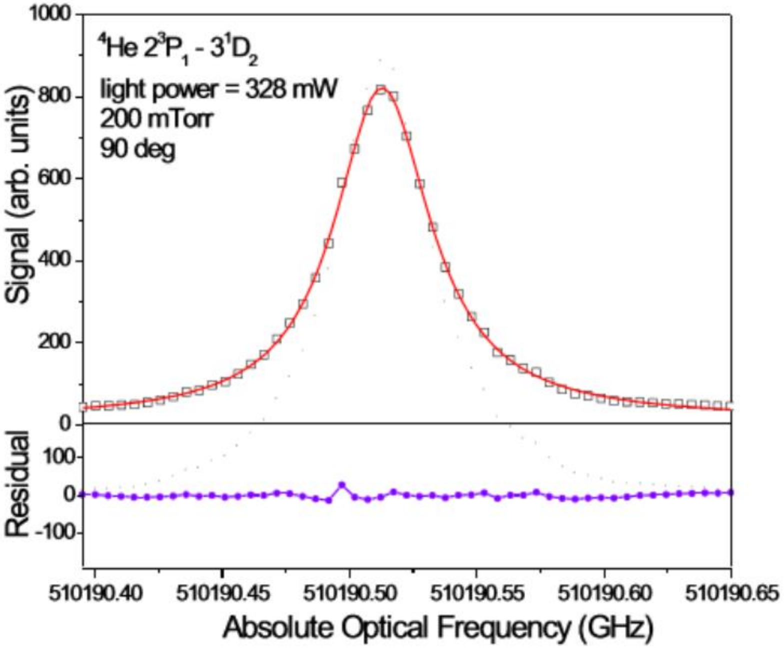
Spin-forbidden transition



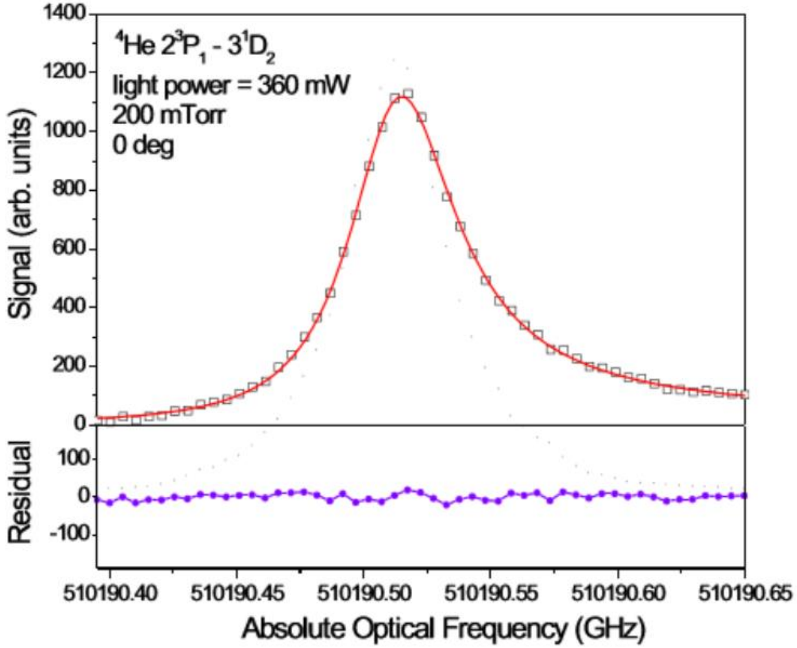
Spin-forbidden transition

^{4}He 2^3P_1-3^1D_2 @ 200 mTorr

$\theta = 90$ deg



$\theta = 0$ deg



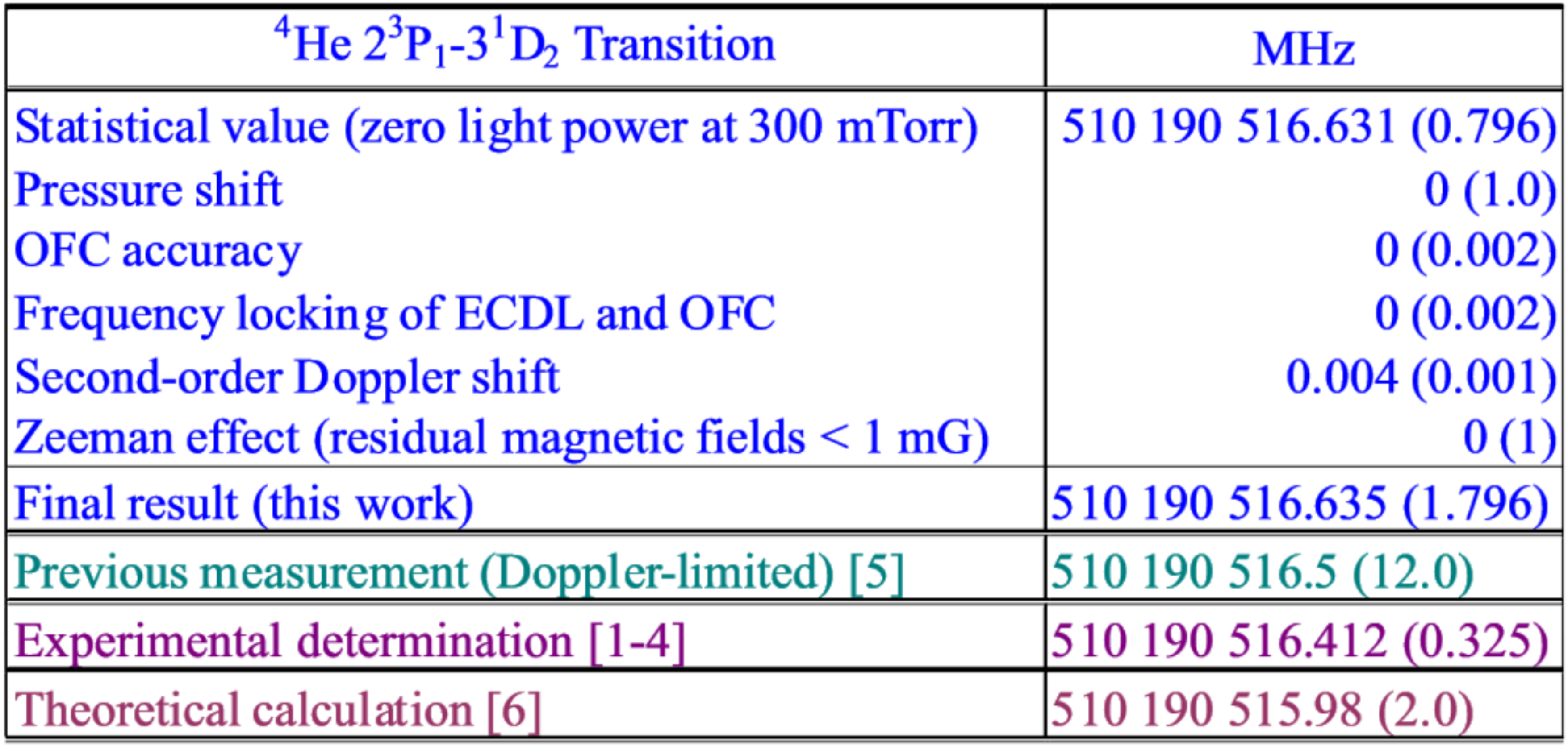
Fitting Formula

$$S = A\left[L(\nu - \nu_C) + B \times D(\nu - \nu_C)\right]$$

Lorentzian

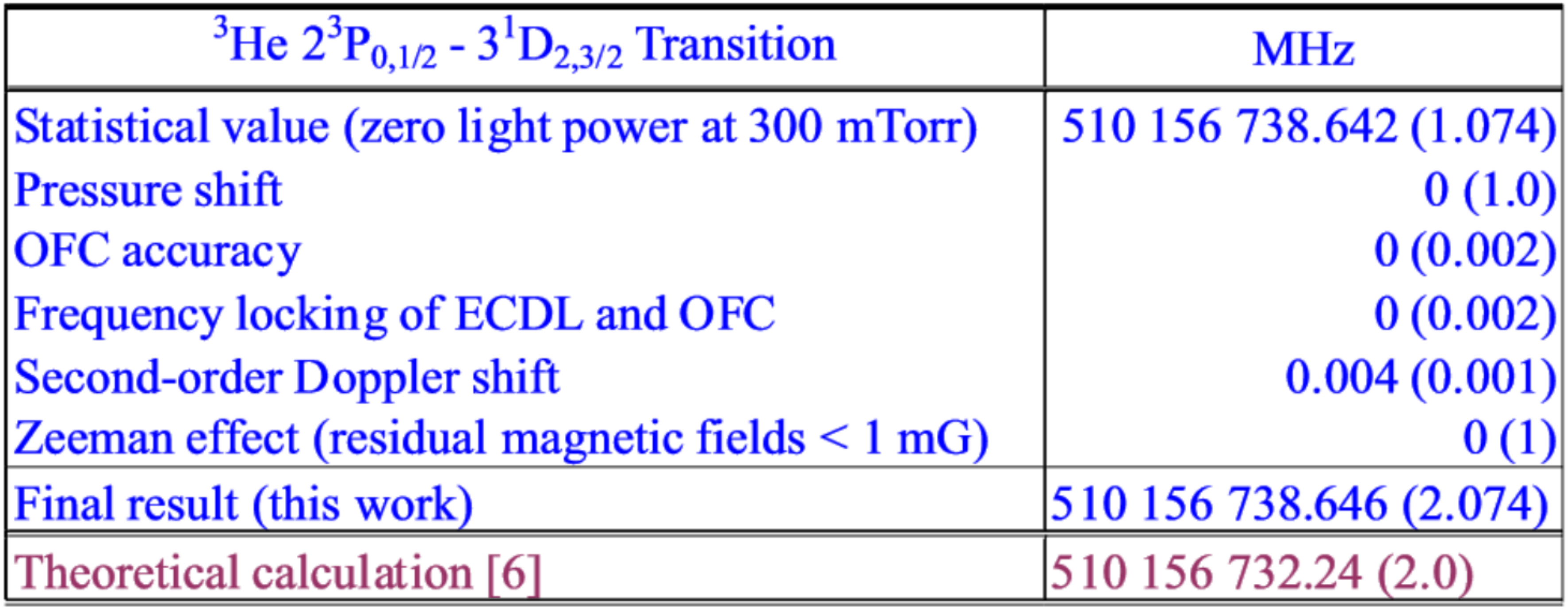
Dispersion

Analysis helium-4

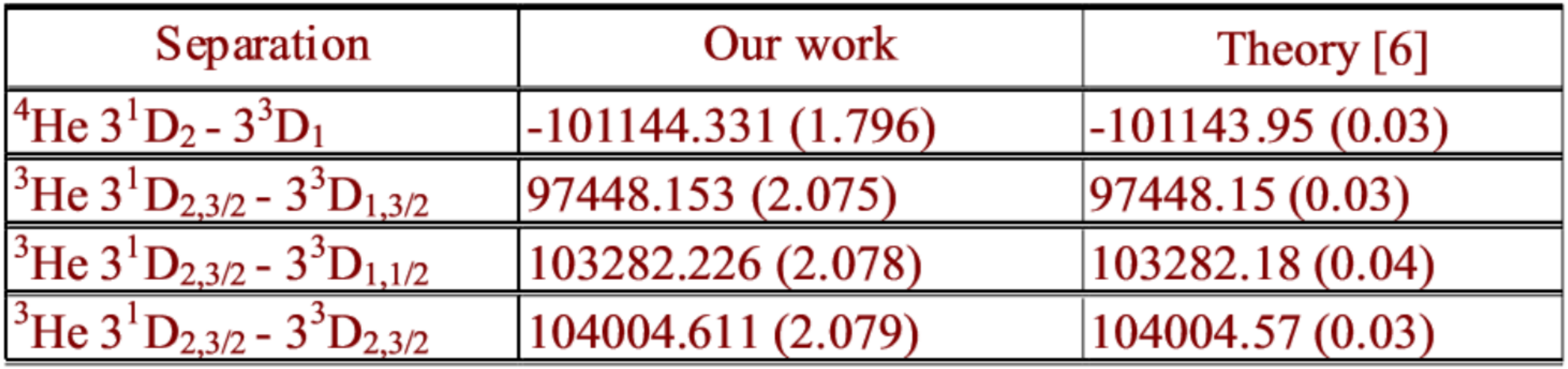


^{4}He $2^{3}P_{1}$-$3^{1}D_{2}$ Transition	MHz
Statistical value (zero light power at 300 mTorr)	510 190 516.631 (0.796)
Pressure shift	0 (1.0)
OFC accuracy	0 (0.002)
Frequency locking of ECDL and OFC	0 (0.002)
Second-order Doppler shift	0.004 (0.001)
Zeeman effect (residual magnetic fields < 1 mG)	0 (1)
Final result (this work)	510 190 516.635 (1.796)
Previous measurement (Doppler-limited) [5]	510 190 516.5 (12.0)
Experimental determination [1-4]	510 190 516.412 (0.325)
Theoretical calculation [6]	510 190 515.98 (2.0)

Analysis helium-3



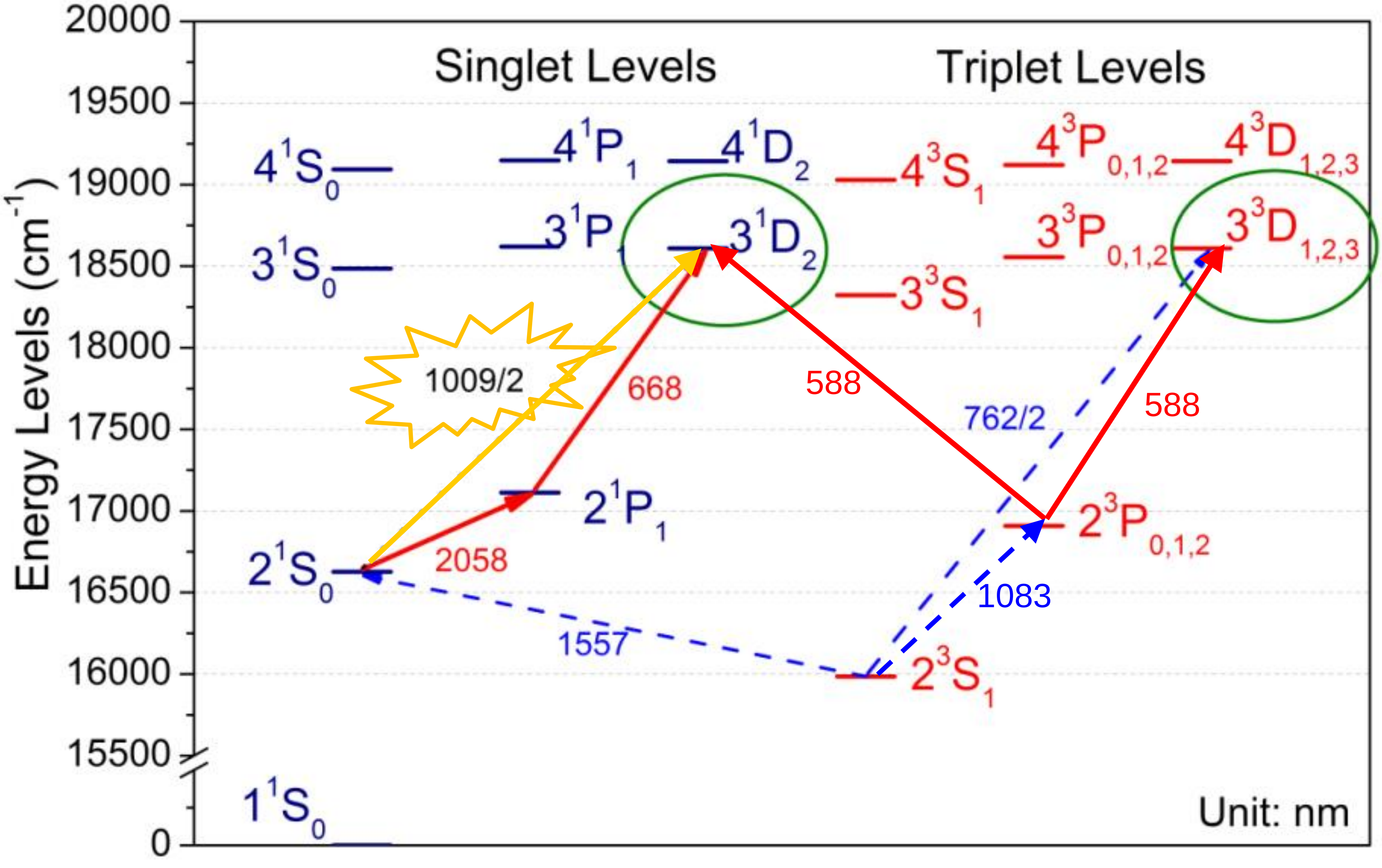
^{3}He $2^3P_{0,1/2}$ - $3^1D_{2,3/2}$ Transition	MHz
Statistical value (zero light power at 300 mTorr) Pressure shift OFC accuracy Frequency locking of ECDL and OFC Second-order Doppler shift Zeeman effect (residual magnetic fields < 1 mG)	510 156 738.642 (1.074) 0 (1.0) 0 (0.002) 0 (0.002) 0.004 (0.001) 0 (1)
Final result (this work)	510 156 738.646 (2.074)
Theoretical calculation [6]	510 156 732.24 (2.0)



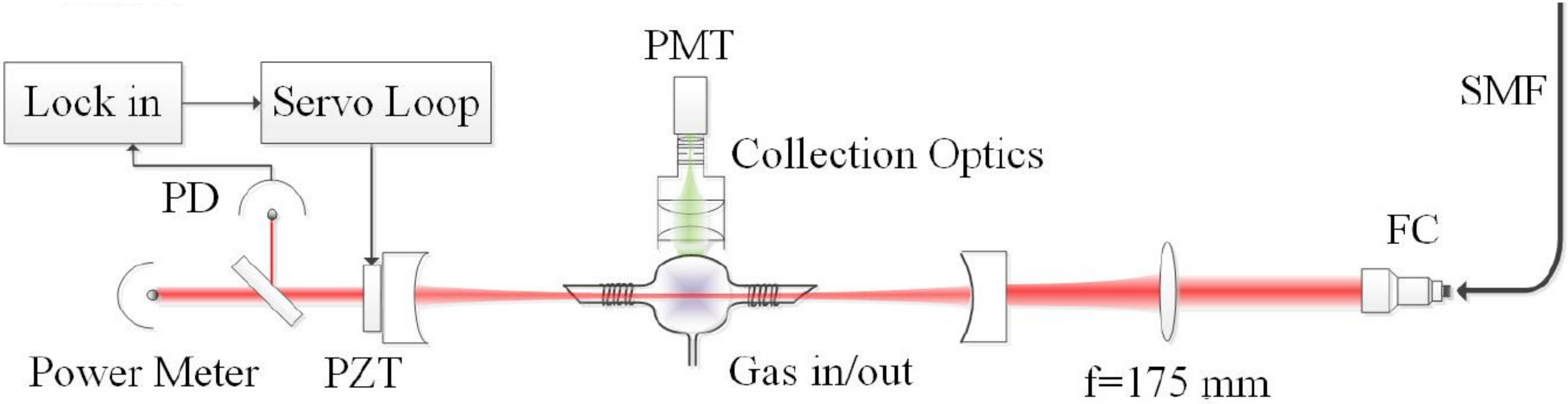
Separation	Our work	Theory [6]
^{4}He 3^1D_2 - 3^3D_1	-101144.331 (1.796)	-101143.95 (0.03)
^{3}He $3^1D_{2,3/2}$ - $3^3D_{1,3/2}$	97448.153 (2.075)	97448.15 (0.03)
^{3}He $3^1D_{2,3/2}$ - $3^3D_{1,1/2}$	103282.226 (2.078)	103282.18 (0.04)
^{3}He $3^1D_{2,3/2}$ - $3^3D_{2,3/2}$	104004.611 (2.079)	104004.57 (0.03)

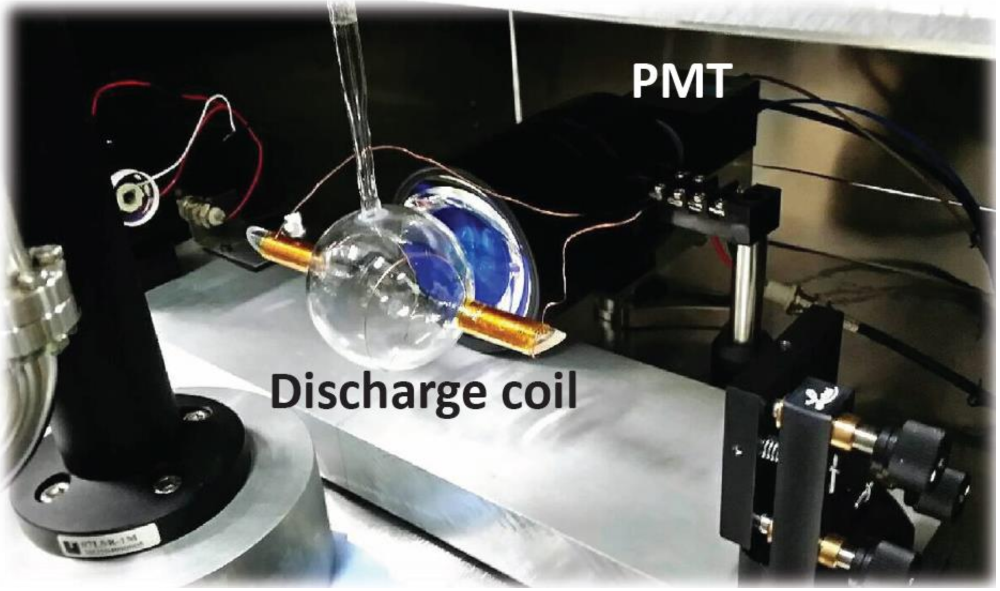
- The 37 MHz (7 σ) discrepancy not seen here.

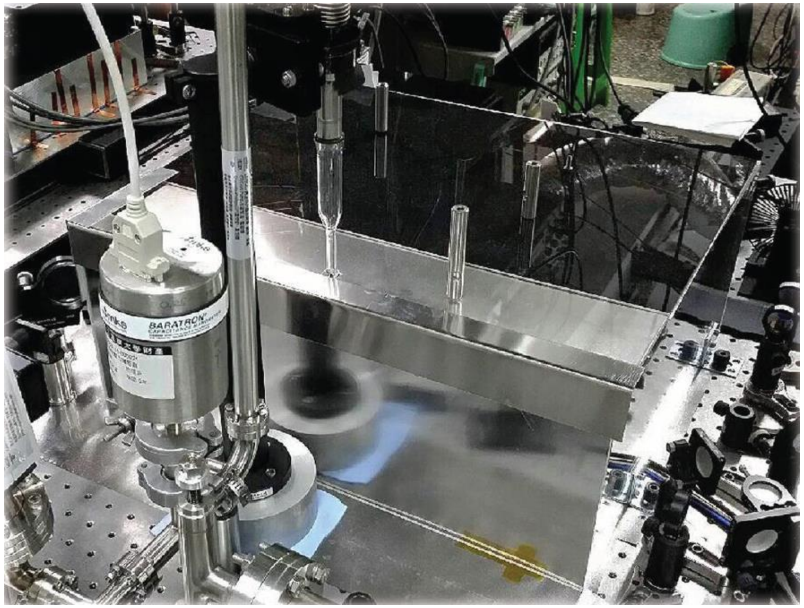
Helium energy levels

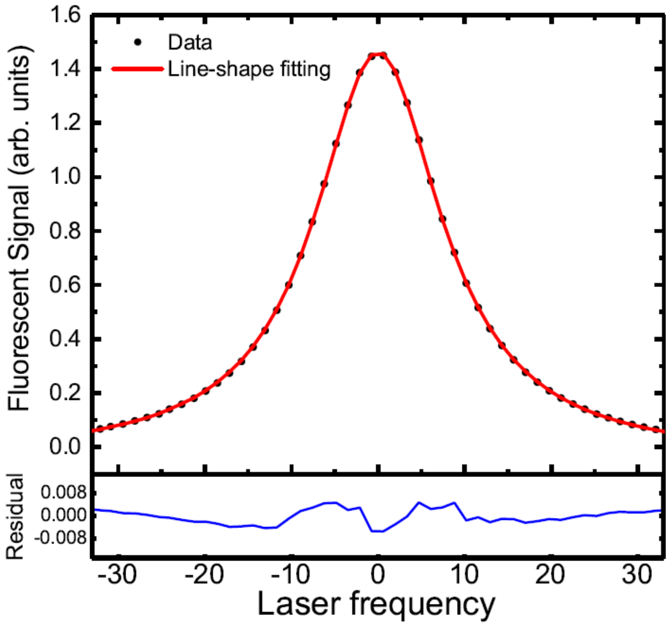


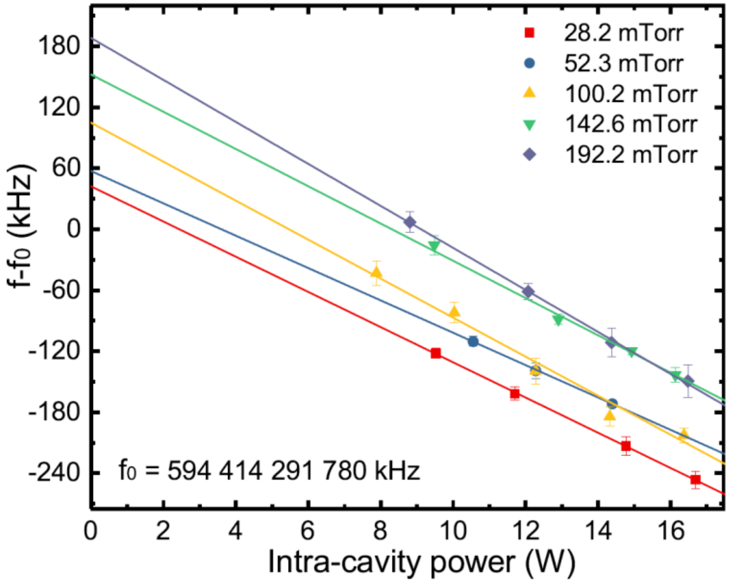
1009 nm two-photon transition

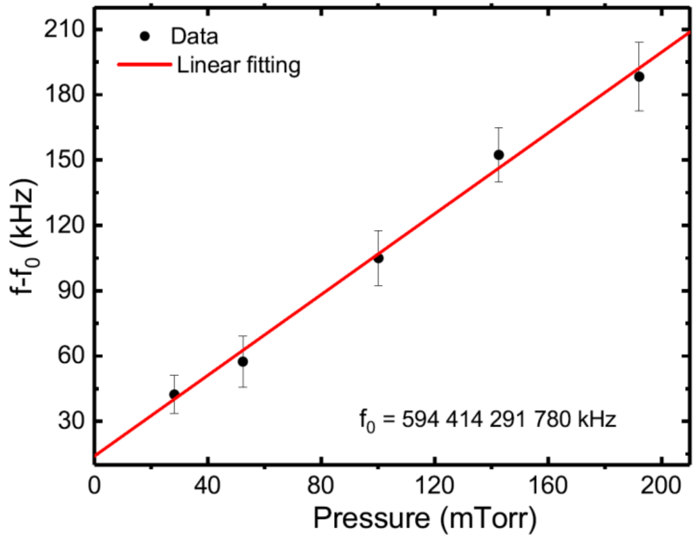


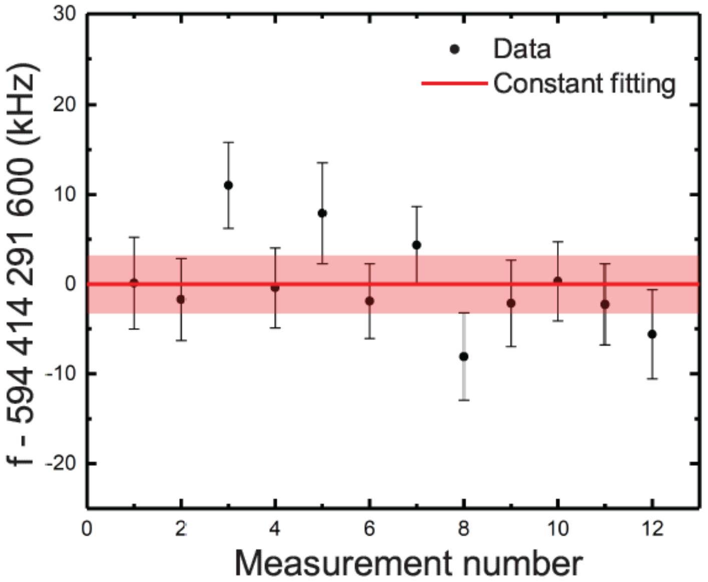




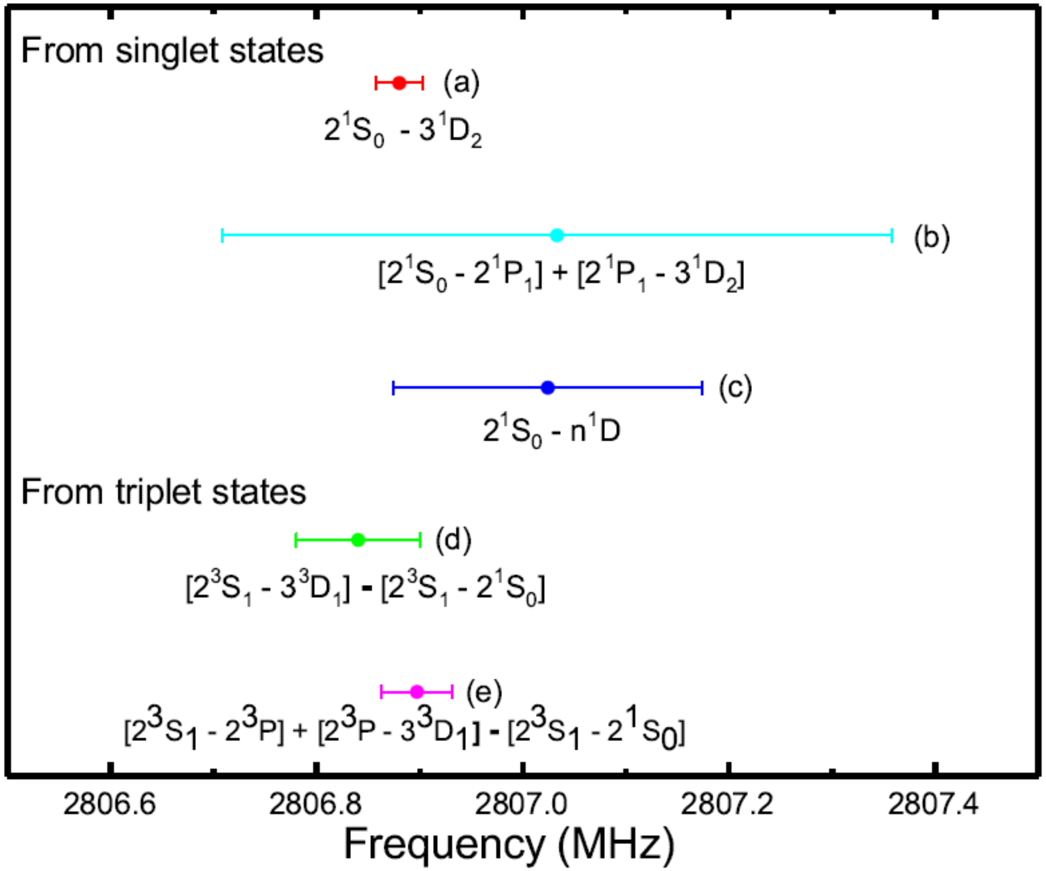

Fluorescent Signal (arb. units)
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
● Data
Line-shape fitting
Residual
0.008
0.000
-0.008
-30
-20
-10
0
10
20
30
Laser frequency


f-f0 (kHz)
180
120
60
0
-60
-120
-180
-240
■ 28.2 mTorr
● 52.3 mTorr
▲ 100.2 mTorr
▼ 142.6 mTorr
◆ 192.2 mTorr
f0 = 594 414 291 780 kHz
0
2
4
6
8
10
12
14
16
Intra-cavity power (W)


f-f0 (kHz)
210
180
150
120
90
60
30
● Data
Linear fitting
f0 = 594 414 291 780 kHz
0
40
80
120
160
200
Pressure (mTorr)

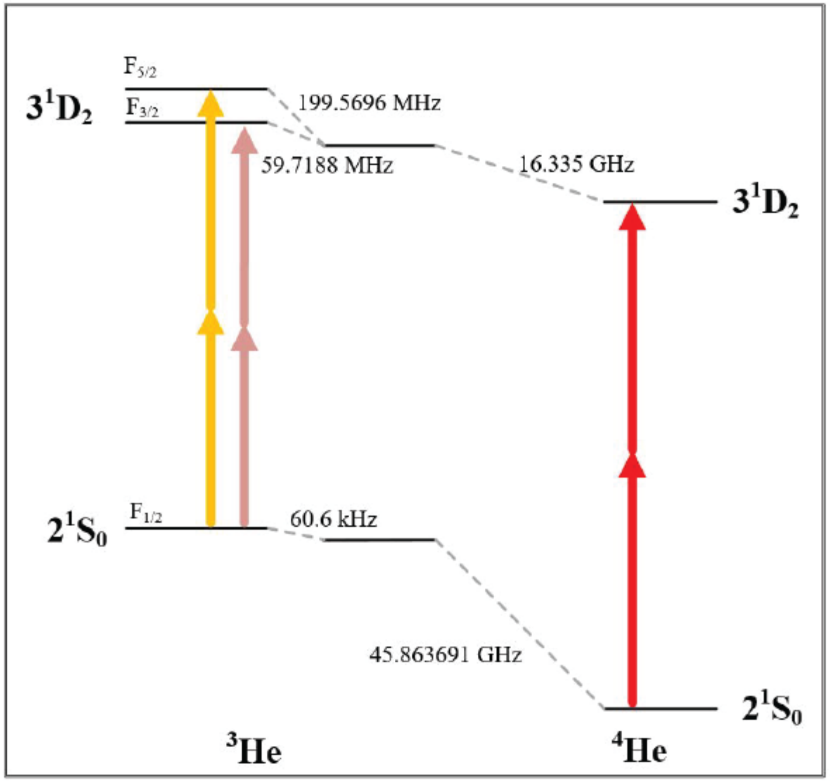

f - 594 414 291 600 (kHz)
30
20
10
0
-10
-20
● Data
Constant fitting
0
2
4
6
8
10
12
Measurement number

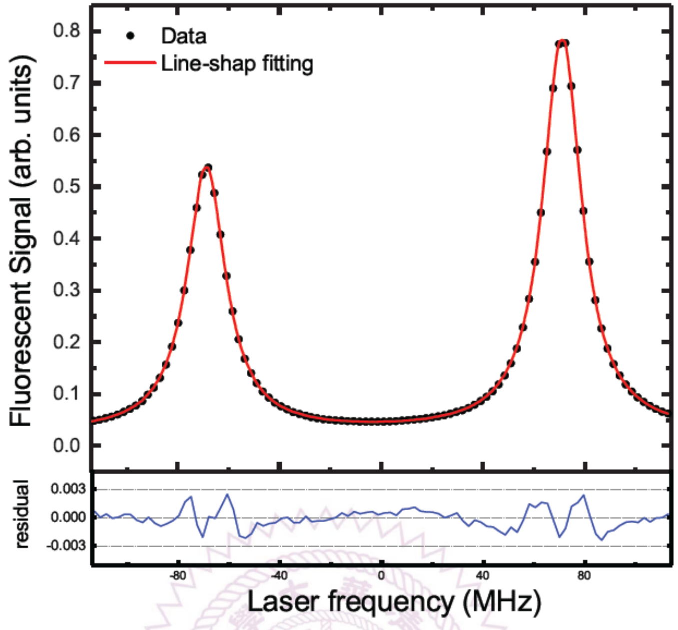
Lamb shift of the singlet 2S state

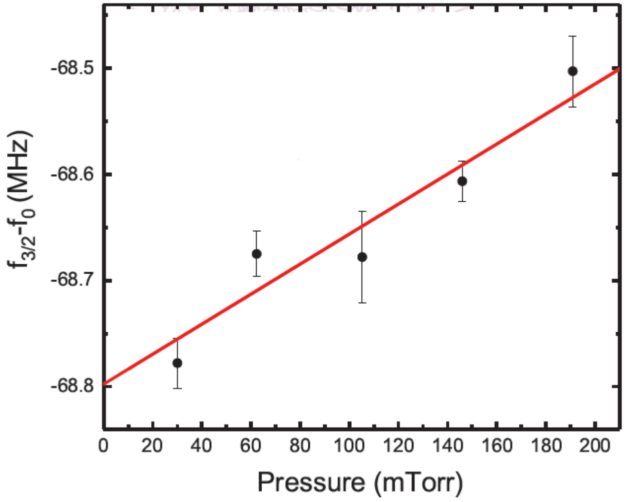


Y.-J. Huang, Y.-C. Guan, Y.-C. Huang, T.-H. Suen, J.-L. Peng, L.-B. Wang, and J.-T. Shy,, *Phys. Rev. A* **97**, 032516 (2018)

Isotope shift and helium-3







Nuclear charge radius

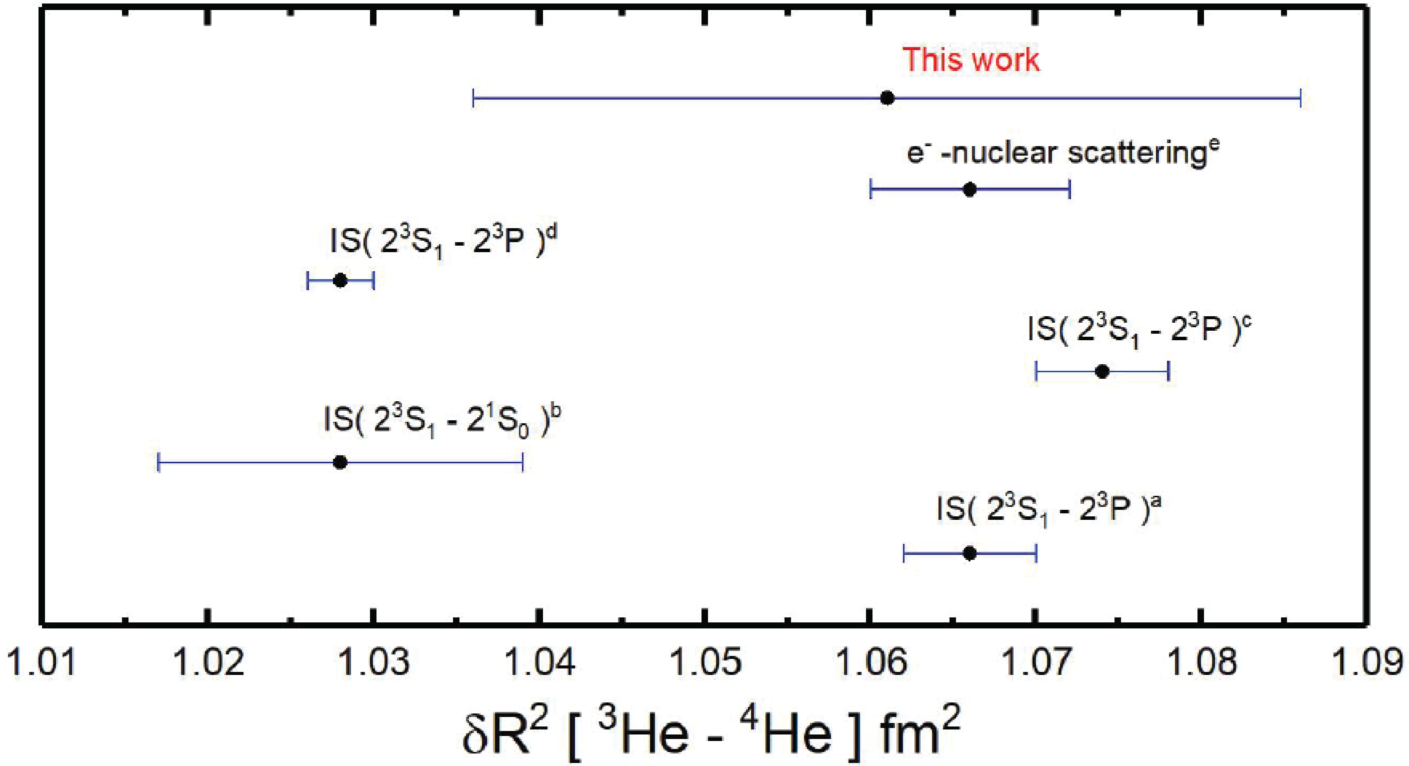


Figure 5.13: The comparison of the determination of the δR^2 from our work and other measurements. (a) Shiner et al. [61]. (b) Cancio Pastor et al. [18, 19, 56]. (c) Van Rooij et al. [42]. (d) X. Zeng et al. [46]. (e) Ingo Sick [62].

To be submitted to *Phys. Rev. A*

Summary

Work completed

- Helium singlet 2S-2P at 2058nm, 2P-3D at 668 nm
- Helium triplet 2P to 3D (singlet and triplet) at 588 nm
- Helium singlet 2S-3D two photon transition
- Either resolve discrepancy or test theories

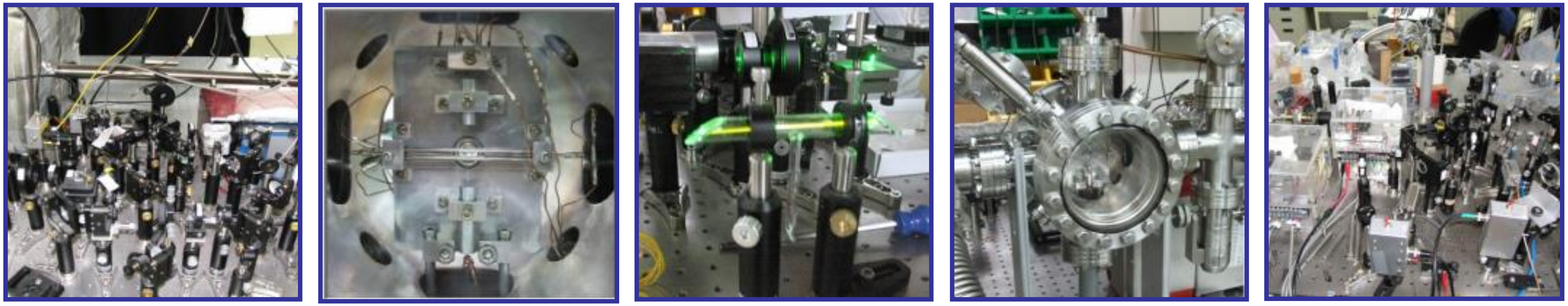
Work in progress and future work

- Helium Rydberg spectroscopy
- Construction of metastable helium beam
- Upgrade of optical frequency comb

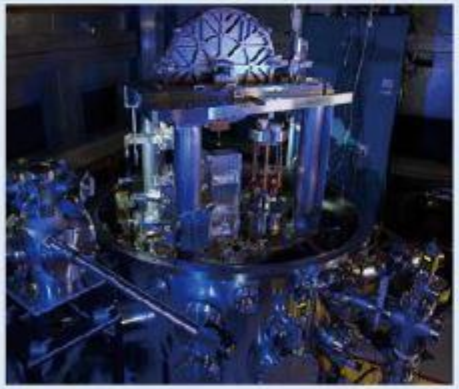
People involved

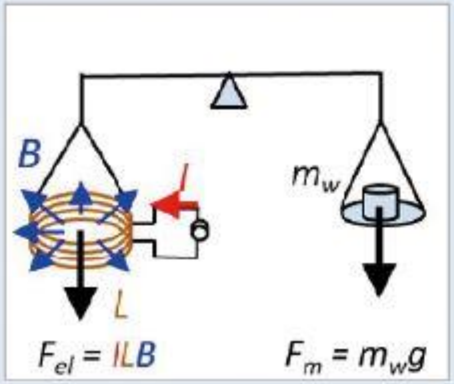
清華大學物理系: 施宙聰老師、孫德輝、官鈺禪、黃一展
原分所:羅佩凌博士
工研院量測中心:彭錦龍博士

$Supported by National Science Council, and Minister of Education

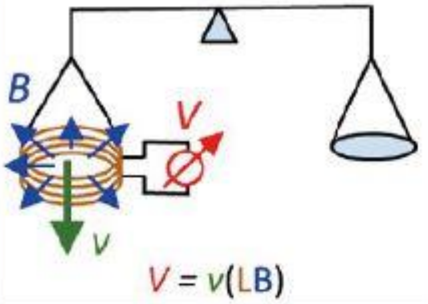


瓦特天平法

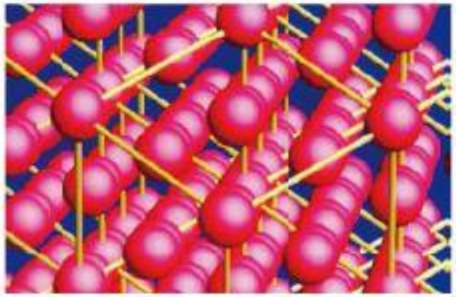




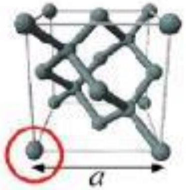
動態模式



矽晶球法



矽原子數量 $N = \frac{V_{core}}{a^3} \times 8$



矽原子質量

$$m_a(^{28}\mathrm{Si}) = \left(\frac{A_r(^{28}\mathrm{Si})}{A_r(e)} \frac{2R_\infty}{c\alpha^2} \right) h$$