Anderson Localization in an oscillating Rydberg-dressed Bose Einstein Condensate with disordered trap

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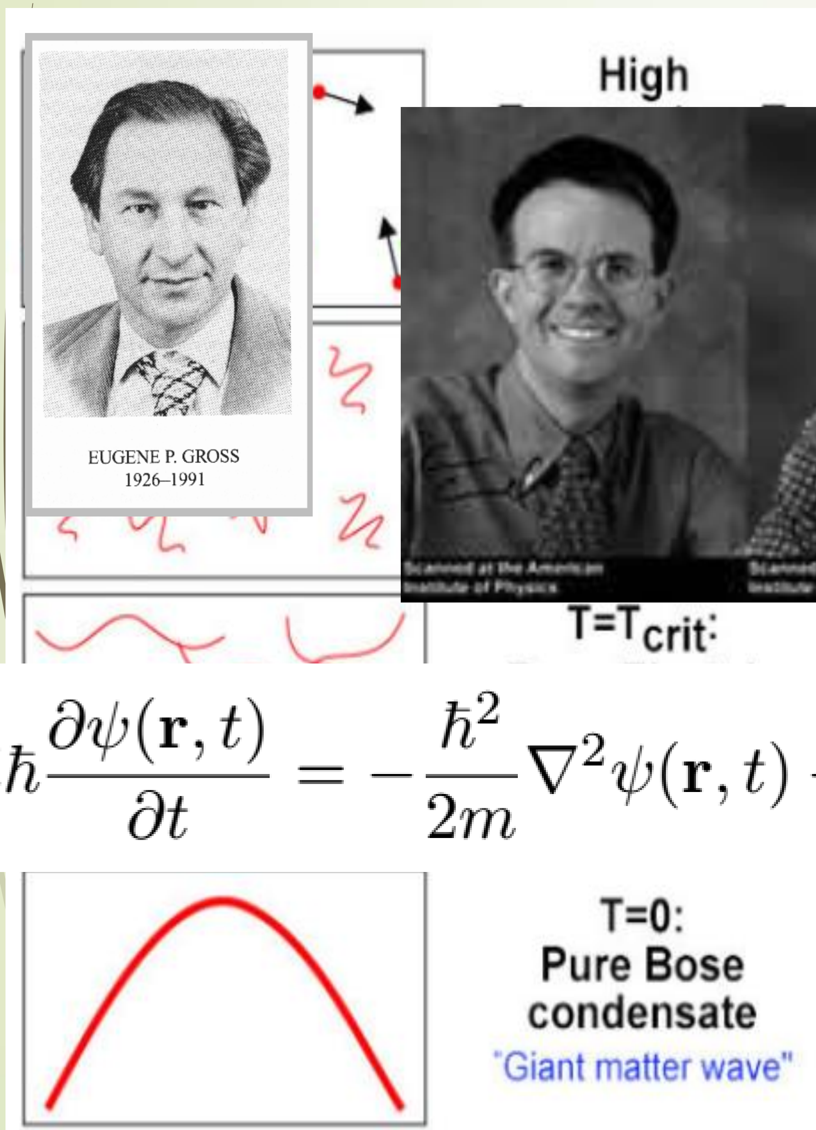
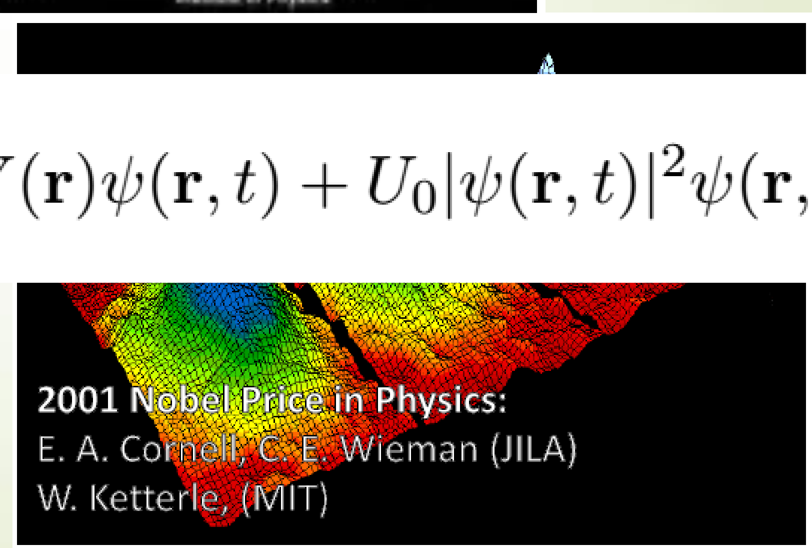
Introduction:
Bose-Einstein Condensation

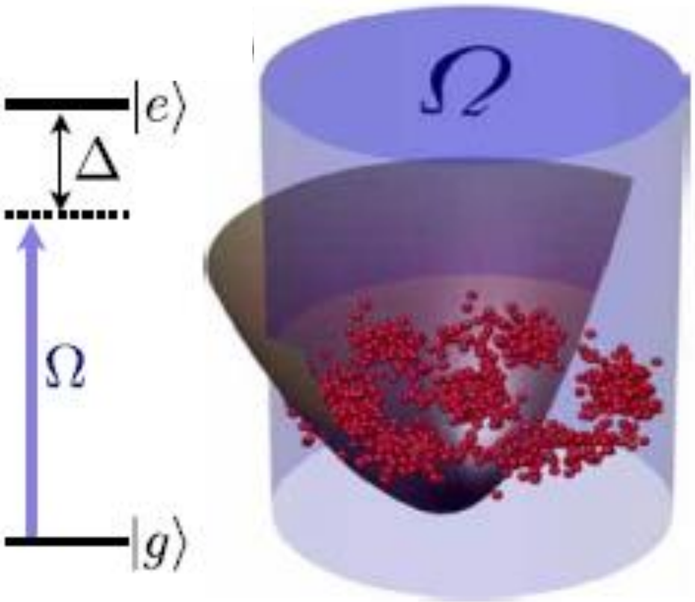




$$i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r},t) + V(\mathbf{r})\psi(\mathbf{r},t) + U_0|\psi(\mathbf{r},t)|^2\psi(\mathbf{r},t),$$

Introduction: Rydberg dressed BEC

A Rydberg-dressed BEC is a gas of Bose-condensed atoms whose ground state |g> is far off-resonantly coupled to a highly excited Rydberg state |e> with a Rabi frequency Ω much less than the corresponding laser detuning Δ.



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Introduction: Rydberg-dressed BEC

- Non-local GPE

$$i\hbar\partial_t\Psi(\mathbf{r},t) = \left[-\frac{\hbar^2\nabla^2}{2M} + V_{\text{ext}}(\mathbf{r}) + g|\Psi(\mathbf{r},t)|^2 + \int U\left(\mathbf{r}-\mathbf{r}'\right)|\Psi(\mathbf{r}',t)|^2 d\mathbf{r}'\right]\Psi(\mathbf{r},t),$$

where

$$U(\mathbf{r}-\mathbf{r}') = \frac{\tilde{C}_6}{R_c^6 + |\mathbf{r}-\mathbf{r}'|^6}$$

U(r-r') = long-range vdW interaction between Rydberg-dressed ground-state atoms.

$\tilde{C}_6$ = Effective coupling constant

R_C = Blockade radius

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Introduction: Rydberg-dressed BEC

- Dimensionless nonlocal-GPE (1-dimension)

Introduce the following parameter :

$$\tilde{t} = \frac{t}{t_s},\ \tilde{z} = \frac{z}{a_o},\ \tilde{\psi}(\tilde{z},\tilde{t}) = a_o^{\frac{1}{2}}\psi(z,t) \quad \tilde{D} - \frac{R_c}{}$$

Together with the scaling length an $\gamma = \frac{4\pi a}{a_o}$ and $\alpha = \frac{\tilde{C}_6 m}{a_o^4 \hbar^2}$.

$$a_o = \sqrt{\frac{\hbar}{m\omega_z}},\ t_s = \frac{1}{\omega_z}$$

We obtain :

$$i\partial_t \Psi(x,t) = \left[-\frac{1}{2}\frac{\partial^2}{\partial z^2} + \frac{1}{2}(z^2) + \gamma|\Psi(z,t)|^2 + \int \frac{\alpha}{(R_c{}^6 + |z-z'|^6)}|\Psi(z',t)|^2 dz'\right]\Psi(z,t)$$

Long range interaction dressed coupling

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Introduction: Rydberg-dressed BEC

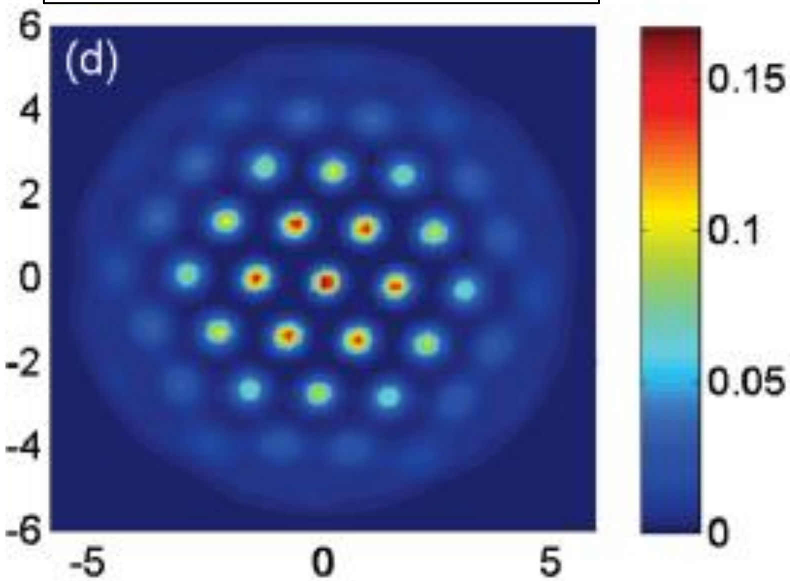
- Superfluid

Exactly same as the short range BEC !!

- Supersolid

Possessing crystalline and superfluid properties simultaneously !!

View of 2-D from the top



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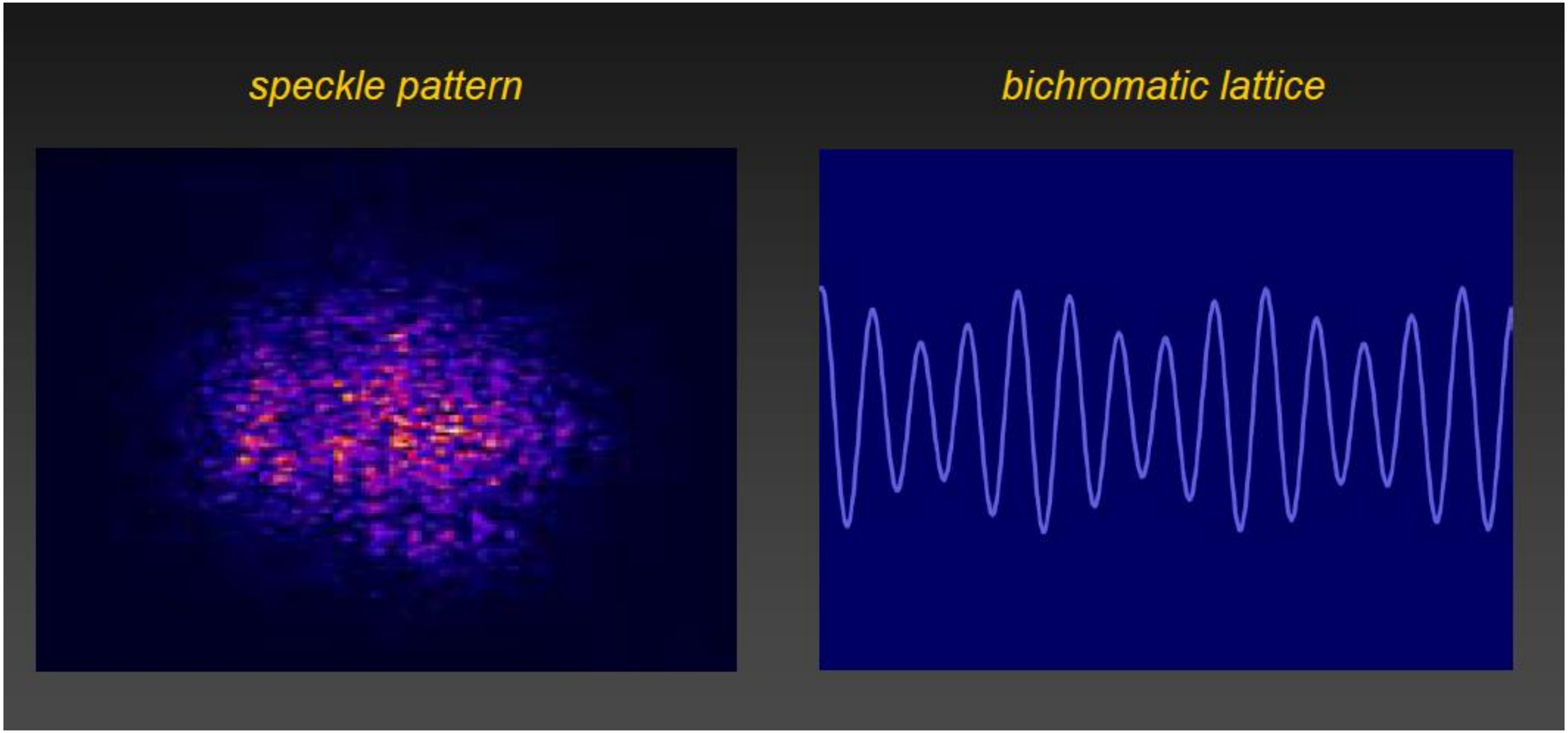
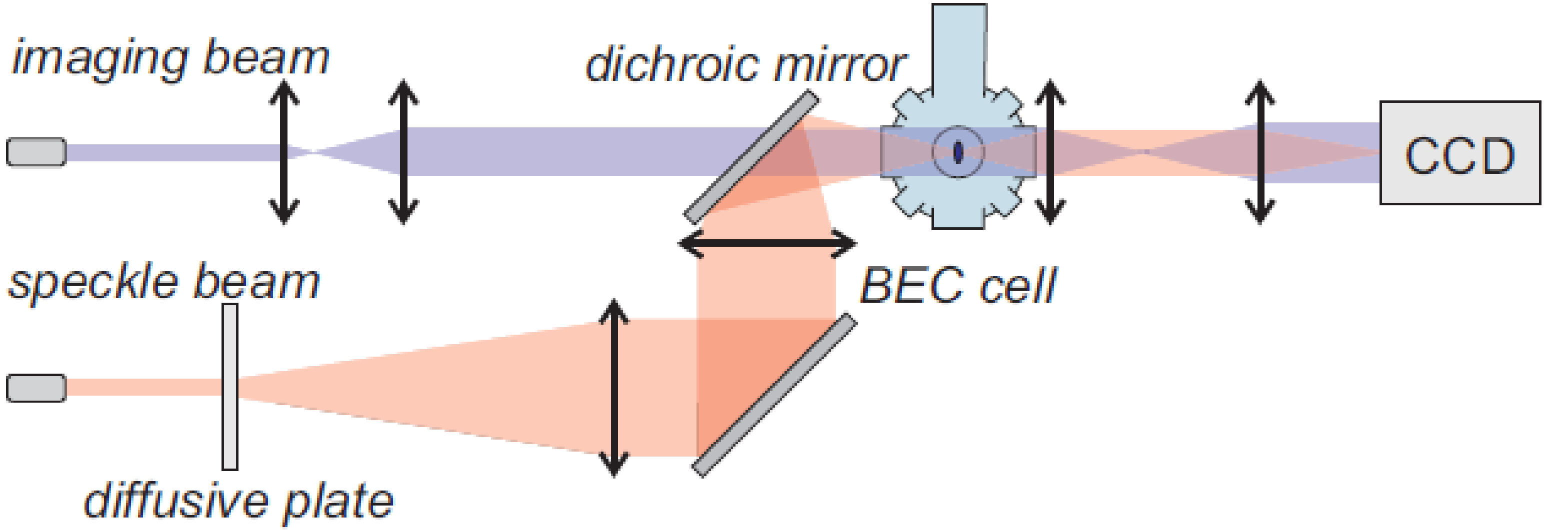
Introduction: Disorder Potential

- How to produce disorder ??



Introduction: Disorder Potential

Experimental setup for the speckle potential and the imaging system for a BEC

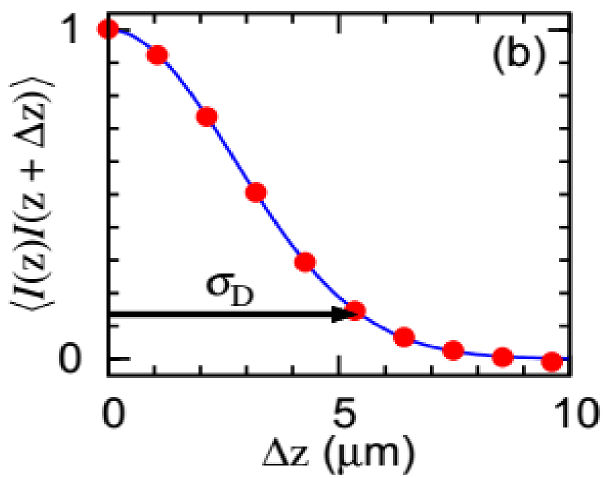


Introduction: Disorder Potential

The disorder potential from speckle potential is defined by the auto-correlation function:

$$\langle V_{dis}(z)V_{dis}(z+\Delta z)\rangle = V_D^2 \exp(-2\Delta z^2/\sigma_D^2)$$

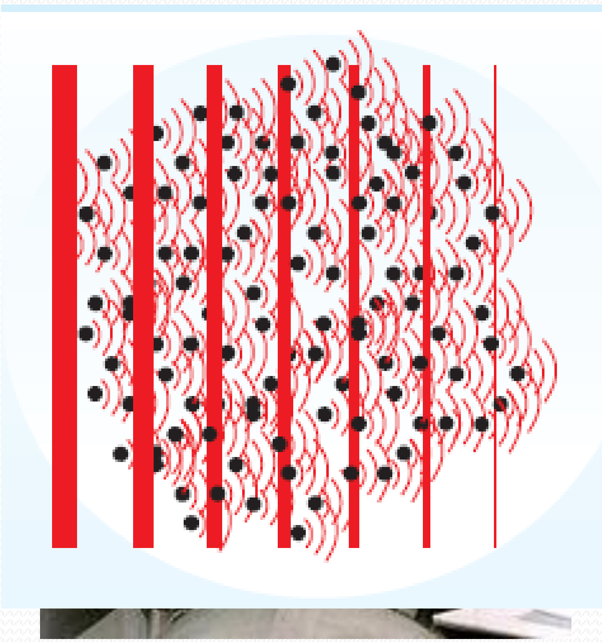
$V_D^2 = strength\ of\ V_{dis}$
$\sigma_D = disorder\ correlation\ length$



Dramatic effects of disorder

- Although this effect is weak, it is not always can be ignored in first approximation.
- A small amount of disorder can strongly effect the properties of physical systems
- Examples:
  - In classical systems: Brownian motion, percolation, etc.
  - In quantum systems: Quantum chaos, superconductor-insulator transition, **Anderson Localization**, etc.

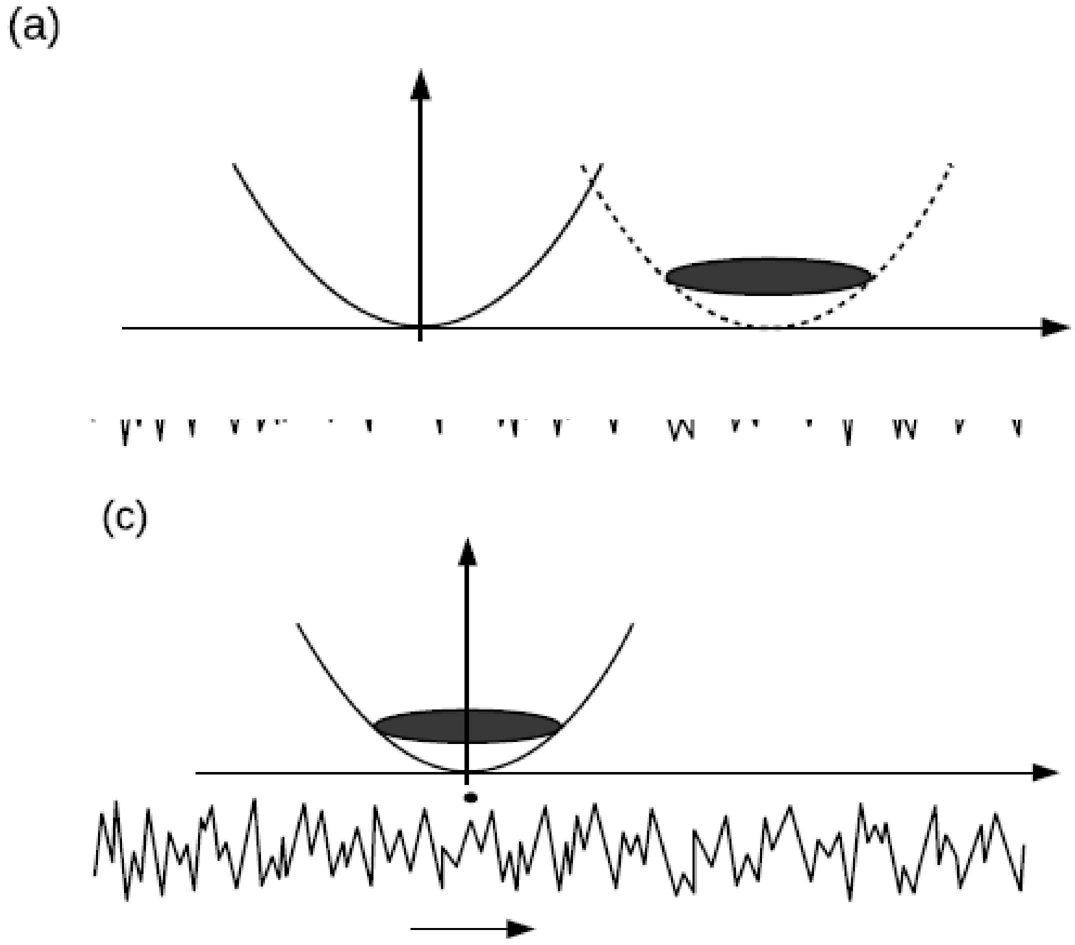
Introduction: Anderson Localization (AL)



- Anderson studied ***the transport of non-interacting electrons in a crystal lattice***. He (1958) reported for first time that in the presence of a critical amount of random impuritie $|\psi| \sim \exp(-|\boldsymbol{r}-\boldsymbol{r}_i|/\xi)$, ion of an electron will come to a halt. This constitutes the idea of electron localization due to a destructive interference of particles (waves).

NUMERICAL APPROACH

Experimental Method (Hulet, *et al.* [Phys. Rev. A 82, 033603]



Numerical approach

Numerical method

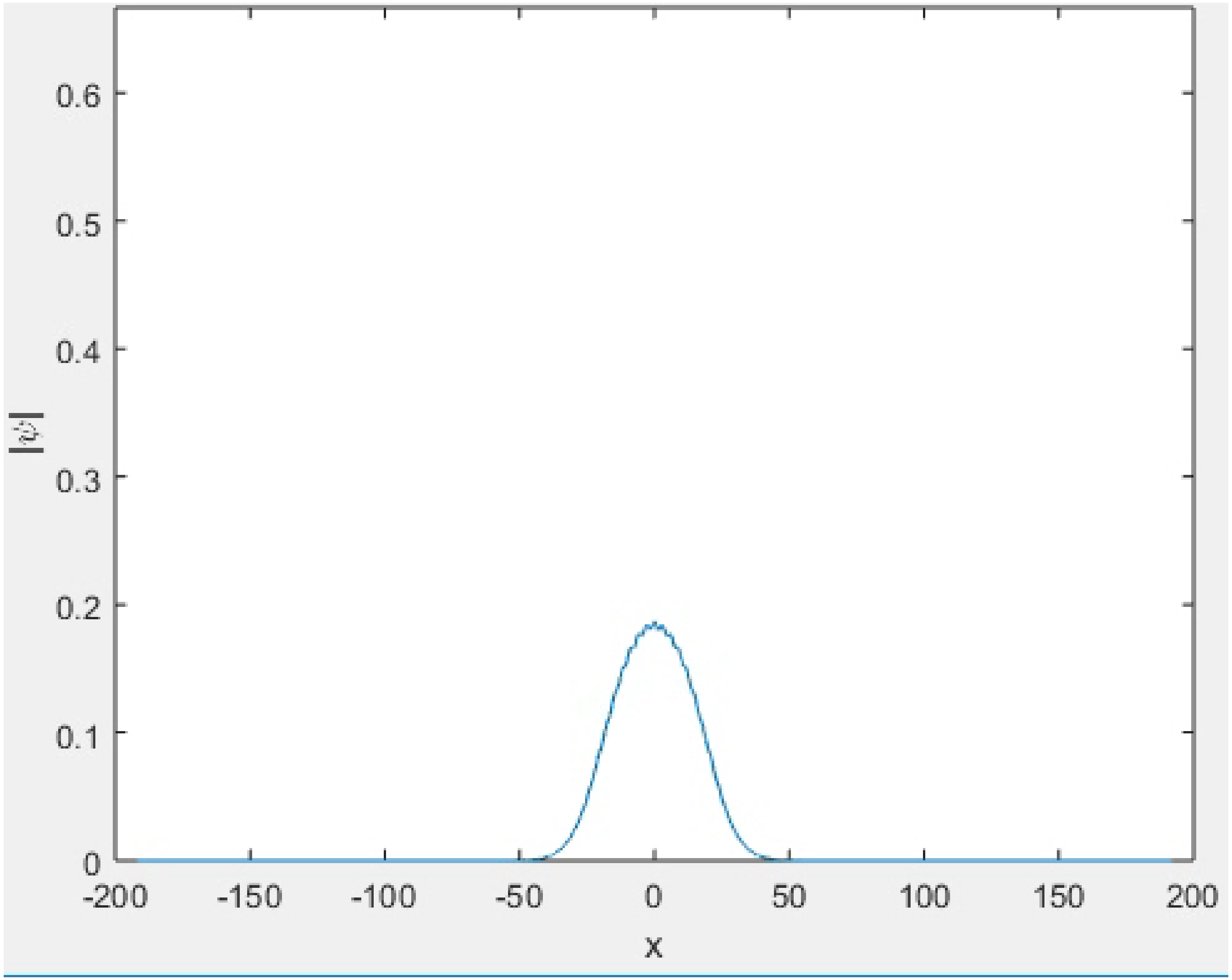
- To obtain an initial wave function with a velocity v_0, we apply the Galilean transformation:

$$\psi = \varphi \exp(imv_0 z)$$

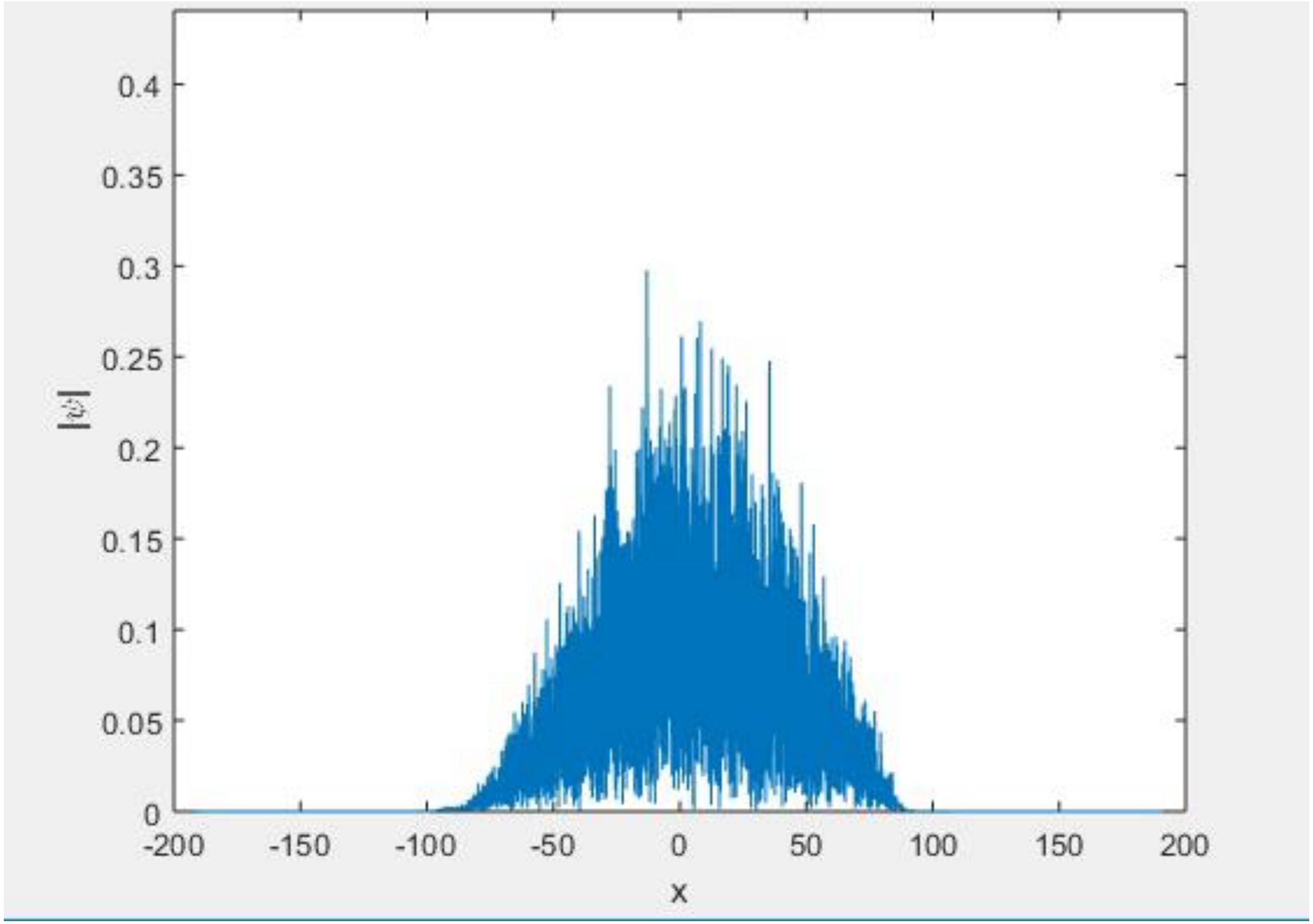
and the corresponding non-local GPE for the residual wave function φ is

$$i\hbar\partial_t\varphi(z,t) = \left[\frac{1}{2}\left(\frac{\hbar}{i}\partial_z - mv_0\right)^2 + \frac{m}{2}(\omega_z^2 z^2) + V_{dis}(z) + g|\varphi(z,t)|^2 + \int \frac{\tilde{C}_6}{({R_c}^6 + |z-z'|^6)}|\varphi(z',t)|^2 dz'\right]\varphi(z,t)$$

THE BEGINNING OF OSCILLATION



The end of oscillation



Brief results for Short Range BEC

(arXiv:1807.11045 [cond-mat.quant-gas])

- Numerical Parameter follow Hulet's parameter:

  $\mu = 200$, $v_o = 37.5$, $V_D = 50.9$, and $\sigma_D = 0.25$.

- With this parameters, then the Healing length of the condensate is:

$$\mu_{TF} = \frac{\hbar^2}{2m\xi^2}$$

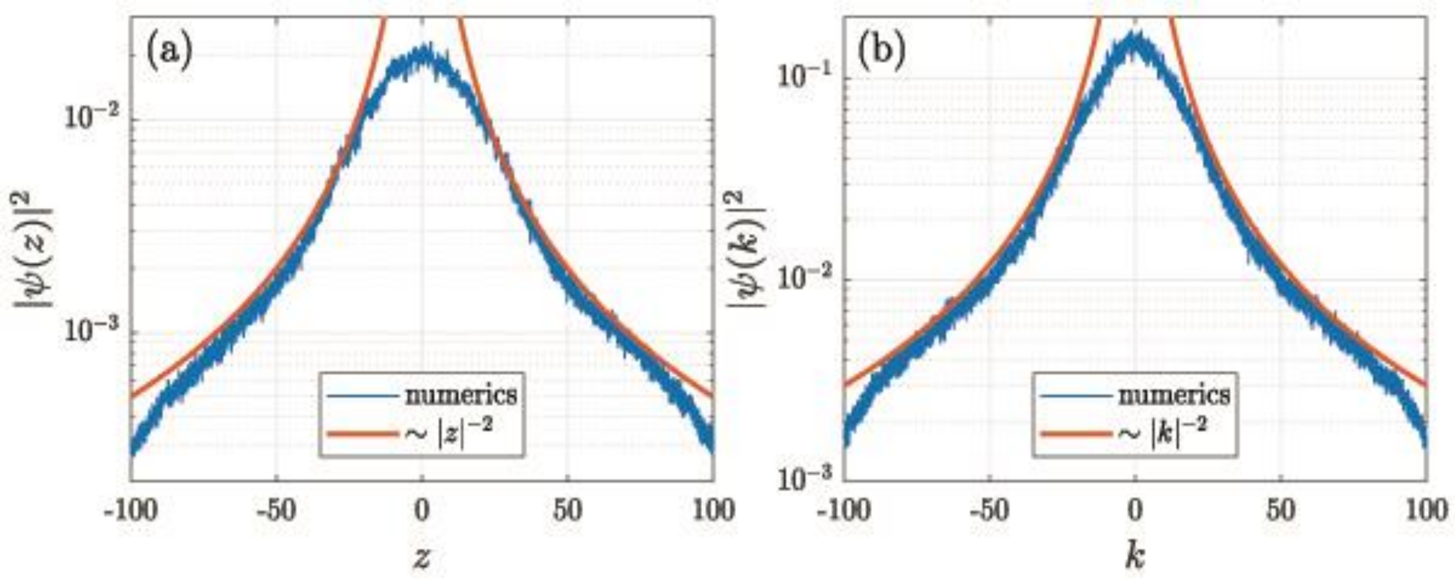
$$200 = \frac{1}{2\xi^2} \rightarrow \xi = 0.05$$

- ξ is typical distance over which the order parameter of the condensate recovers its bulk value when it is forced to vanish at a given point by, for instance an impurity.

Brief results for Short Range BEC

(arXiv:1807.11045 [cond-mat.quant-gas])

Our simulation produce an algebraic localization when ξ is smaller then σ_D



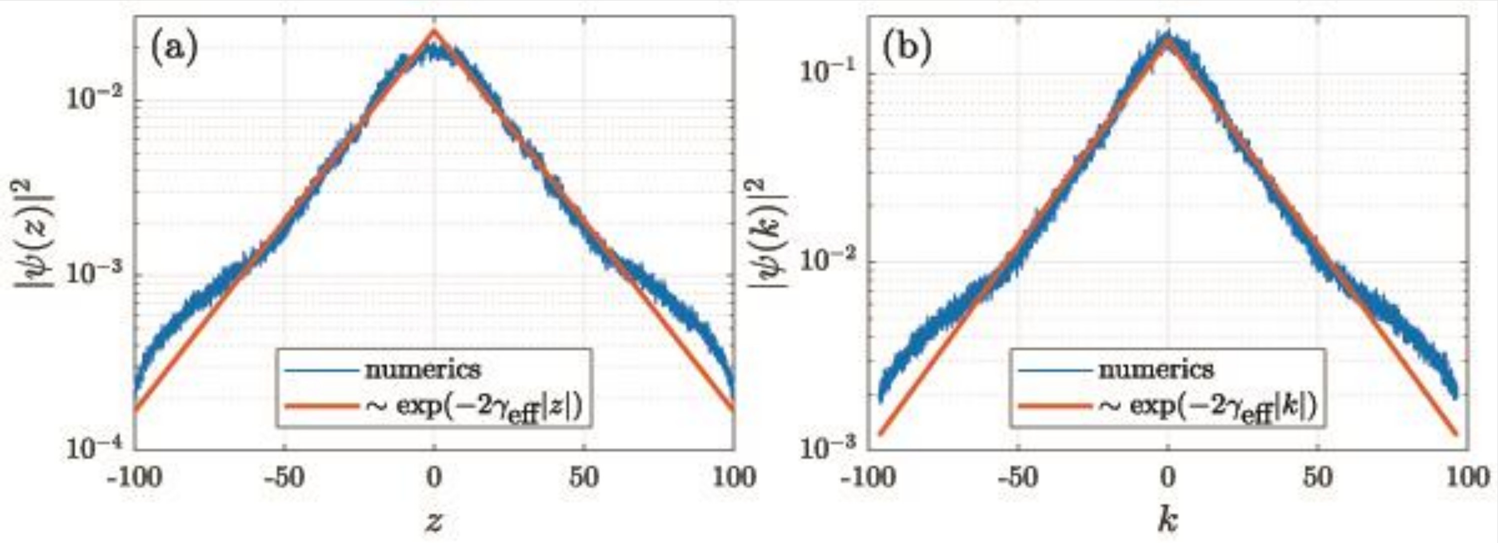
- The result in accordance with the theoretical prediction by Sanchez-Palencia *et al*. [Phys. Rev. Lett. 98, 210401 (2007)] : "**For ξ < σ_D, the localization of the BEC becomes *algebraic* and it is only *partial*.**"

Brief results for Short Range BEC

(arXiv:1807.11045 [cond-mat.quant-gas])

Can the AL be seen in a similar oscillation experiment?

- we perform another simulation for the same parameters except by reducing σ_D to the regime $\xi > \sigma_D$.



The result is also in accordance with the theoretical prediction by Sanchez-Palencia *et al.* [Phys. Rev. Lett. 98, 210401 (2007)] : "**For $\xi > \sigma_D$, the whole BEC wave function is exponentially localize**."

The importance of healing length

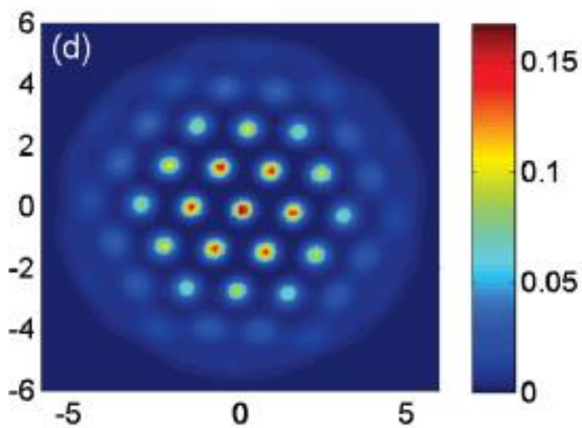
- Healing length is therefore a measure of the importance of the interactions.
- In the regime $\xi > \sigma_{D}$, the particle interaction is relatively weak and the condensate will eventually localize.
- Note: $\xi \rightarrow \infty$ corresponds to a noninteracting limit.

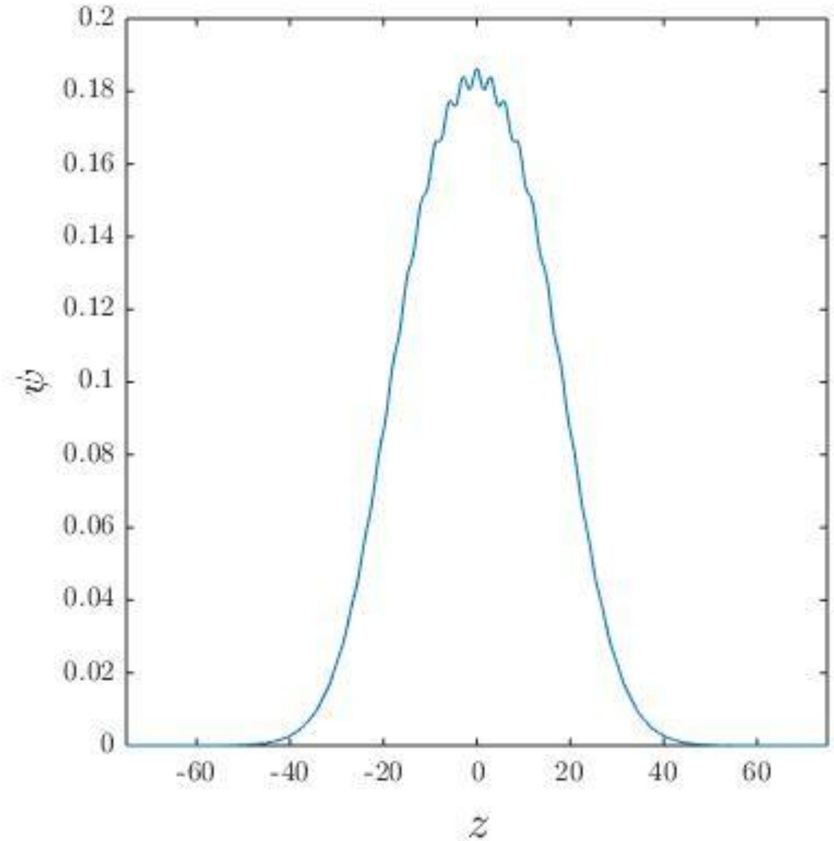
Criteria of our simulation:

- a fixed Thomas–Fermi Radius (L_{TF}) hence the appearance of supersolid or superfluid phase is depend on R_c. In here, **we use L_{TF} = 20.**
- short range interaction (**g**) =**0**
- We limit the values of **σ_D to 0.01, while ξ to 0.05.**

Initial state for supersolid:

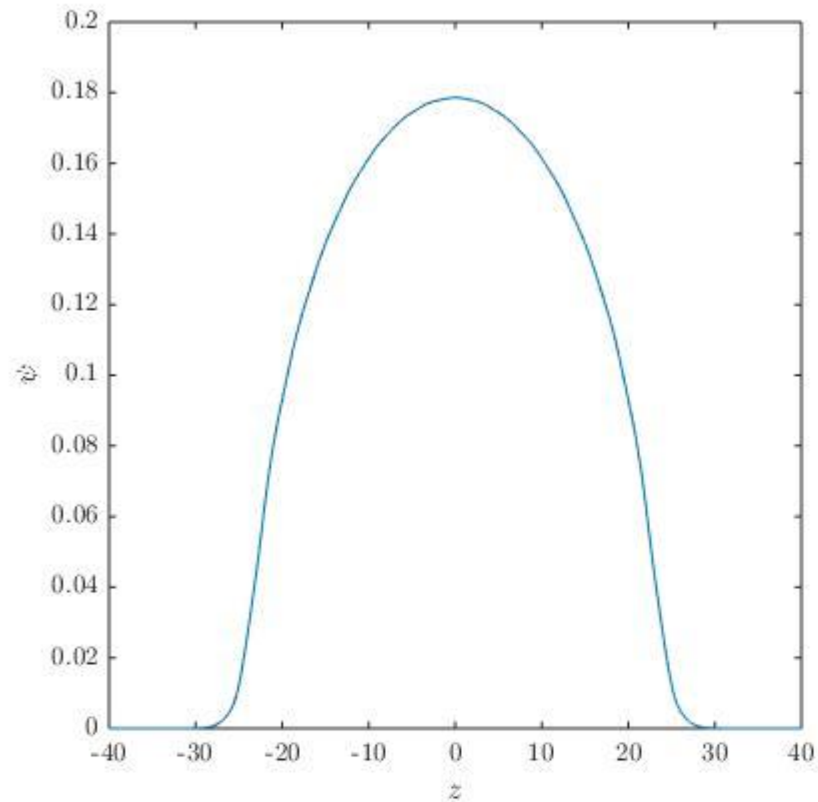
- The condensate enters supersolid state when the value of R_C is relatively large. For example, we use : 2,6.



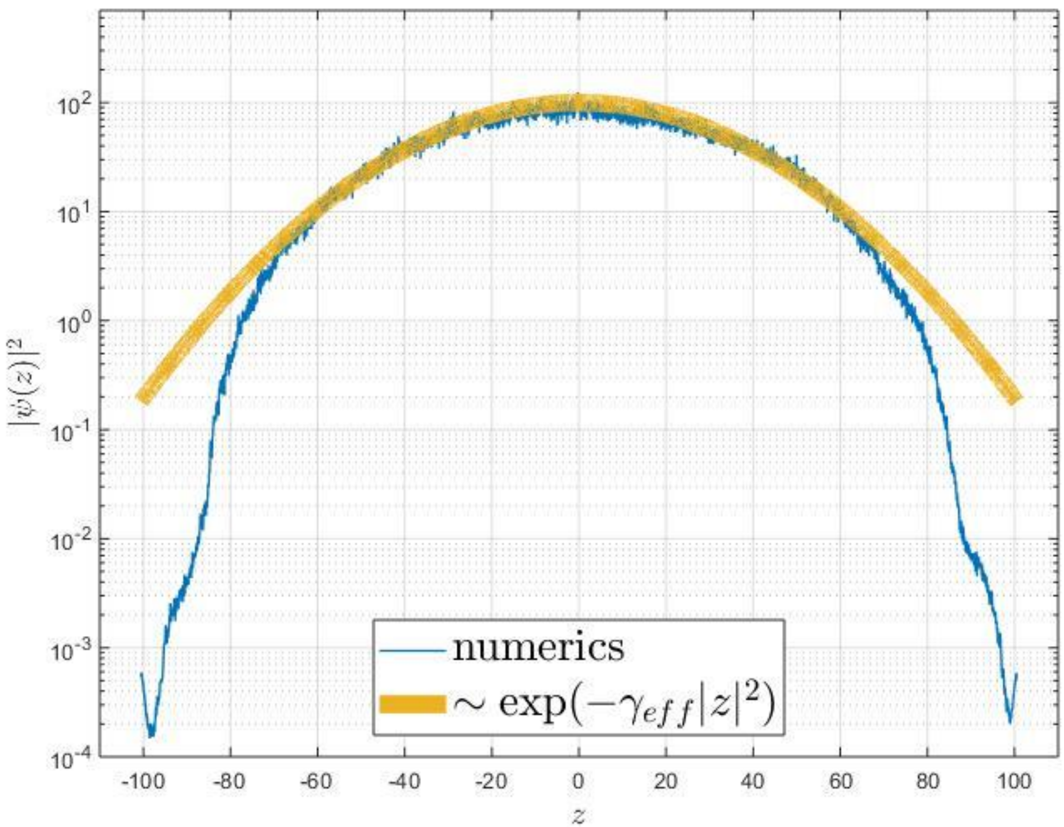


Initial state for superfluid

- The condensate enter superfluid state when the value of R_C is relatively small. For example, we use : 0.01.

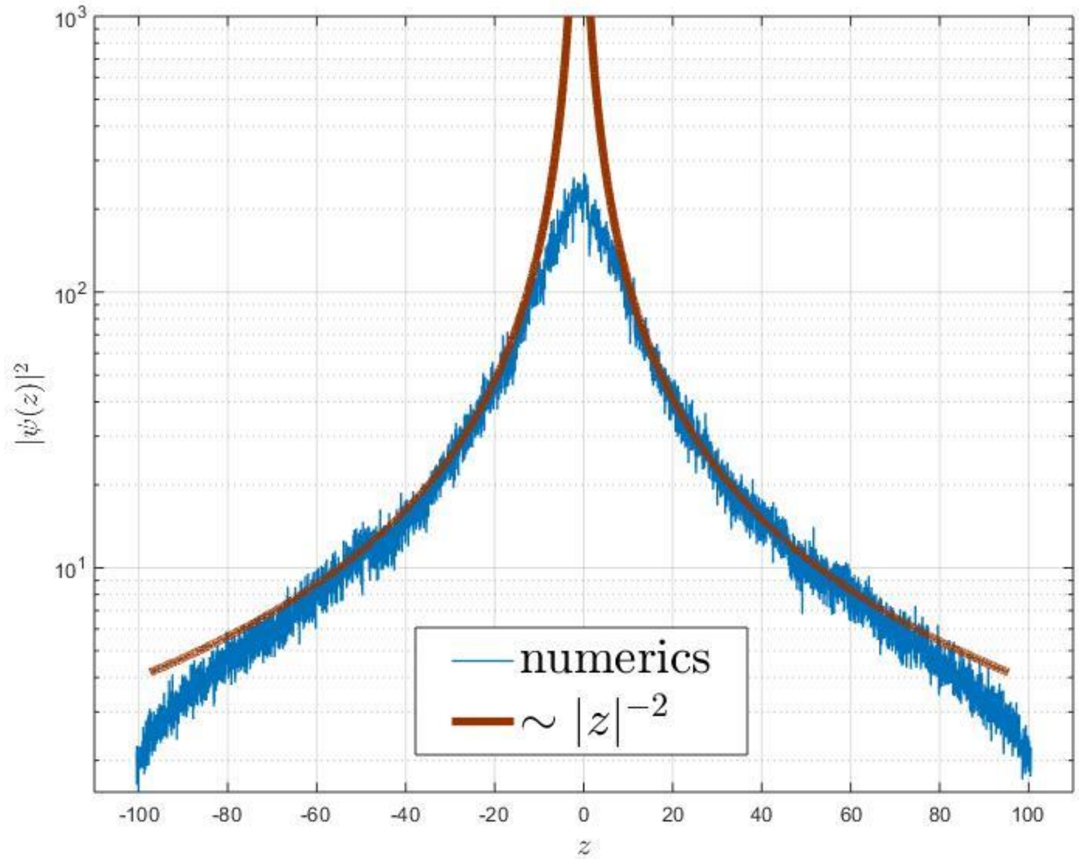


Results: Density Profile(Supersolid)

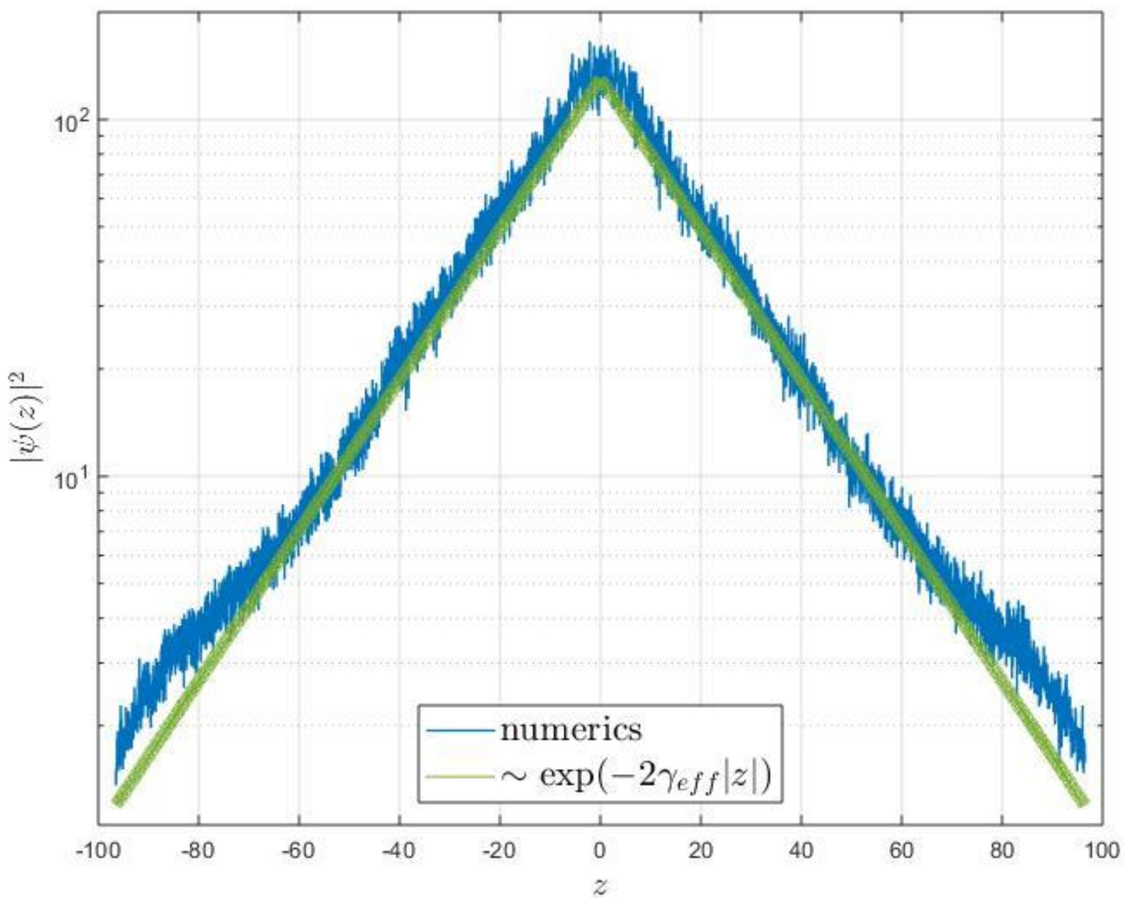


$$\gamma_{eff}(Lyapunov\ exponent) = \left(\frac{\pi}{32\xi}\right)\left(\frac{V_D}{\mu}\right)^2\left(\frac{\sigma_D}{\xi}\right) exp[-(\sigma_D/\xi)^2]$$

Density Profile(Superfluid I)



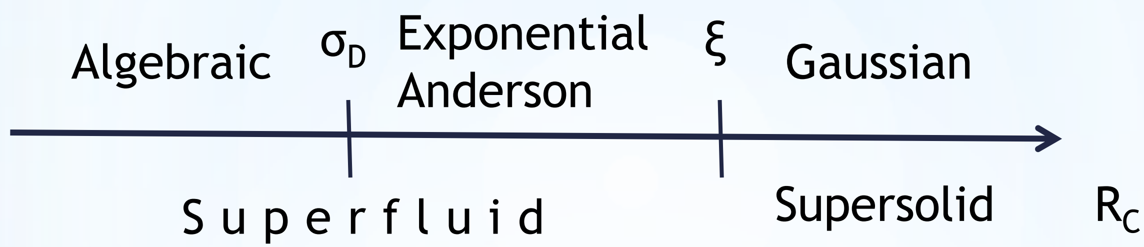
Density Profile(Superfluid II)



Exponential Anderson localization arise when the superfluid initial state has a value of R_c between ξ and σ_D($\sigma_D < R_c < \xi$).

Conclusion

- Oscillation of Rydberg-dressed BEC can also exhibit localization. It can be seen in the following Graph



- The role of ξ in superfluid was taken over by $R_{C,}$

Short Range	Long Range
$\xi > \sigma_D$ (Anderson)	$R_C > \sigma_D$ (Anderson)
$\xi < \sigma_D$ (Algebraic)	$R_C < \sigma_D$ (Algebraic)

- However the rule that full localization exist in $\xi > \sigma_D$ regime still valid.