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2015 AMO Summer School

Quantum Optics with Propagating Microwaves in Superconducting Circuits I

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Io-Chun, Hoi

Outline

1. Introduction to quantum electrical circuits
2. Introduction to superconducting artificial atom
3. Quantum optics with superconducting circuits
4. Single atom scattering

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Introduction to quantum electrical circuits

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Quantum electrical circuits

Macrosopic system

Coherent superposition states:

Charge Q

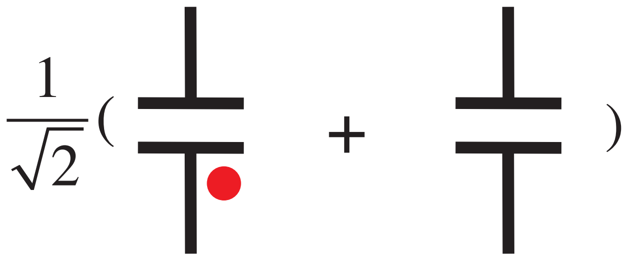
Flux Φ

Properties:

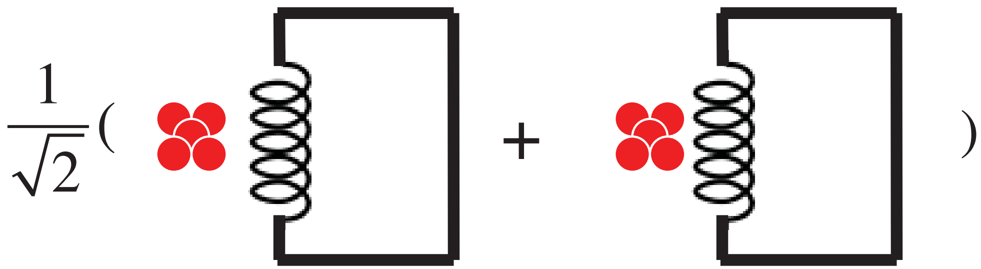
The superposition states collapse when measure.

Probabilistic character.

Charge on a capacitor:



Current or magnetic flux in an inductor:



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Conventional electrical circuits

First transistor 1947

Basic elements:

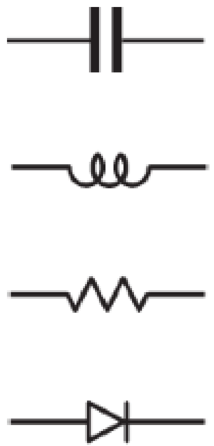
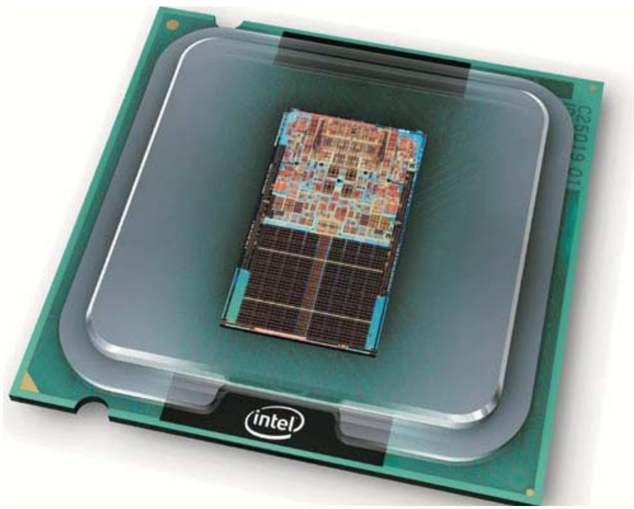


Fig. from Intel

Dual-core Intel processor



Introduced 2007
Clock speed >3GHz
Number of transistors
820million
Manufacturing technology
45nm

Fig. from Intel

Properties:

*Deterministic

*No quantum mechanics

*No superposition principle

*No quantization of fields

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Introduction to superconducting artificial atom

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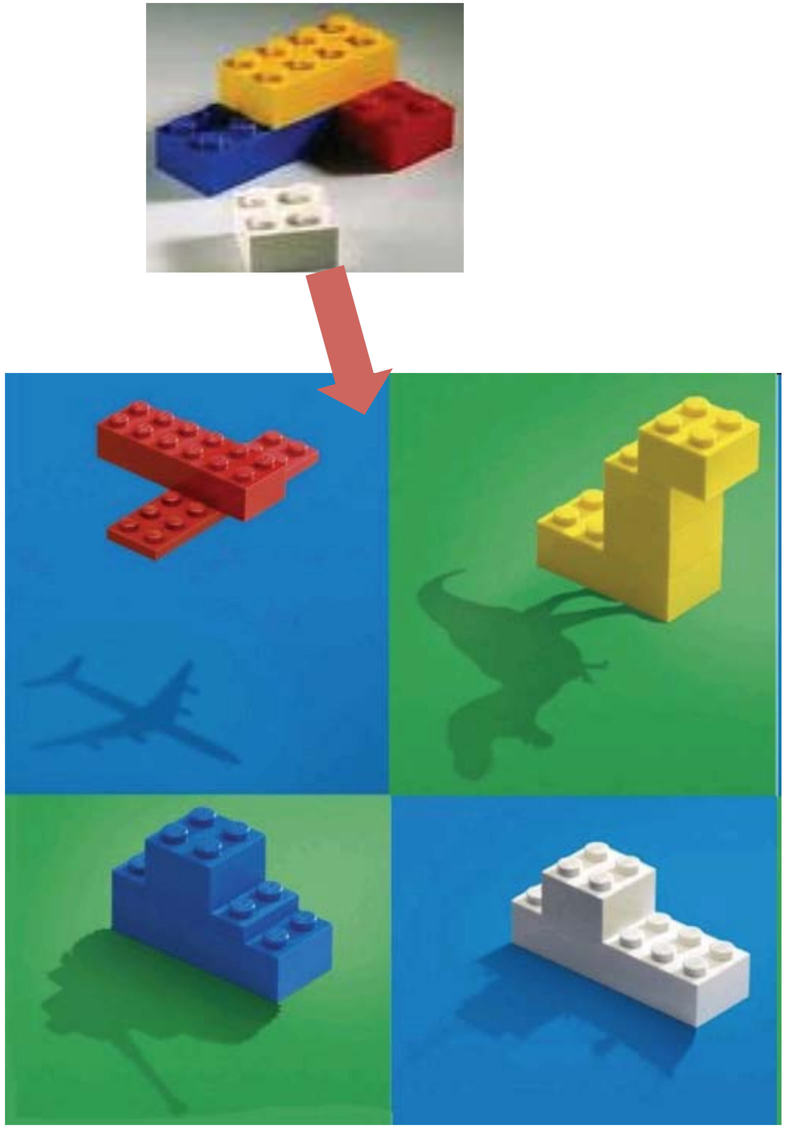
Superconducting circuits are like LEGOS

What's good about circuits?

- **Circuits are like LEGOs!**

a few elementary building blocks,
gazillions of possibilities!

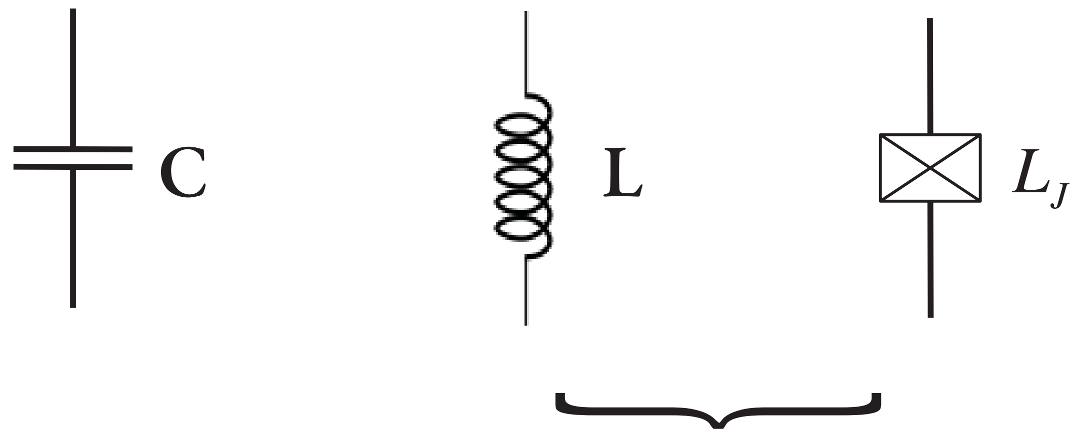


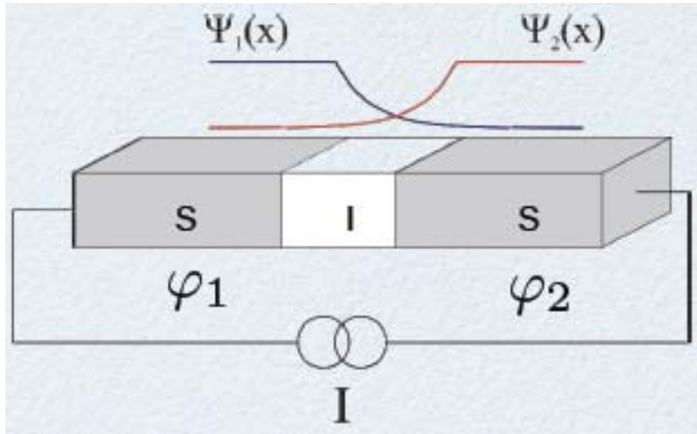
Basic Elements of Superconducting Circuits

Dissipationless!

Josephson Junction:
Non-disspative
nonlinear inductance

Tunnel barrier between
two superconductors



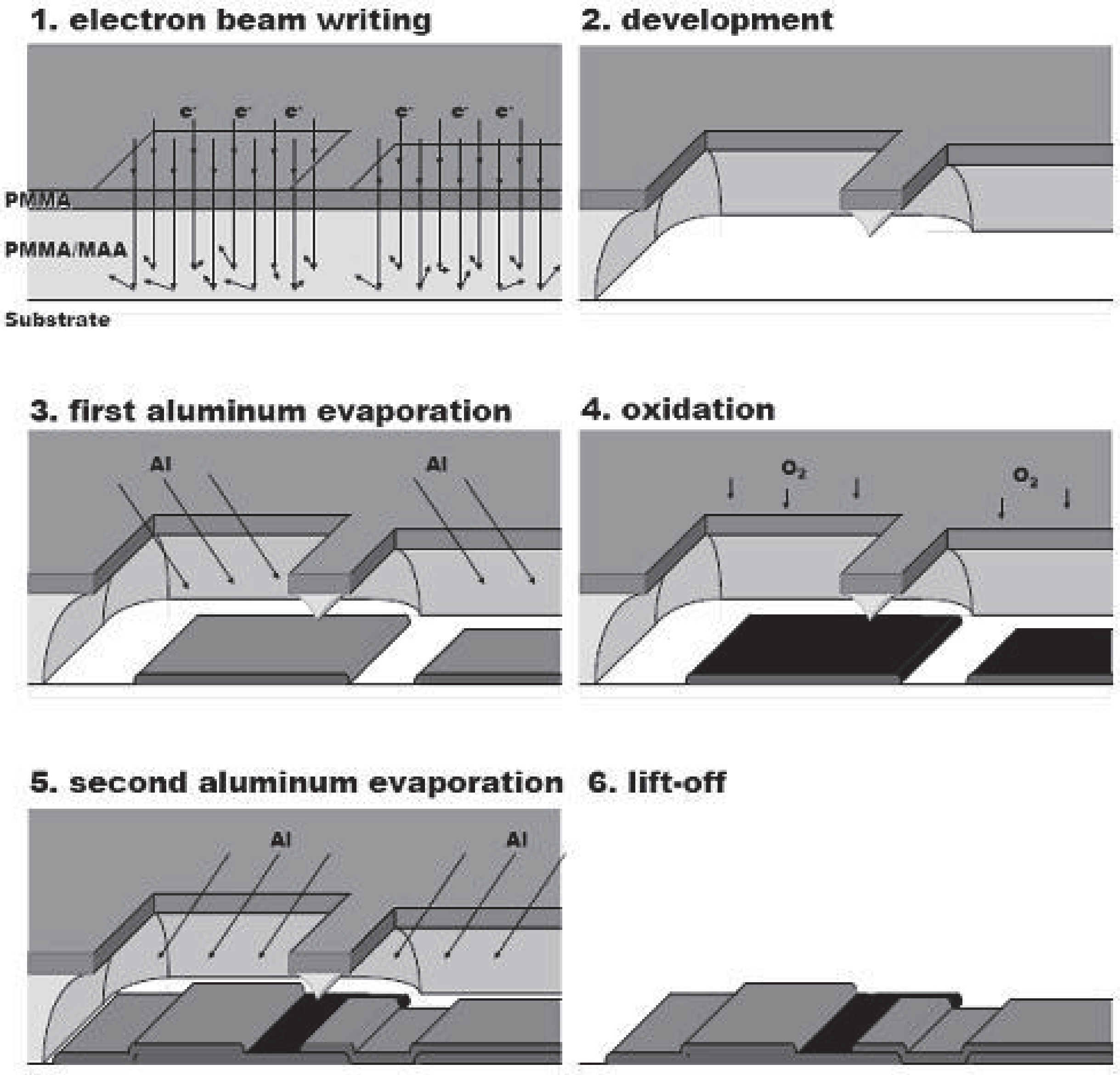


Capacitance

Inductance

$$I = I_c \sin\phi \qquad \frac{d\phi}{dt} = \frac{2e}{\hbar}V$$

Fabrication of Josephson Junction



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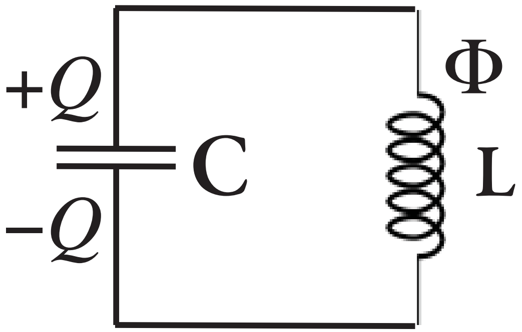
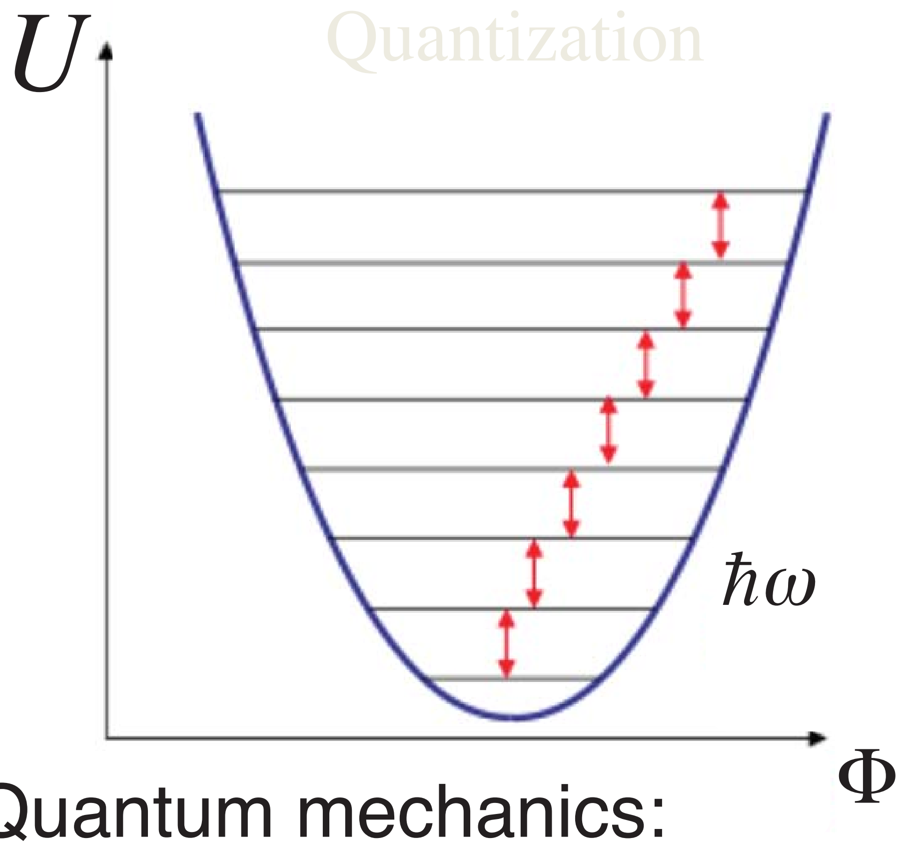
Constructing linear quantum electrical circuits



$$\omega = \frac{1}{\sqrt{LC}} \sim GHz$$



Classical physics:

$$H = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

$$H = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

Analogy with a moving particle in a harmonic potential

Quantum mechanics:

$$H = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L} \quad H = \hbar\omega(a^\dagger a + \frac{1}{2}) \quad [\hat{\Phi}, \hat{Q}] = i\hbar$$

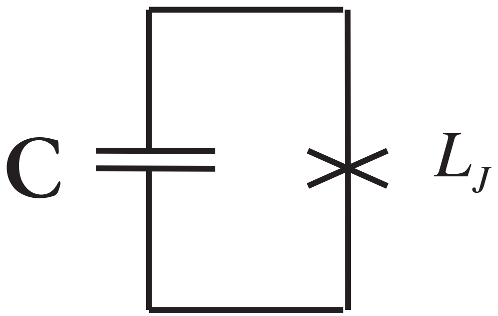
M. H. Devoret, A. Wallraff, and J. M. Martinis. Superconducting qubits: A short review. http://arxiv.org/abs/cond-mat/0411174v1, 2004.

Constructing nonlinear Quantum circuit:

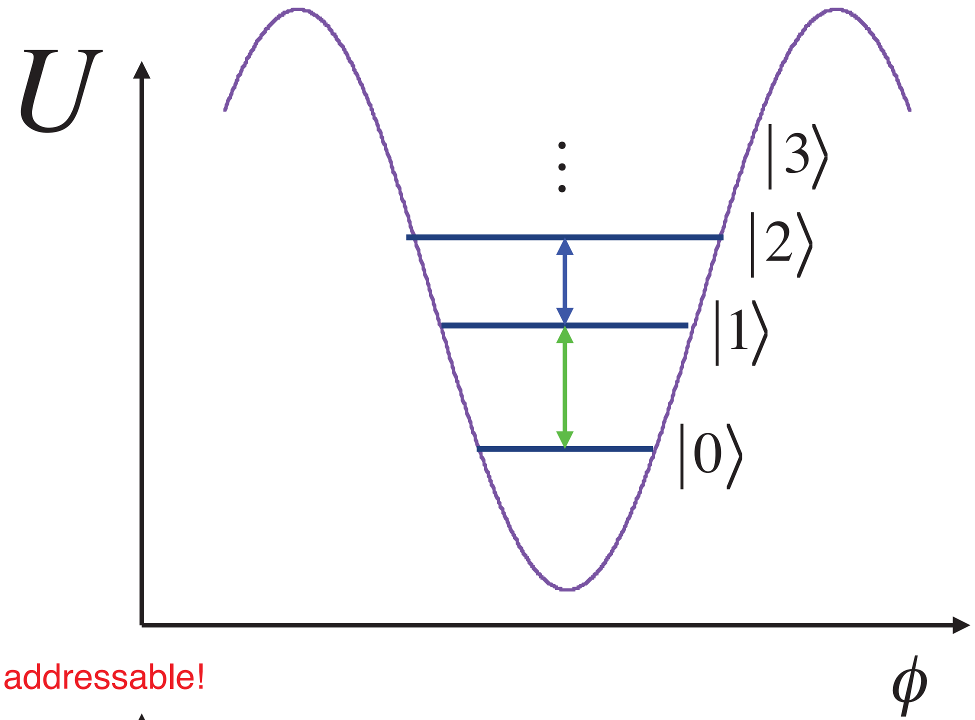
Artificial atom

Replace linear inductance by Josephson junction (Nonlinear inductance)

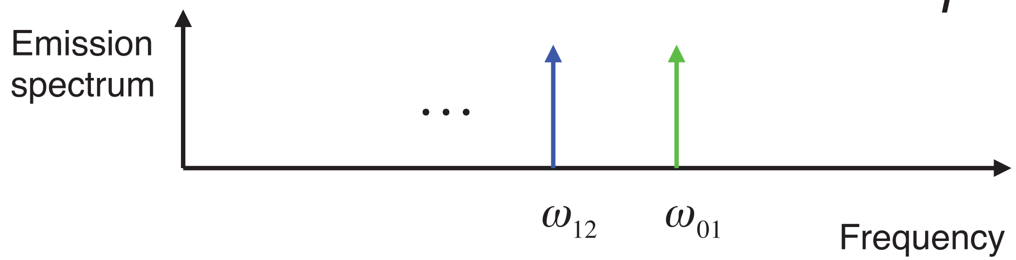
$$U = -E_J \cos\phi$$



$$L_J = \frac{\hbar}{4eI_c \left| \cos\left(\pi \frac{\Phi_{ext}}{\Phi_0} \right) \right|}$$



Transition become addressable!



Anharmonicity:

$$\alpha = \omega_{01} - \omega_{12}$$

How to operate electrical circuits quantum mechanically?

Avoid dissipation

Avoid broaden energy levels

Work at low temperatures

Provide reset of the circuit(Ground state)

$$k_B T << \hbar\omega << \Delta_s$$ Superconducting gap energy

$\omega / 2\pi \sim 4 - 8GHz$ $T @ mK$

Family of superconducting artificial atom

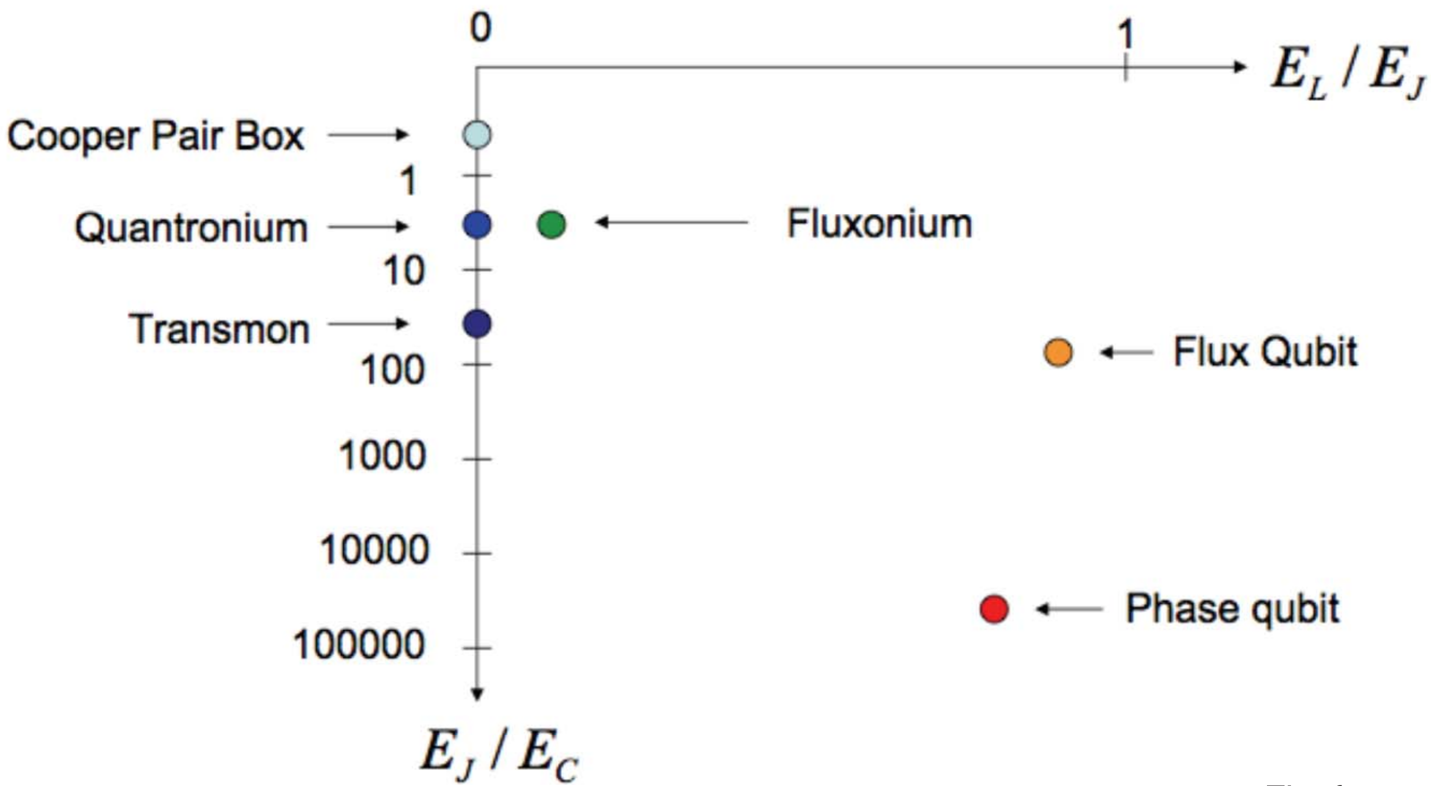


Fig. from
Michel Devoret.
Linneaus summer school
in quantum engineering. 2010.

J. Clarke and F. K. Wilhelm.
Nature, 453:1031–1042, 2008.

G. Wendin and V. S. Shumeiko
Low Temp. Phys., 33(9):724–744, 2007.

Focus on Cooper Pair Box and Transmon!

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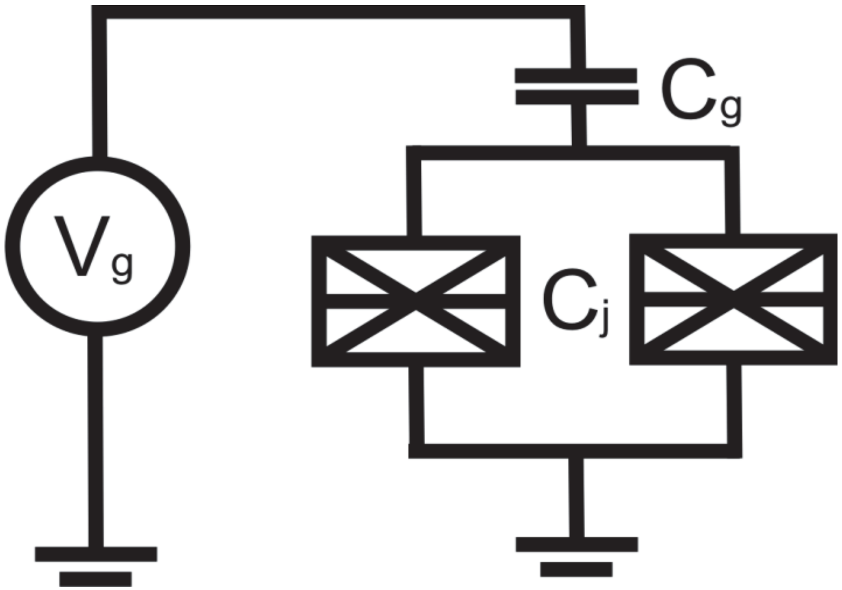
Artificial atom I: The Single-Cooper Pair Box

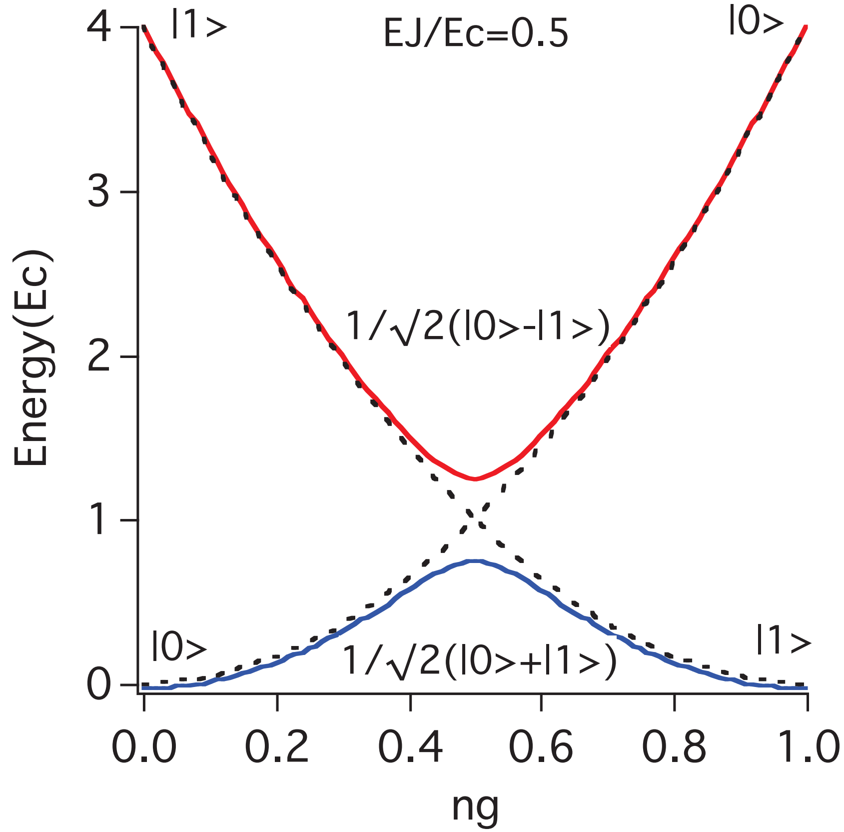
$E_J / E_c < 1$

$$H = -\frac{1}{2}E_{ch}\sigma_z - \frac{1}{2}E_J\sigma_x$$

Map to a spin 1/2 particle in magnetic field.

E_J: Depends on external flux





$$E_J = \frac{\Phi_0 I_c}{2\pi}\left|\cos\frac{\pi\Phi}{\Phi_0}\right| \qquad C_\Sigma = C_g + C_J \qquad \sigma_z, \sigma_x \text{ :Pauli matrix}$$

Coherent oscillations between ground state and excited state in time domain, demonstrated by Y. Nakamura *et al.* Nature, 398:786–788, 1999.

$$E_{ch} = E_Q(1 - 2n_g) \qquad E_Q = 4E_C = \frac{(2e)^2}{2C_\Sigma} \qquad n_g = C_g V_g / (2e)$$

But the coherence time is short (few ns)due to charge noise!

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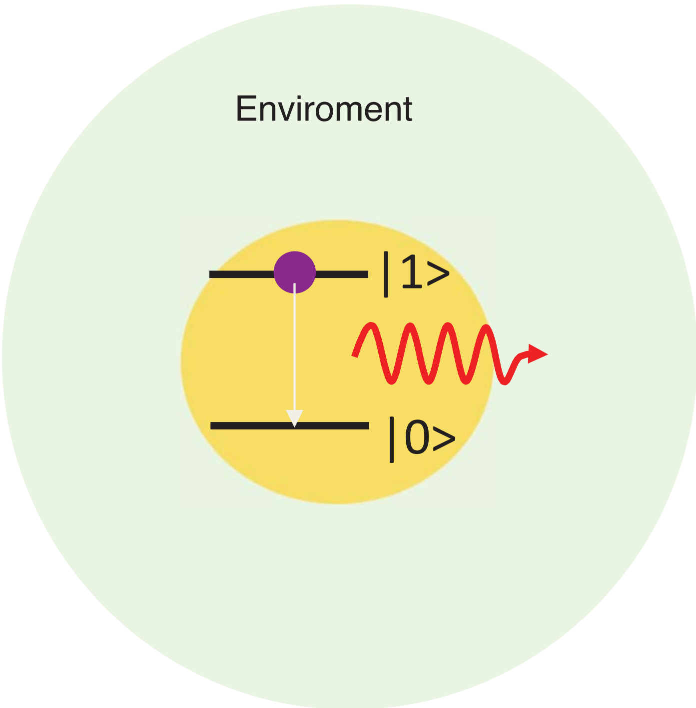
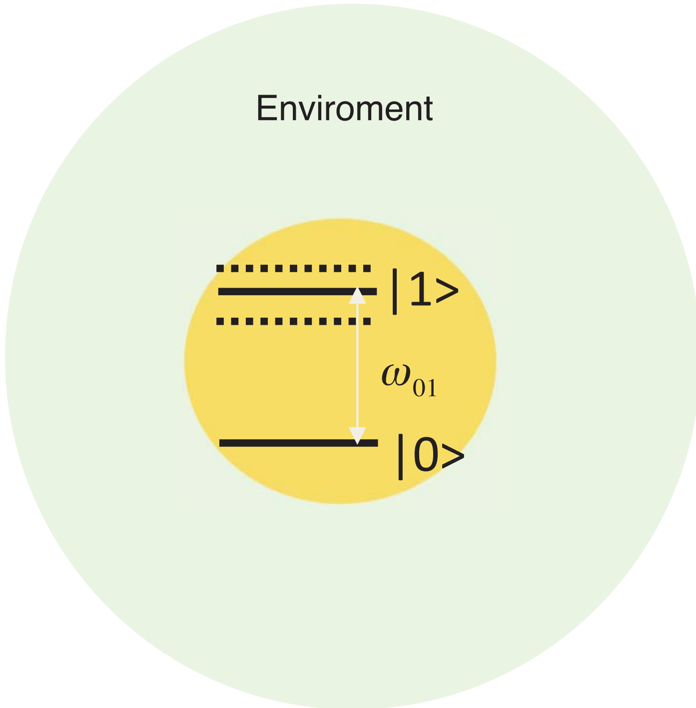
Decoherence of artificial atom

(Effect from the environment)

Relaxation rate Γ_{01}

Pure dephasing rate Γ_{φ}





Random switching

$|1\rangle \rightarrow |0\rangle$

$\omega_{01} \rightarrow \omega_{01} + \delta\omega_{01}(t)$

Phase randomization $e^{-i\omega_{01}t}$

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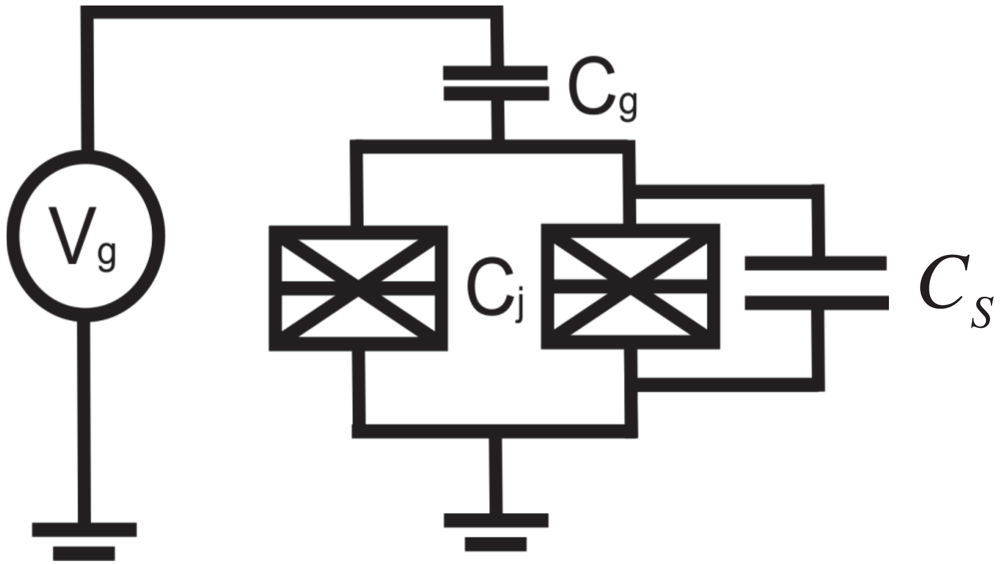
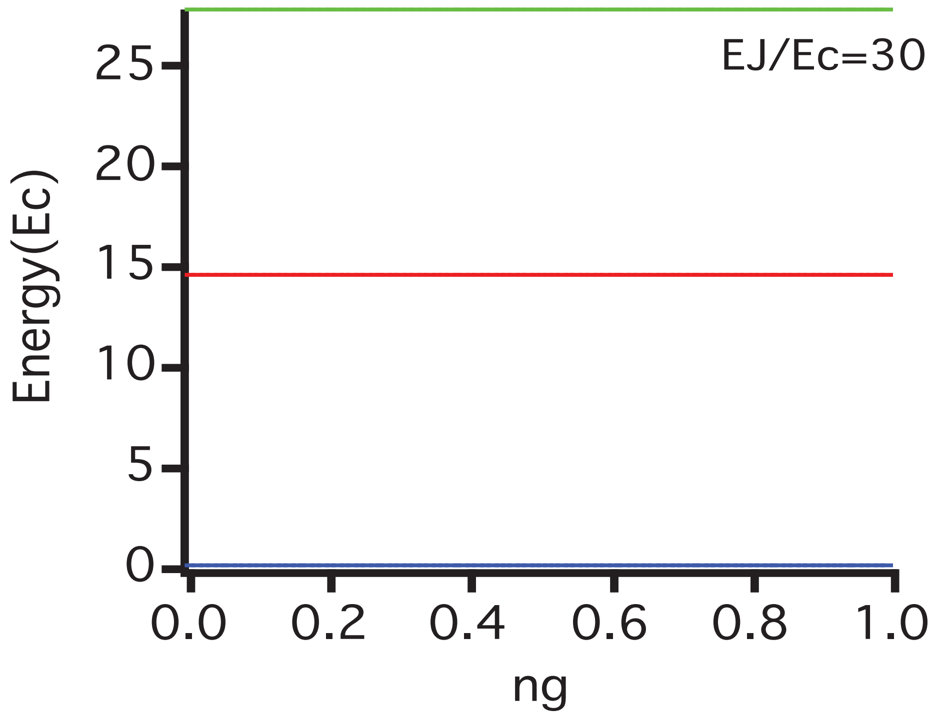
Artificial atom II: The transmon

$$20 < E_J / E_c < 100$$



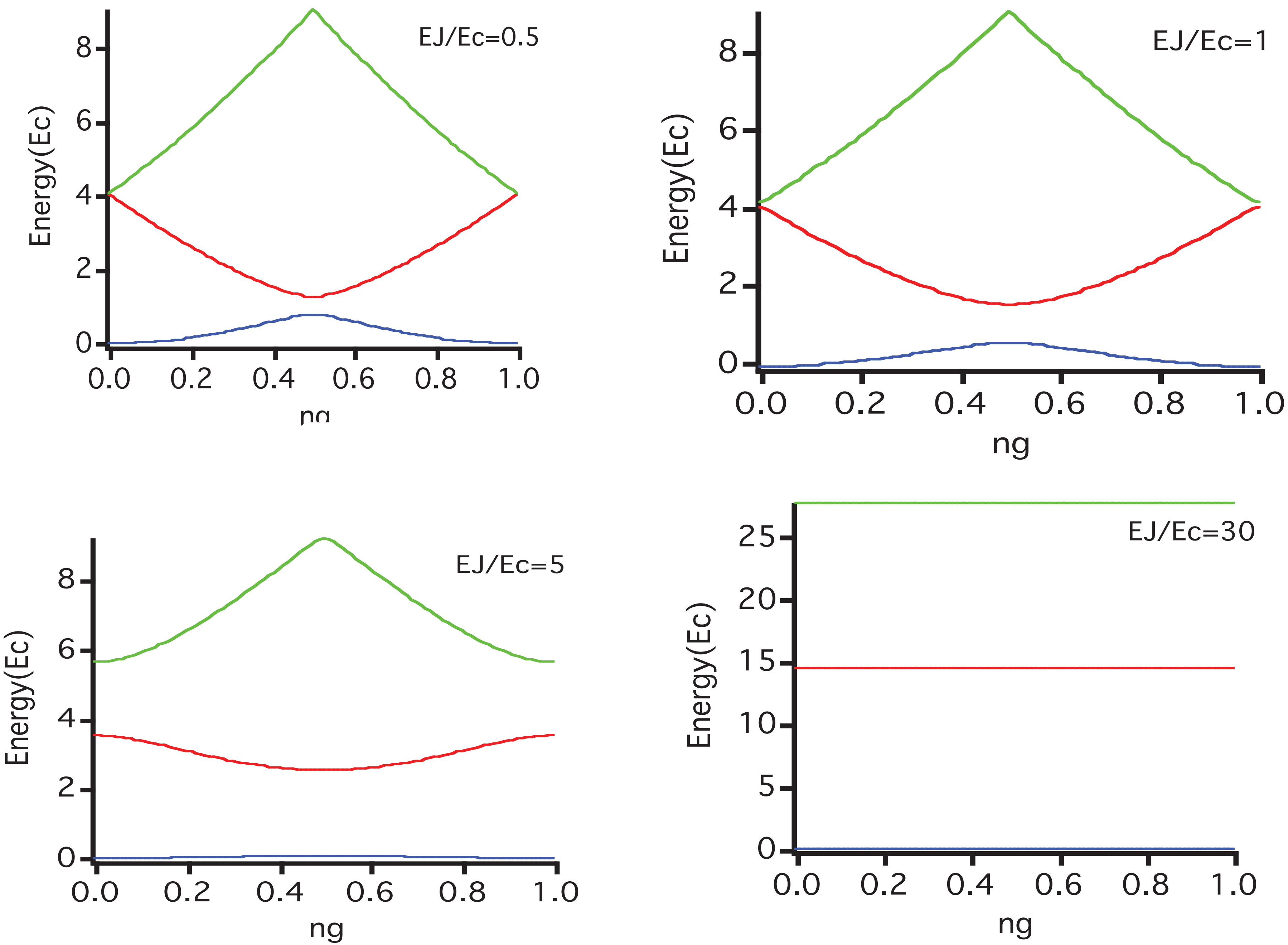


Insensitive to the charge noise

Long coherence time.

Jens Koch *et al.*

Physical Review A, 76(4):042319, 2007.


EJ/Ec=0.5
Energy(Ec)
0
2
4
6
8
0.0
0.2
0.4
0.6
0.8
1.0
ng
EJ/Ec=1
Energy(Ec)
0
2
4
6
8
0.0
0.2
0.4
0.6
0.8
1.0
ng
EJ/Ec=5
Energy(Ec)
0
2
4
6
8
0.0
0.2
0.4
0.6
0.8
1.0
ng
EJ/Ec=30
Energy(Ec)
0
5
10
15
20
25
0.0
0.2
0.4
0.6
0.8
1.0
ng

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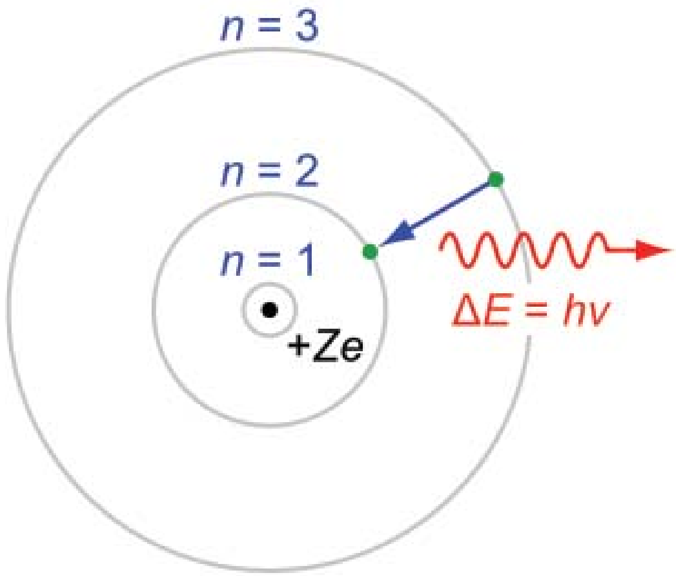
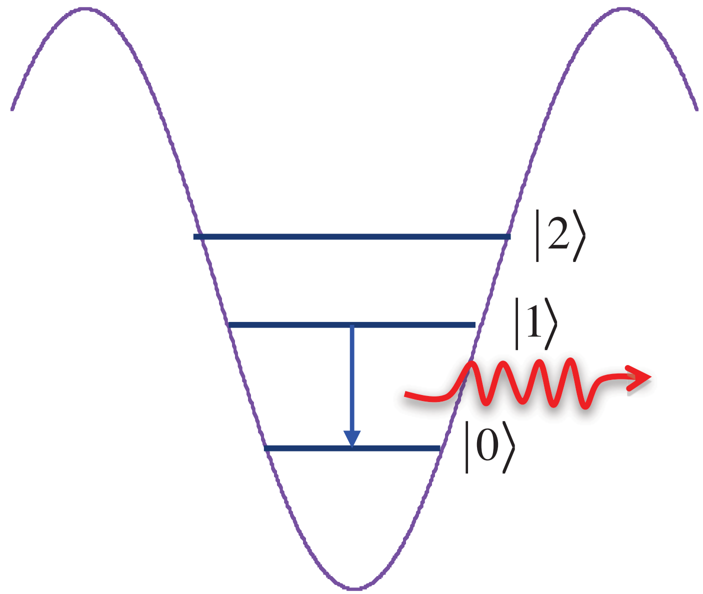
Studying/Engineering the matter-light interaction





Natural atom

Optical photons

Superconducting artificial atom

Microwave photons

Compare with optical photon, the frequency of microwave photon is 10^6 less.

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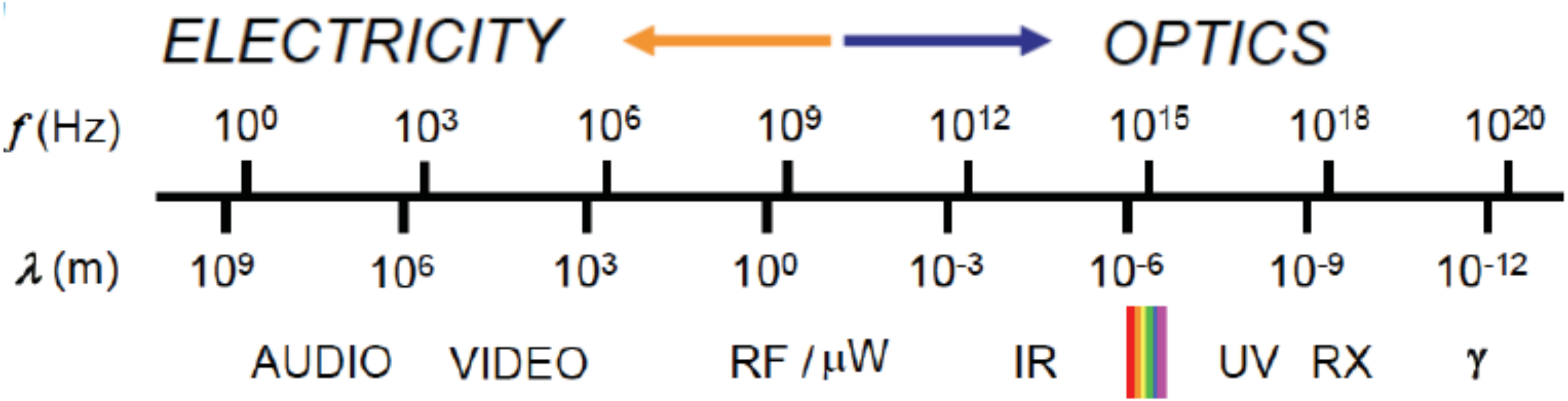
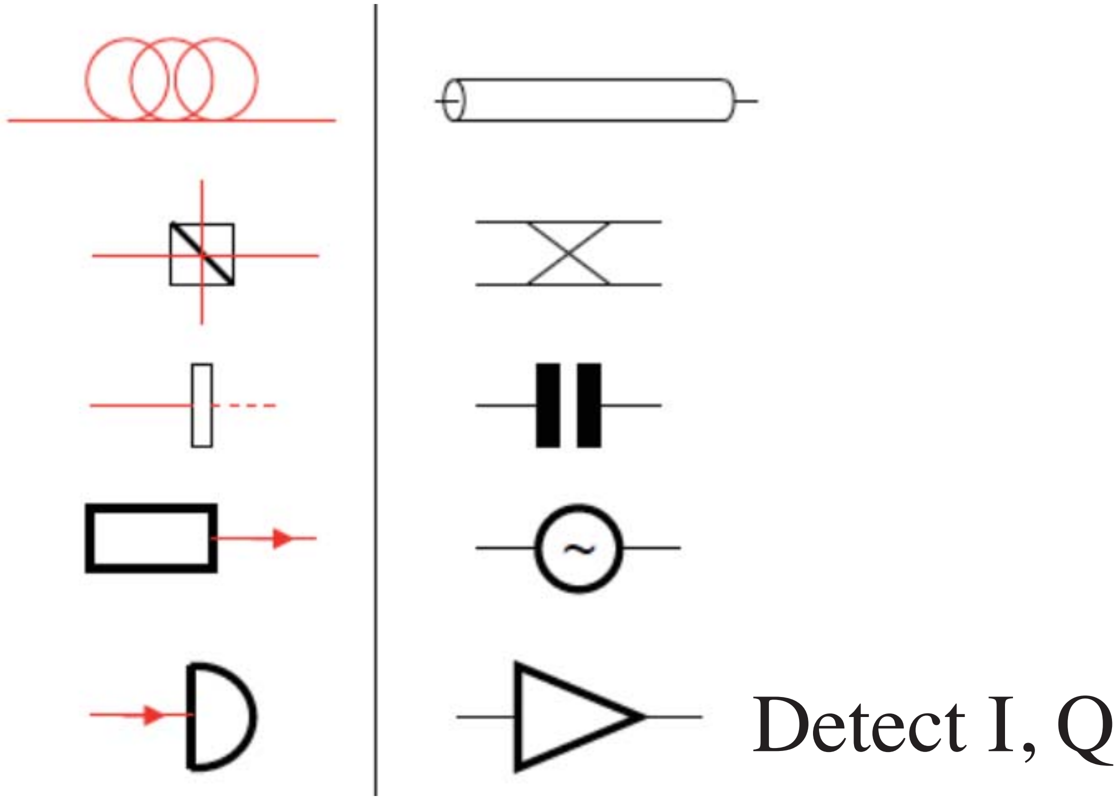

ELECTRICITY
OPTICS
f (Hz)
10^0
10^3
10^6
10^9
10^12
10^15
10^18
10^20
λ (m)
10^9
10^6
10^3
10^0
10^-3
10^-6
10^-9
10^-12
AUDIO
VIDEO
RF / μW
IR
UV
RX
γ

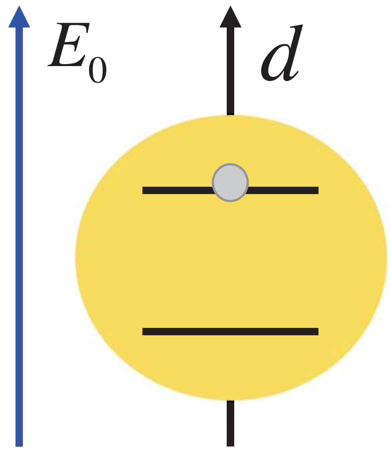
Comparison of the toolboxes

Quantum optics | Superconducting circuits



Optical photons | Microwave photons

Advantages of quantum circuit



Atom-light interaction on single photon level

1. Photons and "atom" interaction can be engineered
2. The photons can be guided by waveguides; beam alignment is not needed.
3. Large vacuum field $E_{0,rms} \simeq 0.2V/m$ due to small mode volume
4. Standard on-chip fabrication technique
5. Tunable transition energy of the "atom"
6. Mechanical stable

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Quantum optics with superconducting circuits

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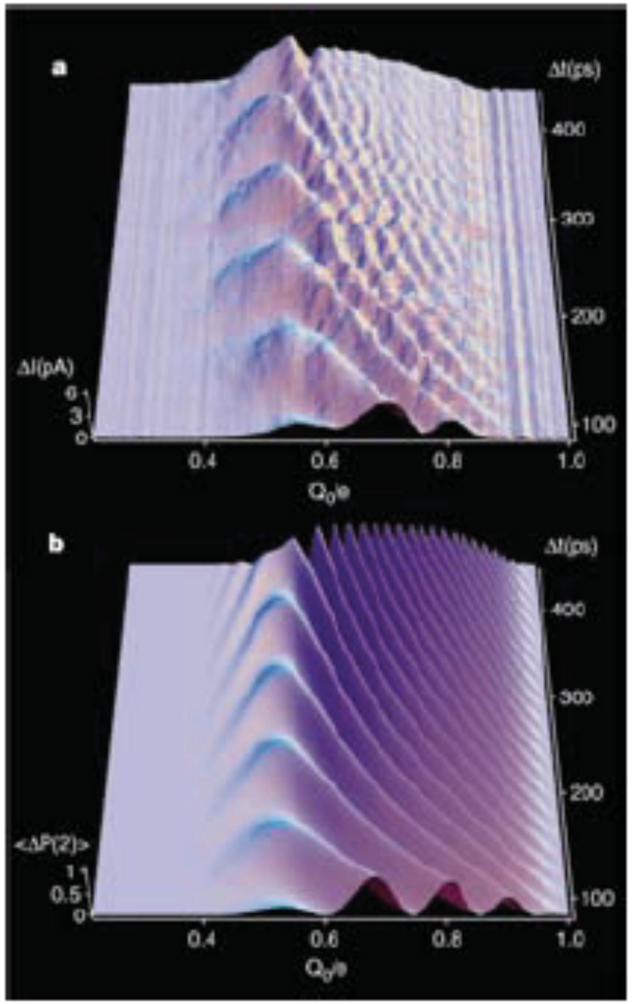
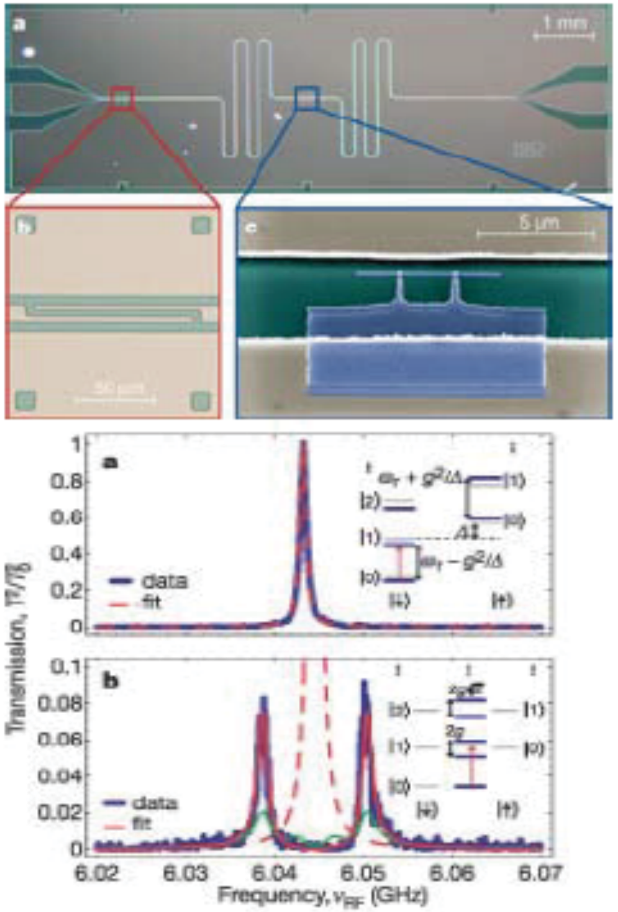
NEC: Nakamura, Pashkin, Yu, Tsai;
Nature 398, 786-788 (1999)

Rabi oscillations



Atoms ⇒ Qubits
3D Cavity ⇒ 1D on-chip resonator

Wallraff *et. al.*; Nature **431** 162 (2004)
Chiorescu *et. al.* Nature **431**, 159 (2004)



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Resonant scattering

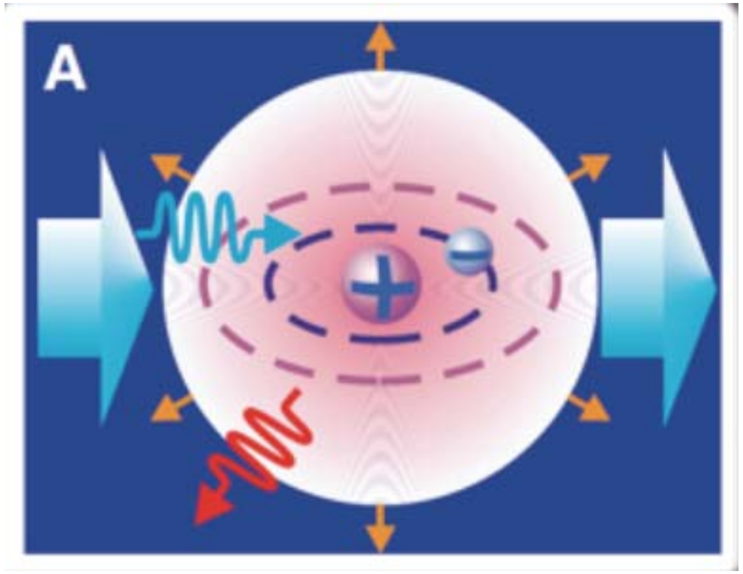

Fig: O. Astafiev, *et al*. **327**, 840 Science (2010)

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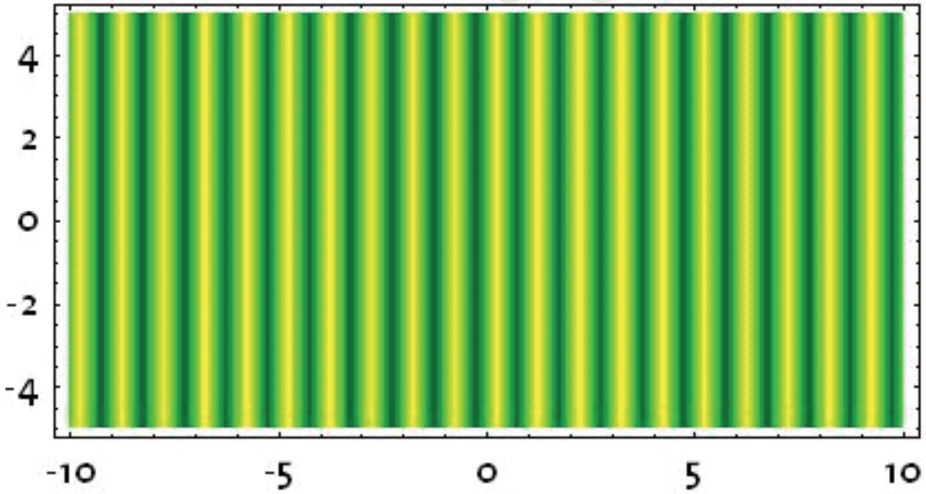
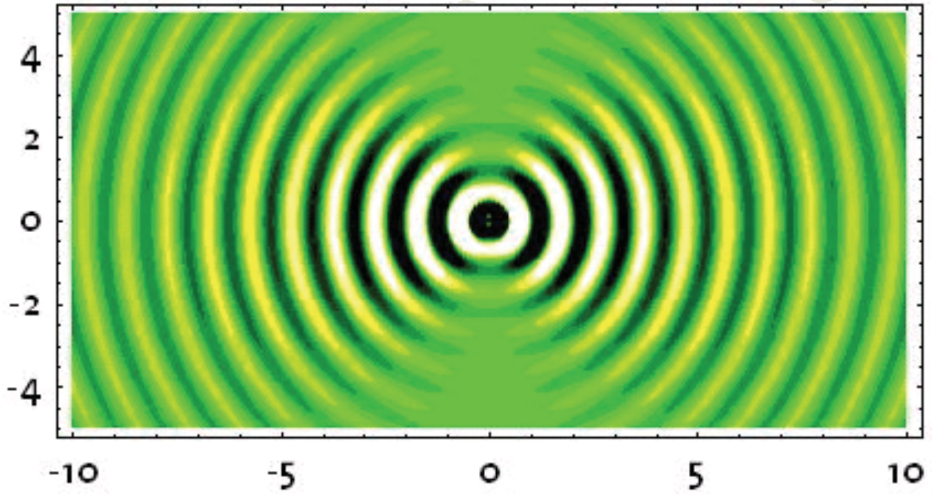
Resonant scattering in 3D space

Incoming light



Atom/dipole emits light



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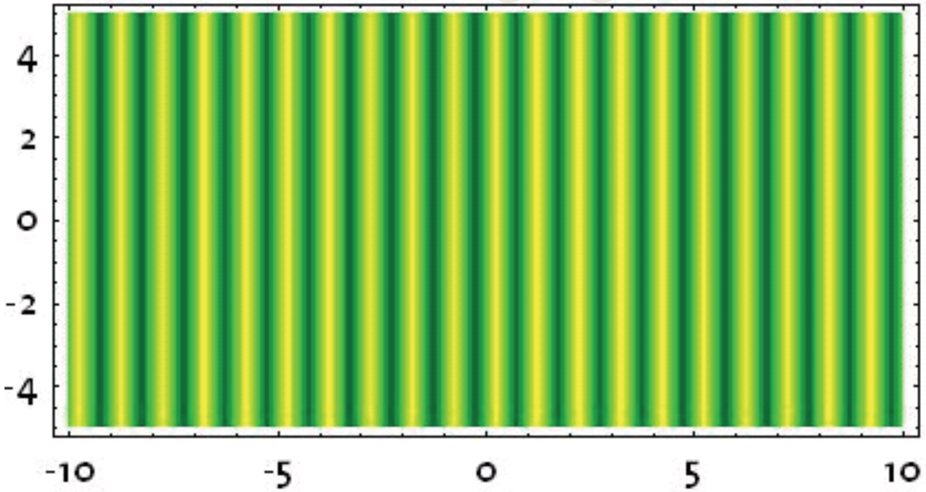
Resonant scattering in 3D space

Incoming light



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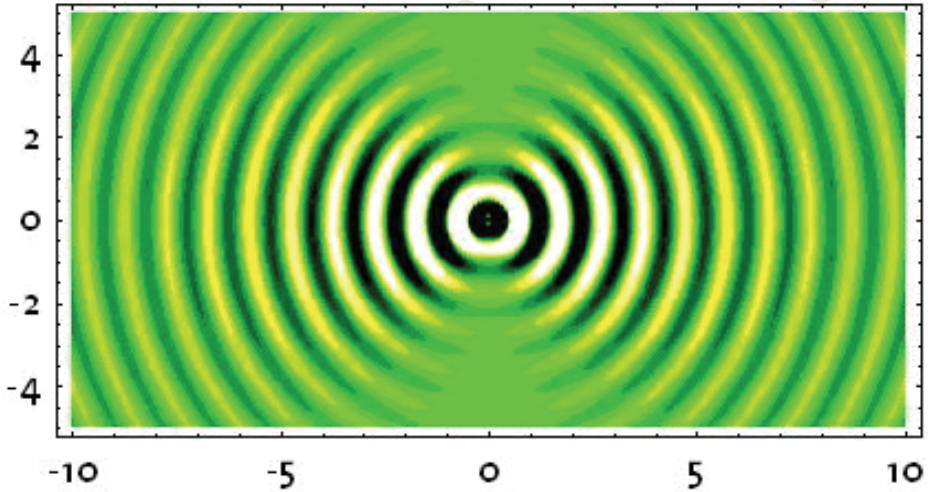
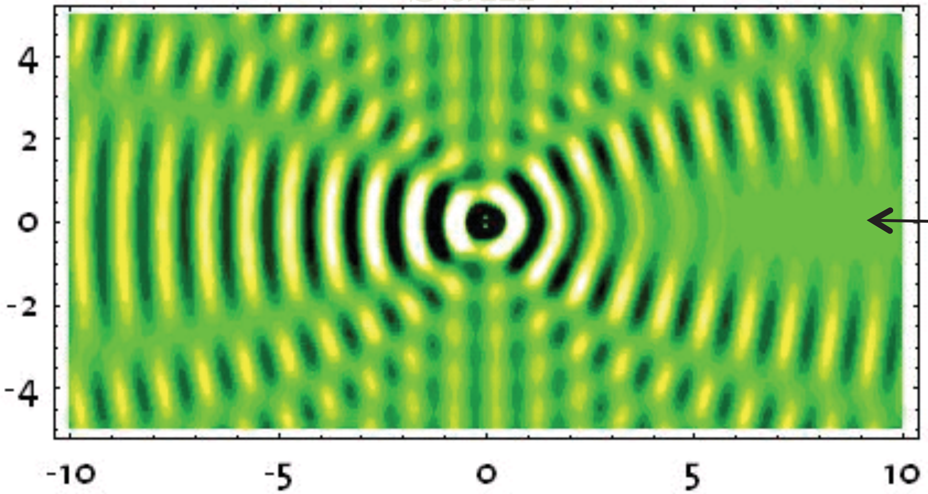


Sum



The extinction signal is due to interference

G. Wrigge *et al*. Nature Phys. **4**, 60 (2008). M. Tey *et al*. Nature Phys. **4**, 924 (2008).

Spatial mode mismatch

Fig. from U. Håkanson

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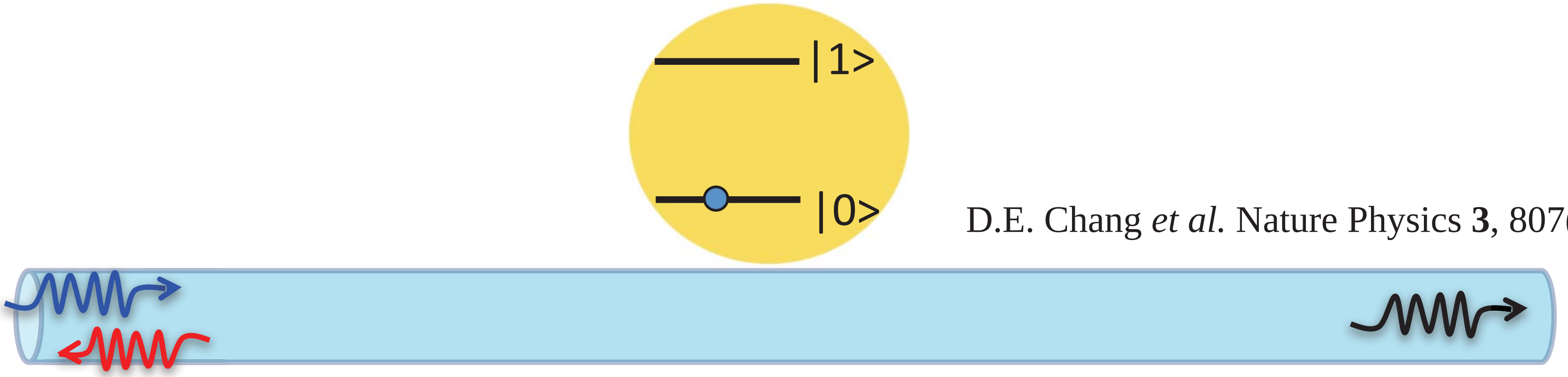
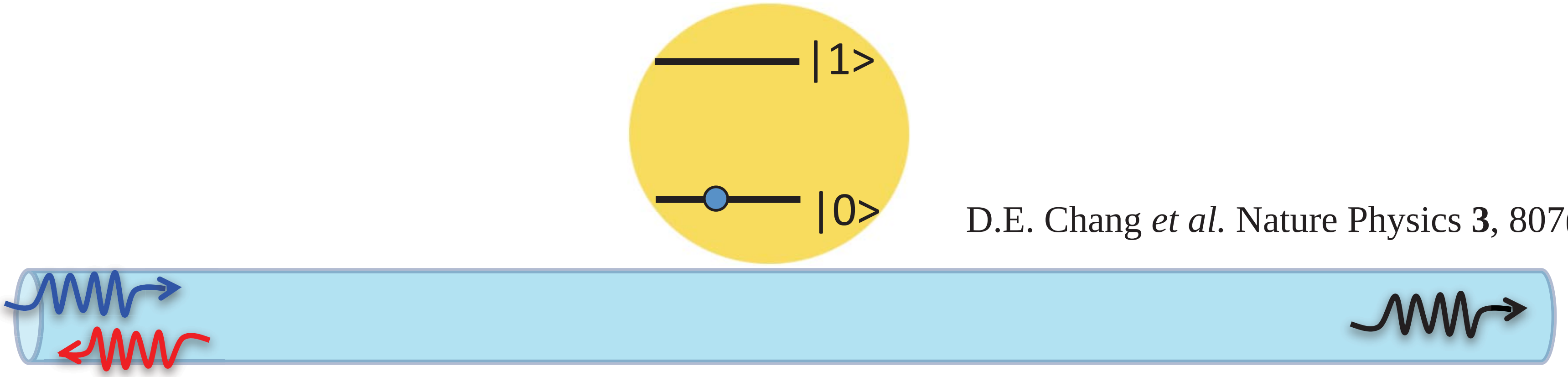
Resonant scattering in 1D waveguide



D.E. Chang *et al.* Nature Physics **3**, 807(2007)

Fully coherent: no transmission, perfect reflection.

Resonant scattering in 1D waveguide



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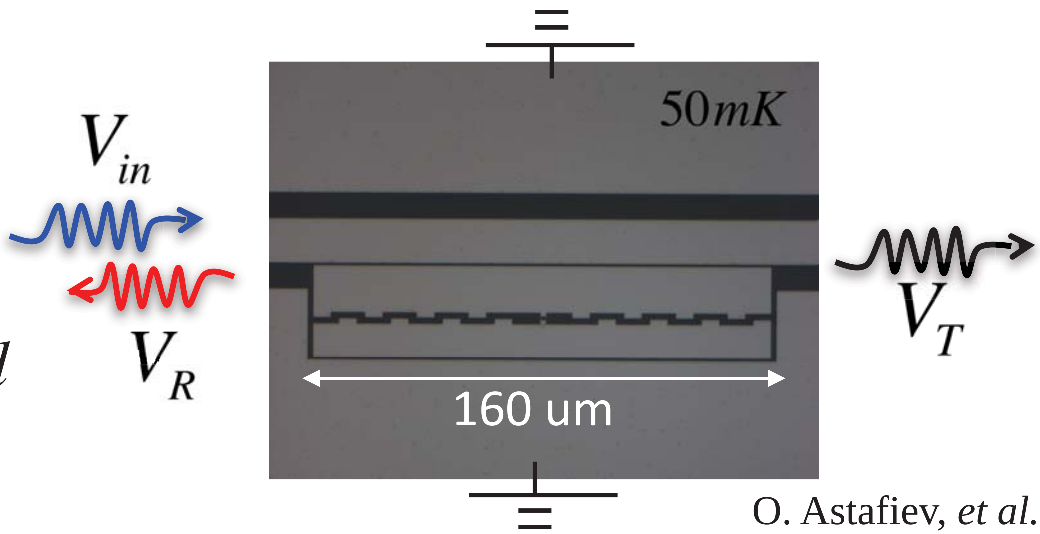


Point like atom/dipole! $\lambda >> d$

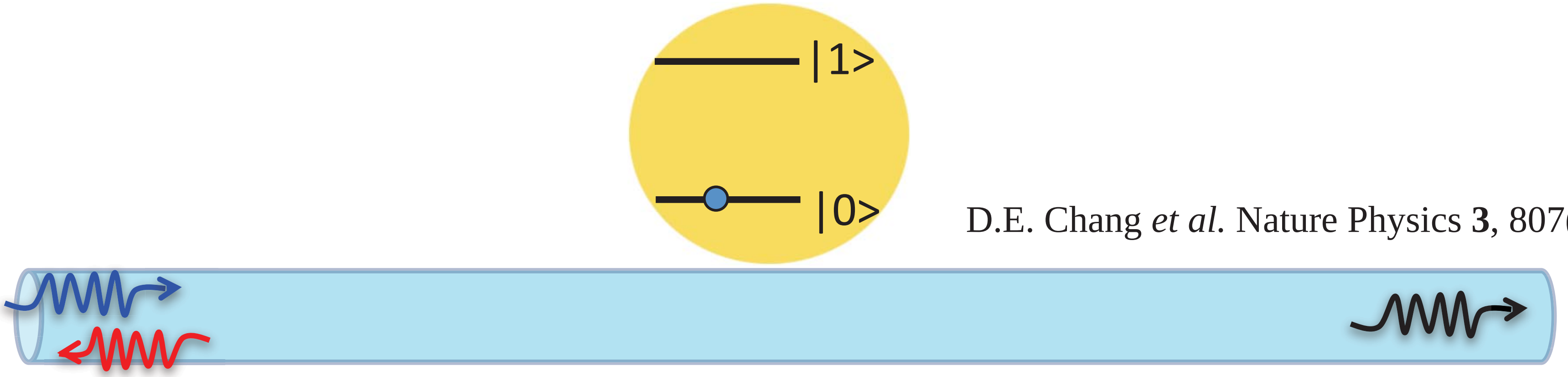
$\lambda \sim cm$ Wavelength of EM field

$d \sim \mu m$ Size of “atom”

Relaxation dominated by transmission line.

O. Astafiev, *et al.* **327**, 840 Science (2010)
IoChun, Hoi *et al.* PRL **107**, 073601 (2011)

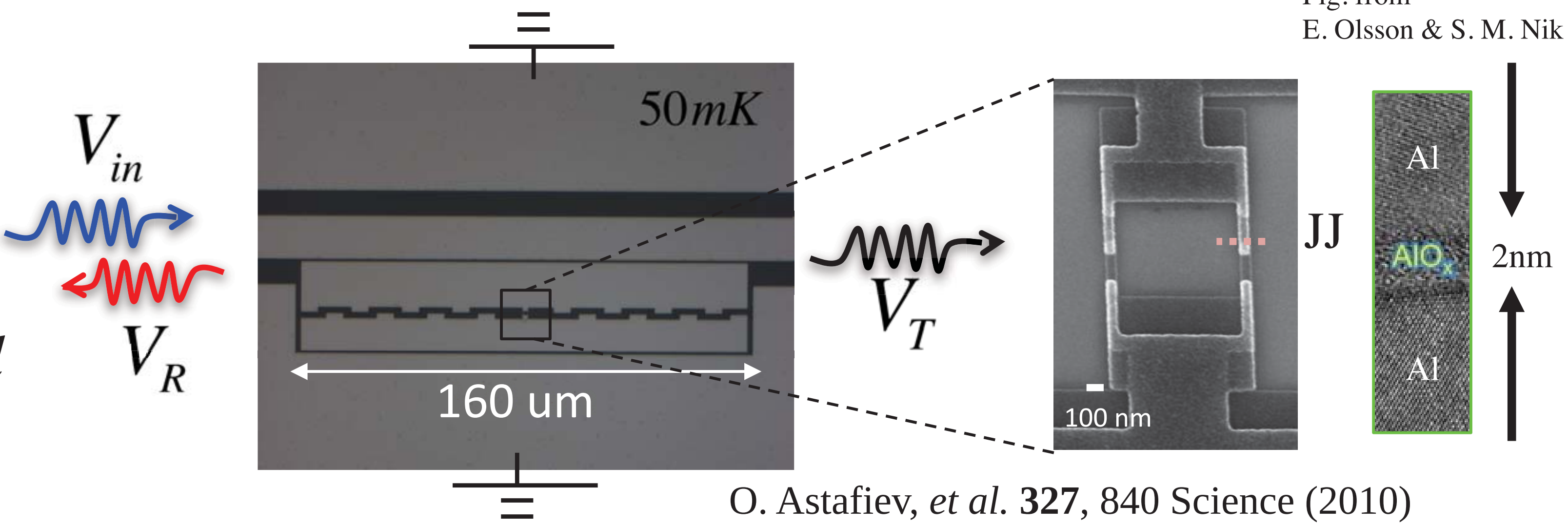
Resonant scattering in 1D waveguide



D.E. Chang *et al.* Nature Physics **3**, 807(2007)

Fully coherent: no transmission, perfect reflection.

Fig. from
E. Olsson & S. M. Nik



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Relaxation dominated by transmission line.

O. Astafiev, *et al.* **327**, 840 Science (2010)
IoChun, Hoi *et al.* PRL **107**, 073601 (2011)

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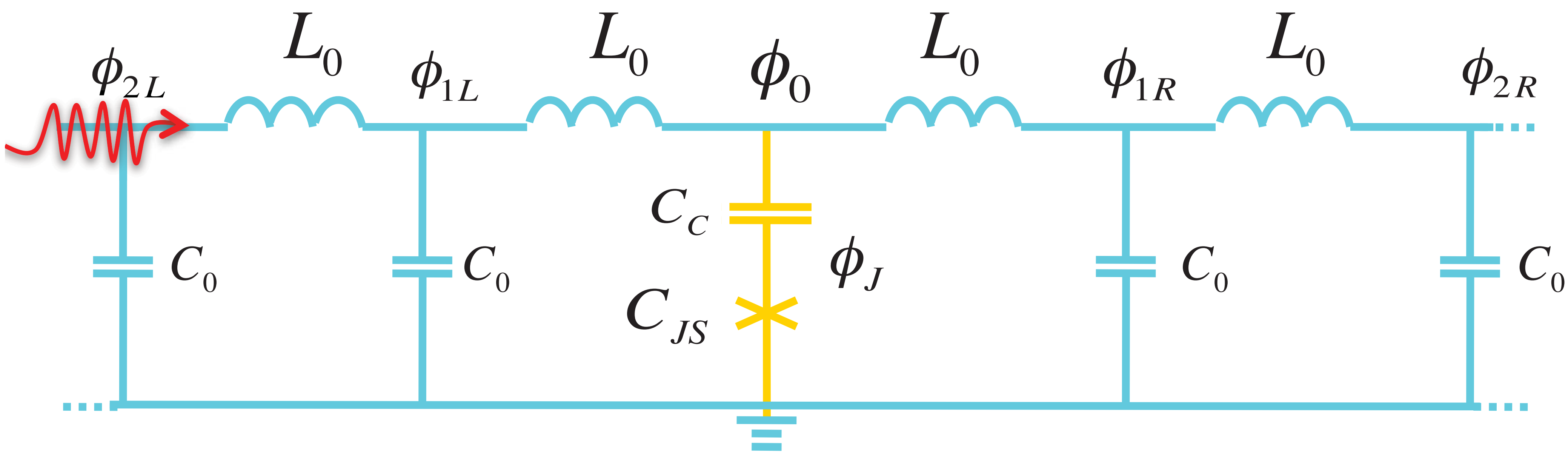
Quantum circuit model



Relaxation rate into 1D transmission line, indicates the strength of coupling!

$$\Gamma_{10} \simeq \frac{\omega_{01}^2 C_c^2 Z}{4C_\Sigma} \qquad C_\Sigma = C_c + C_{JS} \qquad Z = \sqrt{\frac{L_0}{C_0}}$$

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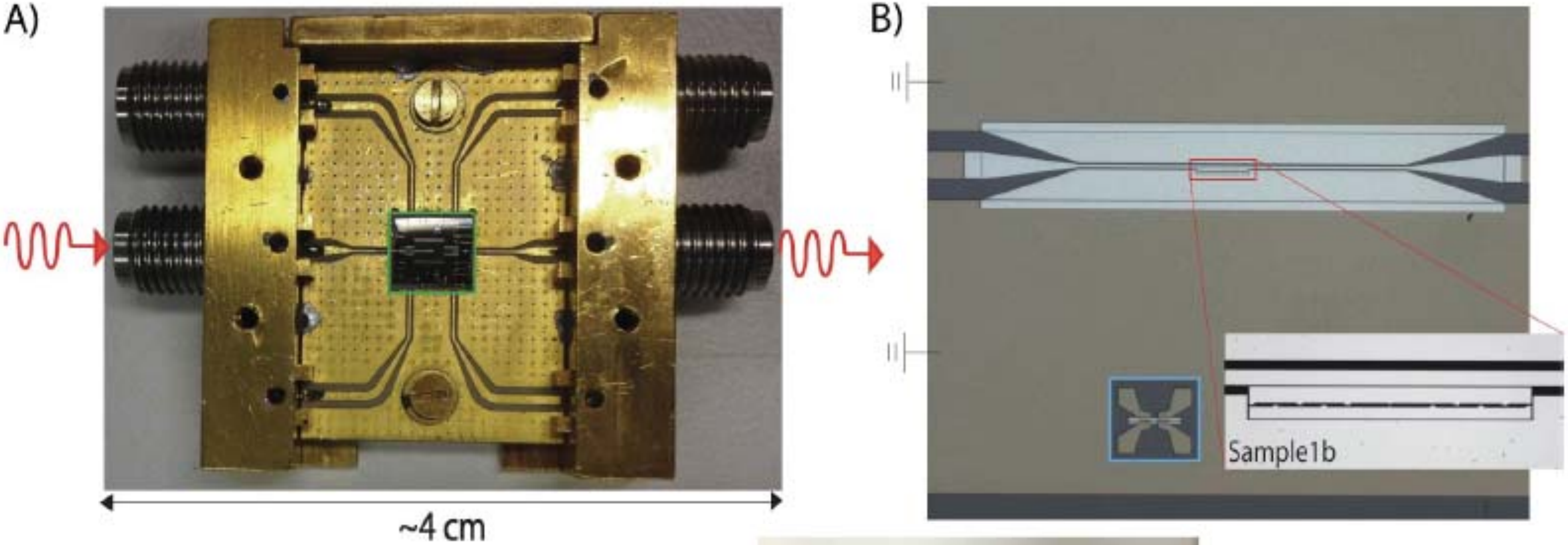
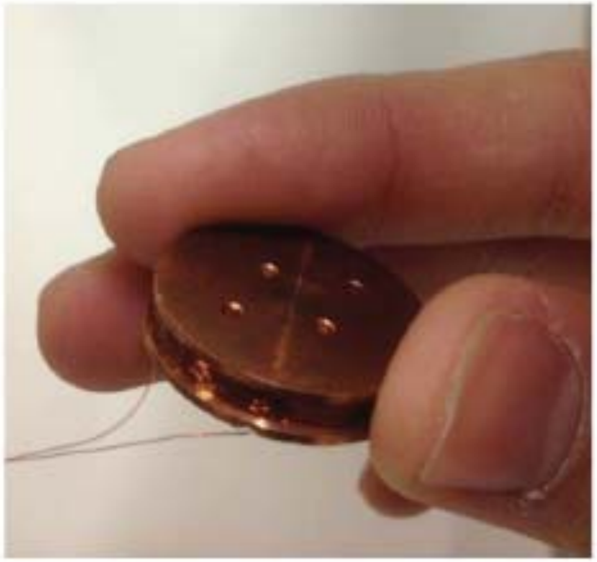
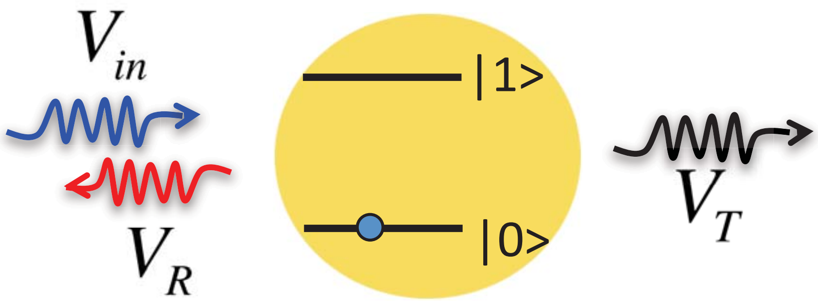




Transmission and reflection

$$r = \frac{\langle V_R \rangle}{\langle V_{in} \rangle}$$



$$t = \frac{\langle V_T \rangle}{\langle V_{in} \rangle}$$

Reflection coefficient

$$r = -\frac{\Gamma_{10}}{2\gamma_{10}}\left[\frac{1 - i\delta\omega_p / \gamma_{10}}{1 + \left(\delta\omega_p / \gamma_{10}\right)^2 + \Omega_p^2 / \Gamma_{10}\gamma_{10}}\right]$$

Transmission coefficient

$$t = 1 + r$$

n resonance, low power

$$\left| r\left(\delta\omega_p = 0, \Omega_p \ll \gamma_{10}\right) \right| = \frac{\Gamma_{10}}{2\gamma_{10}} = \frac{1}{1 + 2\Gamma_\varphi / \Gamma_{10}}$$

$\delta\omega_p$:Detuning
Γ_{10} :Relaxation
Γ_φ :Pure dephasing
$\gamma_{10} = \Gamma_{10} / 2 + \Gamma_\varphi$

Strong interaction limit:

$\Gamma_{10} \gg \Gamma_\varphi$ $\quad \left| r\left(\delta\omega_p = 0, \Omega_p \ll \gamma_{10}\right) \right| \simeq 1$ Fully coherent.

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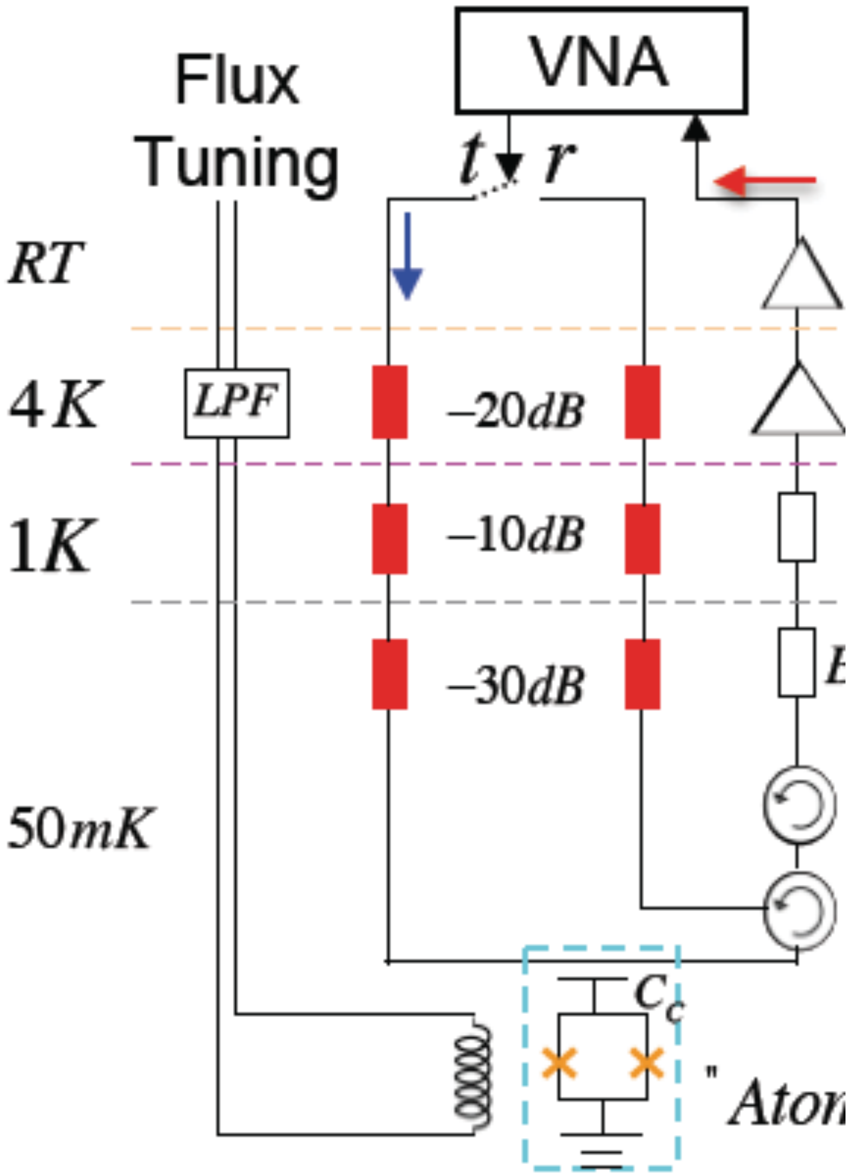
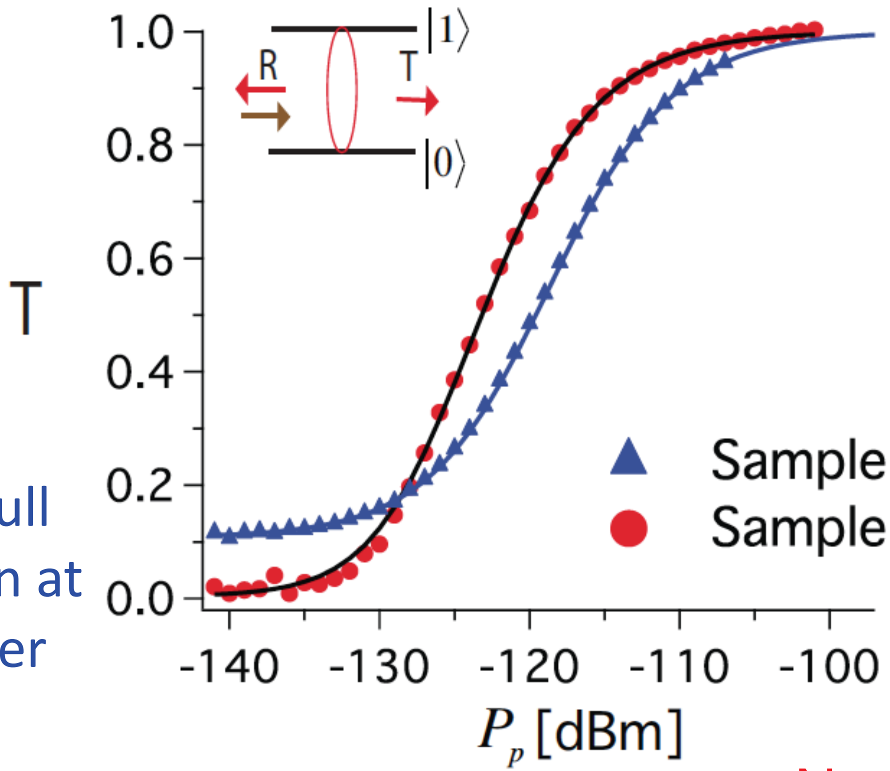

VNA
Flux
Tuning
t
r
RT
4K
LPF
−20dB
1K
−10dB
−30dB
50mK
C_c

Saturation of transmission

$$T = |t|^2$$



Almost full transmission at high power

Almost full reflection at low power

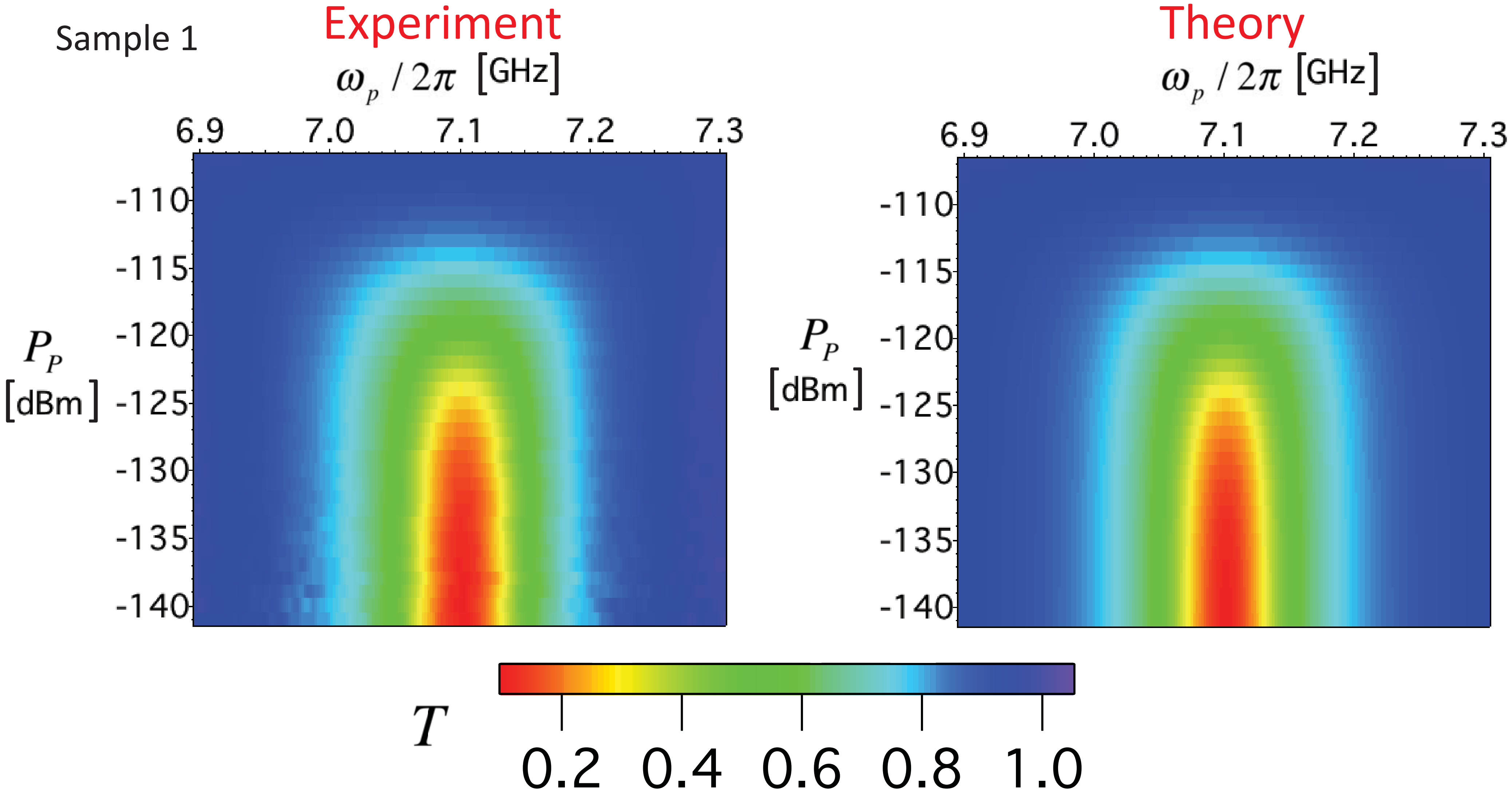
$$\langle N_P \rangle = \frac{P_P}{\hbar\omega_P(\Gamma_{10}/2\pi)}$$

Nonlinear nature of the atom!

Sample	E_J/h	E_C/h	E_J/E_C	$\omega_{10}/2\pi$	$\omega_{21}/2\pi$	$\Gamma_{10}/2\pi$	$\Gamma_\phi/2\pi$	Ext.
1	12.7	0.59	21.6	7.1	6.38	0.073	0.018	90%
2	10.7	0.35	31	5.13	4.74	0.041	0.001	99%

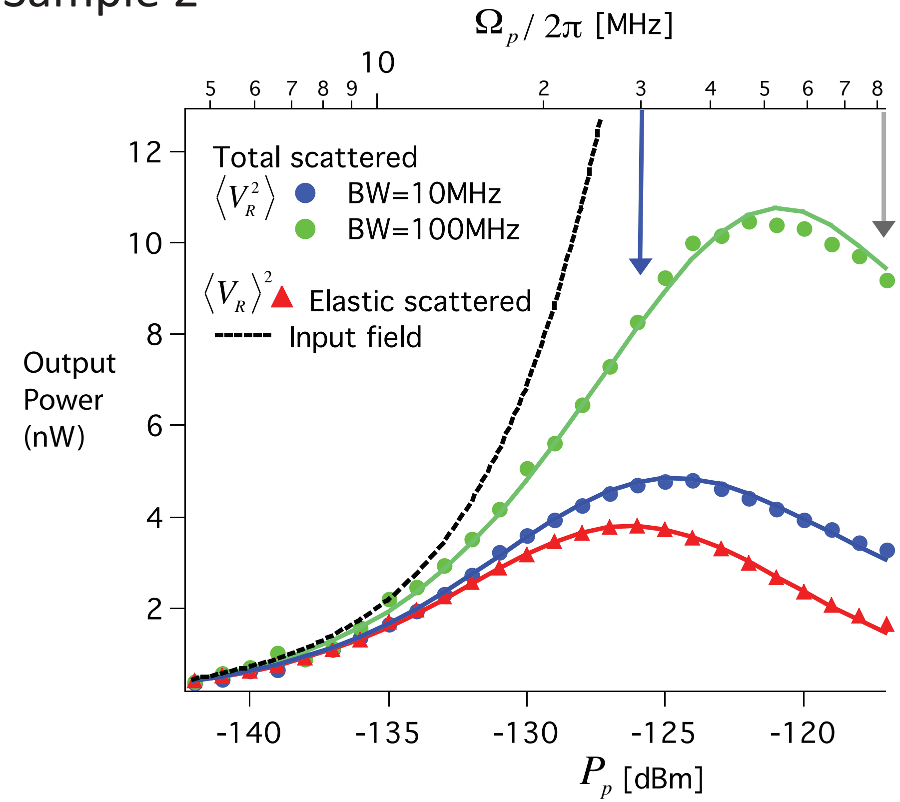
Transmission comparing to theory

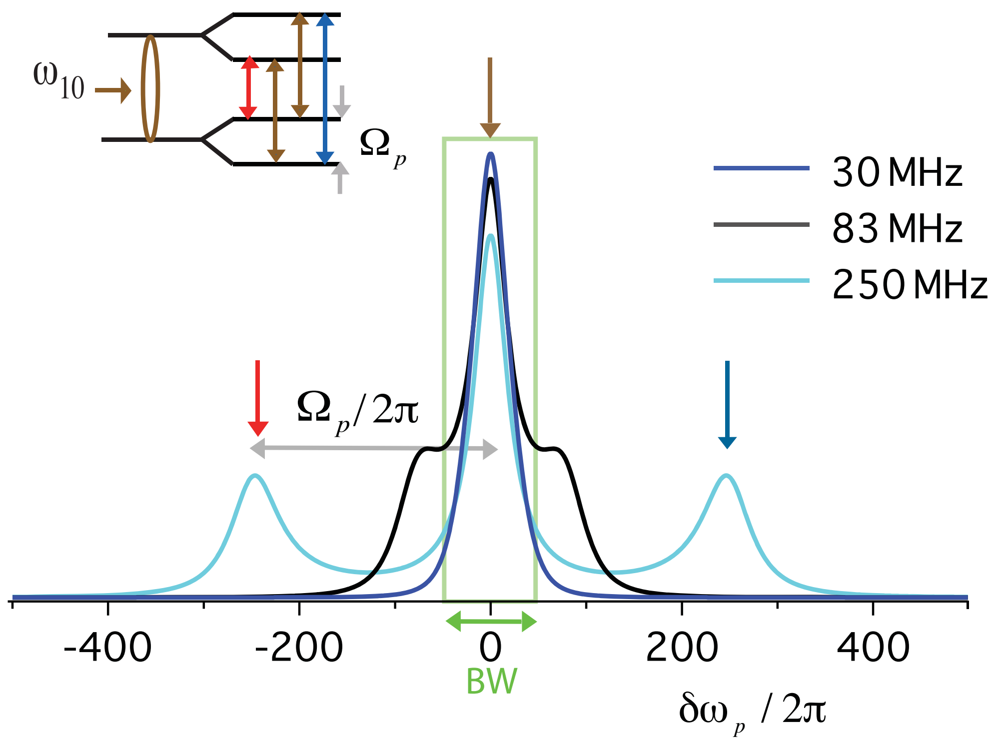
Sample 1



Coherent vs Incoherent scattering

Sample 2





$$\Omega_p \ll \gamma_{10}$$

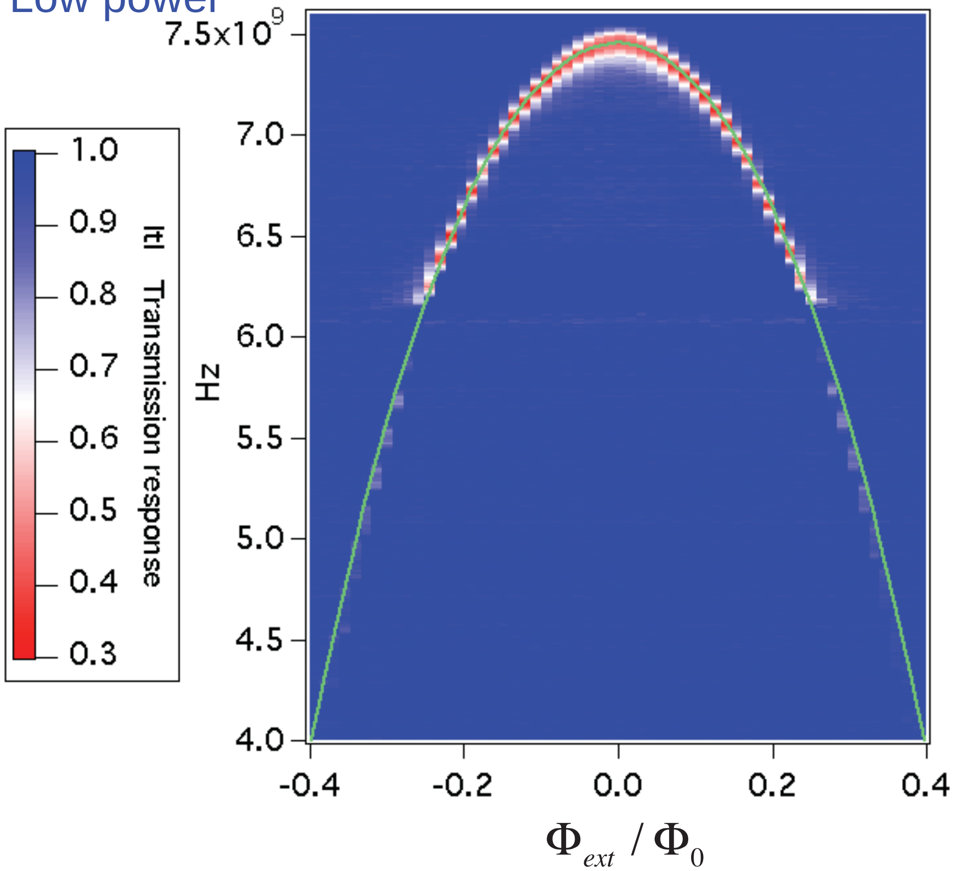
$$\langle V_{in} \rangle^2 \simeq \langle V_R \rangle^2 \simeq \langle V_R^2 \rangle \quad |r_{p,1}| \sim 1$$

I.-C. Hoi *et al.* Phys. Rev. Lett. **108**, 263601(2012)

Tunable artificial atom

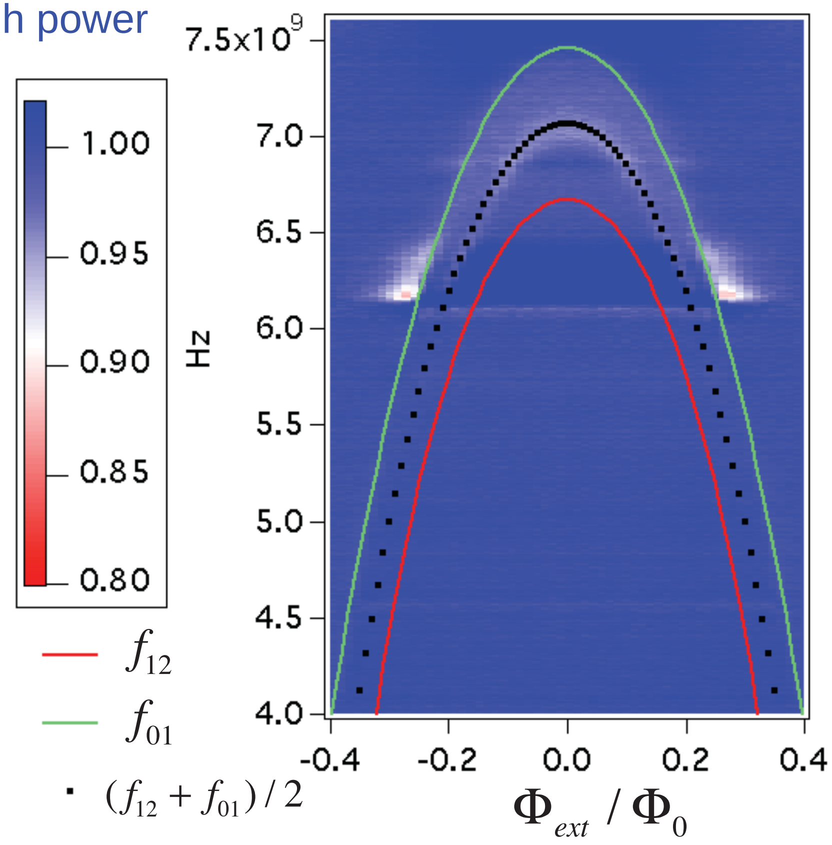
Low power

Only 0-1 transition occurs!



High power

Only two-photon transition occurs!

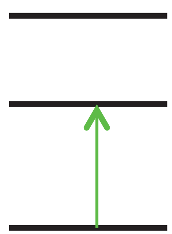


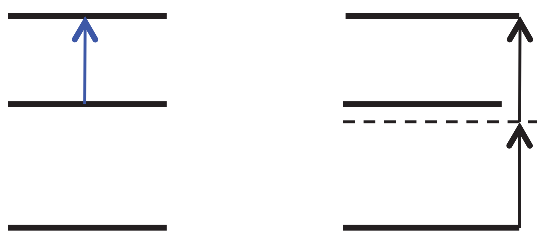
Extract:

$E_{J,Max} = 13GHz$

$E_c = 590MHz$

$E_J / E_c = 23$





Two-Photon Transition

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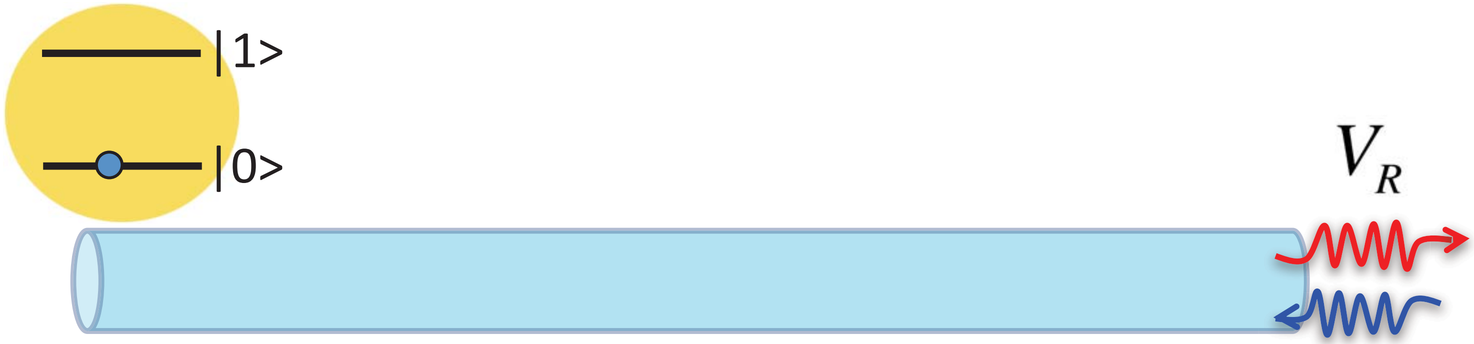
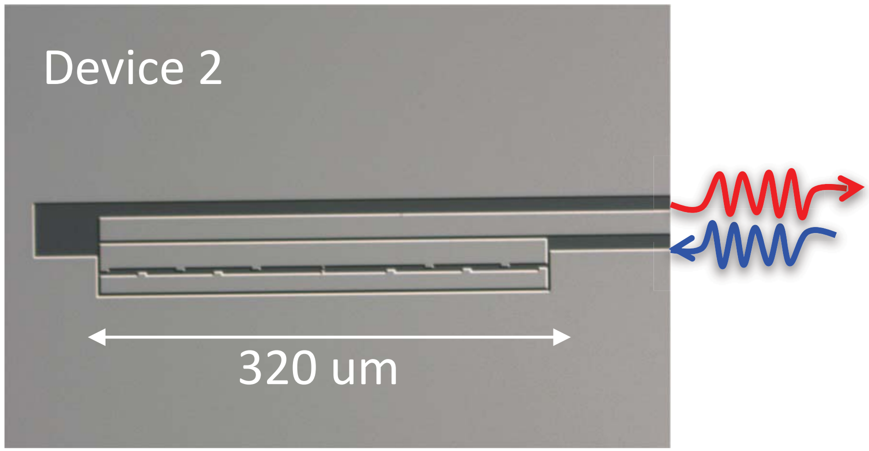


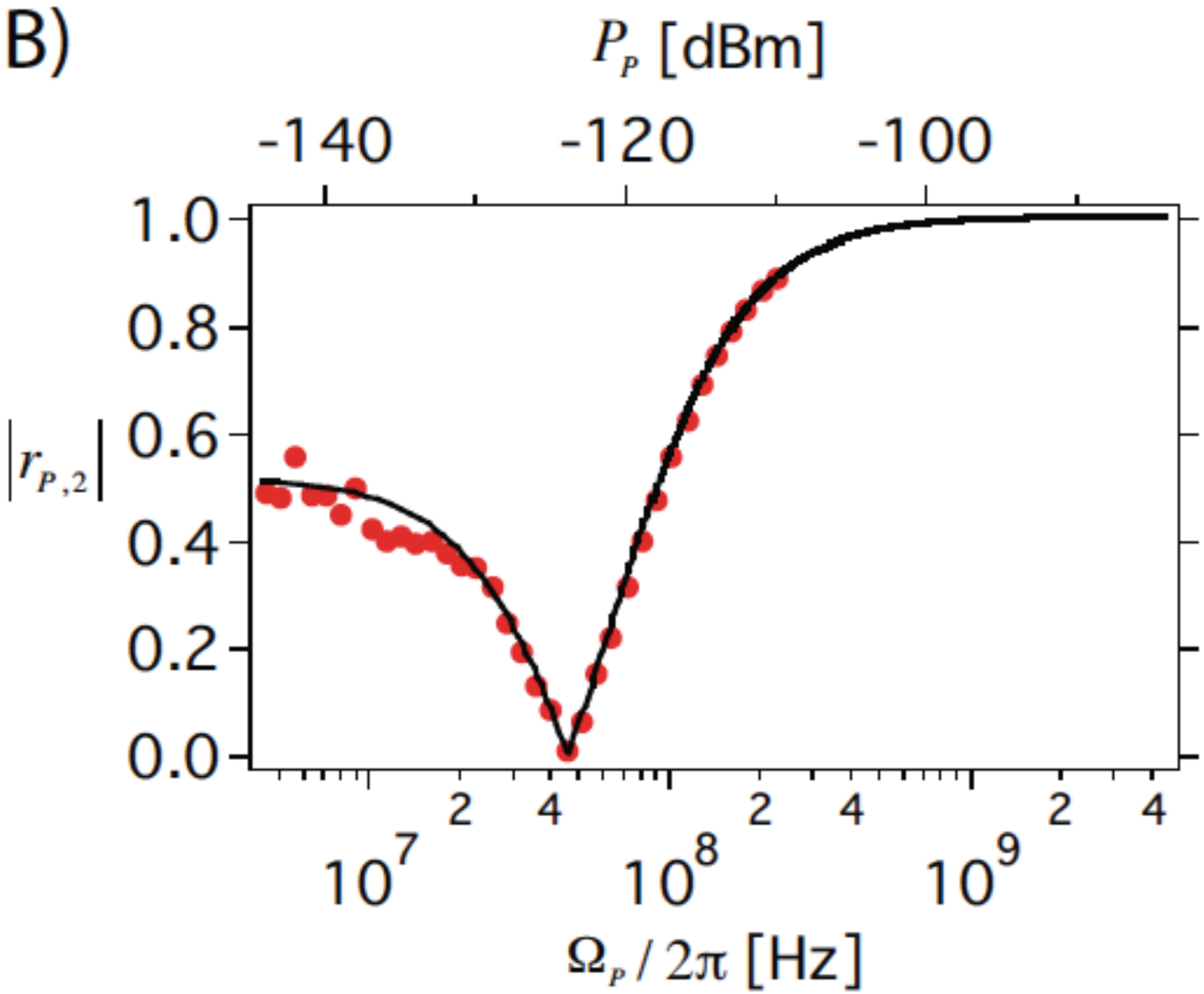
$$r_{p,2} = \frac{\langle V_R \rangle}{\langle V_{in} \rangle}$$

Fully coherent: perfect reflected by the atom.



Emitted fields can propagate
in one directions

r_p measure the phase coherent signal.



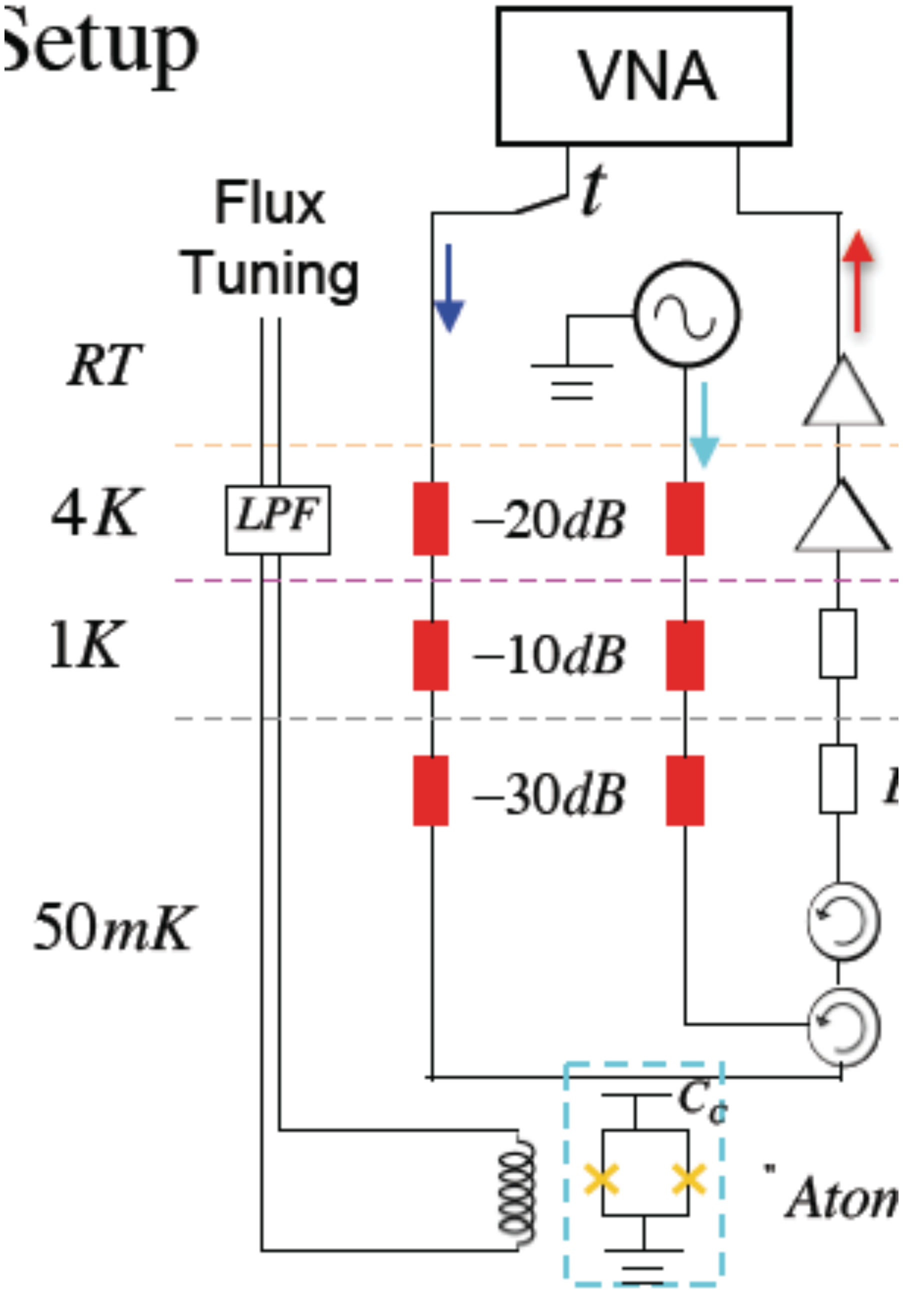
$\Omega_p \ll \gamma_{10}$

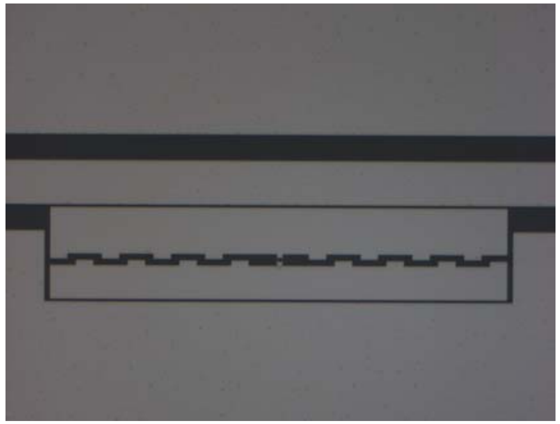
Transmon at the end of transmission line

The missing power are incoherent scattered.

Two-Tone Spectroscopy

Two-Tone Spectroscopy





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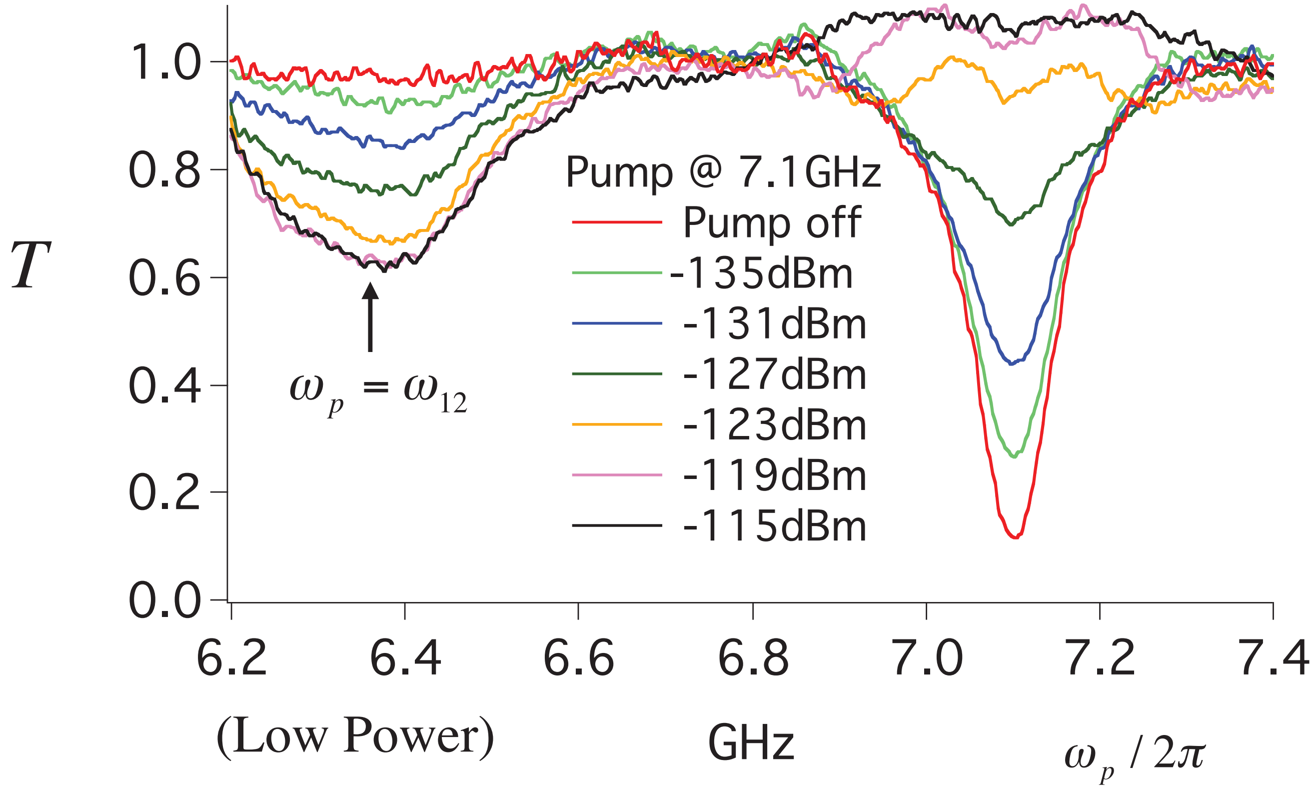
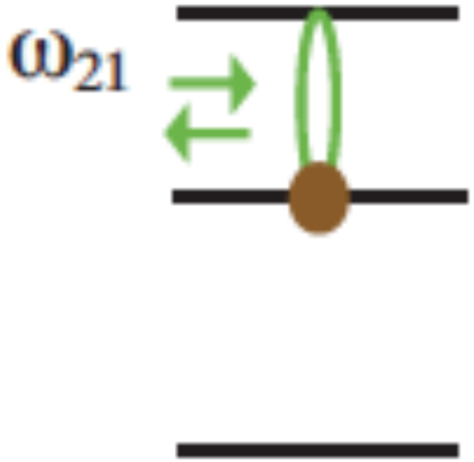
Higher level effect





Anharmonicity:
$\alpha = \omega_{01} - \omega_{12} \simeq 720\text{MHz}$

$\omega_{12} / 2\pi = 6.38GHz$
$\omega_{10} / 2\pi = 7.1GHz$

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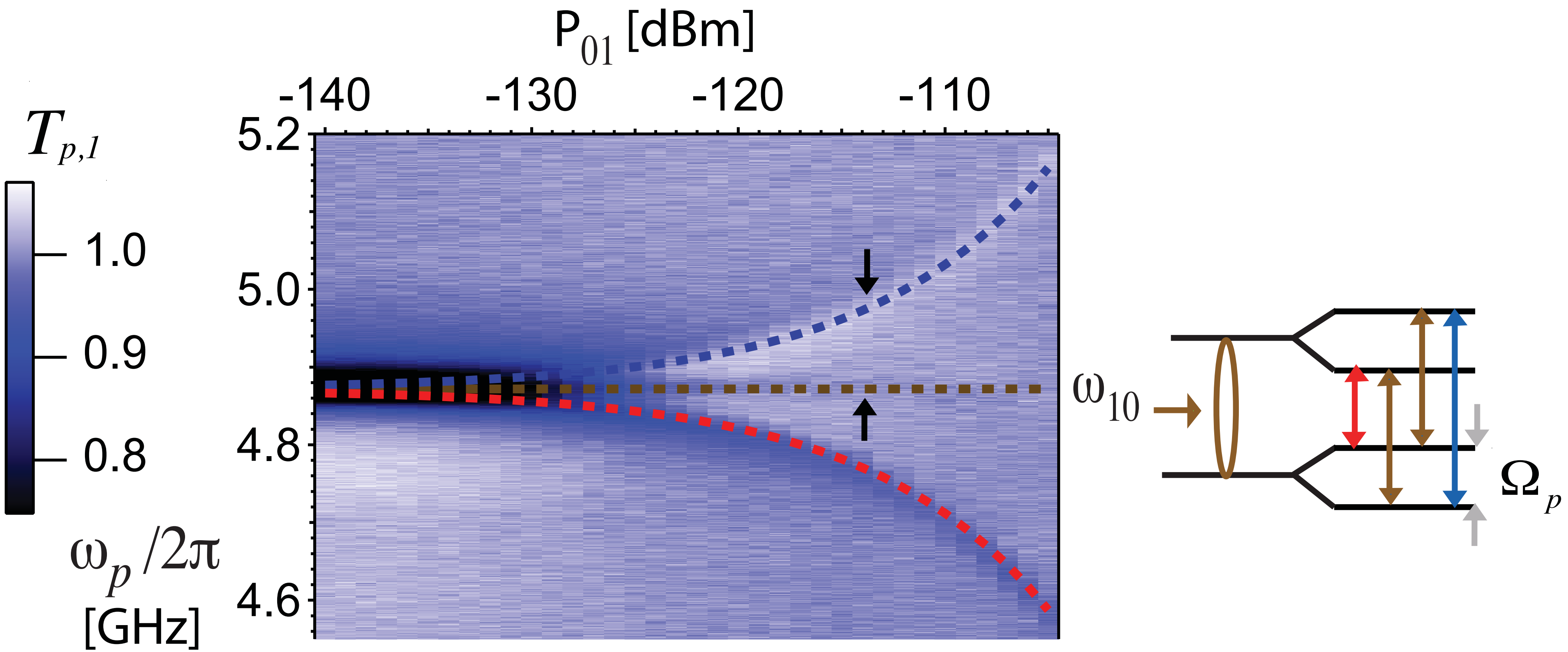
Mollow triplet



B.R. Mollow, Phys.Rev. **188,** 1969 (1969)

O. Astafiev, *et al.* **327**, 840 Science (2010)

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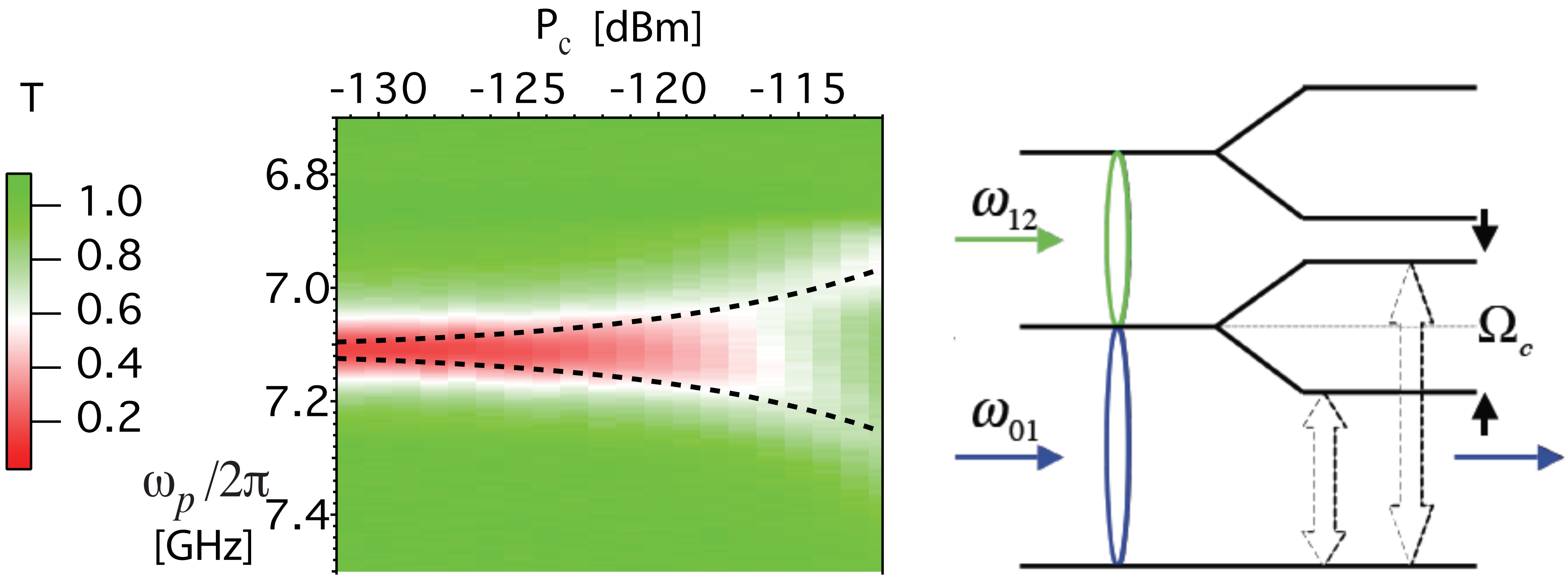
Autler-Townes Splitting



$\Omega_p \ll \gamma_{10}$

A. A. Abdumalikov, Jr *et al.* PRL **104**, 193601 (2010)

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To be continued…