



2015 原子分子與光學物理暑期學校

Quantum Phenomena in High Resolution Laser Spectroscopy

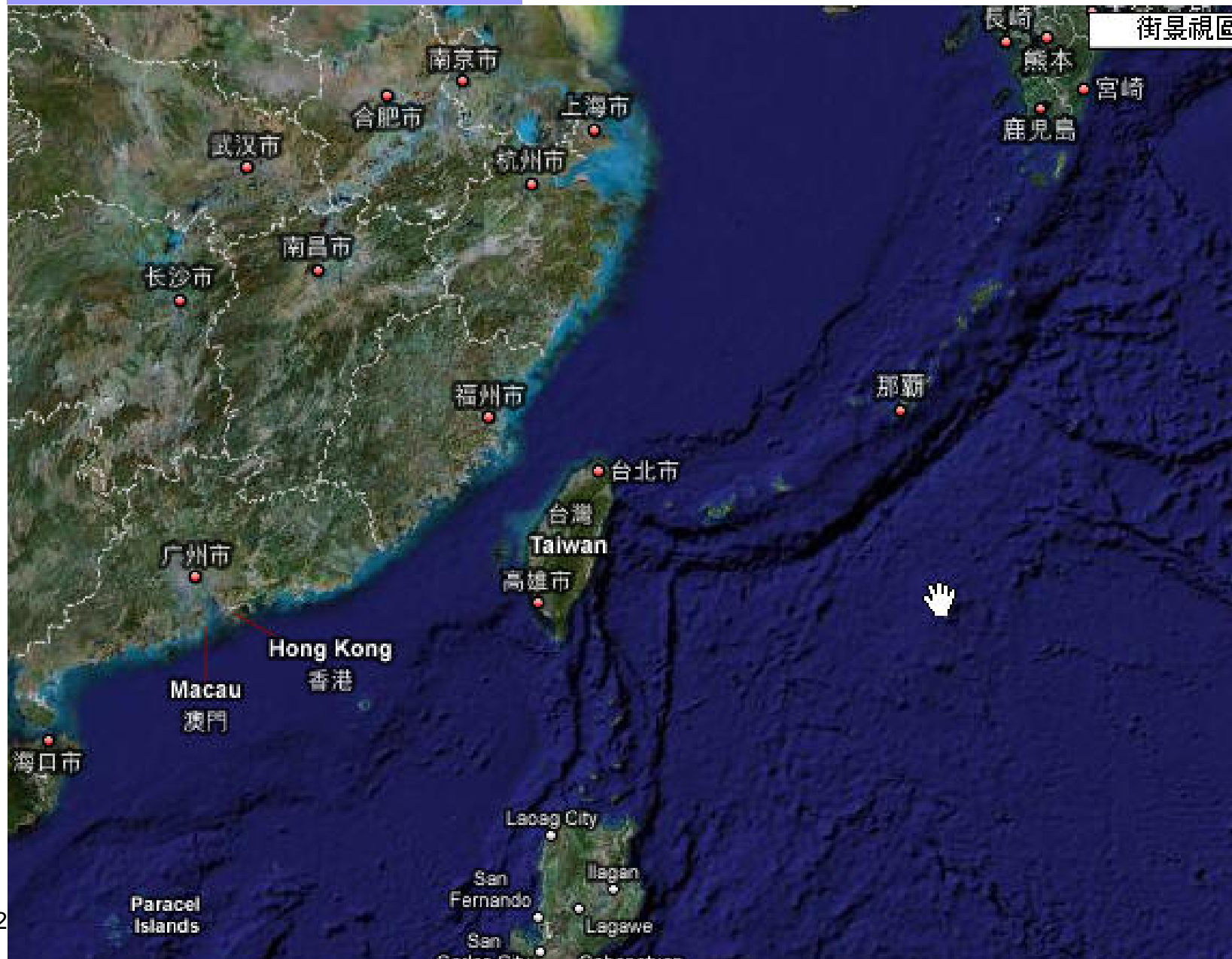
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National Cheng-Kung University

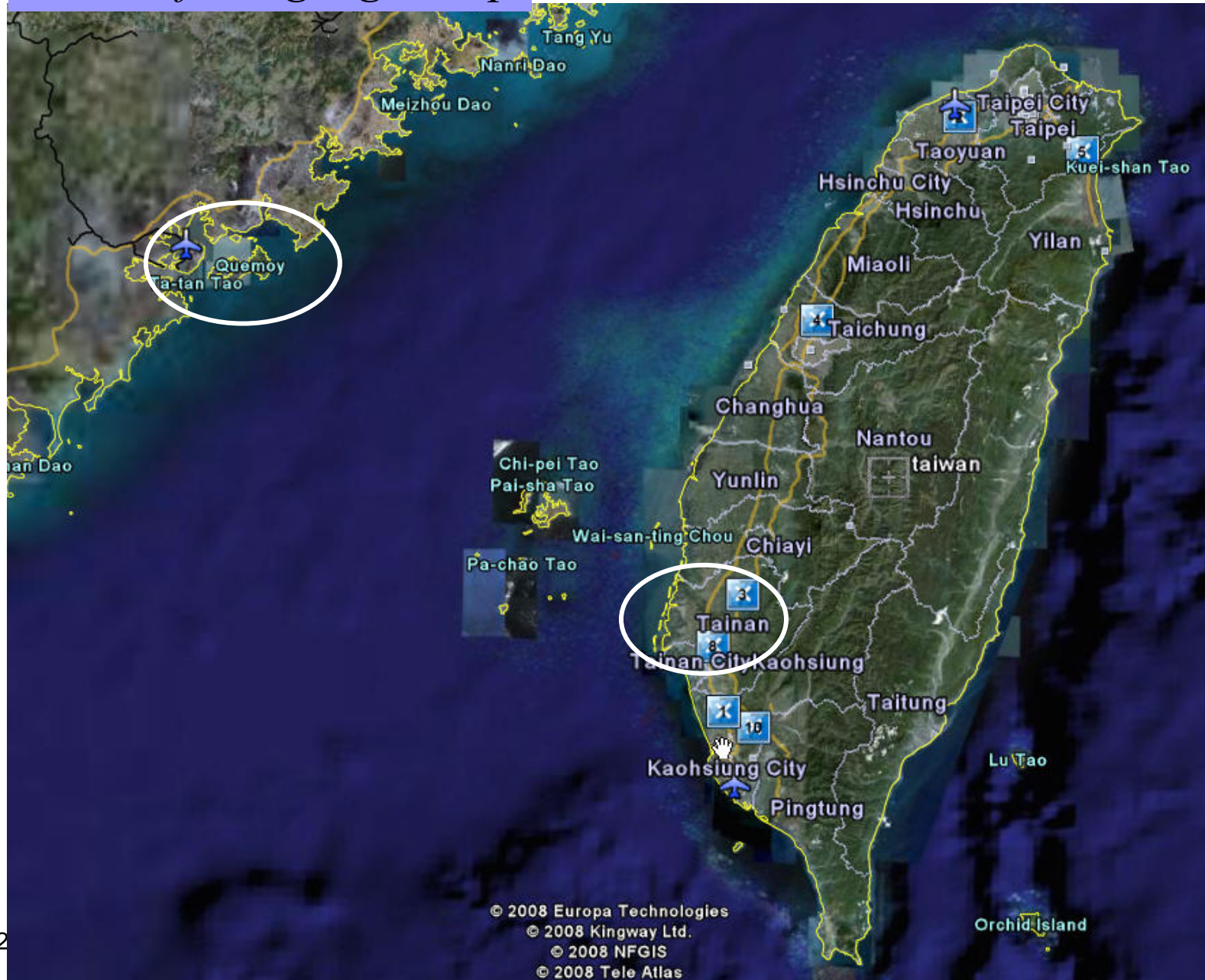


Taiwan from google map





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Outline

* *Introduction*

High resolution laser spectroscopy and the development of Quantum Mechanics

* *Quantum Phenomena in diatomic molecule*

Tunnelling, Avoided-crossing, Fano Resonance

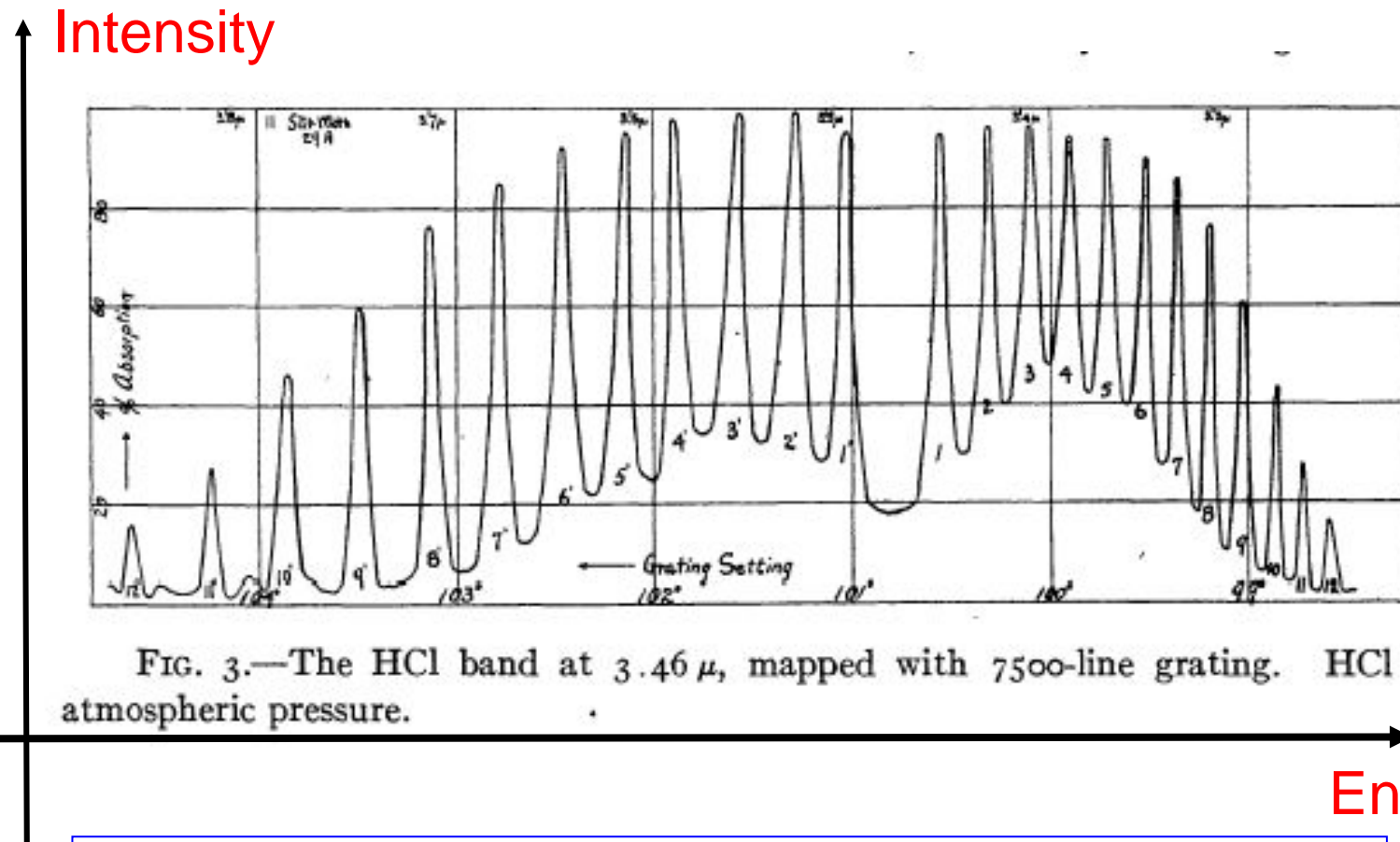
* *Quantum Phenomena in Cold Atoms*

Shape Resonance, Feshbach Resonance, EIT/Decoherence, Pump Probe

* *Summary*



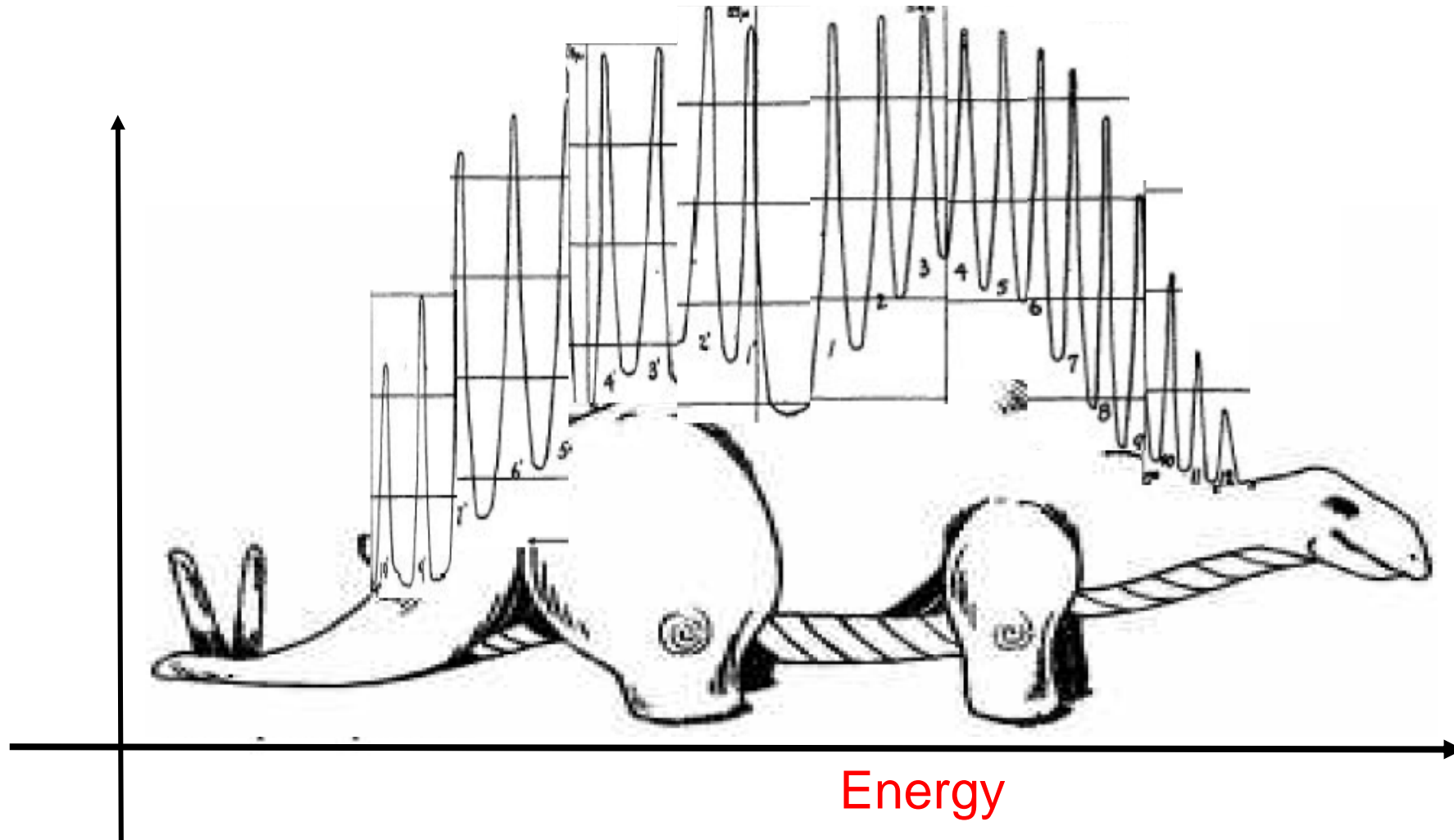
What is Laser Spectroscopy?



What are the importance of a spectrum?
Line position, Intensity and Shape

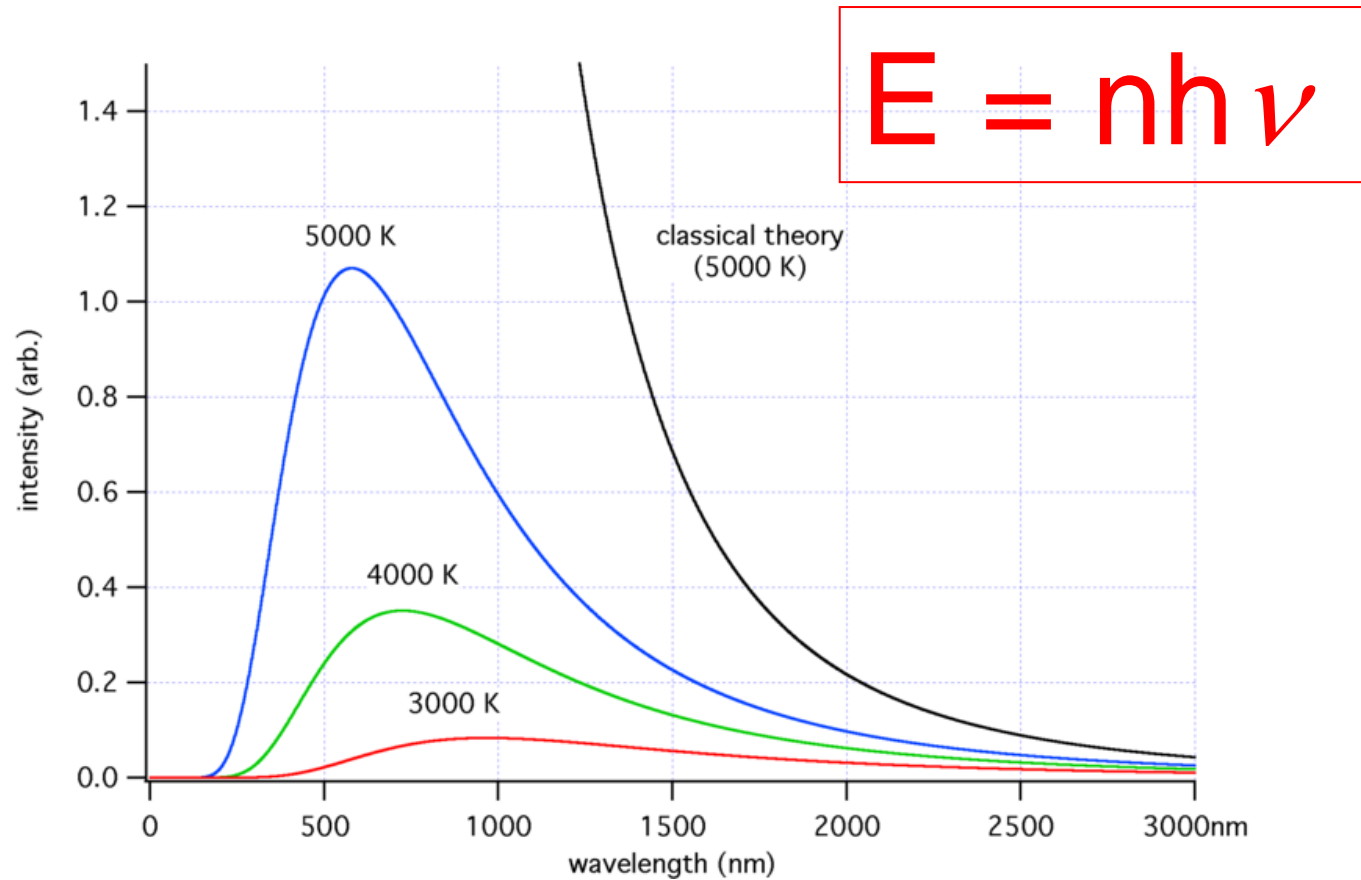


When/Where does it start?





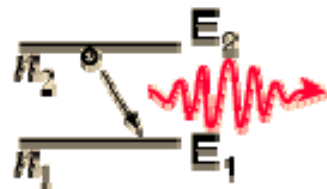
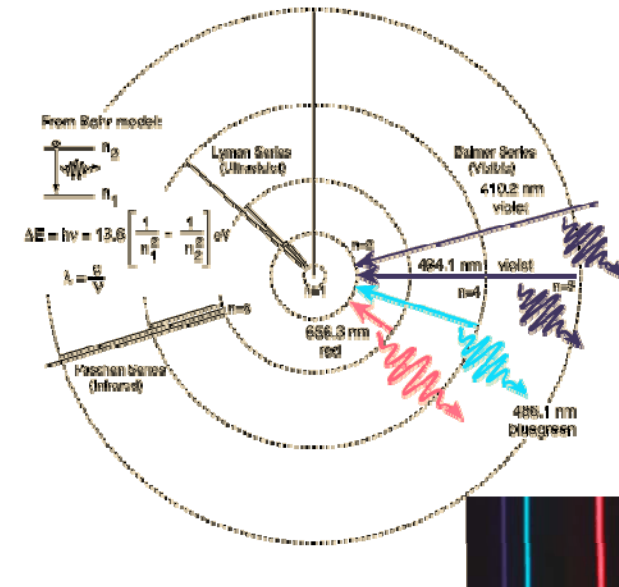
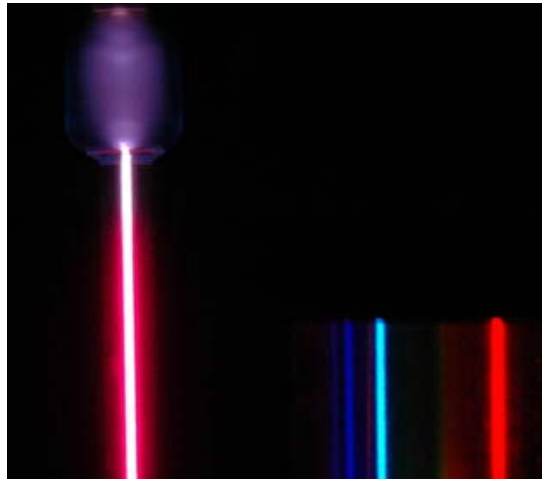
Black body radiation



The dawn of Quantum Mechanics!



Higher resolution emission spectrum of Hydrogen



A downward transition involves emission of a photon of energy:

$$E_{\text{photon}} = h\nu = E_2 - E_1$$

Given the expression for the energies of the hydrogen electron states:

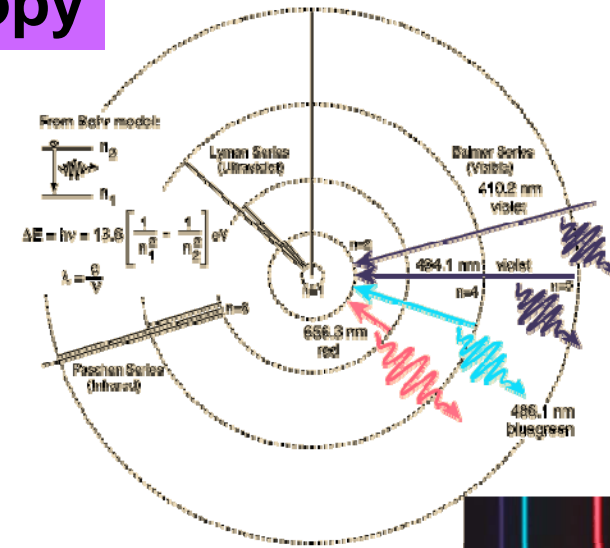
$$h\nu = \frac{2\pi^2 m e^4}{h^2} \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] = -13.6 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \text{ eV}$$

Bohr Model

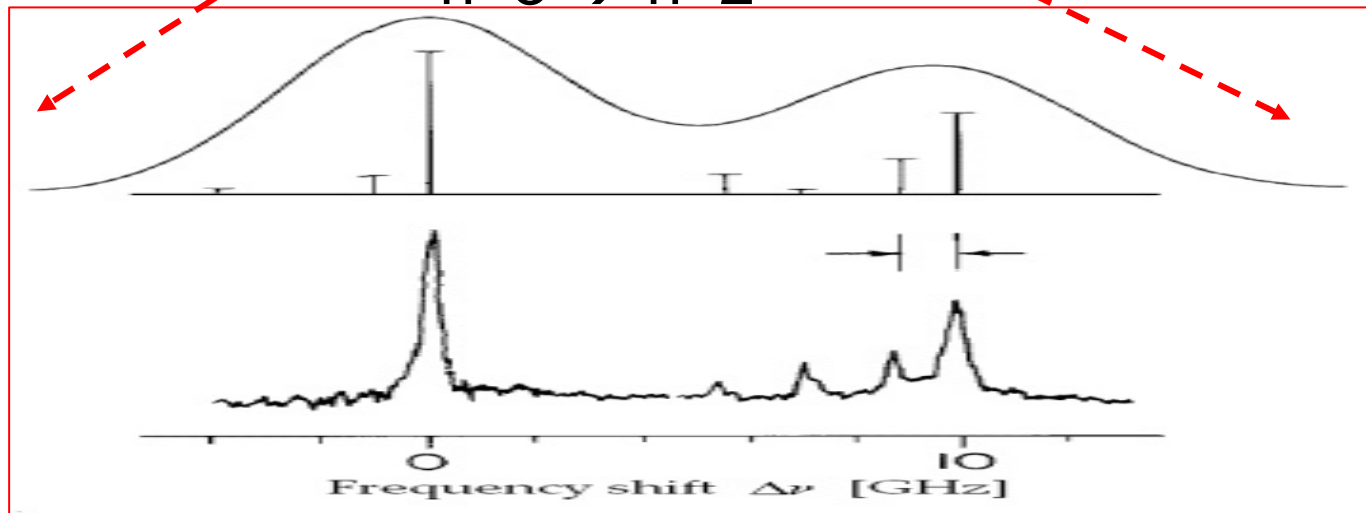
$$L = n\hbar$$



High resolution laser spectroscopy



H_α line
n=3 → n=2





High resolution laser spectroscopy

Schrodinger Equation : $H\Psi = E\Psi \rightarrow$ Wavefunctions and Eigenvalues

$$E = -\frac{Z^2 m e^4}{8 n^2 h^2 \epsilon_0^2} = -\frac{13.6 Z^2}{n^2} \text{ eV}$$

$$r = \frac{n^2 h^2 \epsilon_0}{Z \pi m e^2} = \frac{n^2 a_0}{Z}$$

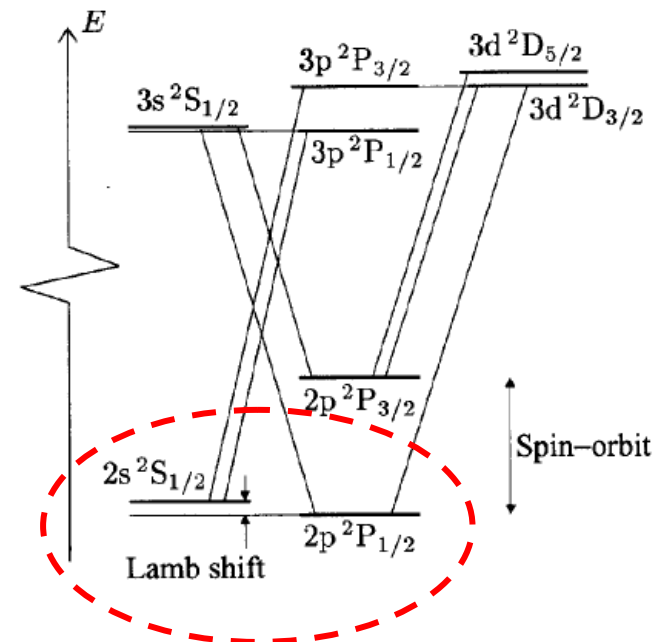
$a_0 = 0.529 \text{ \AA} = \text{Bohr radius}$

Spin Orbital interactions

Transition Probabilities, Selection Rules

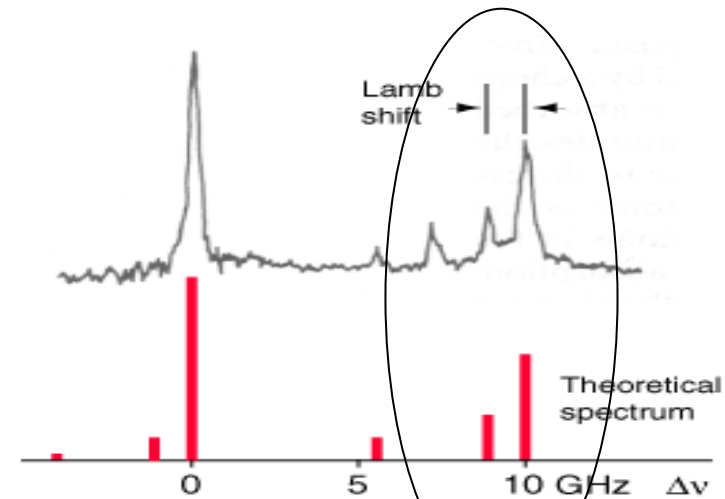
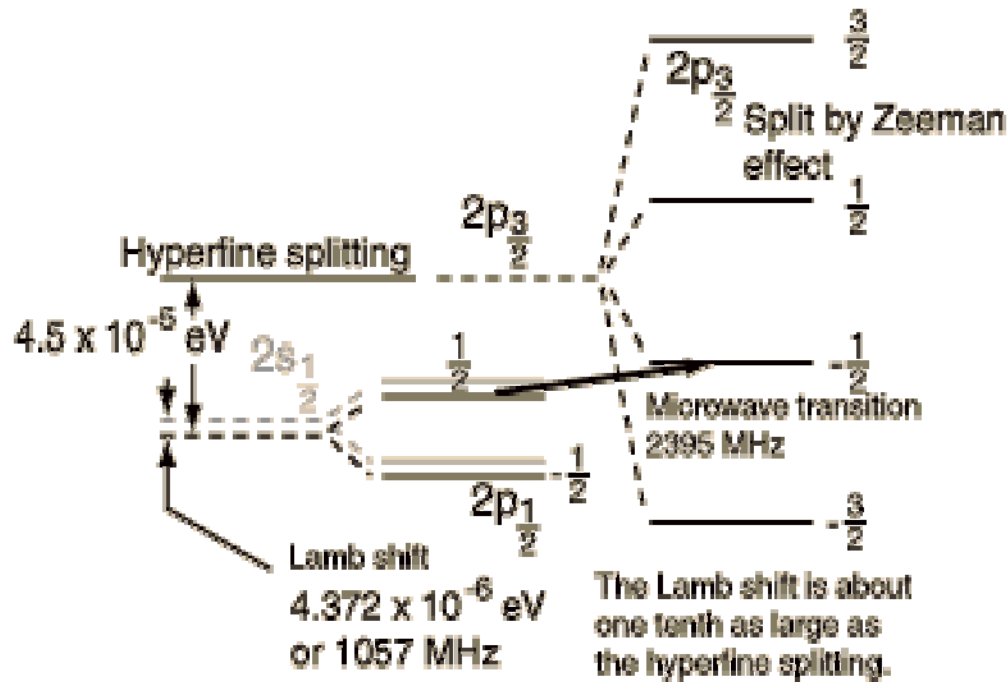
$E(n, j)$

$2s^2S_{1/2}, 2p^2P_{1/2}$ are degenerate.





Lamb Shift → QED



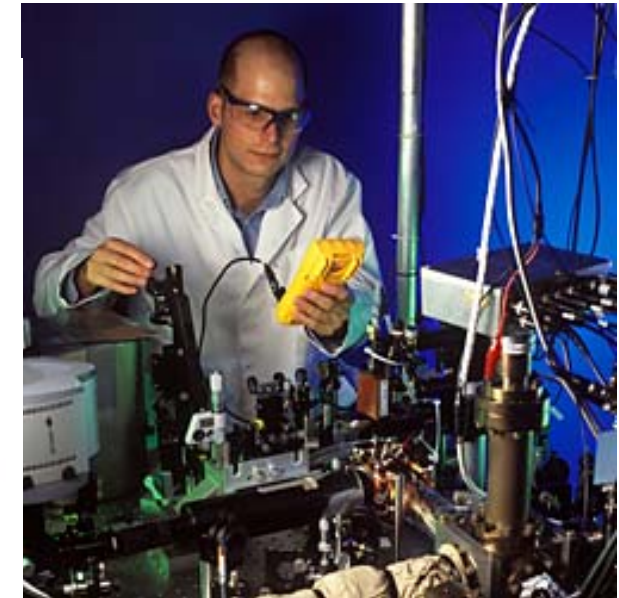
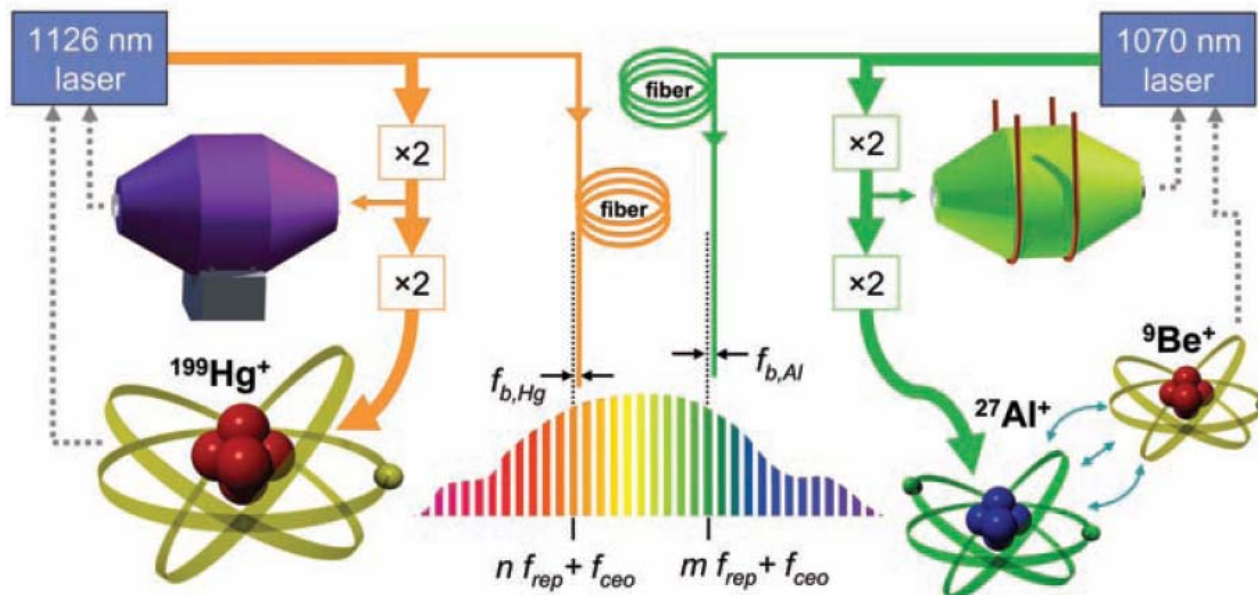
Hydrogen fine structure and hyperfine structure for the $n=3 \rightarrow 2$ transition. (After Ohanian, *Modern Physics*, Ch 7., spectrum from T. W. Hansch, Stanford Univ.)

It provided a high precision verification of theoretical calculations made with the quantum theory of electrodynamics (QED).



High resolution laser spectroscopy

NIST 'Quantum Logic Clock'



The quantum clock frequencies :

$\nu_{\text{Al}^+} / \nu_{\text{Hg}^+}$ is 1.052871833148990438(55);

strontium-87 and ytterbium-171, is 2/1,000,000,000,000,000,000.

Clocks based on the latter exhibit stability greater than
a tenth of a second over the age of the universe.

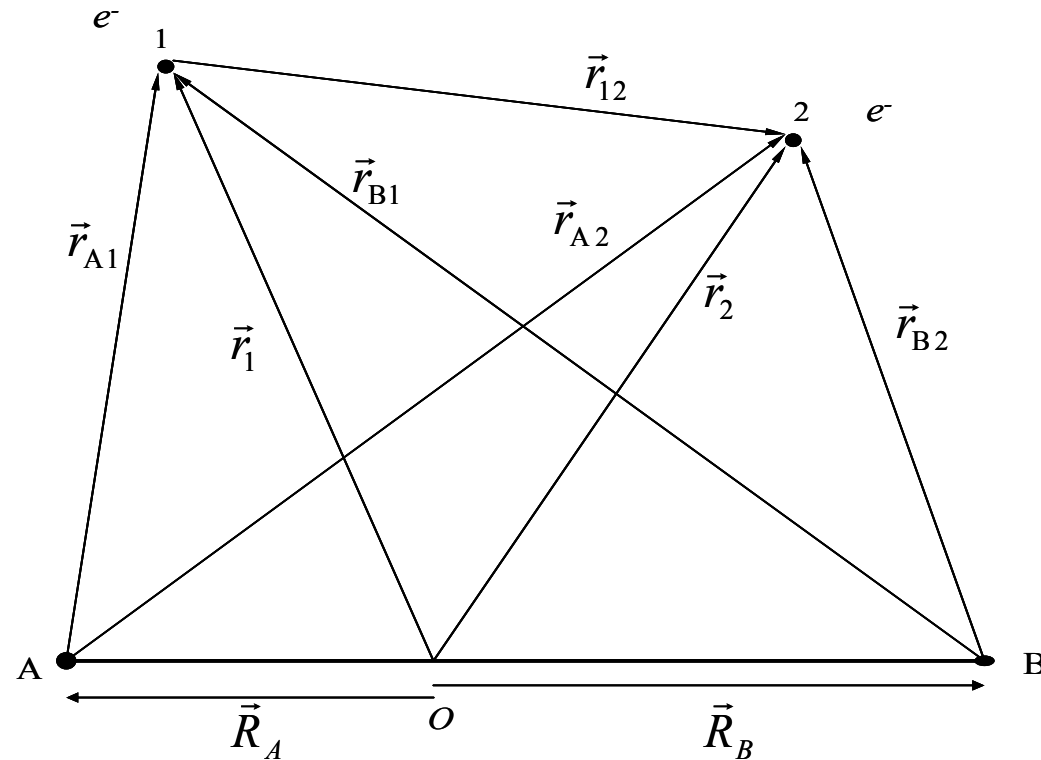


Quantum Phenomena of atom-atom interactions

Molecular Spectroscopy



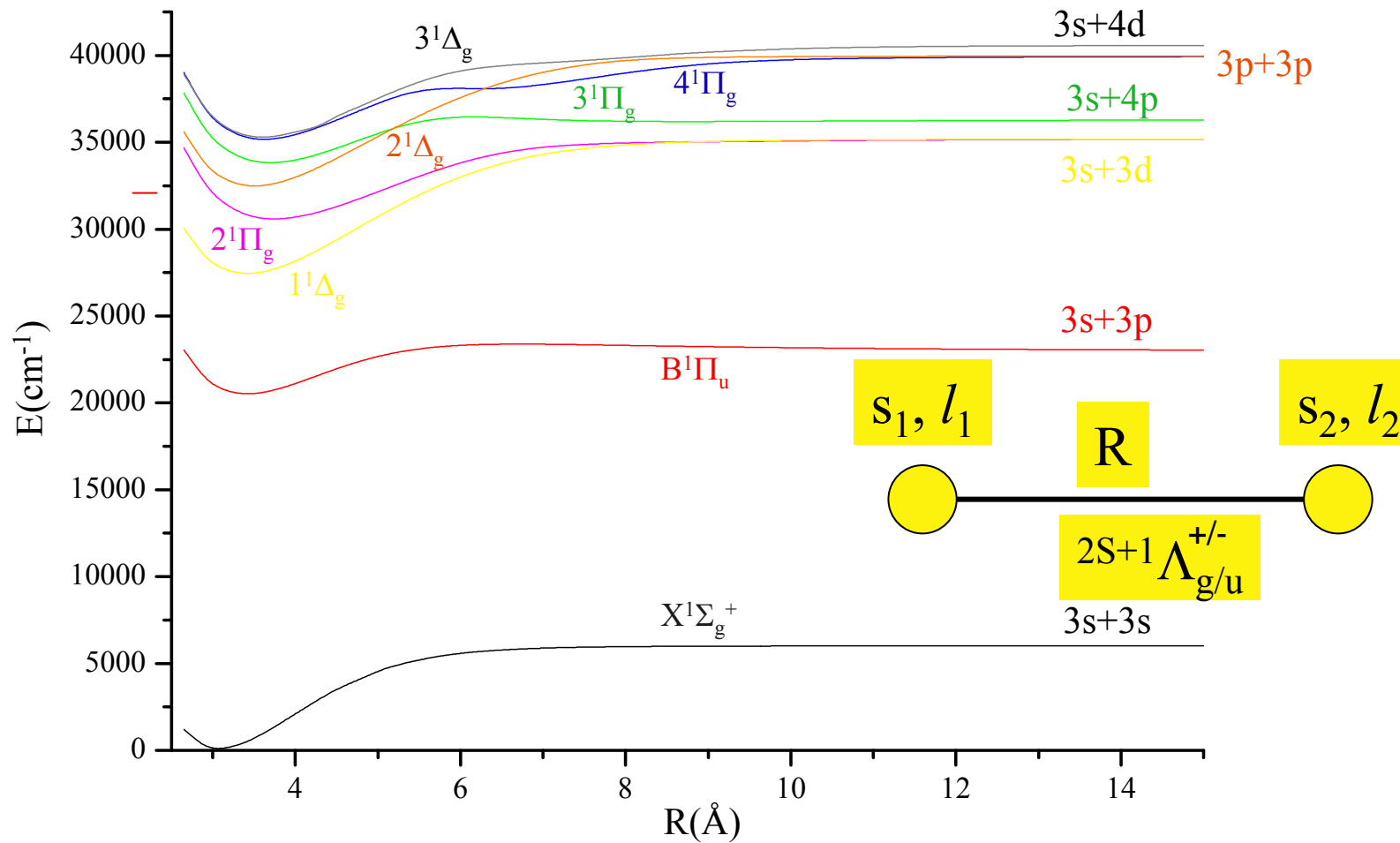
Diatomic molecule



$$H_e \psi_q = (T_e + V) \psi_q = E_q(R) \psi_q$$



Some Potential curves of Na_2 and asymptotic limits

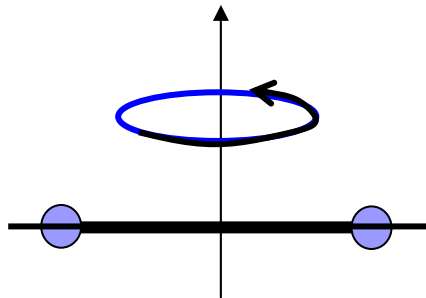




Diatomic molecule



Vibrational Mode



Rotational Mode

Eigenvalues of **Harmonic Oscillator**

$$E_v = \left(v + \frac{1}{2}\right) \hbar \omega$$

Eigenvalues as a **Rigid Rotator**

$$E_J = \frac{J^2}{2I} = \frac{J(J+1)\hbar^2}{2I}$$

Eigenfunctions $\Psi(v, J)$, v : vibration quantum number, J : Rotation quantum number

Eigenvalues : Term(v, J)



Diatomic molecule

Dunham Coefficients

$$T_{v,J} = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} Y_{ij} \left(v + \frac{1}{2} \right)^i [J(J+1) - \Lambda^2]^j$$

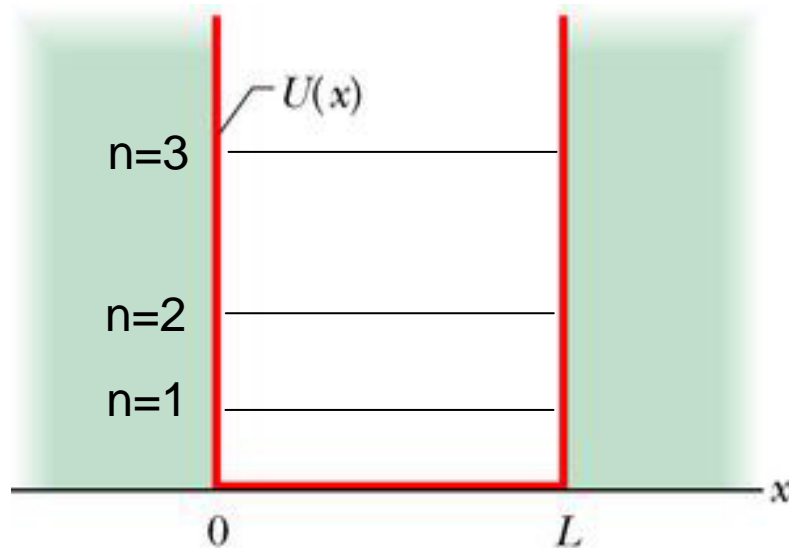
Lower terms of Dunham Coefficients (Y_{ij})

Harmonic Oscillator

$j \setminus i$	0	1	2	3	4
0	T_e	ω_e	$-\omega_e X_e$	$\omega_e Y_e$	$\omega_e Z_e$	
1	B_e	$-\alpha_e$	γ_e	δ_e	..	
2	$-D_e$	$-\beta_e$	
3	H_e Parts of Anharmonicity
4	L_e Parts of Anharmonicity
..



In quantum mechanics, the eigenvalues are discrete, the space is not isotropic.



Boundary conditions :

$$\psi(0) = 0 \quad \psi(L) = 0$$

$$\frac{d^2\psi}{dx^2} = \frac{2m}{\hbar^2} (V_0 - E) \psi = -\frac{2m}{\hbar^2} E \psi = -k^2\psi$$

$$\psi(x) = C_1 \sin kx + C_2 \cos kx$$

$$\psi(0) = C_2 = 0 \quad \longrightarrow \quad \psi(x) = C_1 \sin kx$$

$$\psi(L) = 0 \quad \longrightarrow \quad \psi(L) = C_1 \sin kL = 0$$

$$kL = n\pi$$

$$E_n = \left(\frac{\hbar^2}{8mL^2} \right) n^2$$

$$\phi_n(x) = C_1 \sin\left(\frac{n\pi}{L}x\right)$$

$$\lambda = \frac{2L}{n}$$



Transitions are the Difference between Eigenvalues

Intensity

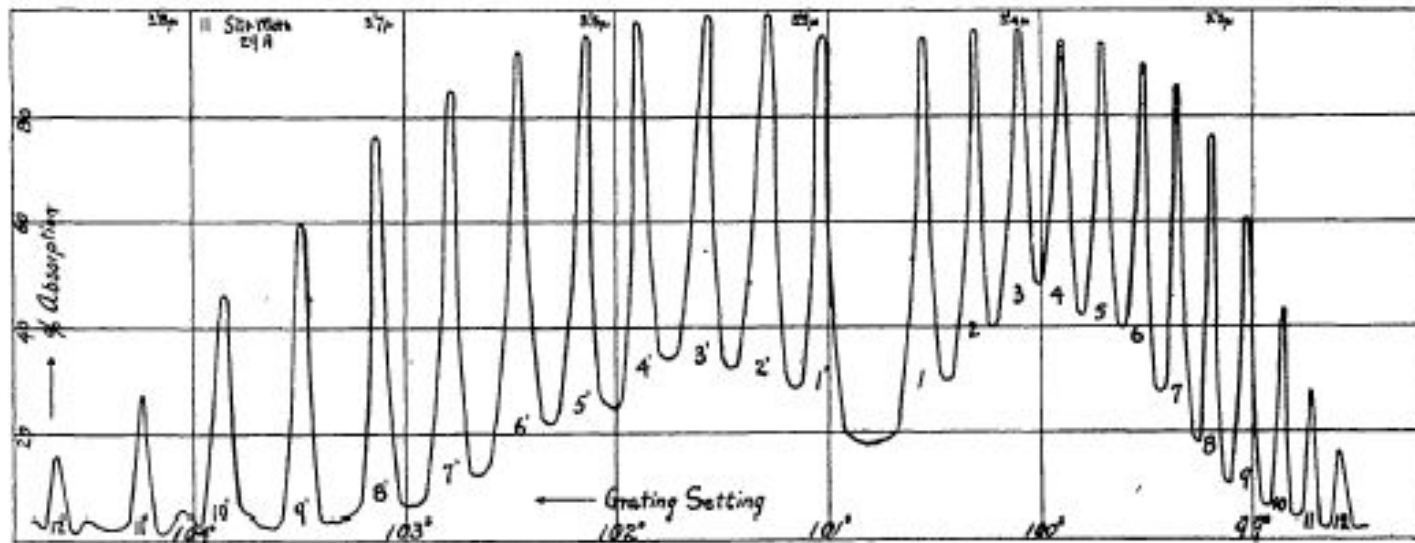


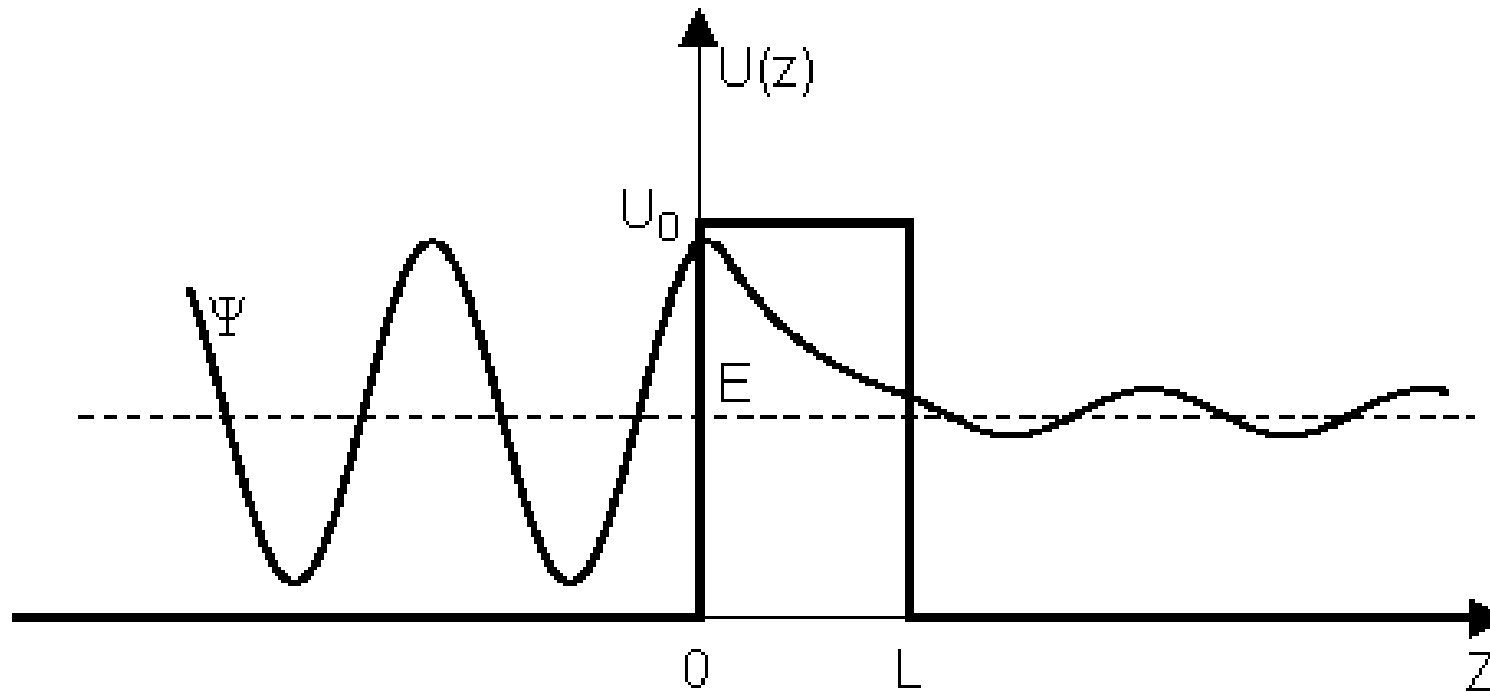
FIG. 3.—The HCl band at 3.46μ , mapped with 7500-line grating. HCl at atmospheric pressure.

Energy

Discrete Eigenvalues



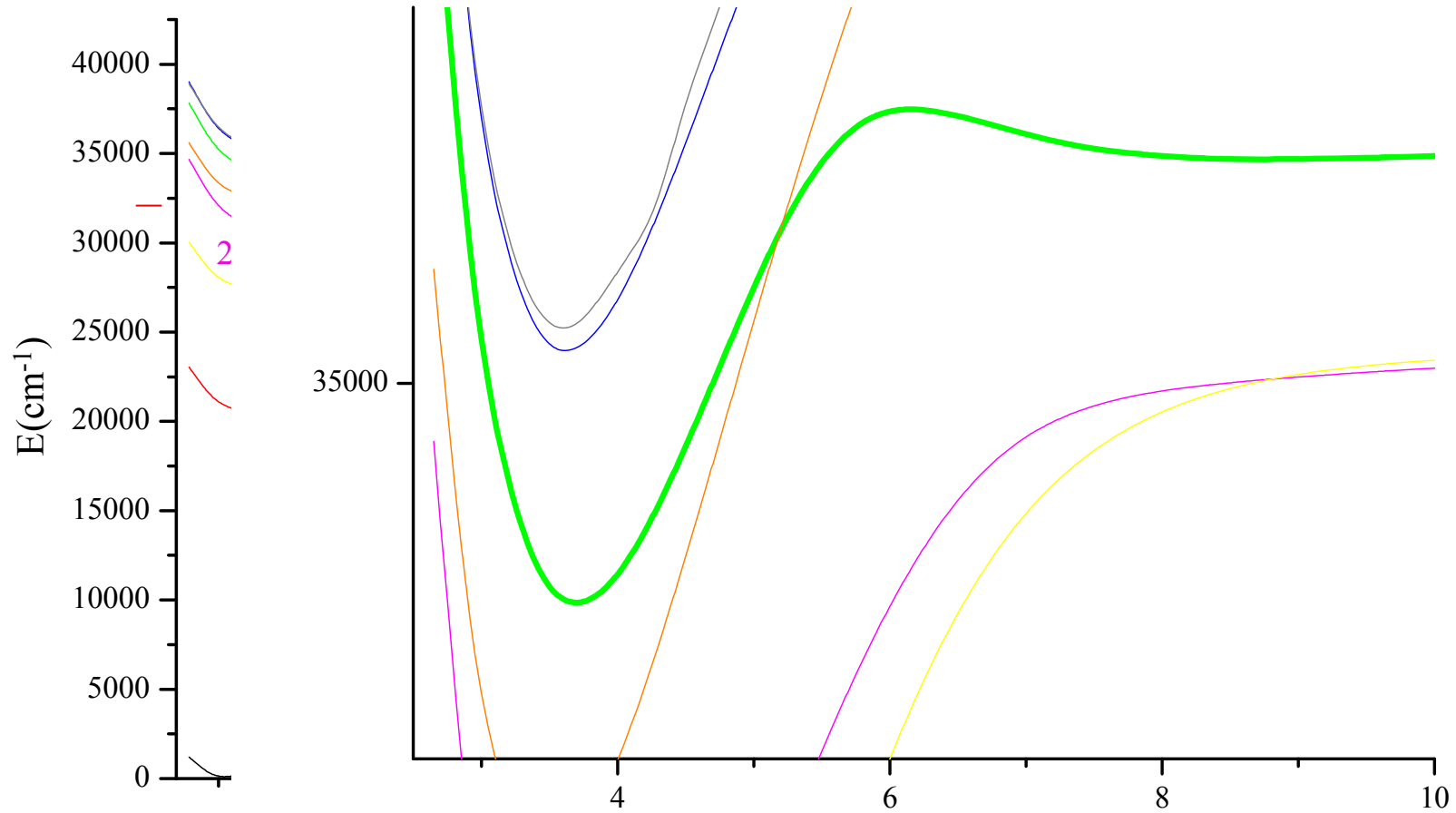
Tunneling Effect



$$D(E) = D_0 \exp \left\{ -\frac{2L}{\hbar} \sqrt{2m(U_0 - E)} \right\}$$

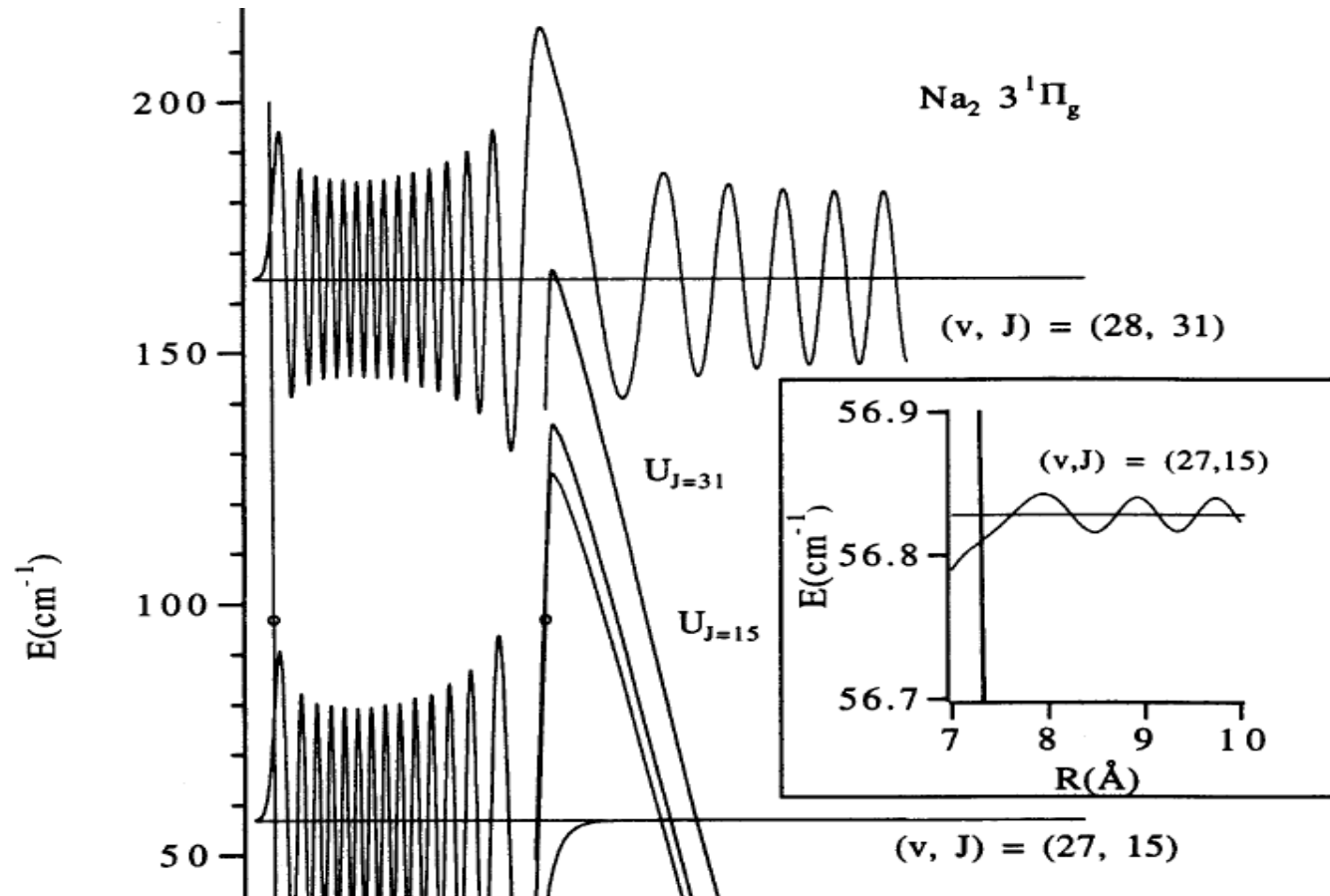


Tunnelling Effect





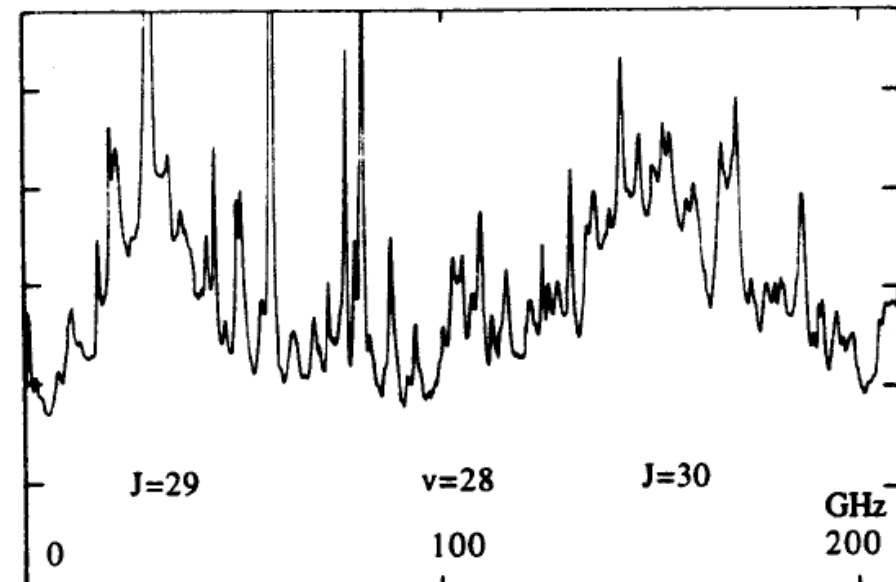
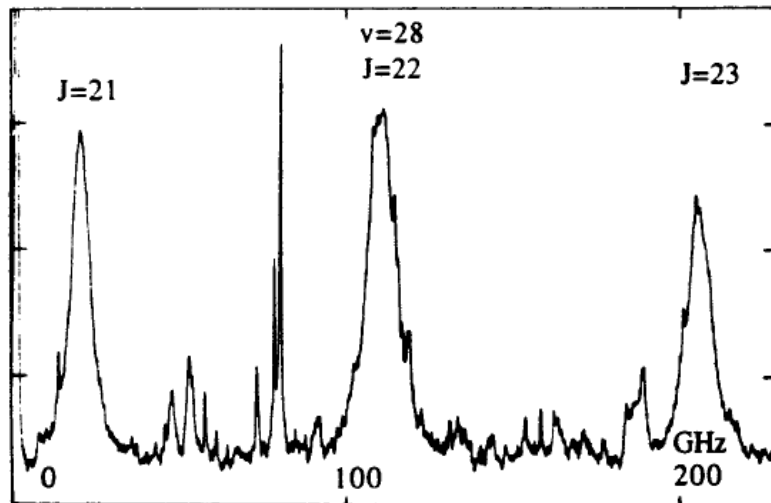
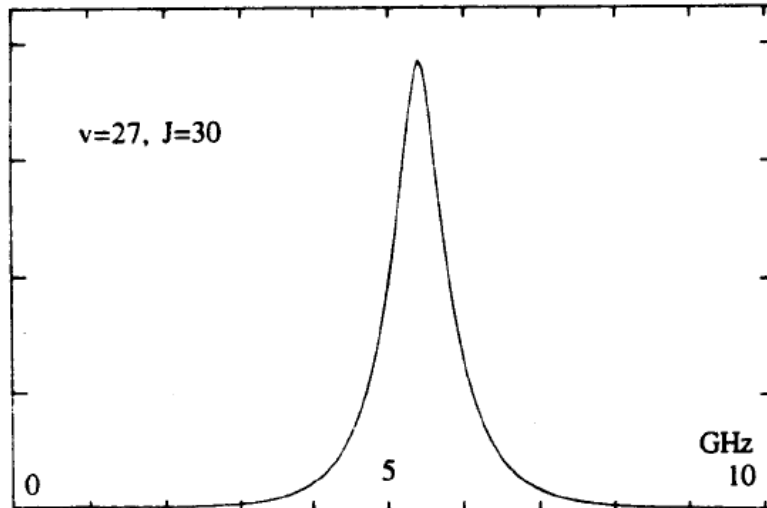
Tunnelling Effect



From WKB Approximation:
$$2\Gamma = \frac{\hbar}{\pi} \left[\frac{1}{2} \tau_0 \exp \left(\frac{4\pi}{\hbar} \int_a^b \sqrt{2\mu(U-E)} dR \right) \right]^{-1}$$

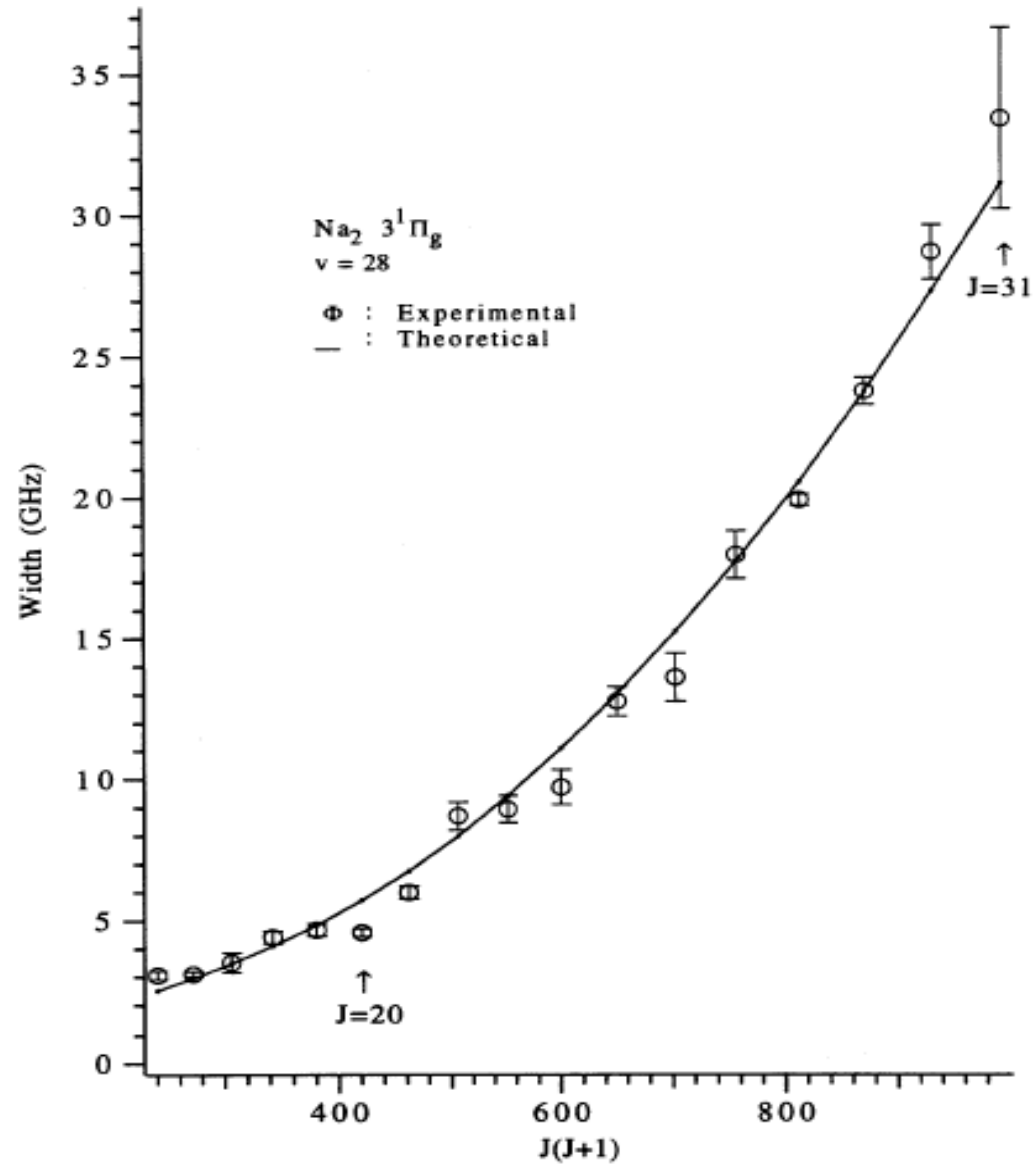


Tunnelling Effect





Tunnelling Effect





Avoided Crossing

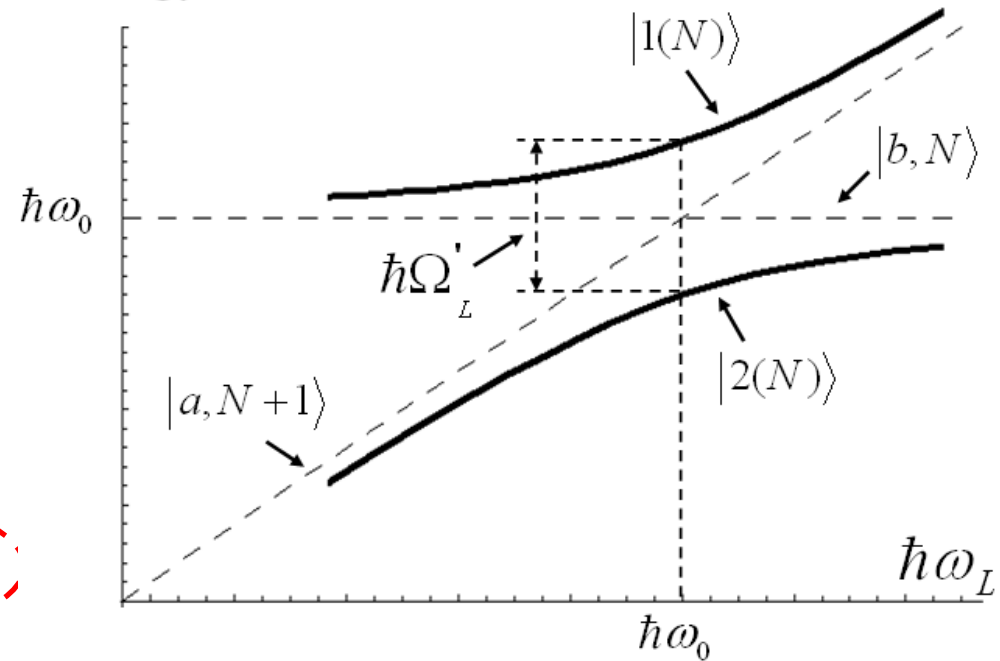
Energy of dressed states :

The eigenvalues of

$$V_{AL} = \hbar \begin{pmatrix} 0 & \Omega_L/2 \\ \Omega_L/2 & \Delta_L \end{pmatrix}$$

$$E_{AL} = \frac{\hbar\Delta_L \pm \hbar\sqrt{\Delta_L^2 + \Omega_L^2}}{2}$$

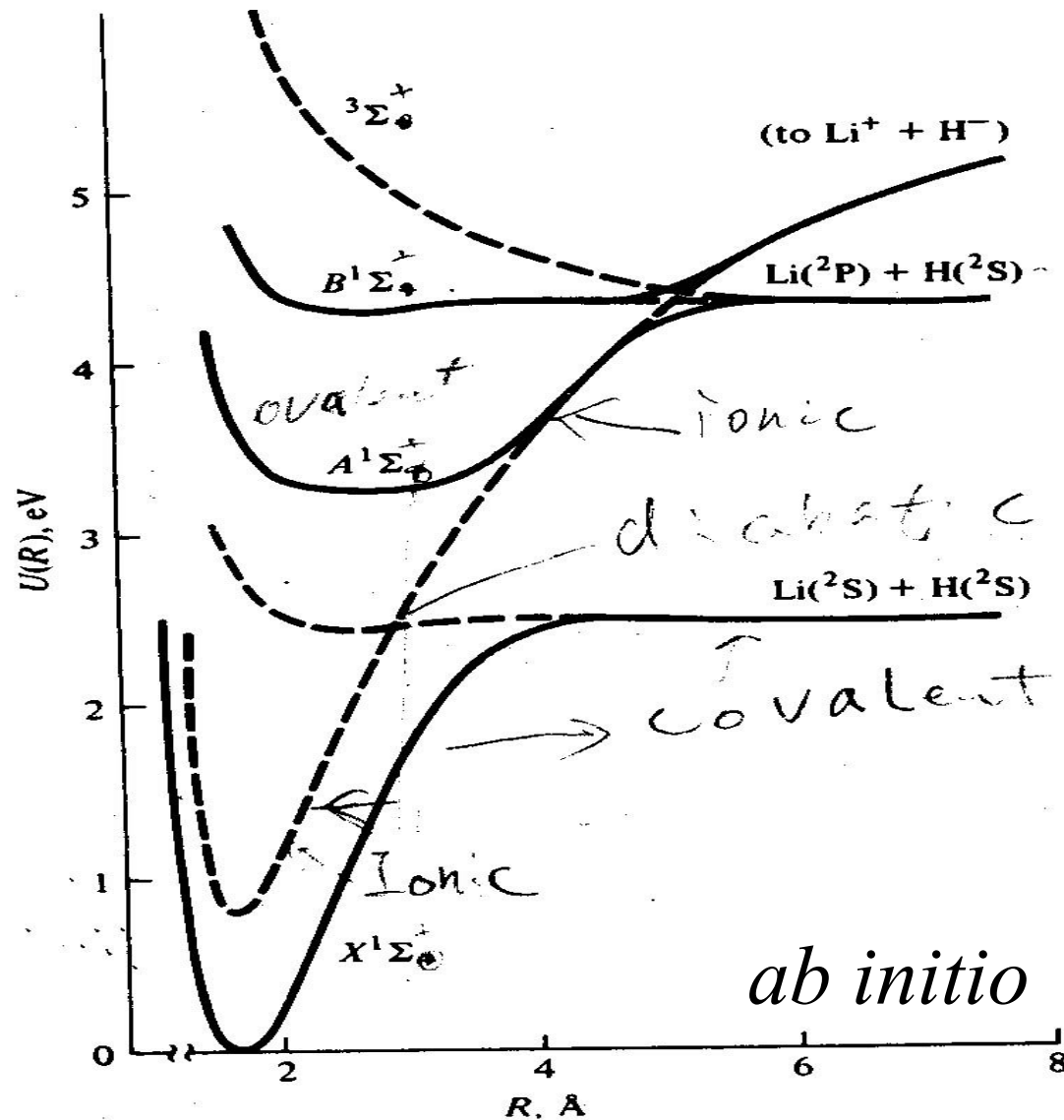
Energy





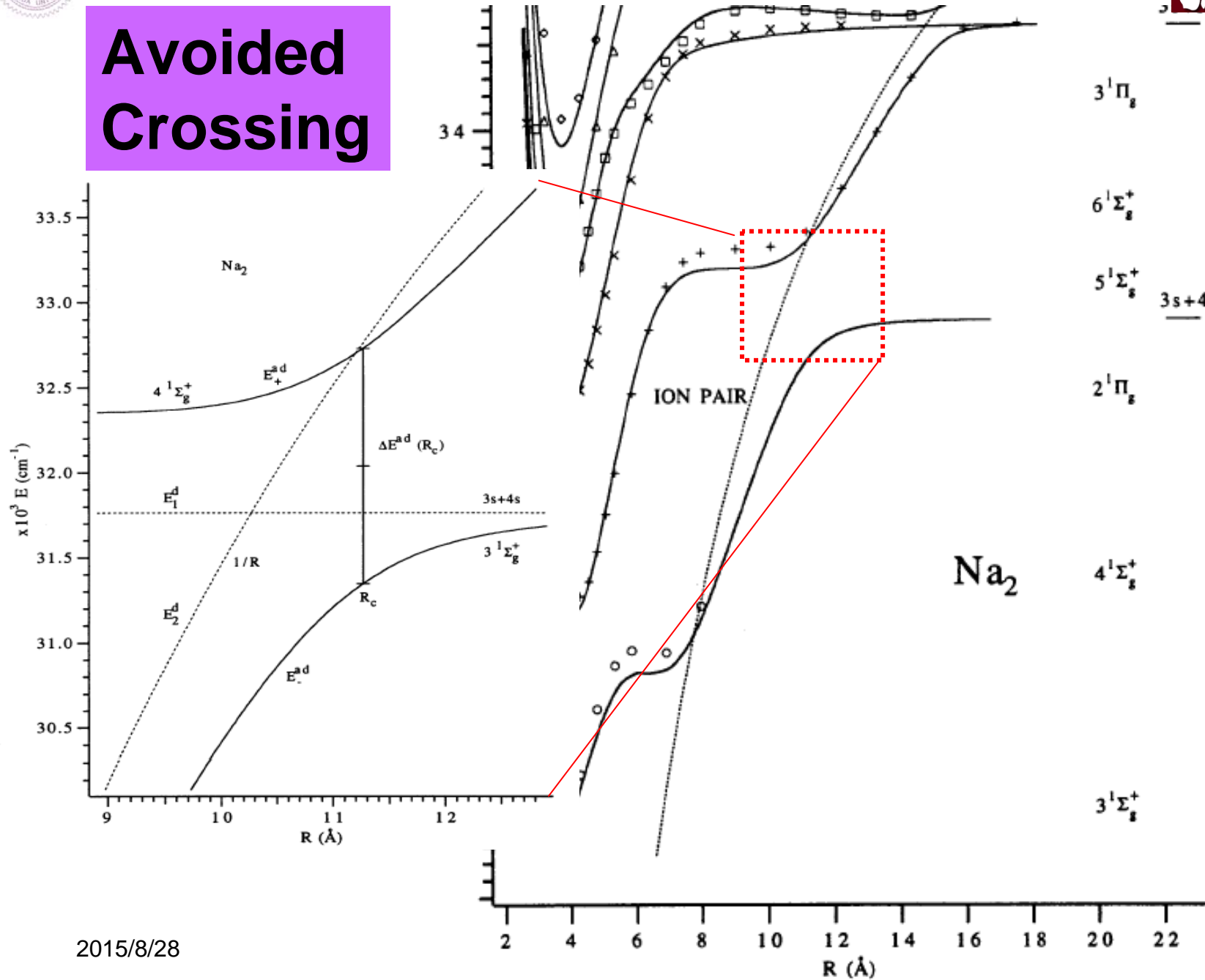
Avoided Crossing

Intermolecular Potentials



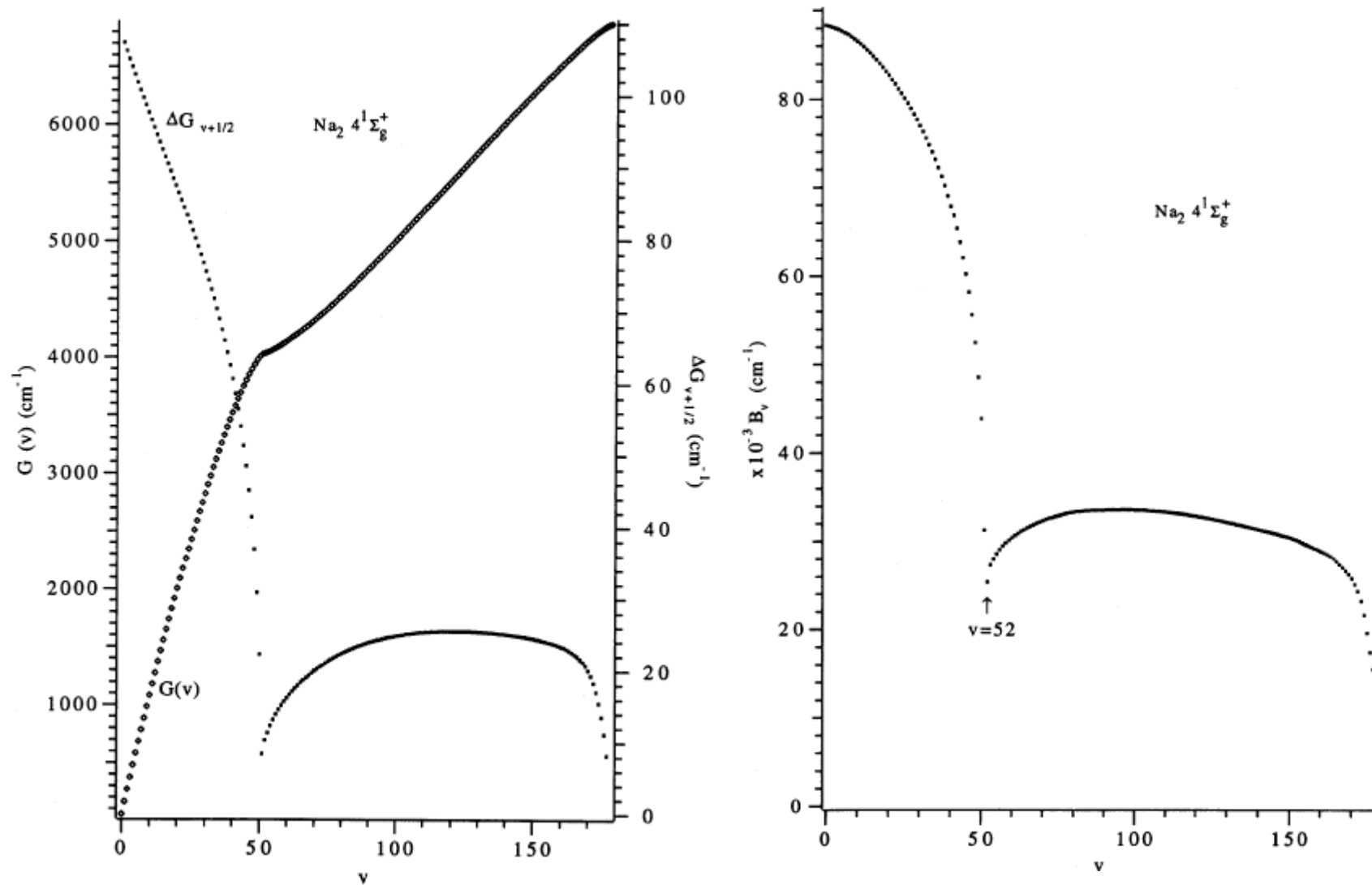


Avoided Crossing



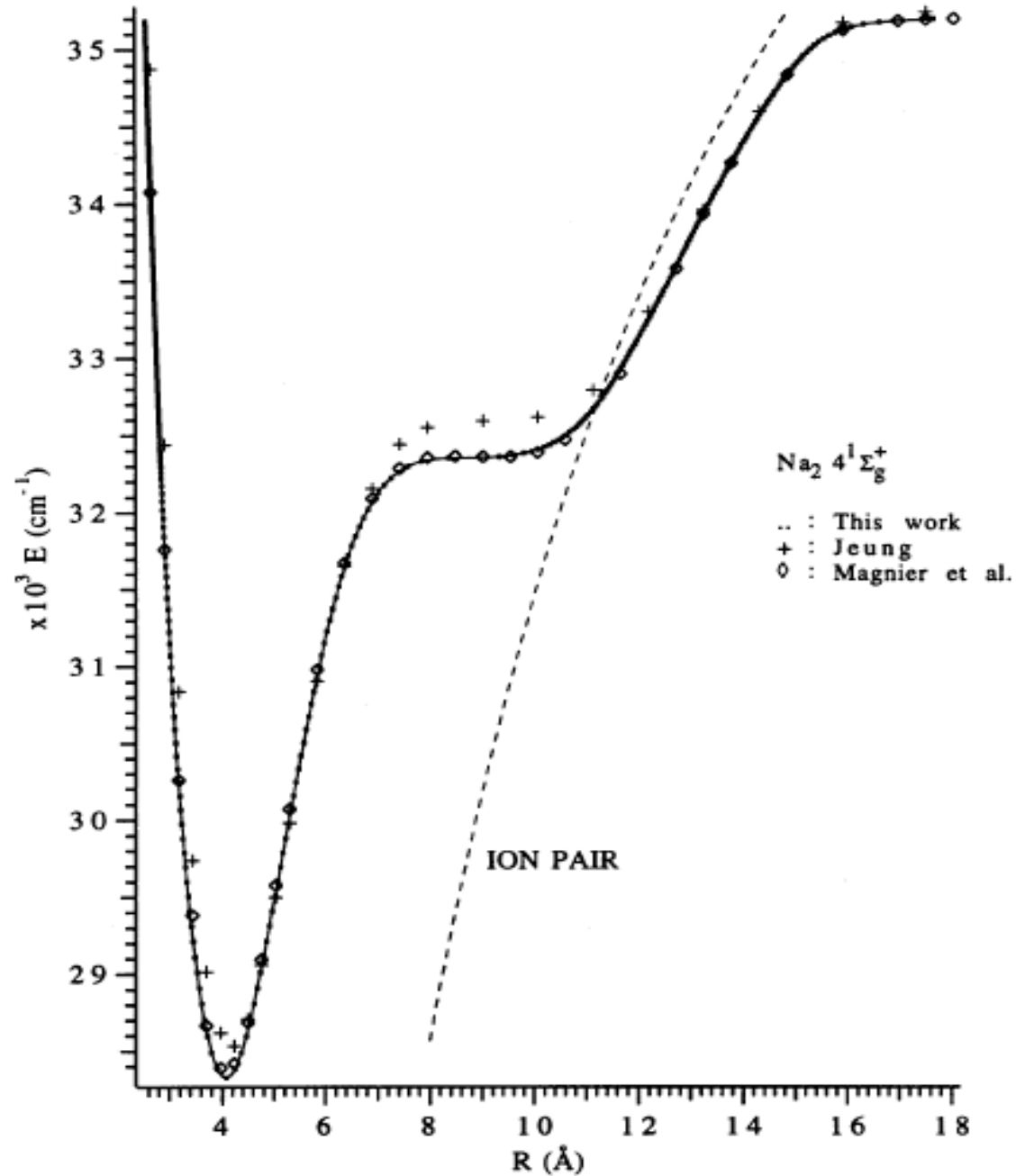


Avoided Crossing



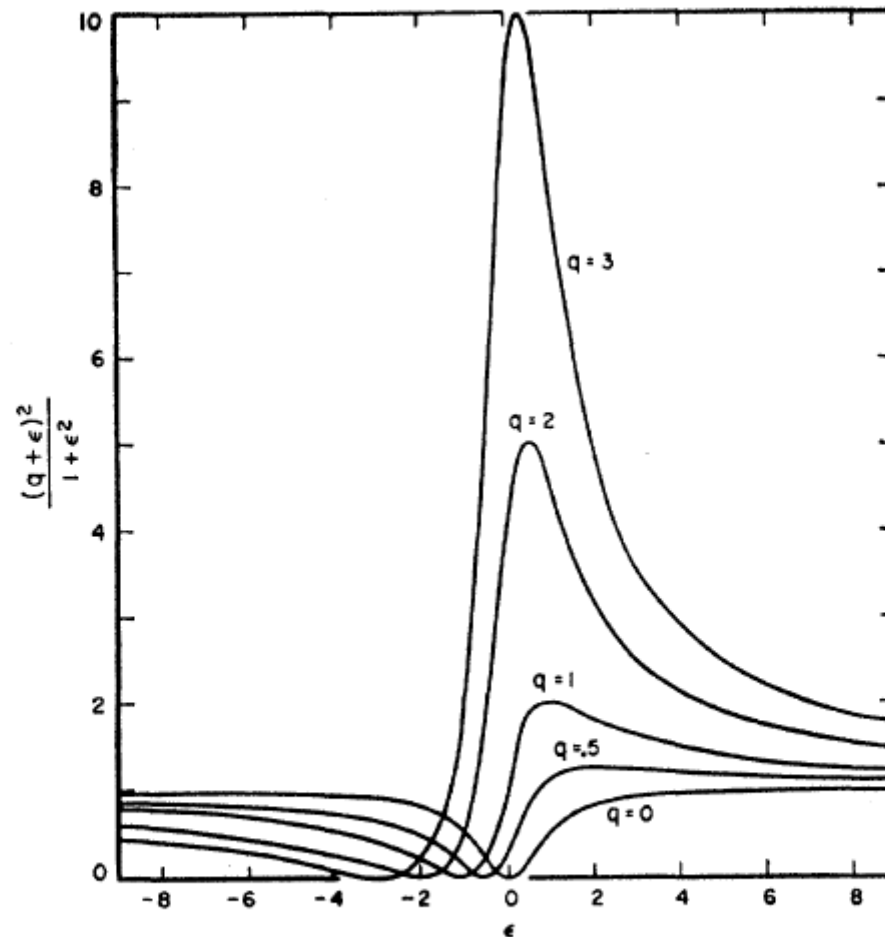


Avoided Crossing





Fano Resonance



$$\frac{(q + \epsilon)^2}{1 + \epsilon^2}$$

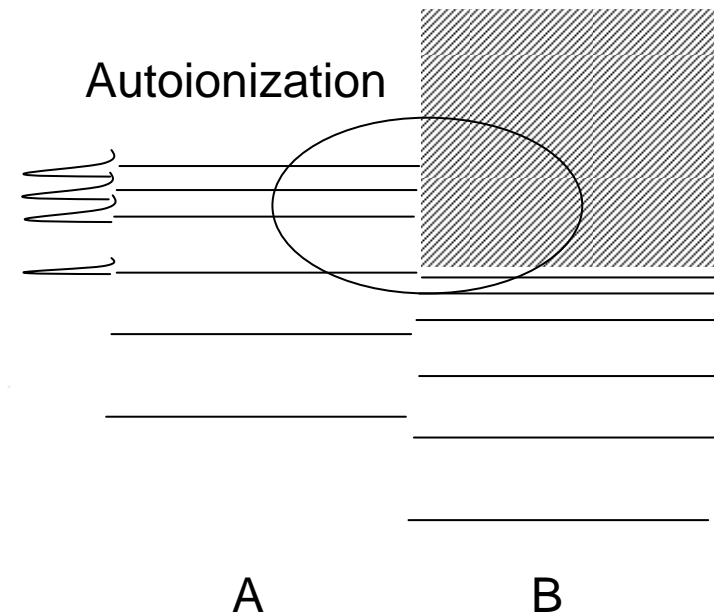
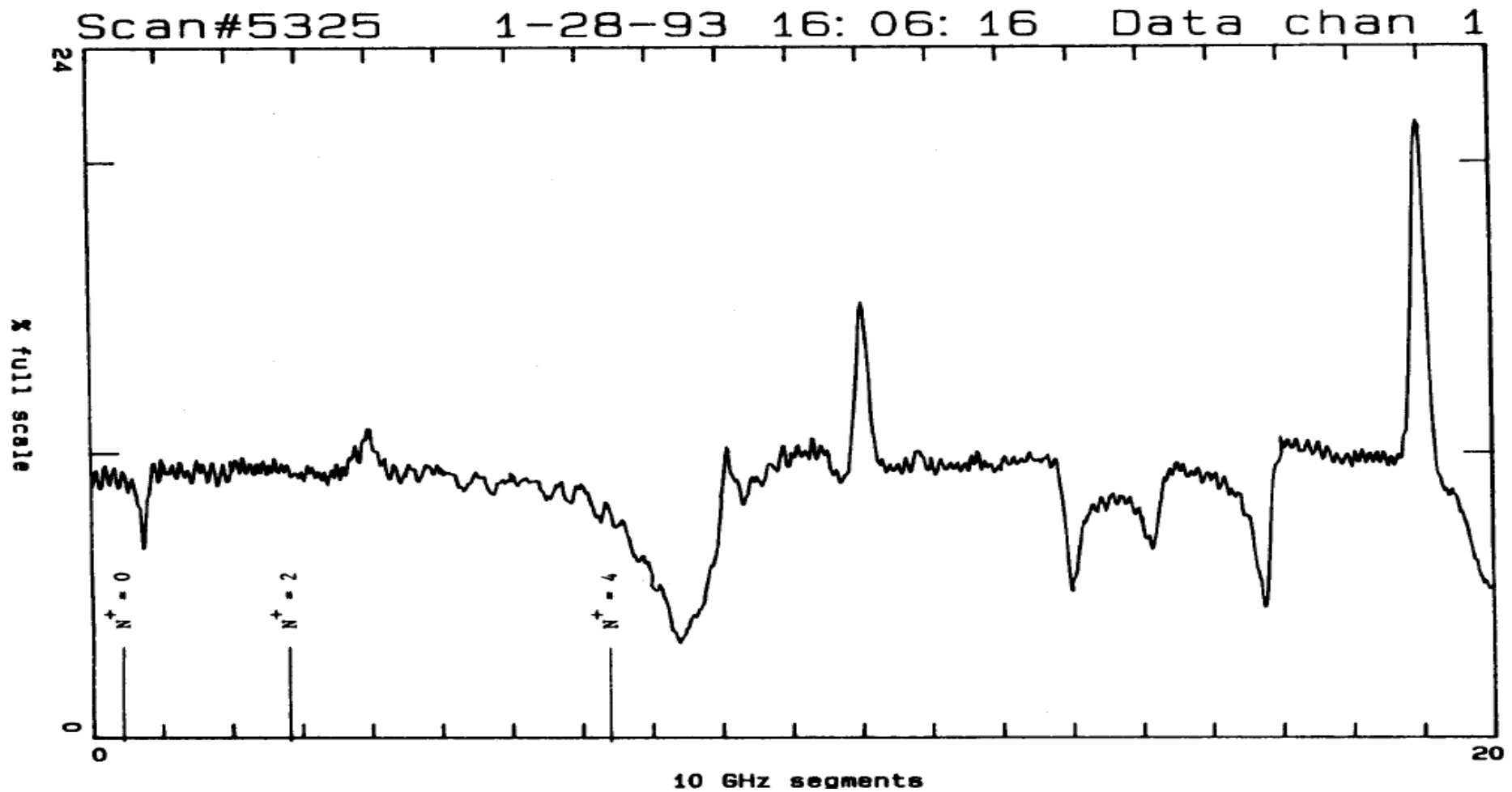
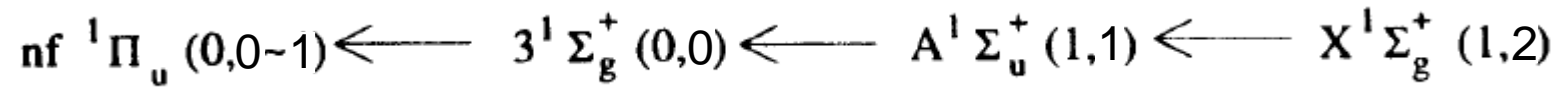


FIG. 1. Natural line shapes for different values of q . (Reverse the scale of abscissas for negative q .)



Fano Resonance





Fano Resonance

Discrete : U (Na 3p + Na 4s), Energy : 42000 cm⁻¹ ~
Continuum : Na₂⁺ + e⁻

$$\frac{(q + \varepsilon)^2}{(1 + \varepsilon^2)}$$

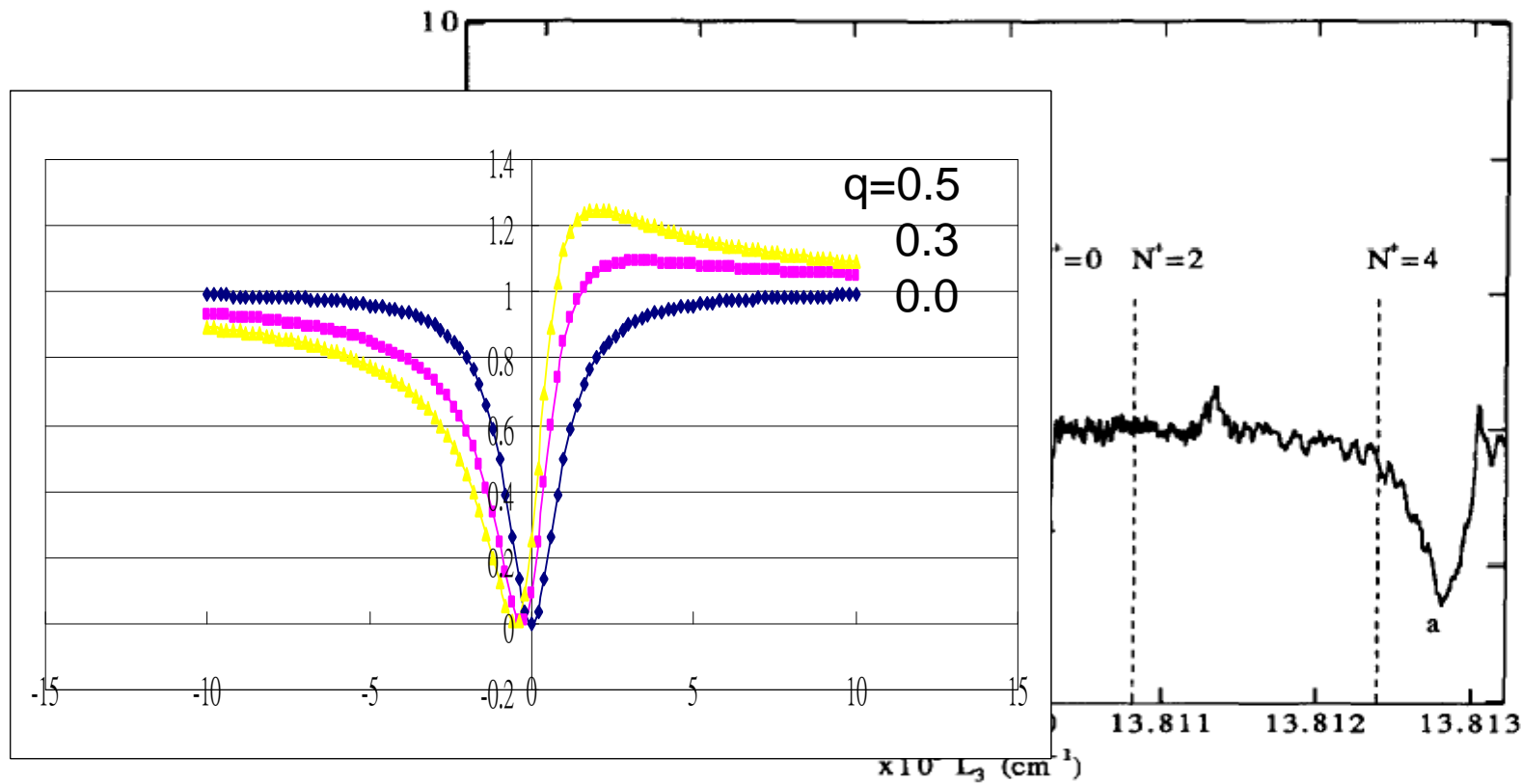


Fig. 3. The AOTR spectrum near the series limit clearly shows the continuum (a) (and quasi-continuum (b)) Fano autoionization profiles. The intermediate level is the Na₂ 3¹Σ_g⁺ (0, 0) level. Line (c) is an experimental artifact.



Quantum Phenomena in Cold Collisions

Photoassociation Spectroscopy in Rb



Motivations for using cold collisions:

Study the atoms free from spectral line broadening and shifts that arise from atomic motions and collisions

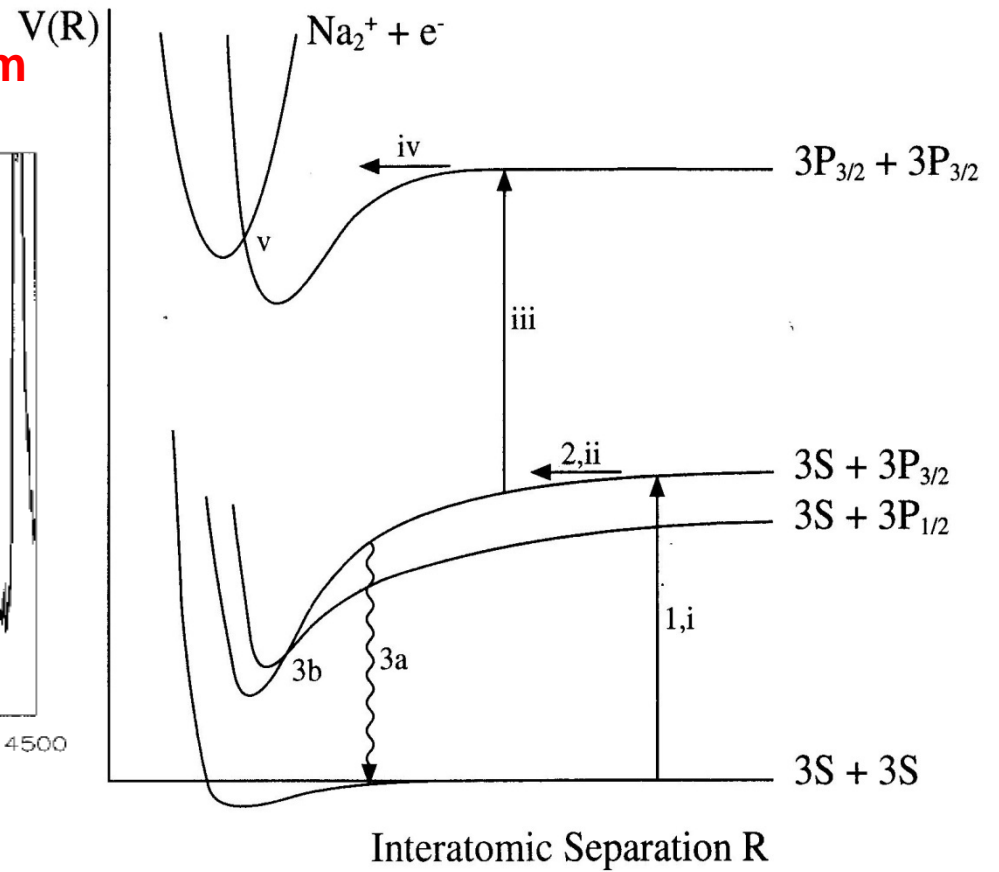
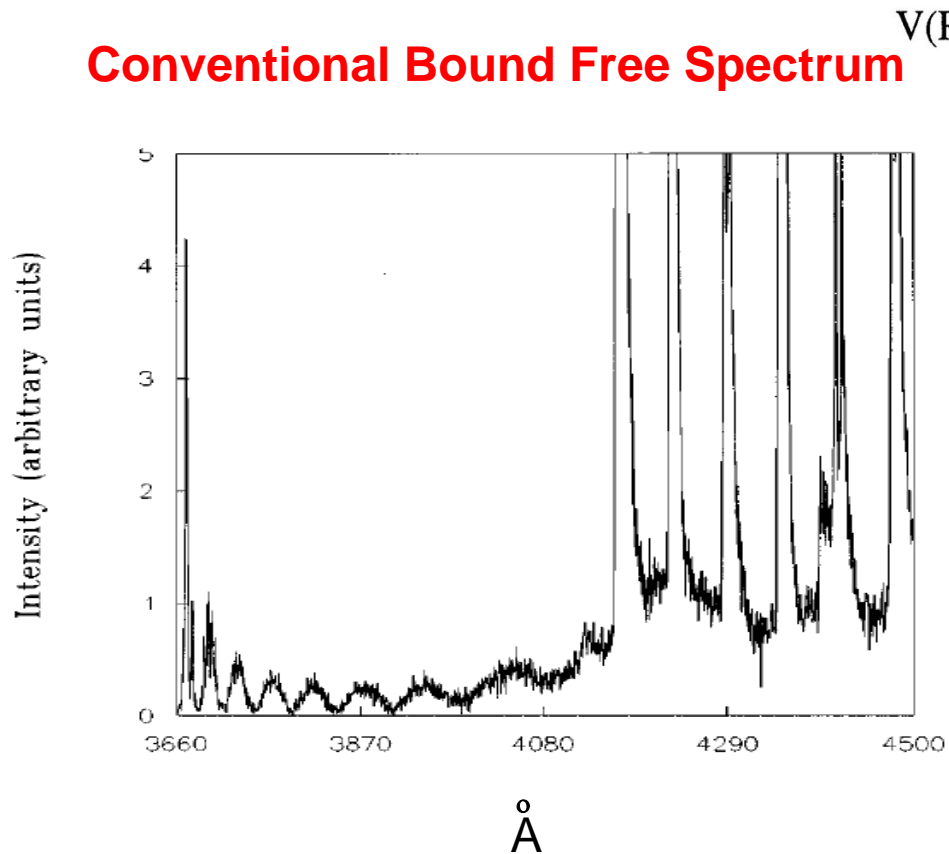
Advantages:

- I. Cold collisions are highly quantum-mechanical in nature
- II. Cold collisions are simple, involving only a few partial waves
- III. Cold collisions are sensitive to long-range interatomic forces
- IV. Long collision times can significantly affect the collision dynamics
- V. Spontaneous emission during the collision may occur to change the collision channels involved.



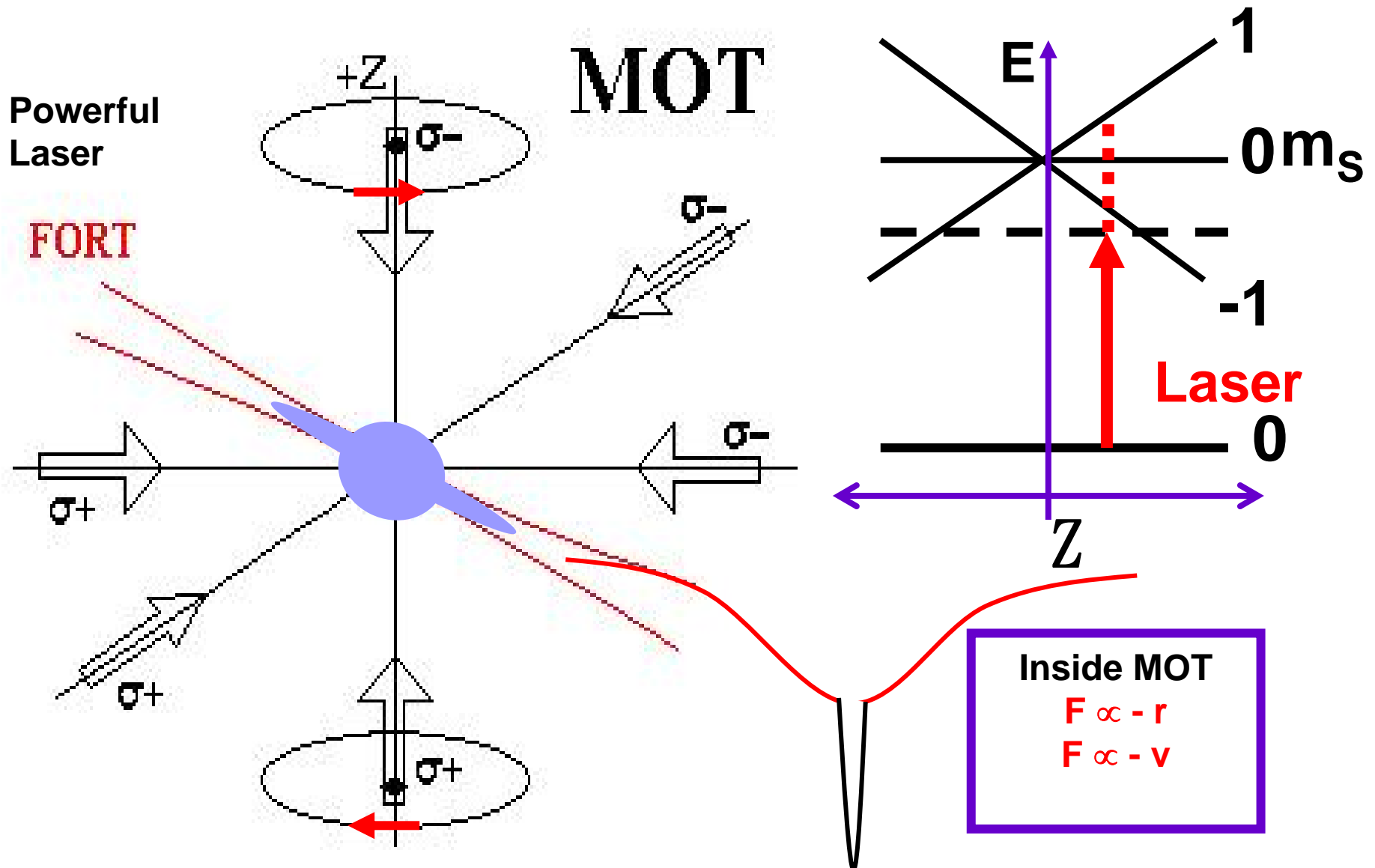
Bound Free Transitions

Conventional Bound Free Spectrum





A. Cold collisions in a far-off resonance trap (FORT)



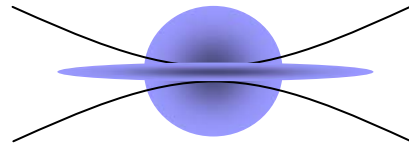


B. Cold collisions under high resolution laser spectroscopy

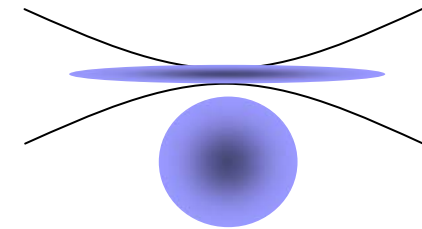
a. MOT



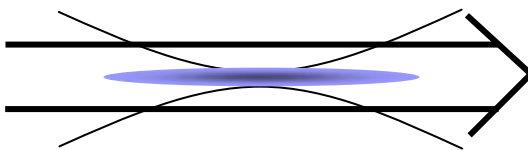
b. Loading into FORT



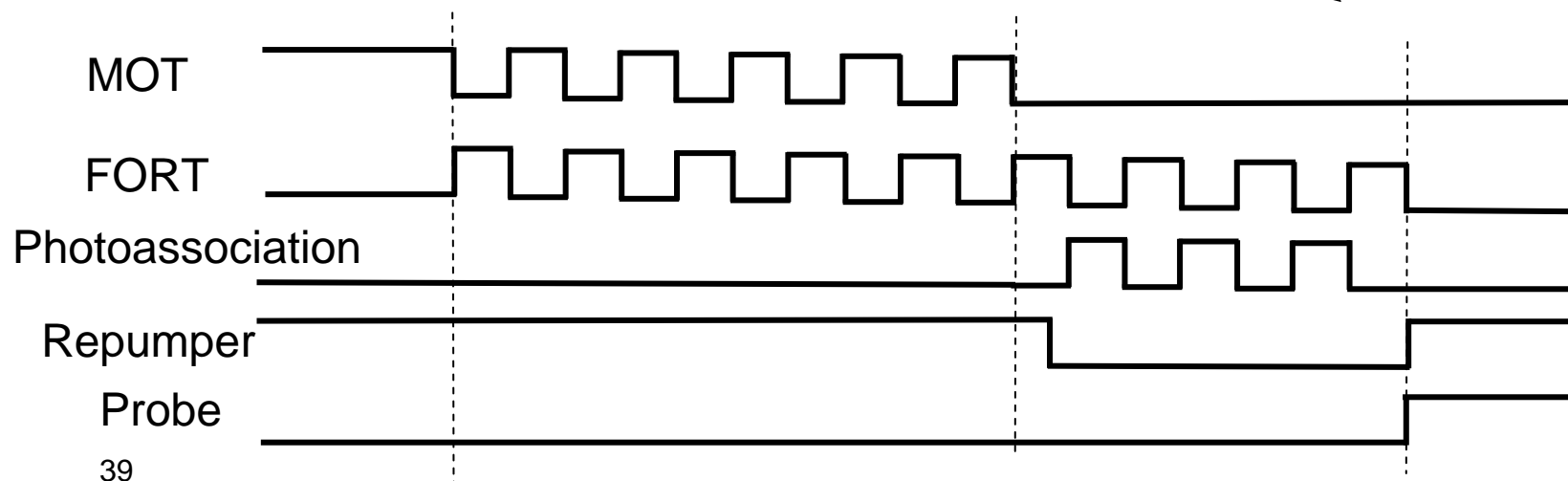
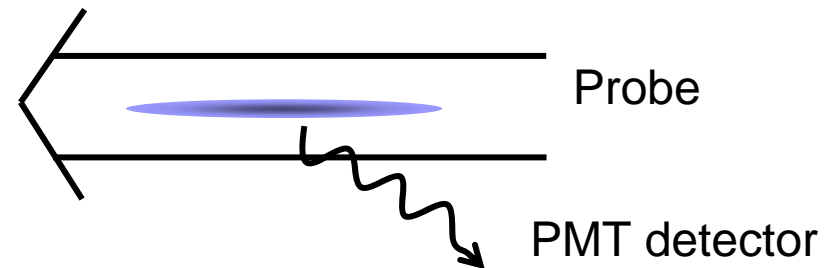
c. MOT off



d. Photoassociation

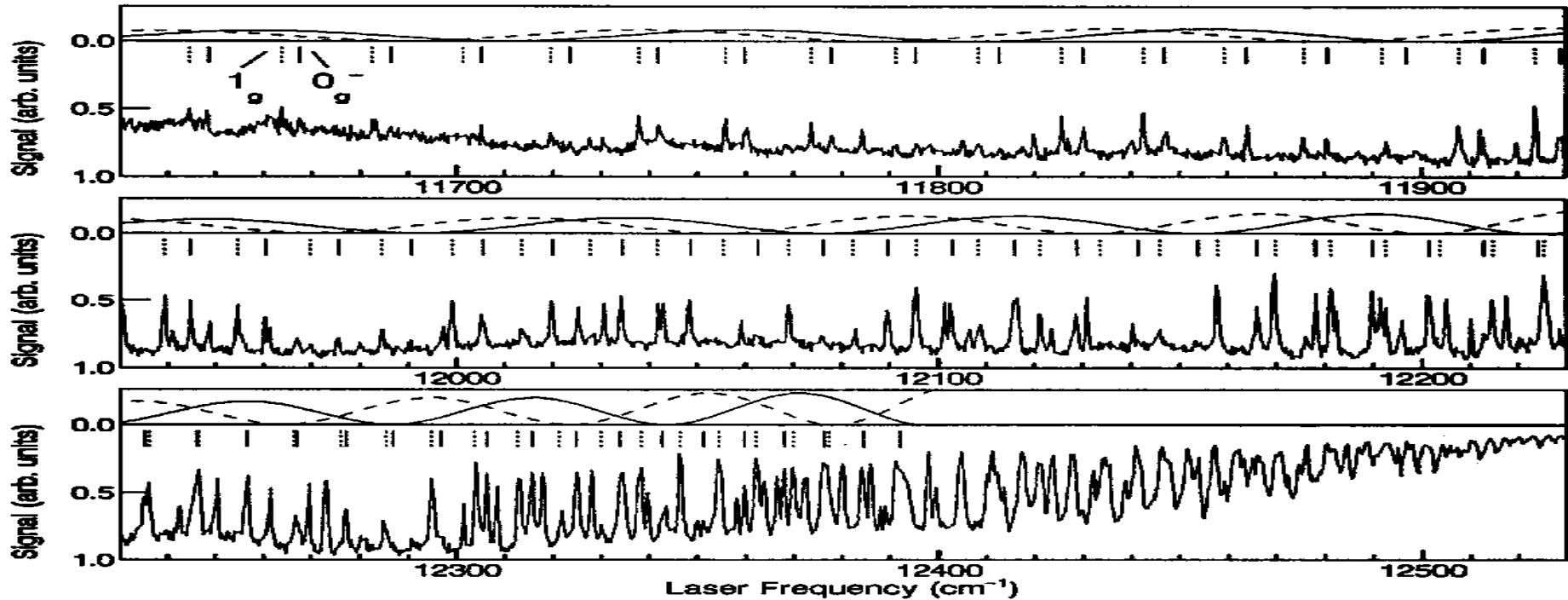
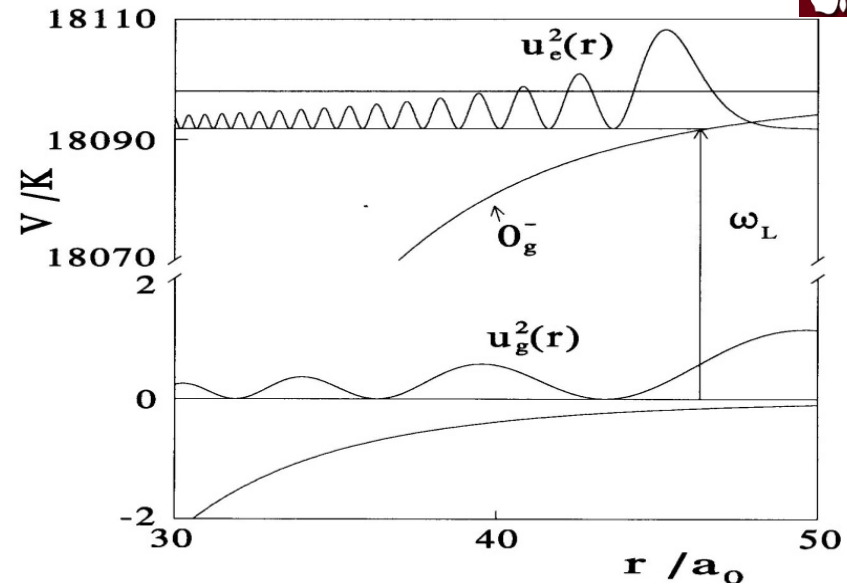


e. Probe and detection





Bound Free Transitions



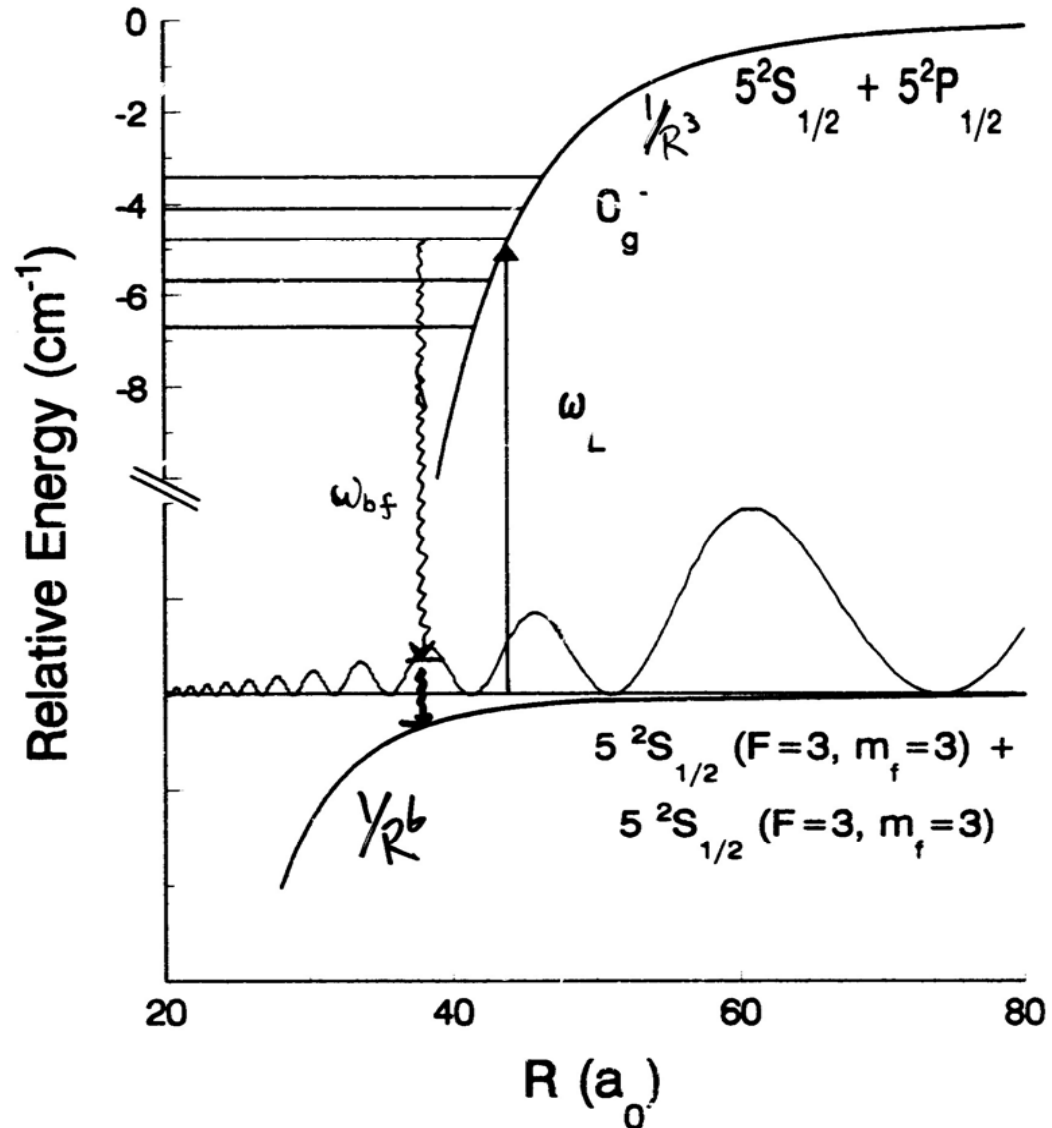


Shape Resonance

Atom trapped in the
MOT or FORT

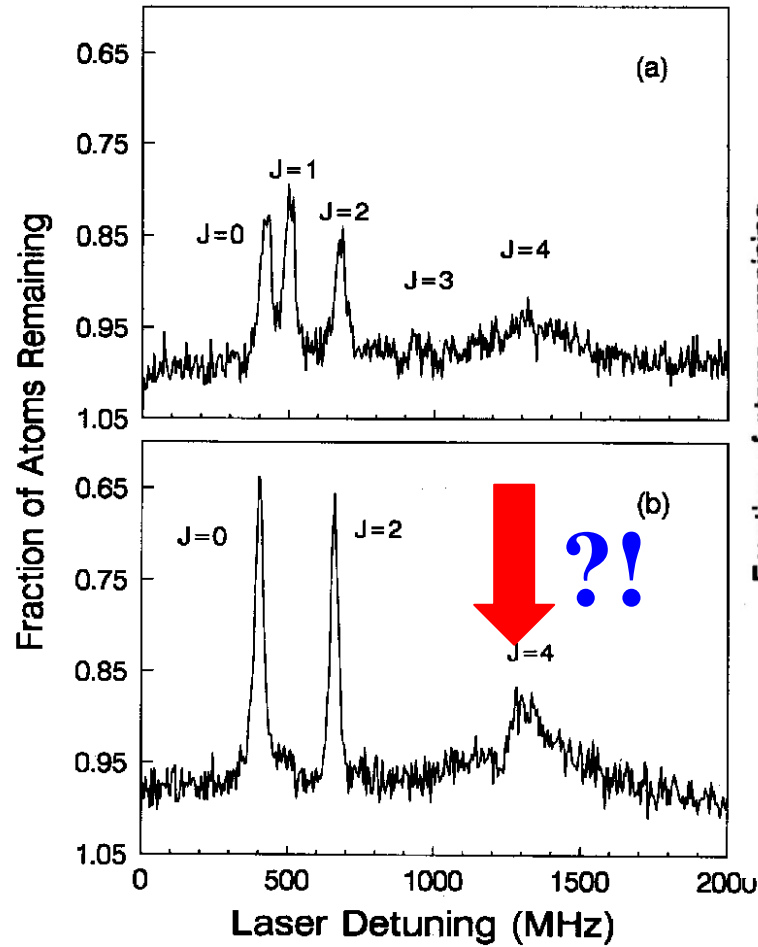
Detecting the trap
loss

Photoassociation of Ultra-Cold Rb Atoms

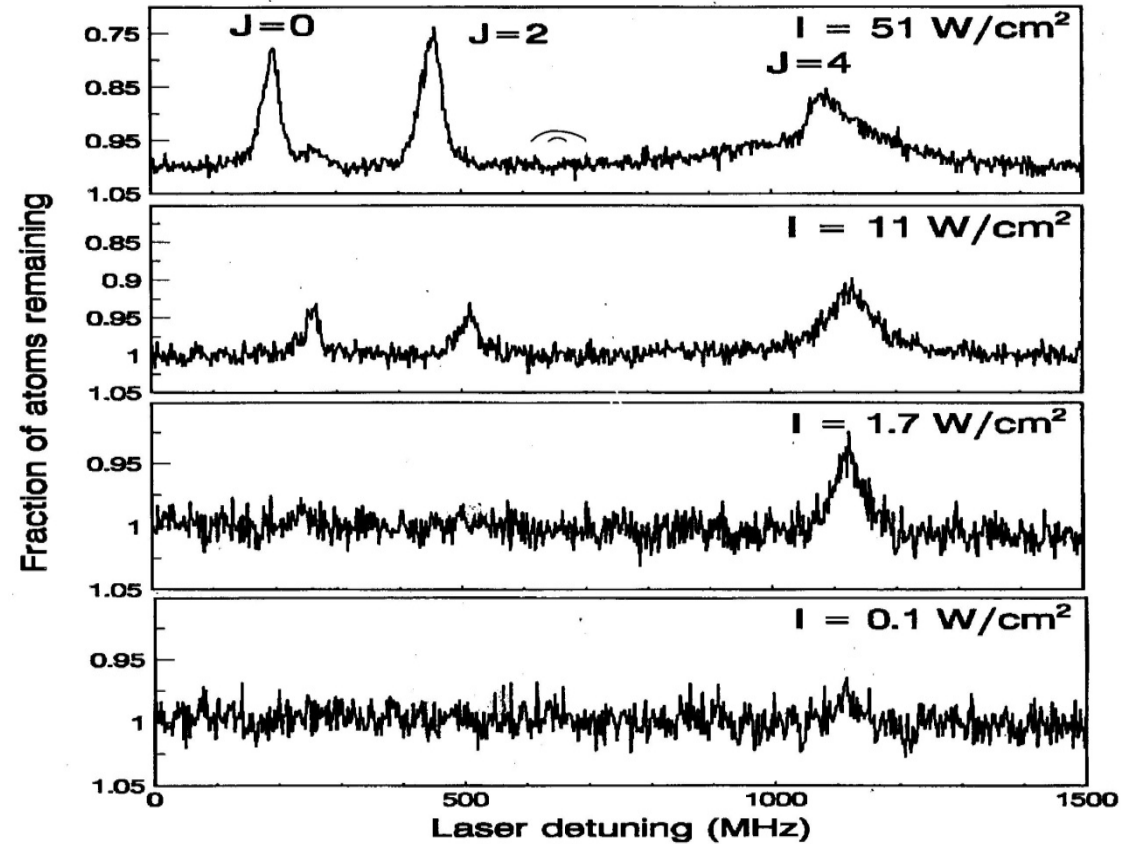




Shape Resonance



$^{85}\text{Rb}_2\text{O}_g^-(S+P_{1/2})$ Rotational Spectra
atoms doubly polarized, level at -5.8 cm^{-1}



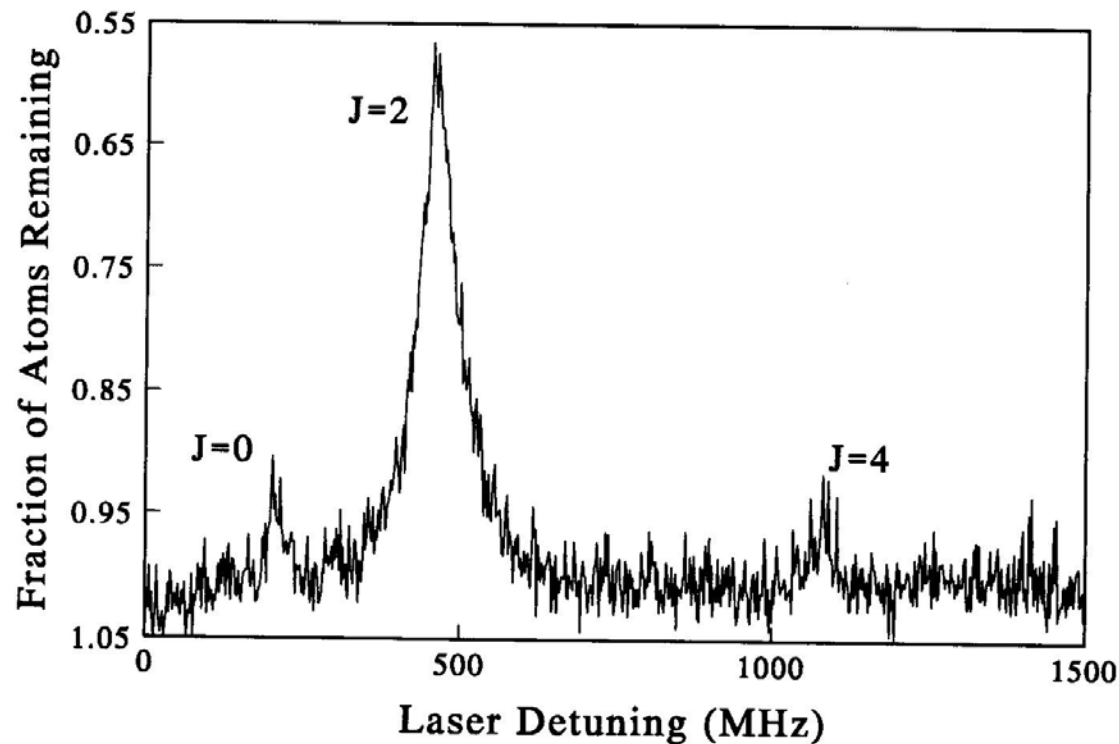
$^{85}\text{Rb}+^{85}\text{Rb}$



Shape Resonance

Cold collisions under **d-wave shape resonance**

How about the PA spectrum of $^{87}\text{Rb}+^{87}\text{Rb}$

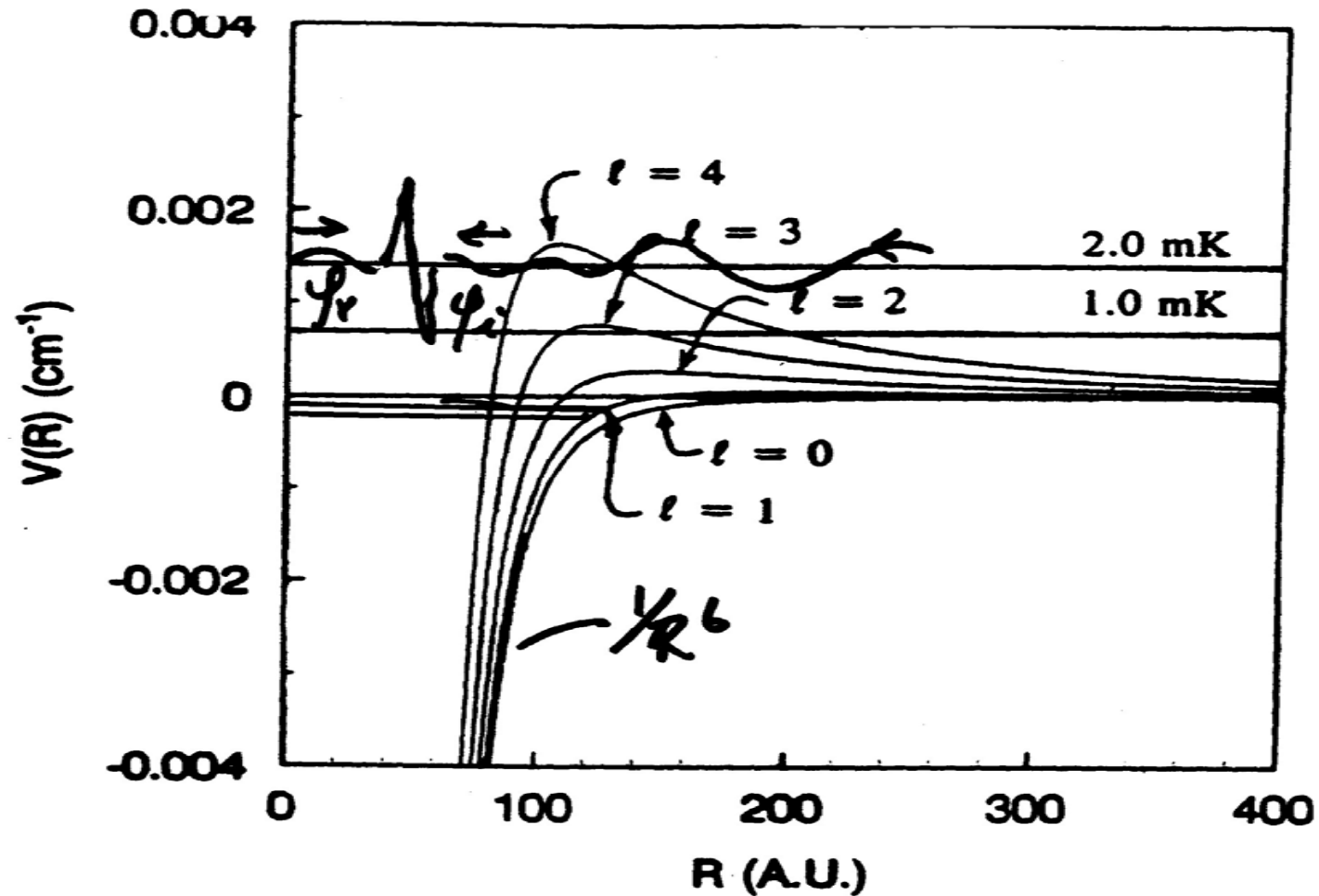


Vibrational level of 0_g^- state at 5.9 cm^{-1} below $5^2\text{S}_{1/2}+5^2\text{p}_{1/2}$



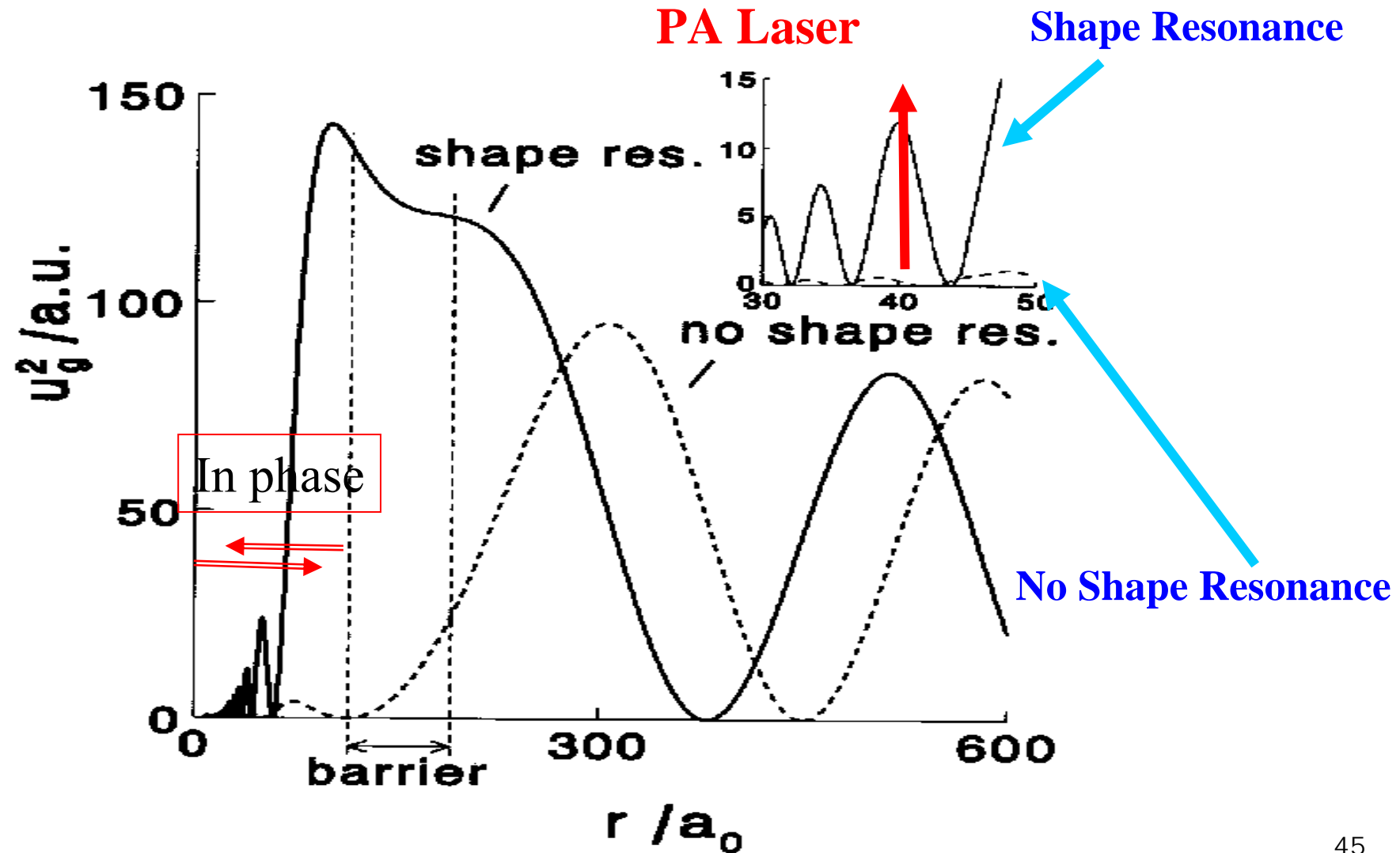
Shape Resonance

Rb₂ Ground State Potentials at Long Range



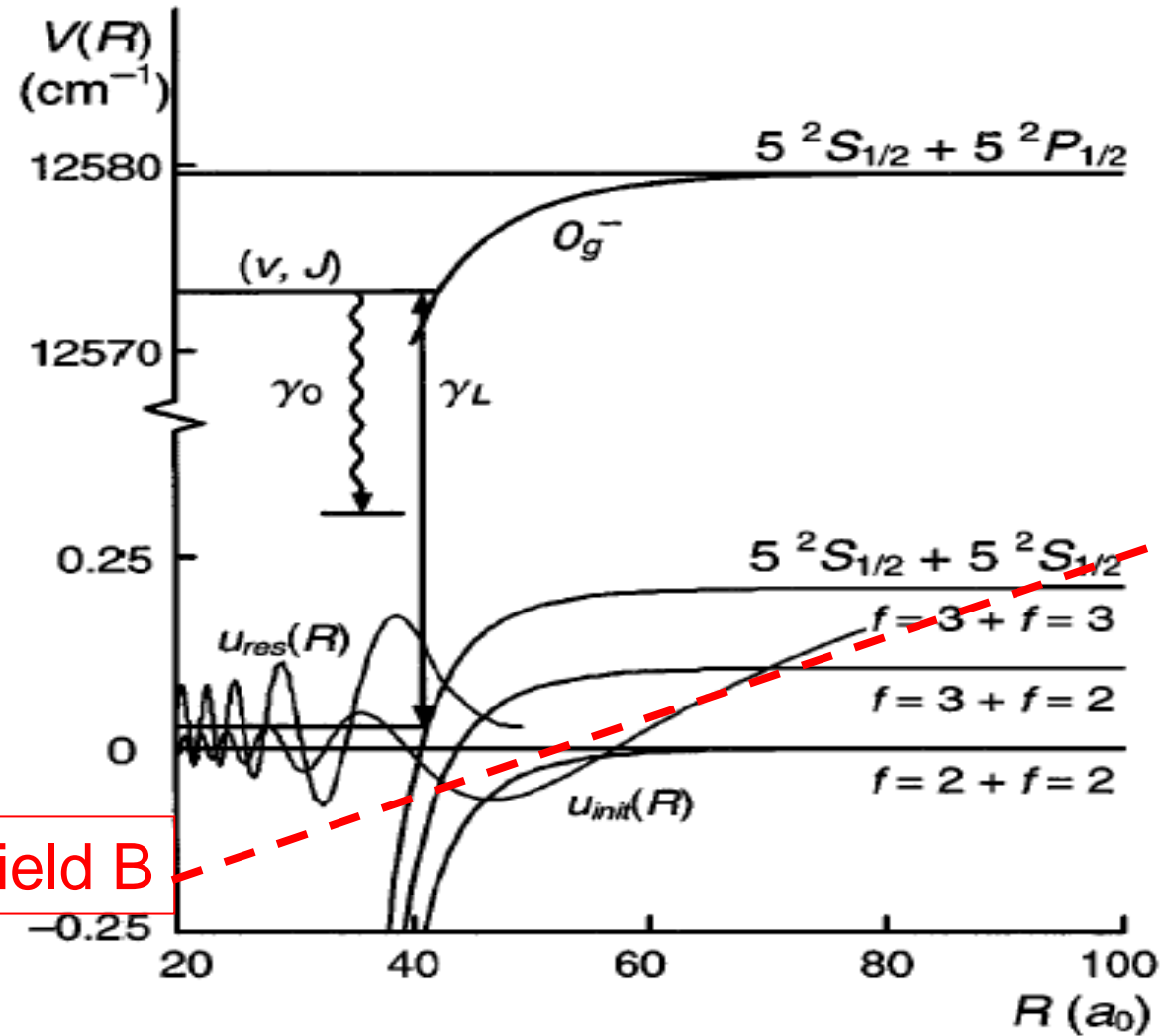


Shape Resonance





Feshbach Resonance



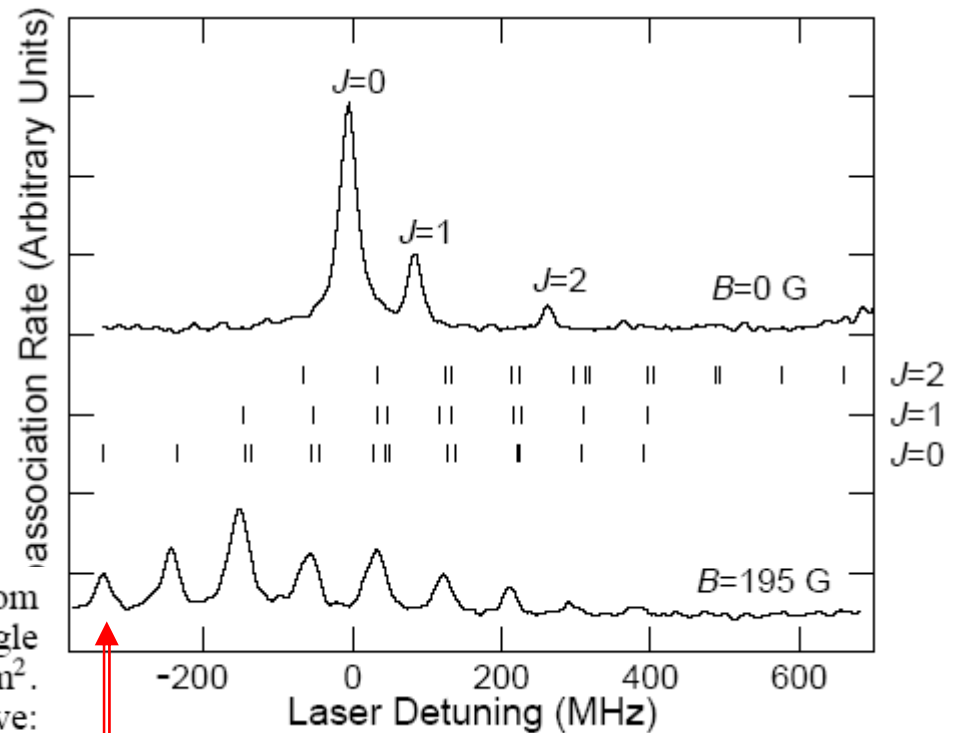
Magnetic Field B



Feshbach Resonance

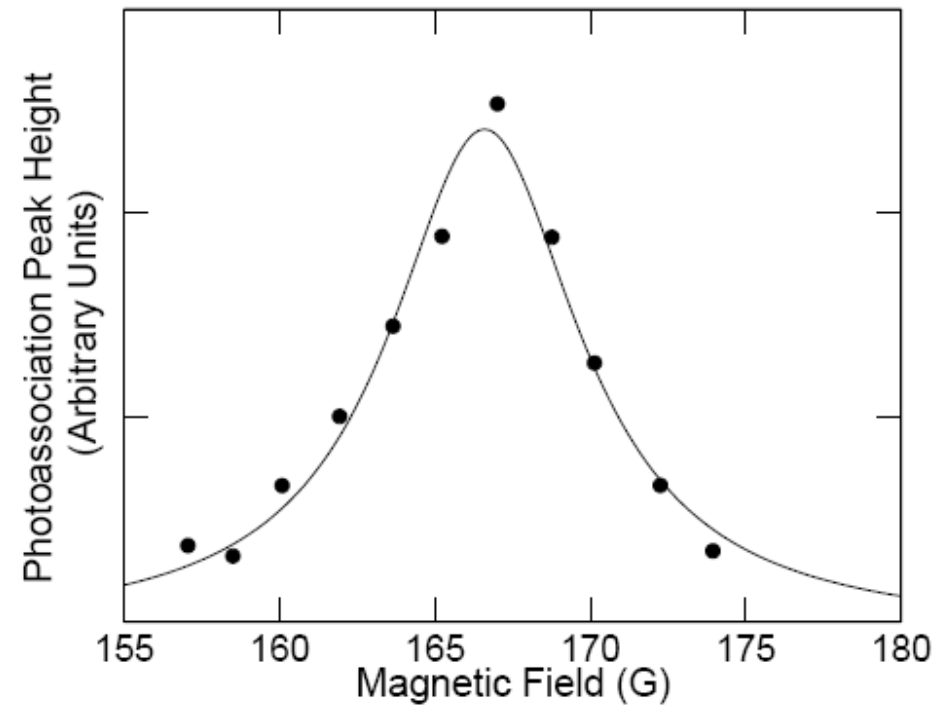
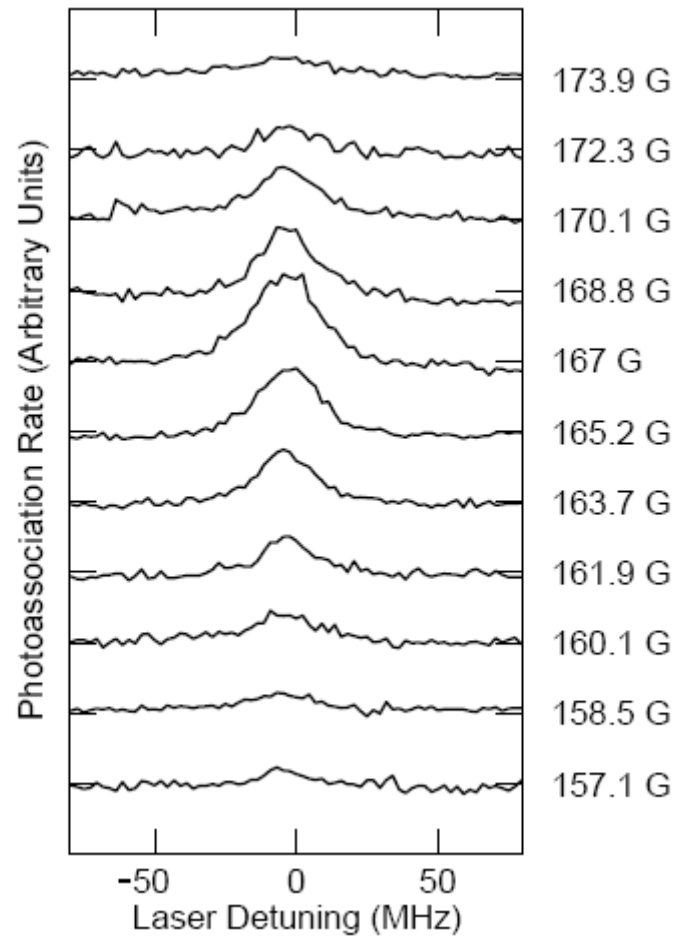
Cold collisions

FIG. 2. $^{85}\text{Rb}_2$ photoassociation spectra for excitation from lower ($f = 2 + f = 2$) hyperfine state collisions to a single excited vibrational level, at a laser intensity of 20 W/cm^2 . Upper curve: spectrum at zero magnetic field. Lower curve: spectrum at a magnetic field of 195 G. Each of the zero field components splits into 10 or 15 distinct components due to Zeeman splitting of the ground state atoms; calculated splittings are shown by the vertical dashed marks. The successive peaks in the lower spectrum correspond mainly to $J = 0$, and (from left) $M_F = -4, -3, -2, -1, 0, 1$, and 2.





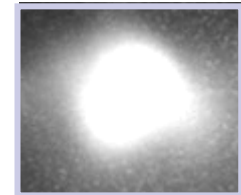
Feshbach Resonance



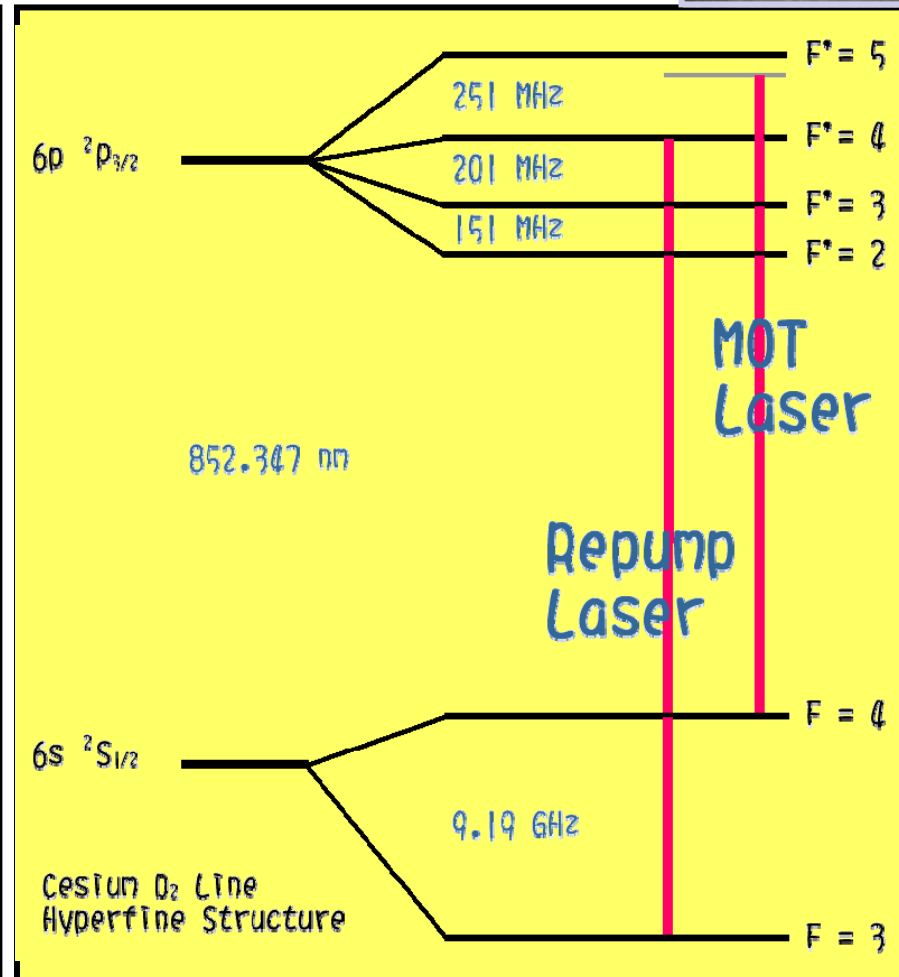
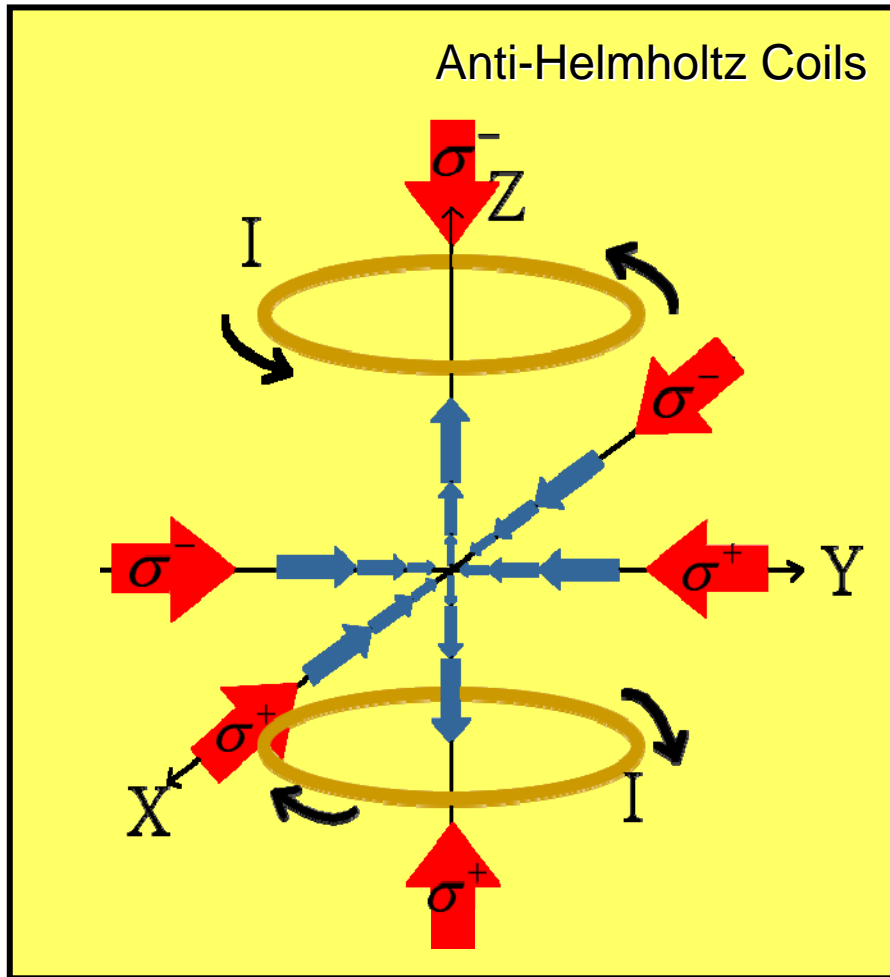


Quantum Interference in Cold Cs

Electromagnetically Induced Transparency
EIT



Magneto-Optical Trap of Cesium atoms

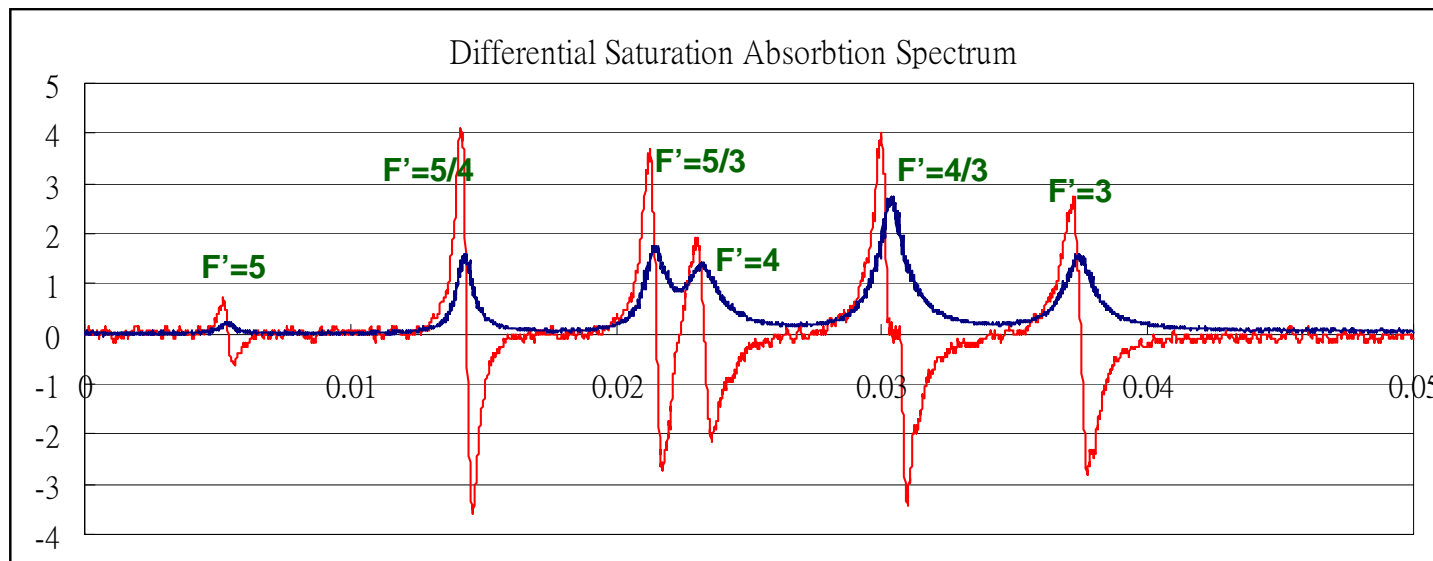




High-Precision and High-Resolution Laser Spectroscopy on Magneto-Optical Trap of Cesium Atoms

Atom number 4×10^9 , Cloud size 5 mm, Density $5 \times 10^{10}/\text{cm}^3$

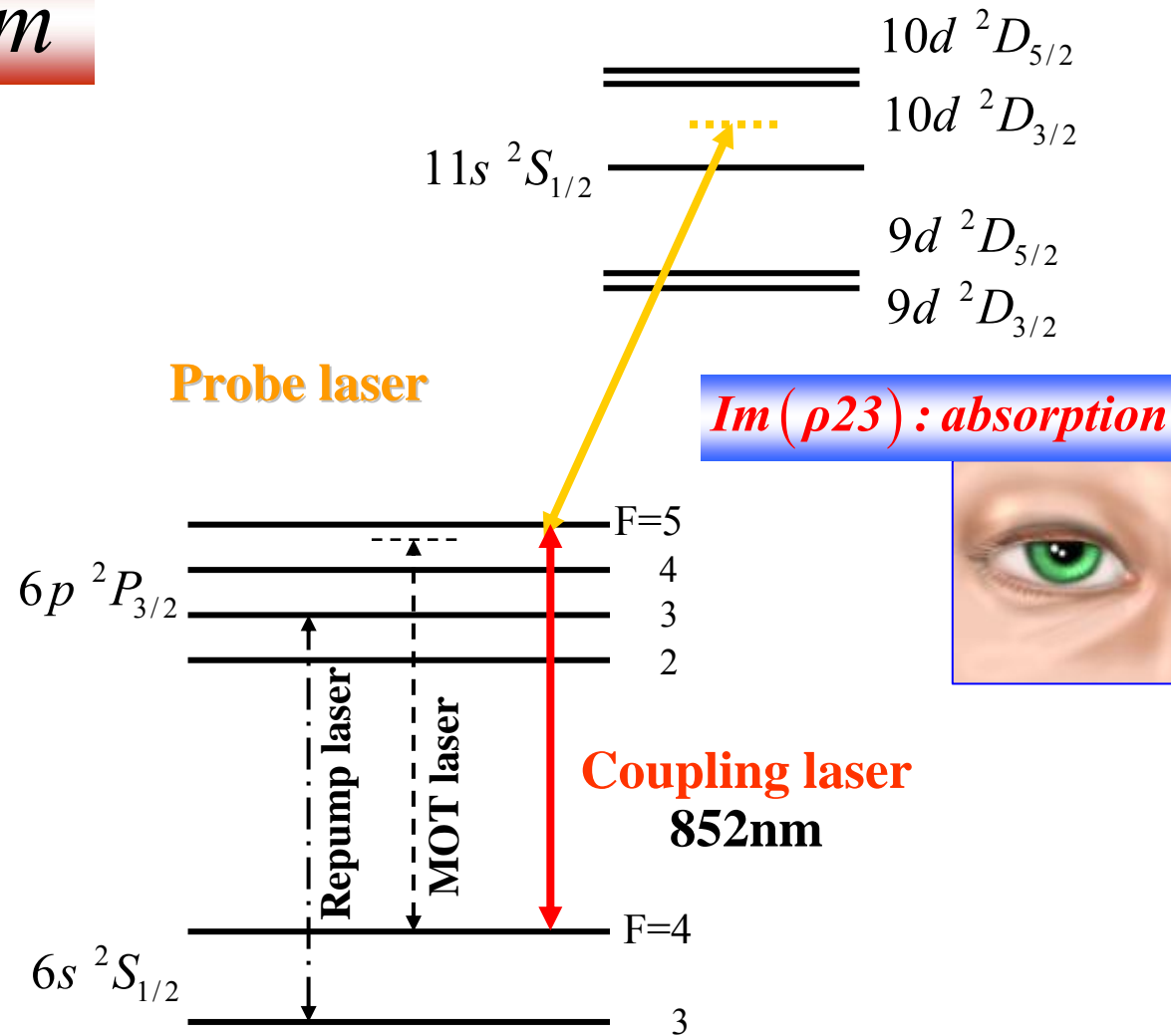
Atom temperature (Time of flight) : $100 \mu\text{K}$





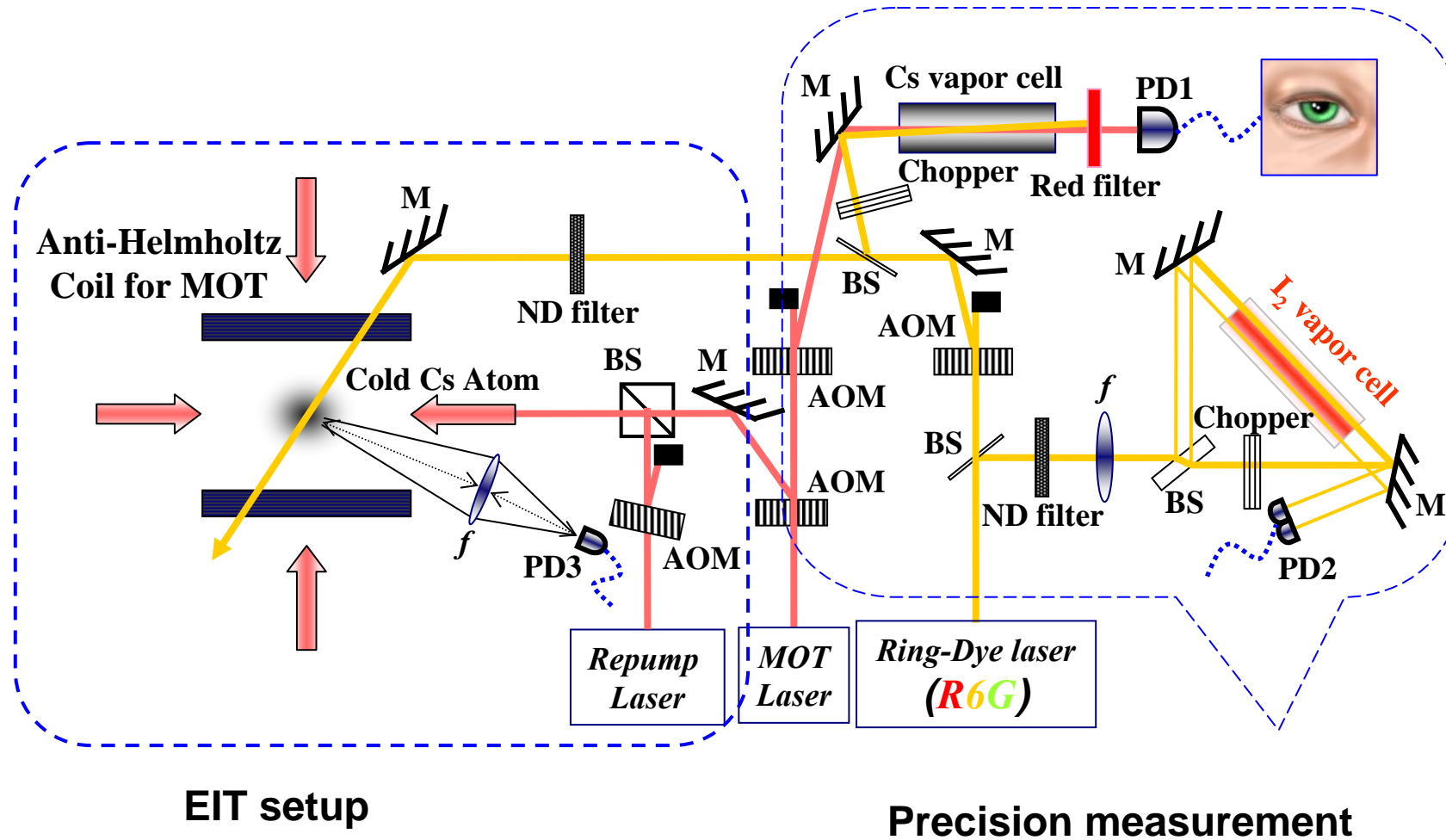
Energy level diagram

¹³³Cesium





Experimental Setup





Visible Cs MOT

Visible Cs MOT :

Probe laser transition

$$|6p \ ^2P_{3/2}\rangle \rightarrow |10d \ ^2D_{5/2}\rangle$$

563.6nm

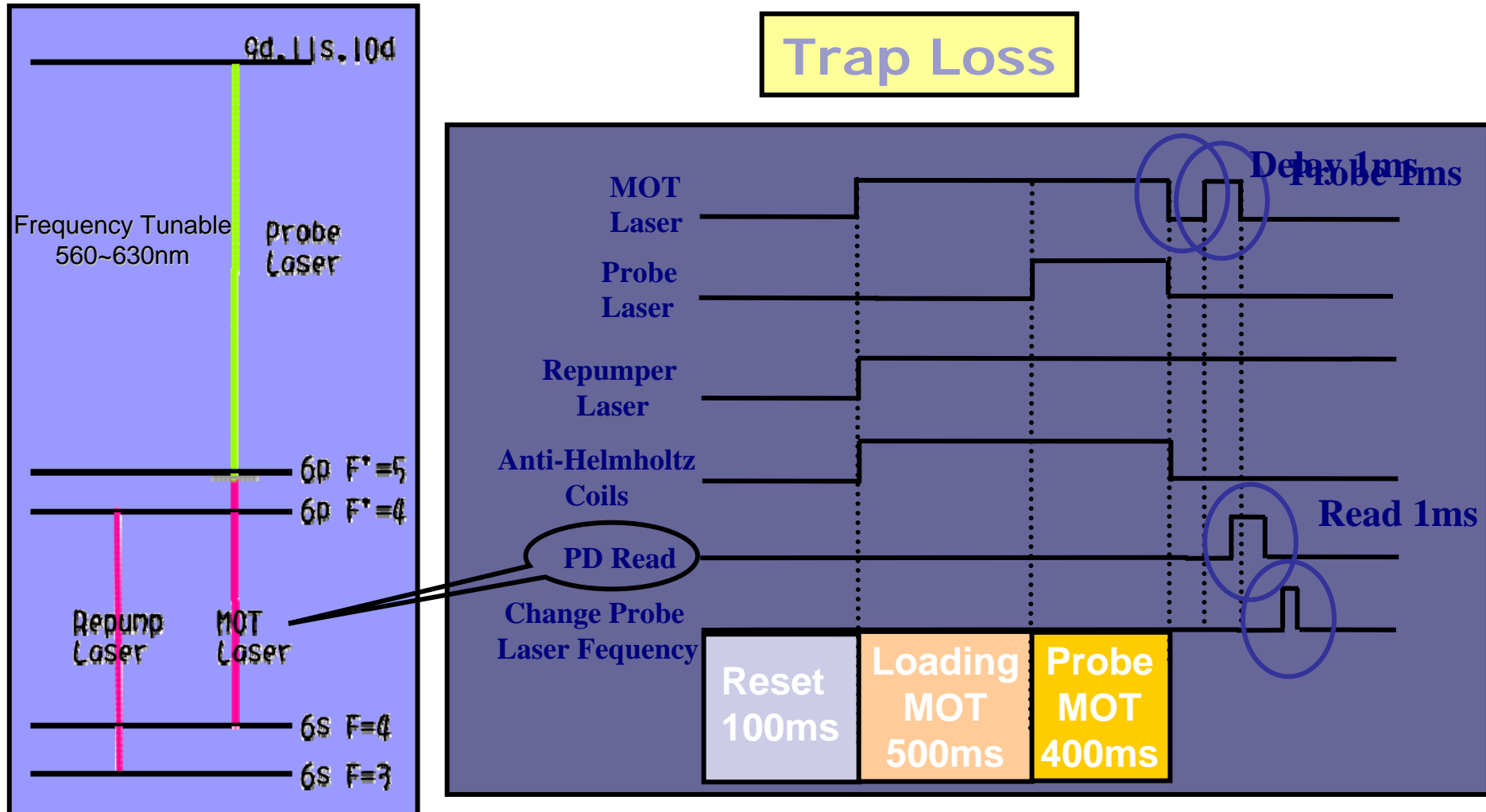
Atom number $\sim 10^8$

Temperature $\sim 200\mu\text{K}$



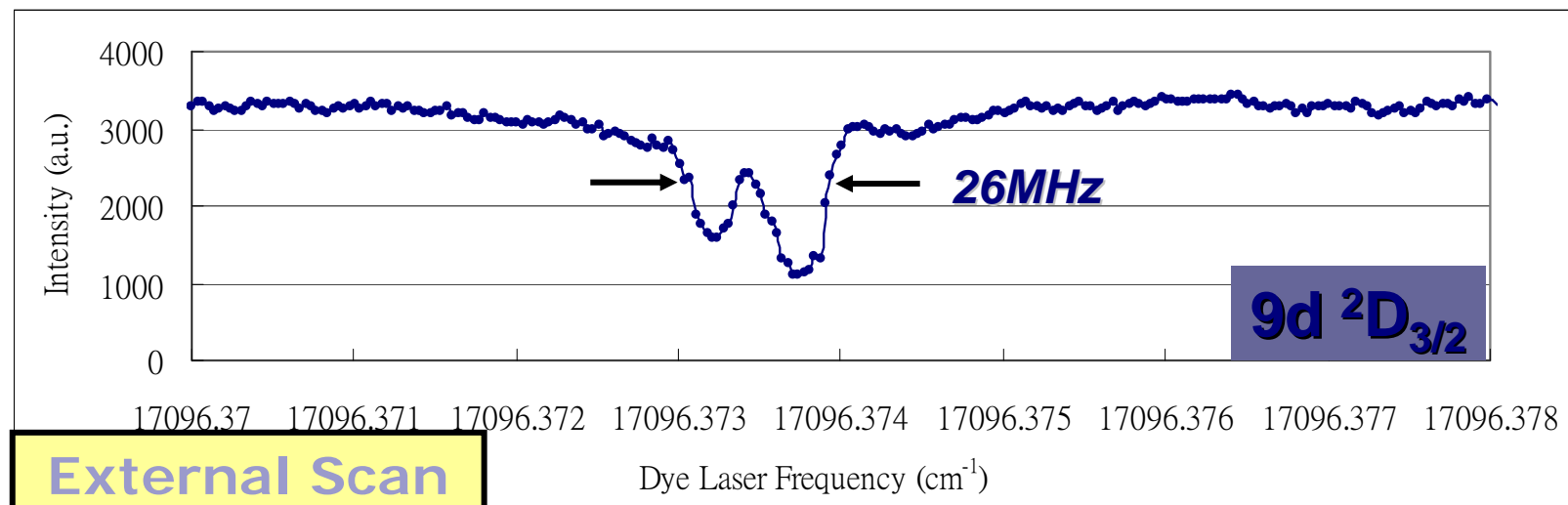
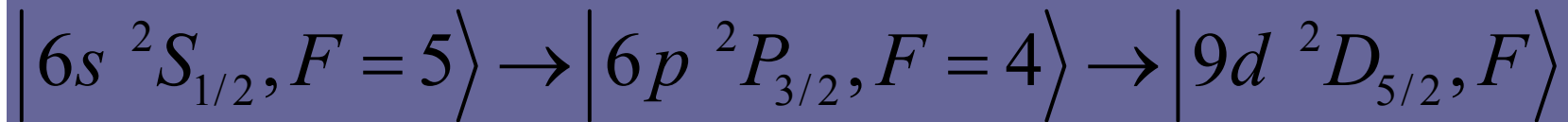


Data Acquisition by External Scan



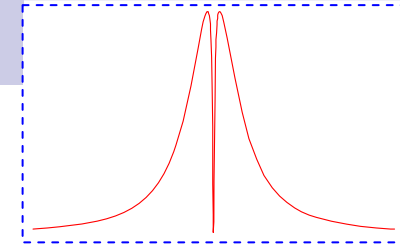


Atomic Transitions



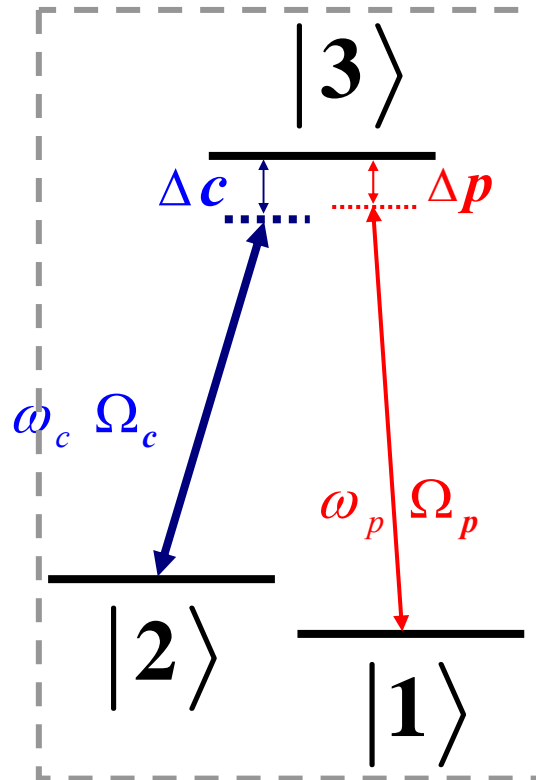


Electromagnetically Induced Transparency

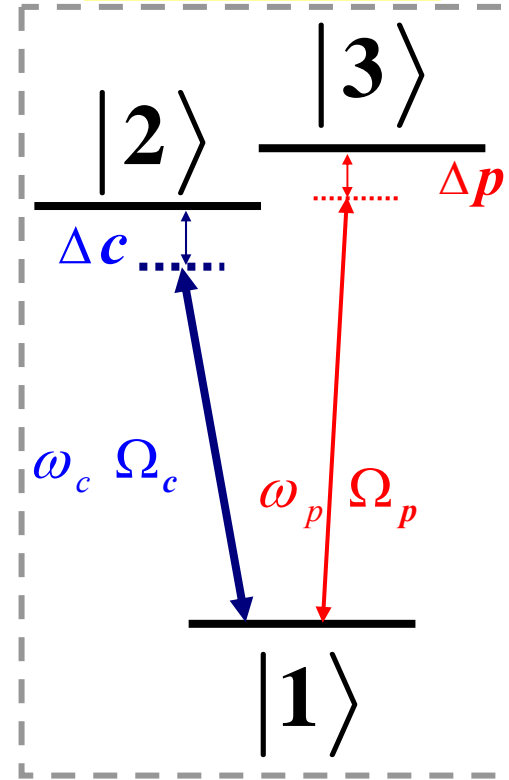


Quantum Interference

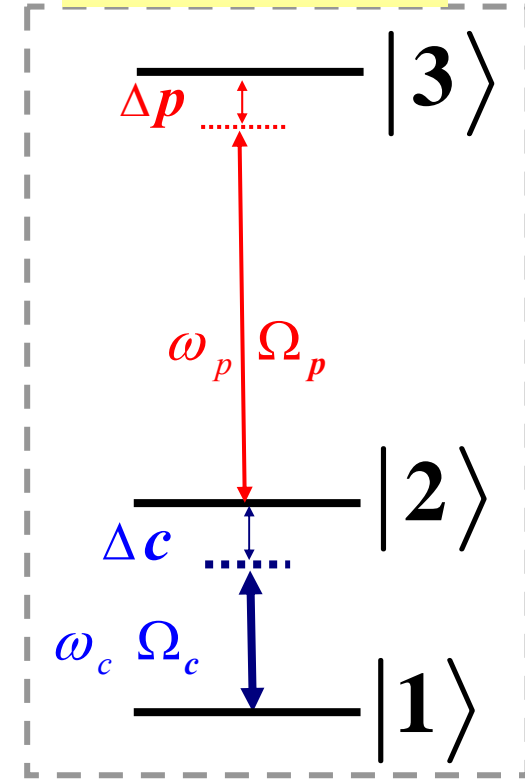
Λ - type



V - type



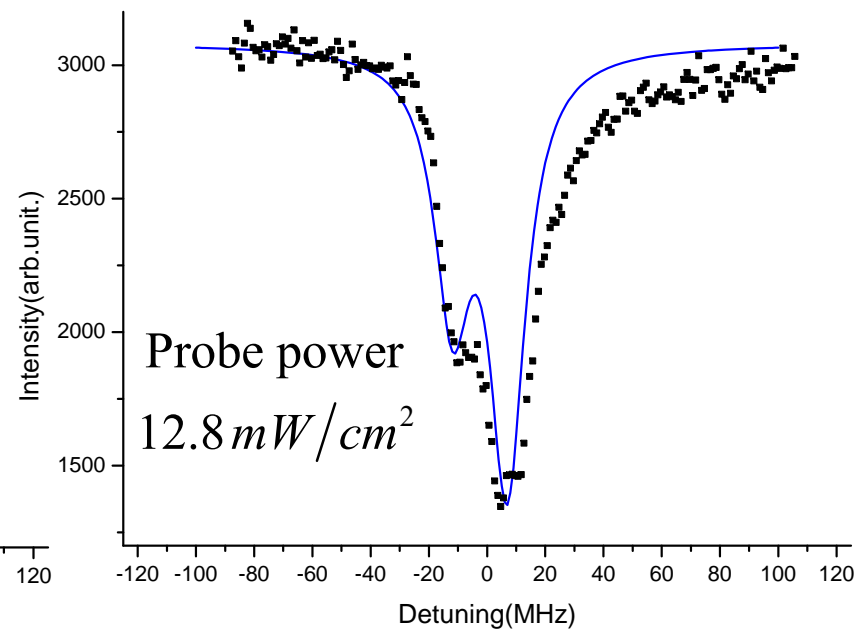
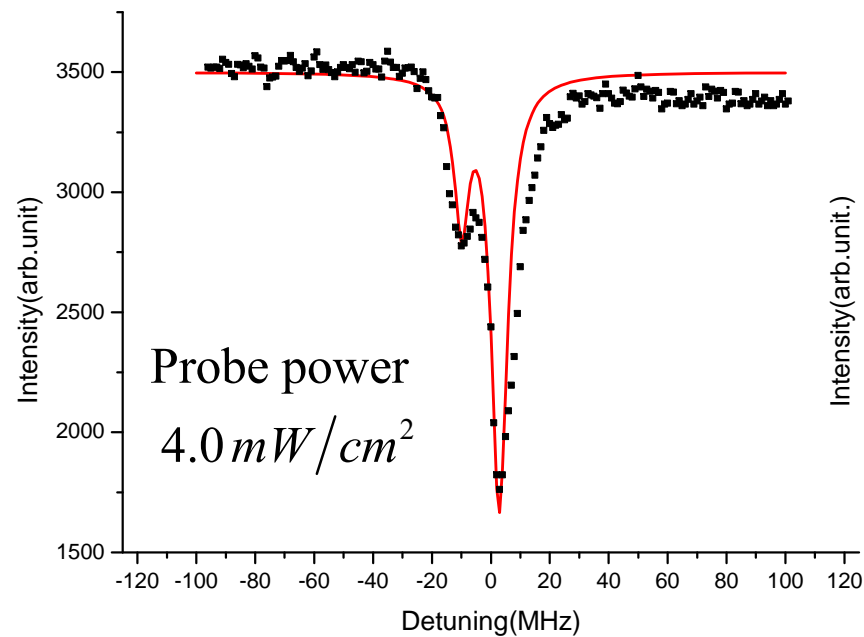
Cascade





Numerical Simulation

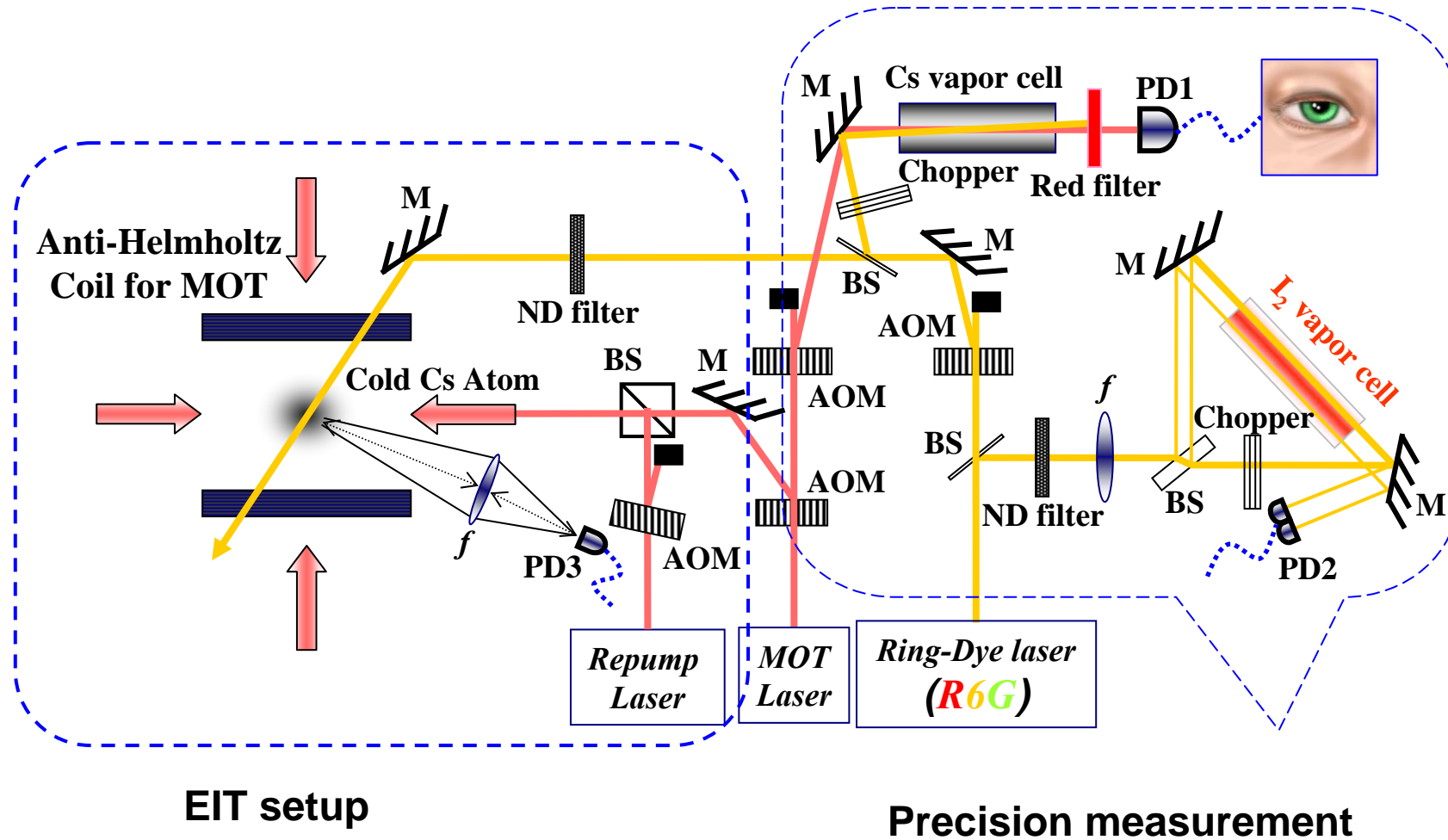
$$|6s^2S_{1/2}, F=5\rangle \rightarrow |6p^2P_{3/2}, F=4\rangle \rightarrow |11s^2S_{1/2}, F\rangle, \quad w_c: 10\text{mW/cm}^2$$



$$\gamma_2=5.2\text{MHz}, \gamma_3=2.5\text{MHz}, w_c=3\text{MHz}, w_p=1\text{MHz}, \Delta_c=-10\text{MHz}, \Omega_c=20\text{MHz}$$



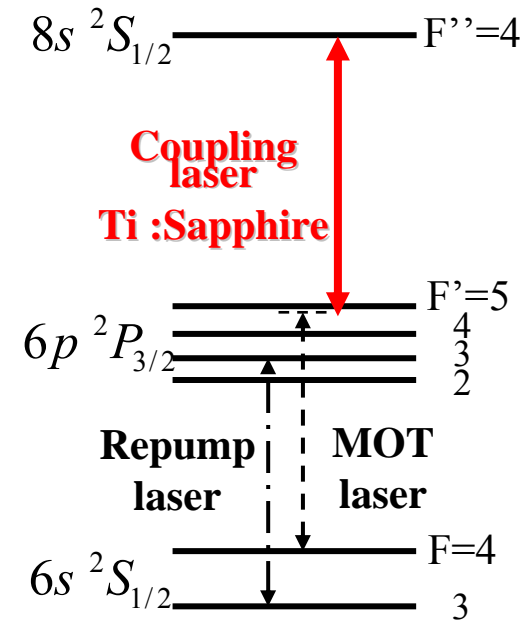
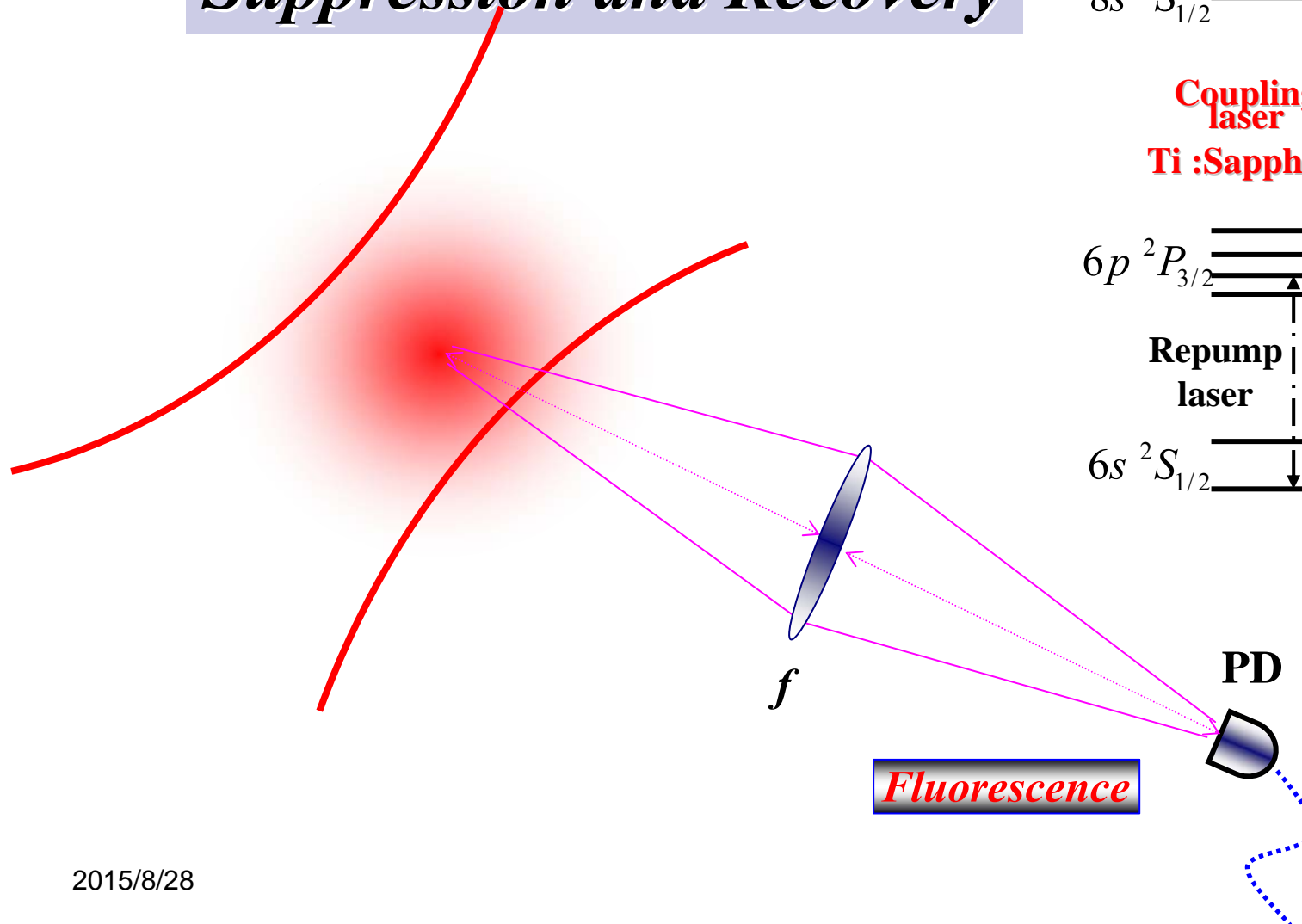
Experimental Setup





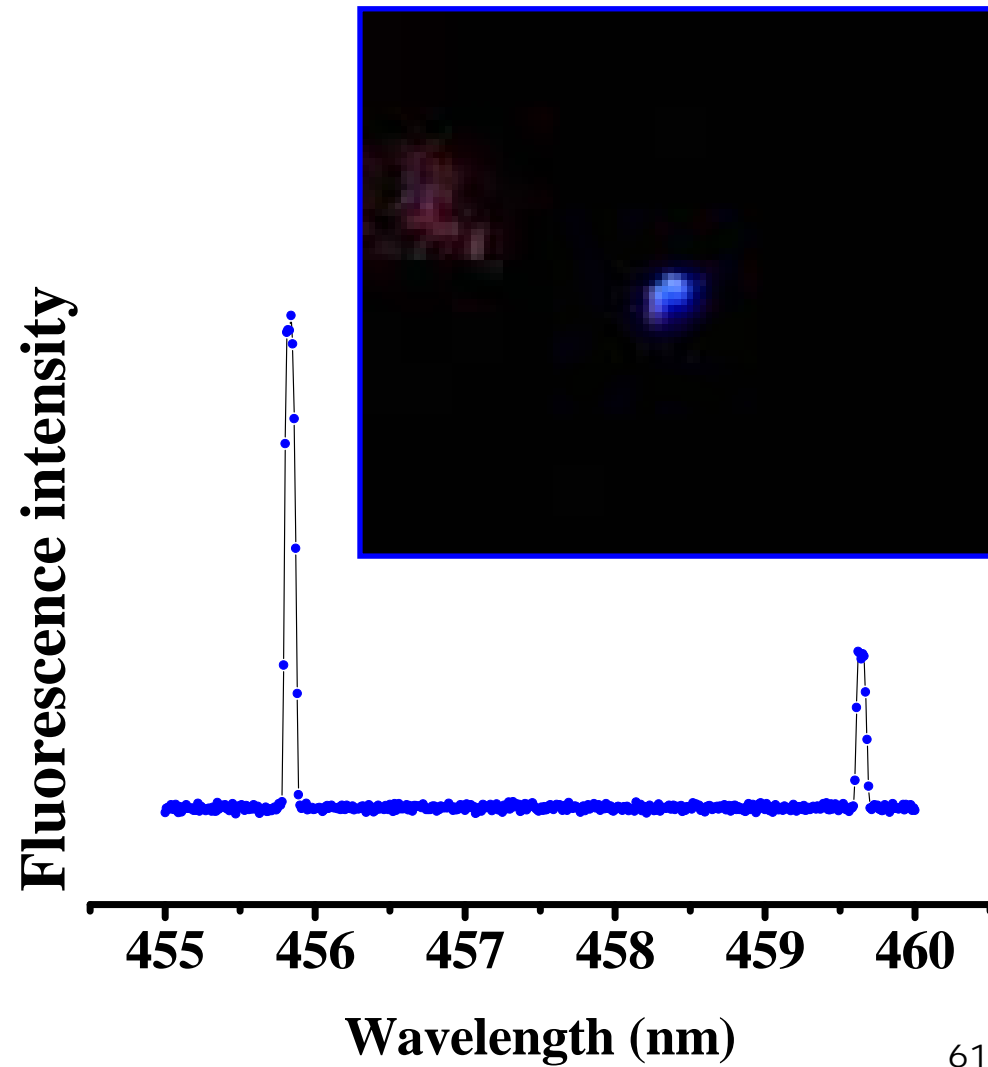
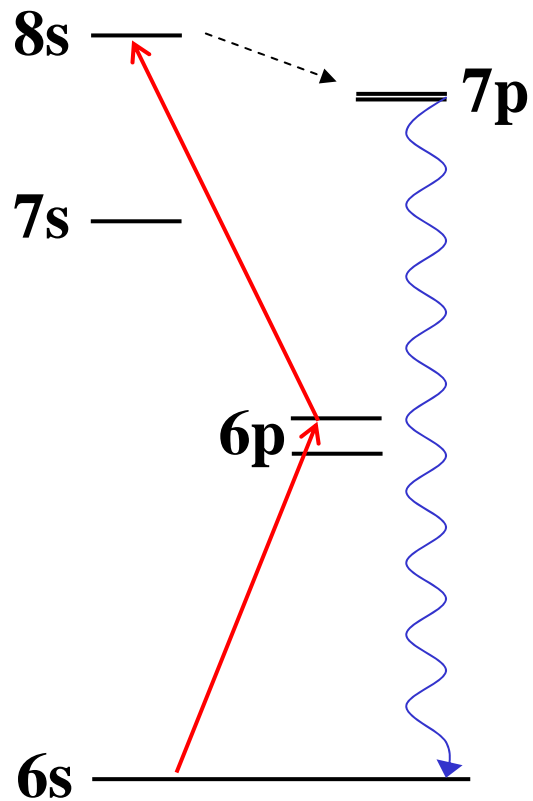
Quantum Decoherence

Suppression and Recovery



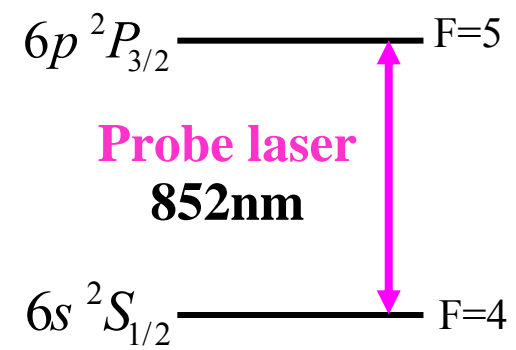
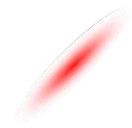


Decay fluorescence



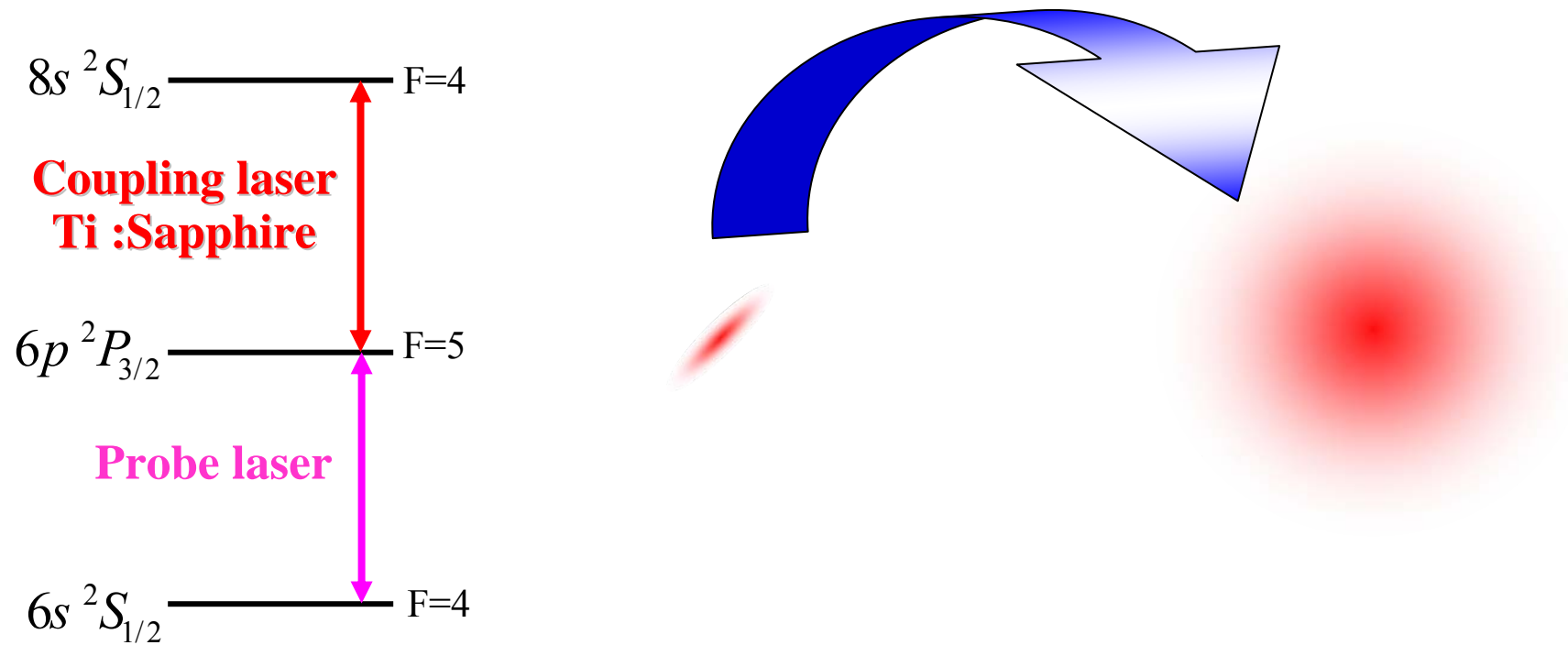


Suppression



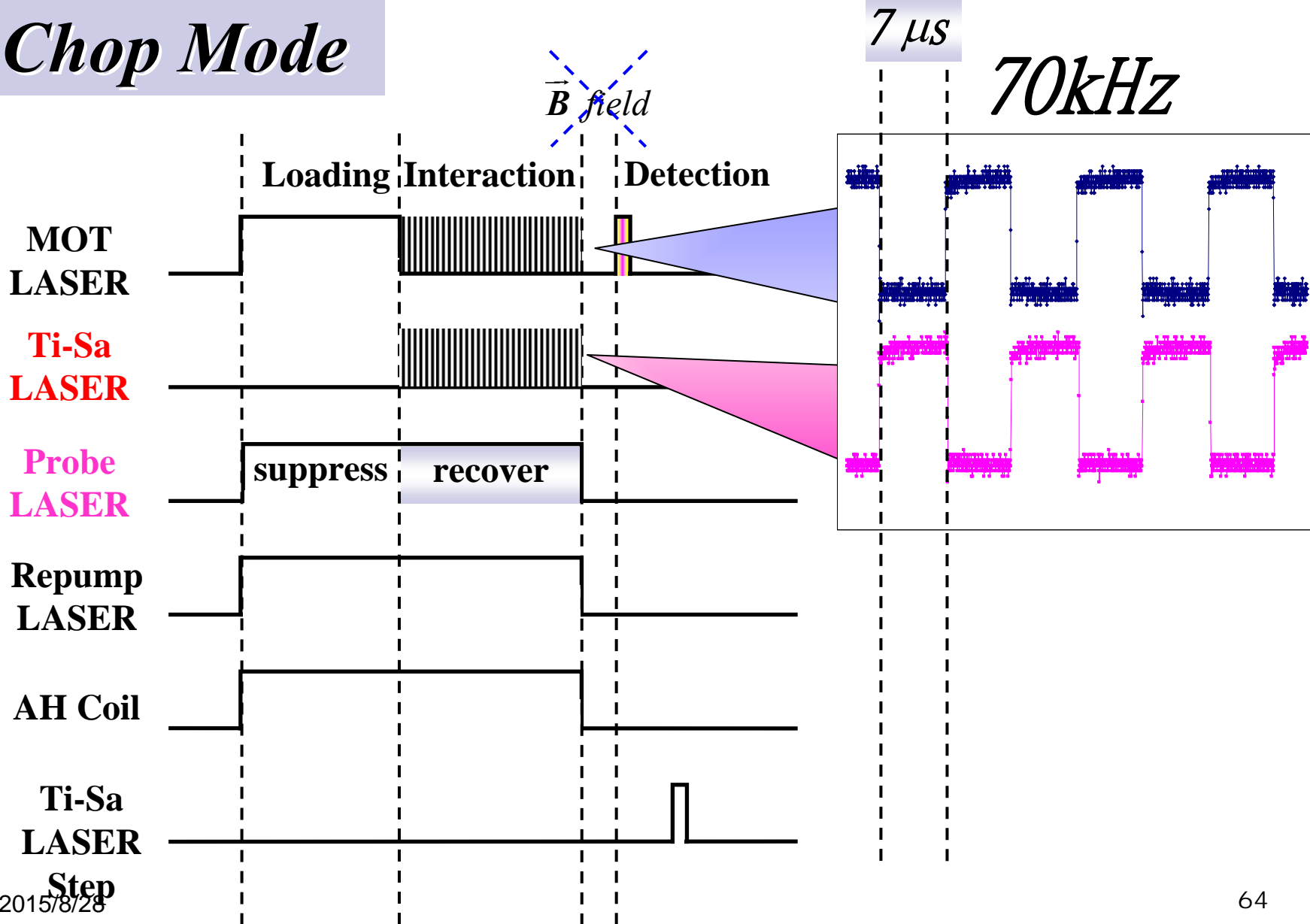


Suppression & Recovery



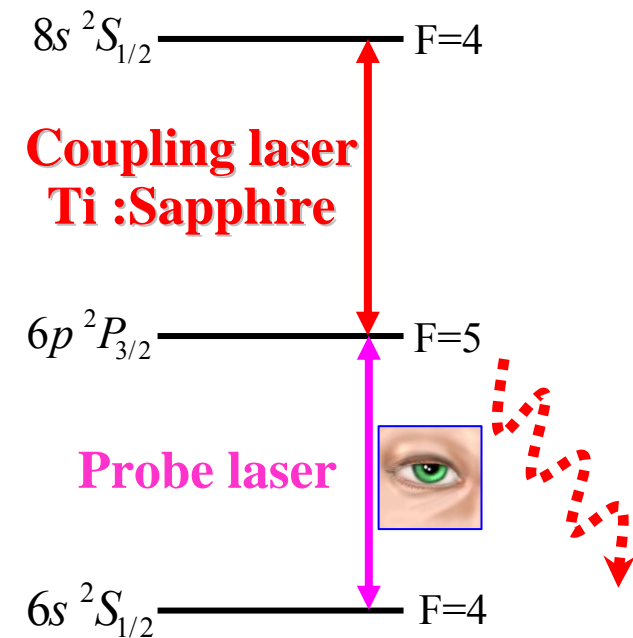
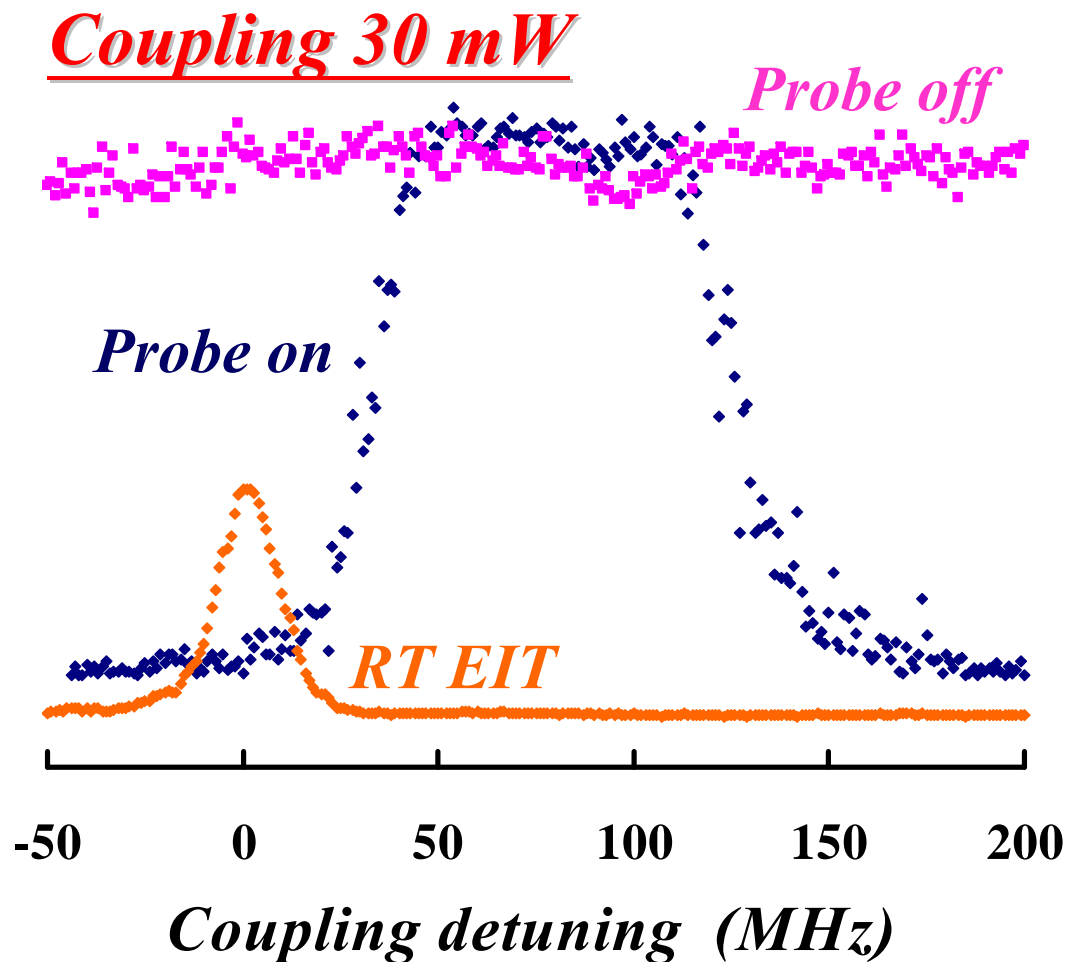


Chop Mode



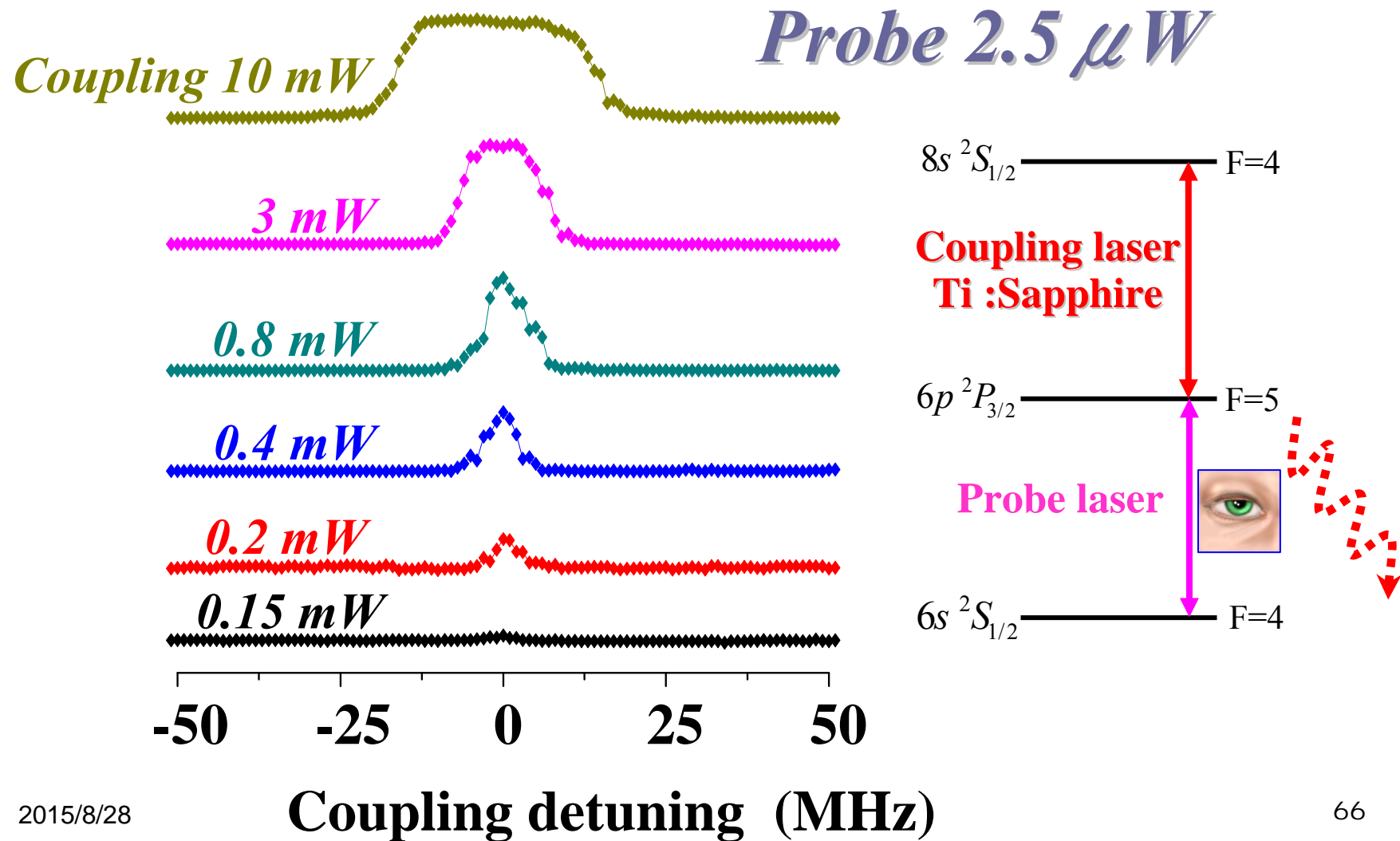


Suppression & Recovery



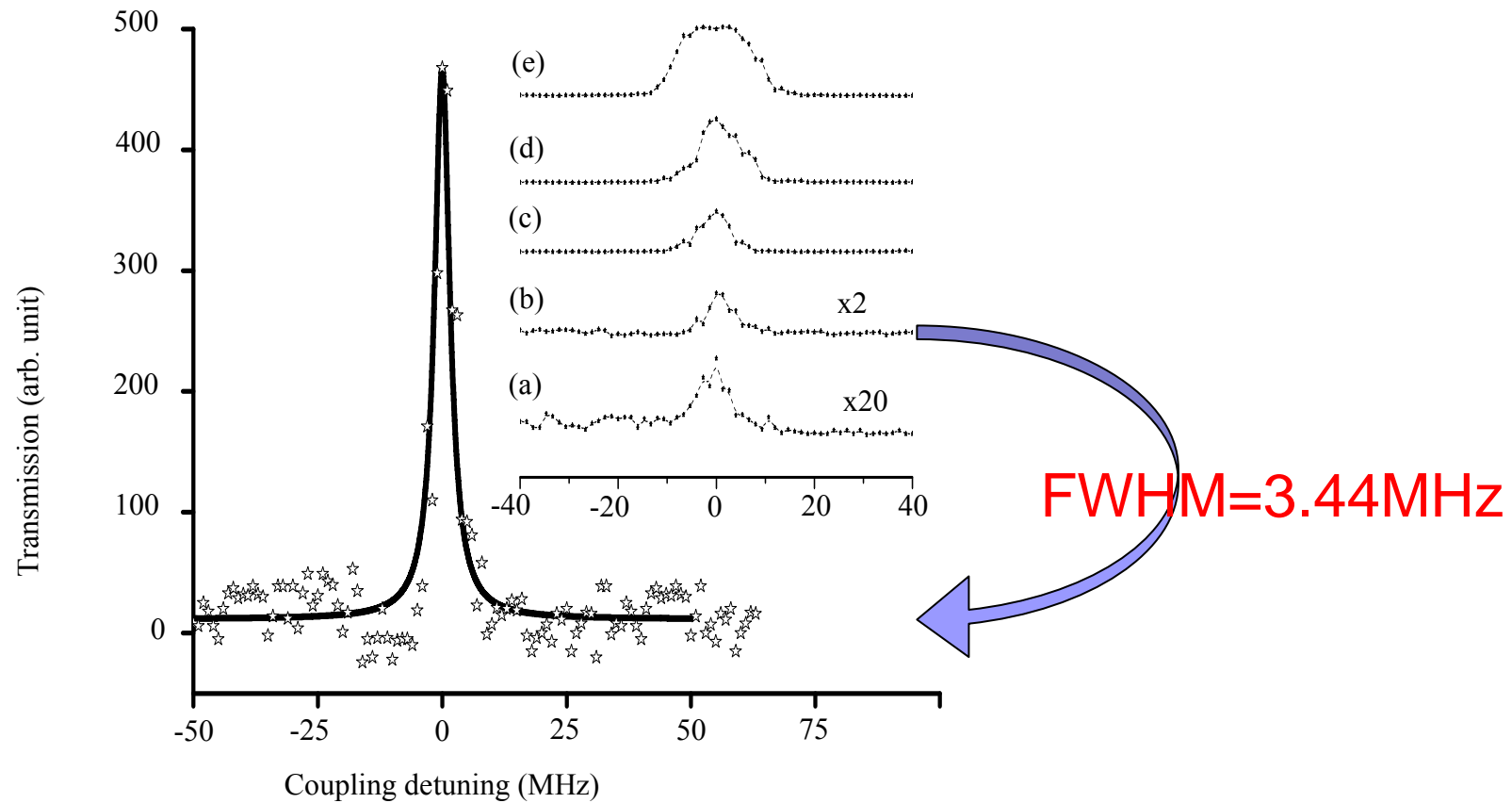


Power dependence



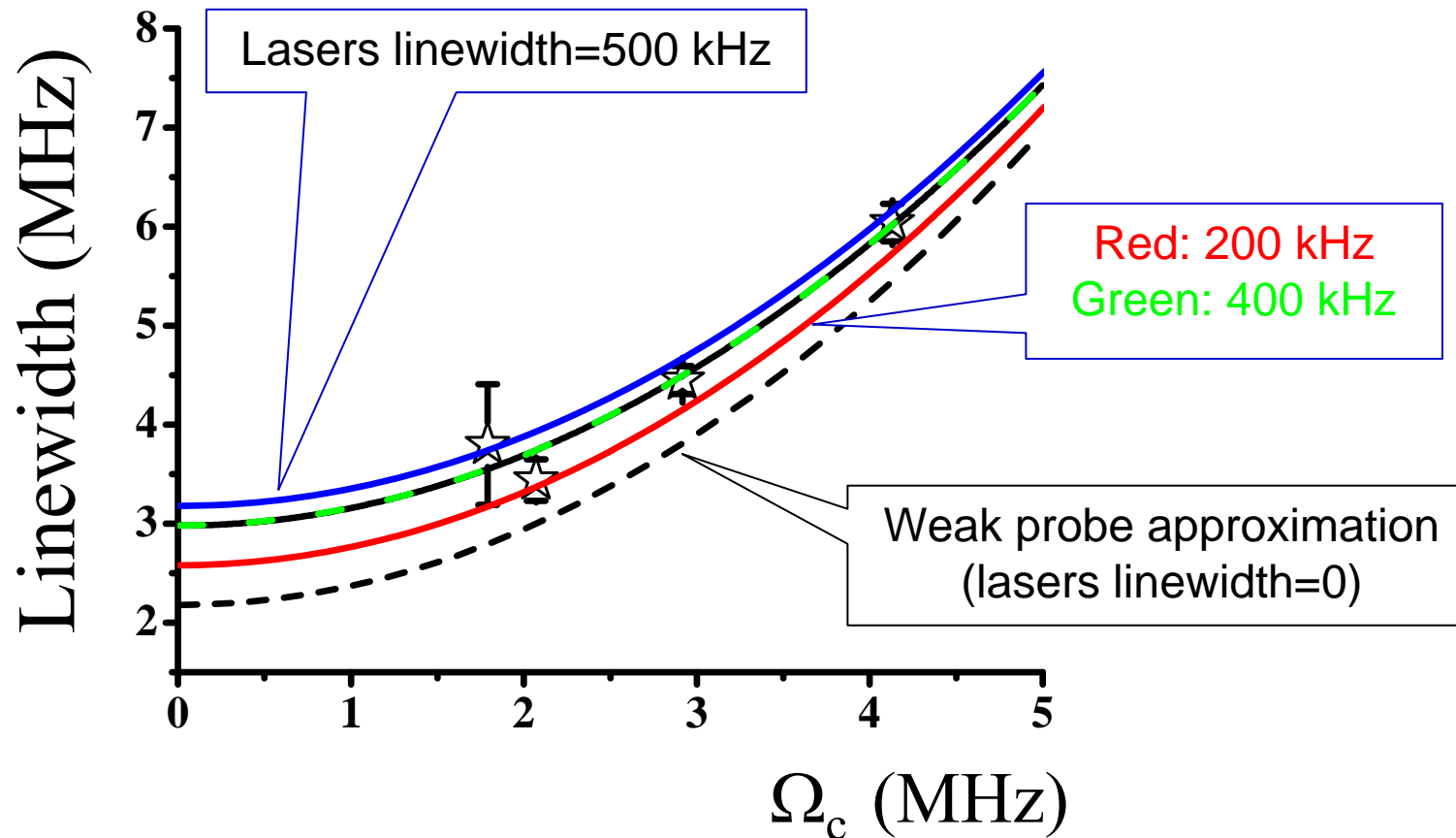


Power dependence





Linewidth vs. coupling Rabi frequency



Laser linewidth is a de-coherence source.

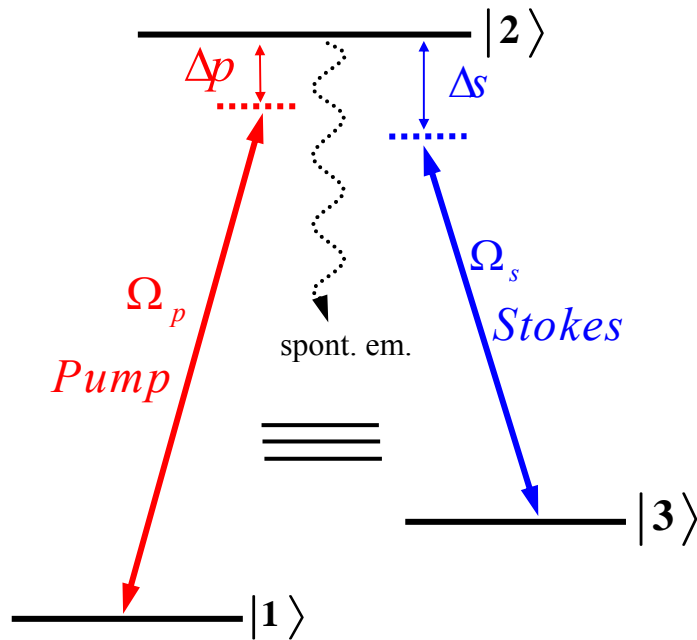
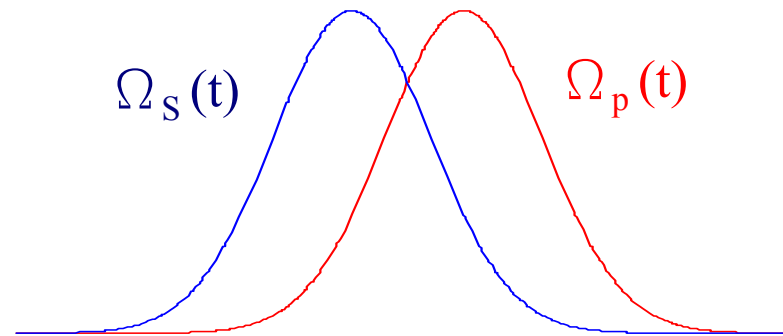


Quantum Interference in Cold Cs

Stimulated Raman Adiabatic Passage
(STIRAP)



STIRAP

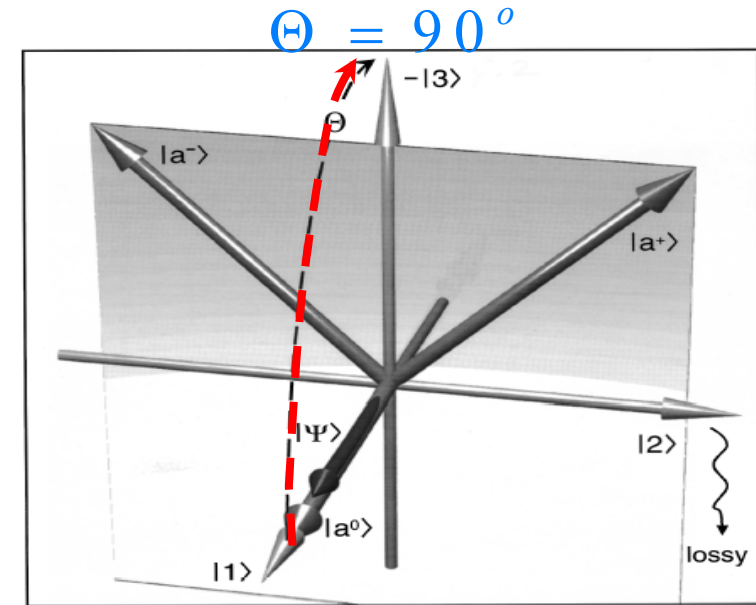
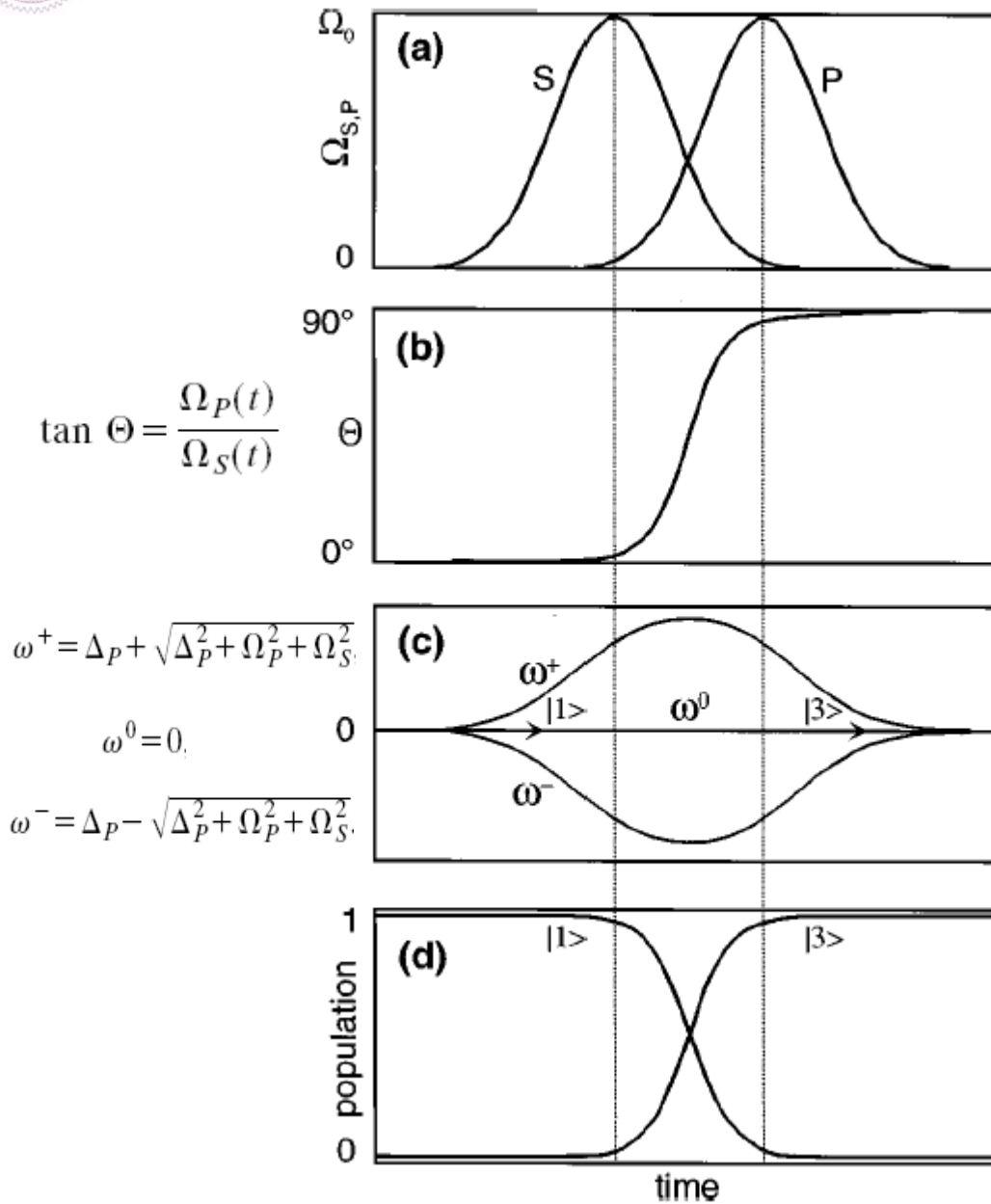


$$H(t) = \frac{\hbar}{2} \begin{bmatrix} 0 & \Omega_P(t) & 0 \\ \Omega_P(t) & 2\Delta_P & \Omega_S(t) \\ 0 & \Omega_S(t) & 2(\Delta_P - \Delta_S) \end{bmatrix}$$

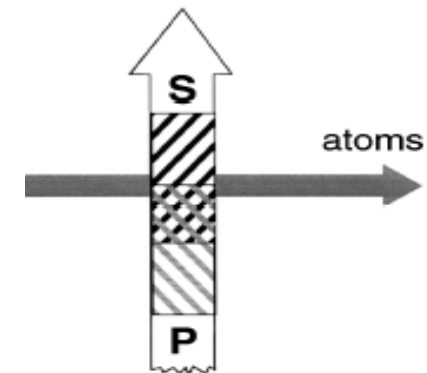
$$\left\{ \begin{aligned} |a^+\rangle &= \sin \Theta \sin \Phi |1\rangle + \cos \Phi |2\rangle + \cos \Theta \sin \Phi |3\rangle, \\ |a^0\rangle &= \cos \Theta |1\rangle - \sin \Theta |3\rangle, \\ |a^-\rangle &= \sin \Theta \cos \Phi |1\rangle - \sin \Phi |2\rangle + \cos \Theta \cos \Phi |3\rangle, \end{aligned} \right.$$

$$\left\{ \begin{aligned} \omega^+ &= \Delta_P + \sqrt{\Delta_P^2 + \Omega_P^2 + \Omega_S^2}, & \omega^0 &= 0, \\ \omega^- &= \Delta_P - \sqrt{\Delta_P^2 + \Omega_P^2 + \Omega_S^2}. \end{aligned} \right. \quad \tan \Theta = \frac{\Omega_P(t)}{\Omega_S(t)}$$

mixing angle

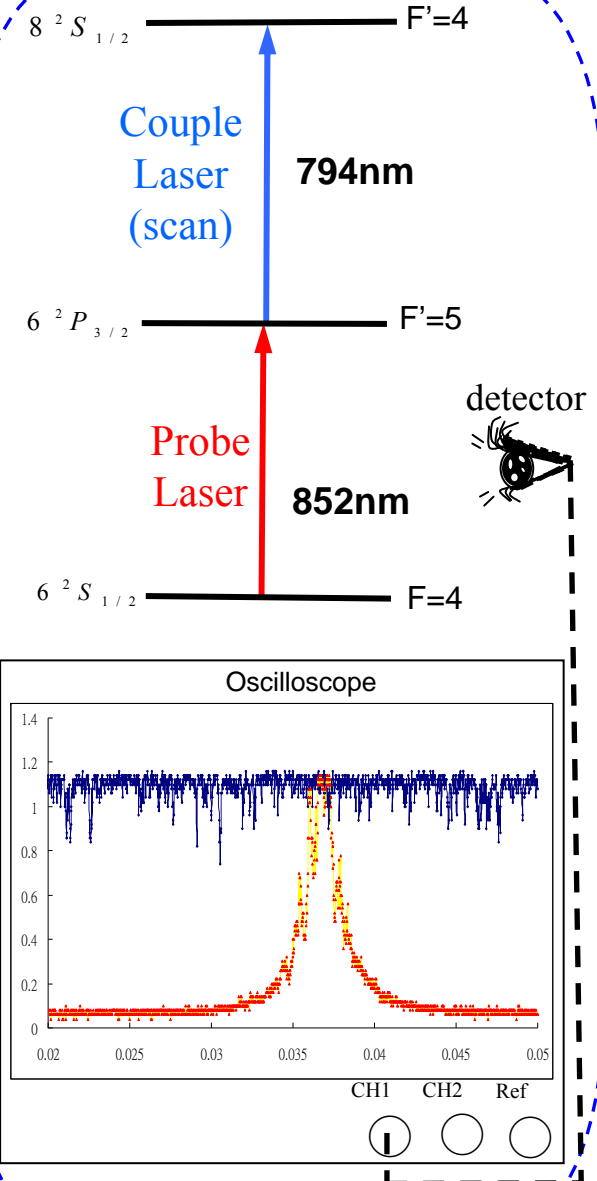
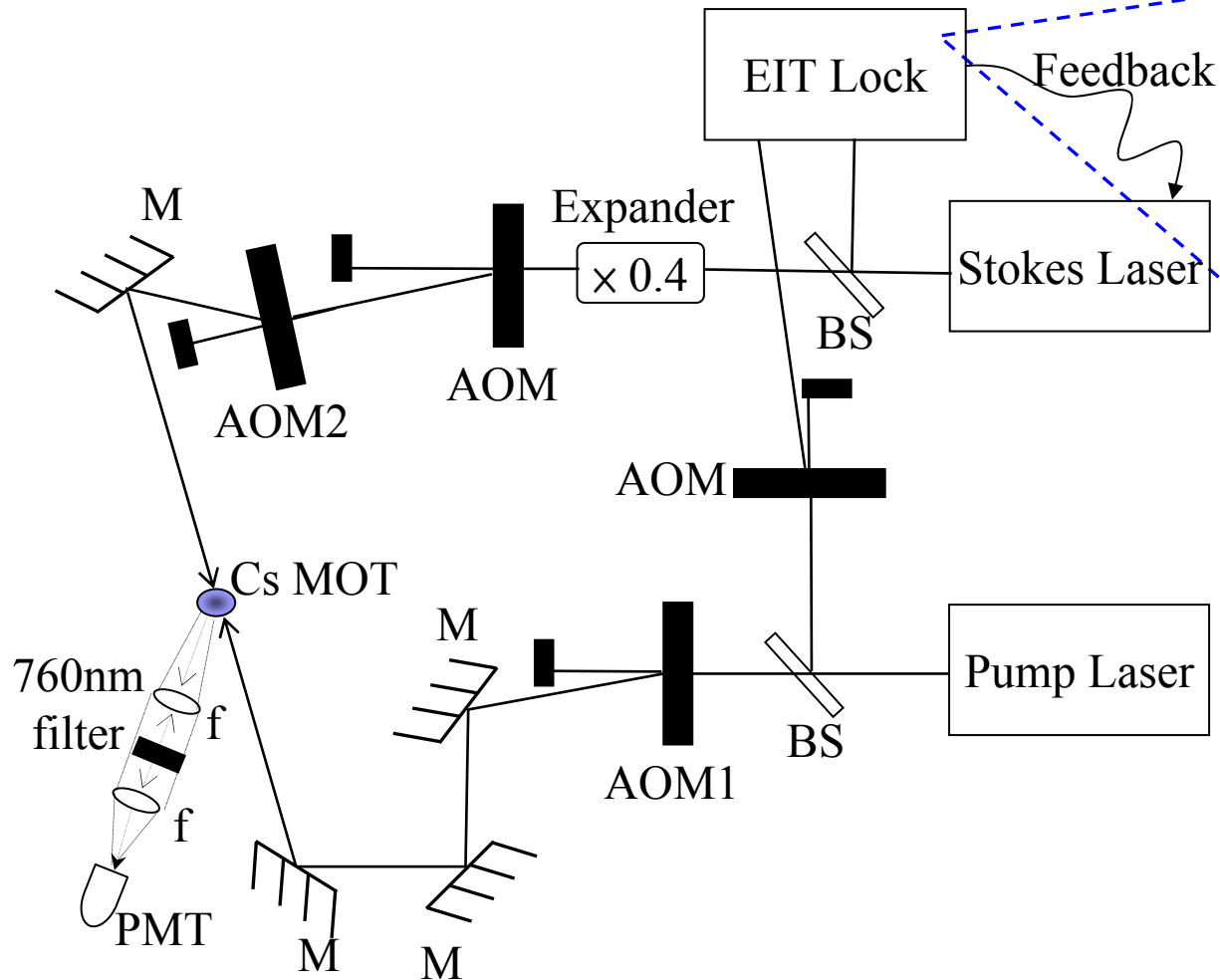


$\Theta = 0$
 pulsed



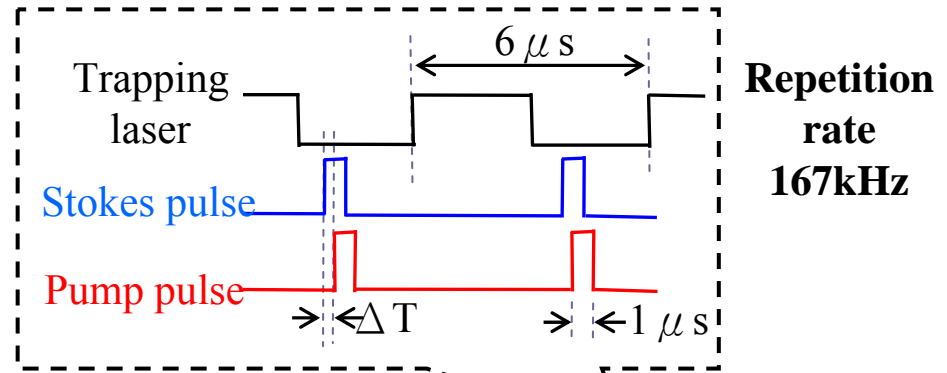
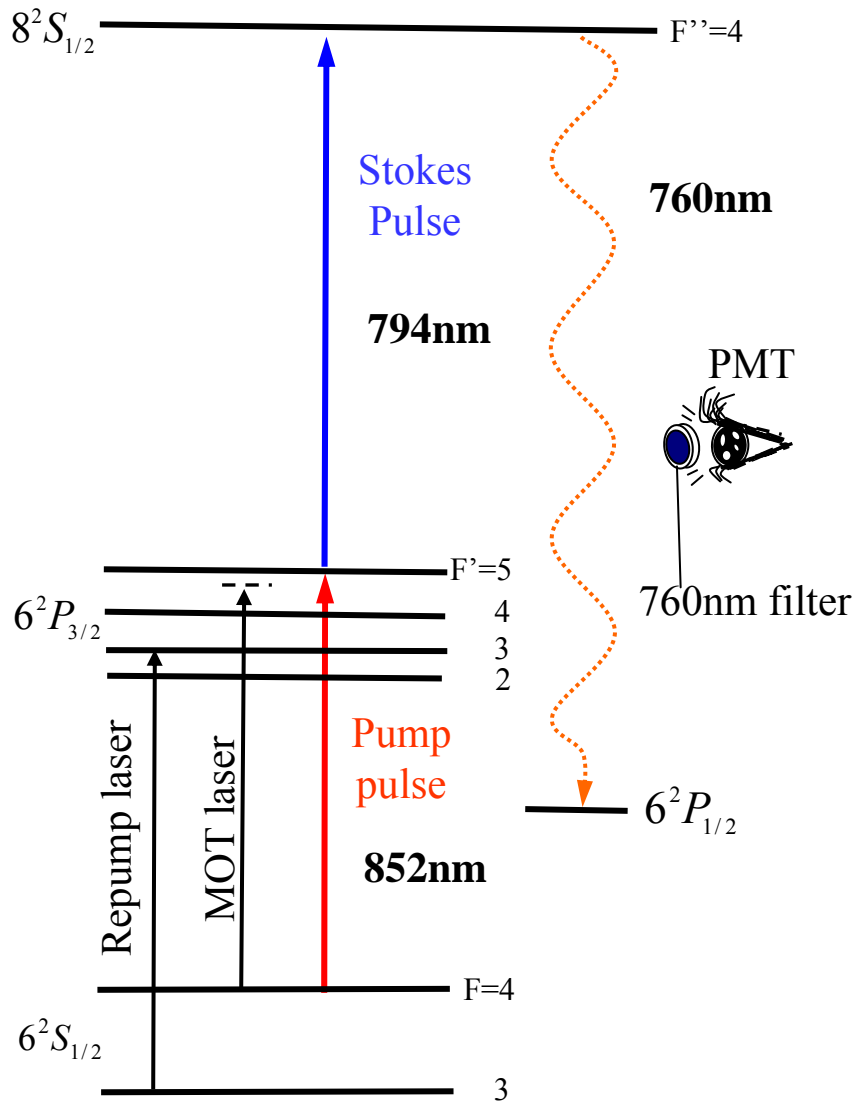


Experimental setup

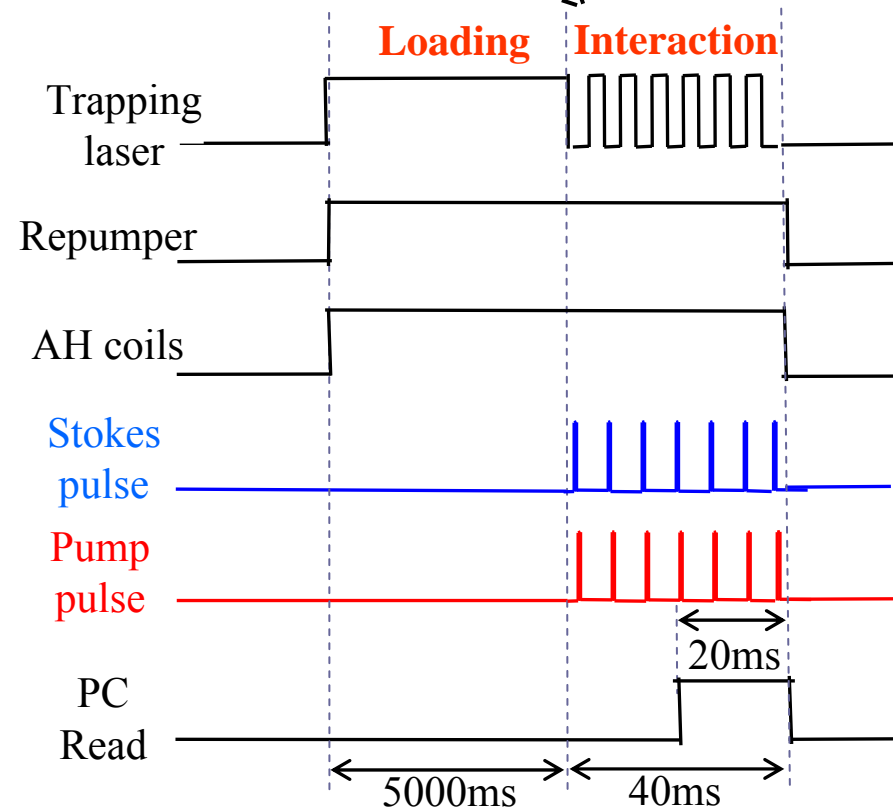




time sequence

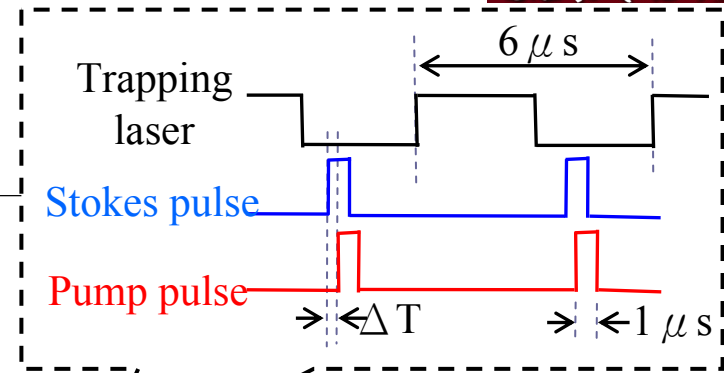
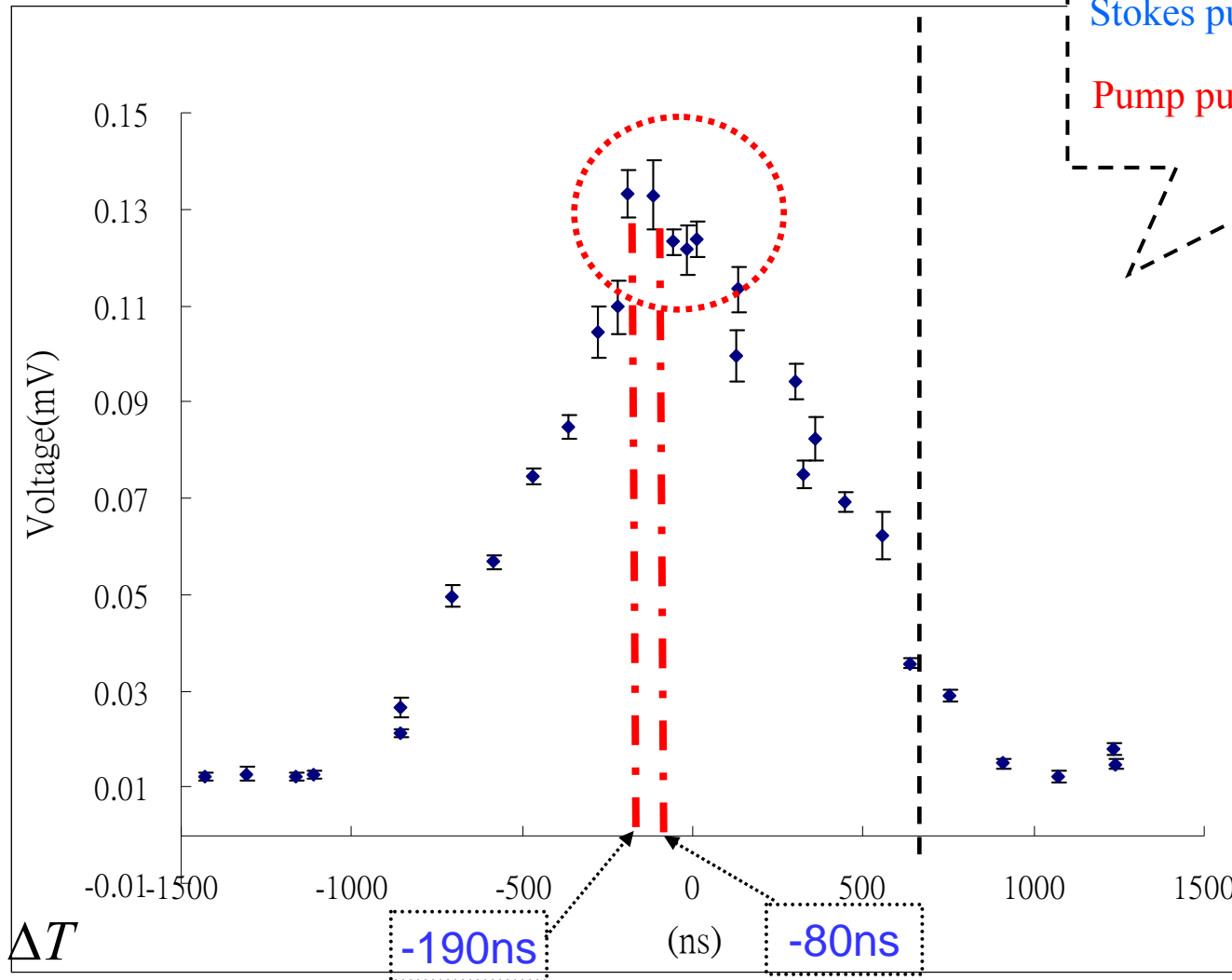


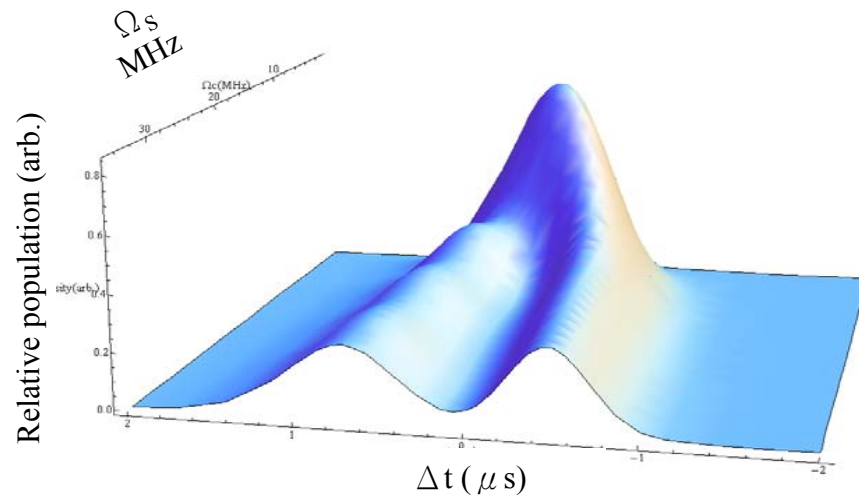
Repetition rate
167kHz



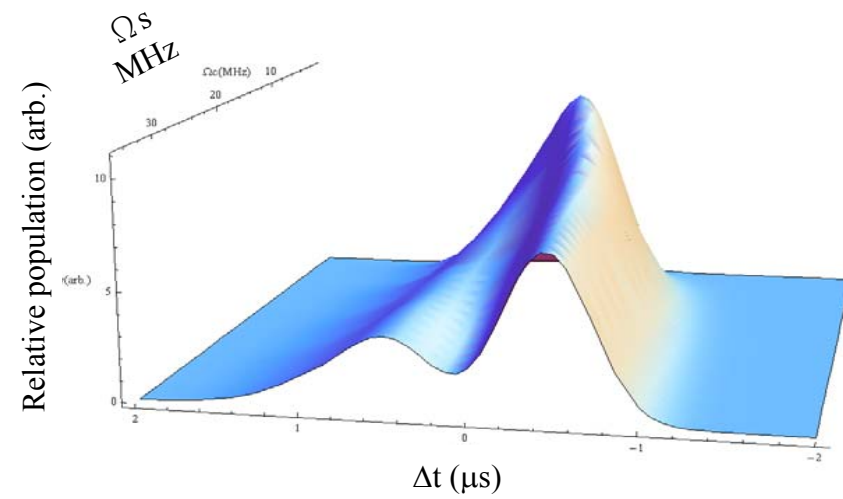


Experimental result





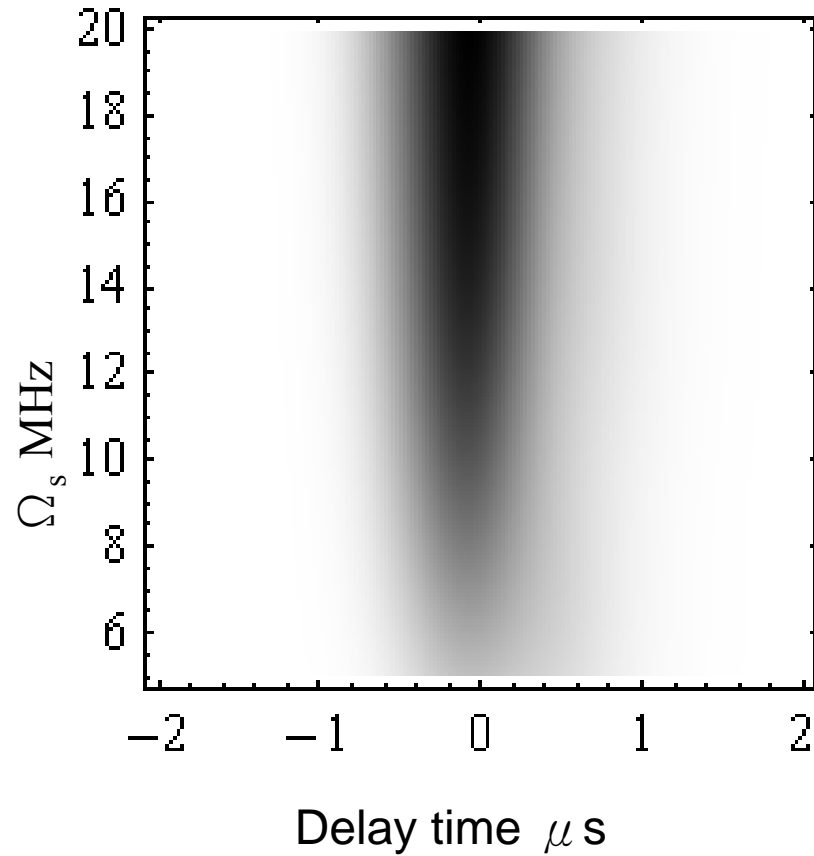
(a) $\Omega_p = 1.6$ MHz



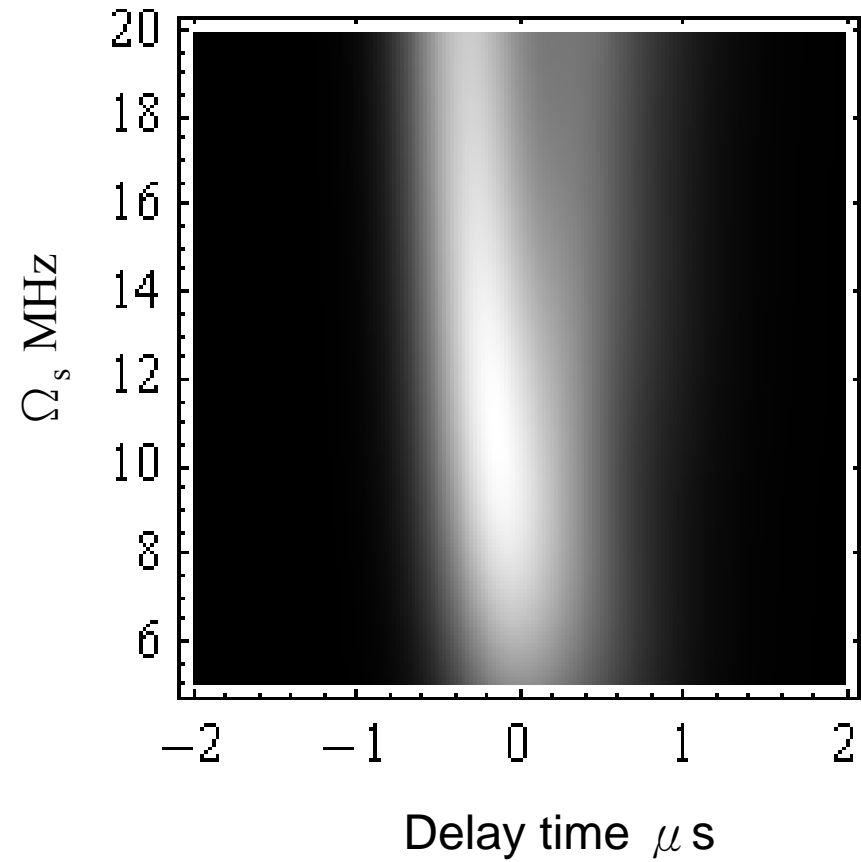
(b) $\Omega_p = 9.5$ MHz

Contour of the relative populations as functions of Ω_s and delay time for

(a) $\Omega_p = 1.6$ MHz and (b) $\Omega_p = 9.5$ MHz,



ρ_{22} population

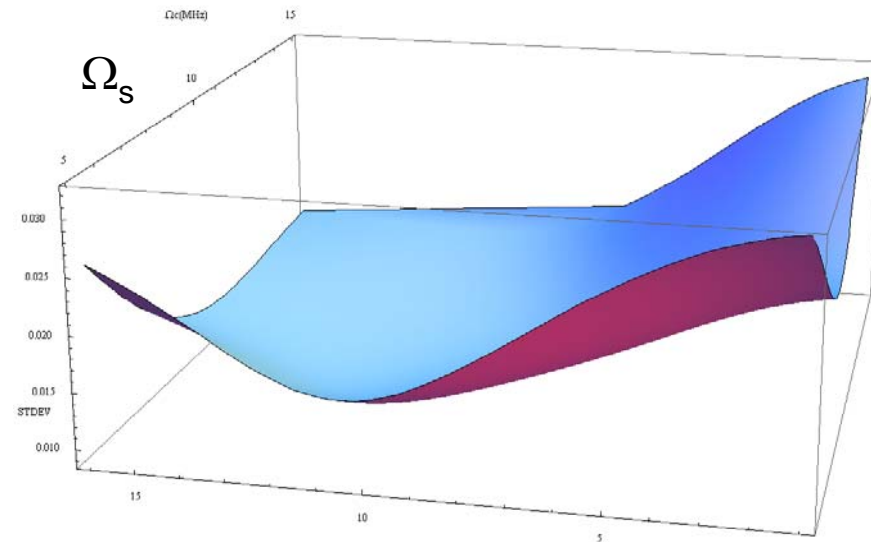


ρ_{33} population

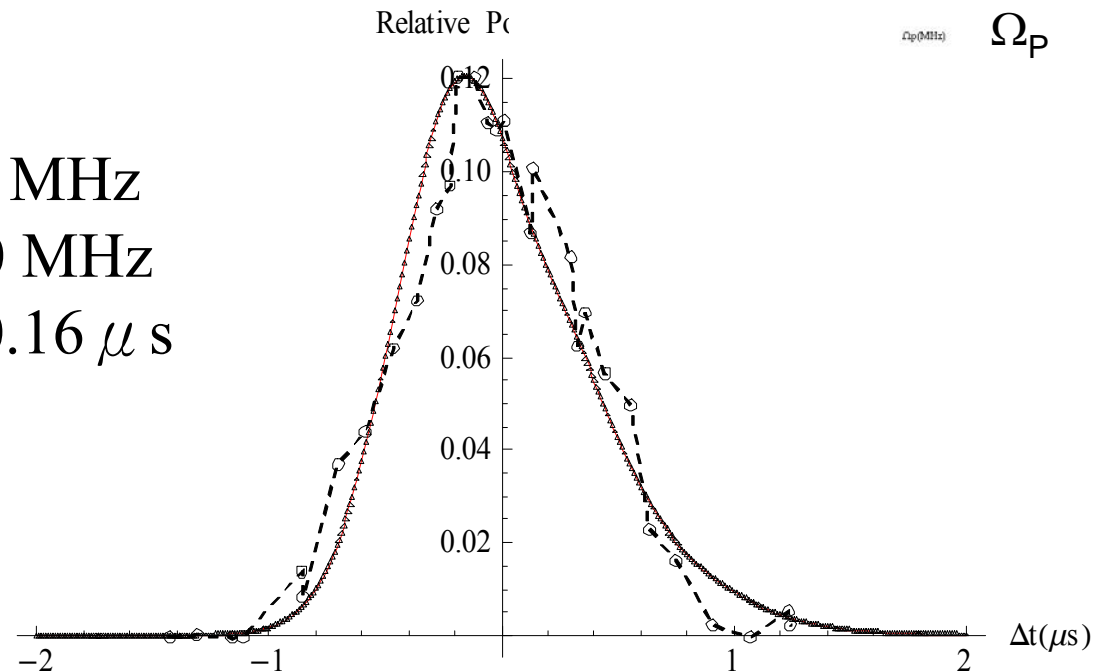


STDEV vs. Ω_s & Ω_p

$$\sigma = \sqrt{\frac{\sum_{n=1}^N (Exp_n - Simu_n)^2}{N - 1}}$$

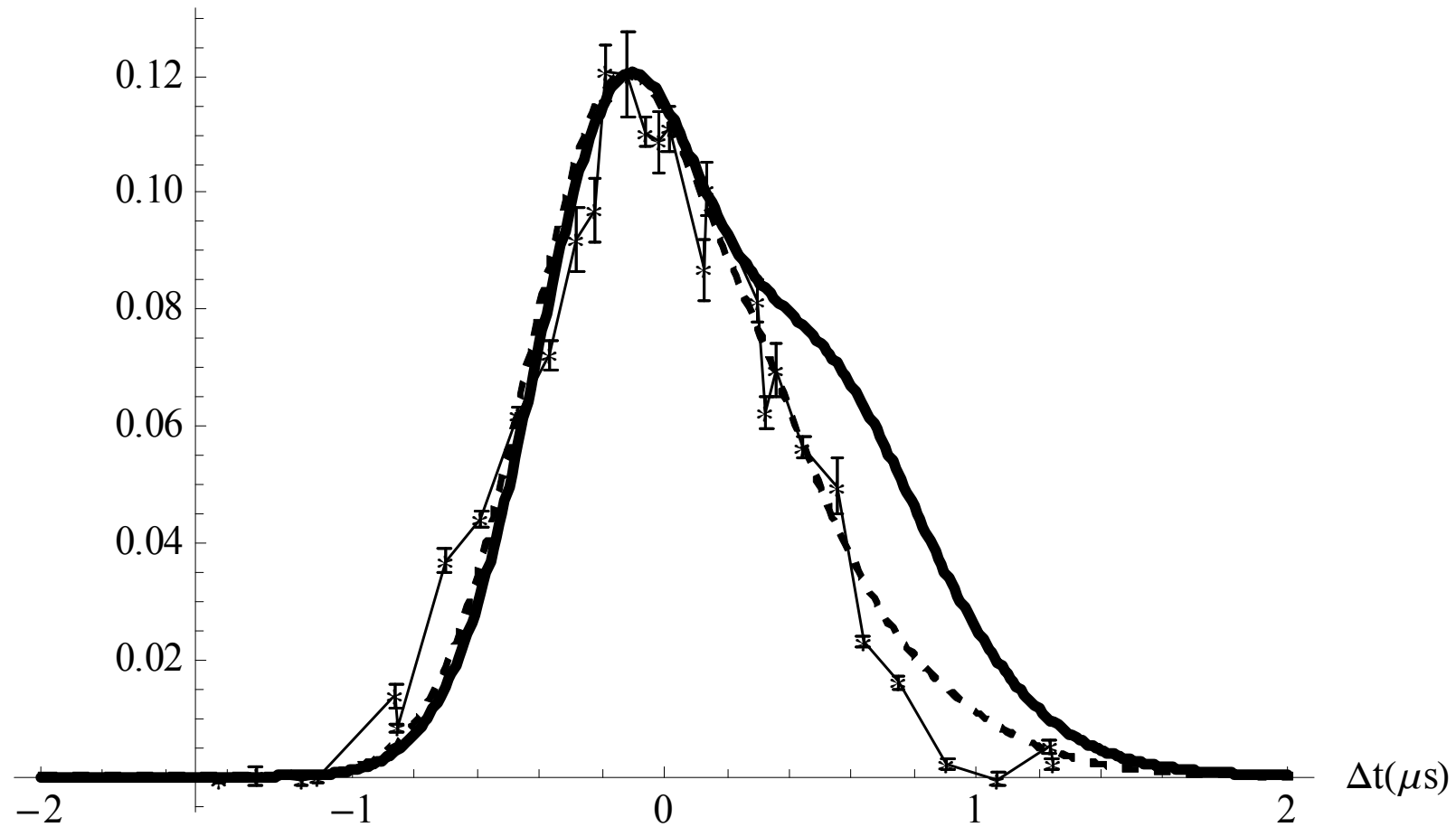


$\Omega_p = 9.5 \text{ MHz}$
 $\Omega_s = 10.0 \text{ MHz}$
 $\Delta t_{\text{mas}} = -0.16 \mu\text{s}$





Relative Population



Minimized STDEV simulation: $\Omega_p = 9.5$ MHz and $\Omega_s = 10.0$ MHz.

The thick line is another simulation: $\Omega_p = 3.1$ MHz and $\Omega_s = 11.5$ MHz.



Summery

* *Quantum Phenomena in diatomic molecule*

Tunnelling, Avoided-crossing, Feno Resonance

* *Quantum Phenomena in Cold Atoms*

**Shape Resonance, Feshbach Resonance,
EIT/Decoherence, STIRAP**



Group Reunion



2015/8/28