Laboratoire Kastler Brossel
Physique quantique et applications

Lectures on Quantum Optics and Quantum Information

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Taiwan-France joint school, Nantou, May 2011

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Quantum Information Science

Over the last 20 years, QIS has developed driven by the prospect to exploit capabilities from the quantum realm to accomplish tasks difficult or even impossible with traditional methods of information processing.

Quantum communication, the art of transferring a quantum state from one place to another. It led for instance to the demonstration of quantum cryptography, an absolutely secure way to transmit information, and even to commercially available quantum key distribution systems and a network deployed in a metropolitan area.

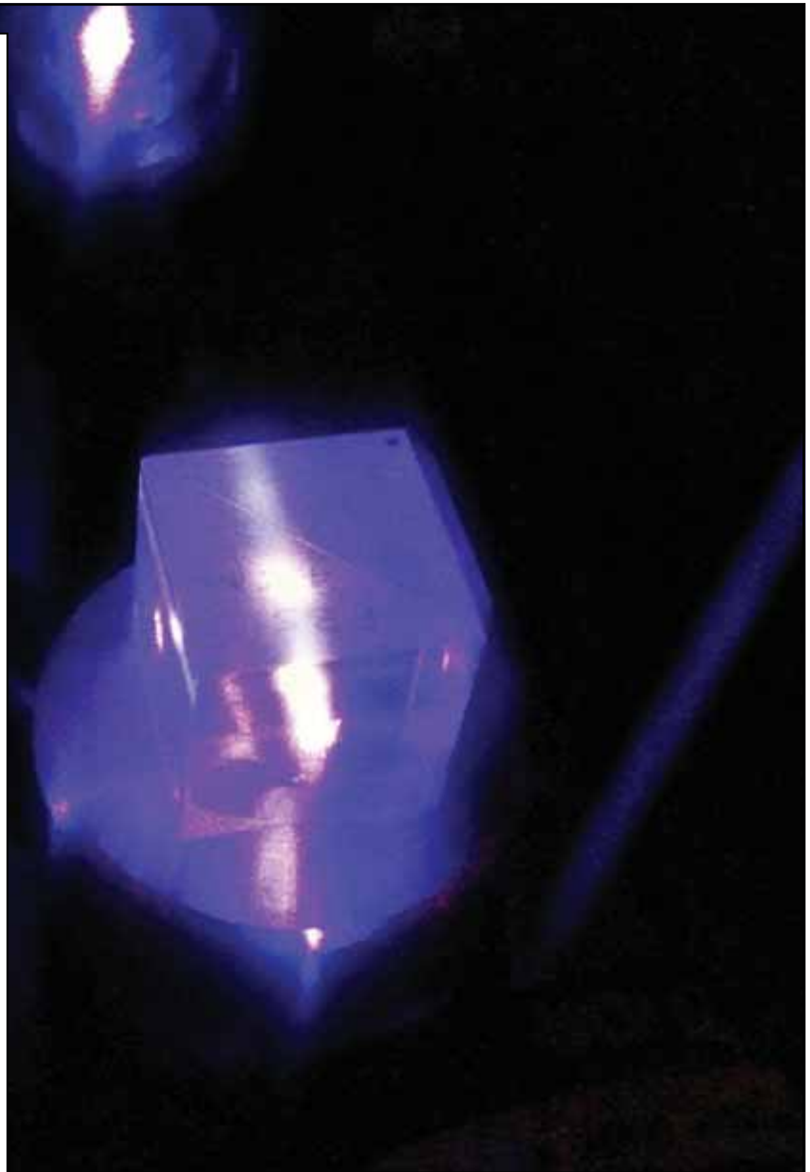
Quantum computation, where bits are replaced by qubits. It detains the promise of computing power beyond the capabilities of any classical computer. For instance, P. Shor showed that a quantum computer can factorize a large number efficiently, i.e. in a polynomial time, while it is an exponentially difficult problem for classical algorithms. Beyond its fundamental interest for understanding quantum complexity, it is of practical importance as the difficulty of factoring numbers is the basis of encryption systems, such as the RSA scheme.

A third direction is **quantum metrology and enhanced sensing**.

M.A. Nielsen, L. Chuang, Quantum computation and Quantum Information, Cambridge Univ. Press

S.M. Barnett, Quantum Information, Oxford Univ. Press

H.A. Bachor,T.C. Ralph, A guide to experiments in quantum optics, Wiley-VCH



Content of the lectures

Lecture 1 Introduction to quantum noise, squeezed light and entanglement generation

Quantization of light, Continuous-variable, Homodyne detection, Gaussian states, Optical parametric oscillators, Entanglement, Teleportation

Lecture 2 Quantum state engineering

Conditional preparation, Non-Gaussian states, Schrödinger cat states, Hybrid approaches, Quantum detectors, POVM and detector tomography

Lecture 3 Optical quantum memories.

Quantum repeaters, atomic ensembles, DLCZ, EIT, Photon-echo, Matter-Matter entanglement

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Lecture 1
Quantum noise, squeezed light and entanglement generation

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Lecture 1

- Light quantization, discrete vs continuous representation

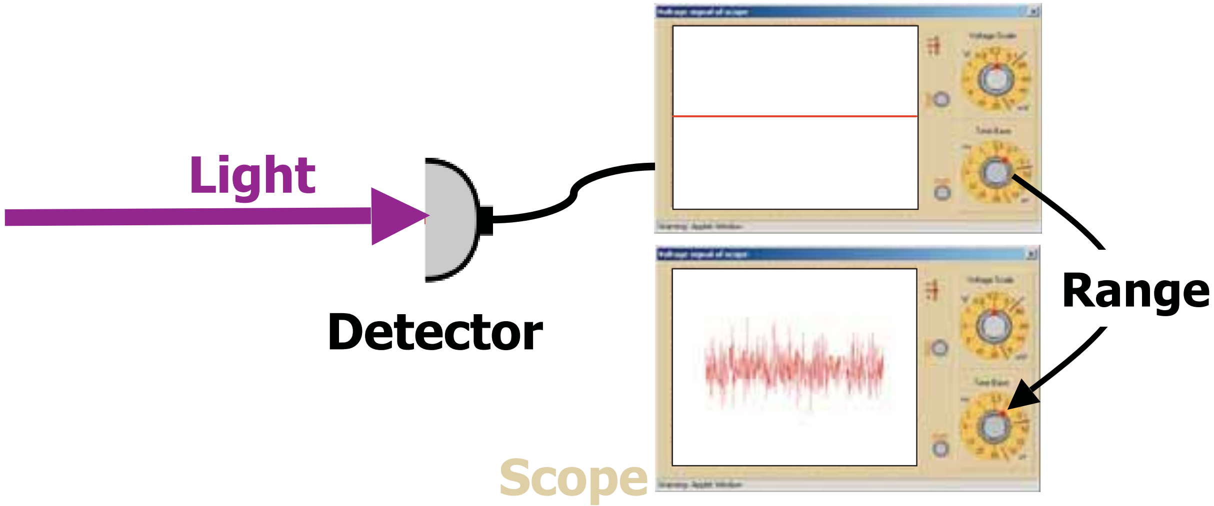
- Measuring and characterizing optical continuous variables

- How to generate squeezed light ?

- Quantum correlations and entanglement in the CV regime

What Are We Speaking About ?

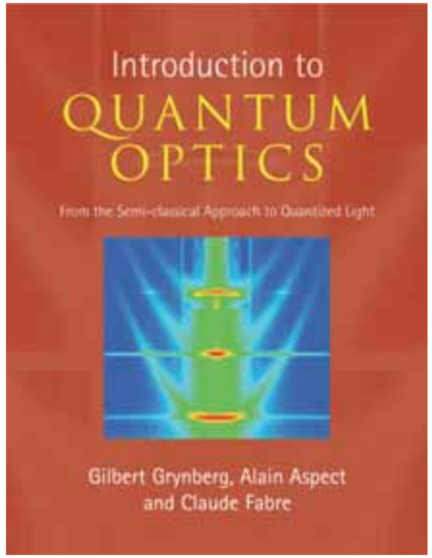
An introductive example : noise in a light beam



In the limit of unity quantum efficiency and no electric noise, the measured fluctuations are only due to the noise of the light: it can be *classical noise* (e.g. 1/f noise) or *quantum noise* (e.g. shot noise).

In this lecture, we will focus on information carried by travelling light fields and we wil be interested in the quantum mechanical nature of light.

Quantization of the Electromagnetic Field



We are not going to detail the quantization procedure for the free electromagnetic field, but give here the basic steps and analogies, which enable to introduce useful notations.
More details can be found for instance in "Introduction to QO".

- Maxwell equations in vacuum (no charges, no currents)

$$\vec{\nabla} \times \vec{\mathcal{E}} = -\frac{\partial}{\partial t}\vec{\mathcal{B}} \qquad \vec{\nabla} \times \vec{\mathcal{B}} = \frac{1}{c^2}\frac{\partial}{\partial t}\vec{\mathcal{E}}$$

$$\vec{\nabla}.\vec{\mathcal{E}} = 0 \qquad \vec{\nabla}.\vec{\mathcal{B}} = 0$$

- Give the Helmholtz equation for the E field: $\Delta\vec{\mathcal{E}} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{\mathcal{E}} = 0$

- Classical Plane wave decomposition (l: 2 polarizations, k: wave vector)

$$\vec{\mathcal{E}}(\vec{r},t) = \frac{1}{(2\pi)^{3/2}}\sum_{l=1}^{2}\int\int\int E_{l,\vec{k}}(t)\ \vec{\epsilon}_{l,\vec{k}}\ e^{i\vec{k}.\vec{r}}\ d^3k$$

- By injecting this decomposition into the Helmholtz equation :

$$\frac{\partial^2}{\partial t^2}E_{l,\vec{k}}(t) + \underbrace{k^2c^2}_{\omega_{\vec{k}}^2}E_{l,\vec{k}}(t) = 0$$

Quantization of the Electromagnetic Field

- For each wave vector: $$\frac{\partial^2}{\partial t^2}E_{l,\vec{k}}(t) + k^2c^2E_{l,\vec{k}}(t) = 0$$

This equation is similar to the one describing the evolution of a harmonic oscillator with frequency ω, which can be described by a vector a, function of p (position) and q (momentum).

$$a(t) = \frac{p + i\,q}{\sqrt{2}}e^{i\omega t}$$

- By analogy, we introduce an operator $\hat{a}_{l,\vec{k}}$ and define a field operator :

$$\hat{E}_{l,\vec{k}} = \sqrt{\frac{\hbar\omega_{\vec{k}}}{2\epsilon_0}}\hat{a}_{l,\vec{k}}$$

We can also define the quadrature operators:

$$\hat{p}_{l,\vec{k}} = \frac{\hat{a}_{l,\vec{k}} + \hat{a}^{\dagger}_{l,\vec{k}}}{\sqrt{2}} \qquad \hat{q}_{l,\vec{k}} = \frac{\hat{a}_{l,\vec{k}} - \hat{a}^{\dagger}_{l,\vec{k}}}{i\sqrt{2}}$$

Where $\hat{a}_{l,\vec{k}}$ and $\hat{a}^{\dagger}_{l,\vec{k}}$ can be identified to the creation and annihilation operators for harmonic oscillators

- By considering Fourier-limited wavepackets, it can be shown finally for one mode (one dimension quantized harmonic oscillator) :

Total number of photons

$$\hat{H} = \hbar\omega\left(\hat{a}^{\dagger}a + \frac{1}{2}\right)$$

$$\left[\hat{a}, \hat{a}^{\dagger}\right] = 1$$

$$\hat{p} = \frac{\hat{a} + \hat{a}^{\dagger}}{\sqrt{2}} \qquad \hat{q} = \frac{\hat{a} - \hat{a}^{\dagger}}{i\sqrt{2}}$$

$$[\hat{p}, \hat{q}] = i$$

Two Possible Descriptions

Quantization of the electromagnetic field
Modes are quantized harmonic oscillators
2 possible observables for the description:
Energy or Electric field

Discrete degree of freedom
number of photons $N=a^{+}a$
(measured by photon counting)

Continuous degree of freedom
quadratures P and Q
(fluctuations measured by homodyning, photodiodes)

Discrete Variables

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Quantum bits or "qu-bits" have been introduced in this description.
e.g. presence/absence of a photon in one mode, orthogonal polarization modes (H/V),...

$$\alpha|0\rangle + \beta|1\rangle \qquad \alpha^2 + \beta^2 = 1$$

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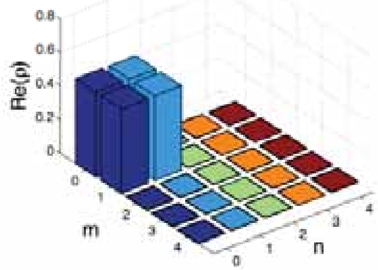
A convenient tool : the density matrix

For a pure state: $\rho = |\psi\rangle\langle\psi|$

For a mixed state: $\rho = \sum_k p_k |\psi_k\rangle\langle\psi_k|$

Example : $|\psi\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$

$$\rho = |\psi\rangle\langle\psi| = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$



Written in the Fock basis, the density matrix is very illustrative for discrete systems.

Quantum Continuous-Variables

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Why using them?

- Perfect detectors, high rates/bandwidth
- Deterministic operations
- Non-gaussian states/cluster states with many potential applications in Q. computing and communication

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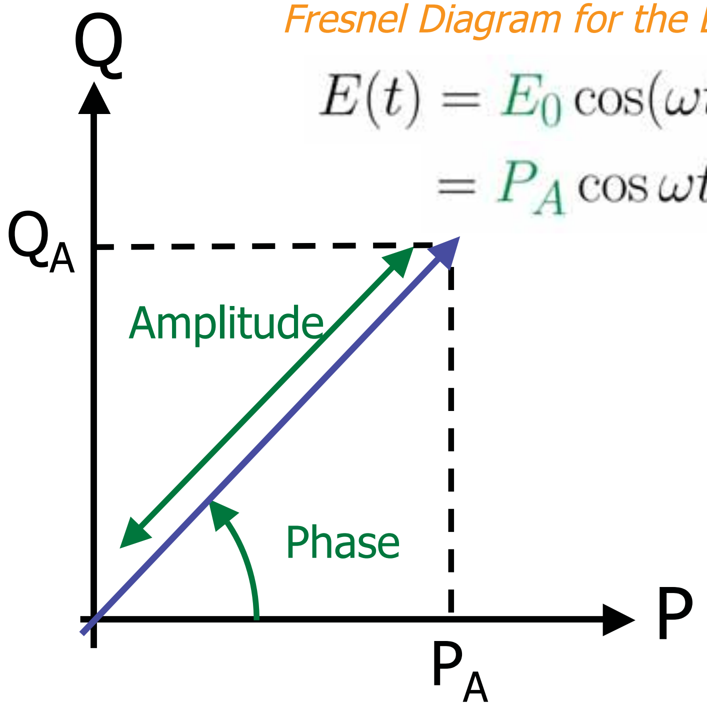
Fresnel Diagram for the Electromagnetic Field

$$E(t) = E_0 \cos(\omega t + \varphi) = P_A \cos \omega t + Q_A \sin \omega t$$



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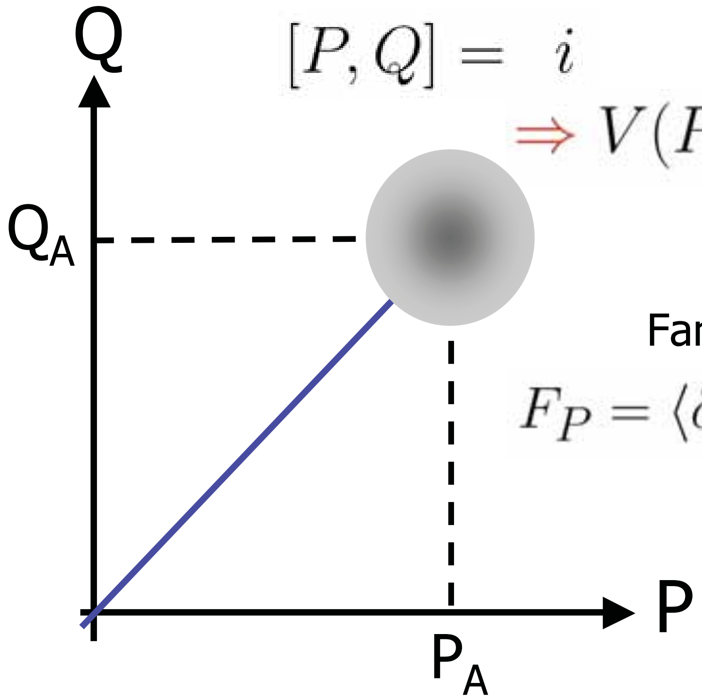
Phasor Diagram



$$[P,Q] = i$$

$$\Rightarrow V(P)V(Q) \geq 1$$

Fano factor F

$$F_P = \langle \delta P^2 \rangle = V(P)$$

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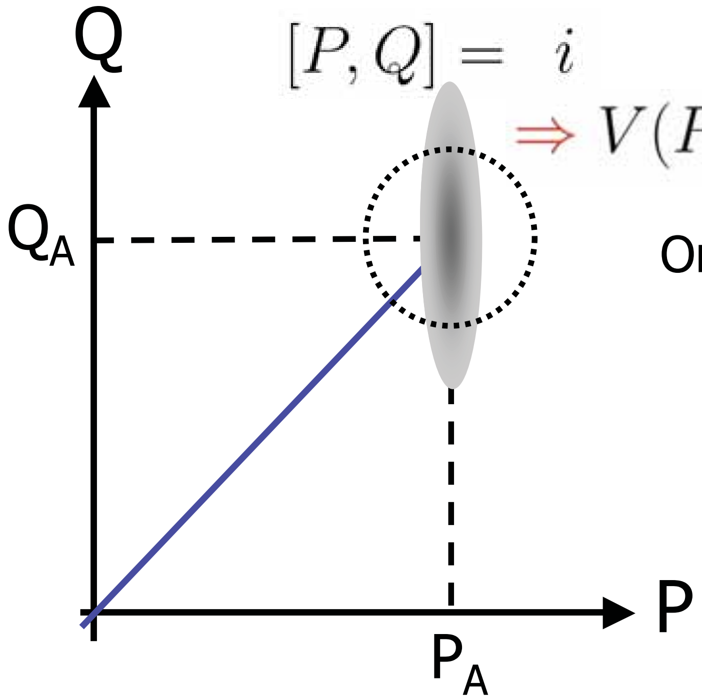
Squeezing

$$[P,Q] = i$$

$$\Rightarrow V(P)V(Q) \geq 1$$



One can have :

$$V(P) \leq 1$$
$$V(Q) \geq 1$$

"Noise reduction" or "Squeezing"

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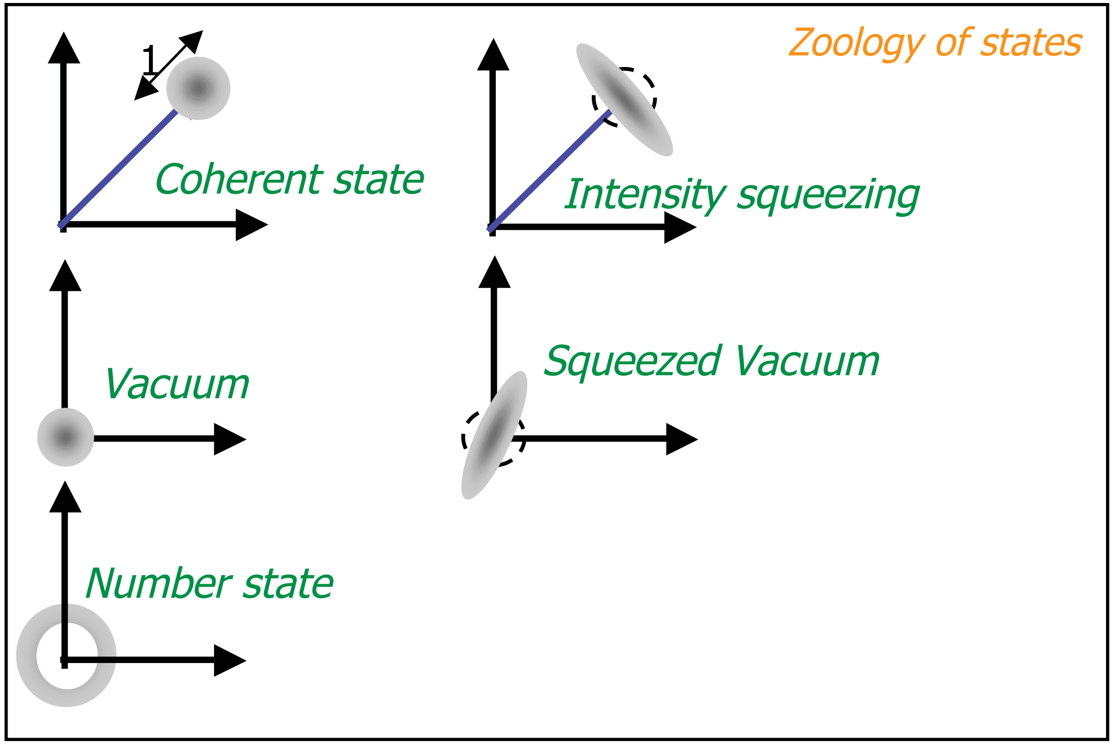

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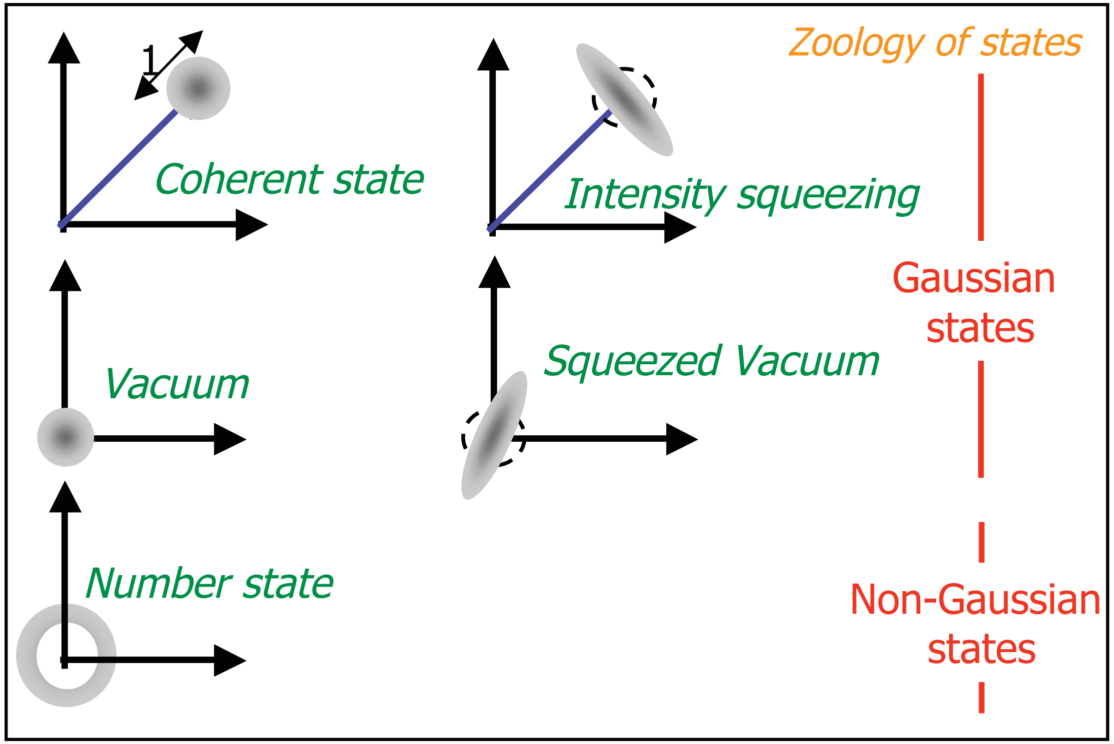

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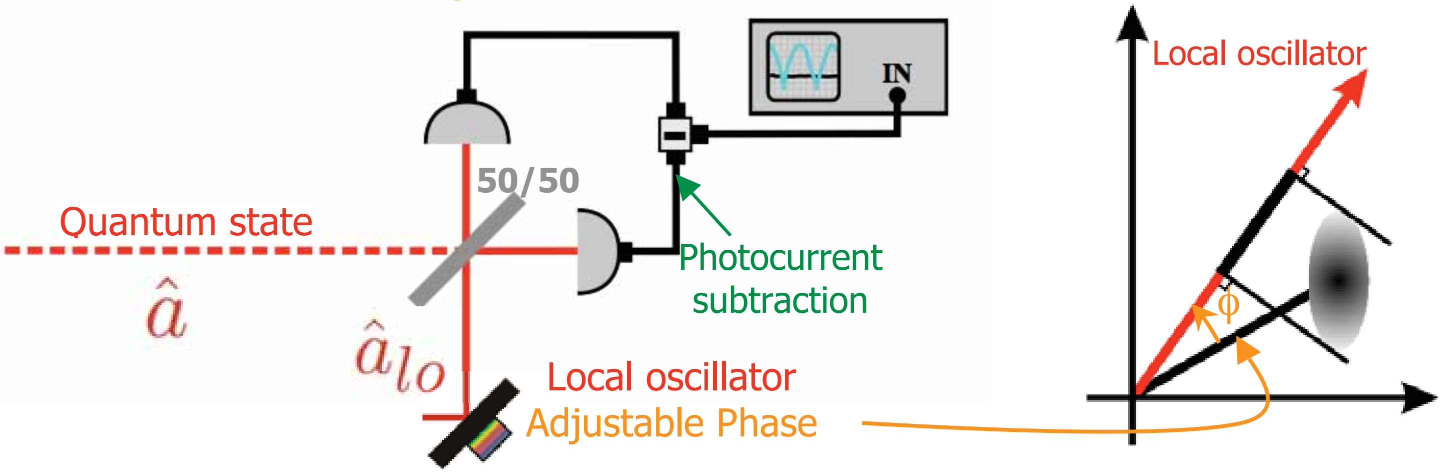
Measuring Optical Continuous-Variables

THE tool : Homodyne detection



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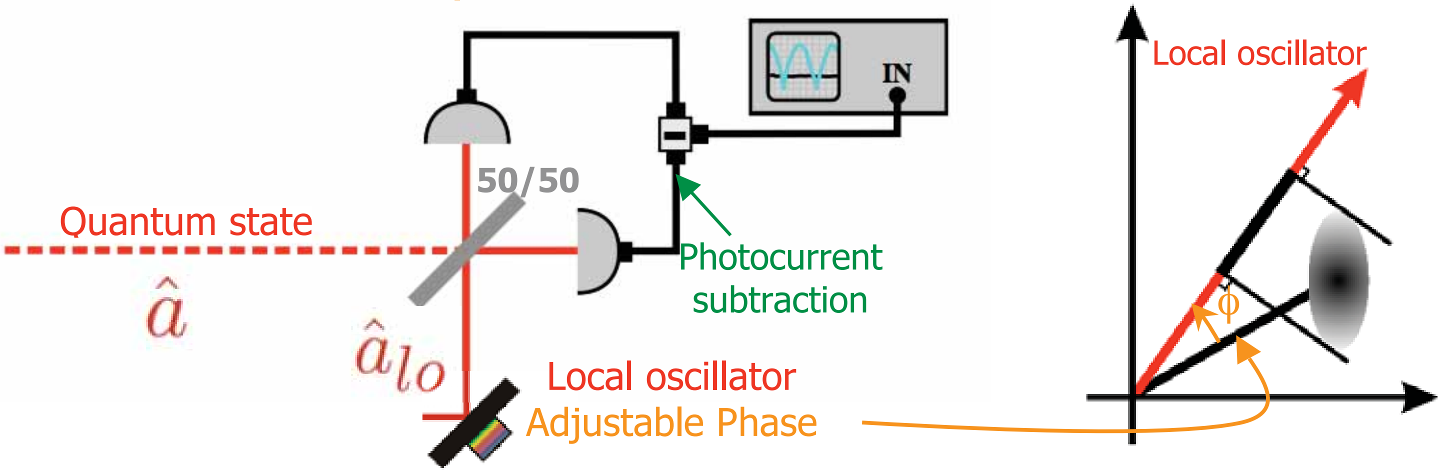

- Annihilation operators of the mixed modes: $\hat{a}_1 = \frac{\hat{a} + \hat{a}_{lo}}{\sqrt{2}}$ $\hat{a}_2 = \frac{\hat{a} - \hat{a}_{lo}}{\sqrt{2}}$
- After subtraction, the resulting photocurrent operator is:

$$\hat{N}_- = \hat{a}_1^\dagger \hat{a}_1 - \hat{a}_2^\dagger \hat{a}_2 = \hat{a}^\dagger \hat{a}_{lo} - \hat{a}_{ol}^\dagger \hat{a}$$

- Mean Value and variance for $|\psi\rangle \otimes |\alpha e^{i\phi}\rangle$:

$$\langle \hat{N}_- \rangle = \alpha \langle \psi | \hat{a}^\dagger e^{i\phi} + \hat{a} e^{-i\phi} | \psi \rangle = \alpha \langle \psi | \hat{p}_\phi | \psi \rangle$$

$$V(\hat{N}_-) = \alpha^2 V(\hat{p}_\phi) + \langle \psi | \hat{a}^\dagger \hat{a} | \psi \rangle$$

For large photon number in lo:

$$V(\hat{N}_-) \simeq \alpha^2 V(\hat{p}_\phi)$$

Measuring Optical Continuous-Variables

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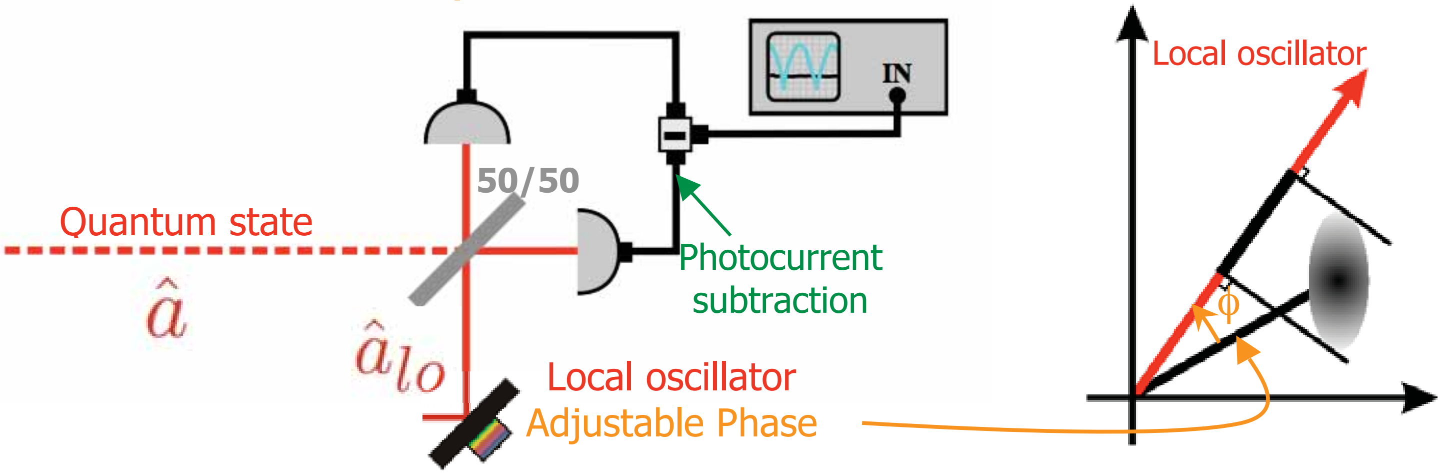

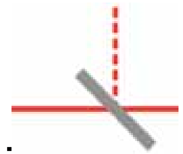

- Overall Efficiency -
Photodiode efficiency η
Interference Visibility V

$\eta_{tot}=\eta.V^2$
(ex: 0.99 x 0.99²=0.97)

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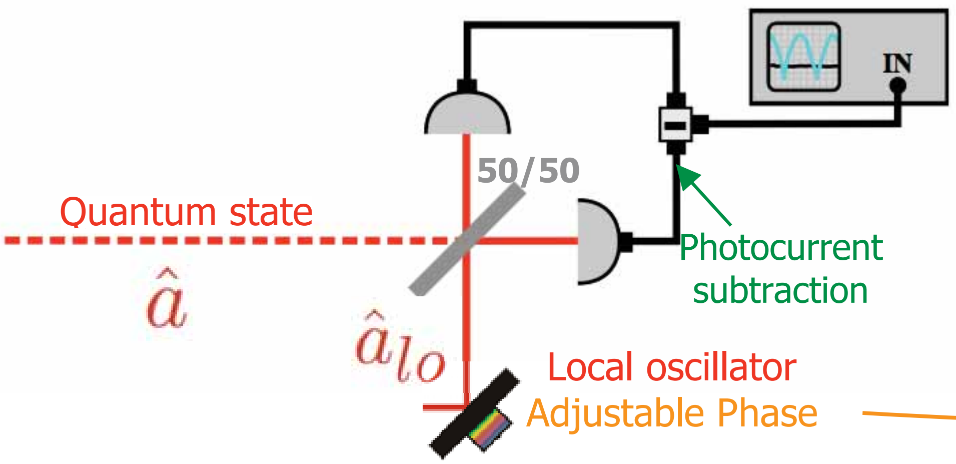

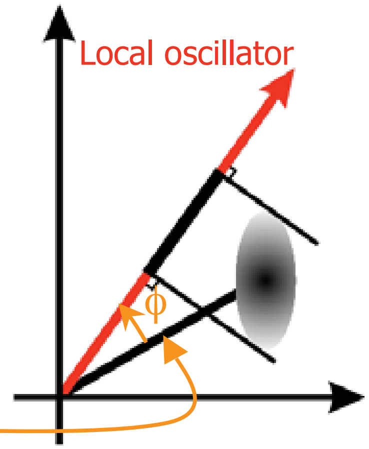

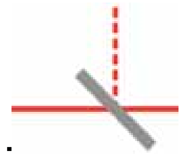

- Overall Efficiency -

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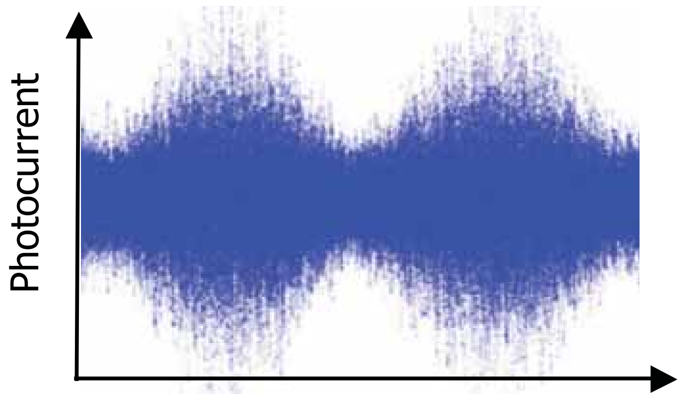
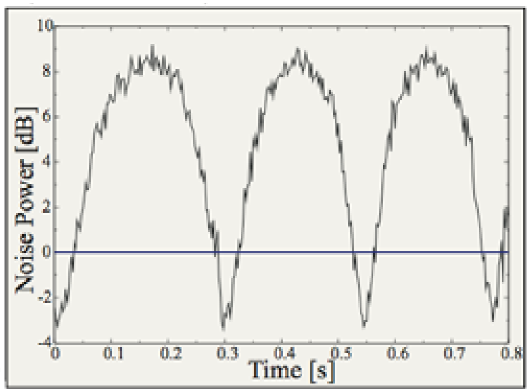
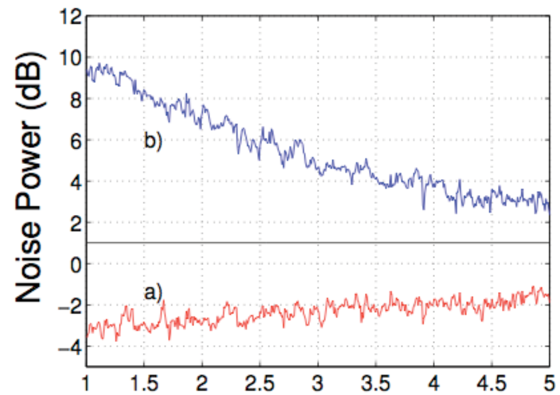
What we obtain ? Example of a squeezed state



Probability distribution $\mathscr{P}(X,\varphi)$



Variance



TF : Noise spectrum

The Wigner Function

For continuous-variable, the density matrix is useful, but not easy to interpret.
Another tool : the Wigner function, which is a quasi-probability distribution.

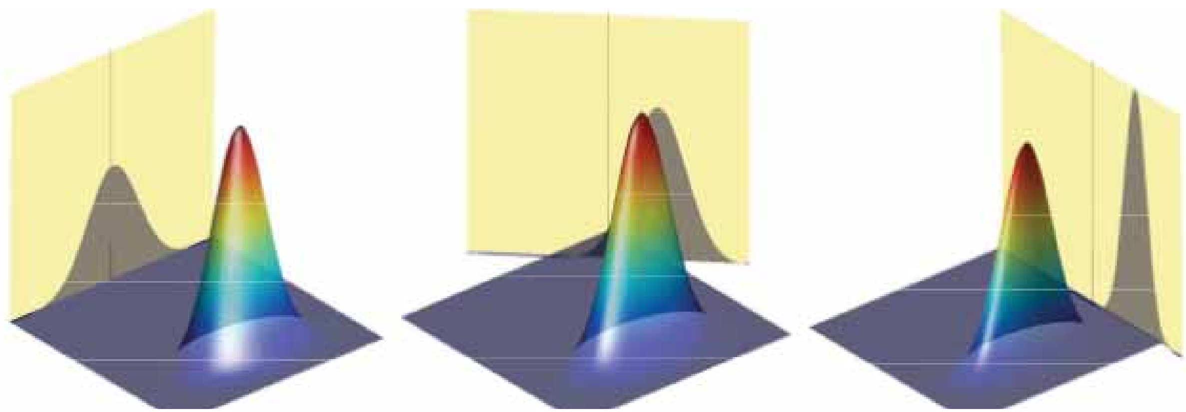
$$W(p,q) = \frac{1}{2\pi}\int e^{i\nu q}\langle p-\nu/2|\hat{\rho}|p+\nu/2\rangle d\nu$$

Marginal distributions for $\mathcal{P}(\hat{p}_\phi)$ (what is measured with homodyne detection) are obtained by projection of the Wigner function on the axis defined by $\hat{p}_\phi$, i.e. by integrating it over the orthogonal direction.

$$\mathcal{P}(\hat{p}_\phi) = \int W(p\cos\phi - q\sin\phi, p\sin\phi + p\cos\phi)dq$$

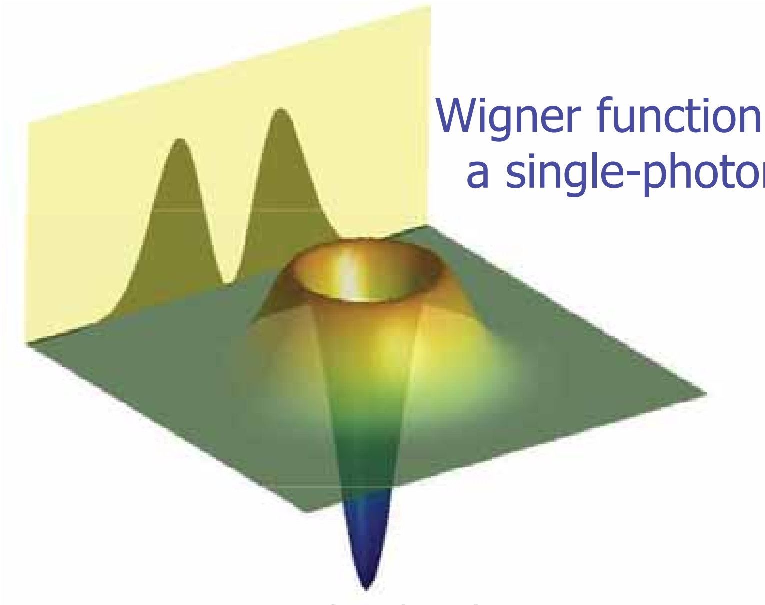
Importantly, one can also obtain ρ and W from the marginal distributions : this is the goal of tomography. It requires to use reconstruction algorithm, such as Radon transform or Maximum-likelihood algorithm. [see A.Lvovsky, RMP 81, 299 (2009)]

Wigner function and some projections for a squeezed state



From A. Ourjoumtsev PhD Thesis

The Wigner Function: Negativity



Wigner function of
a single-photon

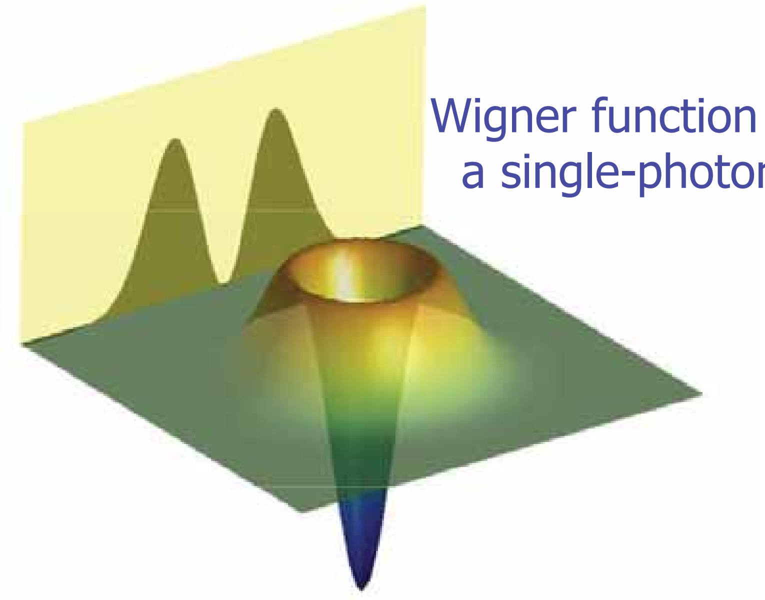
(From A. Ourjoumtsev PhD Thesis)

The Wigner function is a quasi-probability distribution: it can take negative values.

Hudson-Piquet Theorem for pure state:

Gaussian state ⇔ Positive Wigner function
Non-Gaussian state ⇔Negative Wigner function

The Wigner Function: Negativity



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Negativity at the origin, W(0,0)<0 : something special ?

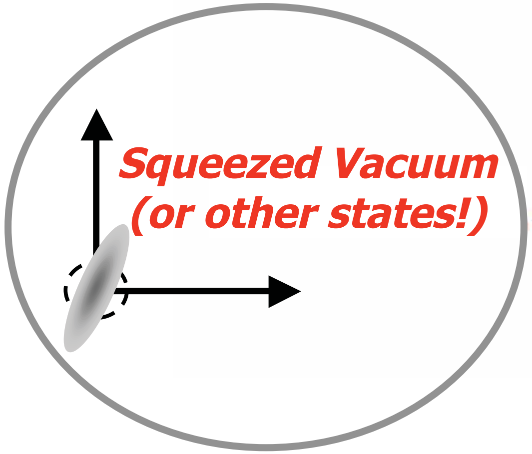
Quantumness, or non-classicality, can manifest by many ways. It is usually considered that the negativity of the Wigner is a very strong signature of non-classicality.

Parity operator for the number of photon: $\hat{P}_n = (-1)^{\hat{n}}$

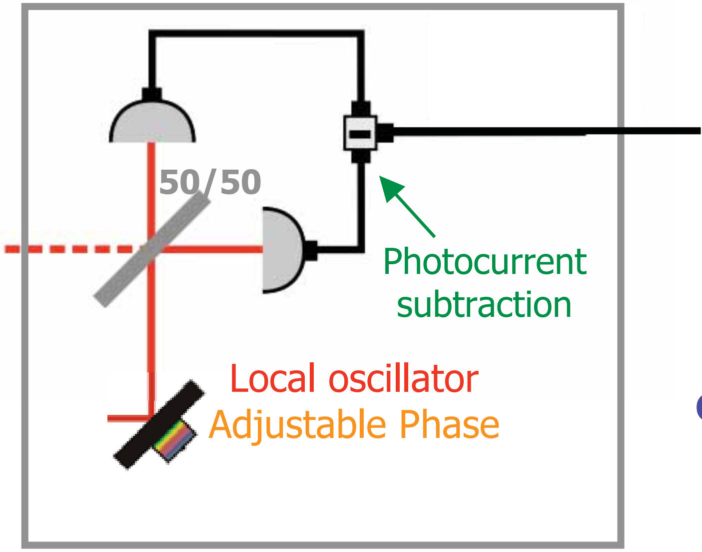
W(0,0) is related to a odd number of photons

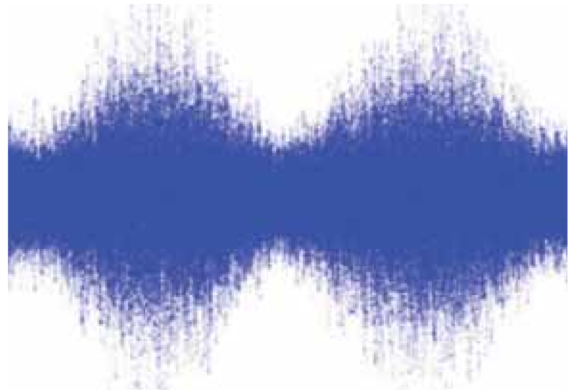
$$W(0,0) = \frac{1}{\pi}\langle \hat{P}_n \rangle$$

Measuring Optical CV : a Summary



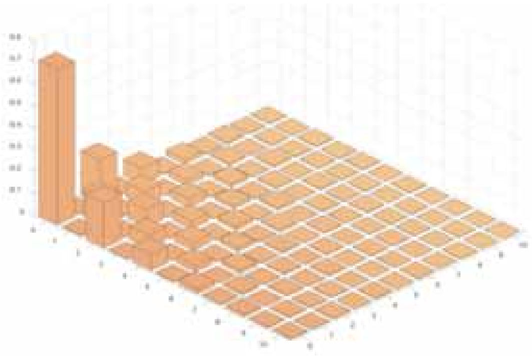
Homodyne detection



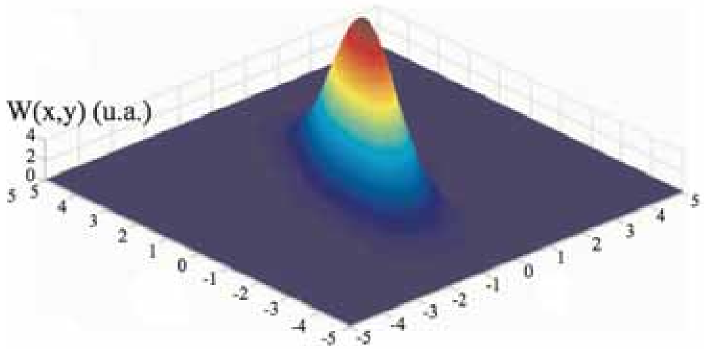


Photocurrent vs phase
Give the marginal distributions

By reconstruction algorithm (Radon, MaxLik)

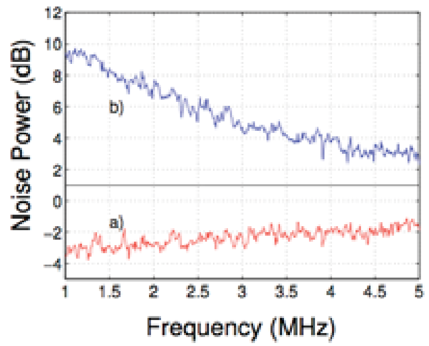


Density matrix



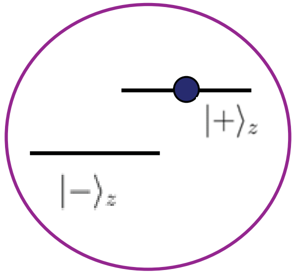
Wigner function

Variance, Fourier Transform



Noise spectrum

Interlude: CV for Atomic Ensembles



N 2-level atoms

N fictitious 1/2 spins described by a collective spin

Collective spin operators

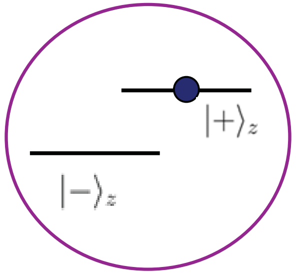
$$J_x = \sum_{i=1}^{N} j_x{}^i \quad J_y = \sum_{i=1}^{N} j_y{}^i \quad J_z = \sum_{i=1}^{N} j_z{}^i$$

Individual spins aligned along Oz $\quad \langle J_z \rangle = \frac{N}{2}$

Heisenberg inequality

$$[J_x, J_y] = i\frac{N}{2} \Rightarrow \Delta J_x \Delta J_y \geq \frac{N}{4}$$

Interlude: CV for Atomic Ensembles



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N fictitious 1/2 spins described by a collective spin

Collective spin operators

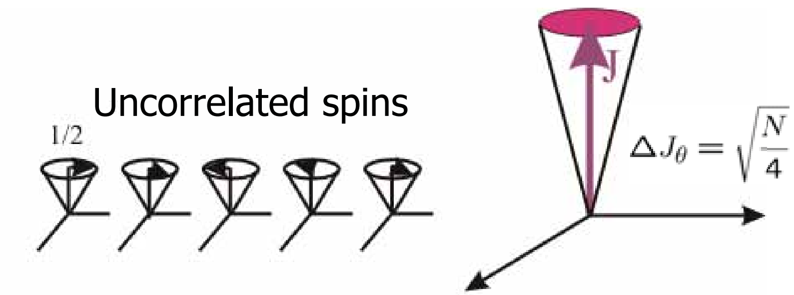
$$J_x = \sum_{i=1}^{N} j_x{}^i \quad J_y = \sum_{i=1}^{N} j_y{}^i \quad J_z = \sum_{i=1}^{N} j_z{}^i$$

Individual spins aligned along Oz $\langle J_z \rangle = \frac{N}{2}$

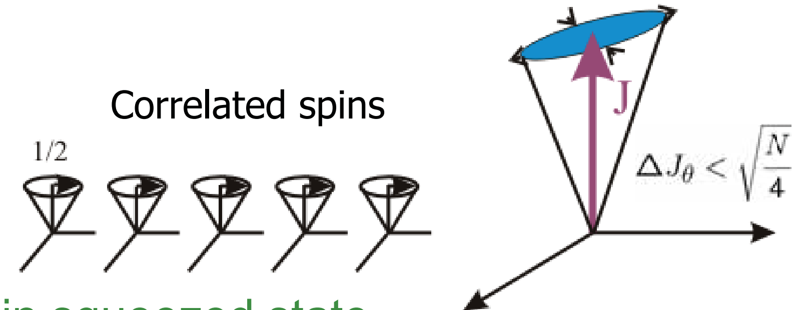
Heisenberg inequality

$$[J_x, J_y] = i\frac{N}{2} \Rightarrow \Delta J_x \Delta J_y \geq \frac{N}{4}$$

Some atomic states…



Spin coherent state



Spin squeezed state

Lecture 1

- Light quantization, discrete vs continuous representation

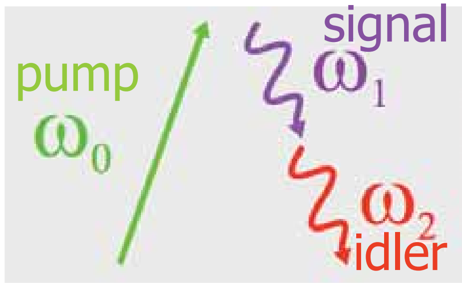
- Measuring and characterizing optical continuous variables

- How to generate squeezed light ?

- Quantum correlations and entanglement in the CV regime

How to Generate Squeezed Light ?

Squeezed light generation requires the use of non-linear effects. We focus here on the case of degenerate parametric interaction in non-linear $\chi^{(2)}$ crystals.



$$\omega_0 = \omega_1 + \omega_2$$
$$\vec{k}_0 = \vec{k}_1 + \vec{k}_2$$

'Degenerate': signal and idler identical (frequency, polarization)

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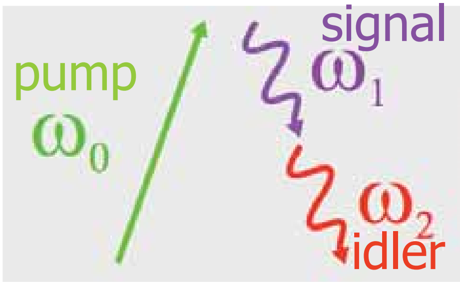


$$\omega_0 = \omega_1 + \omega_2$$
$$\vec{k}_0 = \vec{k}_1 + \vec{k}_2$$

'Degenerate': signal and idler identical (frequency, polarization)

- Hamiltonian associated to this process (down conversion and up conversion):

$$H = i\hbar\chi\left(\hat{a}_s\hat{a}_i - \hat{a}_s^{\dagger}\hat{a}_i^{\dagger}\right) = i\hbar\chi\left(\hat{a}_s^2 - \hat{a}_s^{\dagger 2}\right)$$

Degenerate case

- It leads to the temporal evolution $\hat{a}_s(t) = cosh(\chi t)\hat{a}_s(0) - sinh(\chi t)\hat{a}_s^{\dagger}(0)$

- This gives for the quadratures after the interaction :

$$\hat{p}_s^{out} = e^{-2r}\hat{p}_s^{in}$$
$$\hat{q}_s^{out} = e^{2r}\hat{q}_s^{in}$$

$r = \chi\tau$

One quadrature is amplified while the orthogonal one is deamplified.

Pulsed Parametric Amplification

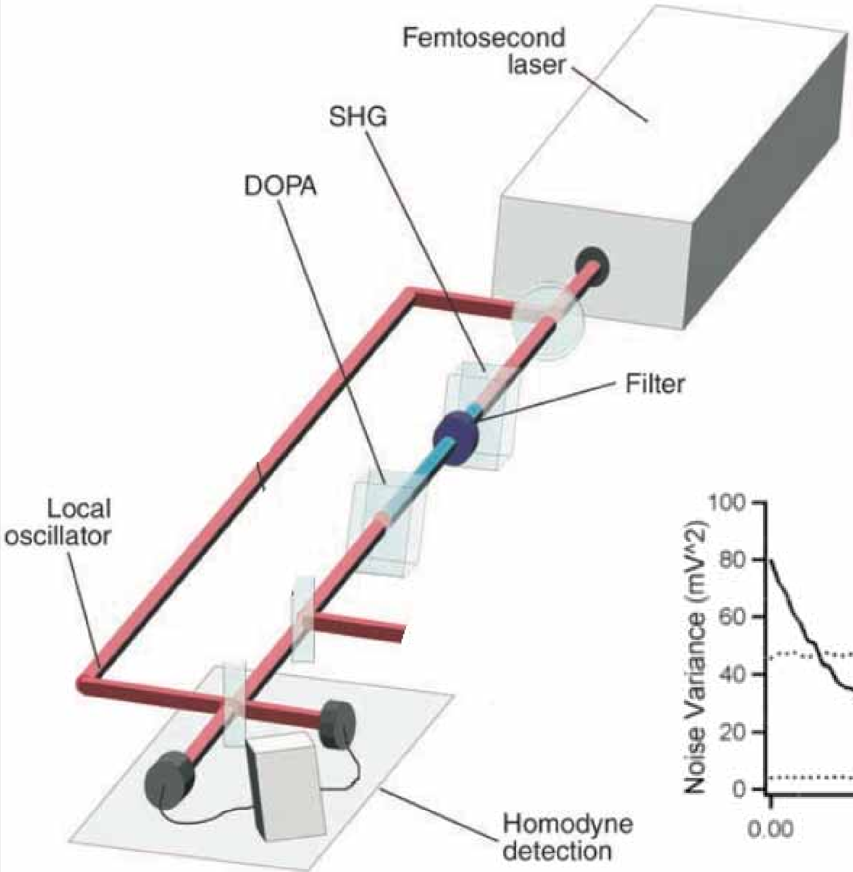
Requires intense pulses.
Squeezing increases with pump power.

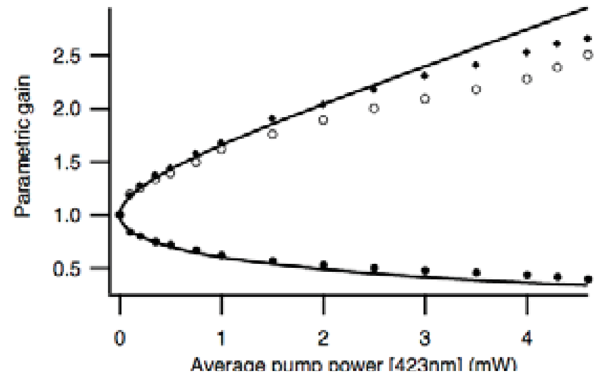
Pulsed homodyne measurements of femtosecond squeezed pulses generated by single-pass parametric deamplification

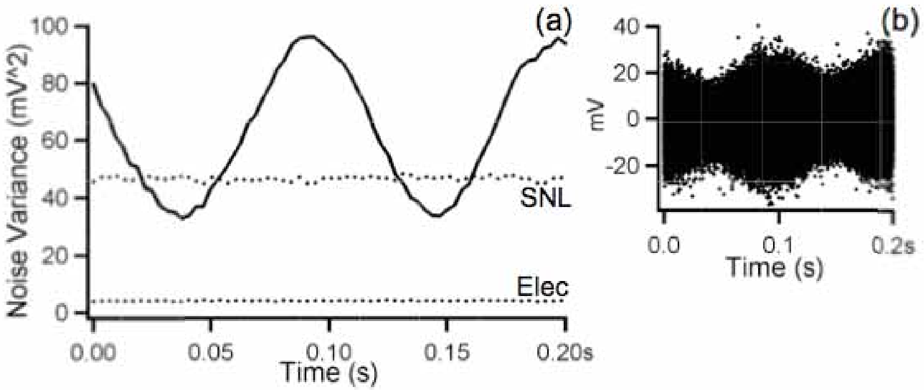
Jérôme Wenger, Rosa Tualle-Brouri and Philippe Grangier
Laboratoire Charles Fabry de l'Institut d'Optique, CNRS UMR 8501, F-91403 Orsay, France.

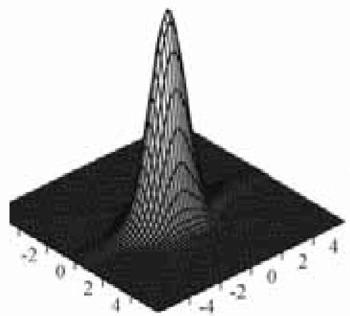
A new scheme is described for pulsed squeezed light generation using femtosecond pulses parametrically deamplified through a single pass in a thin (100 μm) potassium niobate $KNbO_3$ crystal, with a significant deamplification of about -3dB. The quantum noise of each individual pulse is registered in the time domain using a single-shot homodyne detection operated with femtosecond pulses and the best squeezed quadrature variance was measured to be 1.87 dB below the shot noise level. Such a scheme provides the basic ressource for time-resolved quantum communication protocols.

Opt. Lett. 29, 1267 (2004)



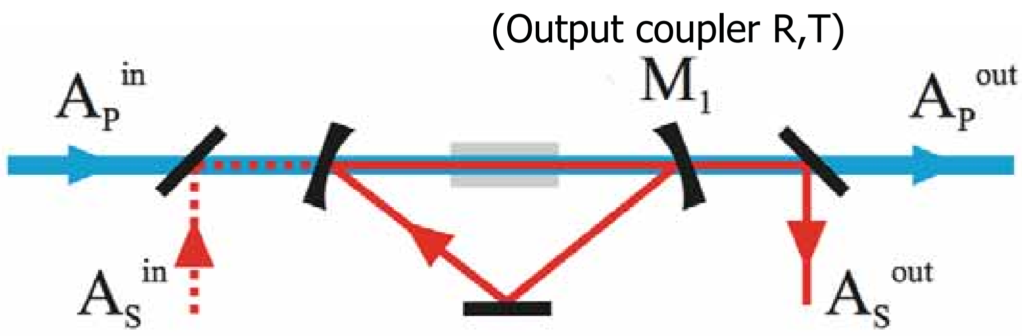






CW Parametric Amplification in a Cavity

Another solution: use a cw less intense pump laser and a cavity which is resonant on the common signal-idler mode and enhances the non-linear interaction. 'Optical Parametric Oscillator', which can oscillate above a given threshold.



- R=1 for all the mirrors, but the output coupler R
- Losses L (absorption, interfaces,..)
- Pump power normalized to the threshold : σ
- Ω_c : cavity bandwidth

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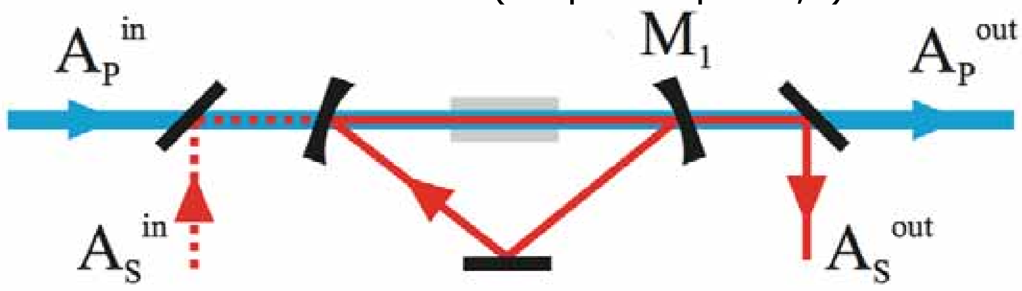

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- Pump power normalized to the threshold : σ
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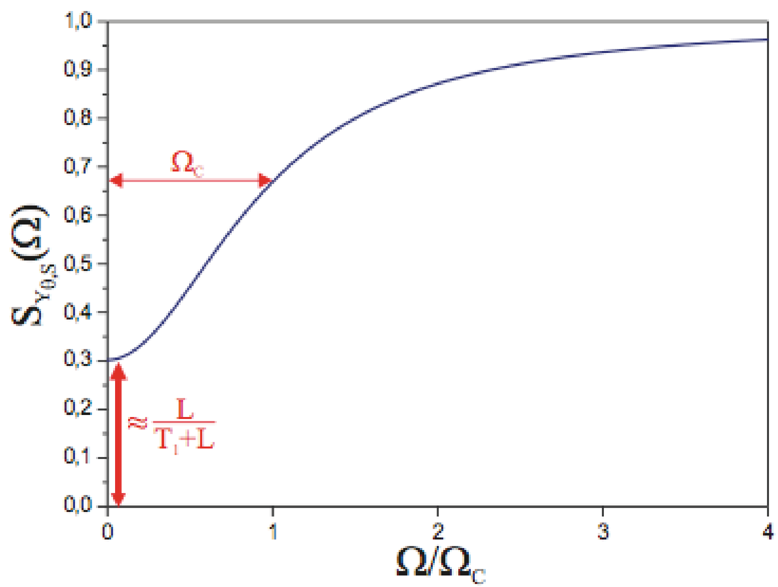
$$\begin{cases} S_{X_{\theta,S}}(\Omega) = 1 + \dfrac{T}{T+L}\dfrac{4\sigma}{(1-\sigma)^2 + 4\frac{\Omega^2}{\Omega_c^2}} \\ S_{Y_{\theta,S}}(\Omega) = 1 - \dfrac{T_1}{T+L}\dfrac{4\sigma}{(1+\sigma)^2 + 4\frac{\Omega^2}{\Omega_c^2}} \end{cases}$$



Maximum squeezing : at threshold, at zero frequency.
Squeezing only depends on the escape efficiency T/(T+L)

Ex. : T=10%, L=1% -->V=1-10/11~0.09 (~10dB)

CW Parametric Amplification in a Cavity

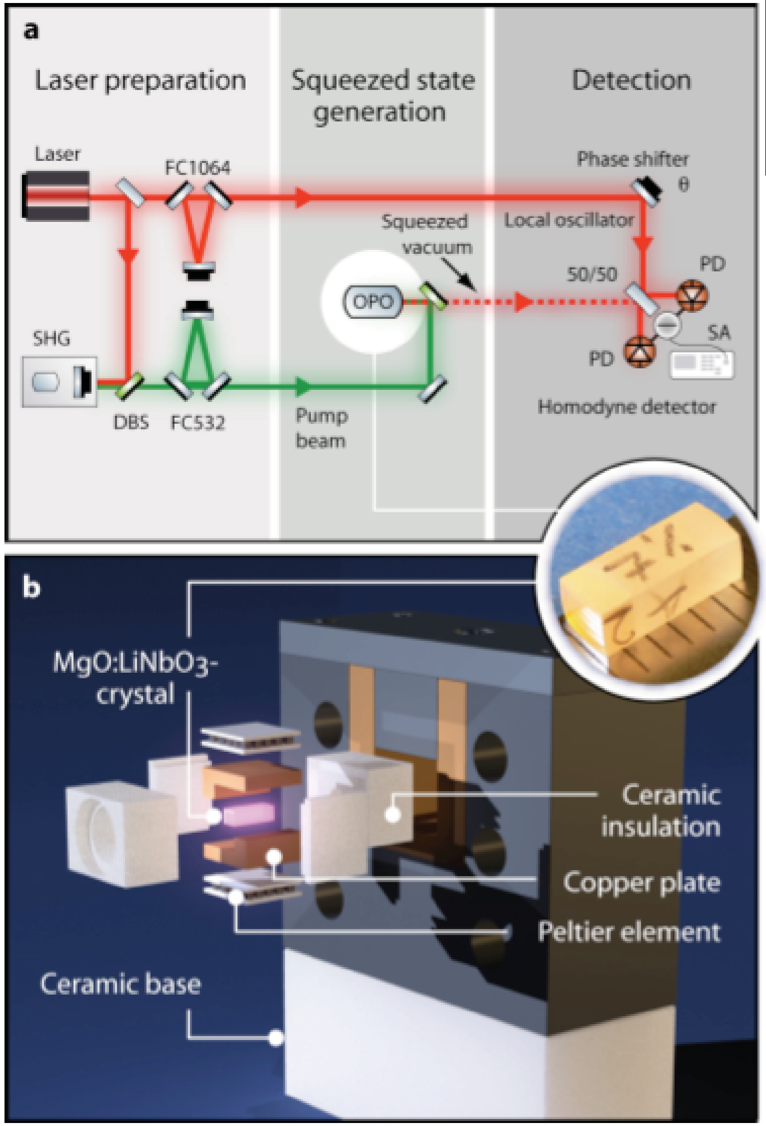
Observation of squeezed states with strong photon number oscillations

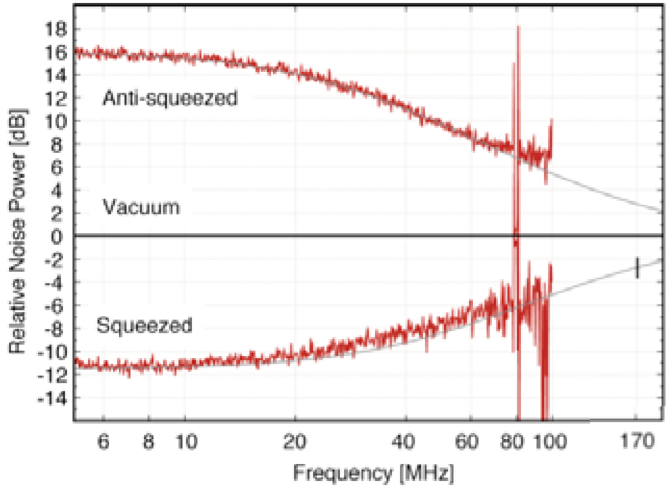
Moritz Mehmet,[1,2] Henning Vahlbruch,[1] Nico Lastzka,[1] Karsten Danzmann,[1] and Roman Schnabel*[1]

[1]*Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut) and Institut für Gravitationsphysik, Leibniz Universität Hannover, Callinstr. 38, 30167 Hannover, Germany*

[2]*Centre for Quantum Engineering and Space-Time Research - QUEST, Leibniz Universität Hannover, Welfengarten 1, 30167 Hannover, Germany*

Phys. Rev. A 81, 013814 (2010)





~12 dB of squeezing (94%)

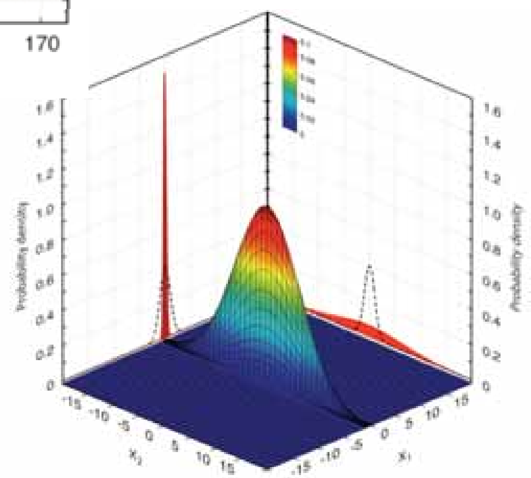


FIG. 3: Wigner function of the squeezed vacuum state produced

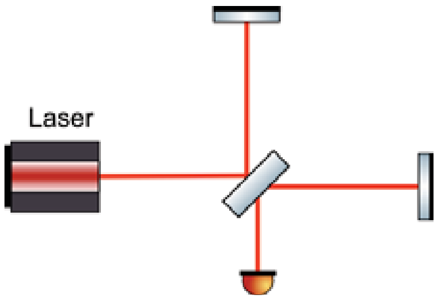
See also works from Paris, Copenhaguen, Tokyo, Naples, Canberra,…

GW Detection and Quantum Imaging

The GEO 600 squeezed light source

Henning Vahlbruch, Alexander Khalaidovski, Nico Lastzka, Christian Gräf, Karsten Danzmann, and Roman Schnabel
Institut für Gravitationsphysik of Leibniz Universität Hannover and Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut), Callinstr. 38, 30167 Hannover, Germany

Class.Quant.Grav.27:084027 (2010)



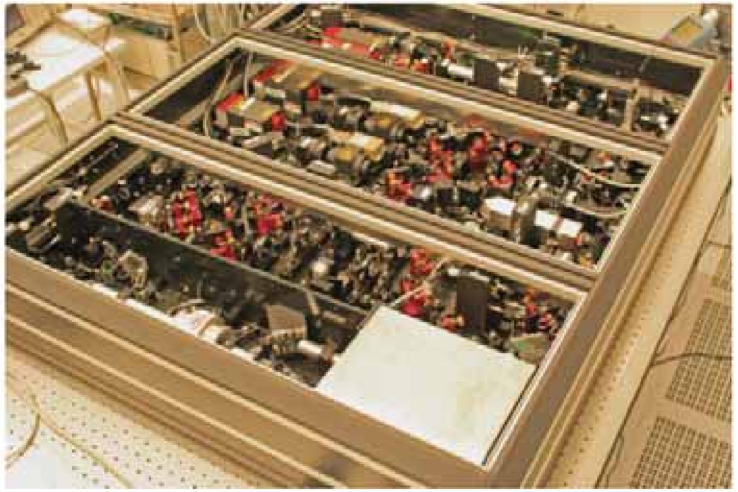
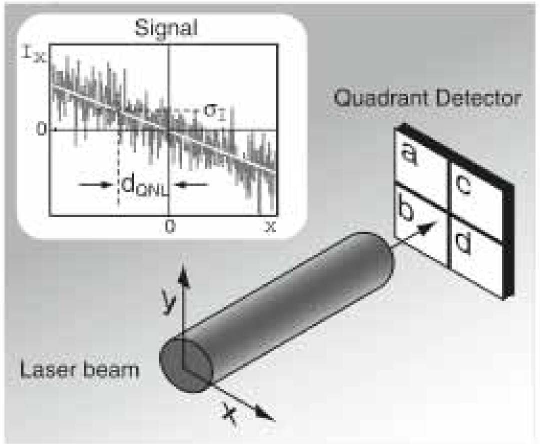


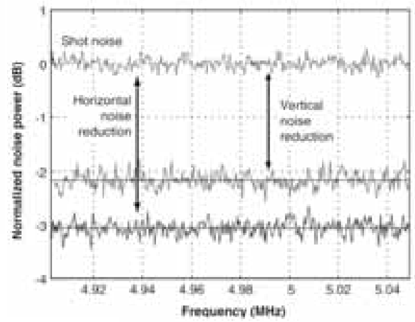
Figure 3. Photograph of the GEO 600 squeezed light source. The breadboard dimensions are 135 cm x 113 cm. The three Nd:YAG Lasers are located on the upper left, at the bottom left the squeezing resonator and on the bottom right the homodyne detector with its covering box is shown. The total weight of the complete system is approximately 130 kg.

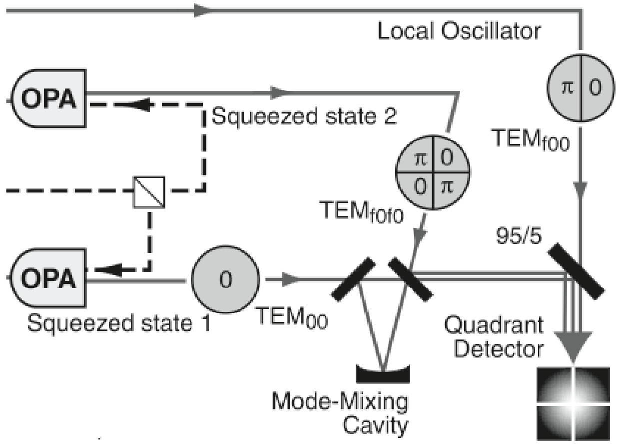
A Quantum Laser Pointer

Nicolas Treps,[1,2*] Nicolai Grosse,[1] Warwick P. Bowen,[1] Claude Fabre,[2] Hans-A. Bachor,[1] Ping Koy Lam[1]

Science 301, 940 (2003)





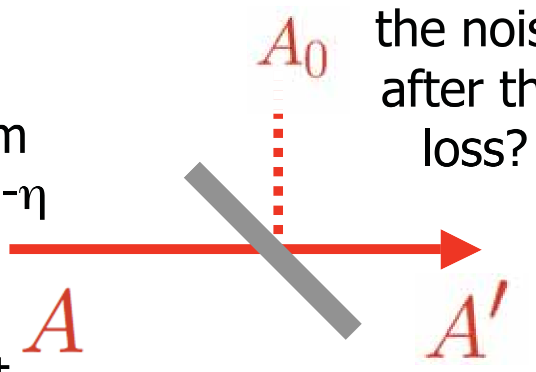


Effect of Losses on Squeezed Light

Model of losses in quantum optics

Intensity loss : 1-η

Can be modelled by a beam splitter with reflectivity R=1-η and transmitivity T= η. Vacuum fluctuations are entering by the empty port.



What is the noise after the loss?

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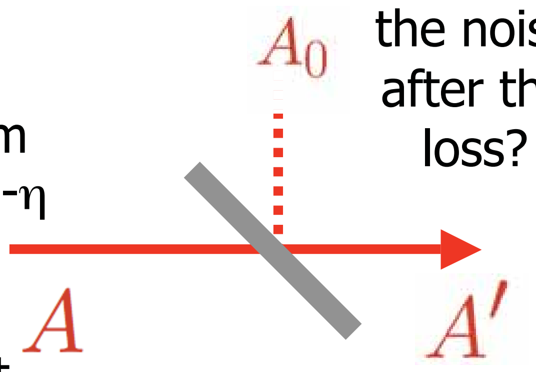

What is the noise after the loss?

Calculations

- The beam-splitter gives: $A' = \sqrt{\eta}A + \sqrt{1-\eta}A_0$
- We linearize the fluctuations : $A = \bar{A} + \delta A$ and $A_0 = \delta A_0$

and obtain for a quadrature P: $\delta A' = \sqrt{\eta}\delta A + \sqrt{1-\eta}\delta A_0 \rightarrow \delta P_{A'} = \sqrt{\eta}\delta P_A + \sqrt{1-\eta}\delta P_{A_0}$

- We calculate the noise variance :

$$\langle|\delta P_{A'}|^2\rangle = \eta\langle|\delta P_A|^2\rangle + (1-\eta)\langle|\delta P_{A_0}|^2\rangle + \sqrt{\eta}\sqrt{1-\eta}\langle\delta P_A \delta P_{A_0}\rangle$$

=V′ ; =V ; =1 (shot) ; =0 (uncorrelated)

V′ goes to 1 (shot) for strong losses (η=0), ok!

$$V' = \eta V + (1-\eta)$$

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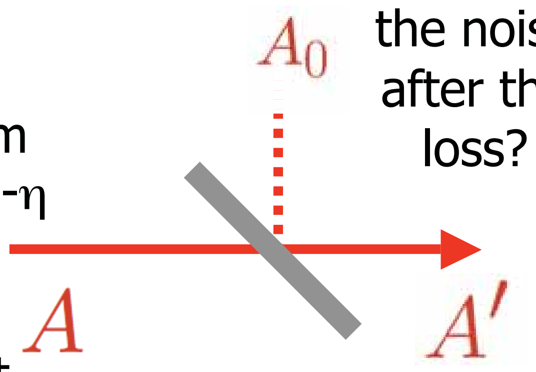

What is the noise after the loss?

Some illlustrative values.....

V=0.1 (10dB squeezing)
10% loss give:
V'=0.2 (7dB squeezing)

Squeezing is very sensitive to losses!

Calculations

- The beam-splitter gives: $A' = \sqrt{\eta} A + \sqrt{1-\eta} A_0$
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$$V' = \eta V + (1-\eta)$$

Lecture 1

- Light quantization, discrete vs continuous representation

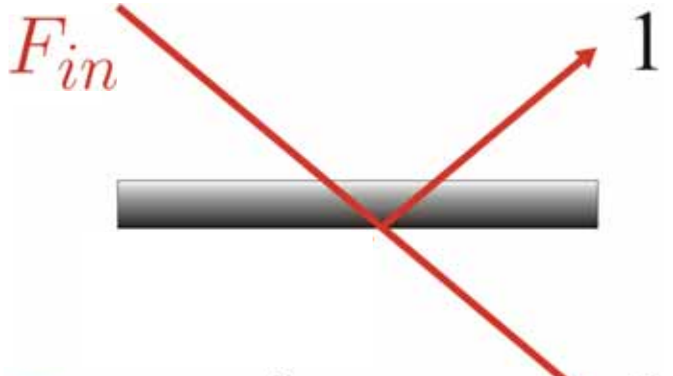
- Measuring and characterizing optical continuous variables

- How to generate squeezed light ?

- Quantum correlations and entanglement in the CV regime

Classical Correlations of Light Beams

A simple example : intensity « correlations » is easy to obtain….



$$F_{in} = \langle \delta P^2 \rangle = V(P)$$

Linear Correlation coefficient

$$C_{12} = \frac{\langle \delta P_1 \delta P_2 \rangle}{\sqrt{F_1 F_2}} \Rightarrow C_{12} = \frac{F_{in} - 1}{F_{in} + 1}$$

For a very noisy incident beam, the correlation goes to 1!

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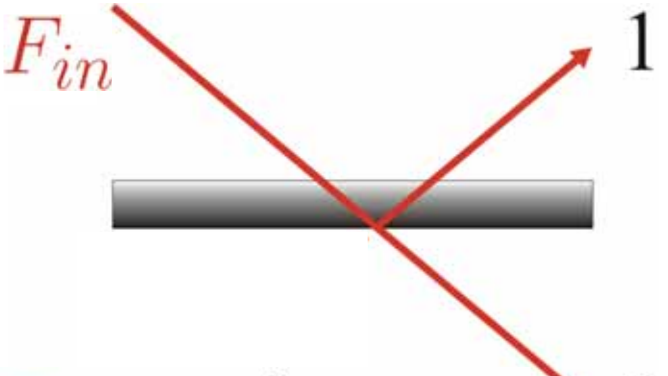


$$F_{in} = \langle \delta P^2 \rangle = V(P)$$

Normalized linear correlation coefficient

$$C_{12} = \frac{\langle \delta P_1 \delta P_2 \rangle}{\sqrt{F_1 F_2}} \Rightarrow C_{12} = \frac{F_{in} - 1}{F_{in} + 1}$$

For a very noisy incident beam, the correlation goes to 1 !

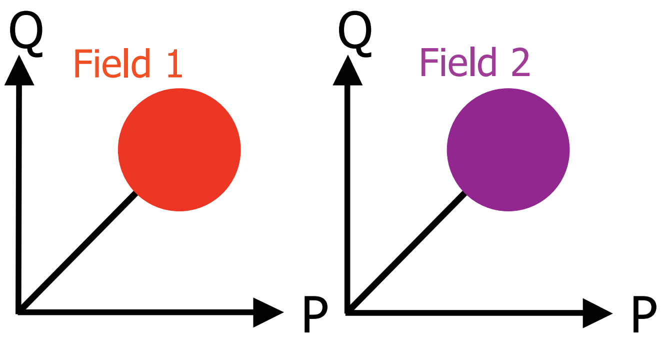
Question : How to define the quantum character of correlations between two beams in the CV regime?

Involving 1 quadrature
Gemellity
QND Correlation

Involving 2 quadratures
Inseparability
EPR Correlations

« One Quadrature » Criteria

Two beams, one quadature



- Difference between the fluctuations

$$P_- = P_1 - P_2$$

- Noise on this difference

$$G = \frac{V(P_-)}{2} = \frac{V(P_1 - P_2)}{2}$$

G=1 for two independent lasers (also for the previous case with noisy beam)

- Conditional Variance $V(P_1|P_2)$

$$V_c = 2G - \frac{G^2}{F} < 1$$

$$G \leq V_c \leq 2G$$

« One Quadrature » Criteria

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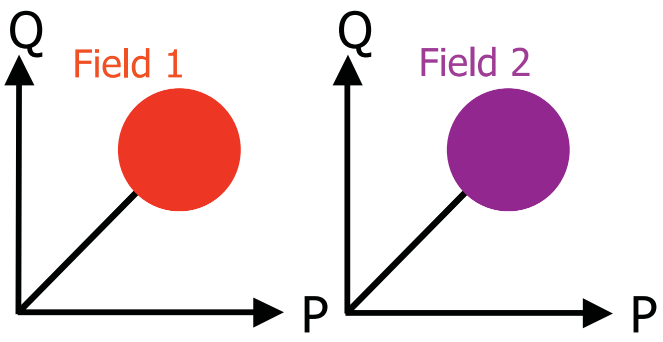

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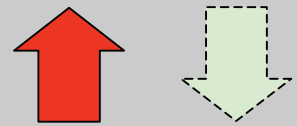
Gemellity : G<1

The correlation cannot be described by a semi-classical model involving classical electromagnetic fields having classical fluctuations.

Ex.: noise on the intensity difference below the shot noise level (Twin Beams)



$G < 0.5$

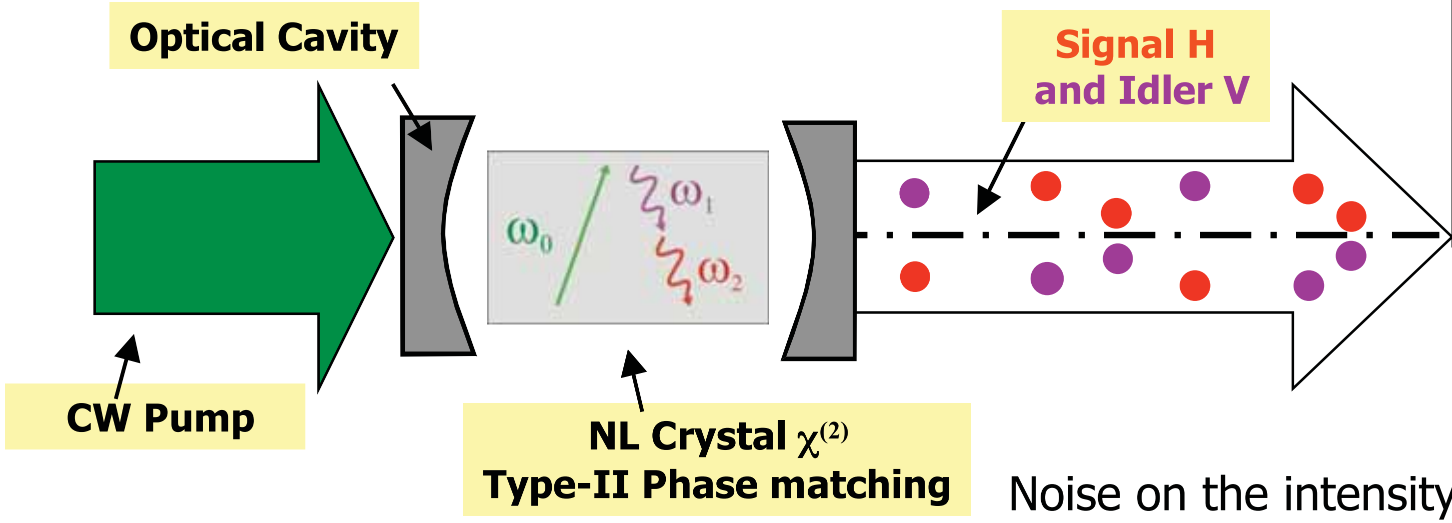
QND Correlation : V_c<1

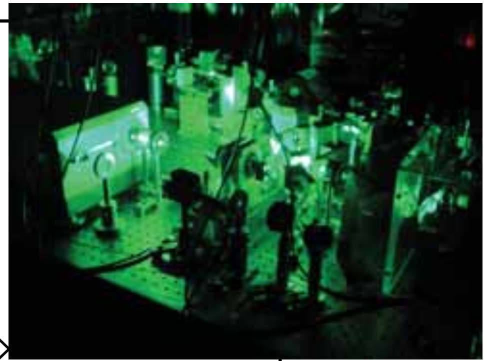
The information extracted from the measurement on one field provides a QND measurement of the other.

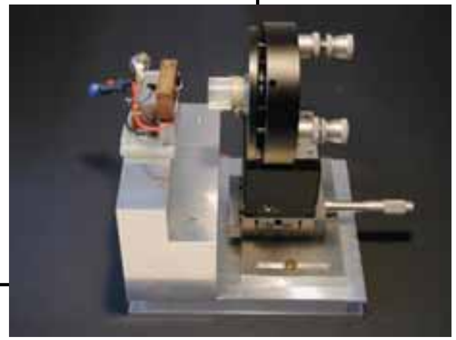
Ex.: in the perfect case, intensity measurement on field 1 gives without uncertainty the value for field 2

Quantum Intensity Correlations

With a type-II optical parametric oscillator







Noise on the intensity difference ?

*J. Laurat et al., Phys. Rev. Lett **91**, 213601 (2003); Optics Lett. 30, 1177 (2005); arXiv:quant-ph/0510063*

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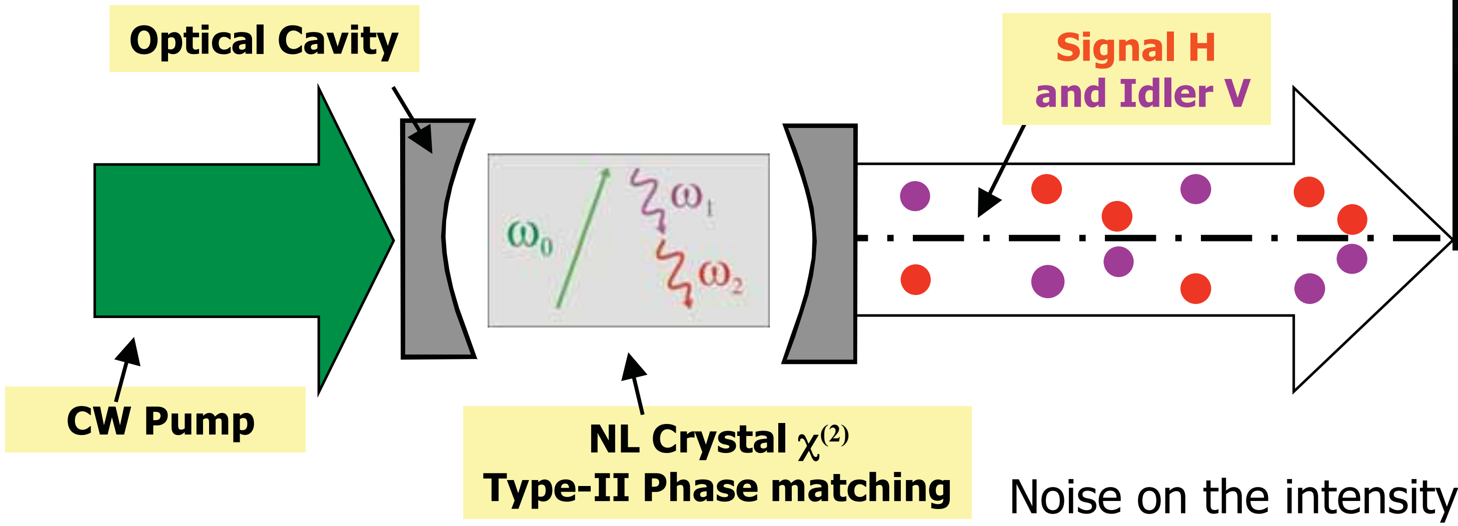
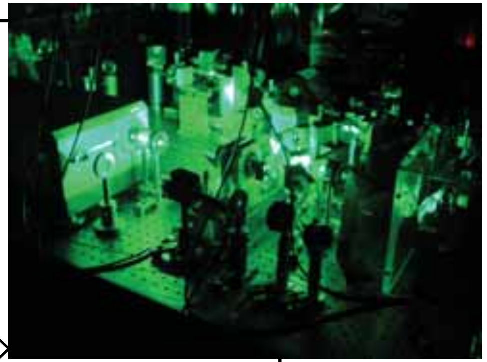
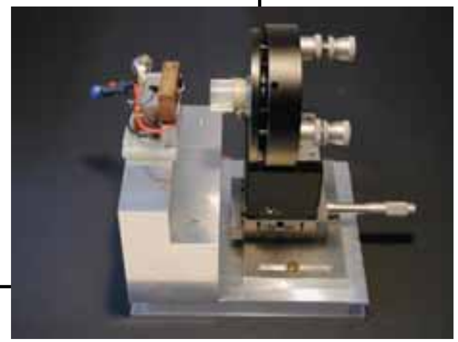
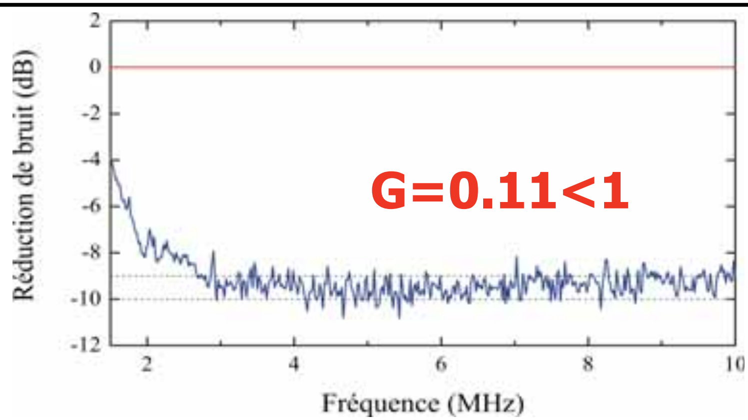


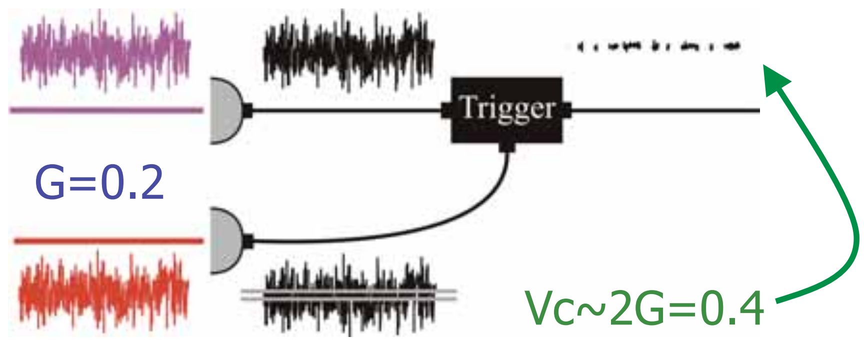




Noise on the intensity difference ?



Noise reduction on the intensity difference

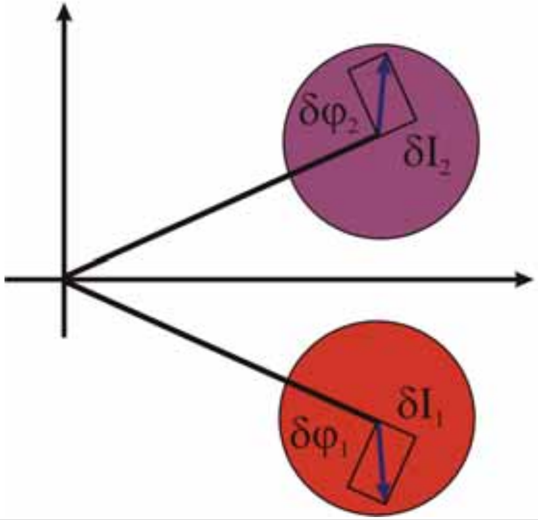


Conditional preparation of a sub-Poissonian beam

J. Laurat et al., Phys. Rev. Lett ***91****, 213601 (2003); Optics Lett. 30, 1177 (2005); arXiv:quant-ph/0510063*

« Two Quadratures » Criteria

A double correlation : « EPR Paradox »



$\delta I_1 = \delta I_2$
$\delta \varphi_1 = -\delta \varphi_2$

$$[P_1 - P_2, Q_1 + Q_2] = 0$$

$$\Rightarrow G_P = 0 \qquad G_Q = 0$$

Phys. Rev. ***47****, 777 (1935)*

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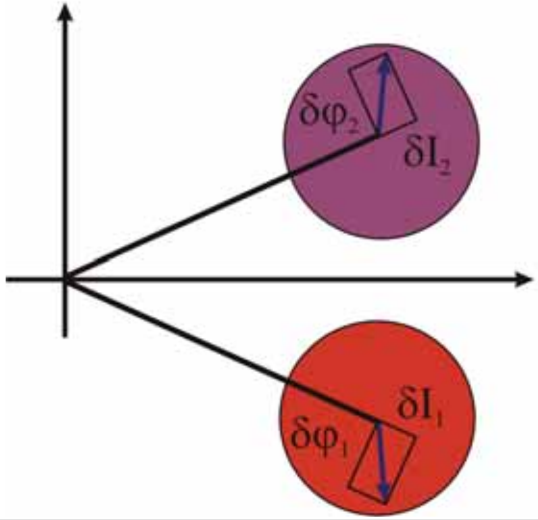

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Inseparabilty (Duan) : $\Sigma<1$

The correlation arises from from a system which can be described only by a non-separable state.

$$\Sigma = \frac{G_P + G_Q}{2} < 1$$

EPR Correlation (Reid) :

Apparent violation of the Heisenberg inequality for the quadatures of beam 1 through measurement performed on beam 2

$$V(P_A|P_B)\,V(Q_A|Q_B) < 1$$

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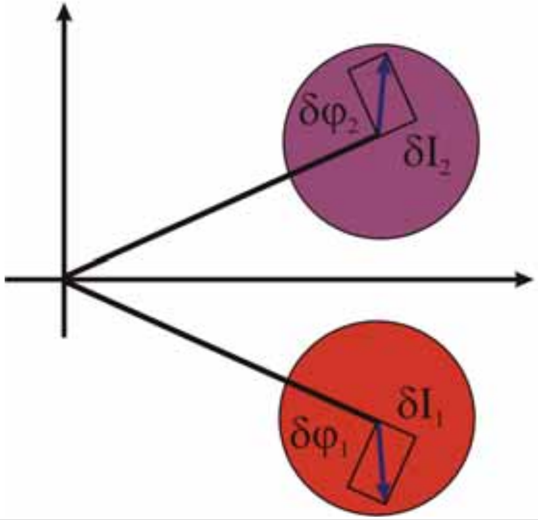

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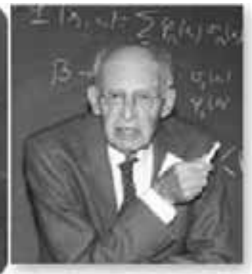

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Apparent violation of the Heisenberg inequality for the quadatures of beam 1 through measurement performed on beam 2

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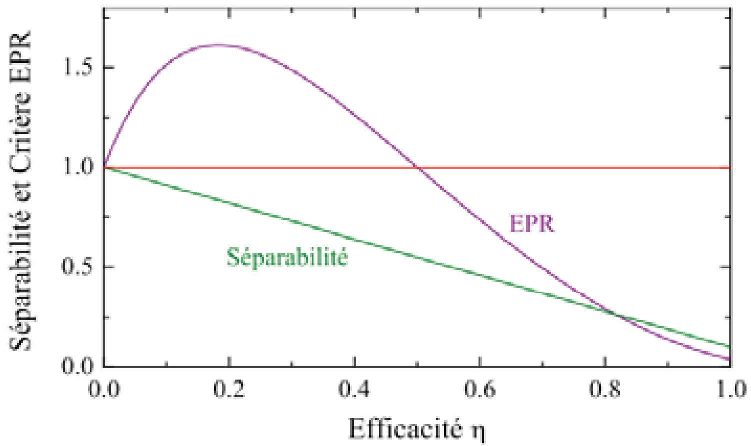
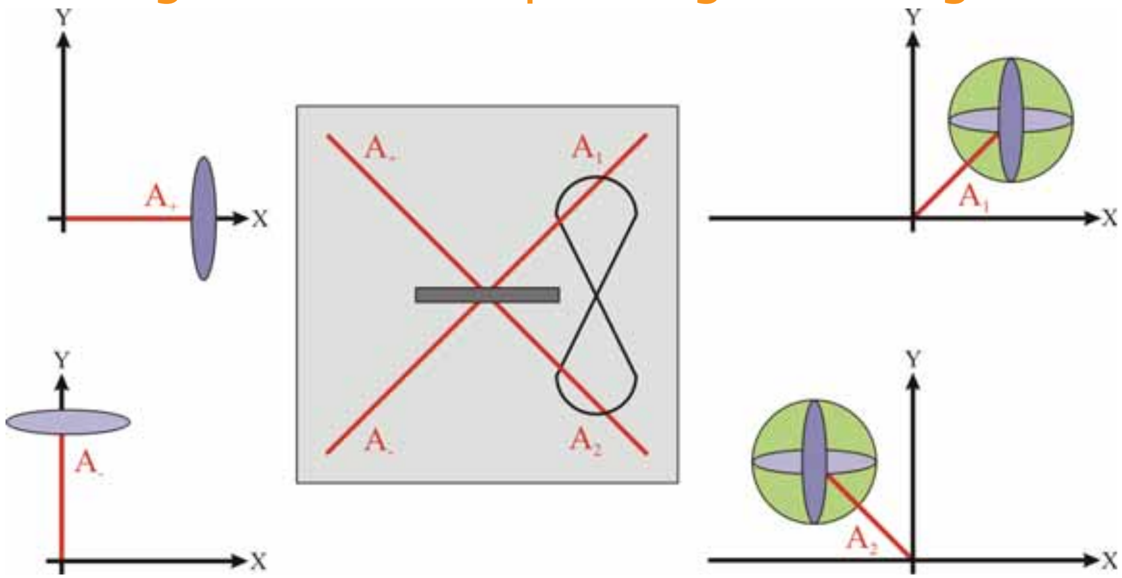


Separability is always preserved with losses
50% loss and EPR correlations disappear

How to Generate CV Entanglement ?

Entanglement and Squeezing : a change in basis

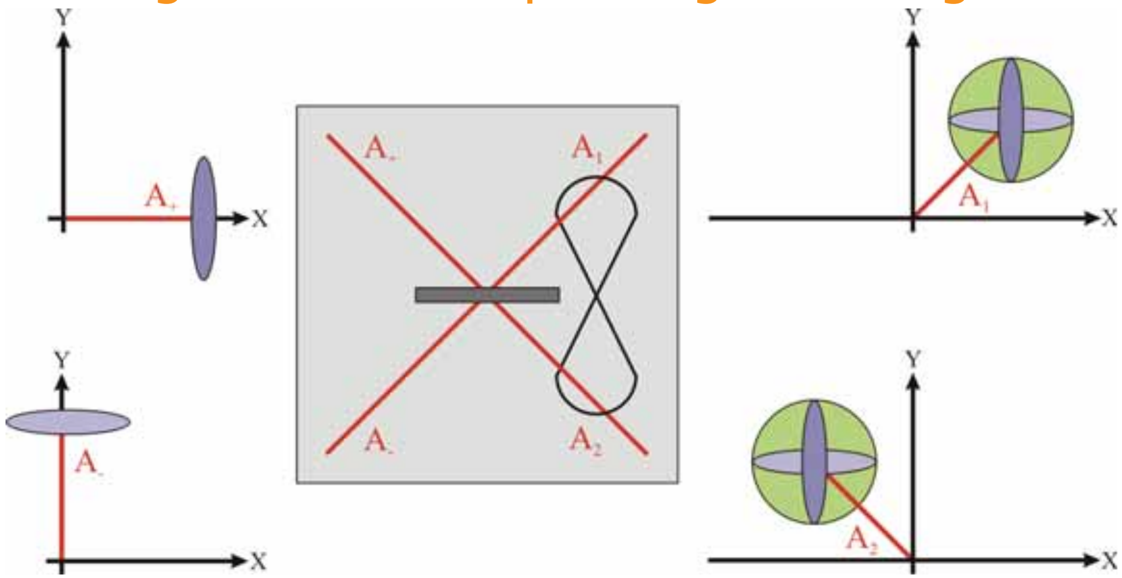


-Using 2 type-I OPO and mixing the two squeezed light on BS

- Using 1 type II OPO

How to Generate CV Entanglement ?

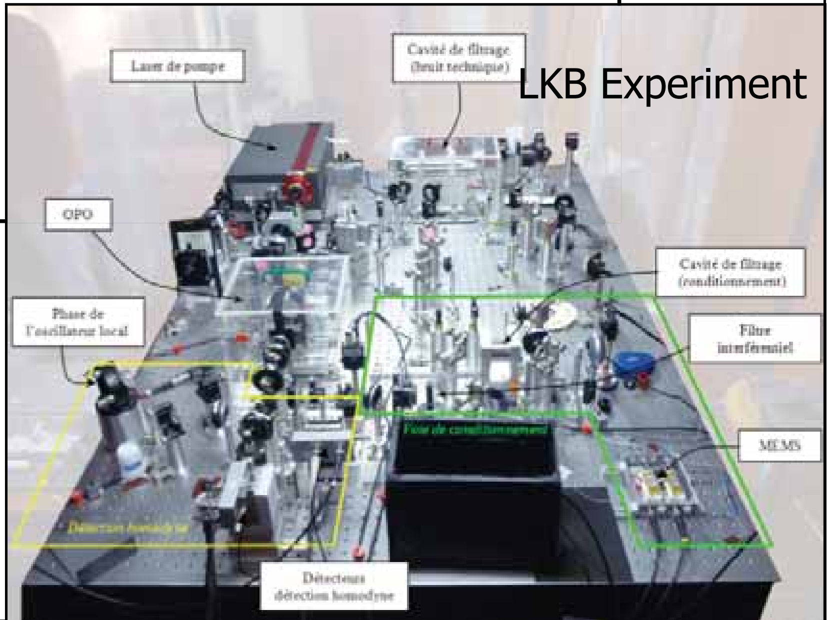
Entanglement and Squeezing : a change in basis

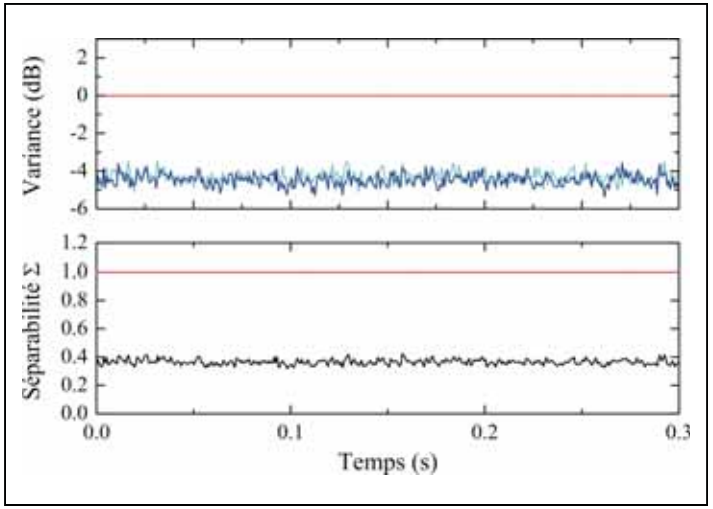


Canberra, Caltech, Copenhaguen, Tokyo,…

-Using 2 type-I OPO and mixing the two squeezed light on BS

- Using 1 type II OPO



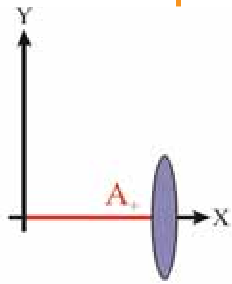


Separability $\Sigma = 0.33 \pm 0.02 < 1$
EPR: $Vc_1.Vc_2 = 0.42 \pm 0.05 < 1$
Teleportation Fidelity = 0.75

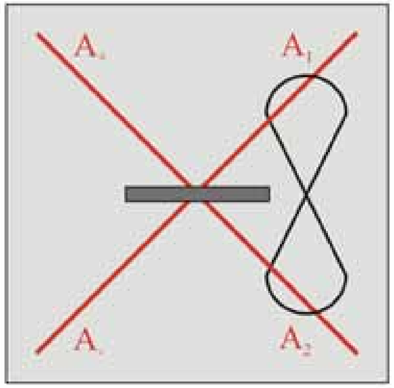
J. Laurat et al., Phys. Rev. A 71, 022313 (2005)

How to Generate CV Entanglement ?

With one squeezed beam?



Squeezing s<1
Antisqueezing
as>1



?

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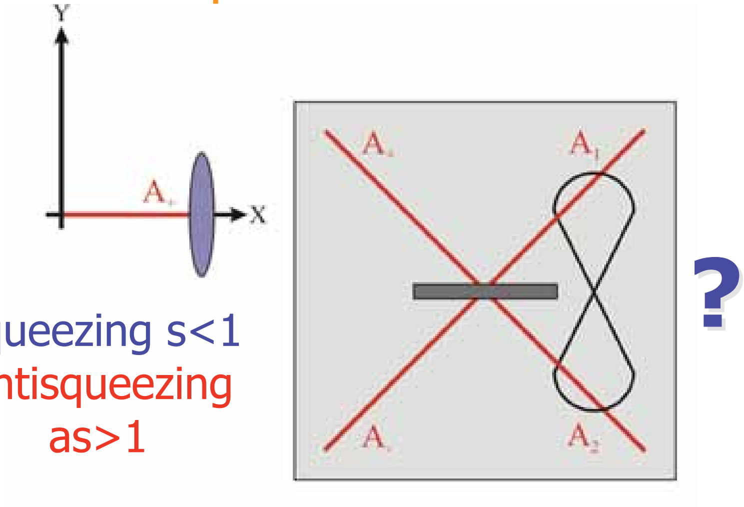


Squeezing s<1
Antisqueezing
as>1

$$\Sigma = \frac{1}{2}(1 + s)$$

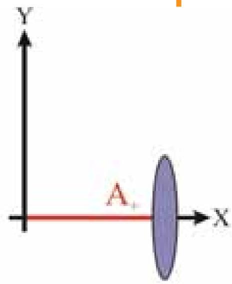
Always entangled…

$$V_{c1}.V_{c2} = \frac{4.(s.as)}{1 + s + as + (s.as)}$$

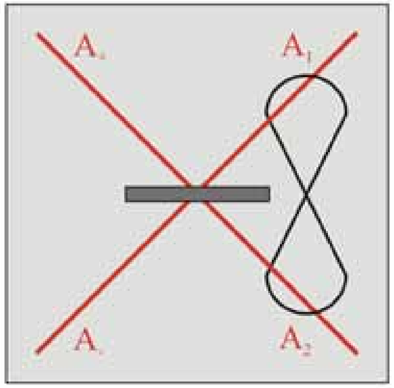
Sometimes EPR entangled : needs good squeezing and importantly good purity

How to Generate CV Entanglement ?

With one squeezed beam?



Squeezing s<1
Antisqueezing as>1



?

$$\Sigma = \frac{1}{2}(1+s)$$ Always entangled…

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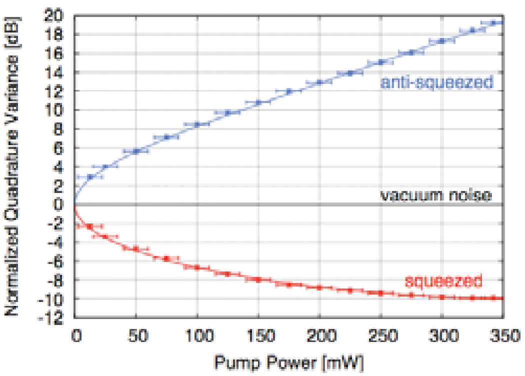
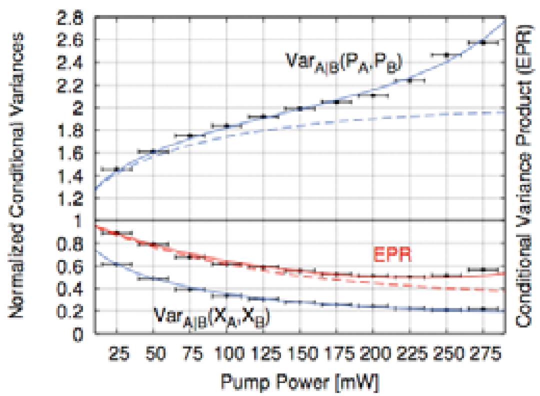




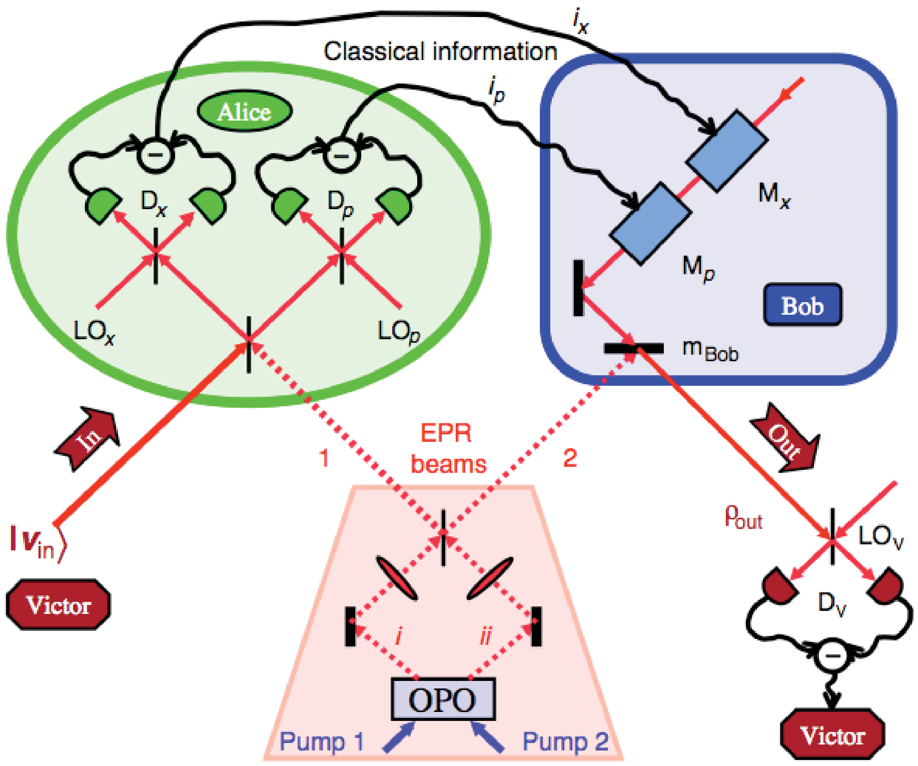
Einstein-Podolsky-Rosen Entanglement in a Vacuum-Class Two-Mode Squeezed State

Tobias Eberle,[1,2] Vitus Händchen,[1] Jörg Duhme,[2,3] Torsten Franz,[3] Reinhard F. Werner,[3] and Roman Schnabel[1,*]

[1] *Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut) and Institut für Gravitationsphysik der Leibniz Universität Hannover, Callinstraße 38, 30167 Hannover, Germany*
[2] *Centre for Quantum Engineering and Space-Time Research - QUEST, Leibniz Universität Hannover, Welfengarten 1, 30167 Hannover, Germany*
[3] *Institut für Theoretische Physik der Leibniz Universität Hannover, Appelstraße 2, 30167 Hannnover*

arXiv:1103.1817

Quantum Teleportation



1- Measurements

$$P = P_{in} - P_1$$

$$Q = Q_{in} + Q_1$$

2- Transmission (classical channel)

3- Modulations

$$P_{out} = P_2 + P = P_{in} + (P_2 - P_1) \quad (P_2 - P_1) \to 0$$

$$Q_{out} = Q_2 + Q = Q_{in} + (Q_1 + Q_2) \quad (Q_1 + Q_2) \to 0$$

First teleporation of a coherent states with Fidelity above 0.5 (1/2=classical strategy)

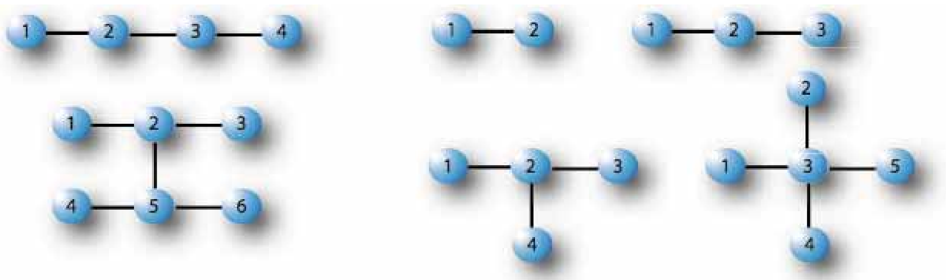
A. Furusawa et al., Unconditional quantum teleportation, Science 282, 706 (1998)

First teleportation of a cat states

N. Lee et al., Teleportation of non-classical wave-packets of light, Science 332, 330 (2011)

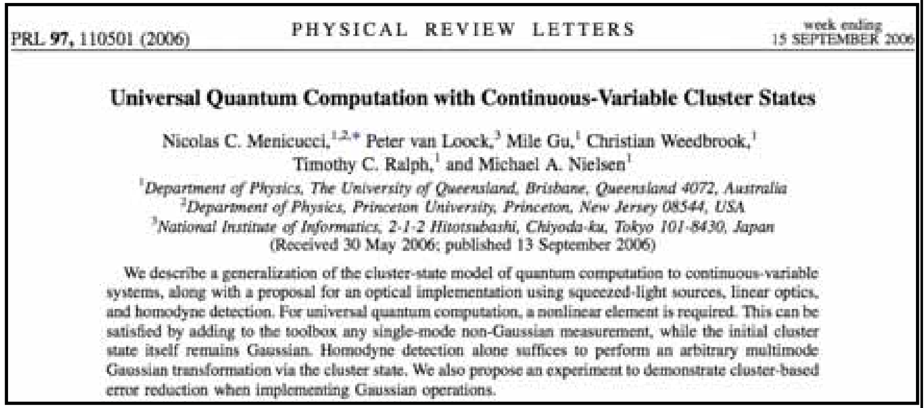
Multimode Entanglement : Cluster States

CV Cluster states



Quadrature correlations verifying:

$$\left(\hat{p}_a - \sum_{b \in N_a} \hat{x}_b\right) \rightarrow 0$$



PRL 97, 110501 (2006) PHYSICAL REVIEW LETTERS week ending 15 SEPTEMBER 2006

Universal Quantum Computation with Continuous-Variable Cluster States

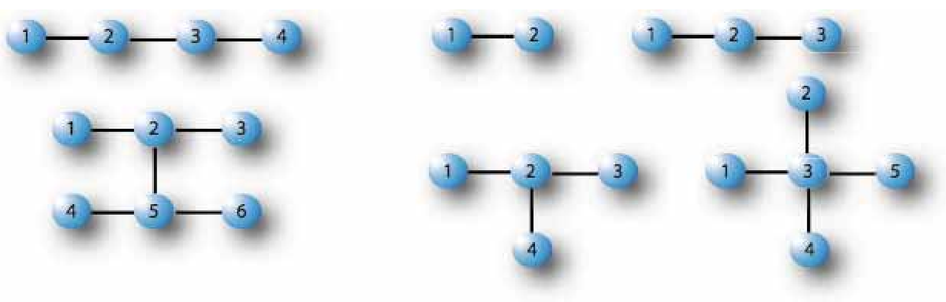
Nicolas C. Menicucci,[1,2,*] Peter van Loock,[3] Mile Gu,[1] Christian Weedbrook,[1] Timothy C. Ralph,[1] and Michael A. Nielsen[1]

[1]*Department of Physics, The University of Queensland, Brisbane, Queensland 4072, Australia*
[2]*Department of Physics, Princeton University, Princeton, New Jersey 08544, USA*
[3]*National Institute of Informatics, 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan*
(Received 30 May 2006; published 13 September 2006)

We describe a generalization of the cluster-state model of quantum computation to continuous-variable systems, along with a proposal for an optical implementation using squeezed-light sources, linear optics, and homodyne detection. For universal quantum computation, a nonlinear element is required. This can be satisfied by adding to the toolbox any single-mode non-Gaussian measurement, while the initial cluster state itself remains Gaussian. Homodyne detection alone suffices to perform an arbitrary multimode Gaussian transformation via the cluster state. We also propose an experiment to demonstrate cluster-based error reduction when implementing Gaussian operations.

Multimode Entanglement : Cluster States

CV Cluster states



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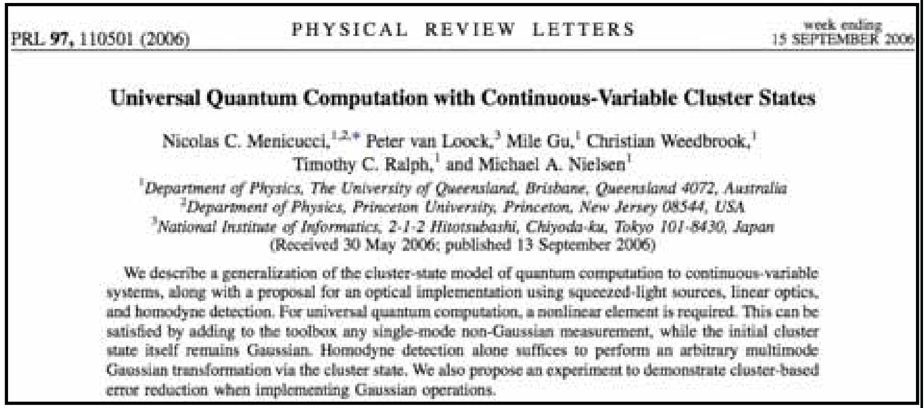

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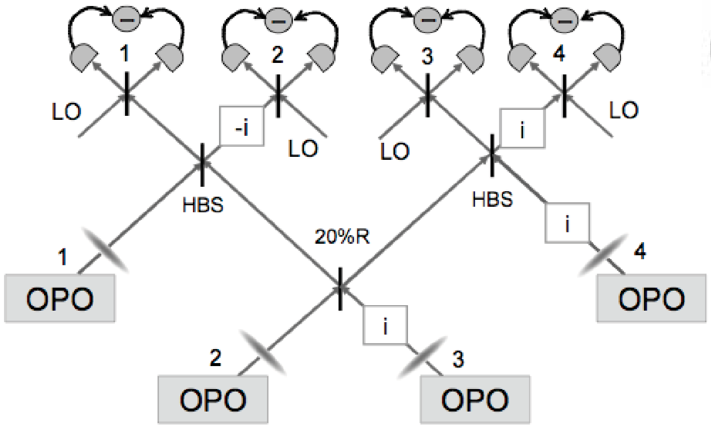
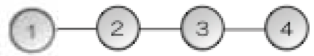
How to build them? With squeezers and beam splitters





With other phase combinations:

M. Yukawa et al., *Experimental generation of four-modes CV cluster states*, Phys. Rev. A **78**, 012301 (2008)

A last Level of Correlations : Non-locality

Violation of a Bell Inequality

The multiple correlations cannot be described by a local hidden variables

Fields with Gaussian statistics (Positive Wigner function) : can always be mapped into stochastic equations for fluctuating fields, which constitute the local « hidden » variables accounting for all the observed quantities…

Non-Gaussian states with Negative Wigner functions

A strongly active field: Schrodinger cat states, CV qubit, distillation

Hybrid Schemes

Requires non-Gaussian measurements or non-Gaussian ressources

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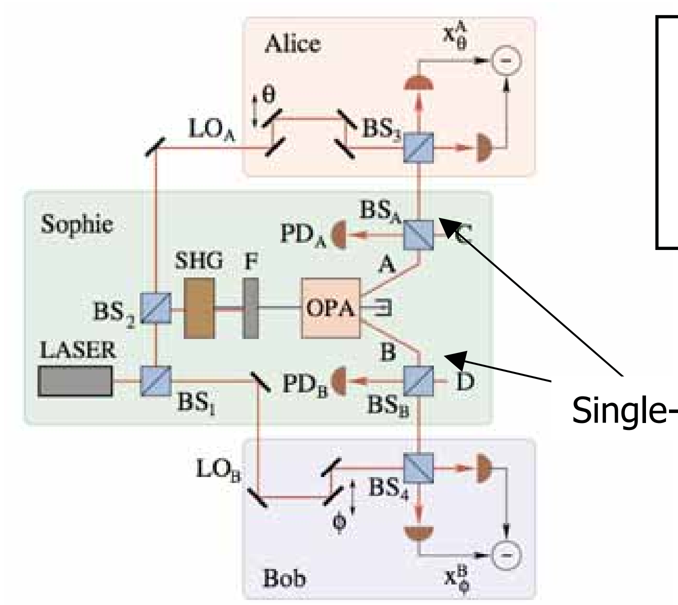


Single-photon subtractions

Proposal for a Loophole-Free Bell Test Using Homodyne Detection

R. García-Patrón,[1] J. Fiurášek,[1,2] N. J. Cerf,[1] J. Wenger,[3] R. Tualle-Brouri,[3] and Ph. Grangier[3]
[1]*QUIC, Ecole Polytechnique, CP 165, Université Libre de Bruxelles, 1050 Bruxelles, Belgium*
[2]*Department of Optics, Palacký University, 17. listopadu 50, 77200 Olomouc, Czech Republic*
[3]*Laboratoire Charles Fabry de l'Institut d'Optique, CNRS UMR 8501, F-91403 Orsay, France*

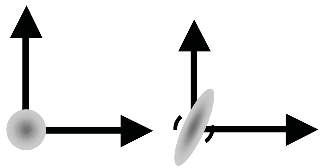
(Received 26 March 2004; published 23 September 2004)

Requires efficiency>95%, 6dB squeezing, high-purity of the initial state…
And gives at most S=2.05

R. Garcia-Patron et al., Phys. Rev. Lett. 93, 130409 (2004); and many other proposals since then…

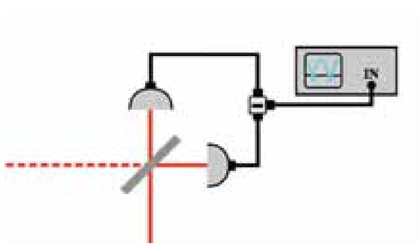
Summary

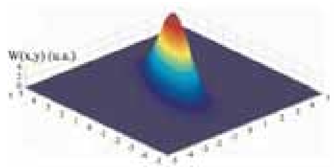
- Continuous-variable regime



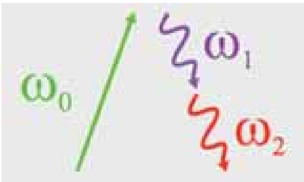
$$\hat{p} = \frac{\hat{a} + \hat{a}^{\dagger}}{\sqrt{2}} \qquad \hat{q} = \frac{\hat{a} - \hat{a}^{\dagger}}{i\sqrt{2}}$$

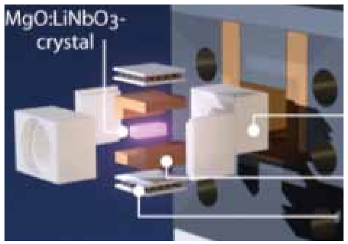
- Measuring optical CV





- *Squezed light generation by parametric amplification*





- *Quantum correlations : 5 levels*

Gaussian: Twins, QND,EPR

Non-Gaussian : Bell-type

Trend: Hybrid schemes (next lecture!)