## Exact probability distribution functions for Parrondo's games

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We consider discrete time Brownian ratchet models: Parrondo's games and the two envelope problem. Using the Fourier transformation, we calculate the exact probability distribution functions for both the capital dependent and history dependent Parrondo's games. We found that in some cases there are oscillations near the maximum of the probability distribution, and after many rounds there are two limiting distributions, for the odd and even total number of rounds of gambling. We assume that the solution of the aforementioned models can be applied to portfolio optimization.