

Activating teleportation power: theory & experiment

Yeong-Cherng Liang

Quantum Nonlocality, Foundations & Information Group

Center for Quantum Frontiers of Research & Technology (*QFort*)

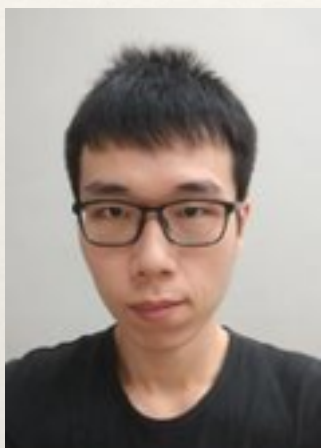
National Cheng Kung University

NCTS Annual Theory Meeting, Hsinchu, Taiwan, 17-19 February 2021



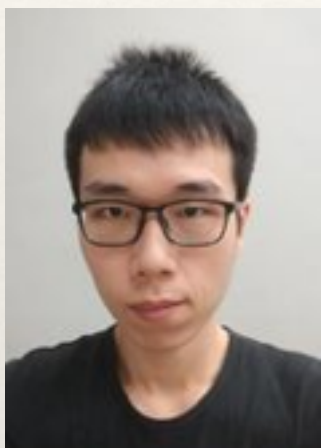
國立成功大學
National Cheng Kung University

Thanks to ...



Jyun-Yi Li

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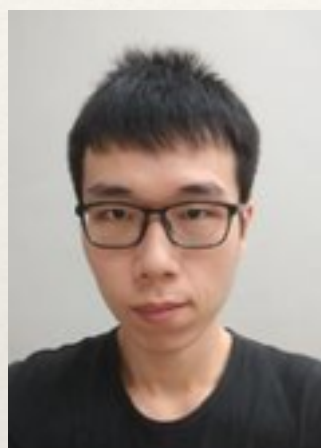


Jyun-Yi Li



Gelo Tabia

Thanks to ...



Jyun-Yi Li



Xiao-Xu Fang



Ting Zhang



Gelo Tabia



He Lu

Ideal Quantum Teleportation

One of the **first** protocols introduced in quantum information processing¹

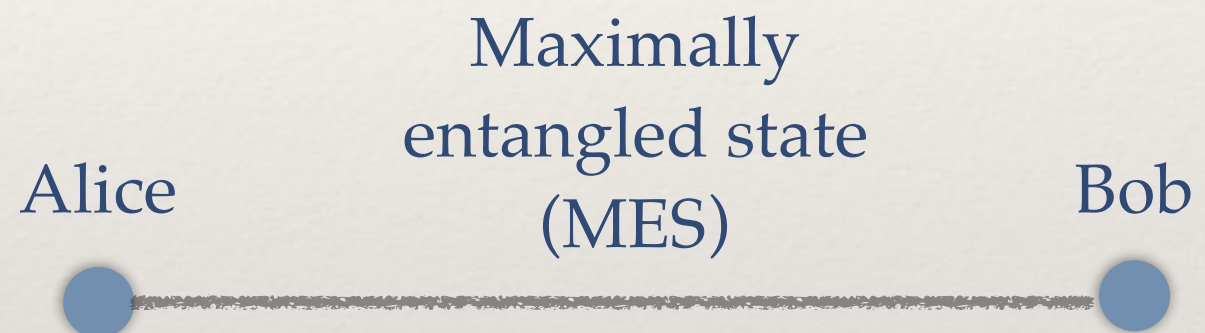
Alice

Bob

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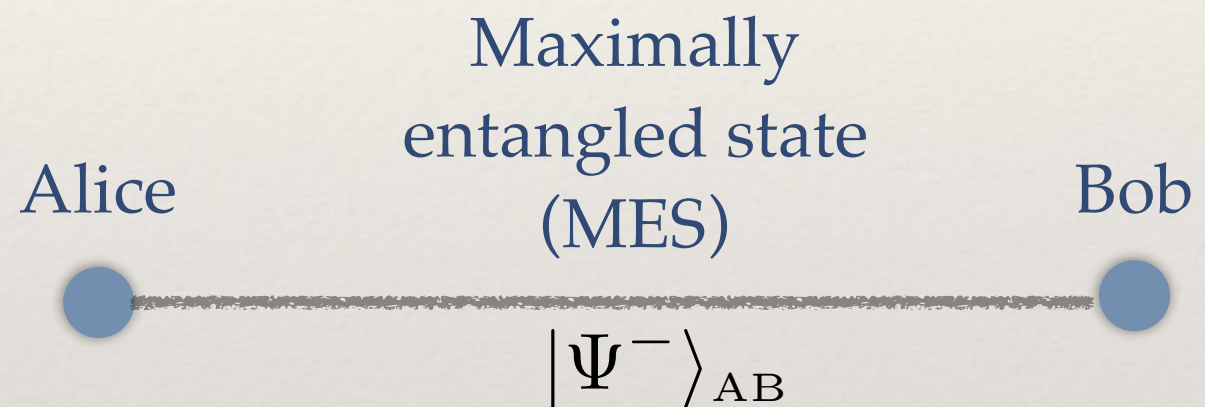
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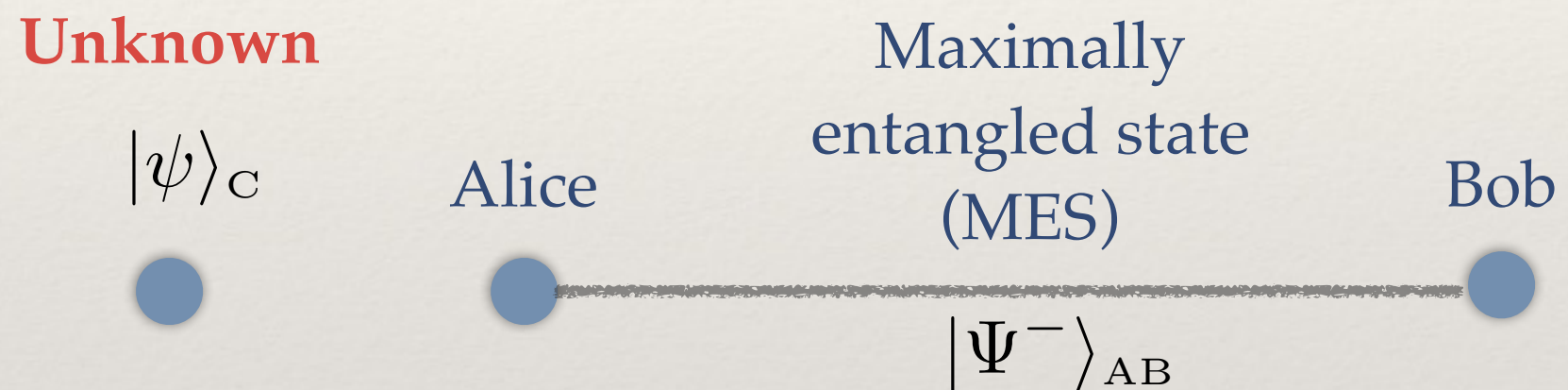
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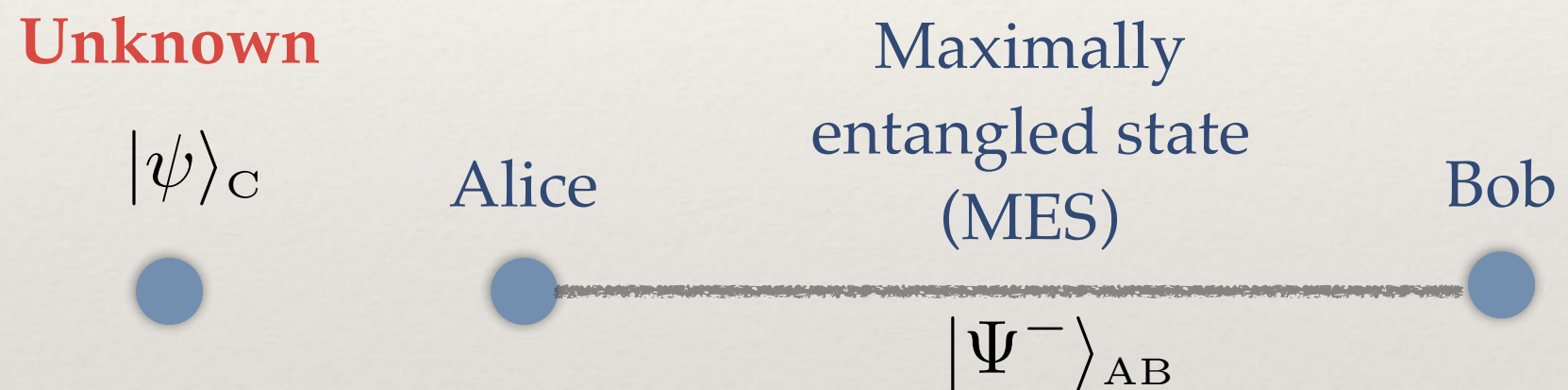
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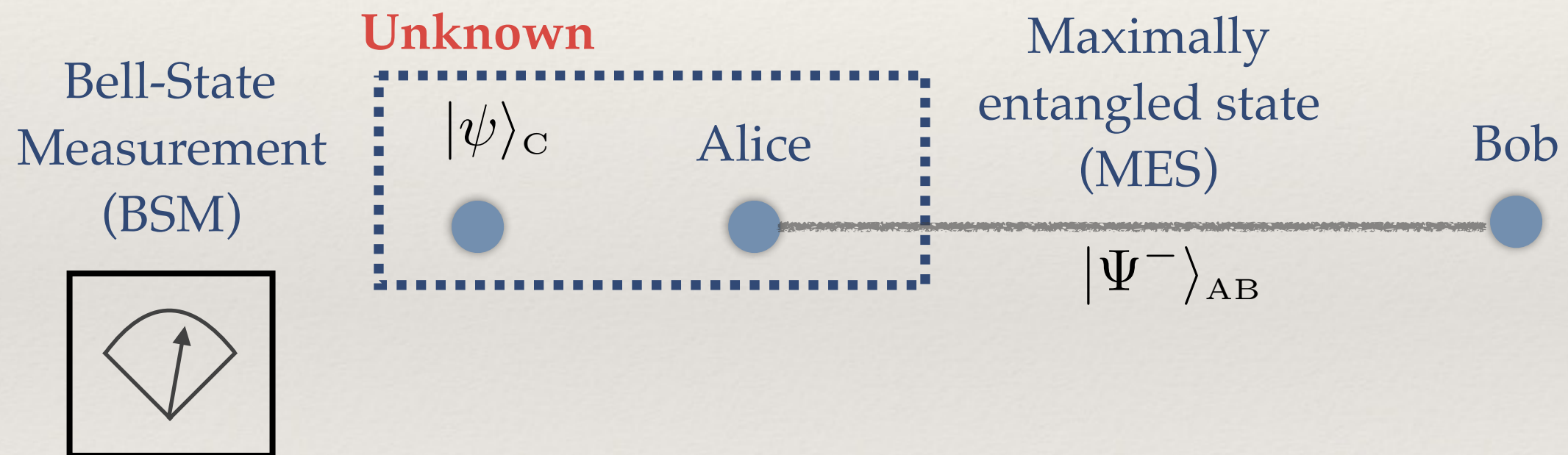
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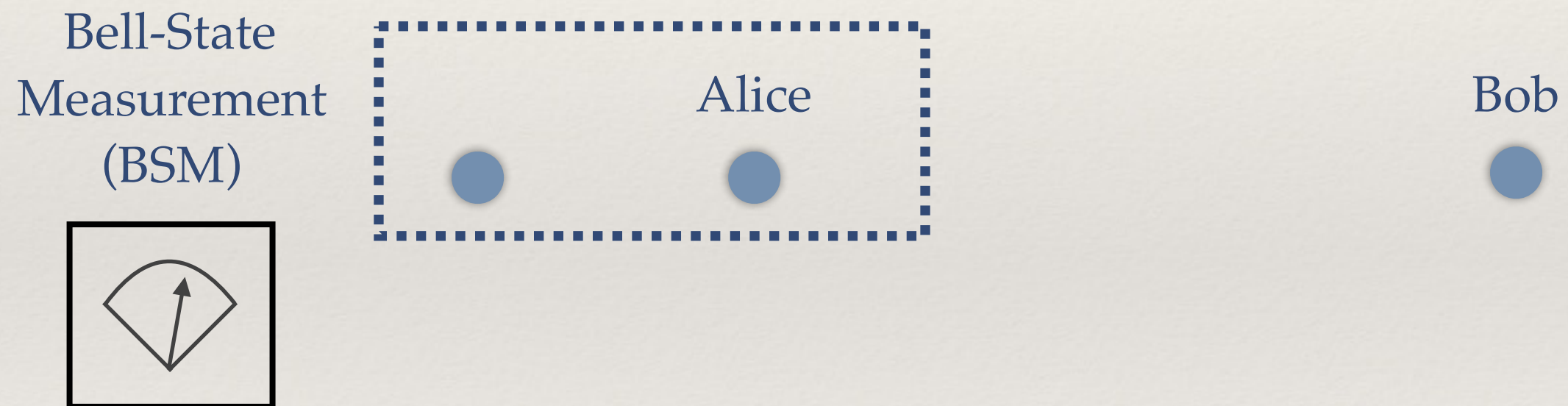
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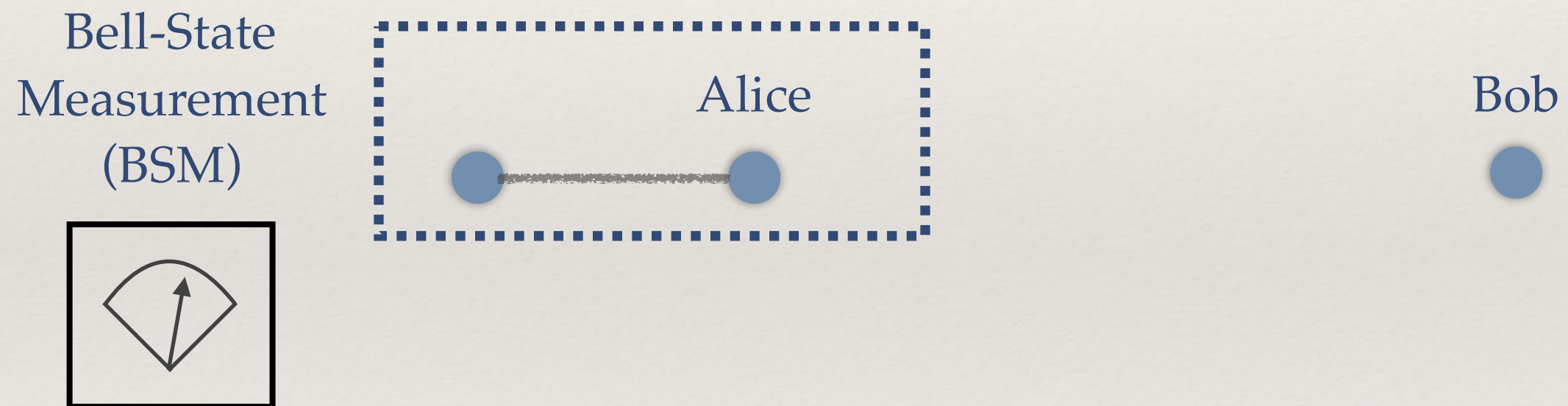
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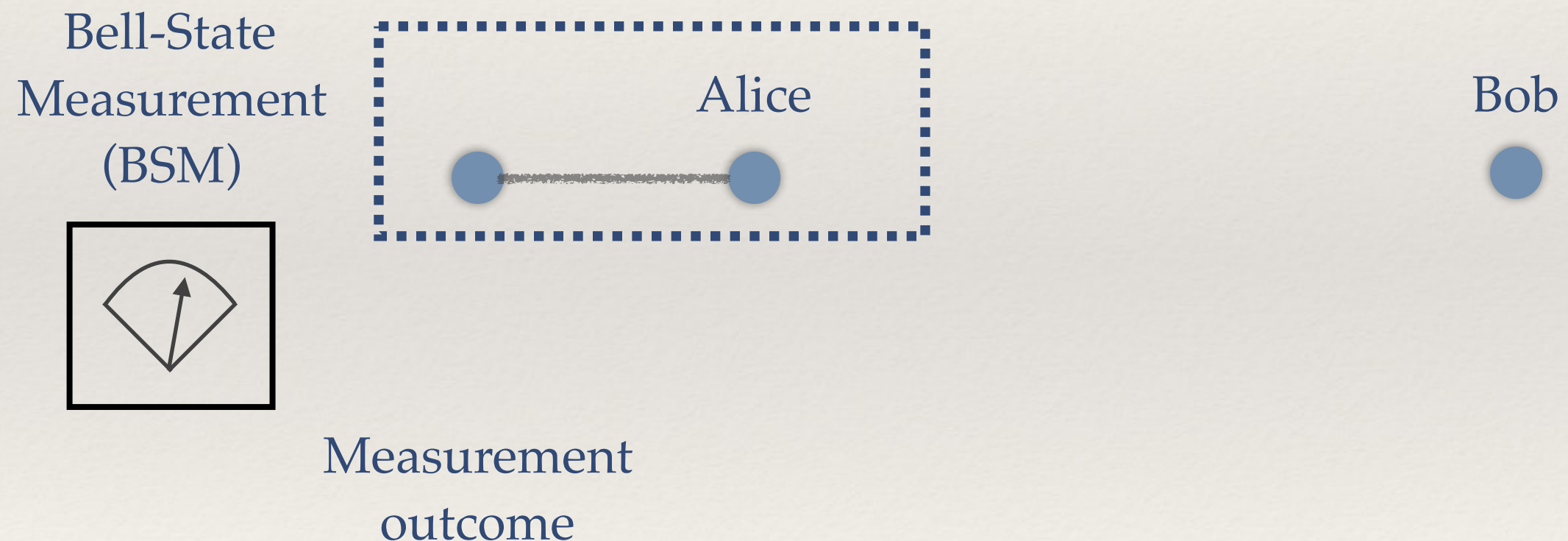
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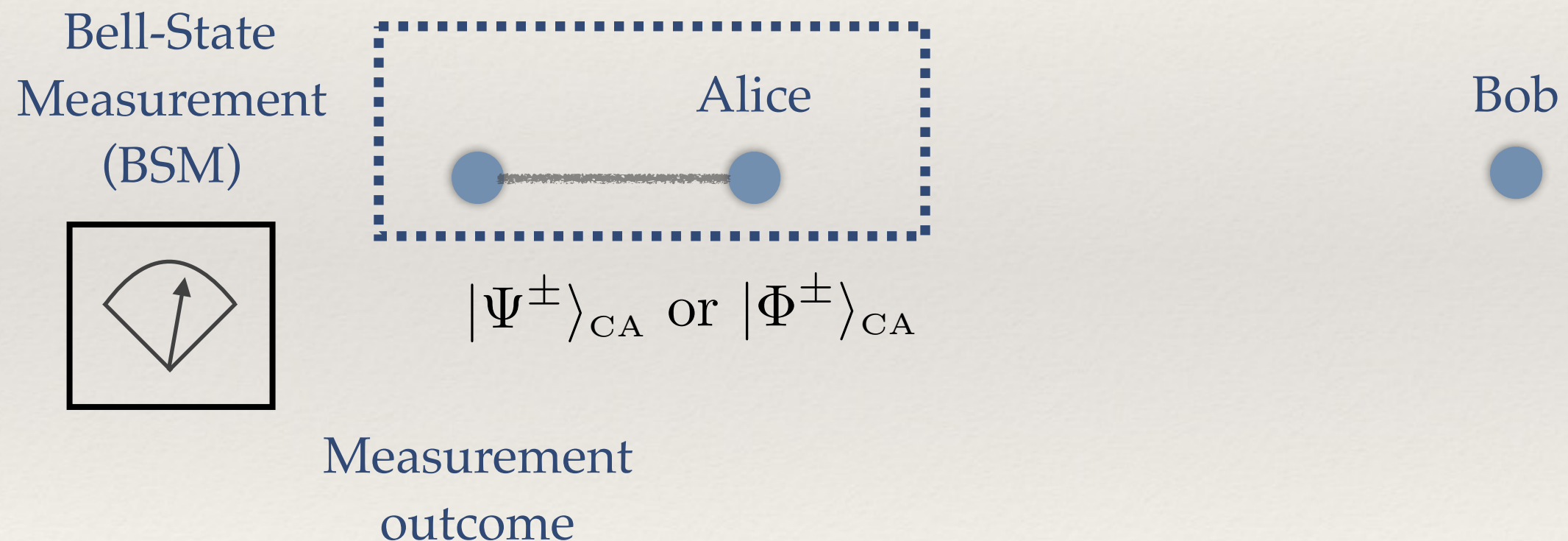
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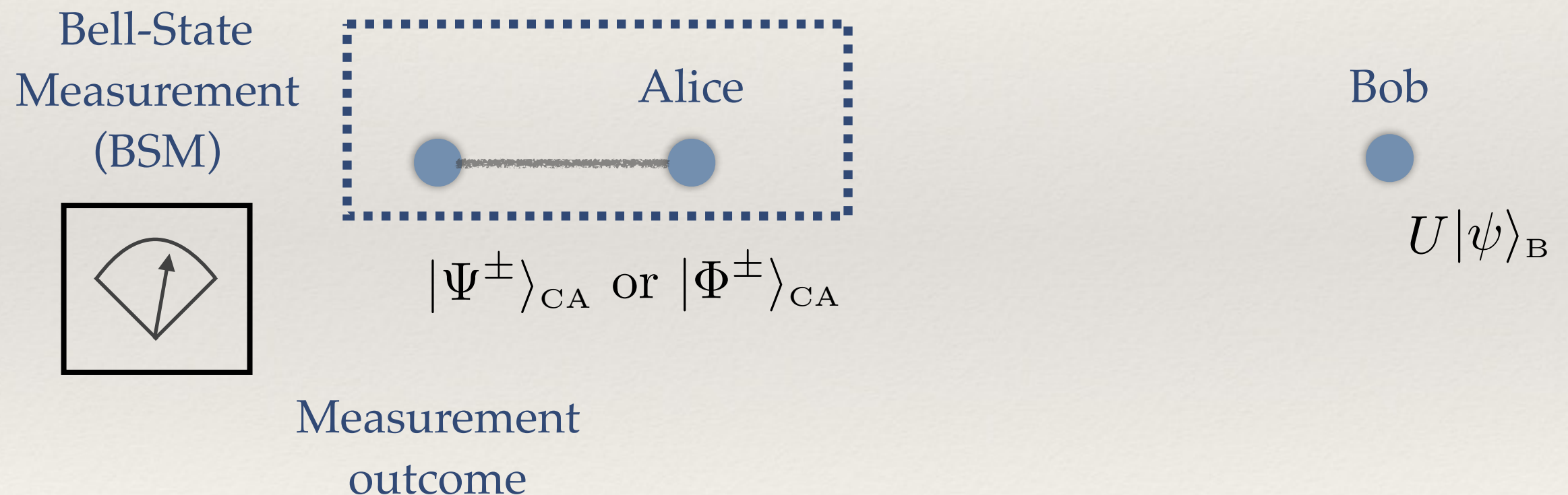
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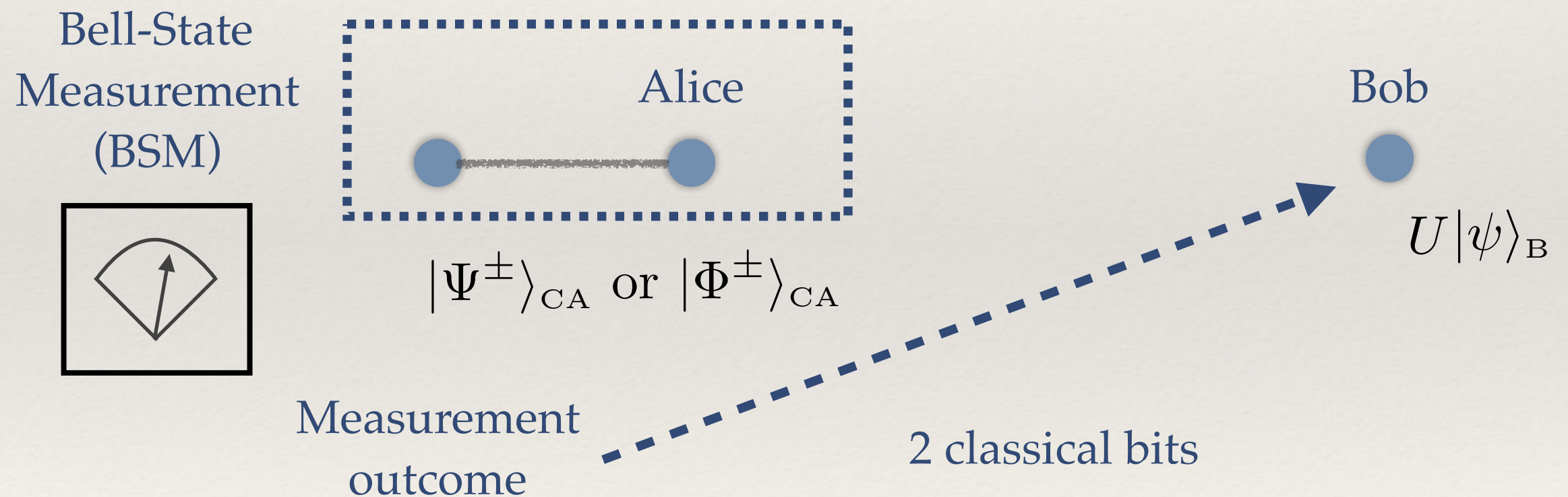
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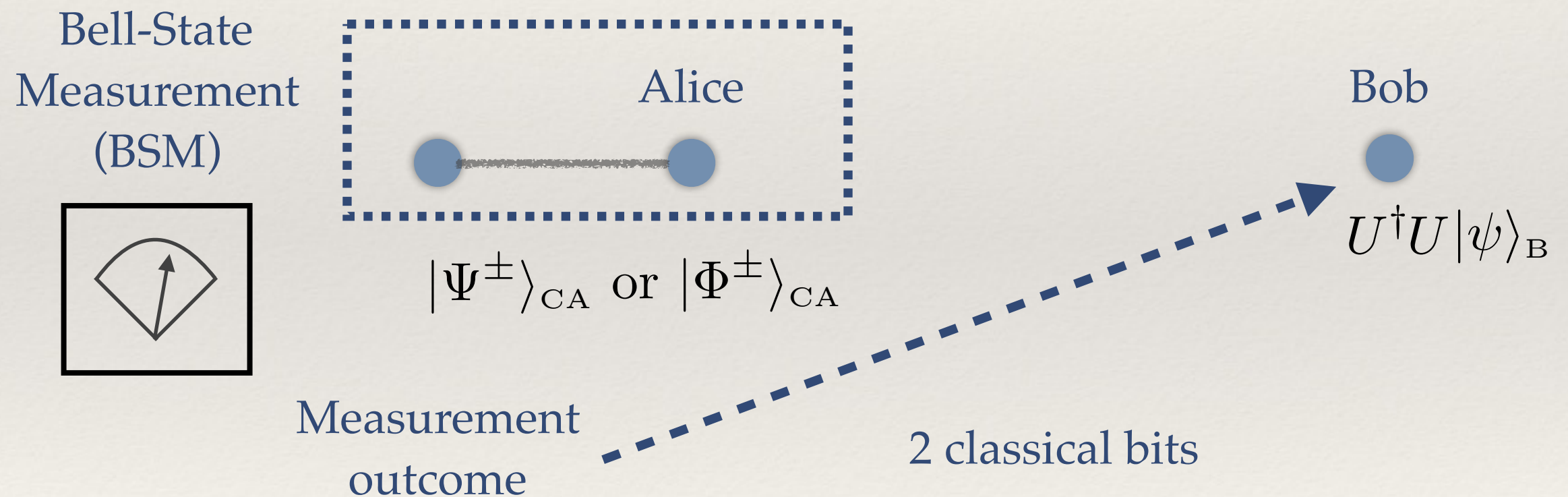
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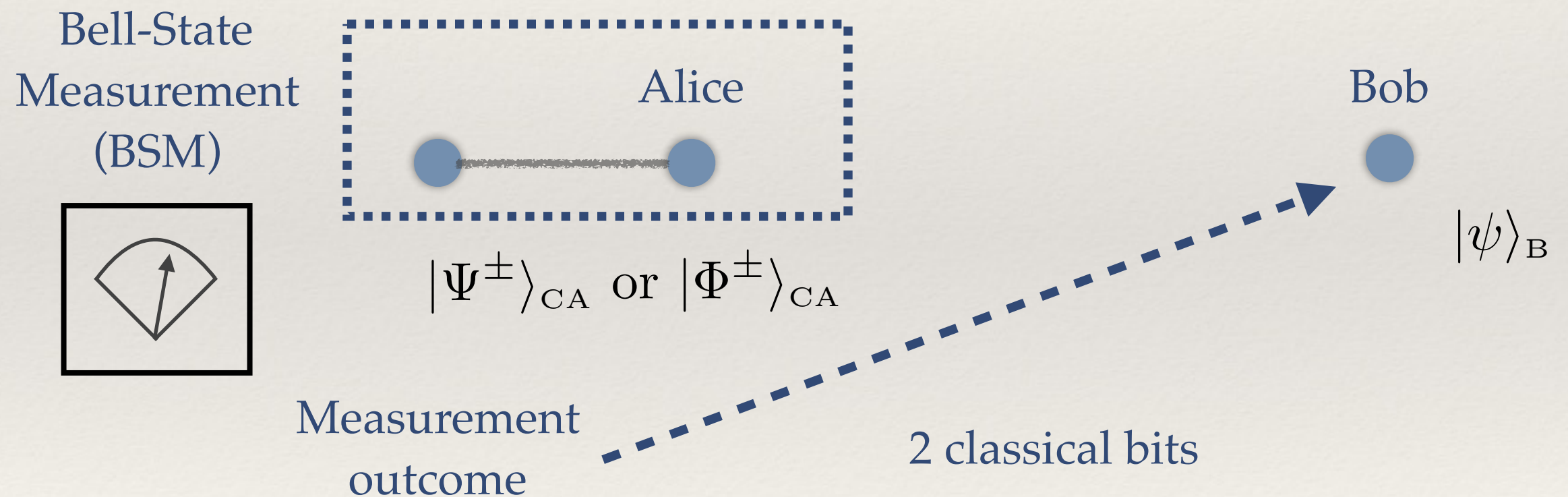
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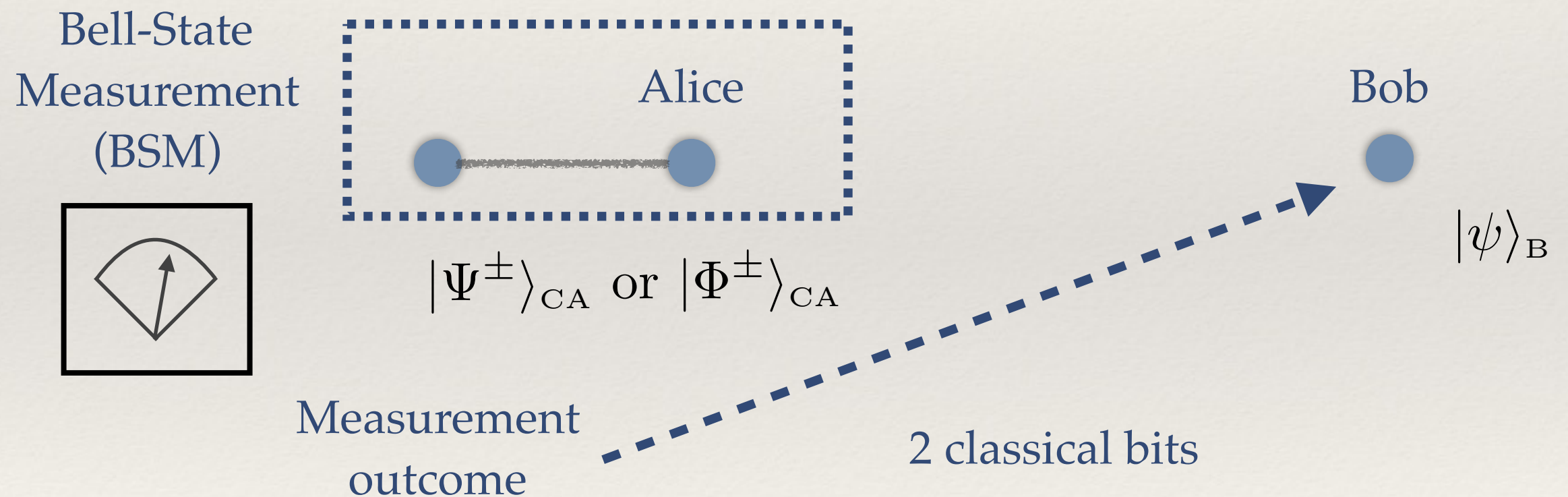
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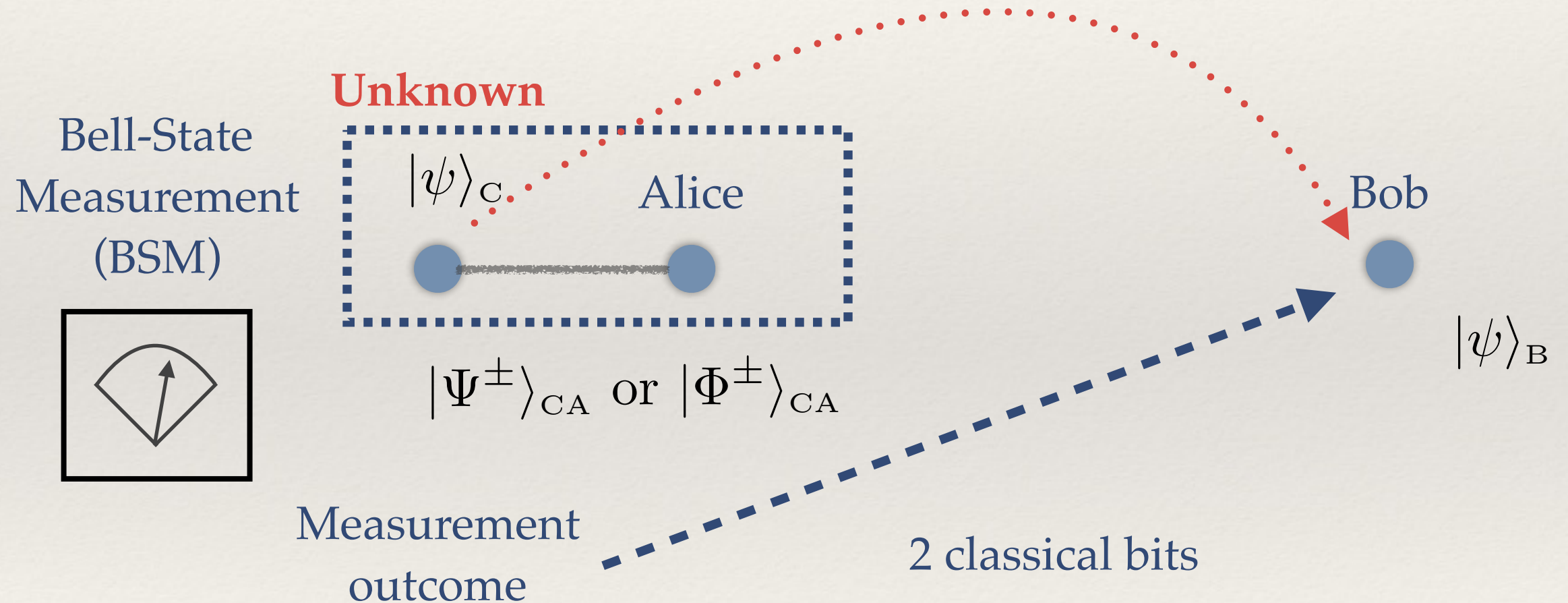
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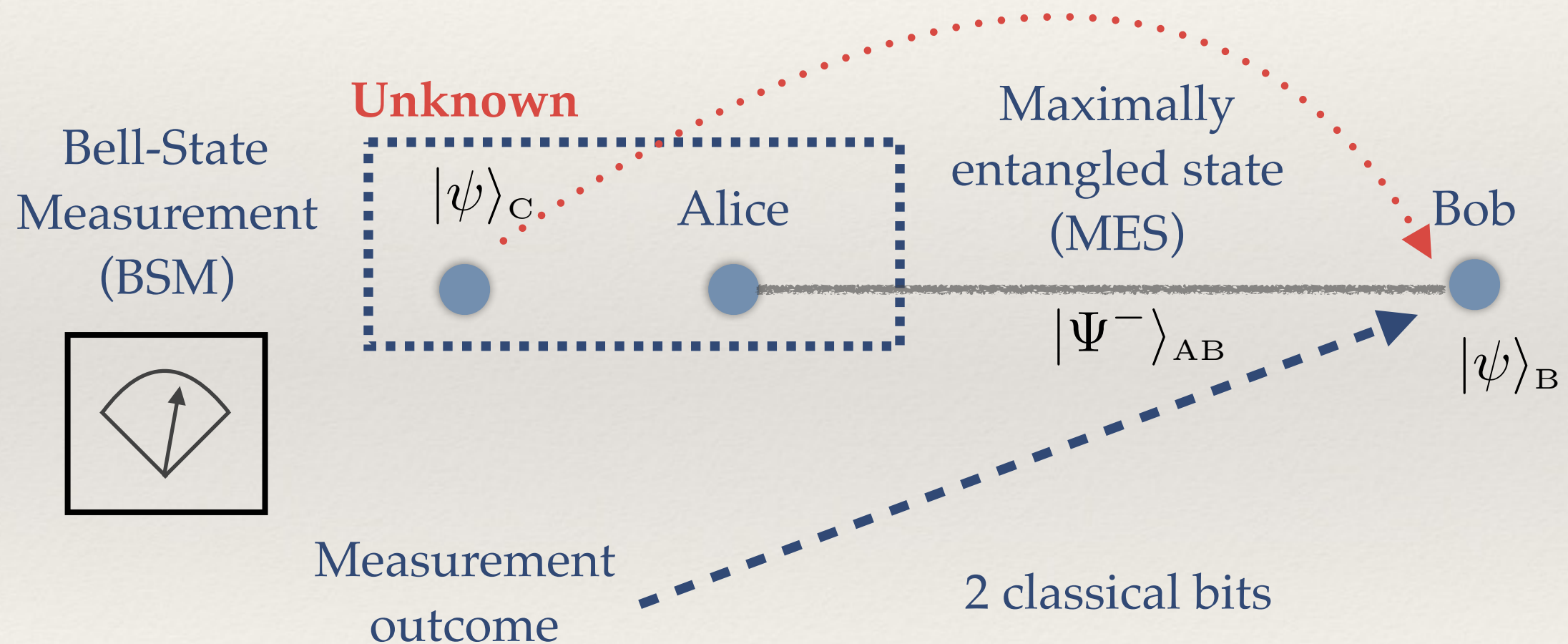
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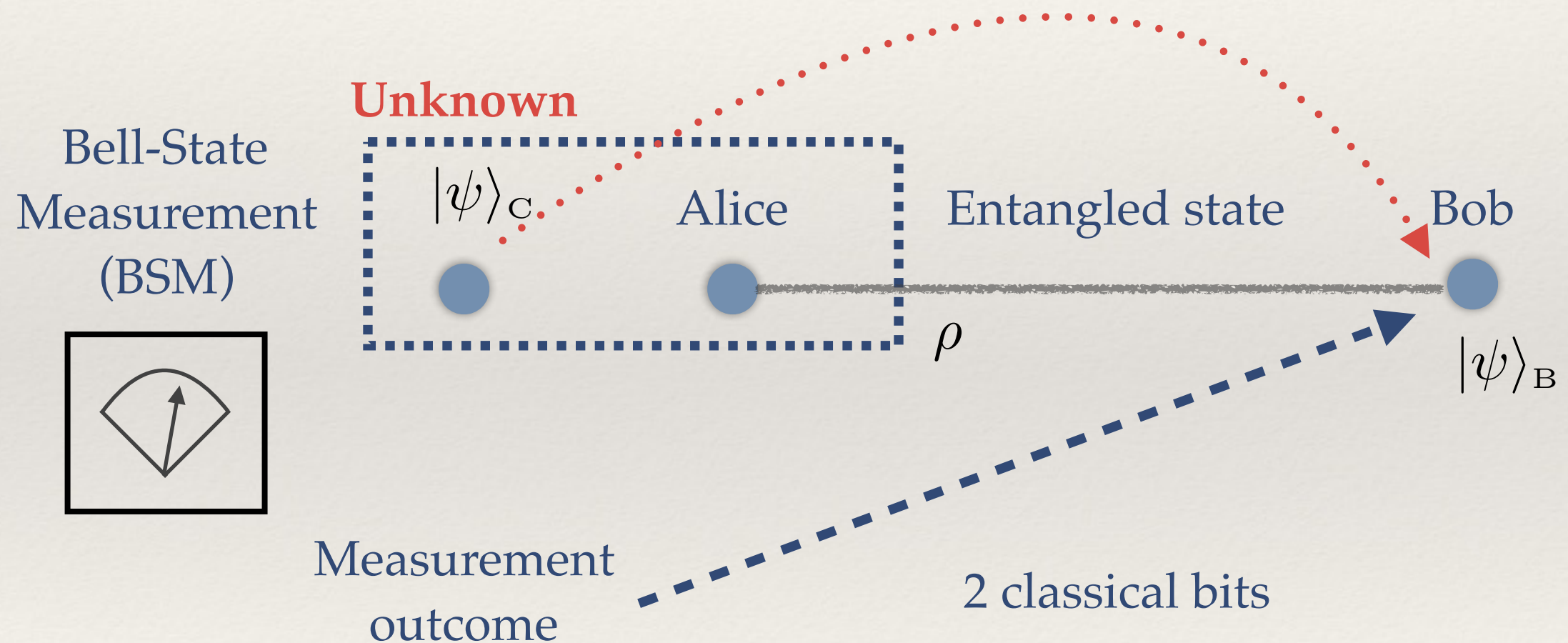
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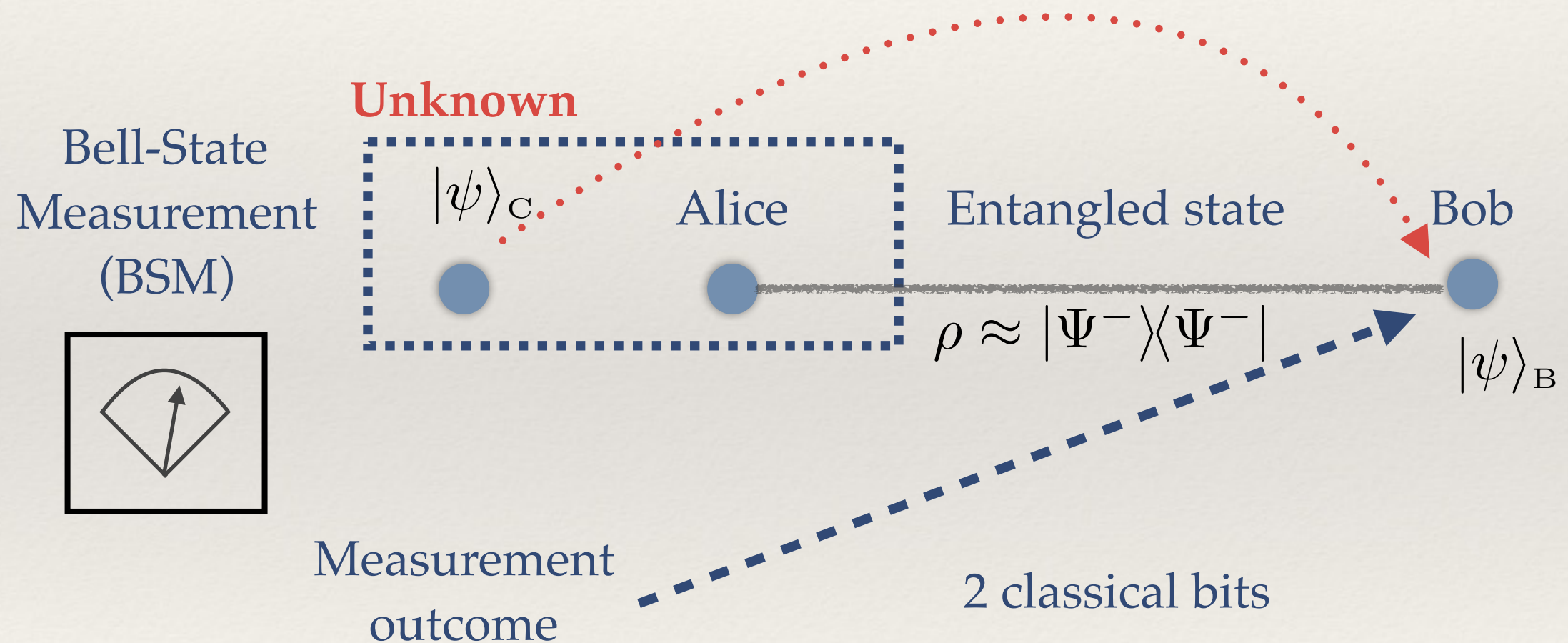
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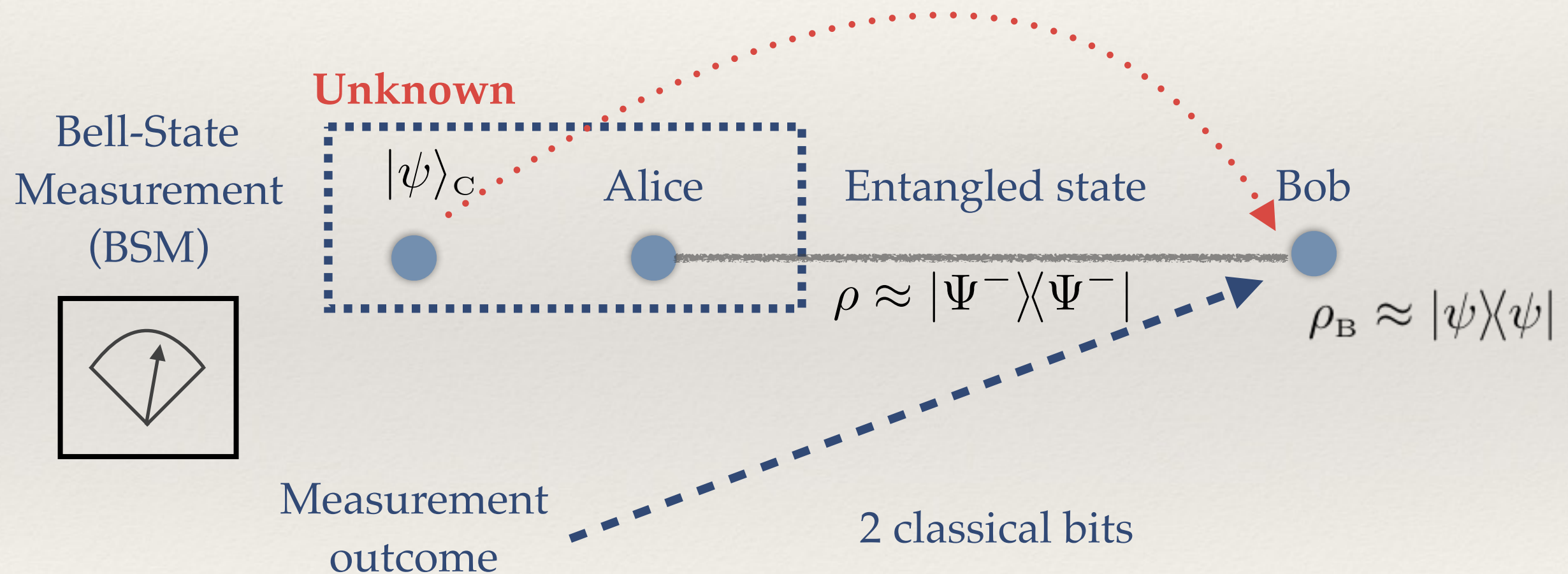
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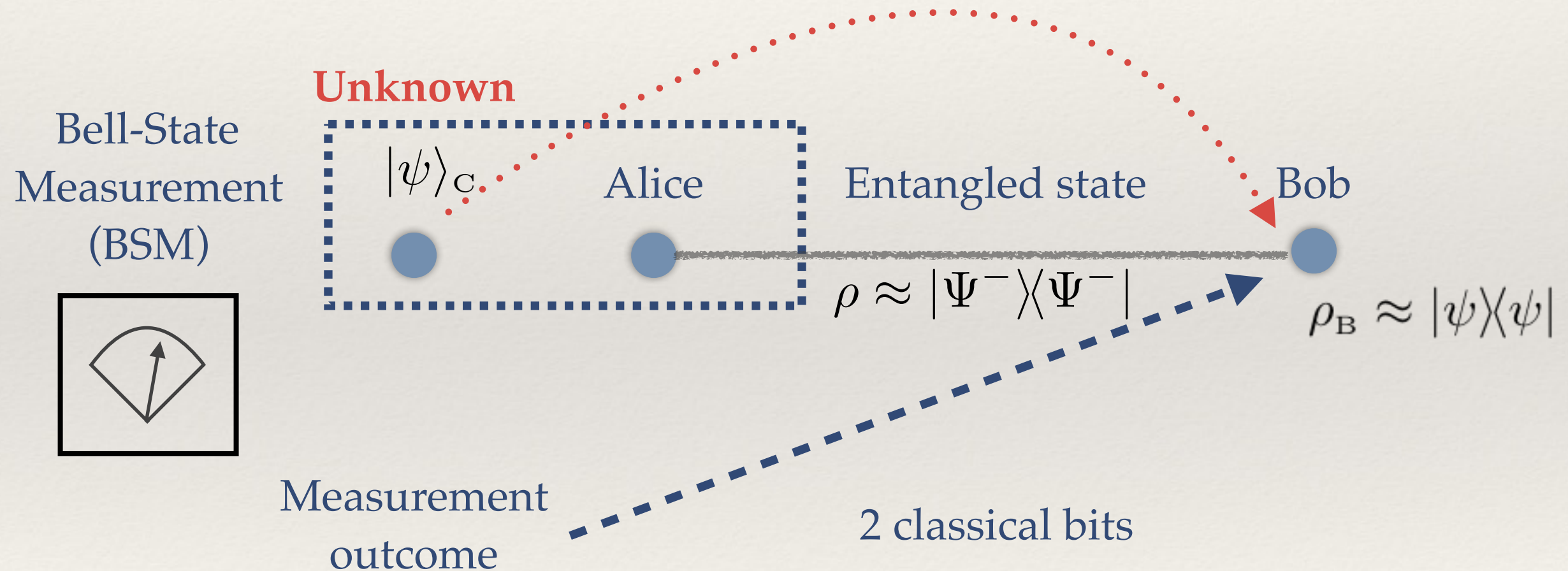
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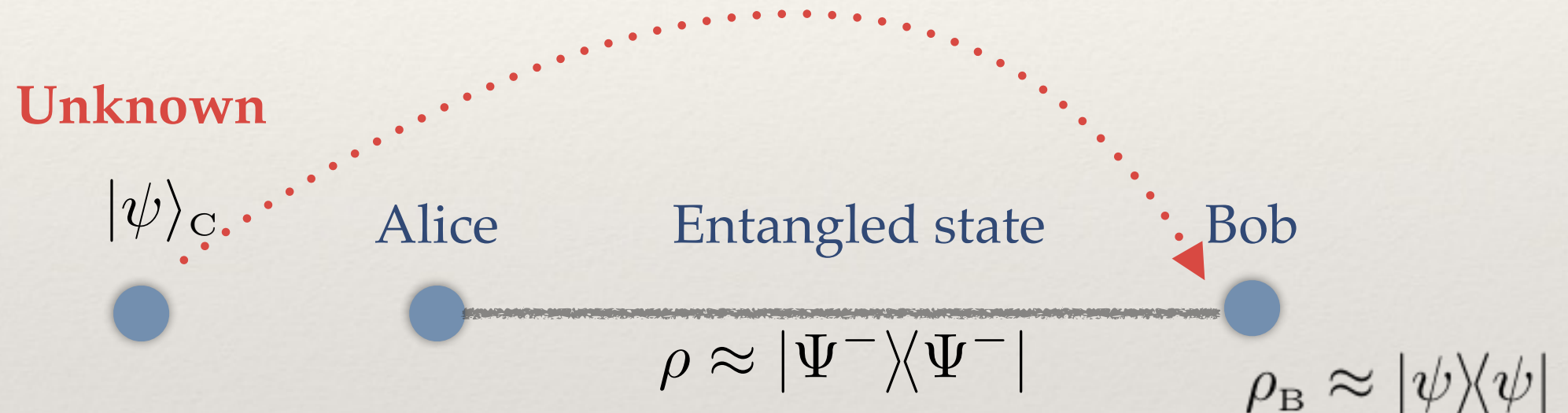
The **quality** of the **output state** can be **measured**² according to the **fidelity**³ between the two states $\mathcal{F}(|\psi\rangle, \rho_B) = \langle\psi|\rho_B|\psi\rangle$



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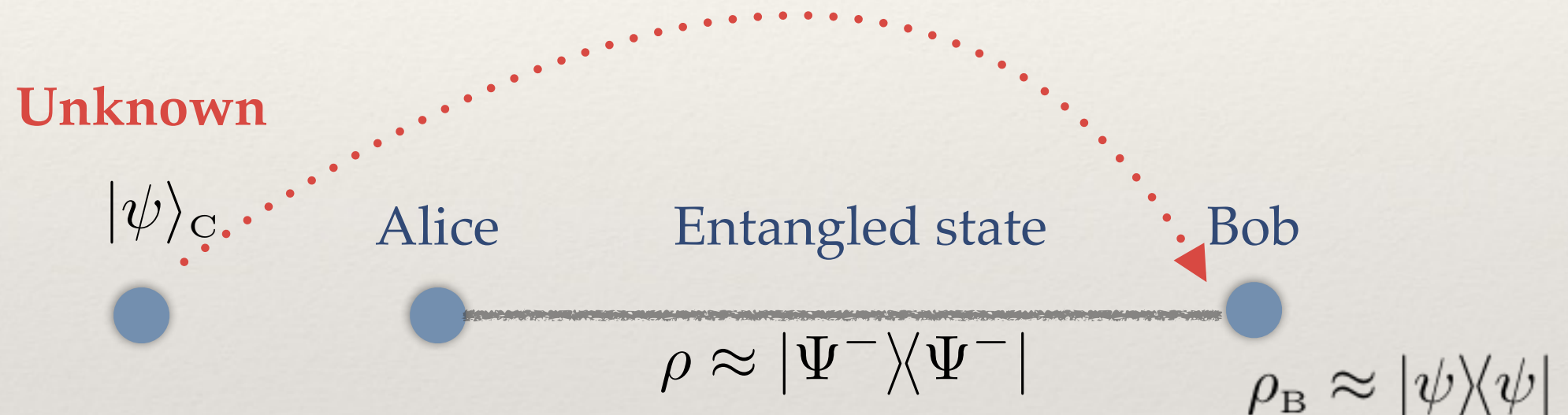
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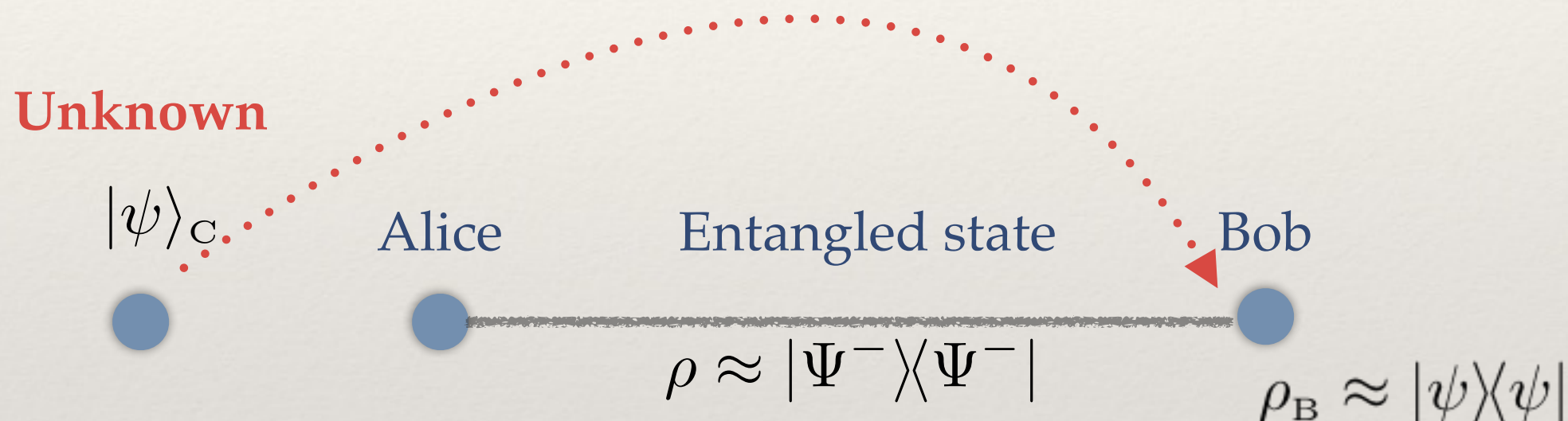
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- ❖ Usefulness of ρ for **teleportation** may be quantified² in terms of the **teleportation fidelity**^{2,4} [**uniform average** over *all* **pure** $|\psi\rangle_C$]

$$f(\rho) = \int d|\psi\rangle \sum_{\text{outcome } k} p_k \mathcal{F}(|\psi\rangle, \rho_{B,k})$$

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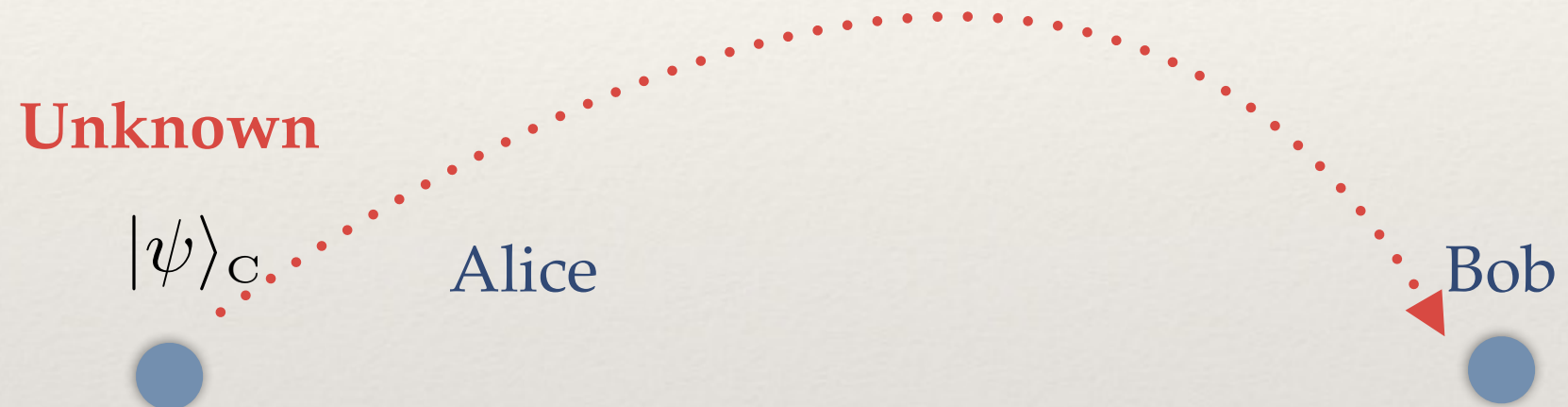
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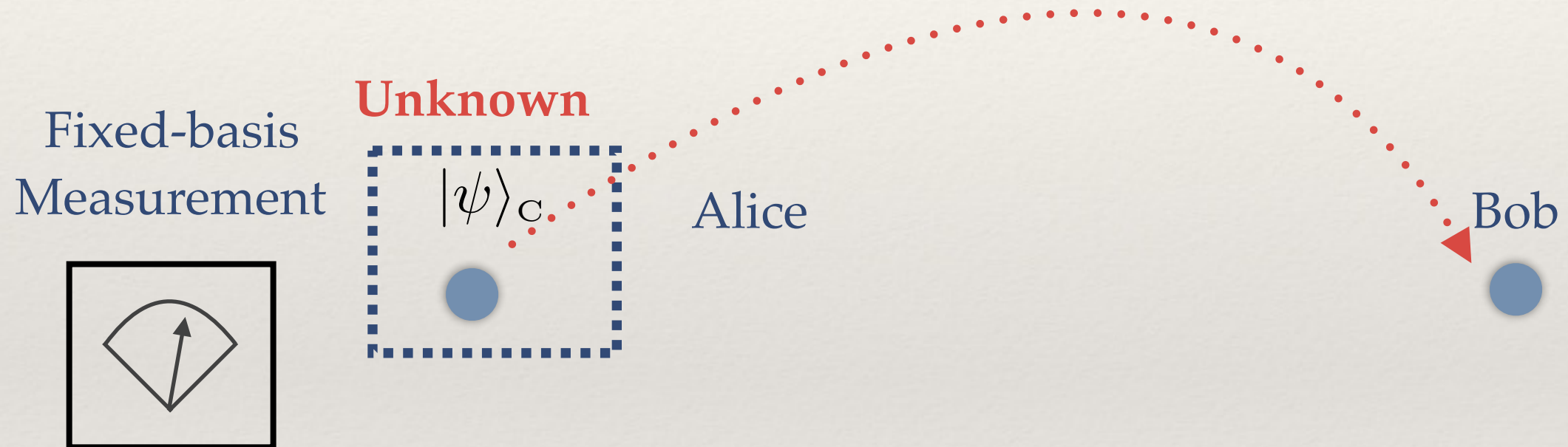
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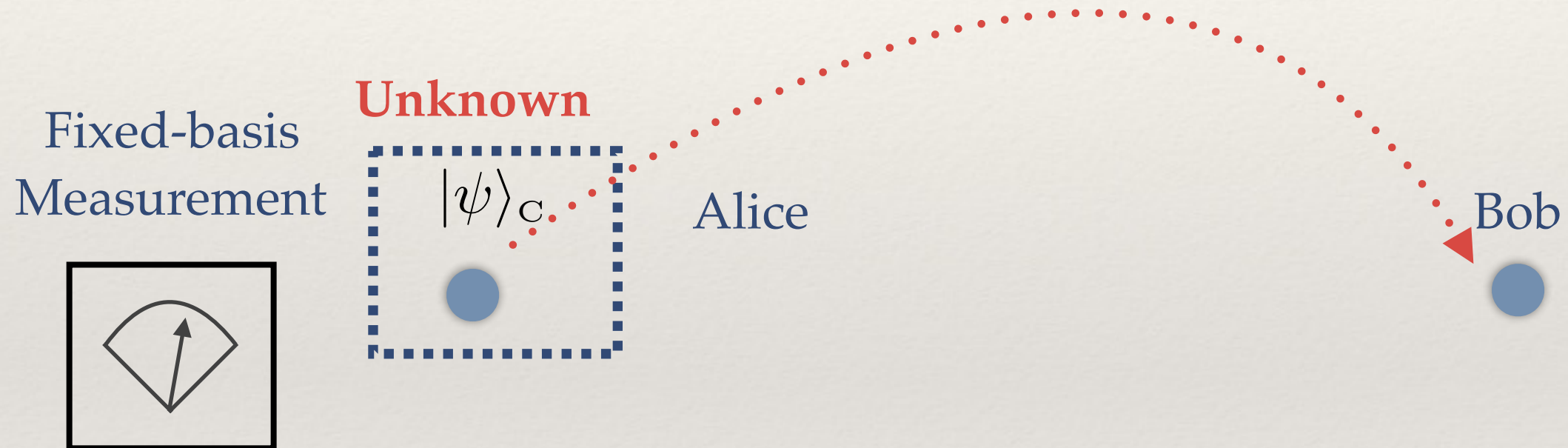
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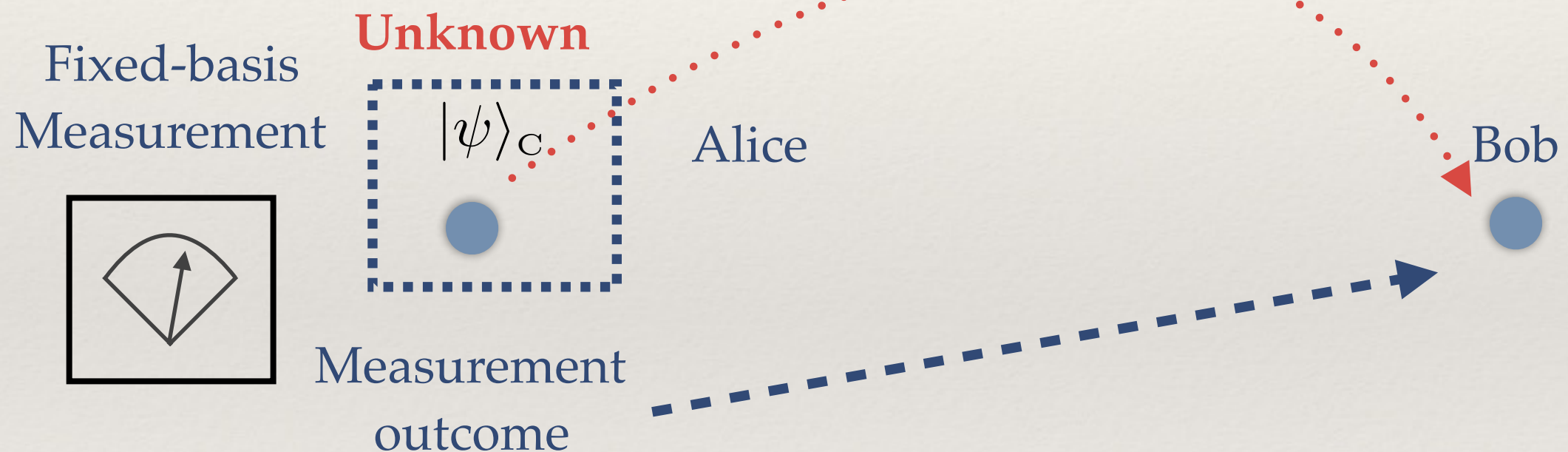
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- ❖ For qubits, the **measure** and **prepare** fidelity is **limited**²: $f_c \leq \frac{2}{3}$
- ❖ More generally, for **qudits**, it can be shown that⁵ $f_c \leq \frac{2}{d+1}$.

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Surviving Local Filtering

- ❖ Some **useless** entangled states were found⁶ to exhibit **teleportation** power after appropriate **local** filtering⁷.

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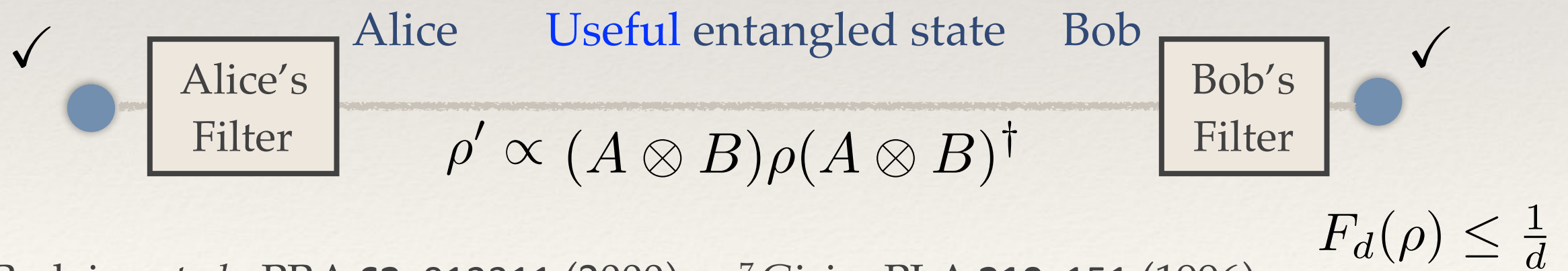
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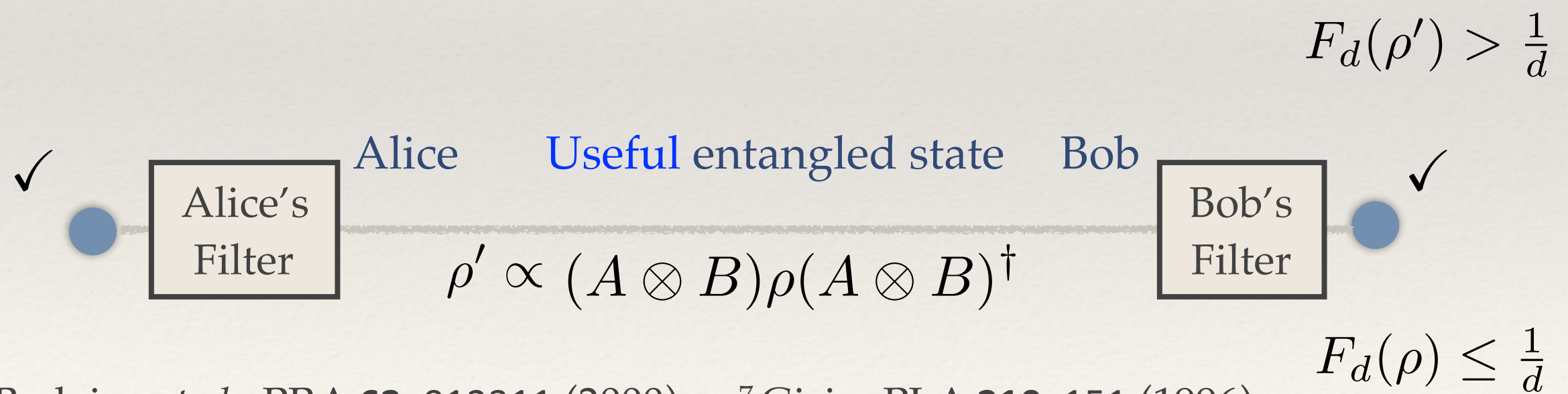


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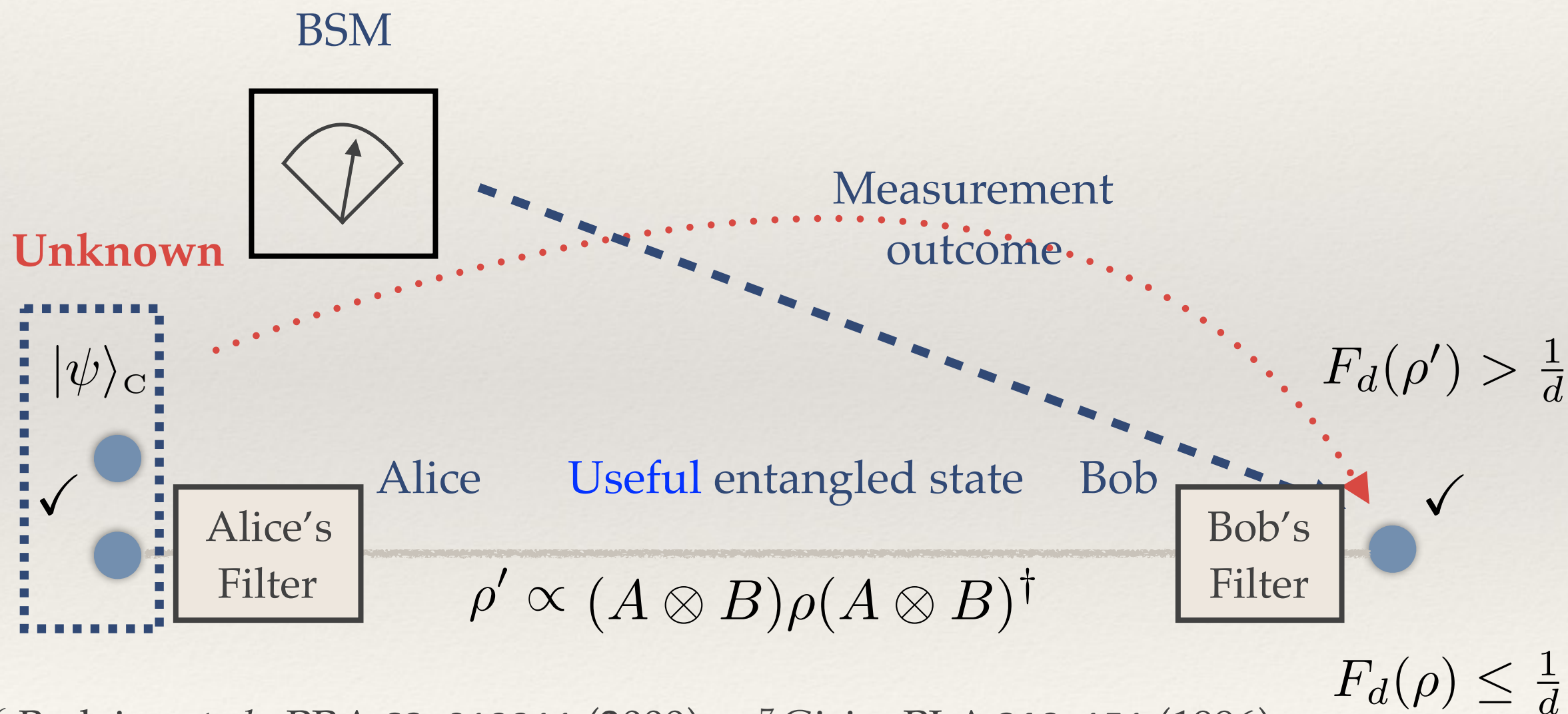


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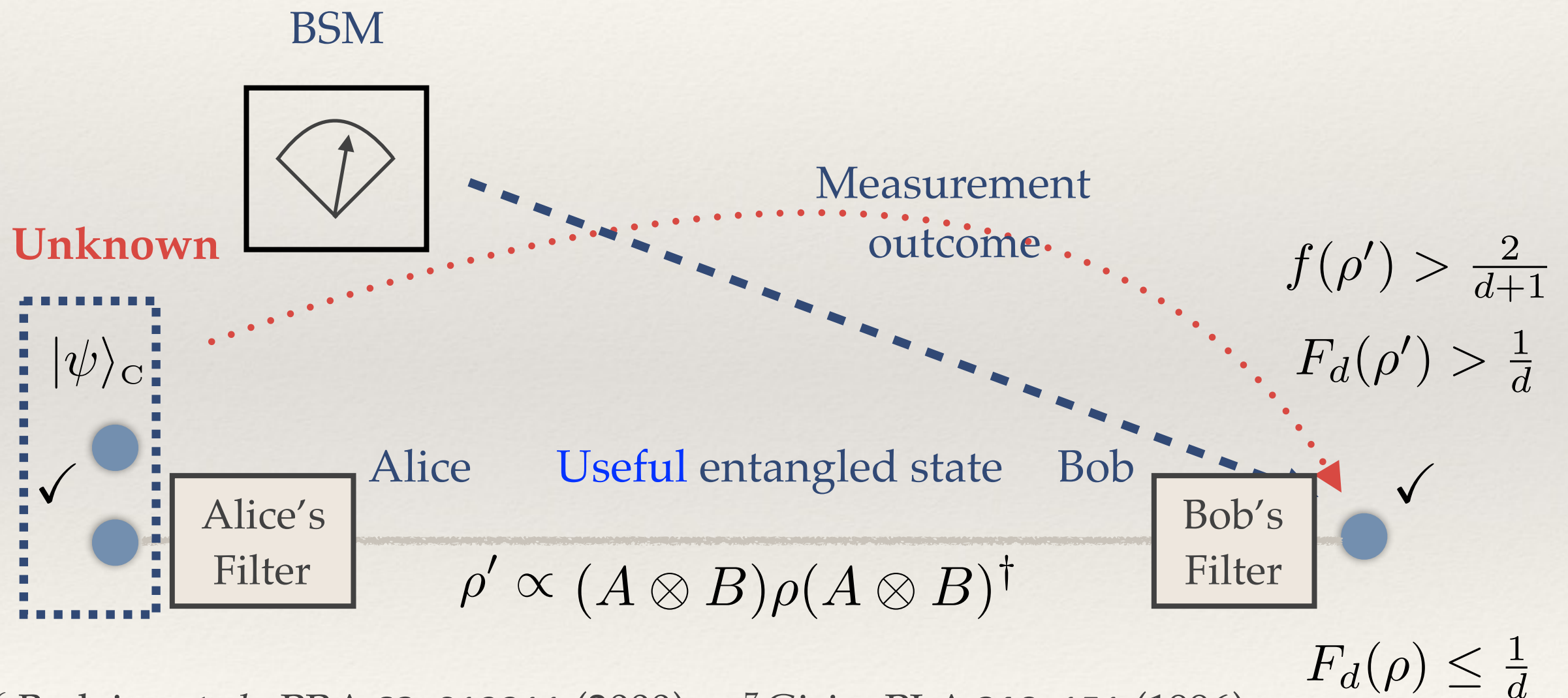


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- ❖ Some **useless** entangled states were found⁶ to exhibit **teleportation power** after appropriate **local filtering**⁷.
- ❖ In analogy with the phenomenon of **hidden nonlocality**^{8,9}, we say that such states exhibit **hidden teleportation power** (HTP).

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Hidden Teleportation Power & Activation

- ❖ Some **useless** **entangled states** were found⁶ to exhibit **teleportation power** after appropriate **local filtering**⁷.
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Werner States

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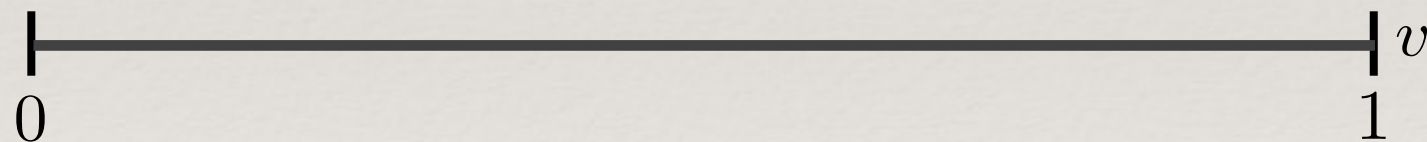
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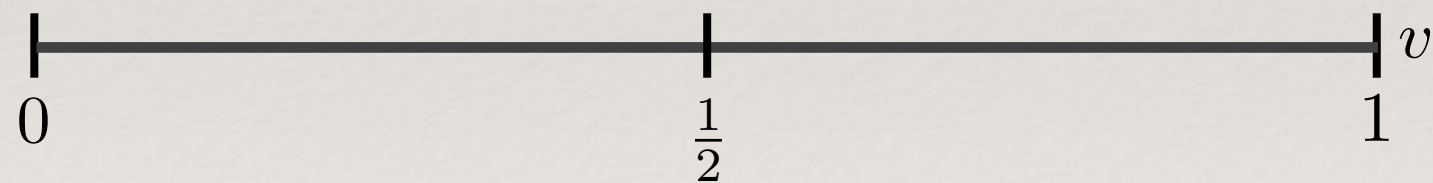
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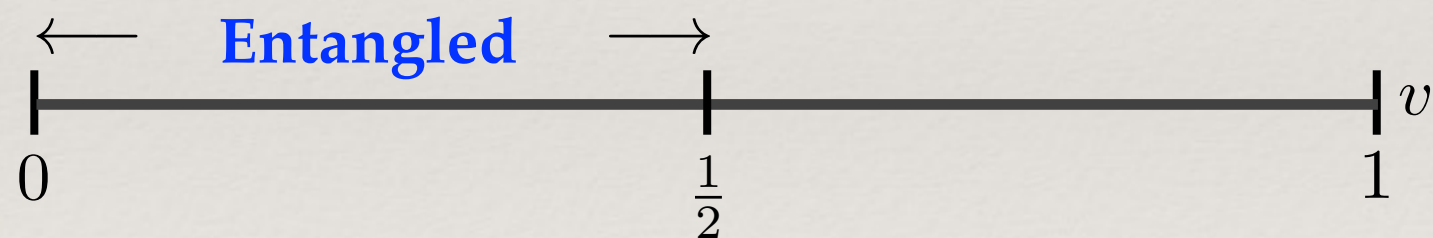
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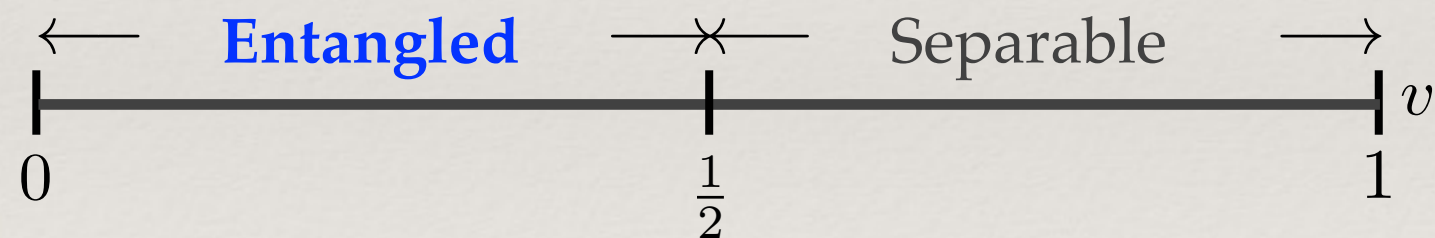
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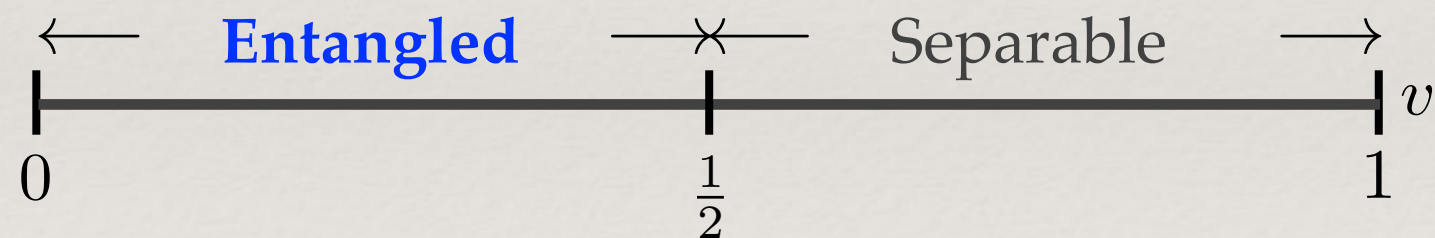
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¹³ Zhao *et al.*, JPA **43**, 275203 (2010)

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- ❖ With the **local filters** $A = \sigma_z \oplus 0_{d-2}$, $B = \sigma_x \oplus 0_{d-2}$, and upon successful **filtering**, the state gets **transformed** to

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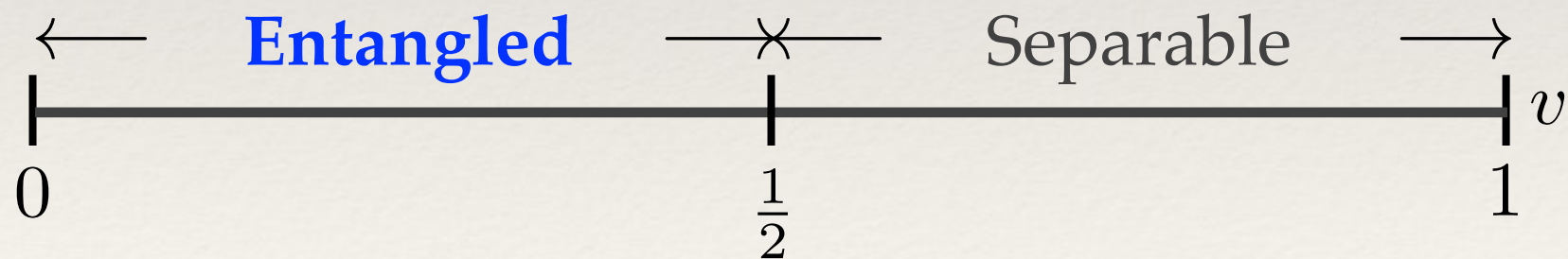
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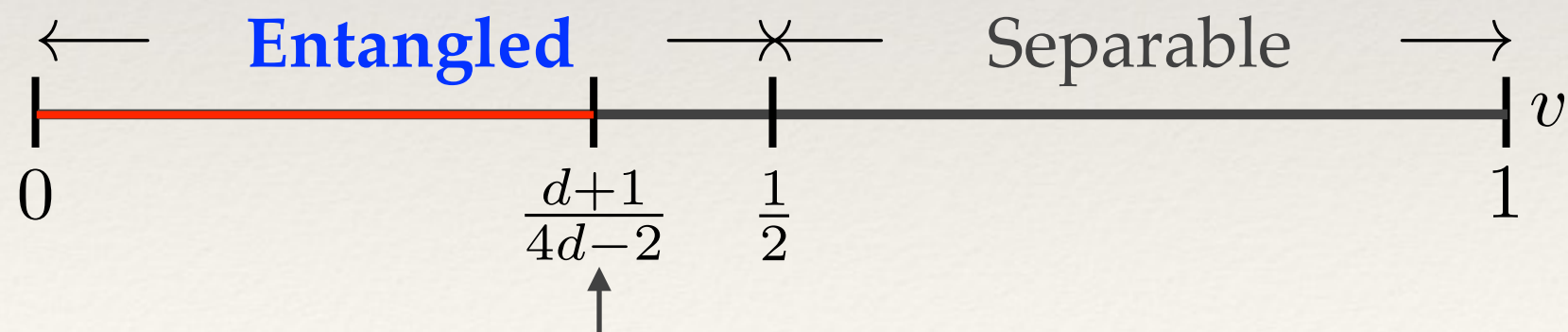
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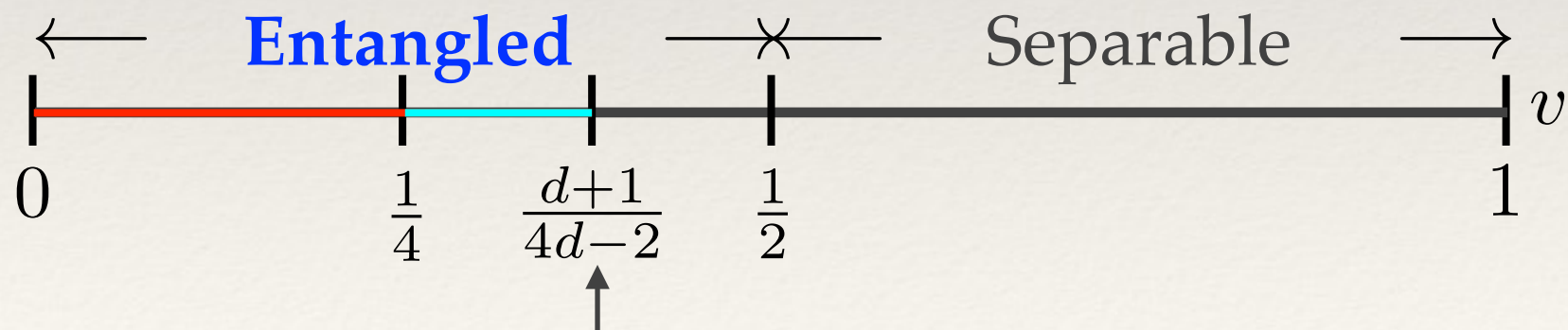
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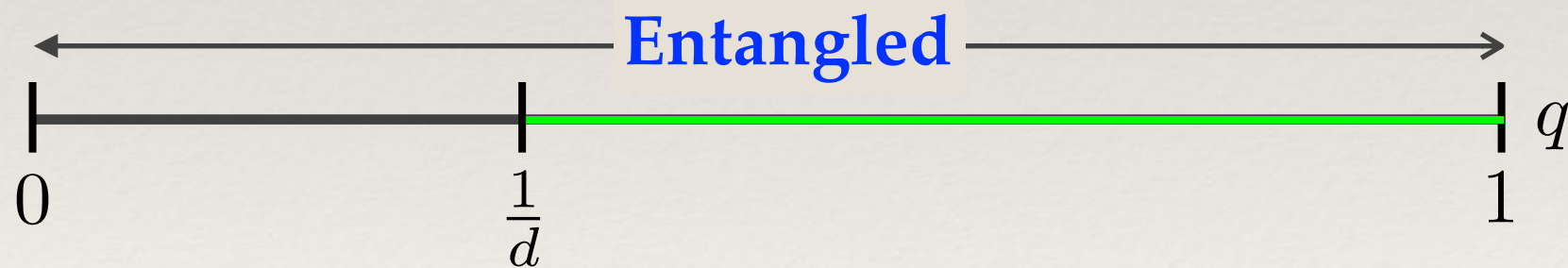
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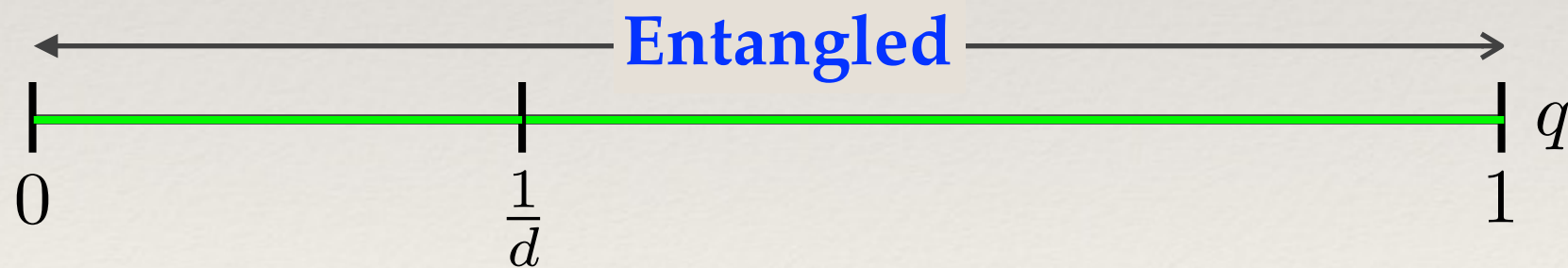
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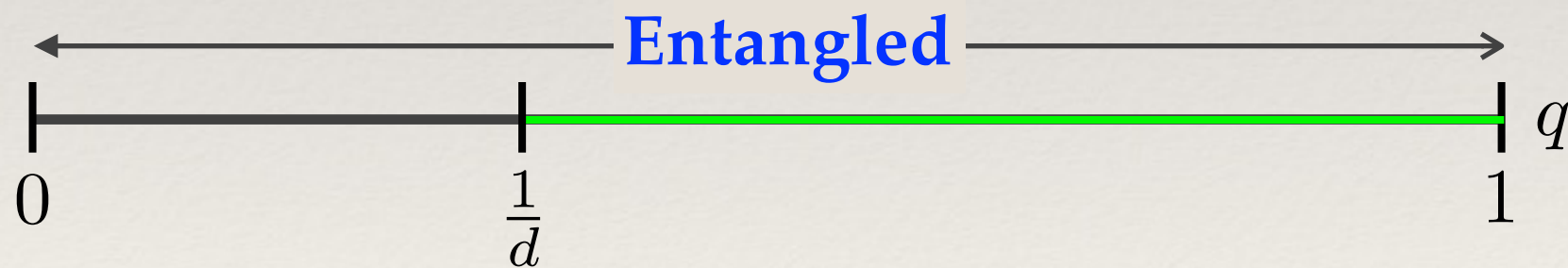
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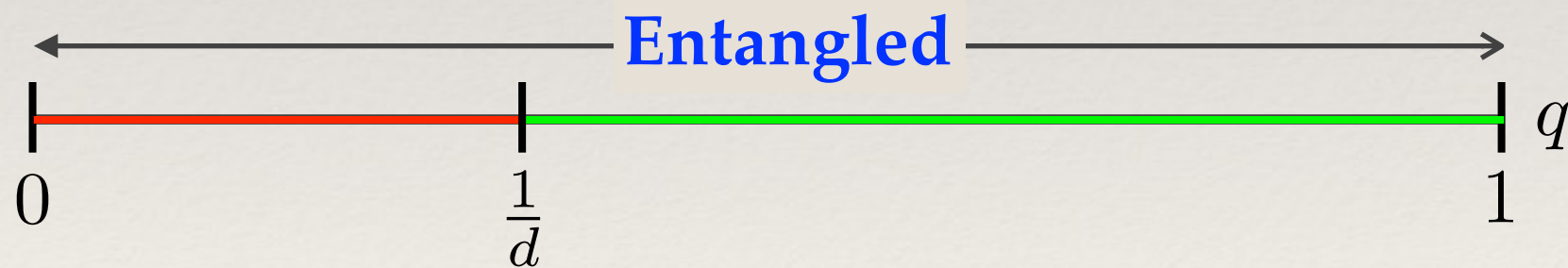
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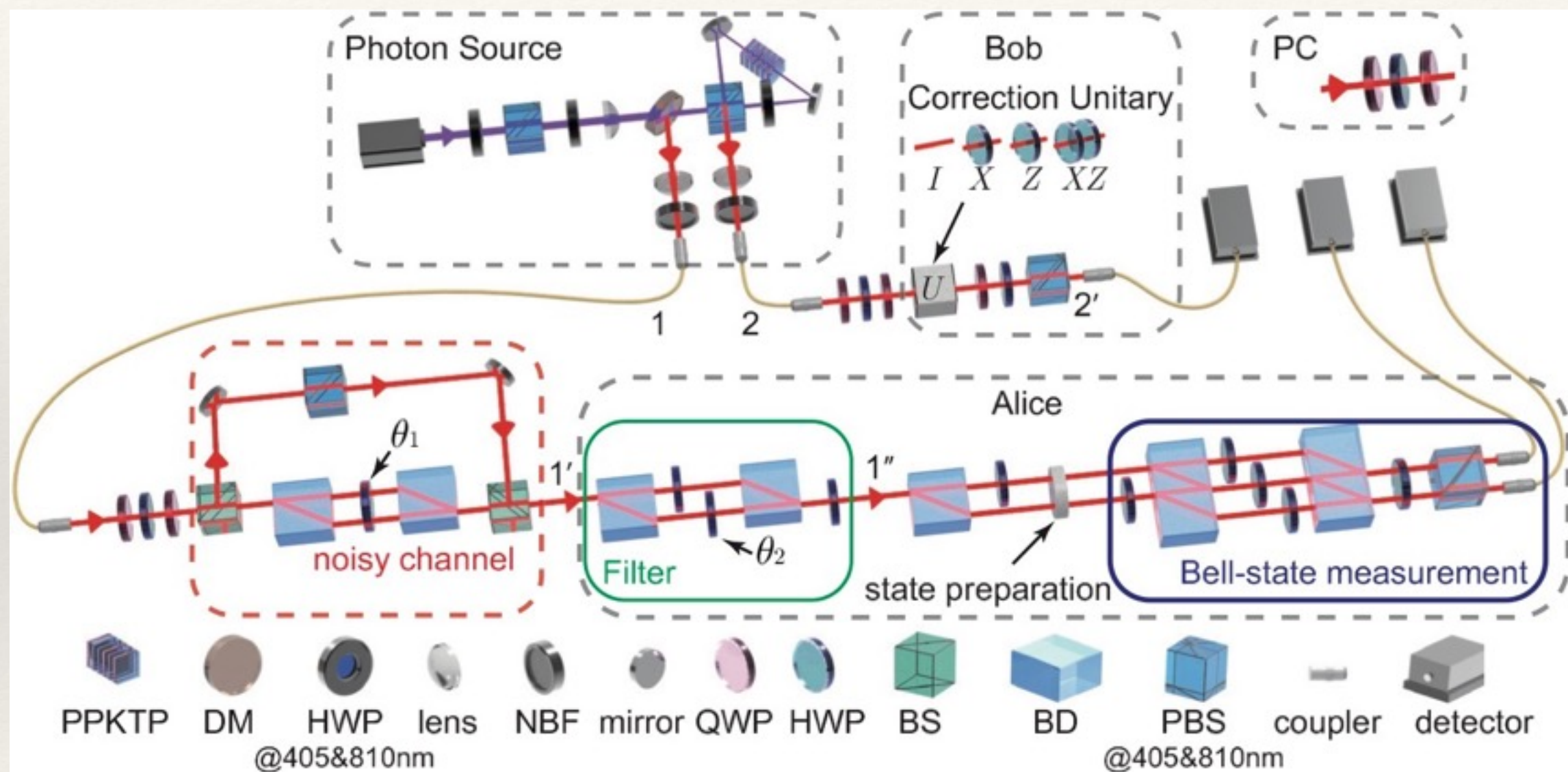
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 - ▶ for $d \leq 3$, **useful** for **teleportation** *iff* $q > \frac{1}{d}$.
- ❖ With **single-side local filter** $A = \kappa|0\rangle\langle 0| + \sum_{i=1}^{d-1} |i\rangle\langle i|$ for some κ , the **teleportation power** of *all* **useless** $\rho(q)$ may be **activated**!

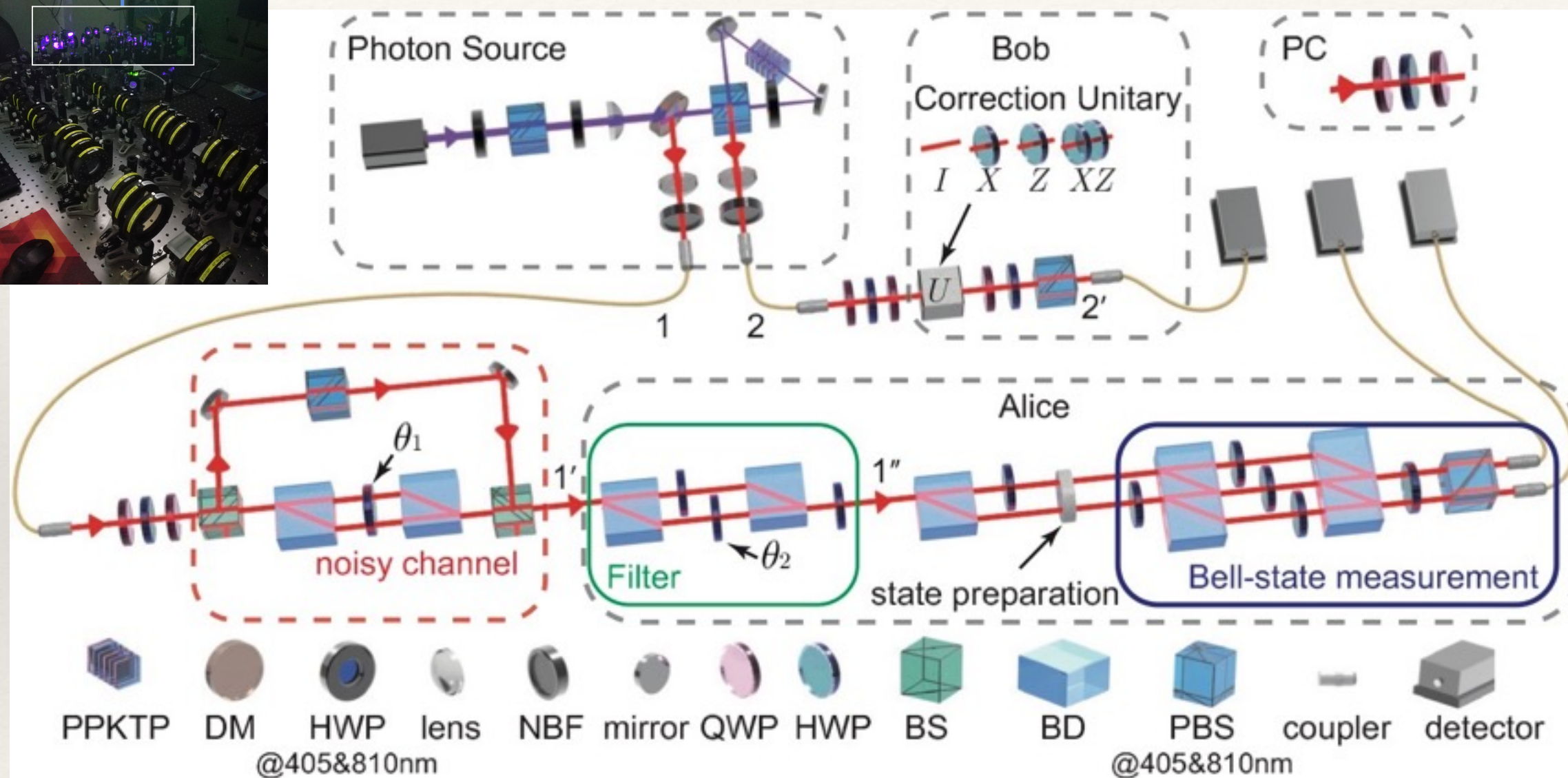
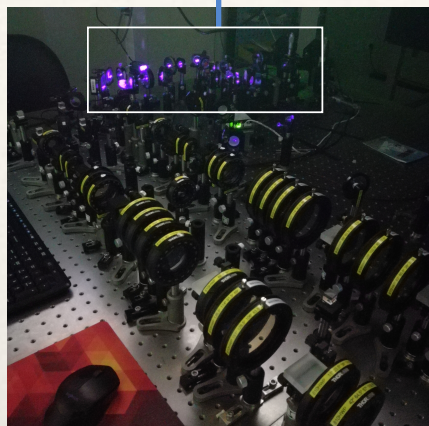
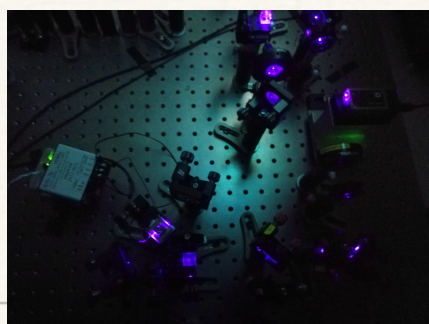


Threshold for showing
hidden teleportation power

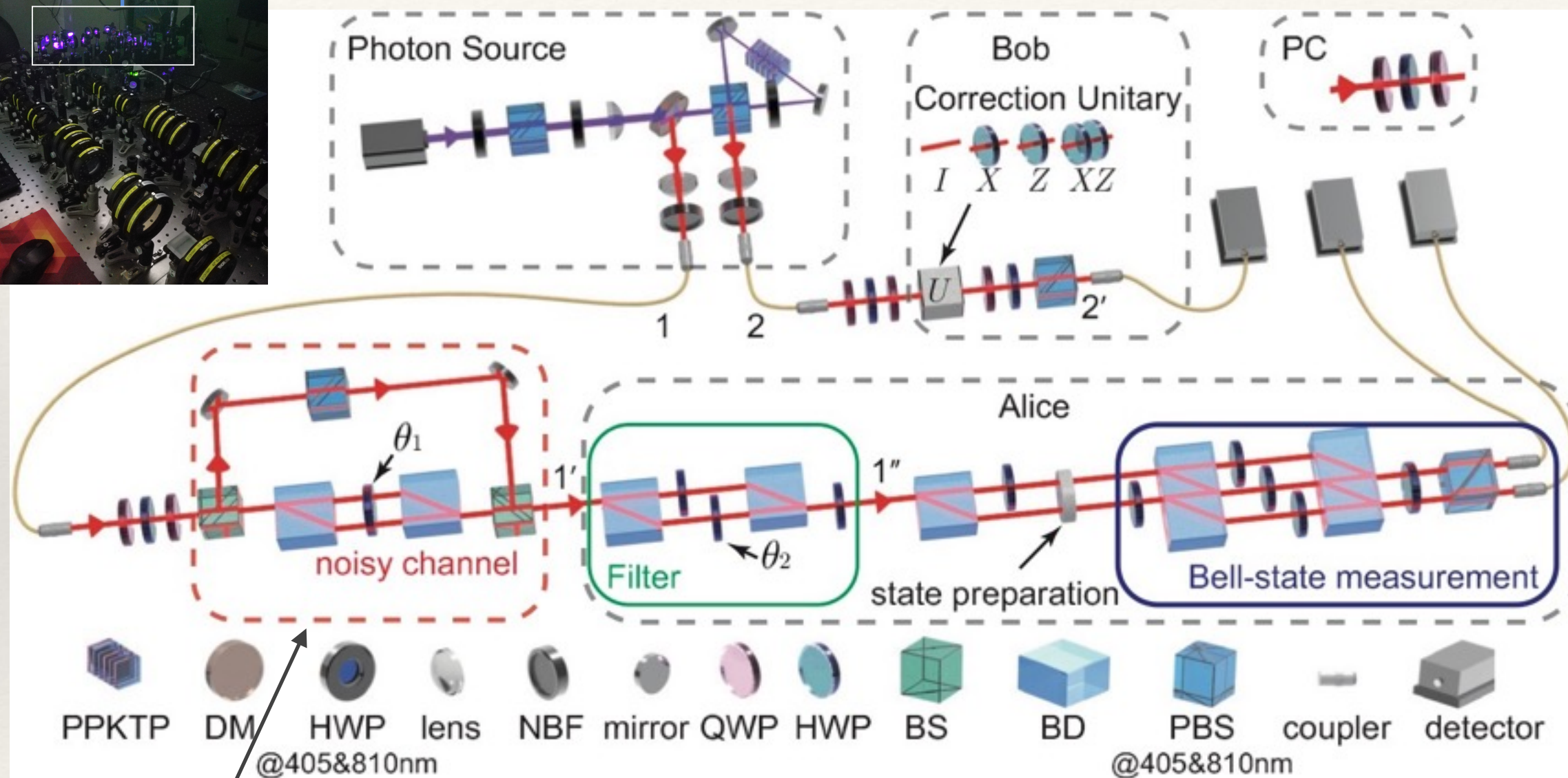
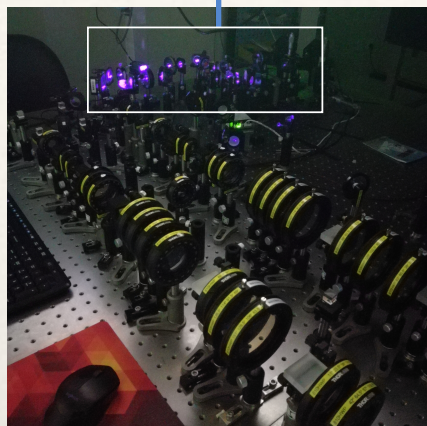
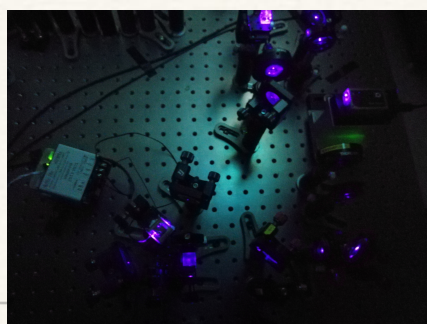
Experimental Setup



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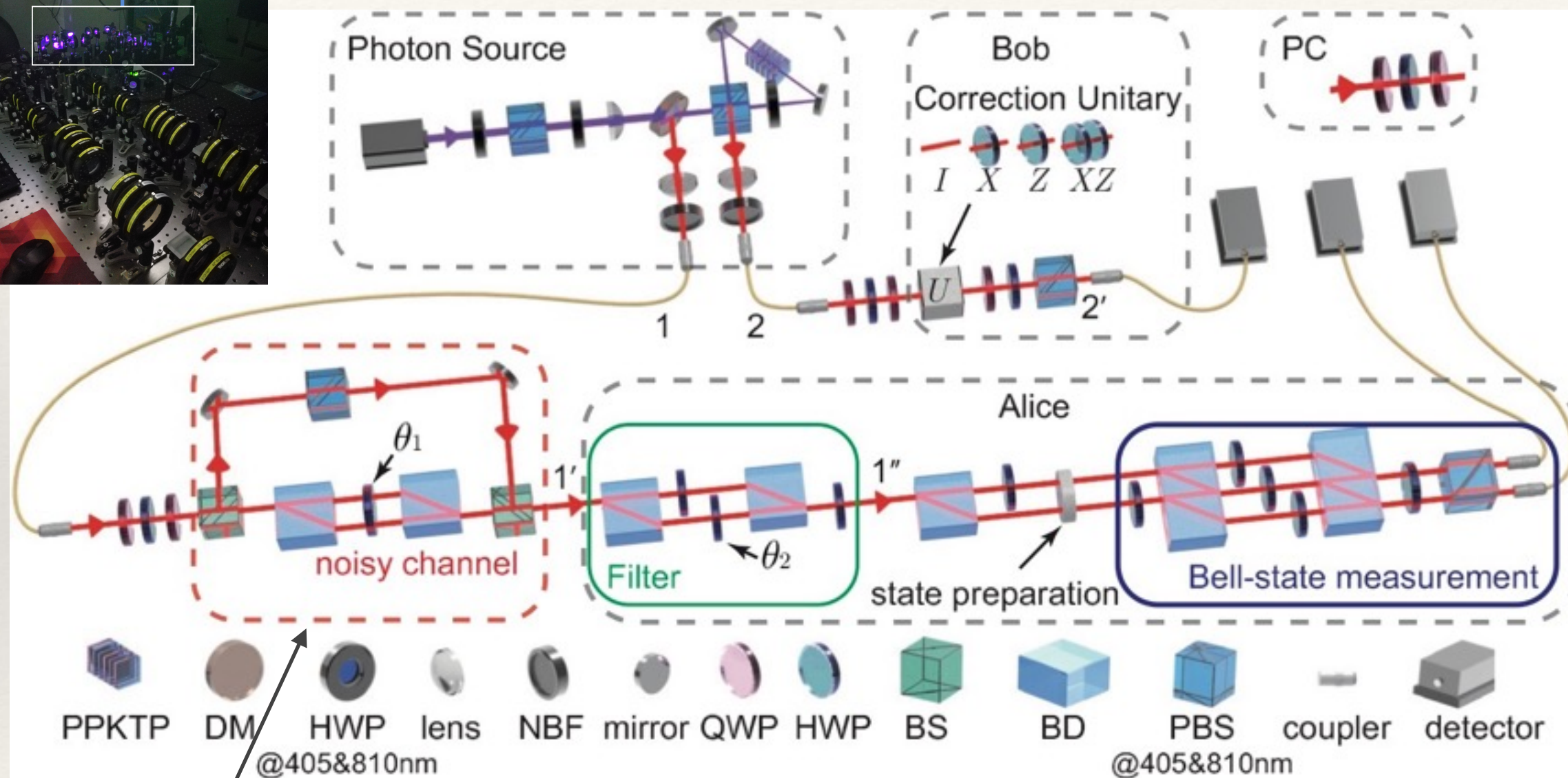
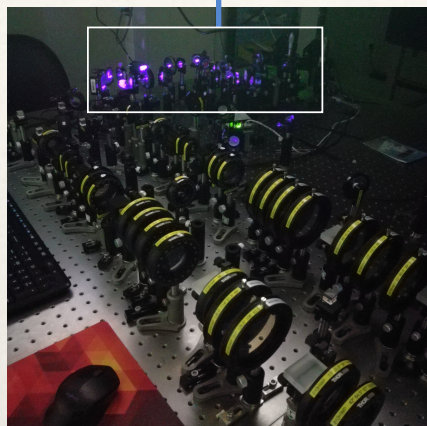
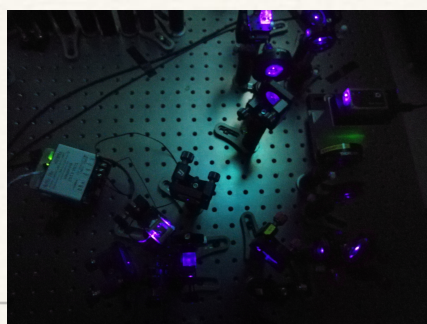


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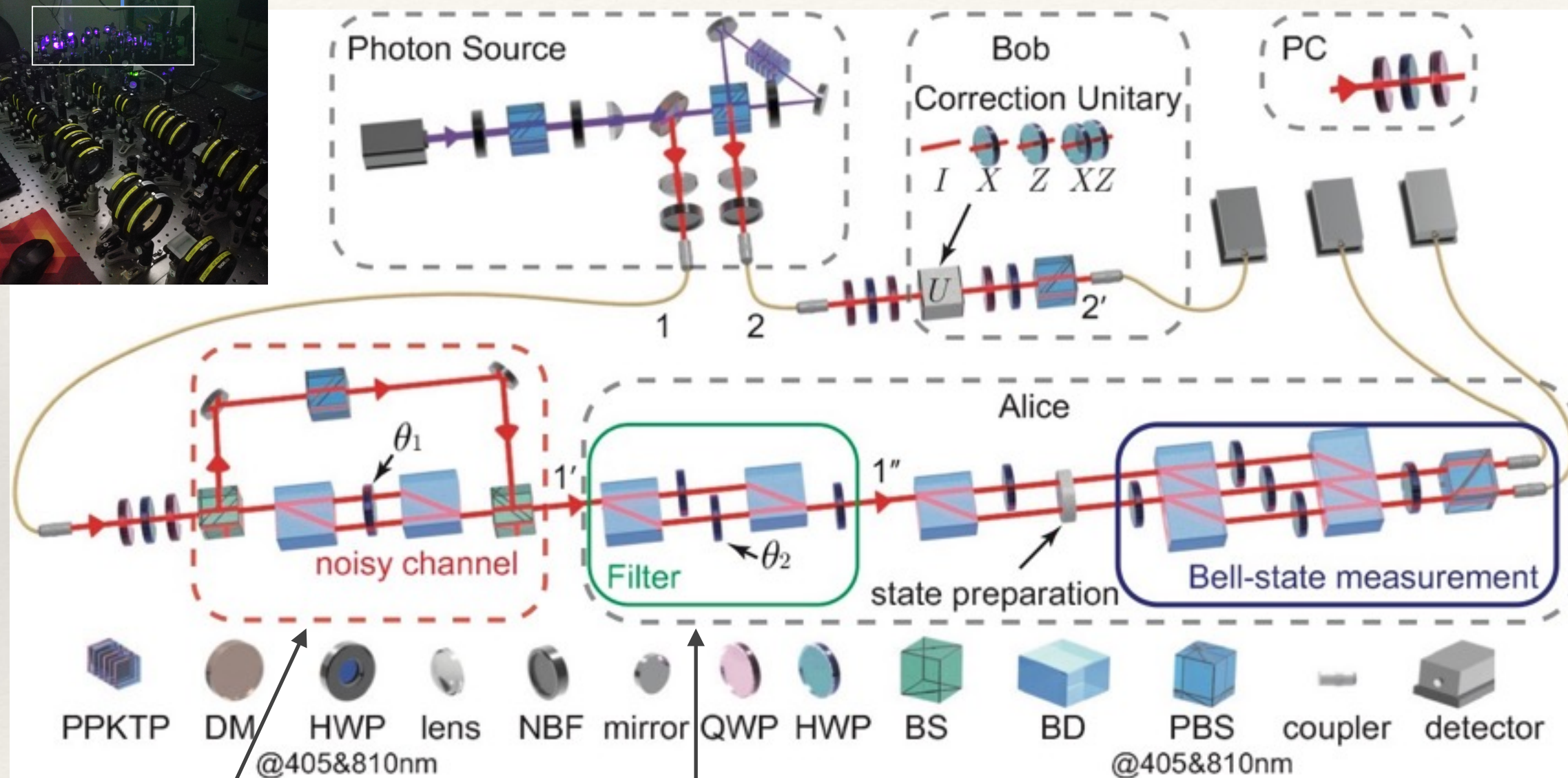
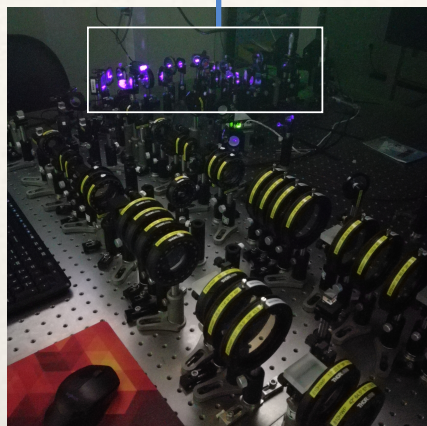
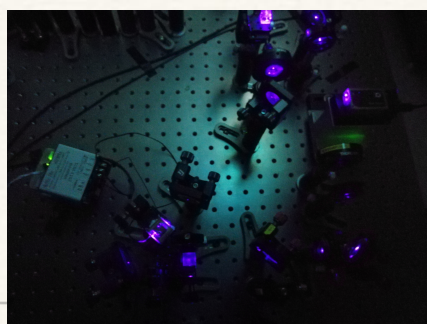
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$$q(\theta_1) = \frac{2 \sin^2 2\theta_1}{1 + 2 \sin^2 2\theta_1}$$

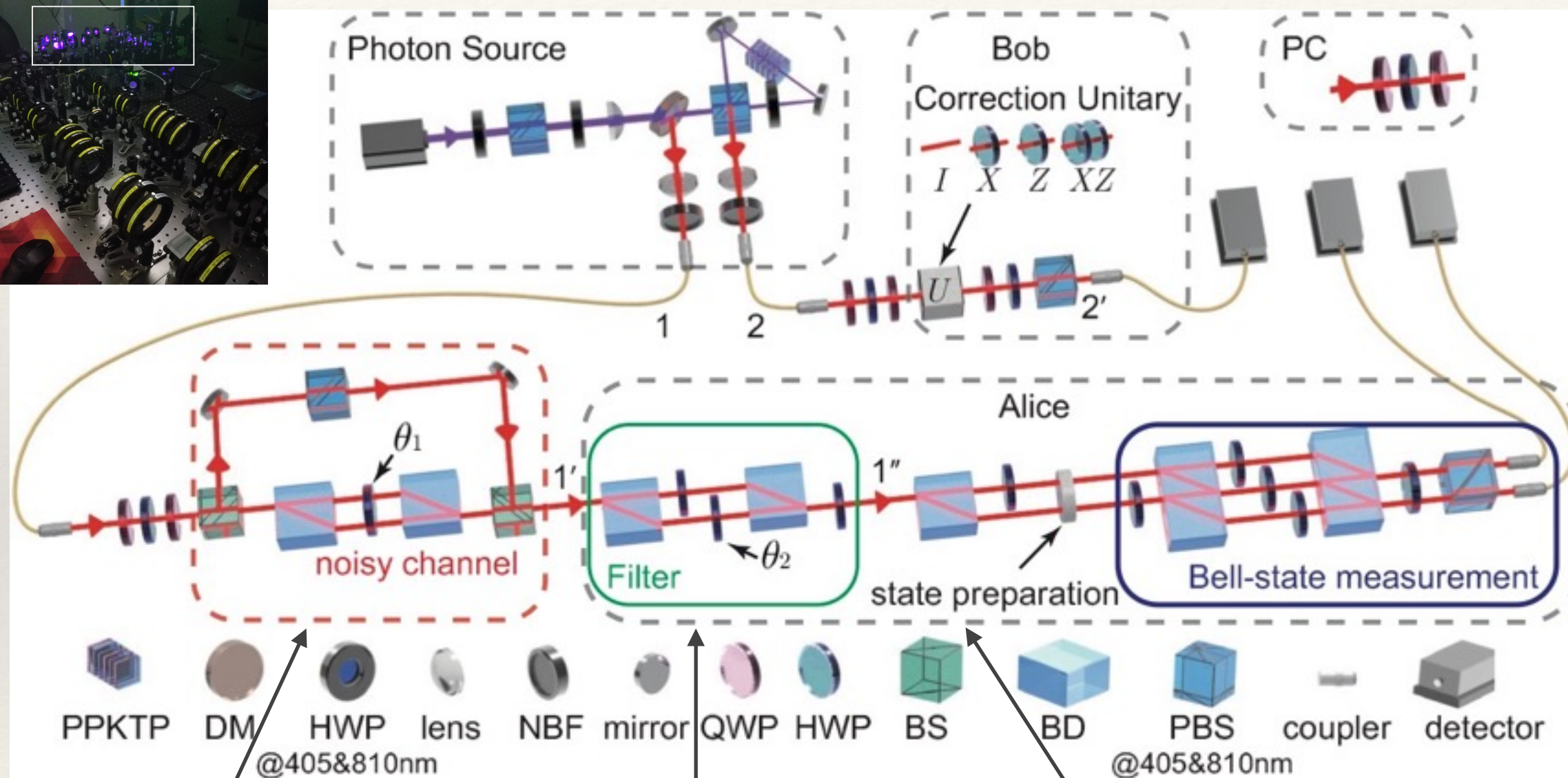
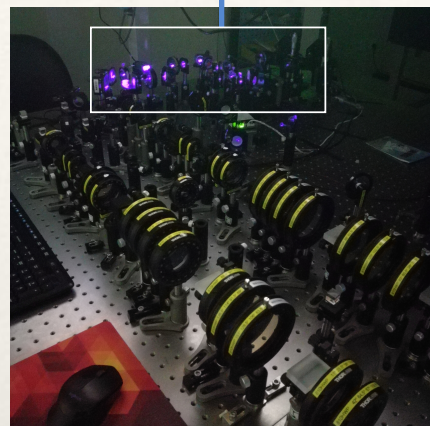
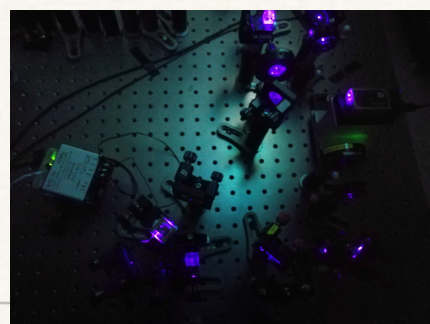
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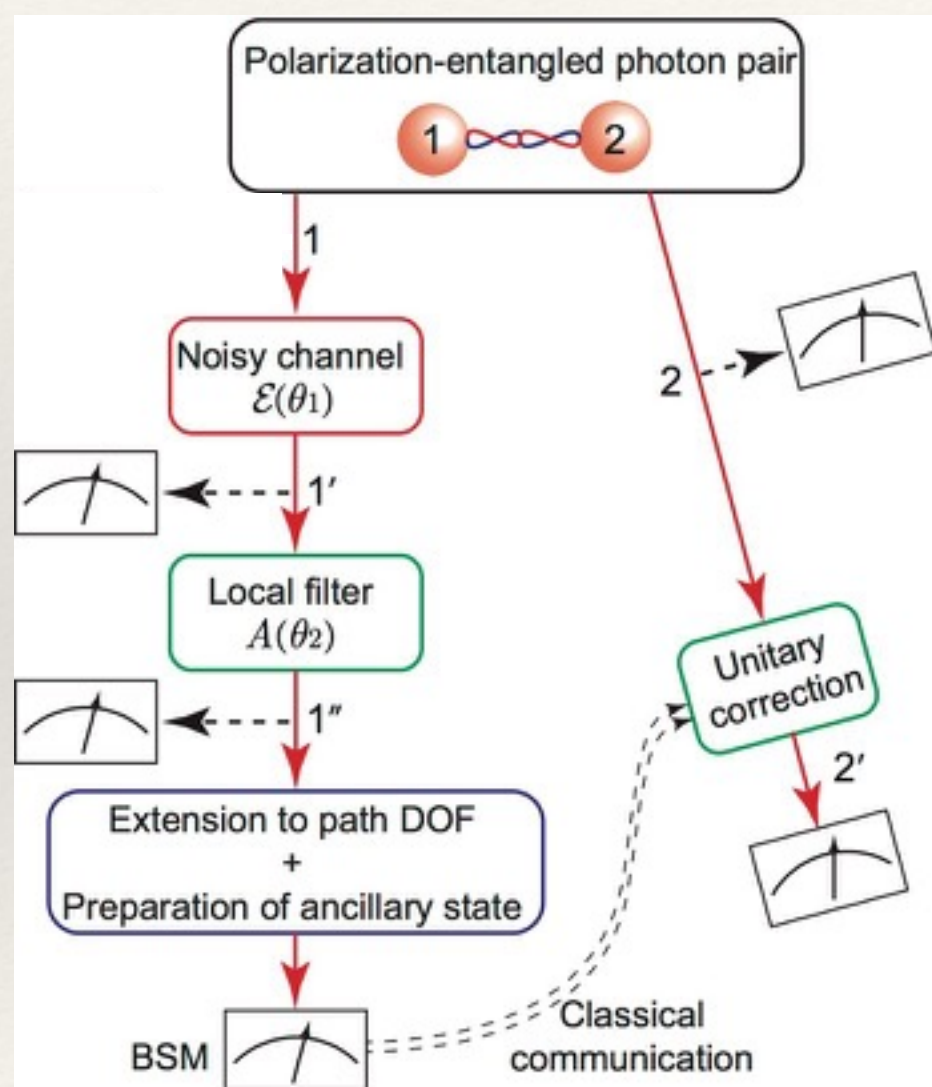


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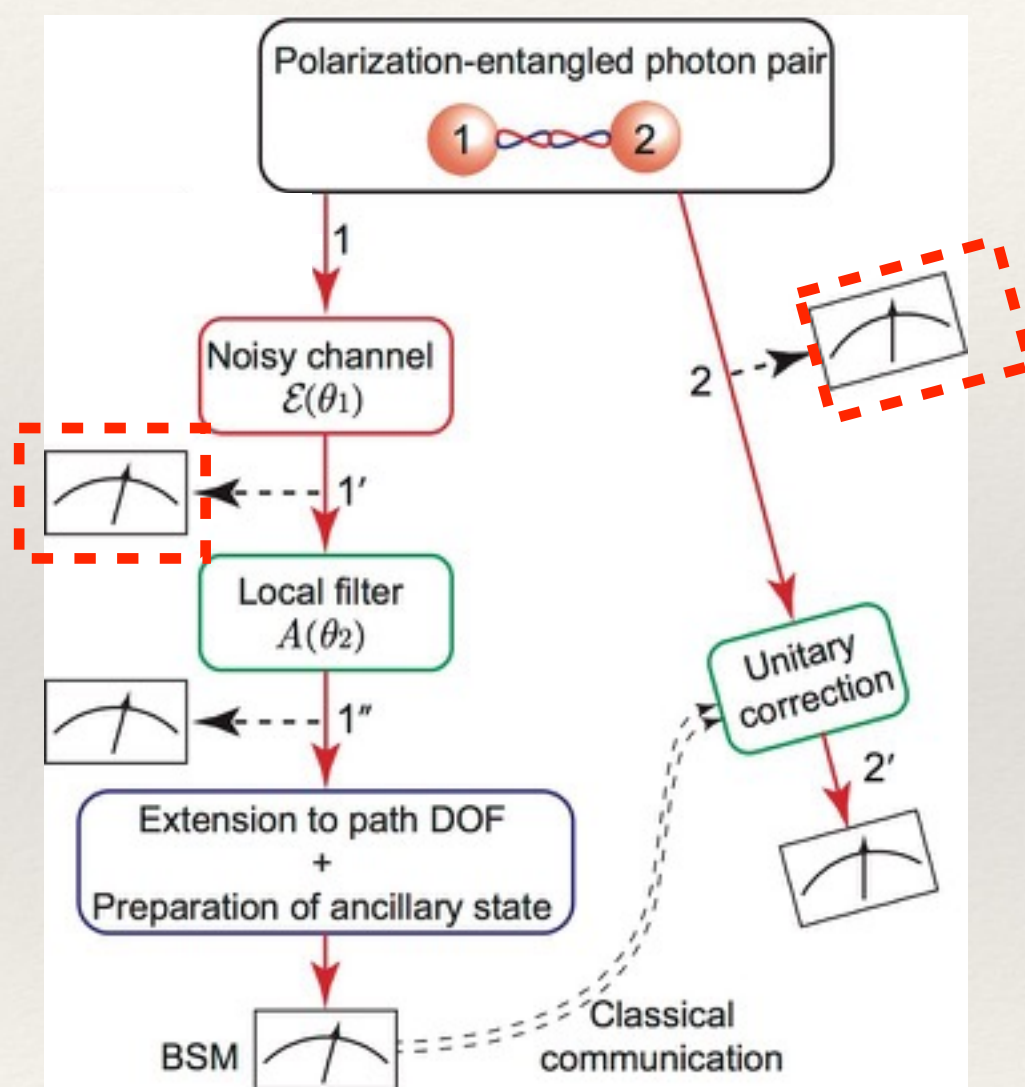
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Convert to **polarization-path** entanglement

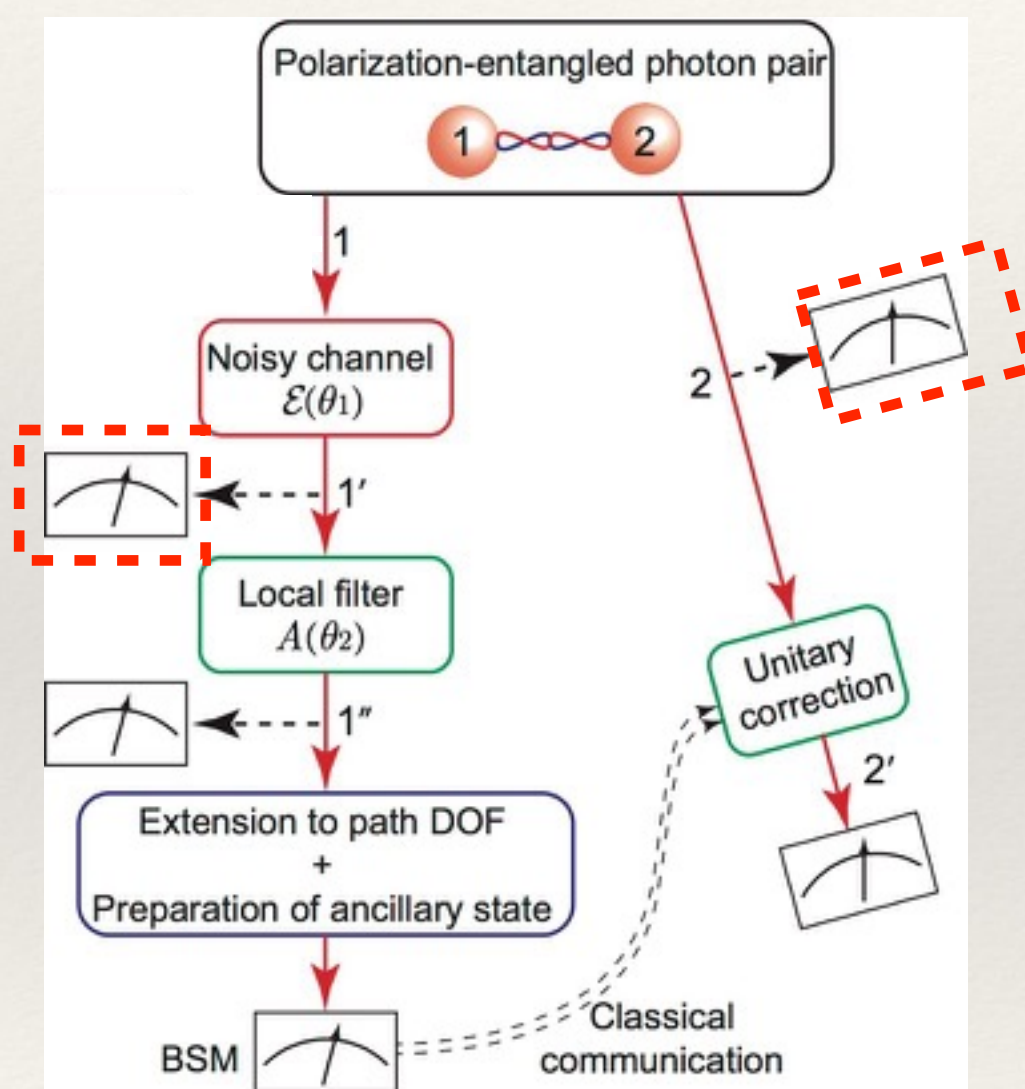
Results: Fully Entangled Fractions



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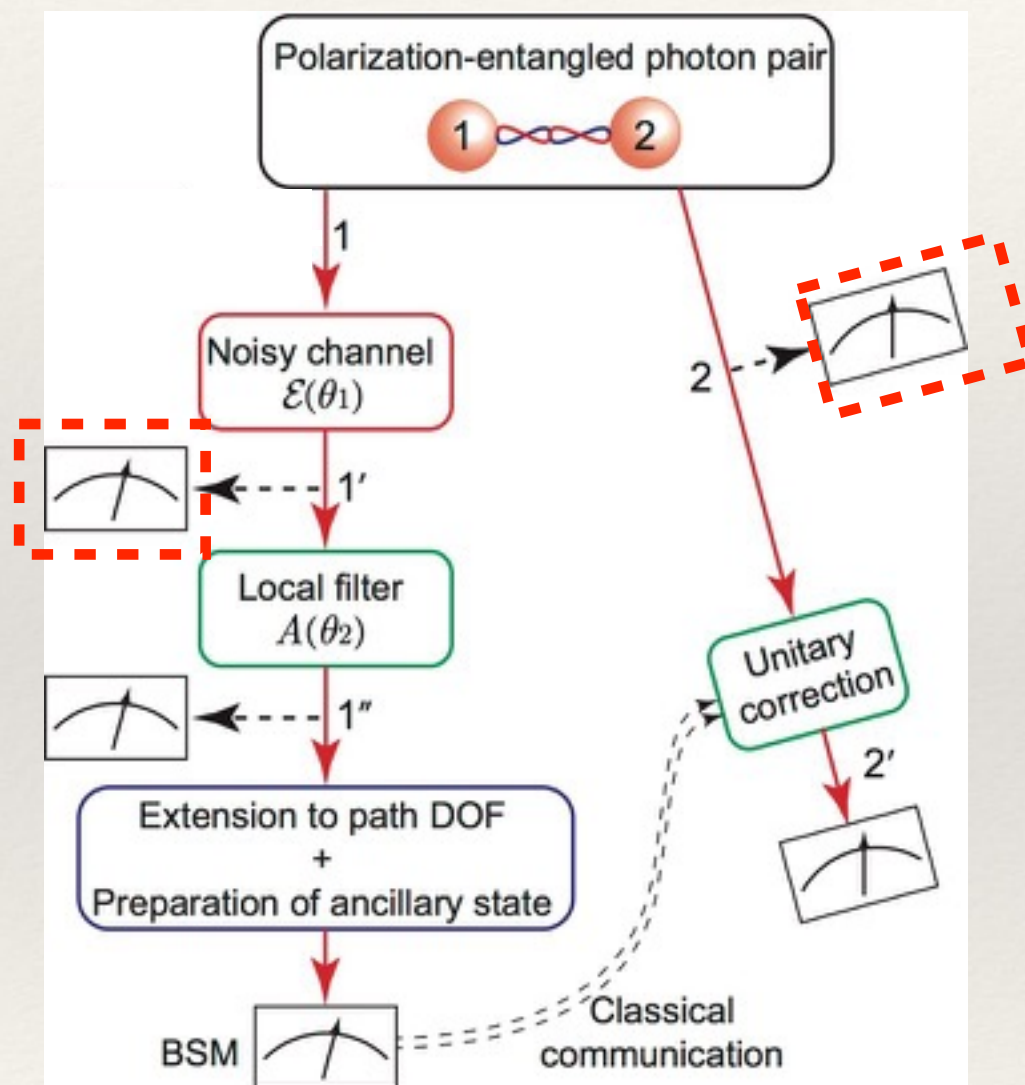


Results: Fully Entangled Fractions



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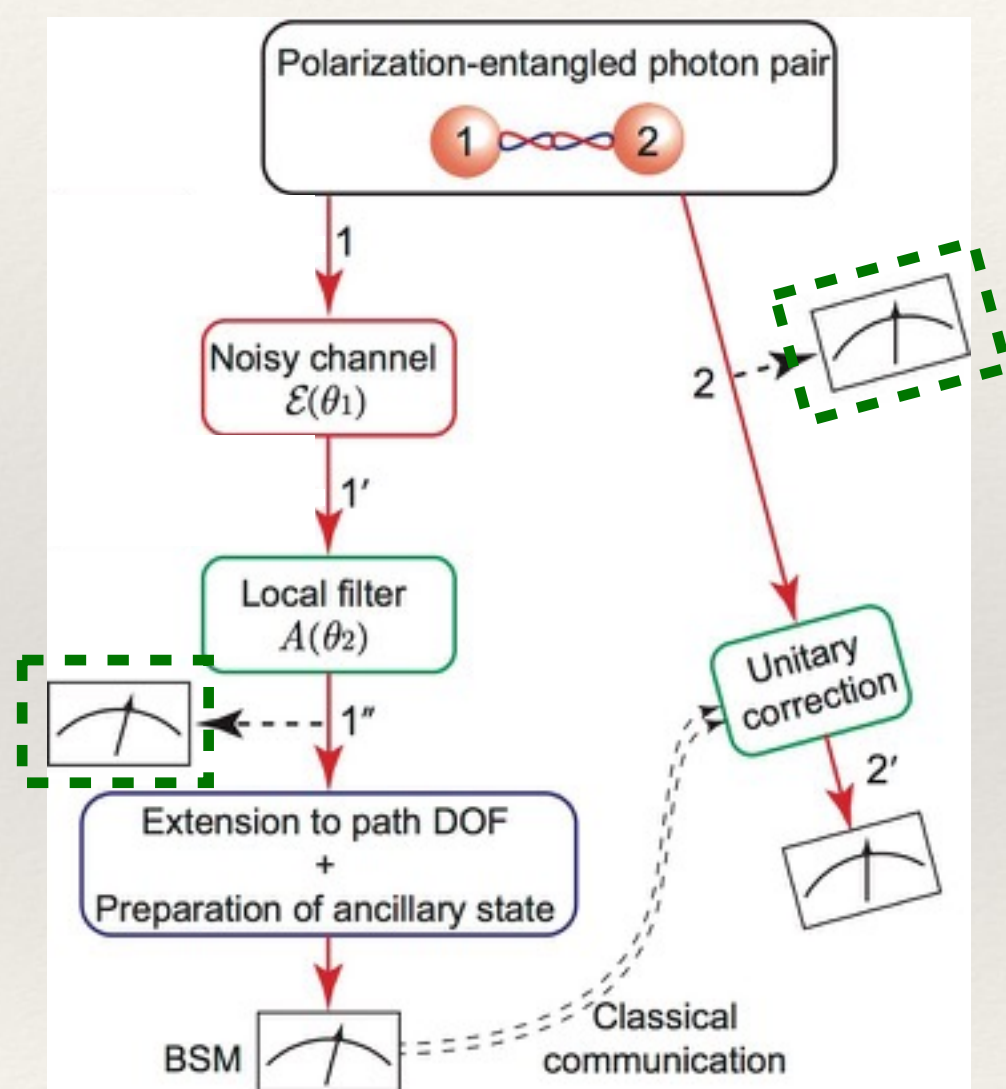
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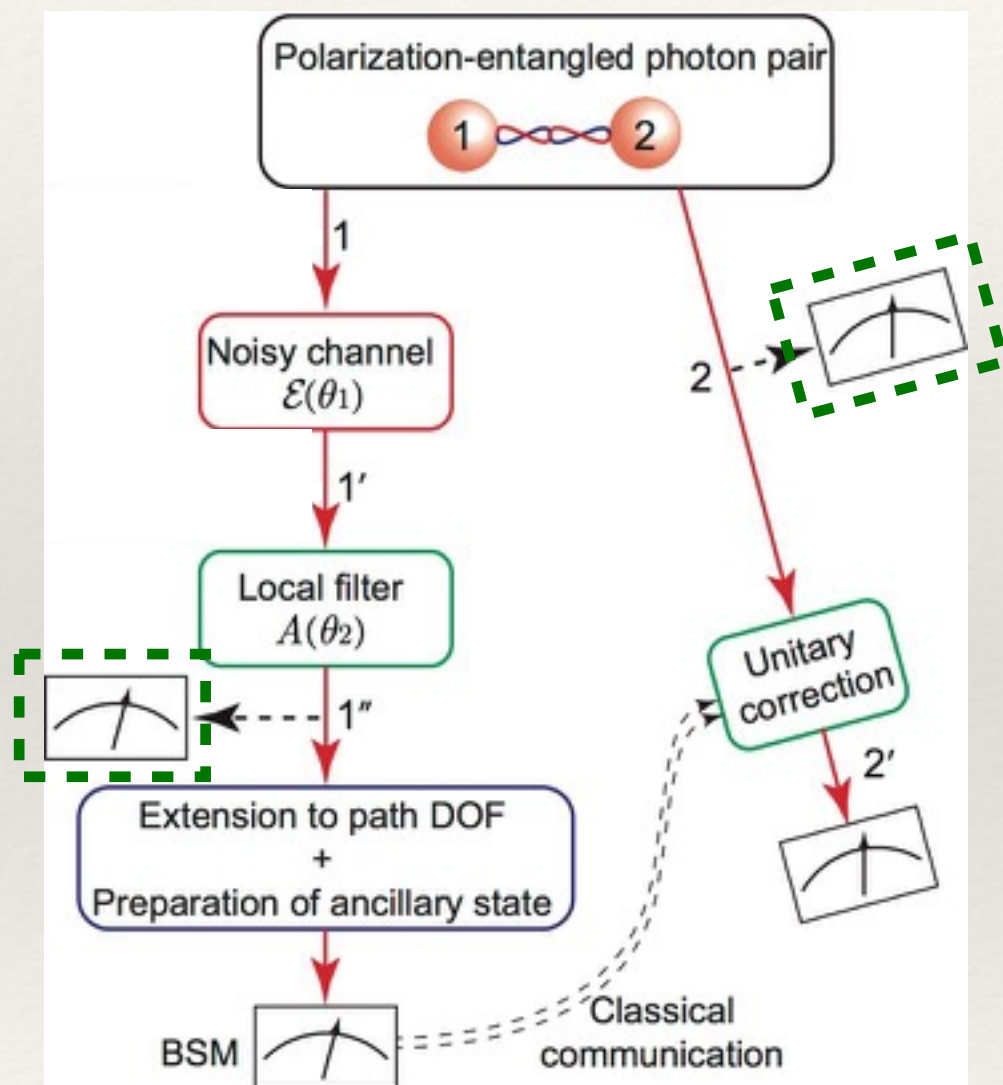
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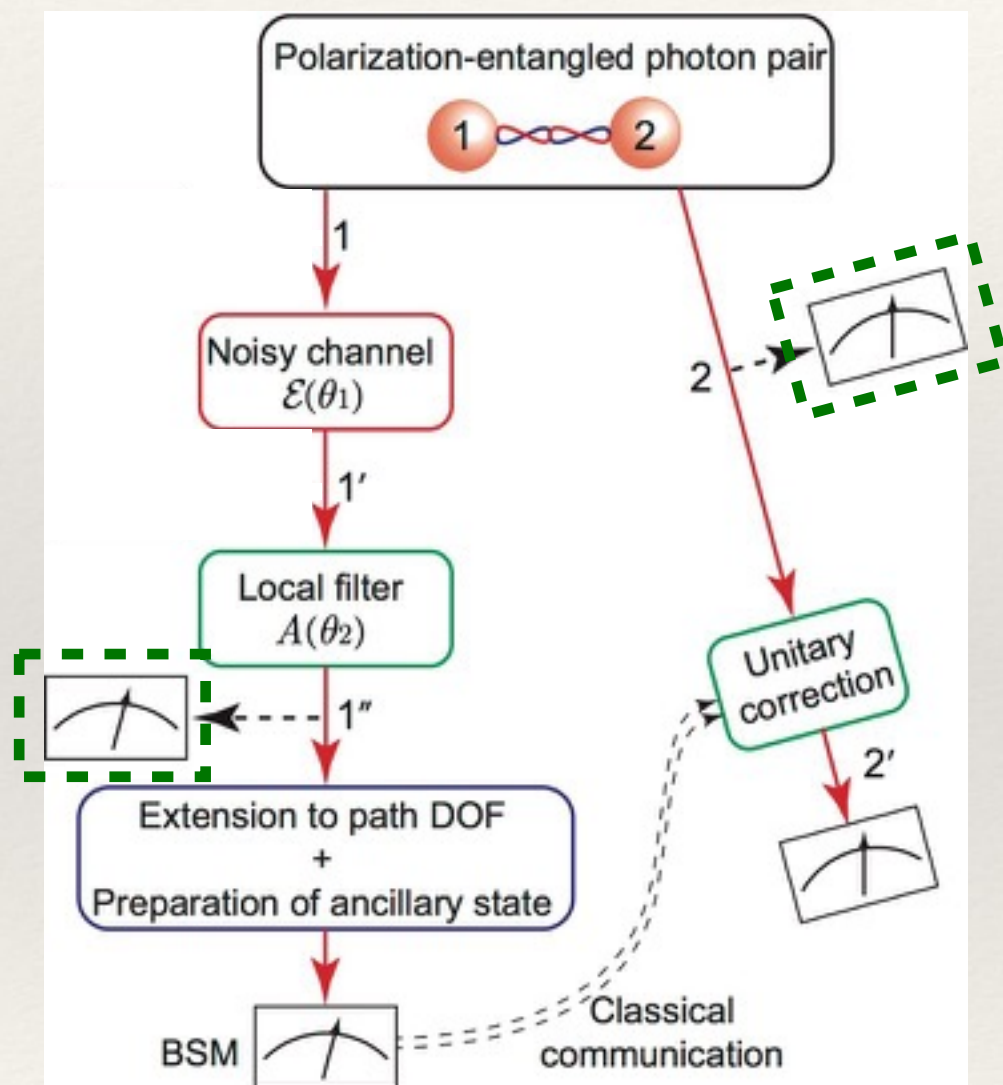


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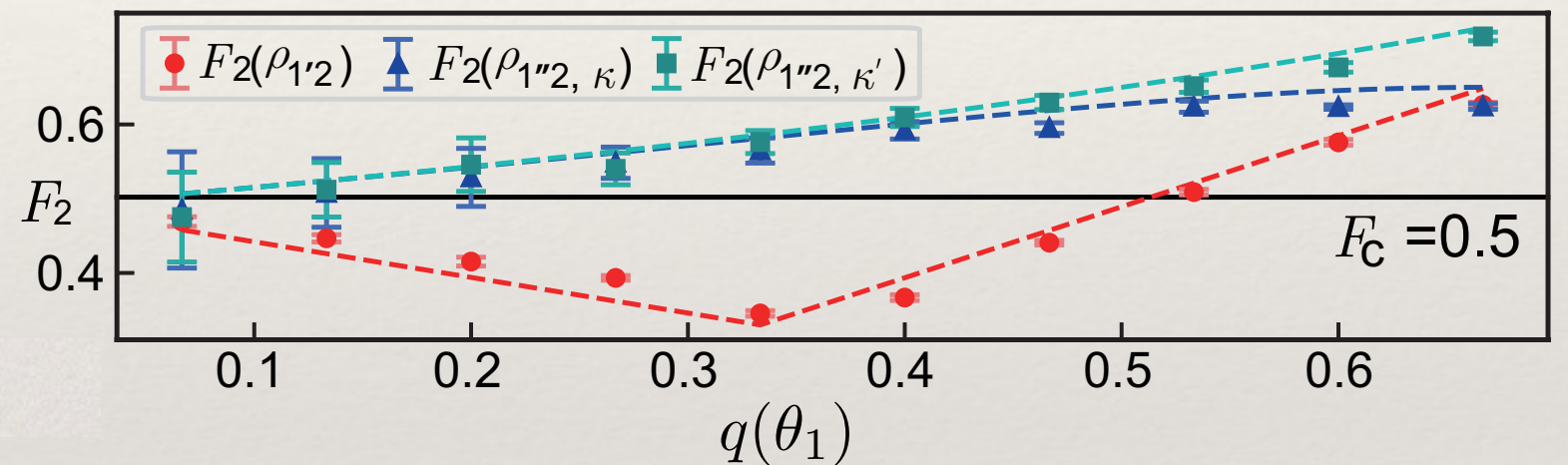
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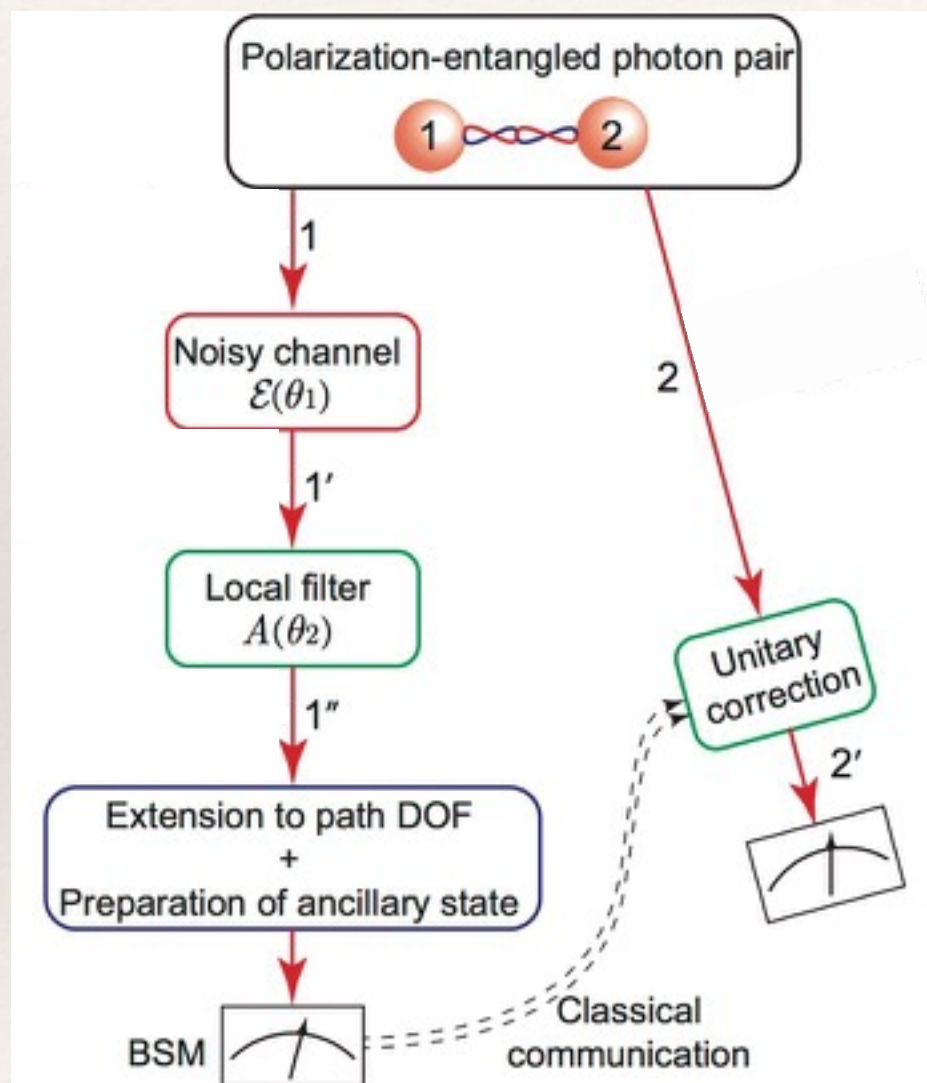
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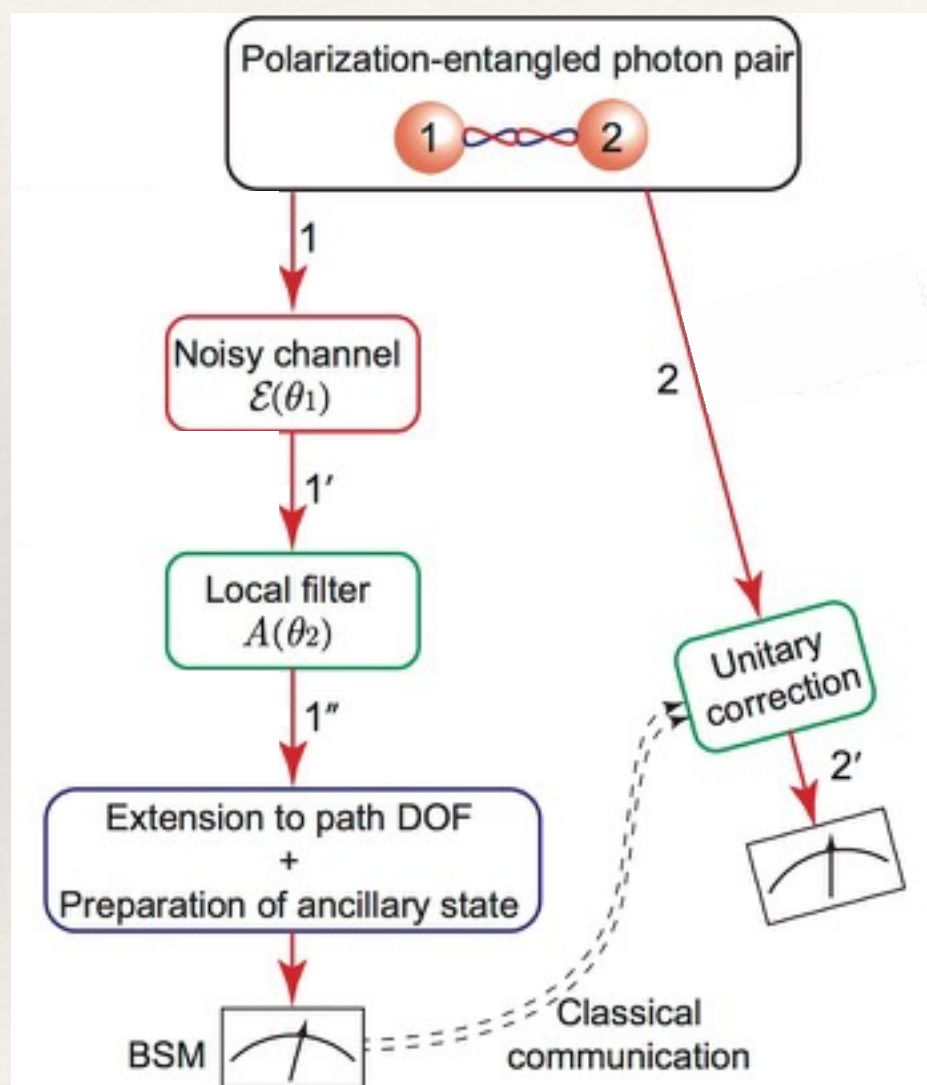
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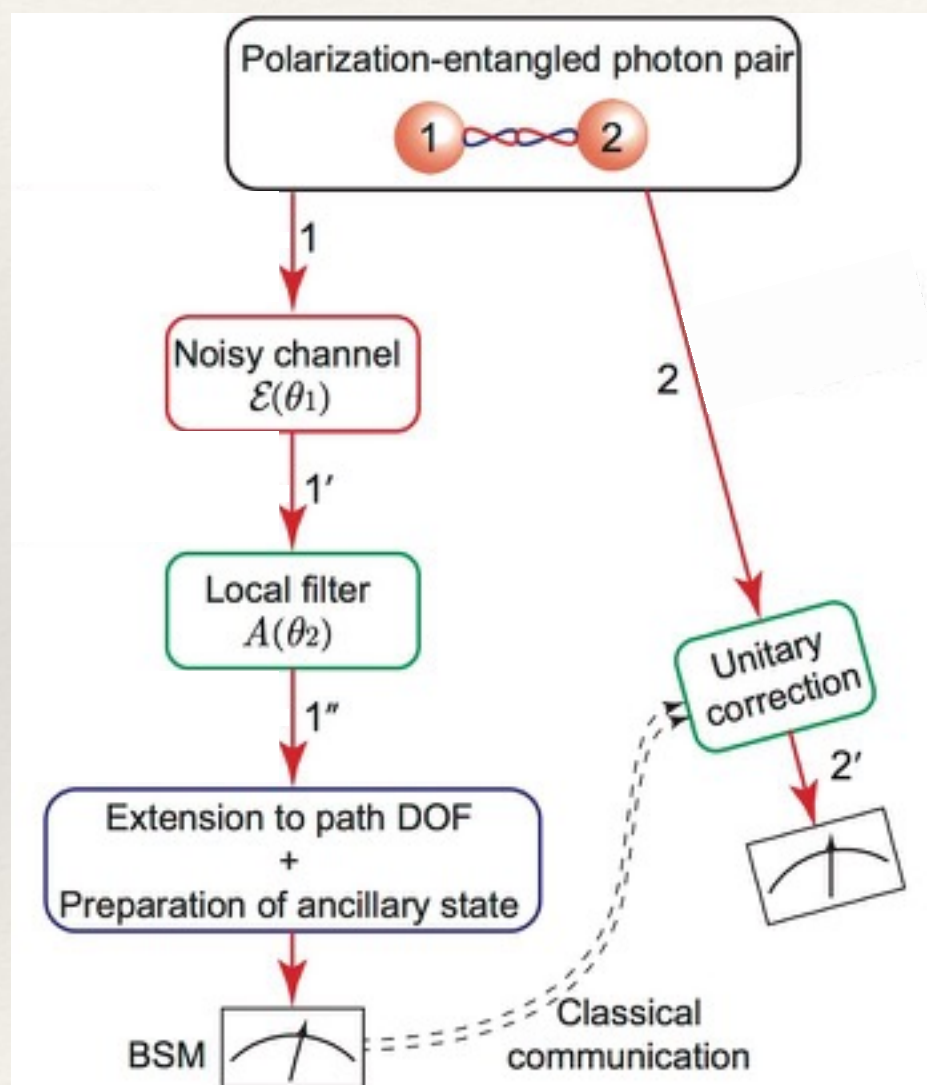


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¹⁵ Nielsen & Chuang, section 8.4.2

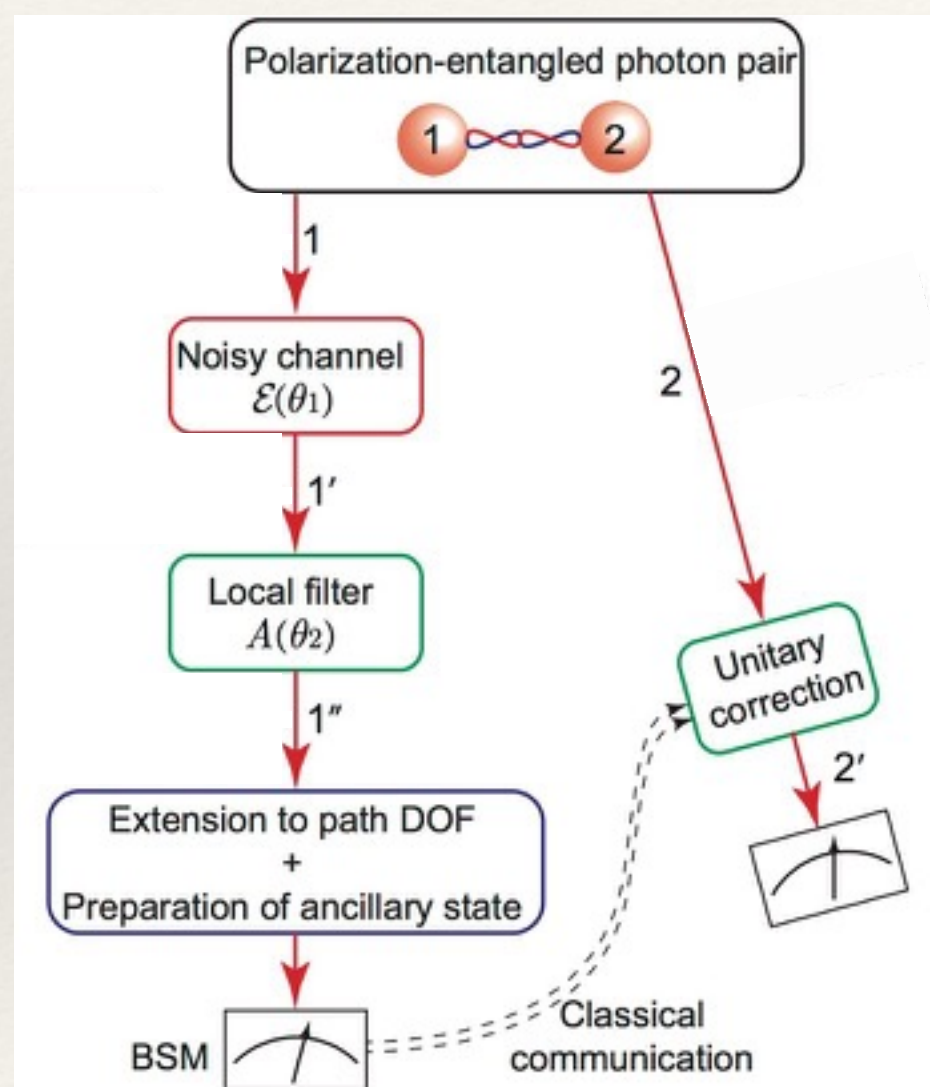
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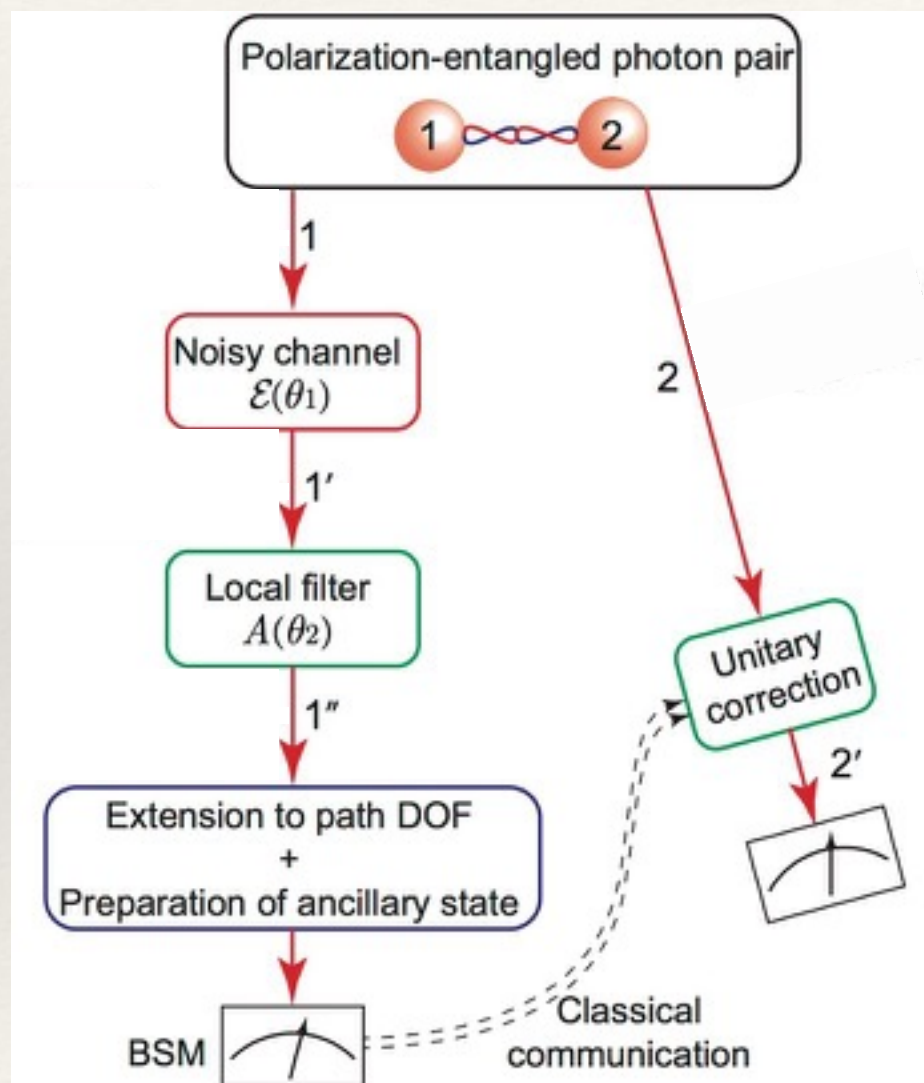
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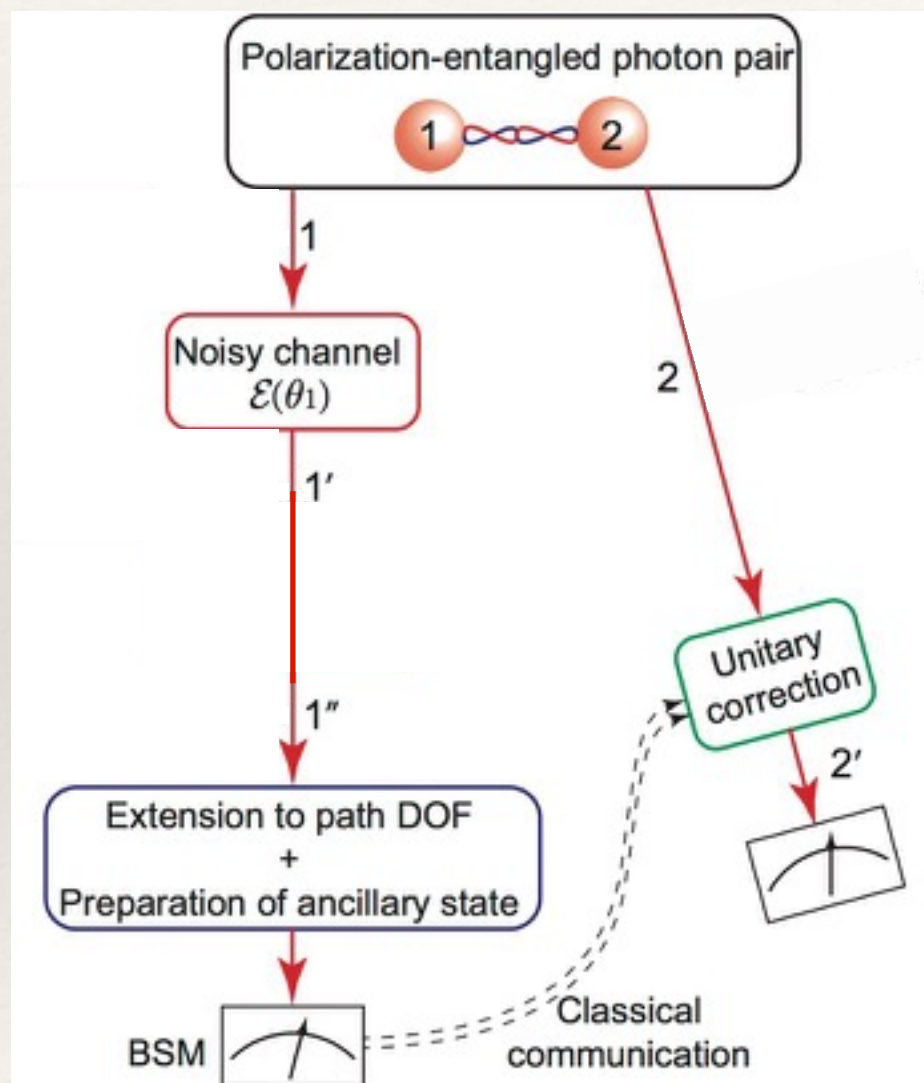
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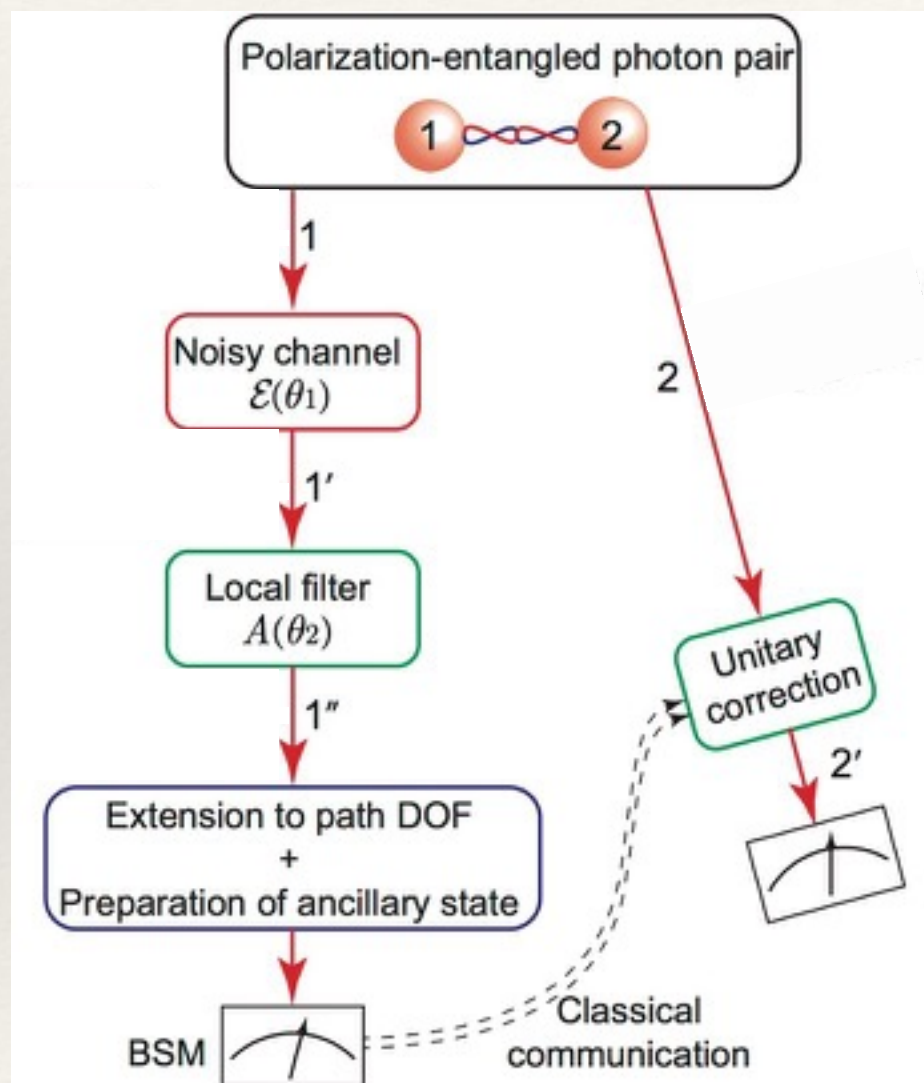
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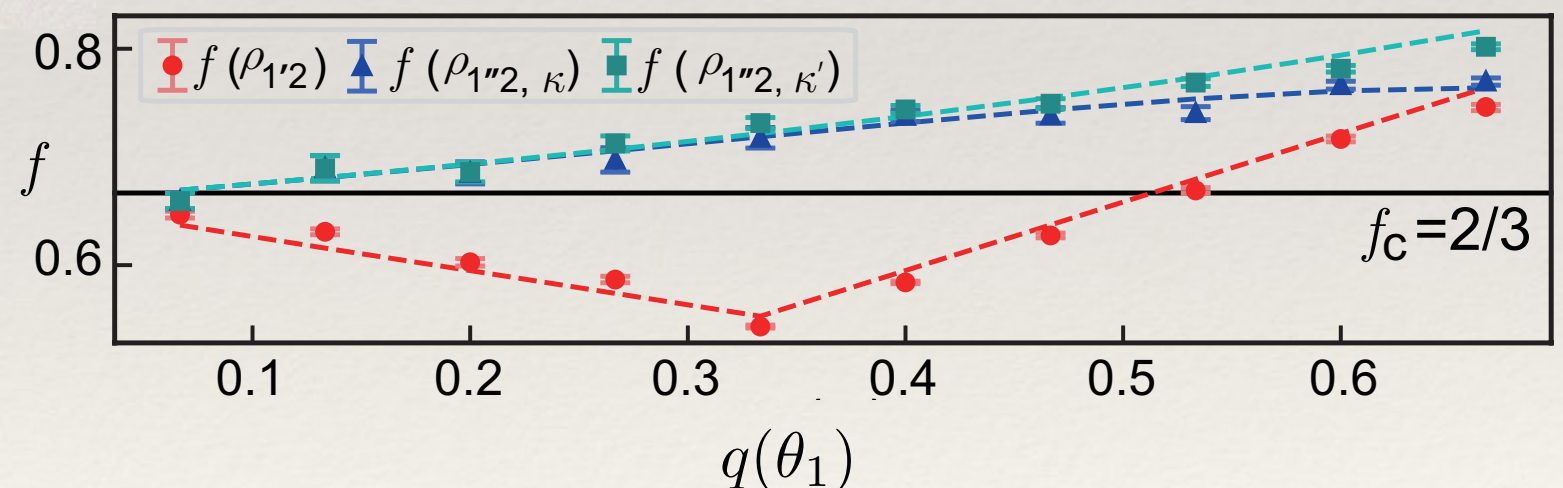
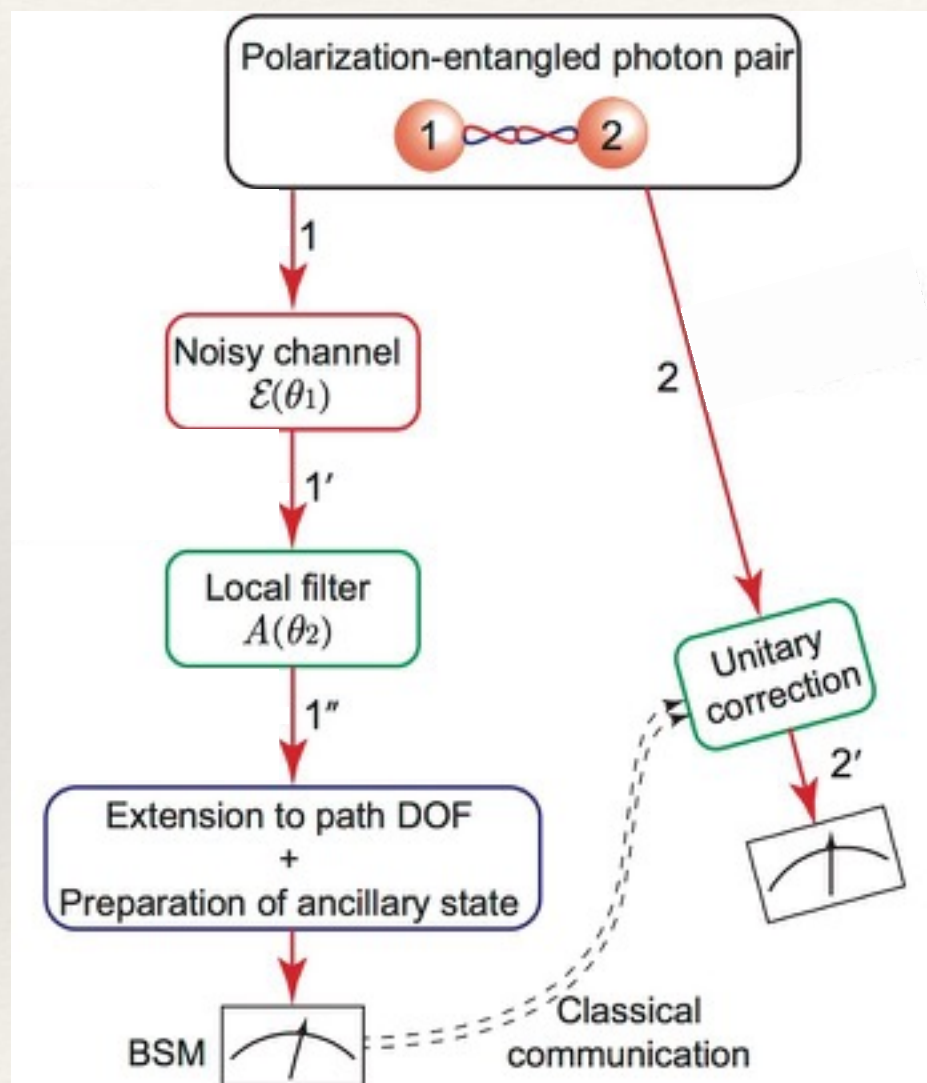
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- ❖ **Experimentally**, we have given a **proof-of-principle** demonstration of the **activation** of the **teleportation power** of some of these **rank-deficient** two-qubit states.

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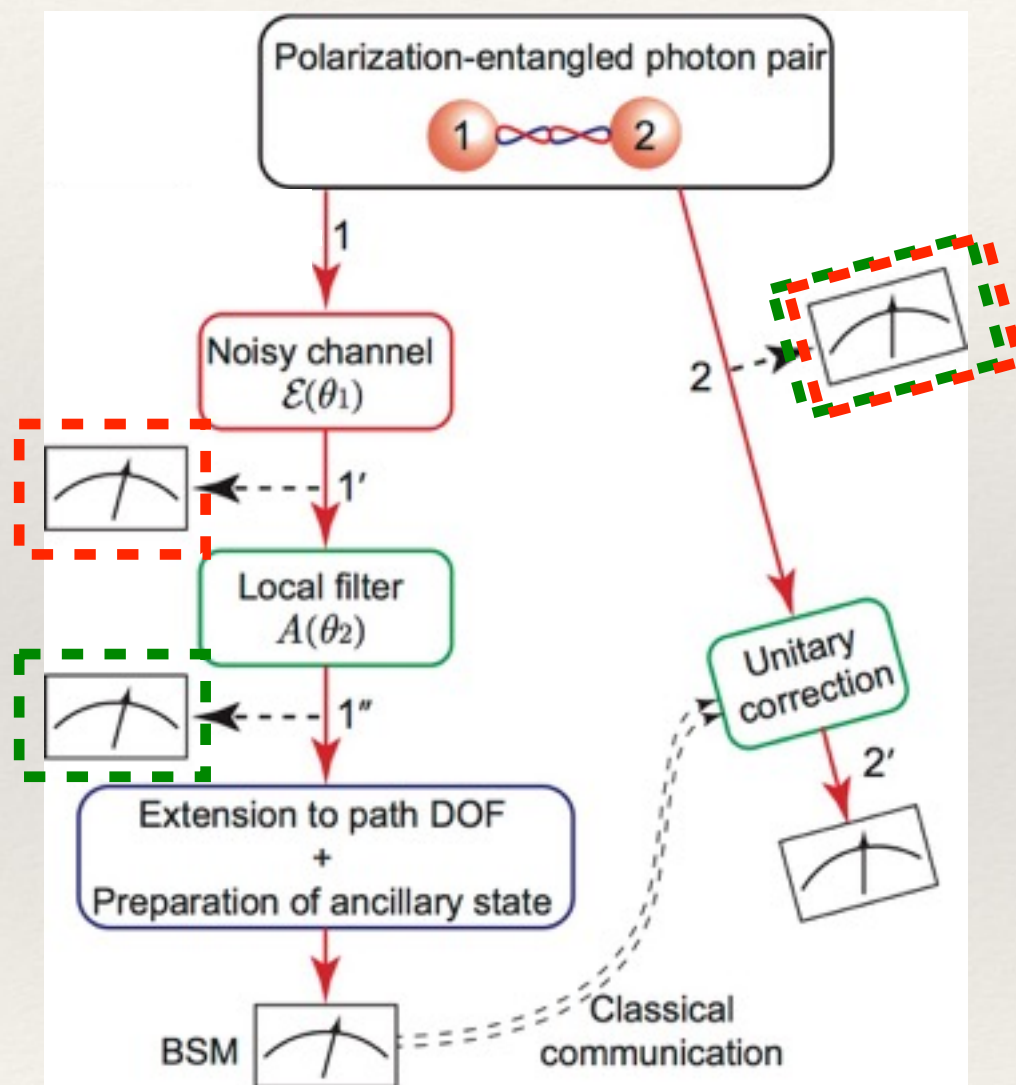
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- ❖ What happens if we allow filtering on **multiple copies** of the **same state**?
- ❖ What happens if allow **joint measurements** other than **generalized Bell measurements**?

Results: FEF vs Teleportation Fidelity



$$f(\rho) = \frac{d F_d(\rho) + 1}{d + 1}$$

