

Certifying quantum randomness with low latency

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Experimental realization



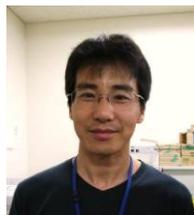
Hsin-Pin Lo



Takuya Ikuta

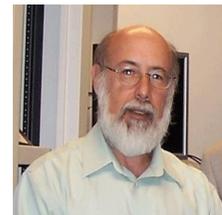


Toshimori Honjo



Hiroki Takesue

*Theoretical model, security analysis
& randomness extraction*



Alan Mink



William J. Munro

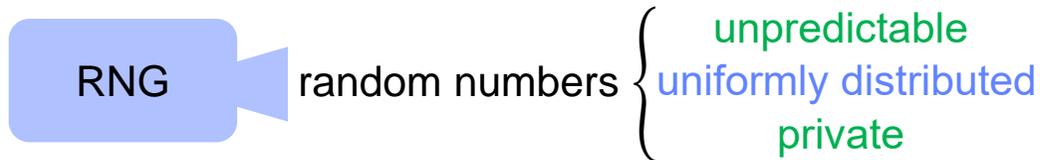
Outline of the talk



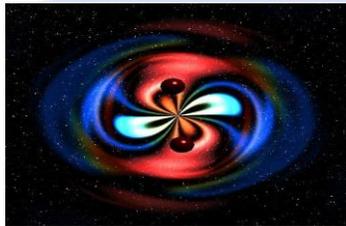
- Introduction to randomness
- Our scheme for quantum randomness generation
- Our method for certifying quantum randomness
- Experimental realization & results

Background: Why randomness is important?

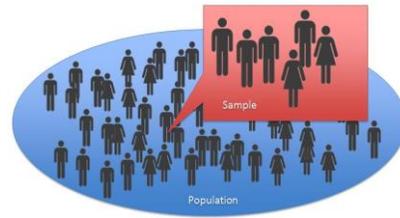
- Random numbers are generated through a process or device called the random number generator (RNG).



Huge amount of uniform randomness



Simulation



Sampling

High-quality, certified, private randomness



Gambling



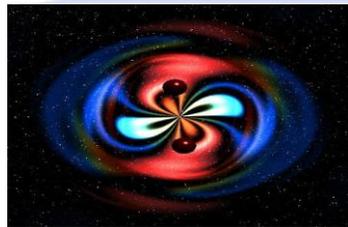
Cryptography

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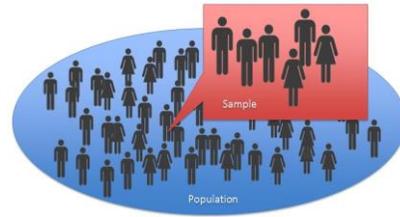
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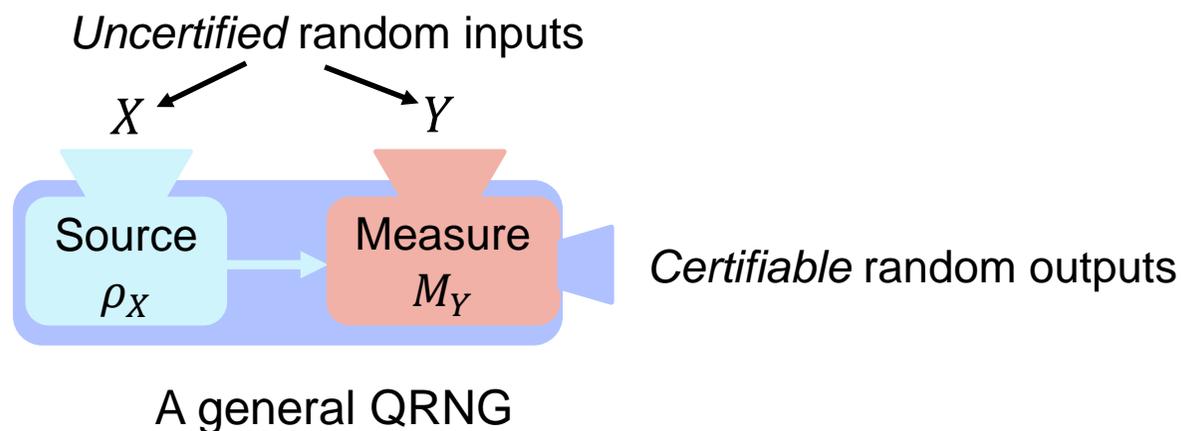
Cryptography

Classical process is deterministic → *No perfect random numbers*

Quantum measure is probabilistic → *Ideal random numbers as needed*

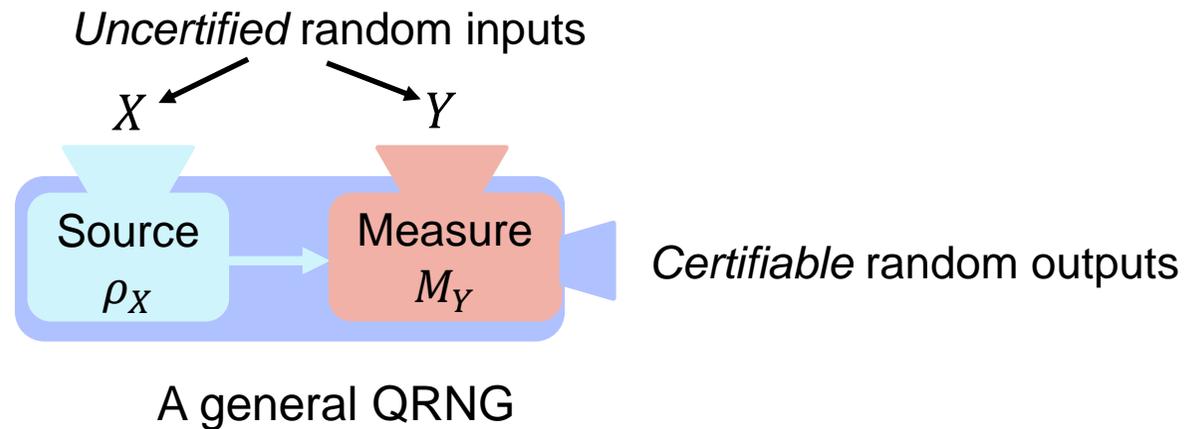
Background: Quantum random number generators

- ❖ **Main idea:** exploits the probabilistic nature of quantum measurements to generate genuine random numbers.



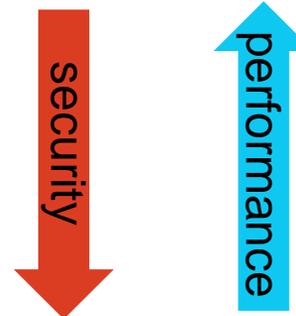
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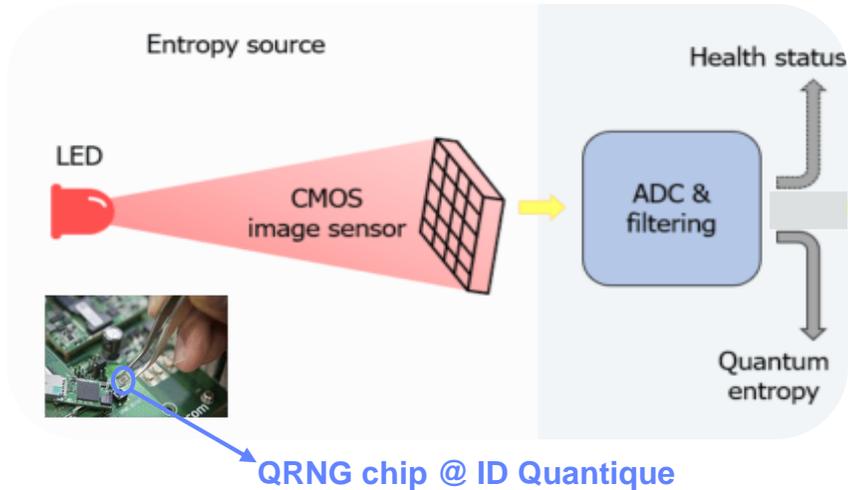


Depending on the amount of characterization on the quantum devices:

- Device-dependent QRNG
- Semi-device-(in)dependent QRNG
- Device-independent QRNG

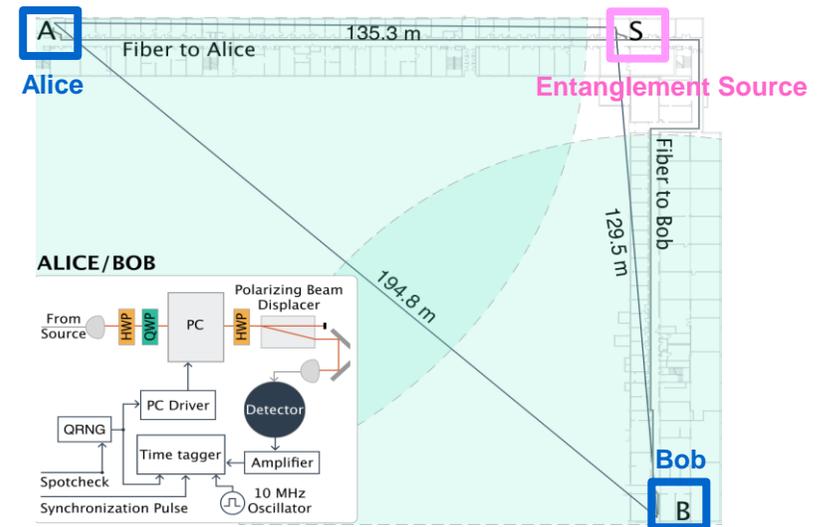


Device-dependent QRNG



- ✓ simple --- a small device
- ✓ high performance --- randomness rate ~250 Kbps (embedded in a smartphone)
- ✗ require **fully characterized** device --- impossible to achieve in practice, so **security is problematic**

Device-independent QRNG



Loophole-free Bell-test setup @ NIST-Boulder

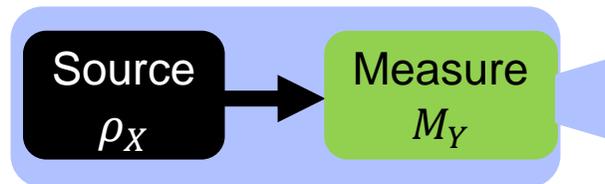
- ✓ high security --- no need to characterize the device
- ✗ complicated --- a large-scale device
- ✗ high latency (time consuming) --- a few minutes or hours delay before generating randomness
- ✗ low performance --- randomness rate ~100 bps

Background: Semi-device-independent QRNG

- Semi-device-independent --- the device is partially characterized.
- Advantage --- can achieve a balance between **performance** and **security**.

Previous works

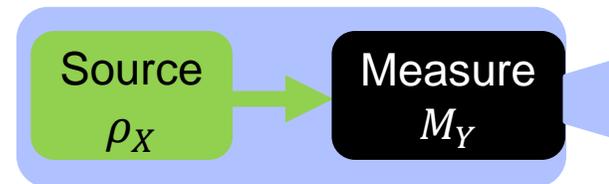
Source-independent QRNG



Uncharacterized *Characterized*

Cao *et al.*, PRX 6, 011020 (2016)
Marangon *et al.*, PRL 118, 060503 (2017)

Measurement-device-independent QRNG



Characterized *Uncharacterized*

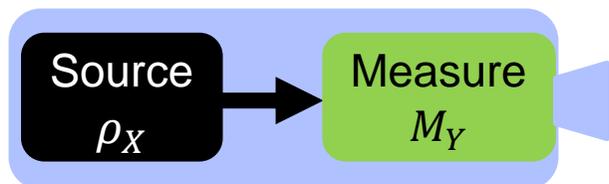
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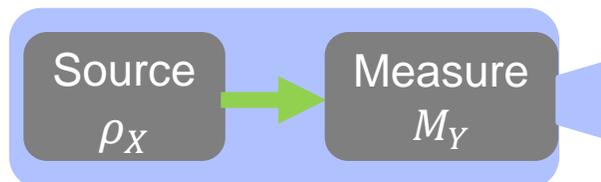
❖ **Current problems: reliability and latency**

Questions:

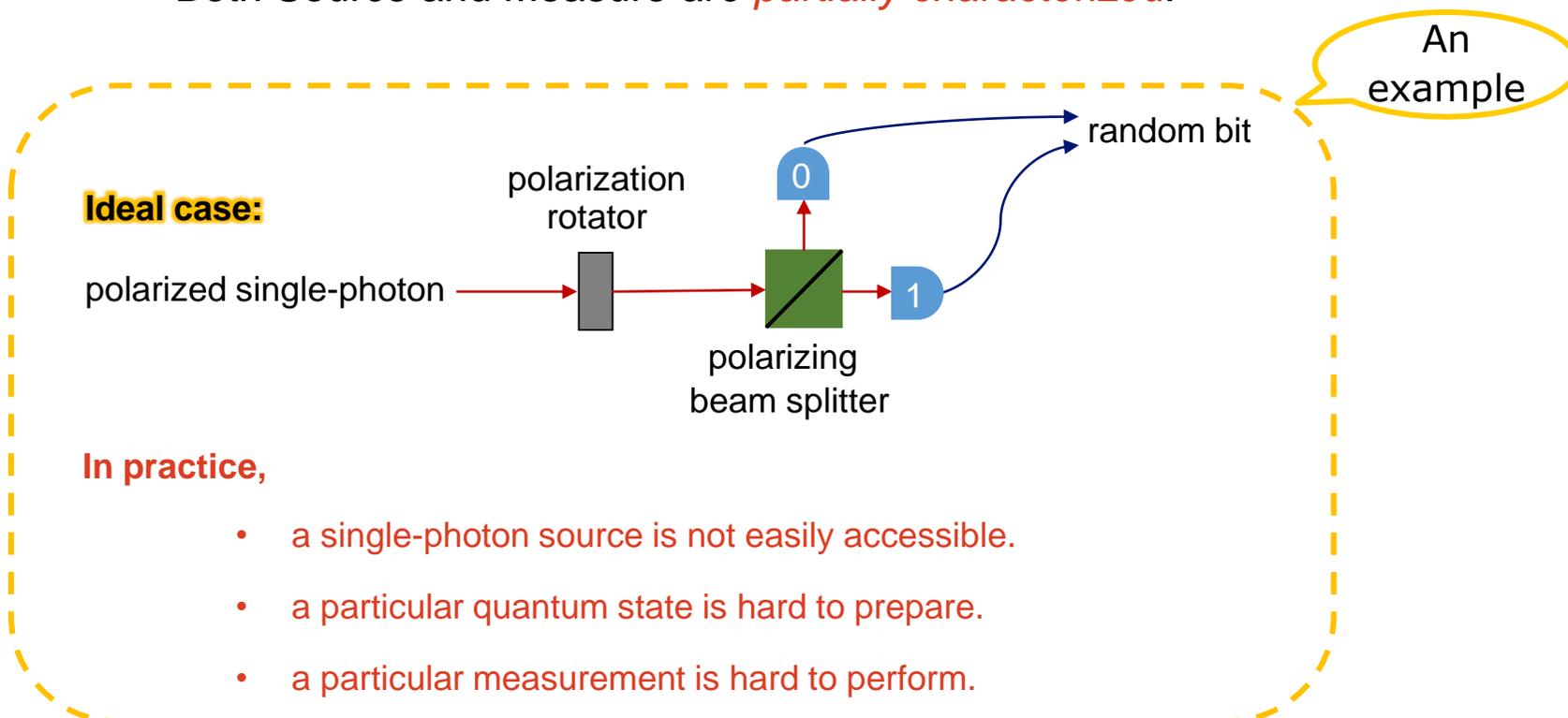
- Increase reliability: can we address imperfections in both source & measure?
- Reduce latency: can we achieve low-latency randomness generation?

Overview of our achievements

- Studied a new semi-device-independent QRNG scheme

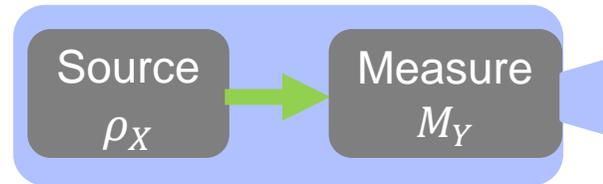


Both Source and Measure are *partially characterized*.



Overview of our achievements

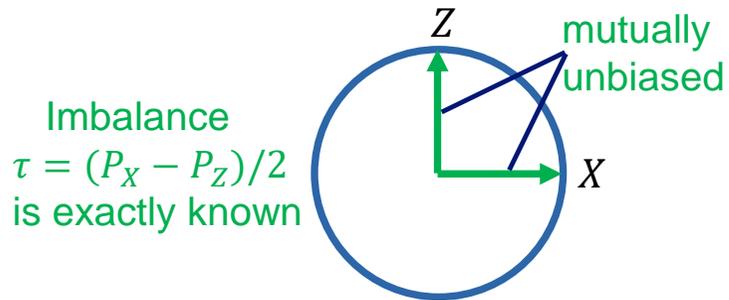
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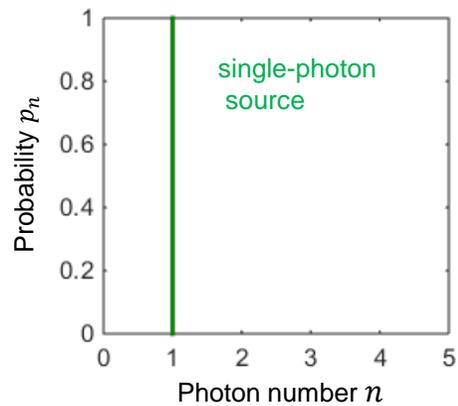
Both Source and Measure are *partially characterized*.

- Developed *efficient* methods for randomness certification in the above scenario
- *Realized* a simple low-latency real-time high-security QRNG

A simple QRNG scheme

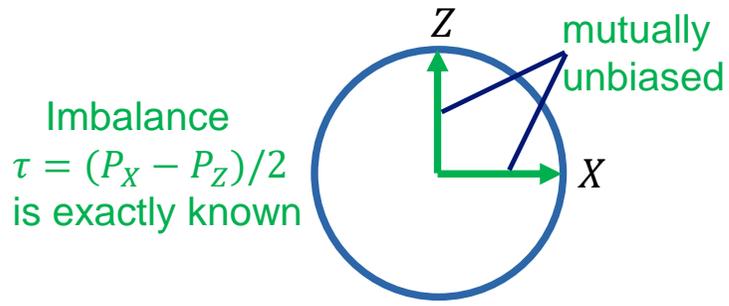


Ideal measure
in the single-photon space

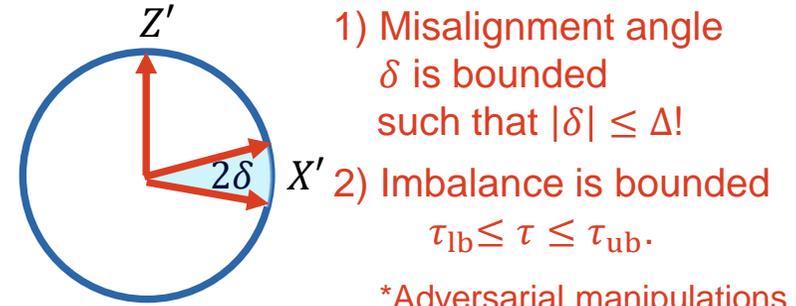
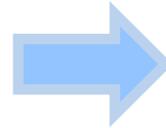


Ideal source

A simple QRNG scheme with imperfections

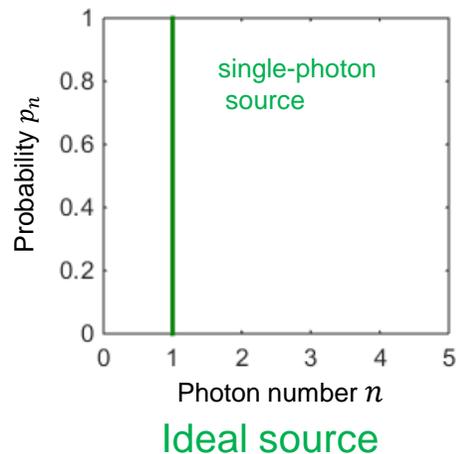


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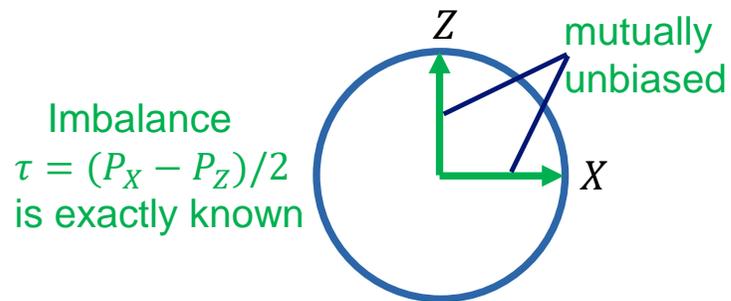


Practical measure
in the single-photon space

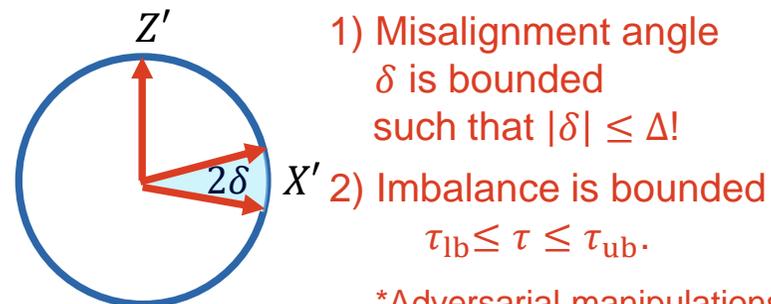
*Adversarial manipulations are allowed.



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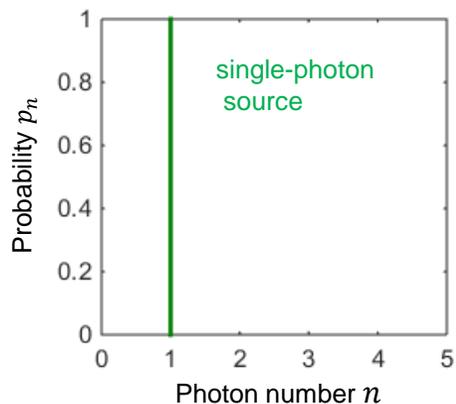


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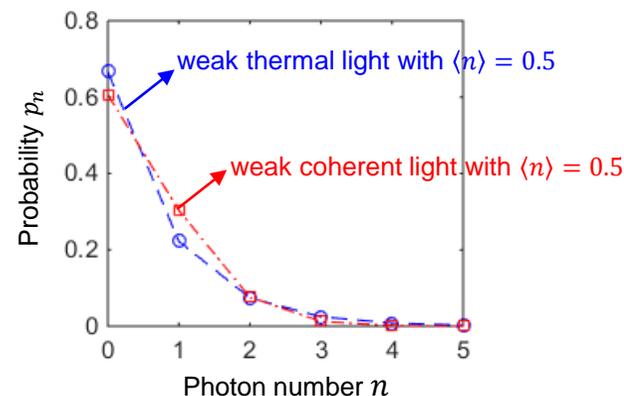
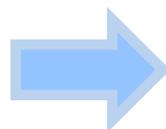


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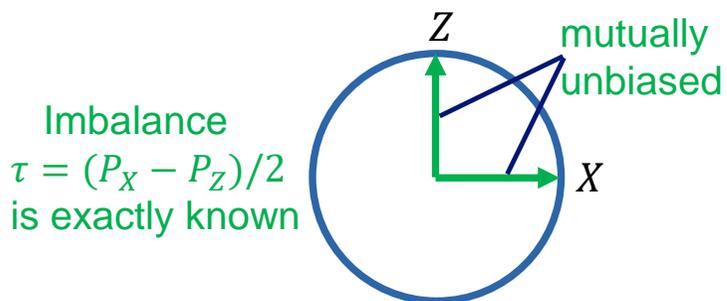


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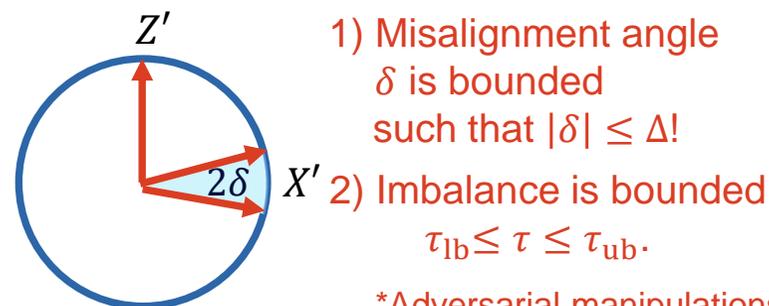


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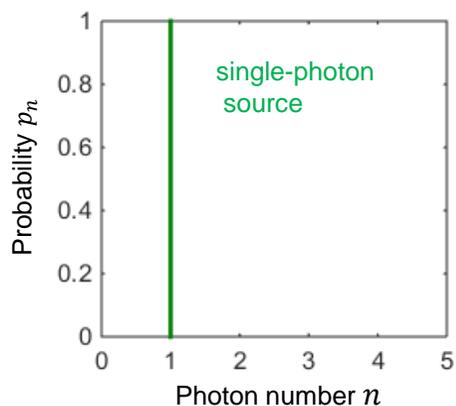


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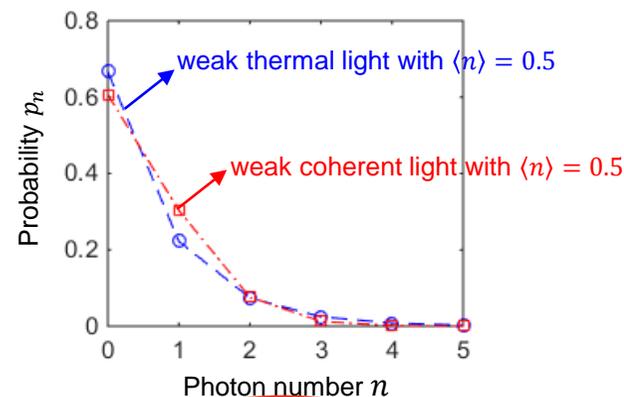
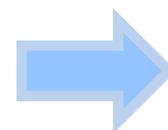


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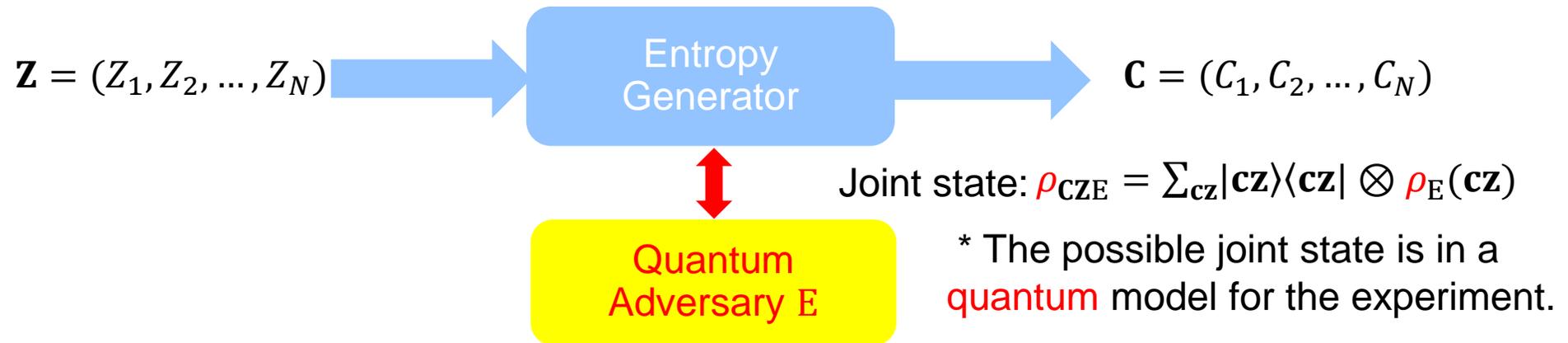
Ideal source



Practical source

- 1) Photon-number dist. is bounded such that $p_{n=1} \geq p_{1,lb}$ or $p_{n=1}/p_{n \geq 1} \geq p_{1,lb}$.
- 2) Measurements are block-diagonal $M = M_{n=1} \oplus M_{n \neq 1}$.

Our method: Quantification of randomness



- Guessing probability: $P_{\text{guess}}(\mathbf{C}|\mathbf{Z}E)_\rho$

- Easily accessible measure of uniform randomness:

$$H_{\min}(\mathbf{C}|\mathbf{Z}E)_\rho = -\log_2[P_{\text{guess}}(\mathbf{C}|\mathbf{Z}E)_\rho]$$

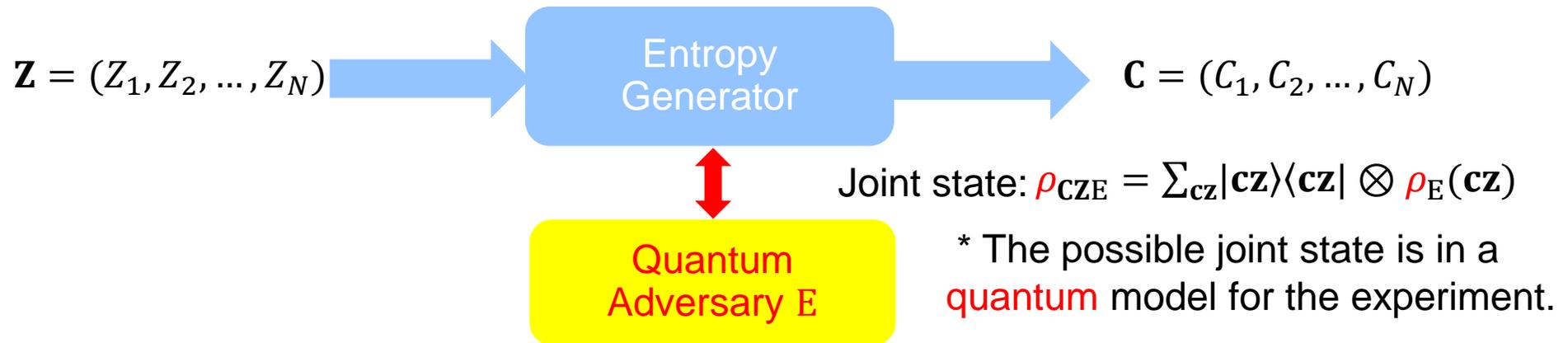
- Flexible measure of uniform randomness:

$$H_{\min}^\varepsilon(\mathbf{C}|\mathbf{Z}E)_\rho = \sup_{\rho'} \{H_{\min}(\mathbf{C}|\mathbf{Z}E)_{\rho'}, P(\rho, \rho') \leq \varepsilon\}$$

R. König, R. Renner, and C. Schaffner, IEEE Trans. Inf. Theory 55, 4337 (2009)

M. Tomamichel, R. Colbeck, and R. Renner, IEEE Trans. Inf. Theory 56, 4674 (2010)

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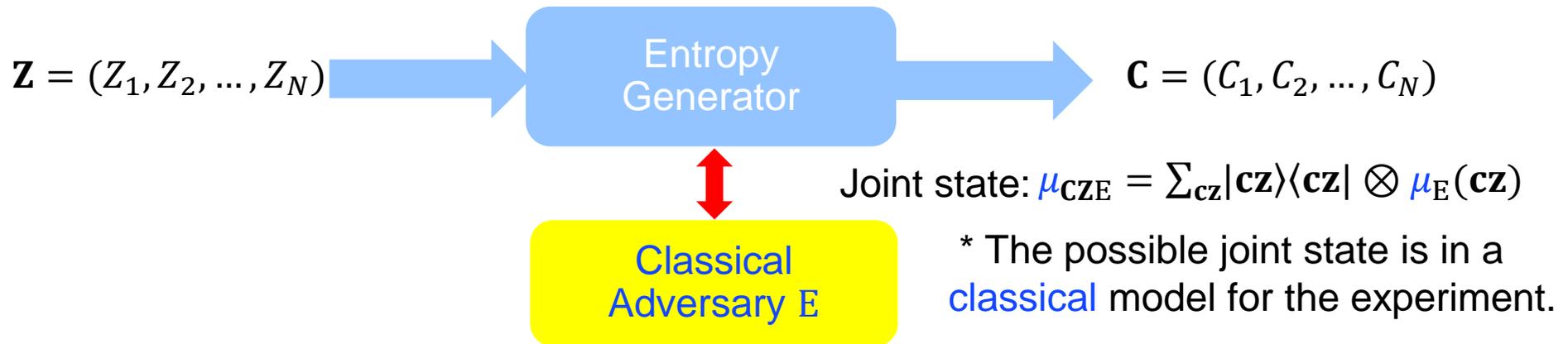
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Goal: Lower-bound smooth conditional min-entropy $H_{\min}^\varepsilon(\mathbf{C}|\mathbf{Z}E)$

Our method: Quantification of randomness

(in the presence of classical side information)



- Guessing probability: $P_{\text{guess}}(\mathbf{C}|\mathbf{Z}\mathbf{E})_{\mu}$

- Easily accessible measure of uniform randomness:

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Our method: Concepts involved



(in the *general* approach of quantum probability estimation)

- **Quantum model** $\mathcal{Q}(\mathbf{CZ})$ --- the set of all possible joint states $\rho_{\mathbf{CZ}E}$ at the end of the experiment.
- **Quantum estimation factor** (QEF) --- a function $F_q(\mathbf{CZ})$ satisfying a set of constraints imposed by each possible $\rho_{\mathbf{CZ}E} \in \mathcal{Q}(\mathbf{CZ})$.

Y. Z., H. Fu, and E. Knill, Phys. Rev. Research 2, 013016 (2020)

Y. Z., L. K. Shalm *et al.*, Phys. Rev. Lett. 124, 010505 (2020)

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Under the **quantum Markov-chain** conditions (natural for time-ordered trials)

$$\mathbf{C}_{<i} \leftrightarrow (\mathbf{Z}_{<i}, E) \leftrightarrow Z_i, \forall i, \text{ [IID assumption is not required]}$$

we need only to construct

- **Model** $\mathcal{Q}(CZ)$ --- the set of all possible joint states ρ_{CZE} at the end of a trial.
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Our method: Concepts involved

(in the *general* approach of probability estimation)

- **Classical model** $\mathcal{C}(\mathbf{CZ})$ --- the set of all possible joint states $\mu_{\mathbf{CZE}}$ at the end of the experiment.
- **Probability estimation factor** (PEF) --- a function $F_c(\mathbf{CZ})$ satisfying a set of constraints imposed by each possible $\mu_{\mathbf{CZE}} \in \mathcal{C}(\mathbf{CZ})$.

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Y. Z., E. Knill, and P. Bierhorst, Phys. Rev. A 98, 040304(R) (2018)

E. Knill, Y. Z., and P. Bierhorst, Phys. Rev. Research 2, 033465 (2020)

Our method: Main theorem

(of quantum probability estimation)

- **Quantum** model $\mathfrak{Q}_i(C_i Z_i)$ and **QEF** $F_{q,i}(C_i Z_i) \geq 0$ with power $\beta_q > 0$ for each trial i .

QEF Def. $\forall \rho_{C_i Z_i E} \in \mathfrak{Q}_i(C_i Z_i), \left\langle F_{q,i}(C_i Z_i) \hat{R}_{1+\beta_q}(\rho_E(C_i Z_i) | \rho_E(Z_i)) \right\rangle \leq 1.$

* Models and QEFs for different trials can be different.

* $\hat{R}_{1+\beta_q}(\rho_E(C_i Z_i) | \rho_E(Z_i))$ is the sandwiched Rényi power of order $(1 + \beta_q)$.

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- The success event $\Phi \triangleq \{\mathbf{c}z: \prod_{i=1}^N F_{q,i}(c_i z_i) \geq t_{\min}\}$.
- κ --- a desired lower bound of the success probability.

Theorem: For each possible state ρ_{CZE} , *either* the success probability satisfies

$$\text{Prob}_{\rho_{CZE}}(\Phi) \leq \kappa,$$

or conditional on success

$$H_{\min}^{\varepsilon}(\mathbf{C} | \mathbf{Z}E)_{\rho_{CZE | \Phi}} \geq \frac{1}{\beta_q} \log(t_{\min}) + \frac{1}{\beta_q} \log\left(\frac{\varepsilon^2}{2}\right) + \frac{1+\beta_q}{\beta_q} \log(\kappa).$$

Our method: Main theorem

(of probability estimation)

- Classical model $\mathbf{e}_i(C_i Z_i)$ and PEF $F_{c,i}(C_i Z_i) \geq 0$ with power $\beta_c > 0$ for each trial i .

PEF Def. $\forall \mu_{C_i Z_i E} \in \mathbf{e}_i(C_i Z_i), \langle F_{c,i}(C_i Z_i) [\mu_E(C_i | Z_i)]^{\beta_c} \rangle \leq 1.$

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Our method: for the scenario considered

Measurements considered:

$$M_X = \begin{pmatrix} M_{X,n=1} & 0 \\ 0 & M_{X,n \neq 1} \end{pmatrix}, \quad M_Z = \begin{pmatrix} M_{Z,n=1} & 0 \\ 0 & M_{Z,n \neq 1} \end{pmatrix},$$

1. $M_{X,n \neq 1}$ and $M_{Z,n \neq 1}$ are arbitrary
2. $M_{X,n=1}$ and $M_{Z,n=1}$ are qubit measurements with $|\delta| \leq \Delta$.

3. M_X and M_Z are randomly selected with bounded probabilities.

States considered: $\rho = \begin{pmatrix} \rho_{n=1} & 0 \\ 0 & \rho_{n \neq 1} \end{pmatrix}$, where $\text{Tr}(\rho_{n=1}) \geq p_{1,\text{lb}}$.

Physical model

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Physical model

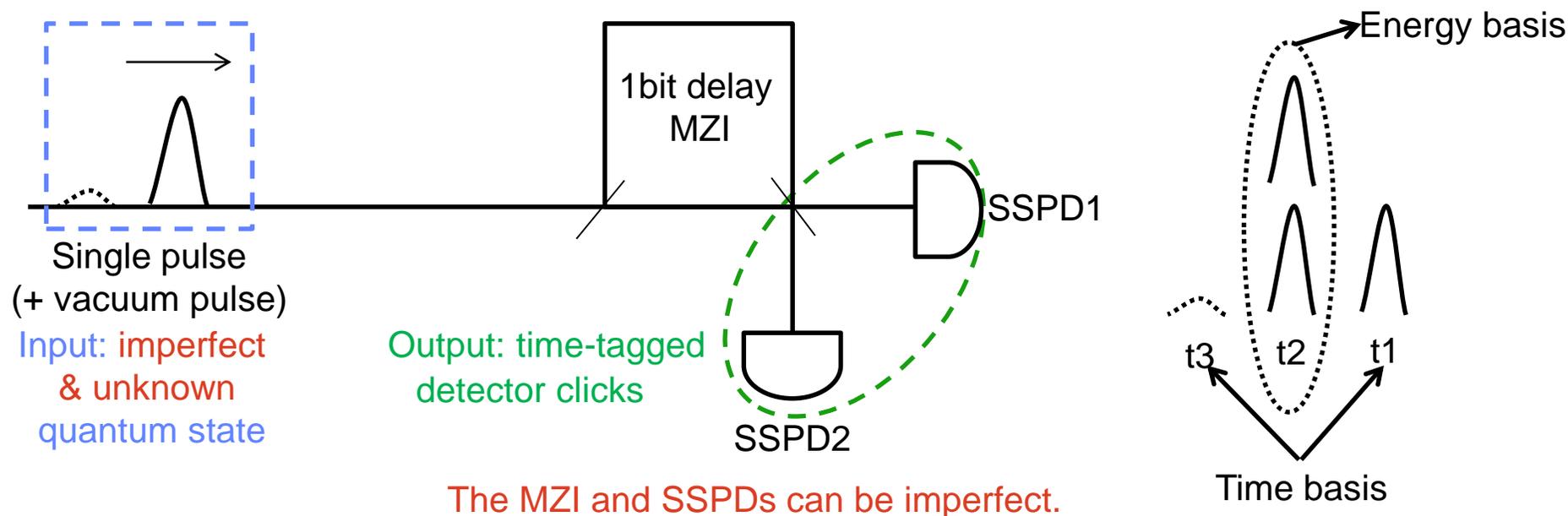
Construct the best QEF and PEF
by convex optimization

Convex polytope $\mathcal{P}_Q \supseteq \mathcal{Q}$
& convex polytope $\mathcal{P}_C \supseteq \mathcal{C}$

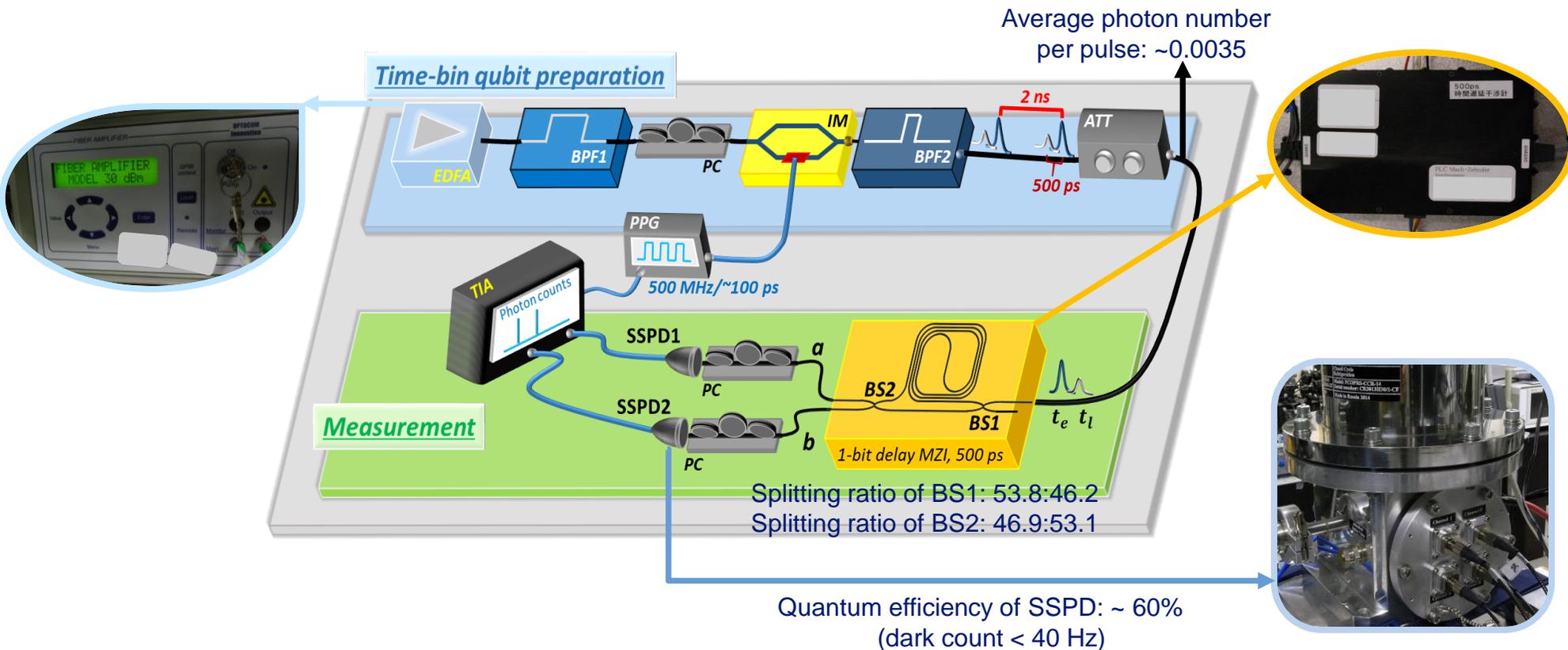
Quantum model \mathcal{Q}
& classical model \mathcal{C}

Our QRNG: Experimental realization

- A laser pulse is inputted into a Mach–Zehnder interferometer (MZI), and outputs are detected by two superconducting single-photon detectors (SSPDs).
- Two orthogonal measurement bases: energy basis & time basis.
 1. Energy basis: **random** (0: click at SSPD1, or 1: click at SSPD2)
 2. Time basis: t1 (almost) t3 (rare event)
- **Advantage --- easily integrated onto a chip.**
- **Our QRNG is semi-device-independent --- allows imperfections in both source & measure.**



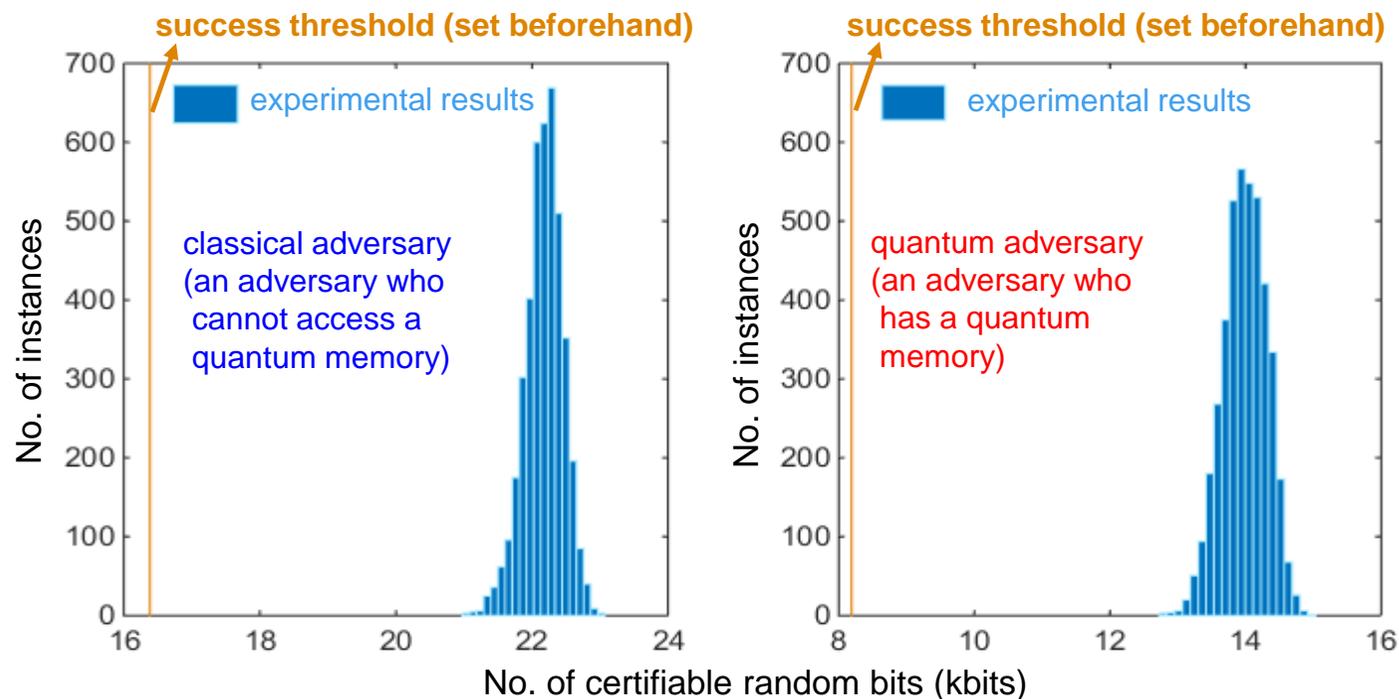
Our QRNG: Experimental realization



Our QRNG allows:

- Imperfect source --- weak optical pulse *rather than* single-photon source.
- Imperfect basis choice --- a basis is selected with an inexact probability.
- Imperfect measure --- measurements are misaligned.

Result 1: Low-latency real-time high-security QRNG



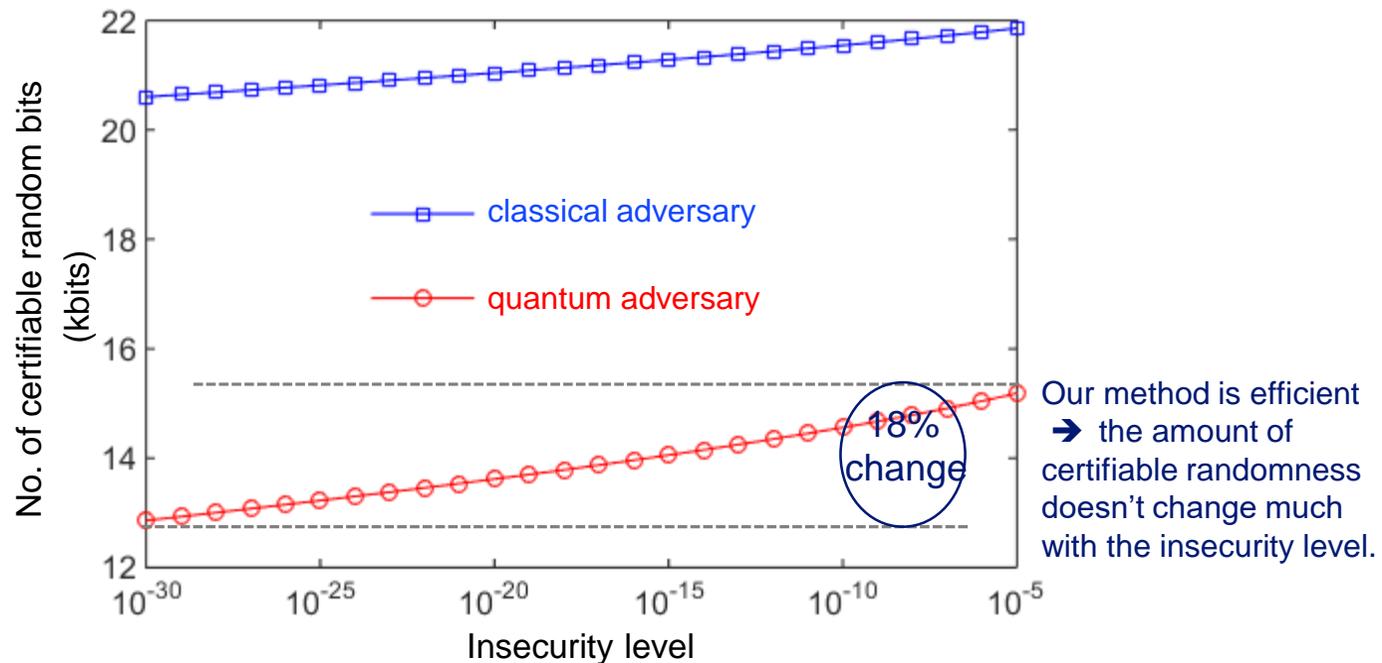
- Each instance generates 8192 (or 2×8192) random bits against quantum (or classical) adversary with insecurity $2^{-64} \approx 5.4 \times 10^{-20} \rightarrow$ high security.
Insecurity --- Adversary's ability to distinguish the generated random bits (real case) from the perfectly random bits (ideal case).
- Each instance takes 0.1 s runtime (which includes the latency 0.047 s) + 0.02 s (or 0.04 s) extraction time \rightarrow real time & low latency.

Result 2: Trade-off between quantity & quality

- Depending on the specific application, we choose the insecurity level beforehand.
E.g.,

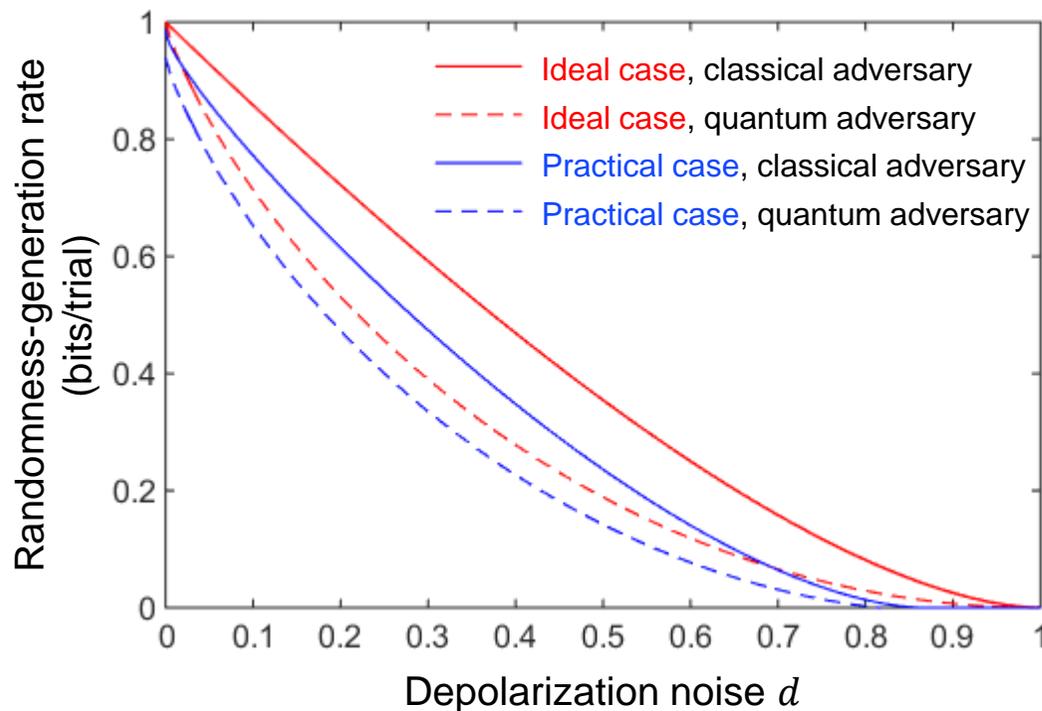
Simulation requires low security --- recommended insecurity level 10^{-5} .

Cryptography requires high security --- recommended insecurity level 10^{-20} .



Expected number of random bits certifiable from the measurement outcomes observed in every 0.1 s runtime.

Result 3: Classical vs Quantum adversary



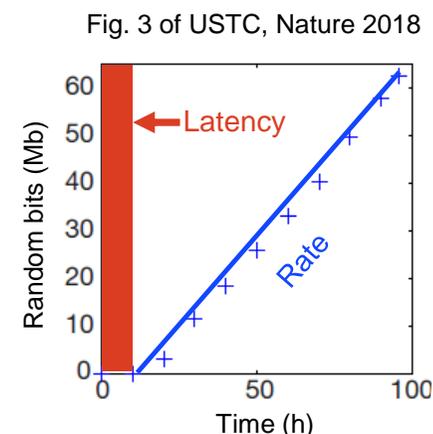
Simulation:
Binary-outcome measurements such that $\langle M_1 \rangle = 0$ and $\langle M_2 \rangle = 1 - d$.

Ideal case: 1-qubit + two mutually unbiased measurements
Practical case: 95% 1-qubit + two misaligned measurements (misalignment angle is 5°)

Clear demonstration of the reduction of the rate w.r.t. quantum adversaries as compared to that w.r.t. classical adversaries.

Comparison with other start-of-art works

	QRNG Type	Latency	Rate (over a long run)	Insecurity
ID Quantique PRX, 2014 arXiv:2011.14129	device dependent	<i>unreported</i>	4.90 Mbps** (the best QRNG chip)	<i>uncertified</i>
USTC Nature, 2018	device independent	13 hours	181 bps	10^{-5} quantum adversary
NIST PRL, 2020	device independent	5 min	55 bps	5.4×10^{-20} quantum adversary
Tsinghua Uvi. PRX 2016	semi device independent	<i>unreported</i>	5 Kbps	1.8×10^{-15} quantum adversary
Our work	semi device independent	47 ms	153 Kbps	5.4×10^{-20} quantum adversary



* **Latency** and **Rate** are two *different* measures of QRNG performance.

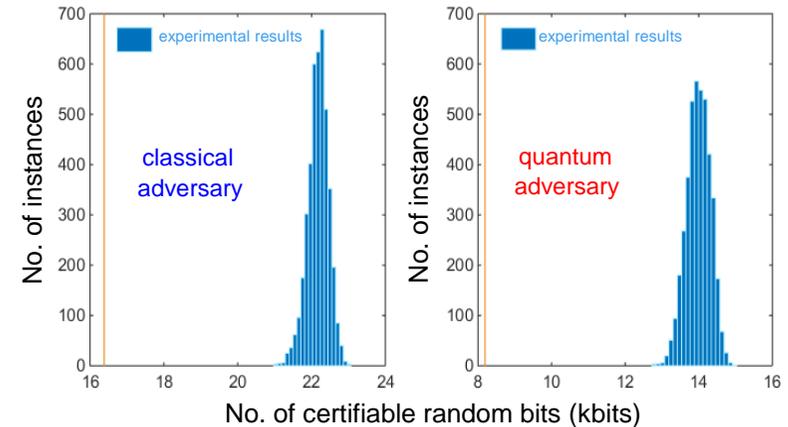
* Previous works focus on the study of the **rate** of a QRNG; however, the **latency** is more relevant for practical applications.

** The rate 250 Kbps in the video presentation is the typical entropy rate of the smallest QRNG chip embedded in a smartphone.

Significance of this work and future developments

Summary

- Simple & reliable QRNG scheme even with imperfections in both source and measure.
- New & efficient method for randomness certification, which is extendable to QKD.
- Low-latency real-time high-security QRNG.
- Advantage of quantum adversary.



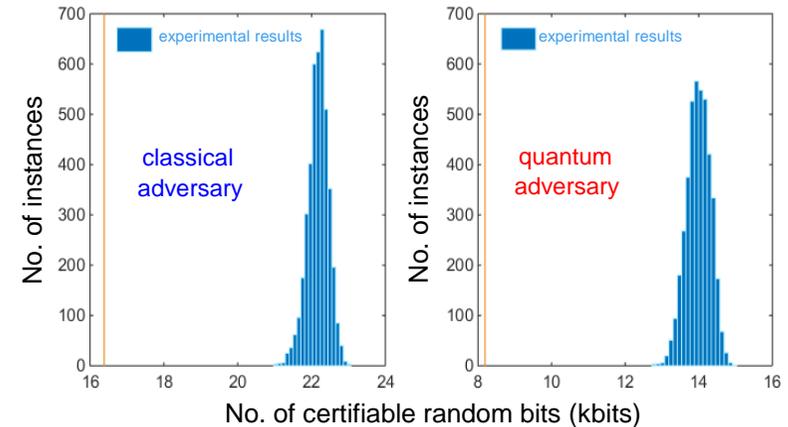
Outlook

- Reduce the size of our QRNG → [Integration into mobile phones.](#)
- Build a continuously-operating, high-security and high-speed [quantum randomness beacon](#) [ongoing efforts at NIST@USA and USTC@China].

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Thank you for your attention!