Quantum Tomography: Theory and Practice

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Ongoing project with

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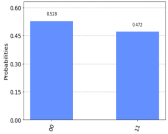

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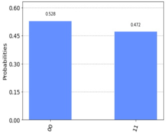

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- How to get the full information?

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- To get complete information, the number of rotations $U_j \rho U_j^\dagger$ is at least

$$(N^2 - 1)/(N - 1) = N + 1.$$

Implementations

Proposition

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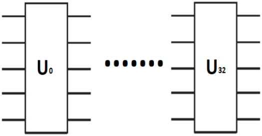

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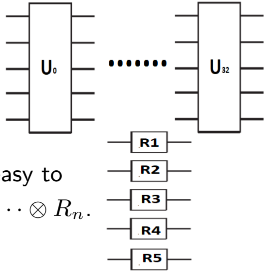

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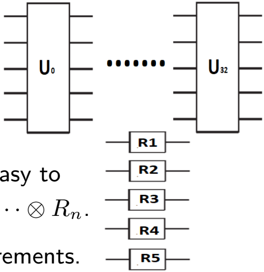

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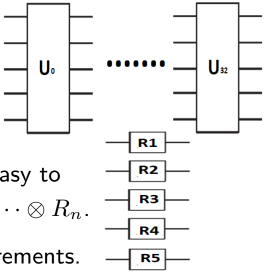

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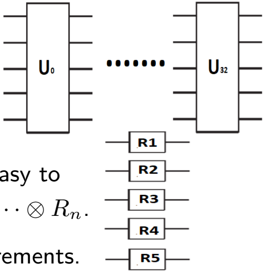

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If $n = 2$, then $3^2 = 9$ steps are optimal.

If $n = 3$, then $3^3 = 27$ steps suffice.

Can we do better?

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Proposition [Ancilla Assisted QST]

Let N be a positive integer. There exist $\sigma \in D_N$, and a unitary $U \in M_{N^2}$ such that a quantum state $\rho \in D_N$ can be determined by the diagonal entries of

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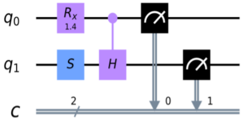

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- Can we make good use of the additional information?

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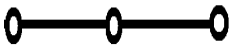

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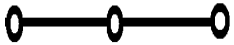
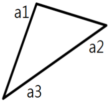


- If three qubits are connected in a triangle, then a measurement yields 24 real data. One can get more then enough information by measuring (1)



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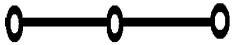

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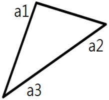

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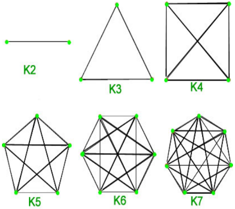

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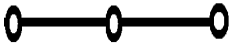

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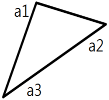

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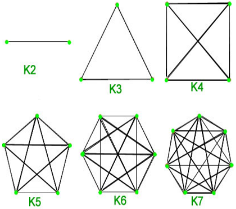

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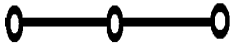

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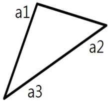

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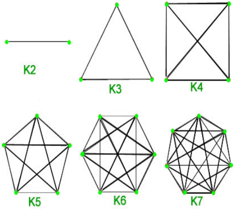

- How to determine optimal qubit connection, σ and unitary U such that one measurement of $U(\sigma \otimes \rho)U^\dagger$ will determine ρ accurately?

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Thank you for your attention!