Probing inequivalent form of Leggett-Garg inequalities in subatomic and $\mathcal{PT}$ symmetric system

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Acknowledgements

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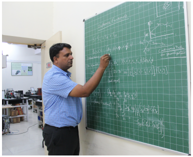

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Publications

1. Probing inequivalent form of LGIs in neutrino and meson systems,
J. Naikoo, S. Kumari, S. Banergee and A. K. Pan, J.Phys.G: Nucl.Part.Phys. 47 095004(2020).

2. Maximal Coherent behaviour at exceptional point in PT symmetric qubit,
J. Naikoo, S. Kumari, A. K. Pan and S. Banergee, arXiv.1912.12030.

Outline

- Macrorealism and Leggett-Garg Inequalities(LGIs)
- Inequivalent Leggett-Garg Inequalities
- Inequivalent Leggett-Garg Inequalities in subatomic systems
- PT symmetric system and violation of LGIs
- Summary

Macrorealism and Leggett-Garg Inequalities(LGIs)

Introduction

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- We are certainly not concerned in which limit the mathematical structure of QM reduces to CM.

- How does the everyday world view about the nature of reality emerge from QM?

Introduction

- Schrondinger's question: When does a macroscopic system (cat) stop existing as a superposition of states and become one (dead) or the other (alive)?

- Heisenberg proposed a bizarre 'cut' but remained silent about how such a 'cut' can be obtained within the very formalism of QM.

Introduction

- Macroscopic quantum coherence

  C_{60} molecule, 720 amu (Arndt et al., Nature, 2000)

  $C_{60}F_{48}$, 1632 amu (Hackermueller, et al.,PRL, 2003)

  $C_{60}[C_{12}F_{25}]_{10}$, 6910 amu (Gerlich, et. al., Nat. Com. 2011)

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Approach within QM:

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- Course-grained measurements (Kofler and Brukner, 2008)

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Approach within QM:

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Realist approach:

Macrorealist model by Legget and Garg (1981): Analogues to Bell's approach

Macrorealism and Legget-Garg inequalities (LGIs)

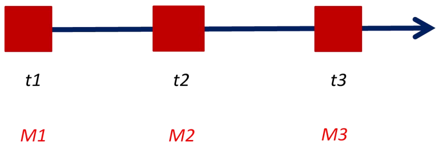
The notion of macrorealism consists of two main assumptions.

- *Macrorealism per se (MRps):* Macroscopic system which has available to it two or more macroscopically distinguishable ontic states remains in one of those states at any instant of time.

- *Non-invasive measurability (NIM):* The ontic state of a macroscopic system can always be determined without affecting the state itself or its subsequent dynamics.

A. J. Leggett and A. Garg, Phys. Rev. Lett. 54, 857(1985).
A. J. Leggett, J. Phys. Condens. 14, R415 (2002).

- Consider the usual LG scenario of the three measurements in a two-level system.



- Let measurement of $\hat{M}$ is performed on the macroscopic system at three different times t_1, t_2 and $t_3 (t_3 > t_2 > t_1)$.

  MRps: The measurement of $\hat{M}$ should produce outcomes $+1$ or -1 at all instant of time.

- NIM tells us that if the measurement of M_2 or M_3 remains unaffected due to the measurement of M_1 and so on.

- Using the MRps and NIM assumptions, the following inequality can be derived,

$$\Delta_s^{LG} = \langle M_1 M_2 \rangle + \langle M_2 M_3 \rangle - \langle M_1 M_3 \rangle \leq 1$$

which is the well-known Leggett-Garg Inequality(LGI).

Here, $\langle M_1 M_2 \rangle = \sum_{m_1, m_2 = \pm} m_1, m_2 P(M_1^{m_1}, M_2^{m_2})$ and similarly for others.

A. J. Leggett and A. Garg, Phys. Rev. Lett. 54, 857(1985).
A. J. Leggett, J. Phys. Condens. 14, R415 (2002).

Quantum violation standard LGIs

- Let the system is prepared in a state $\rho(t_1) = |\psi_{t_1}\rangle\langle\psi_{t_1}|$ at t_1, where

$$|\psi_{t_1}\rangle = \cos\theta|0\rangle + exp(i\phi)\sin\theta|1\rangle$$

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$$|\psi_{t_1}\rangle = \cos\theta|0\rangle + exp(i\phi)\sin\theta|1\rangle$$

- At t_1, we take $M_1 = \hat{\sigma_z}$ and Hamiltonian $\mathcal{H} = \omega\sigma_x$.

- $M_2 = \mathcal{U}_{12} M_1 \mathcal{U}_{12}^{\dagger}$ and $M_3 = \mathcal{U}_{13} M_1 \mathcal{U}_{13}^{\dagger}$.

- $\mathcal{U}_{12} = e^{i\omega\sigma_x(t_2-t_1)}$ and $\mathcal{U}_{23} = e^{i\omega\sigma_x(t_3-t_2)}$. If one takes $\omega(t_2 - t_1) = \omega(t_3 - t_2) = g$,

$$SLG_Q = 2Cos(g) - Cos(2g)$$

$$SLGI_Q^{max} = 1.5 > 1 \text{ at } g = \pi/6.$$

Experimental tests of standard LGIs

- Electron spin

  Knee et al., Nature Comm.3, 606 (2012). (Negative result measurement)

- NV centre

  Waldherr et al., PRL 107, 090401 (2011) (assuming the stationarity of correlations)

  George et al., PNAS, 110, 3777(2013) (classically undetectable wavefunction collapse)

- NMR

  V. Athalye, S.S. Roy and T. S. Mahesh, PRL 107, 130402 (2011) .

- Photons

  Goggin et al., PNAS, 108, 1256(2011).(Weak measurement)

  Avella et al., Phys. Rev. A 96, 052123 (2017). (weak measurement)

- Cesium atom

  Robels et al., Phys. Rev. X, 5, 011003(2015). (Negative result measurement in quantum walks)

Inequivalent LGIs

Swati Kumari and A. K. Pan, J.Phys.A: Math.Theor. 54, 035301 (2021).

Equivalent Bell-CHSH inequalities

- Aurther Fine showed that for a two-party, two measurements per party having two outcomes of each measurement, the only relevant Bell's inequality is the CHSH form.
- Any other form, such as, Wigner and CH forms of inequalities reduce to the CHSH inequality.
- SLGIs are often considered to be the analogues to the CHSH inequalities.
- We showed that Wigner and CH form of LGIs are stronger than standard LGIs.

A. Fine, Phys. Rev. Lett., 48, 291,(1982).

Wigner form of LGIs

- The satisfaction of MR implies the existence of joint probabilities $P(m_1, m_2, m_3)$. The marginals can then be written as

$$P(m_2, m_3) = \sum_{m_1} P(m_1, m_2, m_3)$$

where $m_1, m_2, m_3 = \pm 1$.

Using similar pair-wise joint probabilities, 24 Wigner form of LGIs can be derived are the following;

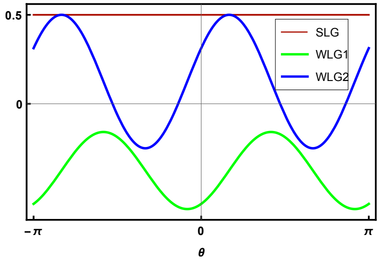
$$P(m_2, m_3) - P(-m_1, m_2) - P(m_1, m_3) \leq 0$$

$$P(m_1, m_3) - P(m_1, -m_2) - P(m_2, m_3) \leq 0$$

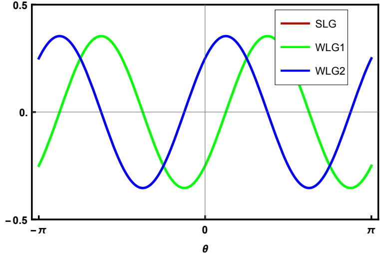
$$P(m_1, m_2) - P(m_2, -m_3) - P(m_1, m_3) \leq 0$$

D. Saha, et al. Phys. Rev. A, 91, 032117 (2015).

Wigner LGI Vs standard LGI



$SLG - 1$ and two WLGIs are plotted against θ by taking $g = \pi/6$.



$SLG - 1$ and two WLGIs are plotted against θ by taking $g = \pi/4$.

Clauser-Horne (CH) form of LGIs

In a macrorealistic theory, single marginal statistics for the measurement of an observable, say for M_2, is $P(m_2) = \sum_{m_1 m_2 = \pm} P(m_1, m_2, m_3)$.

By combining single and pair-wise probabilities, we can derive 24 inequalities are the following;

$$P(m_1, m_2) + P(m_2, m_3) - P(m_1, m_3) - P(m_2) \leq 0$$

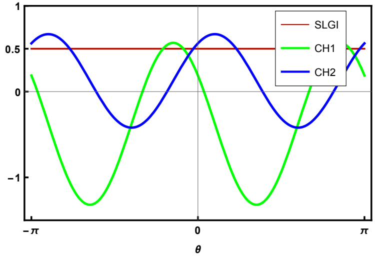
$$P(m_1, m_3) + P(m_2, m_3) - P(m_1, m_2) - P(m_3) \leq 0$$

$$P(m_1, m_3) + P(m_1, m_2) - P(m_2, m_3) - P(m_1) \leq 0$$

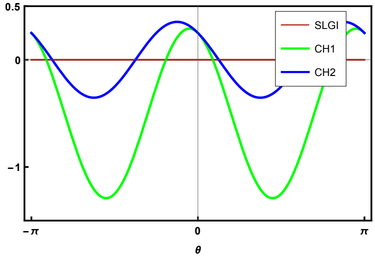
We call them CH form of LGIs.

S. Kumari and A. K. Pan, EPL, 50002, 118 (2017).

Clauser-Horne LGI Vs standard LGI



$SLG - 1$ and two CHLGIs are plotted against θ by taking $g = \pi/6$.



$SLG - 1$ and two CHLGIs are plotted against θ by taking $g = \pi/4$.

Joint probabilities in QM

Three-time probability in terms of correlation functions:

$$P_{123}(m_1, m_2, m_3) = (1/8)(1 + m_1\langle M_1\rangle + m_2\langle M_2^{(1)}\rangle + m_3\langle M_3^{(12)}\rangle$$
$$+m_1 m_2\langle M_1 M_2\rangle + m_2 m_3\langle M_2 M_3^{(1)}\rangle + m_1 m_3\langle M_1 M_3^{(2)}\rangle + m_1 m_2 m_3 D)$$

The pair-wise probabilities are given by

$$\begin{aligned}
P_{13}(m_1, m_3) &= \frac{(1 + m_1\langle M_1\rangle + m_3\langle M_3^{(1)}\rangle + m_1 m_3\langle M_1 M_3\rangle)}{4} \\
P_{23}(m_2, m_3) &= \frac{(1 + m_2\langle M_2\rangle + m_3\langle M_3^{(2)}\rangle + m_2 m_3\langle M_2 M_3\rangle)}{4} \\
P_{12}(m_1, m_2) &= \frac{(1 + m_1\langle M_1\rangle + m_2\langle M_2^{(1)}\rangle + m_1 m_2\langle M_1 M_2\rangle)}{4} \\
P(m_i) &= \frac{(1 + m_i\langle M_i\rangle)}{2} \qquad i = 1, 2, 3
\end{aligned}$$

S. Kumari and A. K. Pan, EPL, 50002, 118 (2017).

WLGIs and CHLGIs are stronger than SLGIs

Using pair-wise and single probabilities, 24 Wigner LGIs can be written as

$$|\langle M_2\rangle - \langle M_2^{(1)}\rangle| + |\langle M_3^{(2)}\rangle - \langle M_3^{(1)}\rangle| + SLG_Q \leq 1$$

where $SLG_Q = m_1 m_2 \langle M_1 M_2\rangle + m_2 m_3 \langle M_2 M_3\rangle - m_1 m_3 \langle M_1 M_3\rangle$.

Wigner LGIs are stronger than standard LGIs.

Similarly, corresponding to 24 CH form of LGIs, we get

$$|\langle M_2\rangle - \langle M_2^{(1)}\rangle| + |\langle M_3\rangle - \langle M_3^{(1)}\rangle| + |\langle M_3\rangle - \langle M_3^{(2)}\rangle| + SLG_Q \leq 1$$

CH form of LGIs are stronger than Wigner form of LGIs.

S. Kumari and A. K. Pan, EPL, 50002, 118 (2017).

Probing inequivalent form of LGIs in subatomic system

J.Phys.G: Nucl.Part.Phys. 47 095004(2020)

Dynamics of neutrino and neutral meson system

For three flavor scenario of neutrino oscillation, one represents a general neutrino state either in the *flavor basis* $\{|\nu_\alpha\rangle\}$ $(\alpha = e, \mu, \tau)$ or in the *mass basis* $\{|\nu_k\rangle\}$ $(k = 1, 2, 3)$

$$|\Psi\rangle = \sum_{\alpha=e,\mu,\tau} \psi_\alpha |\nu_\alpha\rangle = \sum_{k=1,2,3} \psi_k |\nu_k\rangle. \tag{1}$$

The expansion coefficient in the two representations are connected by the so called Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix as follows

$$\psi_\alpha = \sum_{k=1,2,3} U_{\alpha,k} \psi_k. \tag{2}$$

Here, $U_{\alpha,k}$ are the element of the PMNS matrix.

S.M.Bilenky and B. Pontekorvo, Phys.Rep.41(1978).

Dynamics of neutrino system

Later can be parametrized in many ways, one of them is given by,

$$\boldsymbol{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{23}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{13}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}.$$

Here $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, and the parameters θ_{ij} and δ are the mixing angles and the CP violating phase, respectively.

One can now connect the flavor state at time $t = 0$ and some later time t by

$$\psi_f(t) = \boldsymbol{U}\ \boldsymbol{E}\ \boldsymbol{U}^{-1}\ \psi_f(0) = \boldsymbol{U}_f(t)\psi_f(0).$$

We call $\boldsymbol{U}_f(t)$ the flavor evolution operator.

C.Giunti and C W Kim, *Fundamentals of Neutrino Physics and Astrophysics*,(Oxford University Press),2007.
G.Barenboim *et.al.*,, Phys. Lett.B 537,2002.

Dynamics of the neutral meson system

The time evolution of the combined meson system is governed by the unitary operator $U_{SE}(t)$ as follows

$$\rho(t) = U_{SE}(t)\rho(0)U_{SE}^{\dagger}(t).$$

Usually one is interested in the dynamics of the system of interest and the environmental degrees of freedom are traced out

$$\rho_S(t) = Tr_E\{U_{SE}(t)\rho(0)U_{SE}^{\dagger}(t)\}.$$

One may write this reduced state in the following representation

$$\rho_S(t) = \sum_i \mathcal{K}_i(t)\rho_S(0)\mathcal{K}_i^{\dagger}(t).$$

where, $\sum_i \mathcal{K}_i(t)\mathcal{K}_i^{\dagger}(t) = \boldsymbol{I}$.

Inequivalent form of LGIs in neutrino system

With initial neutrino state ν_μ and the dichotomic operator $\hat{A} = 2|\nu_e\rangle\langle\nu_e| - \boldsymbol{I}$, where $\boldsymbol{I} = \sum_{\alpha=e,\mu,\tau} |\nu_\alpha\rangle\langle\nu_\alpha|$, we get,

$$(K_3)_Q = 1 - 4\mathcal{P}_{\mu e}(t) + 4\mathcal{P}_{ee}(t)\mathcal{P}_{\mu e}(2t) + 4\beta(t)$$

where $\beta(t)$ is a non-measurable term.

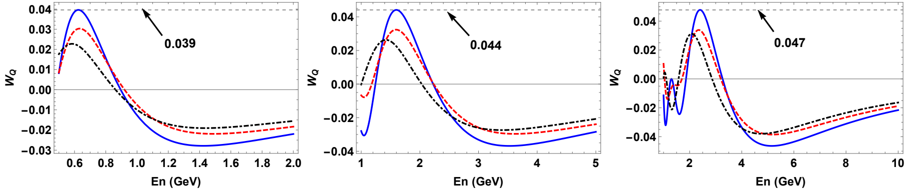
Interestingly, WLGI (W_Q) turns out to be independent of non-measurable terms,

$$W_Q = \mathcal{P}_{ee}(t)\mathcal{P}_{\mu e}(t) - \mathcal{P}_{\mu e}(2t) \leq 0. \quad (3)$$

$\mathcal{P}_{\alpha\beta}(t)$ is the probability of transition from flavor state ν_β to ν_α at time t.

S.F.Huelga, T.W. Marshall and E. Santos, Phys.Rev.A 52, R2497(1995).
J. Naikoo, A.K.Alok,S.Banergee and S.U.Sankar, PRD, 99, 095001(2019).

Inequivalent form of LGIs in neutrino system



WLGI in neutrino system for $T2K$ (left), $NO\nu A$ (middle) and DUNE (right), plotted with respect to the neutrino energy (En) in GeV. The baseline of 295 km, 810 km and 1300 km are used respectively. The CP violating parameter $\delta = 0$ and the matter density parameter $A \approx 1.01 \times 10^{-13}$ eV. The solid (blue), dashed (red) and dot-dashed (black) correspond to the cases with $\delta = 0$, 45^o, and 90^o, respectively.

CHLGIs in neutrino system

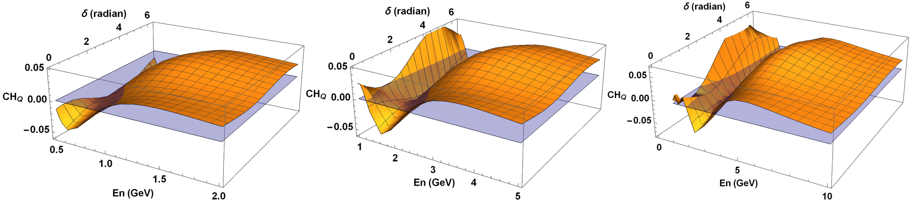
The suitable CHLGI, can be found from the Ineq. (1) for the values of $m_1 = +1$, $m_2 = m_3 = -1$ and is denoted by CH_Q

$$CH_Q = -\mathcal{P}_{\mu e}(t) + \mathcal{P}_{ee}(t)\mathcal{P}_{\mu e}(2t) \leq 0. \tag{4}$$

Another useful CHLGI, CH'_Q, can be obtained for the values of $m_1 = m_3 = -1$, $m_2 = +1$,

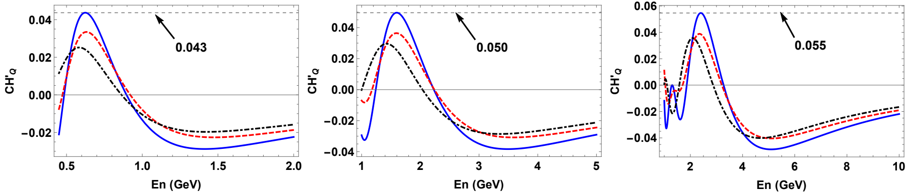
$$CH'_Q = \mathcal{P}_{\mu e}(t) - \mathcal{P}_{\mu e}(2t)[\mathcal{P}_{\mu e}(t) + \mathcal{P}_{\tau\mu}(t)] + \mathcal{P}_{\mu\mu}(2t) + \mathcal{P}_{\tau\mu}(2t) - 1 \leq 0 \tag{5}$$

Inequivalent form of LGIs in neutrino system



CHLGI in neutrino system for different experimental set ups vz., $T2K$ (left), $NO\nu A$ (middle) and DUNE (right). The quantity CH_Q is plotted with respect to the neutrino energy E_n and the CP violating phase δ.

Inequivalent form of LGIs in neutrino system



CHLGI (CH'_Q), Ineq. (5), is depicted with respect to the neutrino energy E_n in $T2K$ (left), $NO\nu A$ (middle) and DUNE (right) setups. The presence of term $\mathcal{P}_{\tau\mu}$ makes the experimental verification of this quantity difficult in contrast to the scenario depicted by Ineq. (4).

Inequivalent form of LGIs in K-meson system

We consider the initial meson state $|K^0\rangle$ and the dichotomic operator $\hat{O} = 2|K^0\rangle\langle K^0| - \boldsymbol{I}$, where $\boldsymbol{I} = |K^0\rangle\langle K^0| + |\bar{K}^0\rangle\langle\bar{K}^0| + |0\rangle\langle 0|$.

The most suitable WLGI is found for the values of $m_1, m_2, m_3 = +1$.

$$\begin{aligned} W = & P(m_2 = +1, m_3 = +1) - P(-m_1 = -1, m_2 = +1) \\ & - P(m_1 = +1, m_3 = +1) \leq 0. \end{aligned}$$

Further, the most suitable CHLGI is found for the values of $m_1, m_3 = -1$ and $m_2 = +1$,

$$\begin{aligned} CH \quad = & P(m_1 = -1, m_2 = +1) + P(m_2 = +1, m_3 = -1) \\ & -P(m_1 = -1, m_3 = -1) - P(m_2 = +1) \leq 0 \end{aligned}$$

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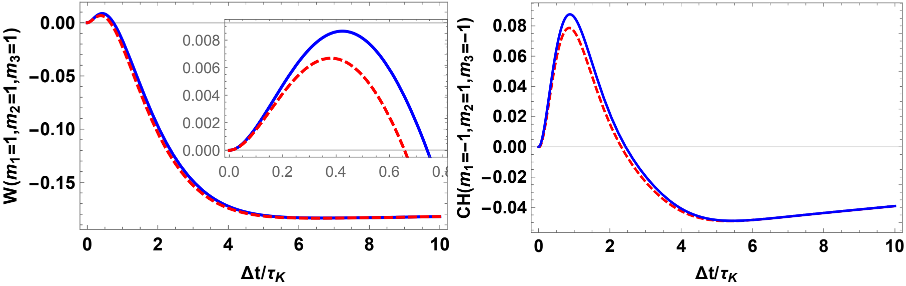
Inequivalent form of LGIs in K-meson system



WLGI (left) and CHLGI (right), are plotted with respect to the dimensionless parameter $\Delta t/_K$, where $_K$ is the average lifetime of K^o meson and Δt is the time interval between two successive measurements. Solid (blue) and dashed (red) curve corresponds to the case without and with decoherence, respectively.

$\mathcal{PT}$ symmetric evolution, maximum coherence and violation of LGIs

arXiv.1912.12030.

$\mathcal{PT}$ symmetric system

Let us consider Hamiltonian of the system is given by,

$$H = \begin{pmatrix} i\gamma & J \\ J & -i\gamma \end{pmatrix},$$

Here, γ is gain/loss parameter and $J = |1 - \exp(-i\phi)|$ is the coupling strength between the two levels. $J > \gamma$, $J < \gamma$ and $J = \gamma$ corresponds to $\mathcal{PT}$ symmetry, broken and exceptional point(EP) respectively.

The time evolution of the states $\rho_k(t) = |\psi_k(t)\rangle\langle\psi_k(t)|$ $(k = 1, 3)$, from time s to t (with $t > s$), is given by the Schrodinger equation $\rho_k(t) = U(t-s)\rho_k(s)U^\dagger(t-s)$. The normalized state after time t is,

$$\tilde{\rho}_k(t) = \frac{U(t-s)\rho_k(s)U^\dagger(t-s)}{\mathrm{Tr}\left[U(t-s)\rho_k(s)U^\dagger(t-s)\right]}. \tag{6}$$

Degree of coherence in terms of l_1 norm

Coherence can be defined in terms of the off-diagonal elements of the density matrix as,

$$C(\rho) = \sum_{i,j(i \neq j)} |\rho_{ij}|,$$

such that $0 \leq C(\rho) \leq 1$.

Let us consider, $\rho(0) = \mathcal{I}/2$, then $C(\tilde{\rho}(t))$ can be obtained as,

$$C(\tilde{\rho}(t)) = 2 \left| \frac{\alpha \sinh^2\left(\tau\sqrt{\alpha^2 - 1}\right)}{\alpha^2 \cosh\left(2\sqrt{\alpha^2 - 1}\right) - 1} \right|.$$

In the limit $\alpha \to 1$, for large τ, $C(\tilde{\rho}(t)) \to 1$, the coherence reaches maximum value at the exceptional point.

A. Strelsov *et.al.*, PRL, 119, 140402(2017); H.L.Shi *et.al.*, Sci.Rep.7, 14806(2017)

Maximal Coherent behaviour of $\mathcal{PT}$ symmetric system

For an arbitrary state $\rho \in \mathbb{C}^d$

$$\frac{C^2(\rho)}{(d-1)^2} + \mu(\rho) \leq 1,$$

where $\mu(\rho) = \frac{d}{d-1}(1 - \mathrm{Tr}[\rho^2])$ is the mixedness parameter.

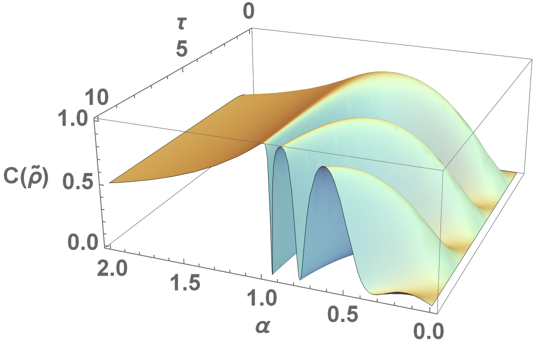
For the state $\tilde{\rho}(t)$, we have

$$\mu(\tilde{\rho}) = \frac{\left(\alpha^2 - 1\right)^2}{\left(\alpha^2 \cosh\left(2\tau\sqrt{\alpha^2 - 1}\right) - 1\right)^2}.$$

In the limiting case with $\alpha \rightarrow 1$, the mixedness parameter $\mu(\tilde{\rho}) = 0$. Thus, the maximally mixed state subjected to the $\mathcal{PT}$ symmetric dynamics becomes a pure state at the EP.

U. Singh, M.N.Bera,H.S.Dhar and A.K.Pati, PRA, 91, 052115(2015).

Maximal coherent of $\mathcal{PT}$ symmetric system



Coherence is plotted as a function of dimensionless parameters $\alpha = \gamma/J$ and $\tau = Jt$. The coherence attains maximum value at the EP, i.e., at $\alpha = 1$.

Quantum violation of LGIs and $\mathcal{PT}$ symmetric system

We choose qubit observable $\hat{M} = \sigma_y$ and the state $\rho(0) = \mathcal{I}/2$, the state evolves according to Eq. (6).

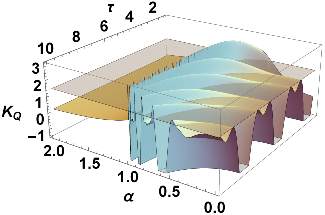
The two-time correlations can be computed as,

$$\langle \sigma_y(t_i)\sigma_y(t_j)\rangle = \sum_{a,b=\pm 1} ab \; p(^a t_i, ^b t_j).$$

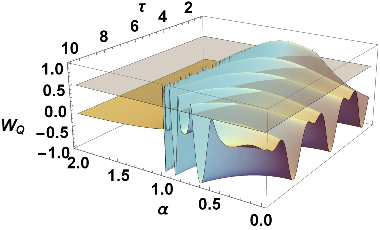
We choose a WLGI for which $m_1 = +1$, $m_2 = -1$ and $m_3 = -1$, that is,

$$W = P(m_2 = -1, m_3 = -1) - P(-m_1 = -1, m_2 = -1) \\ - P(m_1 = +1, m_3 = -1) \leq 0.$$

Quantum violation of LGIs and $\mathcal{PT}$ symmetric system



SLGI is plotted with respect to the dimensionless parameter $\tau = Jt$ and α.



WLGI is plotted with respect to the dimensionless parameter $\tau = Jt$ and α.

Conclusion

- We probe the inequivalent form of LGIs in subatomic system, viz., neutrino and meson system. These inequalities exhibit maximum quantutm violation around the energies roughly corresponding to maximum neutrino flux.

- Decoherence is observed to reduce the degree of violation for meson system and hence the nonclassical nature of the system .

- We studied LGIs in PT symmetric system. Coherence is found to be maximum at exceptional point(EPs), as a consequence SLGI and WLGI acheive their respective algebraic maximum at EPs.

Thank You