Physics of 1D Kondo Lattices

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Kondo chain

$$H = \sum_n \Big[-t\Big(\psi^+_{\alpha,n}\psi_{n+1,\alpha} + H.c.\Big) + J_K \psi^+_n \boldsymbol{\sigma} \mathbf{S}_n \psi_n \Big]$$

We study a **dense regular** array of magnetic moments interacting with conduction electrons.

Single moment is screened by the electrons if $J_K > 0$. The **sign matters!**

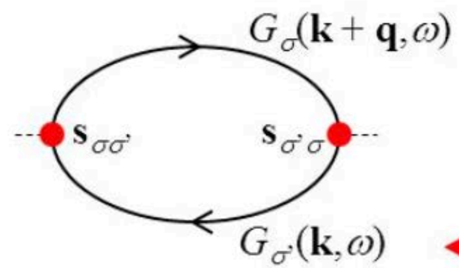
Already 2 spins interact through polarization of the electron cloud – Ruderman-Kittel-Kasya-Yosida (RKKY) interaction ~ $(J_K)^2$. **Sign does not matter!**

Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction

- localized impurity spin $\mathbf{S} \longrightarrow$ acts like magnetic field $\mathbf{B}(\mathbf{q}) \sim \mathbf{S}$
- induces hole magnetization $\mathbf{m}(\mathbf{q}) = \chi(\mathbf{q})\, \mathbf{B}(\mathbf{q})$
- $\chi(\mathbf{q})$ from perturbation theory of 1st order for eigenstates (complicated integral over $\mathbf{k}$ vector of states $|\psi_{\mathbf{k}\sigma}\rangle$)

diagramm:

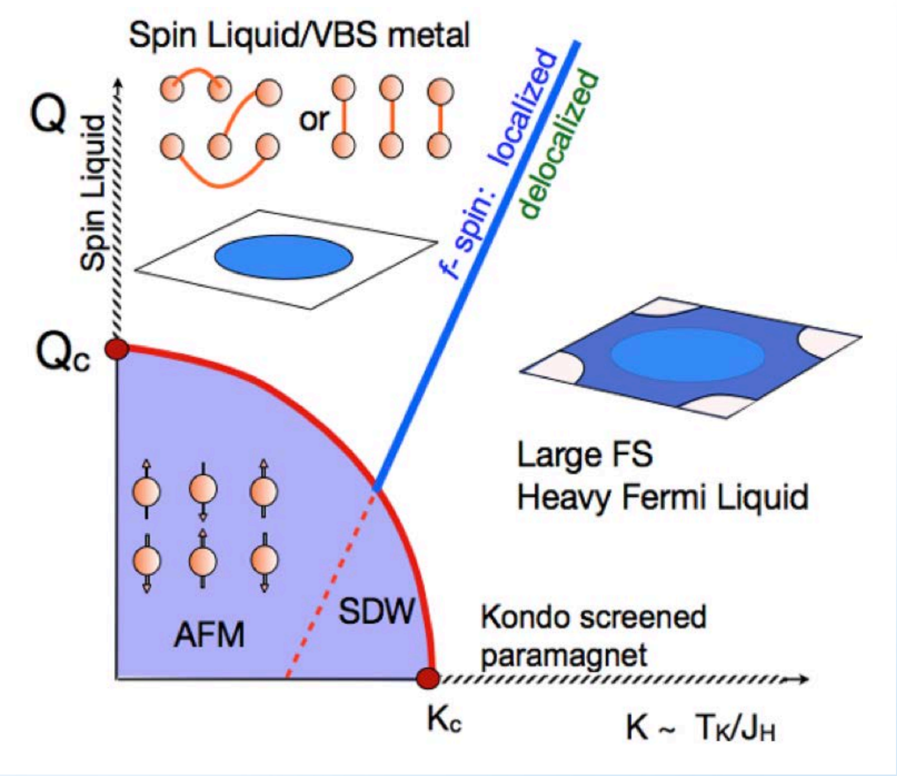
$\chi(\mathbf{q})$ =



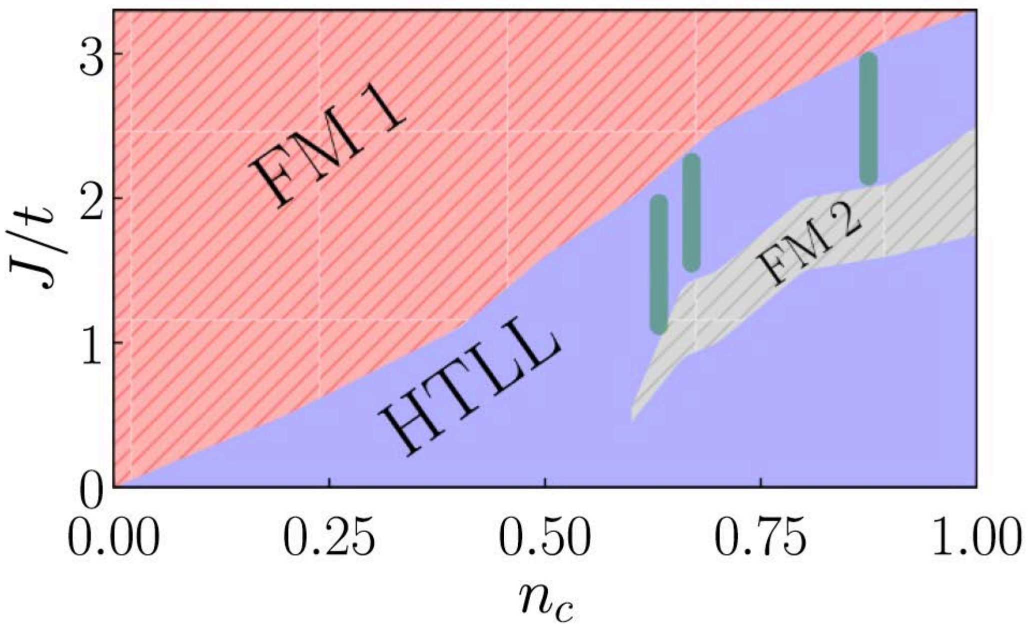
unperturbed Green function

Standard Phase Diagram

- Modified Doniach's phase diagram: Kondo screening (multiple scattering on the same spin) competes with RKKY interaction.
- Q stands for quantum frustration



1D Kondo lattice: RKKY always wins



Schematic phase diagram from Khait et.al. 2018
HTLL stands for Heavy Tomonaga-Luttinger liquid.

From Khait et.al. 2018

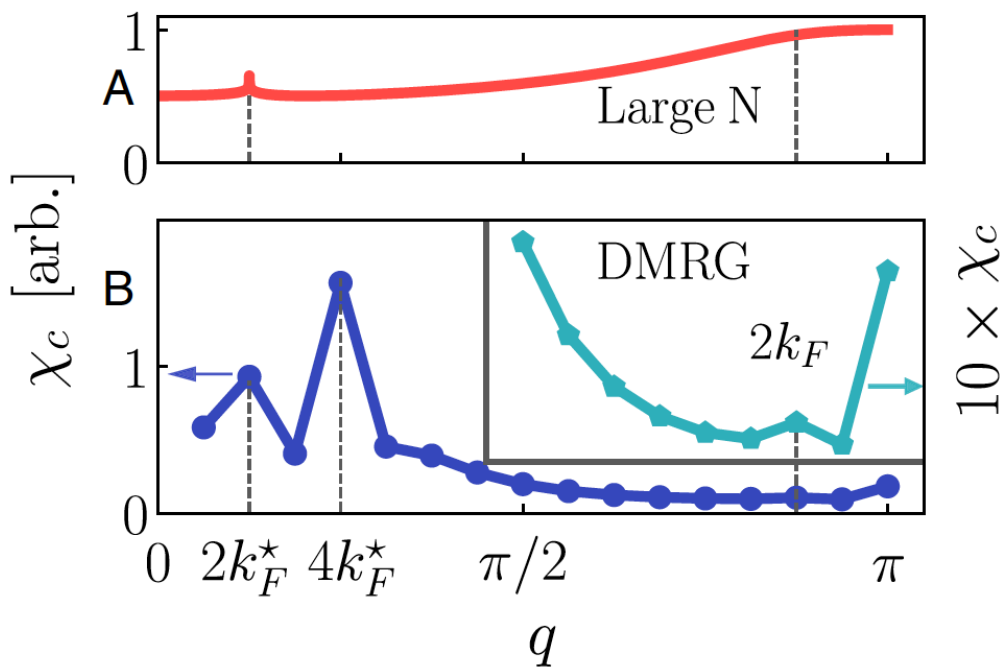


Fig. 2. Charge susceptibility vs. wave vector for $n_c = 0.875$, $J/t = 2.5$. (*A*) large-*N* approximation. (*B*) DMRG calculation. $k_F^\star$ denotes the large Fermi surface wave vector. Finite-field scaling of DMRG reveals divergent peaks at $2k_F^\star$ and $4k_F^\star$ as expected for a TLL. *B*, *Inset* shows a nondivergent peak at twice the small Fermi wave vector $2k_F$, which is attributed to the inverse of the hybridization gap $2r_0$ depicted in Fig. 6. arb, arbitrary units.

The large-N approximation – the SU(2) symmetry is extended to SU(N), the slave boson approach is used.

Our results

In 1D the KL phase diagram is very rich, richer than it has been envisaged before.

It depends on

A.) ***Symmetry***: **SU(2)** *vs*. SU(N) or *vs*. U(1). Large symmetry pulls towards Fermi liquid, small one – to short range spin order.

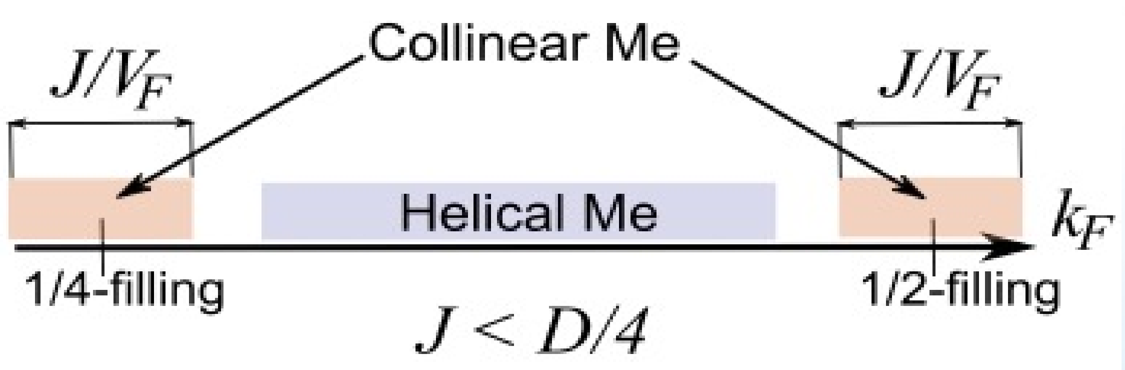
B.) ***Band filling***. At ½, ¼, ¾ it is insulating spin liquid, otherwise for SU(2) it either a metal or $4k_F$ CDW.

C.) ***Direct Heisenberg*** exchange J_H:
for $J_H \gg J_{kondo}$ we have a fractionalized spin liquid with odd-frequency pairing.

Numerics (McCulloch et.al. 2002, Khait et.al. 2018) was done for J_K /t ~ 1.

For smaller J_K /t we suggest an analytic approach.

Our Results for SU(2)



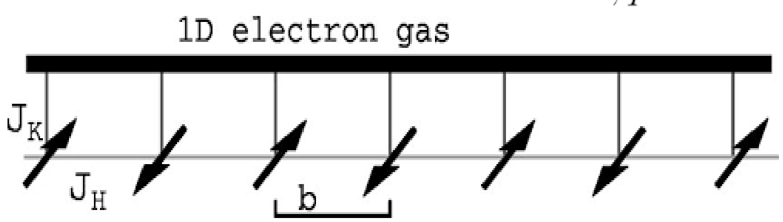
Strictly at n= ½, ¼, ¾ - **insulator,** in the vicinity it is TLL with gapped charge and spin excitations.
Further on –
“helical metal” – 4k_F CDW with gapped spin sector.

More interesting physics emerges at strong Heisenberg exchange.

Kondo-Heisenberg chain:

Spin S=1/2 chain interacting with 1D electron gas.

$$H = \sum_k \epsilon(k)\psi^+_{k\alpha}\psi_{k\alpha} + \frac{J_K}{2}\sum_{k,q}\psi^+_{k+q,\alpha}\vec{\sigma}_{\alpha\beta}\psi_{k,\beta}\mathbf{S}_q + J_H\sum_n \mathbf{S}_n\mathbf{S}_{n+1}.$$



$J_K << J_H$

Zachar, Tsvelik, 2003

The continuum limit: **J_K << J_H** and Fermi energy.

The electron band is *incommensurate* with the lattice: **k_F not equal** $\pi/2$.

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Our approach – semiclassical approximation

- Separate fast from slow degrees of freedom and integrate out the fast ones.

- The formalism: path integral for spins and fermions.
- Under the path integral spins are treated as vector fields with the Wess-Zumino Lagrangian:

$$\mathcal{L}_{\mathrm{WZ}} = is\int_0^1 \mathrm{d}u\,(\boldsymbol{N},[\partial_u \boldsymbol{N}\times\partial_\tau \boldsymbol{N}])\,;\ \boldsymbol{N}(u=0)=(1,0,0),\ \boldsymbol{N}(u=1)=\boldsymbol{S}_n/s;$$

Semiclassical approximation (continued)

- First step: at low T spins are *almost* ordered.
- Choose spin configuration which minimizes energy.

- Integrate over fluctuations around this configuration.
- Result: Ginzburg-Landau action for slow variables.

Spin configuration

$$\boldsymbol{S}_n/s = \boldsymbol{m} + b\Big(\boldsymbol{e}_1\cos(\alpha)\cos(qx_n+\theta) + \qquad (\\ \boldsymbol{e}_2\sin(\alpha)\sin(qx_n+\theta)\Big)\sqrt{1-\boldsymbol{m}^2}\,.$$

This is the form of spin field in the path integral. We will integrate over

$\mathbf{S}_n(t)$ with Berry phase. It is assumed that $[\mathbf{S}_n(t)]^2 = s^2$,

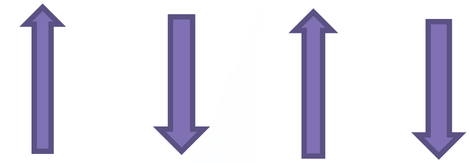
and this field is smooth.

To *satisfy this condition* we need

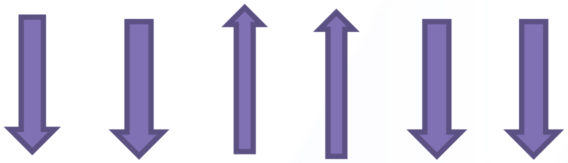
$\mathbf{m}^2 << 1$, $(\mathbf{e}_i, \mathbf{e}_j) = \delta_{ij}$ - unit vector fields, $q = 2k_F$.

Acceptable spin configurations

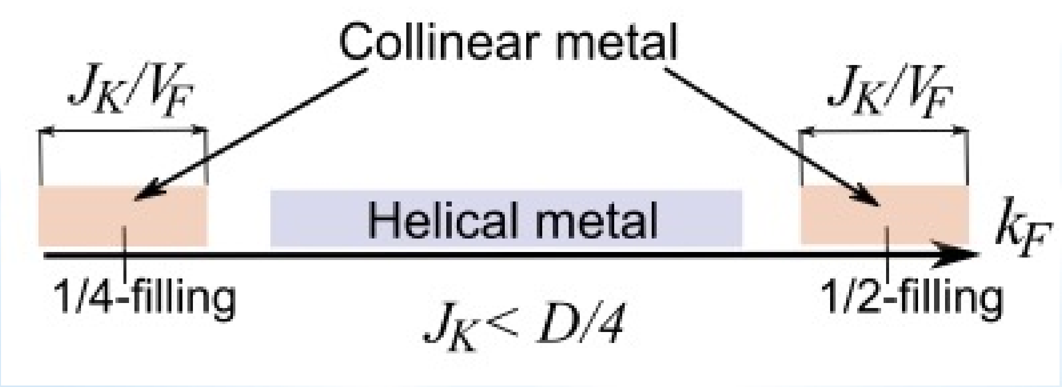
1. Collinear antiferromagnet – ½ filling.

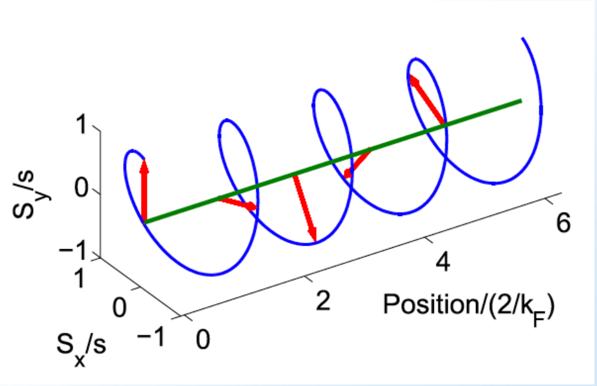


2. 2 spins up 2 – down -1/4 filling.

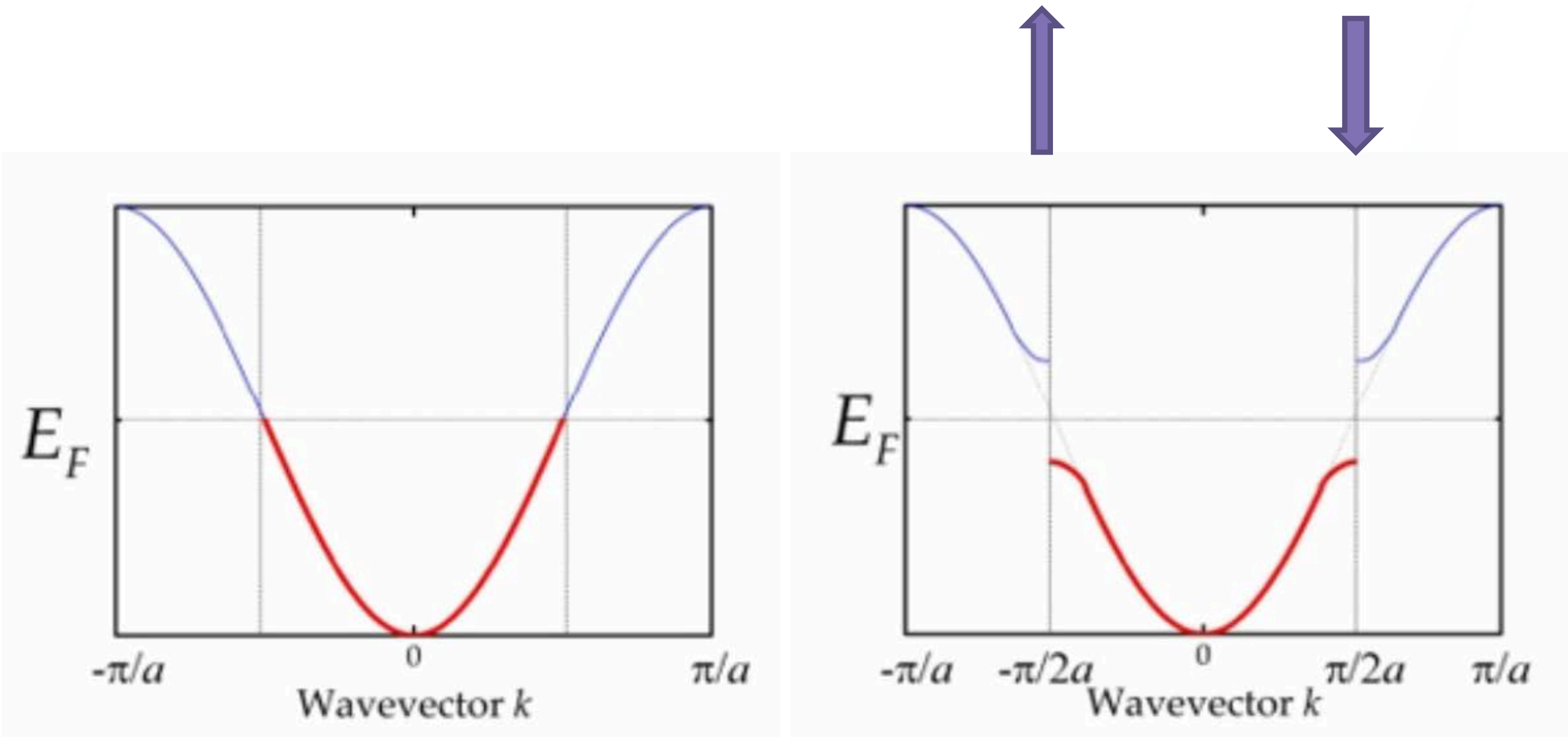


3. Non-collinear spiral – general filling:





Simplest case: ½ filling



Antiferromagnetic spin configuration acts as a periodic potential and opens a spectral gap at $k_F = \pi/2$.

Our approach: semiclassical approximation for spins

Preparatory step: linearization of the band spectrum:

$$\mathcal{L}_F[\psi_\pm] = \sum_{\nu=\pm} \psi^\dagger_\nu \partial_\nu \psi_\nu\,; \quad \partial_\pm \equiv \partial_\tau \mp i v_F \partial_x\,.$$

The most serious part of the interaction is backscattering:

$$\mathcal{L}^{(n)}_{\rm bs} = J_K \left[R^\dagger_n(\boldsymbol{\sigma}, \boldsymbol{S}_n) L_n e^{-2ik_F x_n} + h.c\right];$$
$$R \equiv \psi_+,\ L \equiv \psi_-\,;\ x_n \equiv n\xi.$$

The oscillations must be absorbed into the spin configuration:

$$\boldsymbol{S}_n/s = \boldsymbol{m} + b\Big(\boldsymbol{e}_1 \cos(\alpha)\cos(qx_n+\theta) + \boldsymbol{e}_2 \sin(\alpha)\sin(qx_n+\theta)\Big)\sqrt{1-\boldsymbol{m}^2}\,.$$

The derivation: ½ filling and its vicinity

$$\mathbf{S}_j = S\left[\mathbf{m}(x) + (-1)^j \mathbf{n}\sqrt{1-\mathbf{m}^2}\right], \quad \mathbf{n}^2 = 1.$$

We use non-Abelian bosonization procedure where the action of free s=1/2 fermions is represented as a sum of the Gaussian model and SU_1 (2) Wess-Zumino-Novikov-Witten model of SU(2) matrix field *h(t,x):*

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi_c)^2 + W_1[h] + \mathrm{i}J_K S\sqrt{1-\mathbf{m}^2}\mathrm{Tr}\left[\mathrm{e}^{\mathrm{i}\sqrt{2\pi}\Phi_c} h g^+ - H.c.\right] + A[S].$$

$$g = i(\sigma \mathbf{n})$$

$$A[\mathbf{S}] = \mathrm{i}S\Big(\mathbf{m}[\mathbf{n} \times \partial_\tau \mathbf{n}]\Big) + 2\pi \mathrm{i}S \times (\text{top.-term}).$$

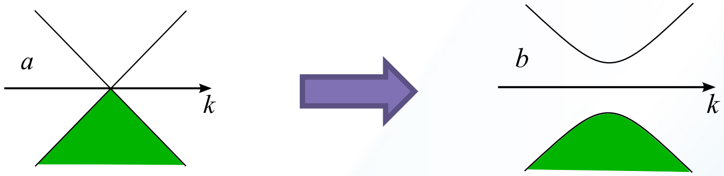
This is the spin Berry phase.

The derivation: ½ filling and its vicinity

- The Polyakov-Wiegmann identity:

$$W_k[hg] = W_k[h] + W_k[g] + \frac{k}{4\pi}\mathrm{Tr}[g^{-1}(\partial_\tau - \mathrm{i}\partial_x)gh(\partial_\tau + \mathrm{i}\partial_x)h^{-1}],$$
$$J_R = -\frac{k}{4\pi}g(\partial_\tau + \mathrm{i}\partial_x)g^{-1}, \quad J_L = \frac{k}{4\pi}g(\partial_\tau - \mathrm{i}\partial_x)g^{-1}.$$

Use it and refermionize:



$$L_f = r^+(\partial_\tau - \mathrm{i}\partial_x)r + l^+(\partial_\tau + \mathrm{i}\partial_x)l - \mu(r^+r + l^+l) + \mathrm{i}J_K S\sqrt{1-\mathbf{m}^2}(r^+l - l^+r) + \frac{\mathrm{i}}{2}(r^+\boldsymbol{\sigma}r)[\mathbf{n}\times(\partial_\tau - \mathrm{i}\partial_x)\mathbf{n}].$$

Now integrate over these **massive fermions** and massive field **m** :

Sigma model for the spin excitations

$$S[\mathbf{n}] = \int \mathrm{d}\tau \mathrm{d}x \Big\{ \Big[\frac{S}{2J_{RKKY}} + \frac{1}{4\pi v_F} \Big] (\partial_\tau \mathbf{n})^2 + \frac{v_F}{4\pi} (\partial_x \mathbf{n})^2 \Big\} + 2\pi(S + 1/2) \times (\text{top. term}).$$

Or in the canonical form:

$$S[\mathbf{n}] = \int \mathrm{d}\tau \mathrm{d}x \frac{1}{2g} \Big[\frac{1}{c} (\partial_\tau \mathbf{n})^2 + c(\partial_x \mathbf{n})^2 \Big] + 2\pi(S + 1/2) \times (\text{top. term}),$$

$$c = v_F \Big(1 + \frac{2\pi v_F S}{J_{RKKY}} \Big)^{-1/2}, \quad g^{-1} = \frac{1}{2\pi} \Big(1 + \frac{2\pi v_F S}{J_{RKKY}} \Big)^{1/2},$$

$$J_K \langle r_\sigma^+ l_\sigma \rangle = \frac{J_K^2}{\pi v_F} \ln \Big[W / \sqrt{(v_F k_F^*)^2 + \Delta^2} \Big] \equiv J_{RKKY}, \quad \Delta = J_K S.$$

Exact solution

- O(3) sigma model is one of the most beautiful field theories.

- Here strong interactions come solely from geometrical constraint on the field: $\mathbf{n}^2 = 1$.
- The result is *dynamical mass generation .* The spectrum is coherent triplet with gap Δ.

The energy scales

$$\Delta = \Lambda g^{-1} \exp(-2\pi/g).$$

$$g \approx \frac{2|J_K|}{v_F}(2S \ln W/|J_K|)^{1/2}$$

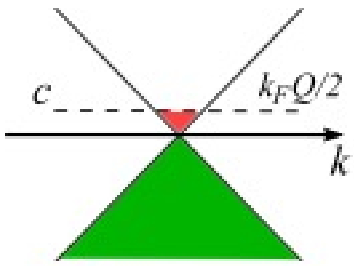
$$\Delta = v_F (\ln W/|J_K|)^{-1/2} \exp\left[-\frac{\pi v_F}{|J_K|(2S \ln W/|J_K|)^{1/2}}\right].$$

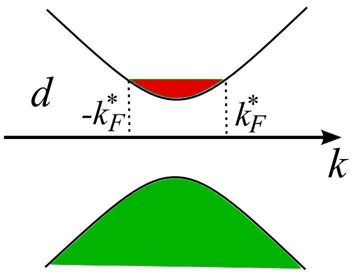
We see that formally the gap is exponentially small in $1/|J_K|$, like Kondo temperature, but it is independent of the sign!

At ½-filling we have insulator with charge gap ~J_K and short range

AF correlations (spin liquid).

Vicinity of ½-filling





$$\langle\langle r r^+\rangle\rangle = \frac{vq+E}{2E}\frac{1}{\mathrm{i}\omega+\mu-E} + \frac{-vq+E}{2E}\frac{1}{\mathrm{i}\omega+\mu+E}, \quad E=\sqrt{(vq)^2+\Lambda^2}.$$

we see that if $v|q| << \Lambda = J_K S$ the Dirac fermions can be approximated by nonrelativistic ones:

$$l \approx r \approx \frac{1}{\sqrt{2}}\chi, \quad H = \chi^+_\alpha\Big(-\frac{v_F^2}{2\Lambda}\partial_x^2 - \mu\Big)\chi_\alpha.$$

This limits our approach to doping

$$x < \frac{\Lambda}{\pi(v_F/a_0)} \sim \frac{J_K S}{W}.$$

Then the coupling between the fermions and the $\mathbf{n}$ field comes from

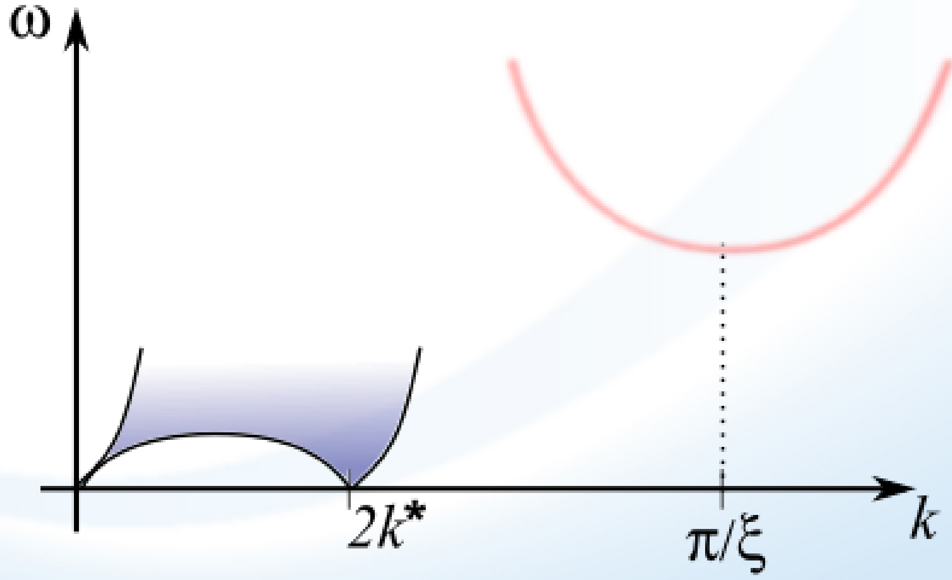
$$\frac{\mathrm{i}}{2}(r^+\boldsymbol{\sigma} r)[\mathbf{n}\times(\partial_\tau - \mathrm{i}v_F\partial_x)\mathbf{n}] \approx \frac{1}{4}(\chi^+\boldsymbol{\sigma}\chi)[\mathbf{n}\times(\mathrm{i}\partial_\tau + v_F\partial_x)\mathbf{n}]$$

Integrating over field **n** we get the Hamiltonian of repulsive Fermi gas:

$$H = \chi^+_\alpha\Big(- \frac{v_F^2}{2\Lambda}\partial_x^2 - \mu\Big)\chi_\alpha + 3(\pi v_F/16)\chi^+_\uparrow\chi_\uparrow\chi^+_\downarrow\chi_\downarrow$$

Fermi momentum: $k^*_F = \pi/2 \square k_F$ - large Fermi surface.

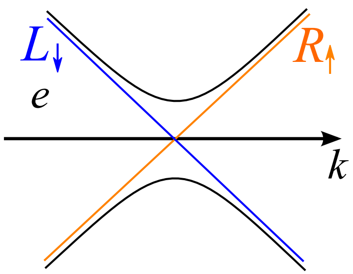
Excitations – all gapless.



Generic filling

$$(\boldsymbol{\sigma S})/S = \mathbf{m} + \mathrm{i}\sqrt{1-\mathbf{m}^2}\, g^{-1}(\mathrm{e}^{2\mathrm{i}k_F x}\sigma^- + \mathrm{e}^{-2\mathrm{i}k_F x}\sigma^+)g,$$

Following the same steps we derive a massive sigma model for SU(2) g-matrix field:



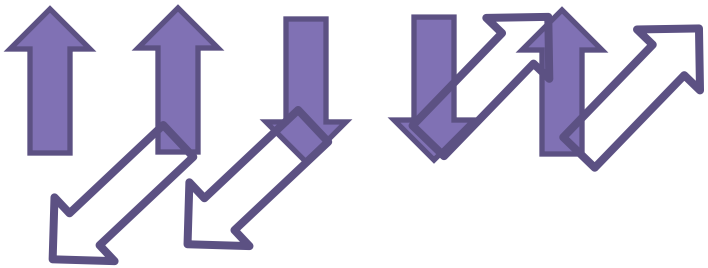
$$\mathcal{L}_{eff} = \frac{1}{2J_{RKKY}}[\Omega_\tau^z]^2 + \frac{1}{4\pi}\mathrm{Tr}\Big(v_F^{-1}\partial_\tau g^+\partial_\tau g + v_F\partial_x g^+\partial_x g\Big),$$
$$\Omega_\mu^a = \mathrm{i}\mathrm{Tr}\Big(\sigma^a g^+\partial_\mu g\Big).$$

This is a version of anisotropic Principal Chiral Field model, the excitations are massive tensor particles $\Psi_{\alpha,\sigma}$, $\alpha,\sigma = +1/2,-1/2$ (Polyakov, Wiegmann 1983).

Gapless modes – “rotated” or dressed fermions $(\underline{R,L})_\alpha = g_{\alpha\beta}\,(R,L)_\beta$ with particular helicity.

¼ filling

$$\mathbf{S}/S = \mathbf{m} + \sqrt{2}\Big[\mathbf{X}_1 \sin\Big(\frac{\pi}{2}(j+1/2)\Big) + \mathbf{X}_2 \cos\Big(\frac{\pi}{2}(j+1/2)\Big)\Big]\sqrt{1-\mathbf{m}^2},$$
$$\mathbf{X}_1^2 + \mathbf{X}_2^2 = 1, (\mathbf{X}_1\mathbf{X}_2) = 0.$$



- It is an insulator with a tendency to dimerization. The sigma model:

$$\mathcal{L}_{mag} = \frac{1}{2g}\Big[(\partial_\mu \mathbf{X}_1)^2 + (\partial_\mu \mathbf{X}_2)^2\Big], \quad \mathbf{X}_1^2 + \mathbf{X}_2^2 = 1, (\mathbf{X}_1\mathbf{X}_2) = 0,$$

$$\delta\mathcal{L} = -\lambda\Big(\mathbf{X}_1^2 - \mathbf{X}_2^2\Big)^2$$

Excitations. ¼-filling

- 1/N-approximation: excitations are vector particles.
-
- It is possible that the system dimerizes (Xavier et.al. 2003 – numerics).

Conclusions

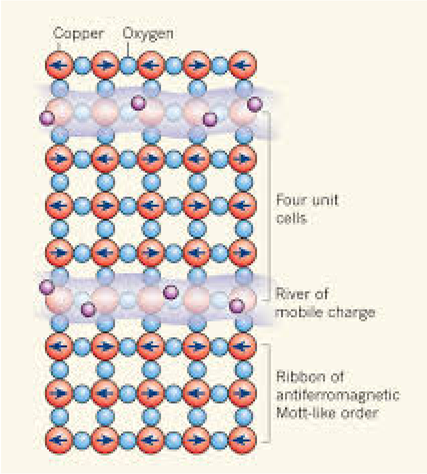
- Although Kondo chain is described by very simple model, its phase diagram is complicated even when one assumes SU(2) symmetry.
- It includes insulators, para- and ferromagnetic metals, charge density waves.

- When direct Heisenberg exchange is added there is a phase with composite CDW and SC quasi long range order.

The problem

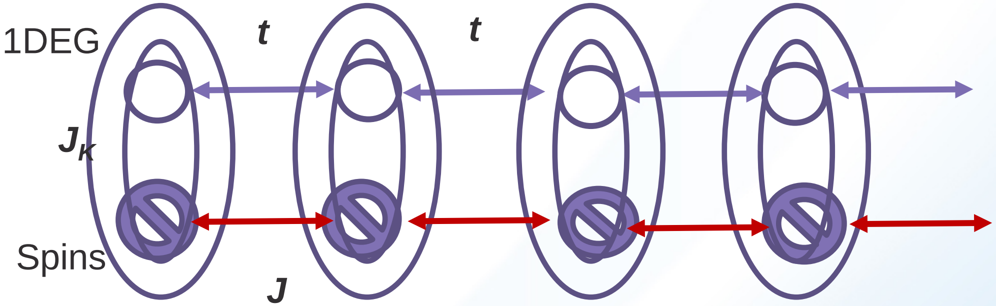
- May we have a metallic state in **D>1** where the Fermi surface volume is not related to the electron density, as it appears to be in the pseudogap phase of the cuprates?
- Senthil, Sachdev and Vojta (2005): *yes*, but the GS must have a nontrivial topology and fractionalized excitations.
- Their approach: gauge theories. Alas, too many uncontrollable steps.
- My approach: consider a *quasi-1D* model, treat the strongest interactions nonperturbatively in 1D and the rest of them approximately in controlled steps.

D >1 array of Kondo-Heisenberg chains



This would be the most realistic arrangement, like in $La_{2-x}\ Ba_x\ CuO_4$ (x=1/8).

I will discuss a less realistic model first (Tsvelik, 2016):



It gives us answers to all questions posed in the beginning.

The core model: Kondo-Heisenberg chain

$$H = \sum_k \epsilon(k)\psi^+_{k\alpha}\psi_{k\alpha} + \frac{J_K}{2}\sum_{k,q}\psi^+_{k+q,\alpha}\vec{\sigma}_{\alpha\beta}\psi_{k,\beta}\mathbf{S}_q + J_H\sum_n \mathbf{S}_n\mathbf{S}_{n+1}.$$

This model constitutes an elementary block for a 2D or 3D model of fractionalized FL.
I'll derive its continuum limit using non-Abelian bosonization.

The 1st step is to linearize the spectrum of 1DEG:

$$\epsilon(k) \approx \pm v_F(k \mp k_F)$$

$$\psi(x) = e^{-ik_F x}R(x) + e^{ik_F x}L(x)$$

Bosonization of 1DEG

$$F_R^a = \frac{1}{2} R^+ \sigma^a R, \; F_L^a = \frac{1}{2} L^+ \sigma^a L$$

Belong to the SU_1 (2) Kac-Moody algebra for spin currents and

$$I_R^z = R_\alpha^+ R_\alpha, \quad I_R^+ = R_\uparrow^+ R_\downarrow^+, \quad I_R^- = R_\downarrow R_\uparrow$$

belong to the SU_1 (2) Kac-Moody algebra for charge currents:

$$[j_R^a(x), j_R^b(x')] = i\epsilon^{abc} j_R^c(x)\delta(x-x') + \frac{i}{4\pi}\delta_{ab}\delta'(x-x')$$

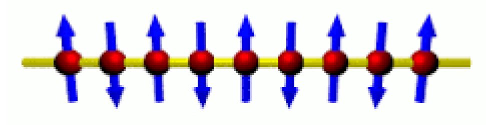
The Hamiltonian density of the 1DEG (free fermions) is

$$\mathcal{H}_{charge} = \frac{2\pi v_F}{3}\Big(: \mathbf{I}_R \mathbf{I}_R : + : \mathbf{I}_L \mathbf{I}_L : \Big)$$

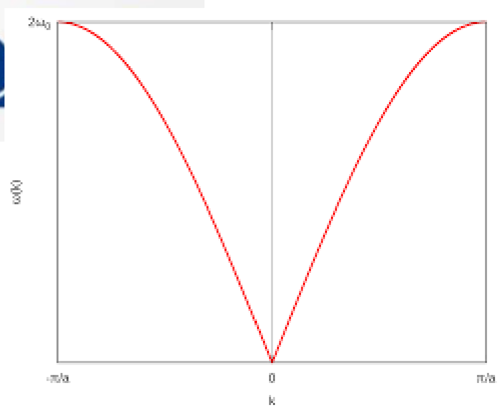
$$\mathcal{H}_s = \frac{2\pi v_F}{3}(: \mathbf{F}_R \mathbf{F}_R : + : \mathbf{F}_L \mathbf{F}_L :),$$

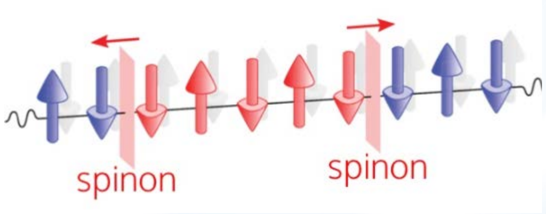
Heisenberg antiferromagnetic S=1/2 chain

At high energies we see individual spins.
But it is not them who is active at low energies.



At energies << J_H we see **collective** excitations - **spinon** waves traveling in opposite directions:





Each spinon carries spin ½,
The spectrum can be split into
Right and **Left** movers.

Bosonization of the S=1/2 Heisenberg chain

$$\mathcal{H}_H = \frac{2\pi v_H}{3}(:\mathbf{j}_L\mathbf{j}_L: + :\mathbf{j}_R\mathbf{j}_R:).$$

$$v_H = \pi J_H/2$$

$$\mathbf{S}_n = [\mathbf{j}_R(x) + \mathbf{j}_L(x)] + (-1)^n\mathbf{N}_s(x) + ..., \quad x = na_0$$

$$\frac{1}{2}\psi^+\vec{\sigma}\psi(x) = \mathbf{F}_R + \mathbf{F}_L + \left[e^{2ik_F x}\mathbf{s} + H.c.\right] + ...,$$

Formation of the spin liquid

- Since 1DEG and Heisenberg chain are incommensurate, the staggered components of the magnetizations do not couple.

$$\frac{1}{2}\psi^+\vec{\sigma}\psi(x)\vec{S} \to (\mathbf{F}_R + \mathbf{F}_L)(\mathbf{j}_L + \mathbf{j}_R)$$

The *strictly marginal* interaction of currents of same chirality can be neglected.

$$\mathcal{H}_{eff} = \mathcal{H}_{charge} + \mathcal{H}_s^{(Rl)} + \mathcal{H}_s^{(Lr)},$$

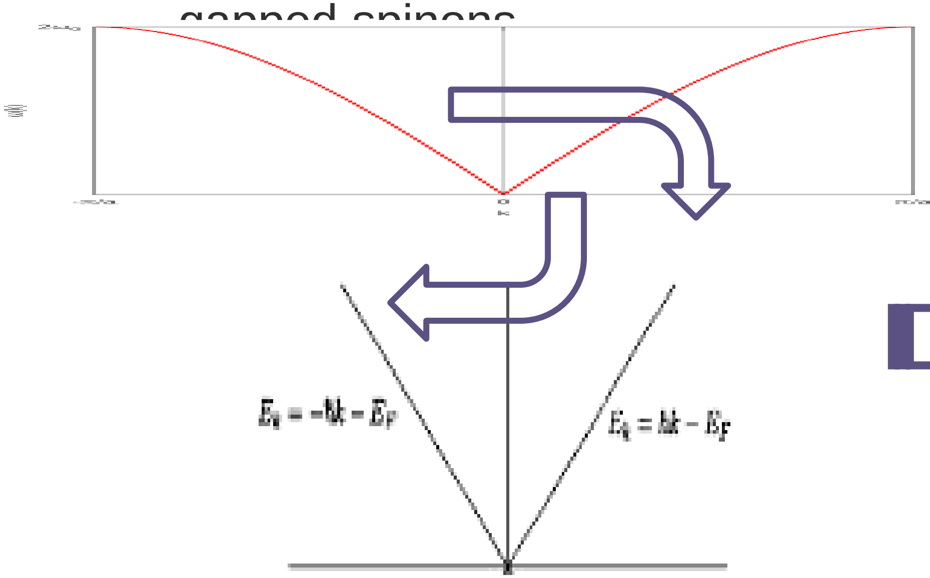
$$\mathrm{H}_s^{(Rl)} = \frac{2\pi v_F}{3} : \mathbf{F}_R\mathbf{F}_R : + \frac{2\pi v_H}{3} : \mathbf{j}_L\mathbf{j}_L : + J_K\mathbf{F}_R\mathbf{j}_L,$$

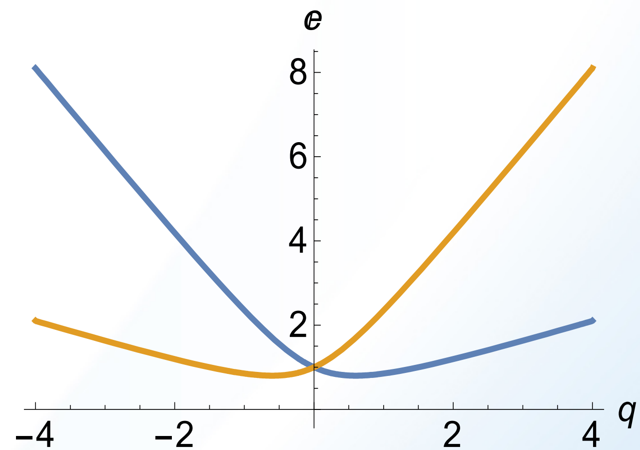
$$\mathrm{H}_s^{(Lr)} = \frac{2\pi v_F}{3} : \mathbf{F}_L\mathbf{F}_L : + \frac{2\pi v_H}{3} : \mathbf{j}_R\mathbf{j}_R : + J_K\mathbf{F}_L\mathbf{j}_R,$$

These models are exactly solvable (N. Andrei, 1980).

Spin gap formation in a single chain.

- When *k_F not equal to $\pi/2$*, spinons from 1DEG pair with spinons of opposite chirality from spin chain. The result is *TWO branches* of gapped spinons





Exact solution, N. Andrei, 1980

$$E(k)_{\pm} = \pm k(v_H - v_F)/2 + \sqrt{k^2(v_F + v_H)^2/4 + \Delta^2},$$

$$\Delta = C\sqrt{J_K J_H}\exp[-\pi(v_F + v_H)/J_K],$$

Order parameters of KH chain

$$\mathcal{O}_{cdw} = \psi^+(x)\Big[(\mathbf{S}_x\mathbf{S}_{x+a_0})\hat{I} + i(\vec{\sigma}\mathbf{S}_x)\Big]\psi(x)e^{i(\pi/a_0+2k_F)x}$$

$$\mathcal{O}_{sc} = i(-1)^{x/a_0}\psi(x)\sigma^y\Big[(\mathbf{S}_x\mathbf{S}_{x+a_0})\hat{I} + i(\vec{\sigma}\mathbf{S}_x)\Big]\psi(x)$$

$$\hat{\mathcal{O}} = \begin{pmatrix} \mathcal{O}_{cdw} & \mathcal{O}_{sc}^+ \\ -\mathcal{O}_{sc} & \mathcal{O}_{cdw}^+ \end{pmatrix} = A\hat{g},$$

A is a numerical amplitude and g is an SU(2) matrix.

Few facts about WZNW models

The action of the $SU_1(2)$ WZNW model can be written in terms of SU(2) matrix field:

$$W[g] = \frac{1}{16\pi}\int d\tau dx \mathrm{Tr}(\partial_\mu g^+ \partial_\mu g) - \frac{i}{24\pi}\int_0^\infty d\xi \int d\tau dx \epsilon^{\alpha\beta\gamma}\mathrm{Tr}(g^+\partial_\alpha g g^+ \partial_\beta g g^+ \partial_\gamma g).$$

However, it can also be written in terms of the free bosonic field:

$$\mathcal{H}_{charge} = \frac{v_F}{2}\left[(\partial_x \Theta_c)^2 + (\partial_x \Phi_c)^2\right]$$

$$[\Phi_c(x), \partial_x \Theta_c(x')] = i\delta(x - x')$$

Then important objects are holomorphic (dependent on $z = \tau$ □ix/v_F) and antiholomorphic fields:

$$\varphi = (\Phi + \Theta)/2, \quad \bar{\varphi} = (\Phi - \Theta)/2.$$

Robustness against local perturbations

- All local primary fields both for 1DEG and Heisenberg chains can be factorized.
- Chiral parts of spin operators pair with parts with opposite chirality from 1DEG. Therefore the perturbations cannot acquire a vacuum average and thus lift the ground state degeneracy.

The operators can be factorized:

$$z_\sigma = (2\pi a_0)^{-1/4} \exp[i\sigma\sqrt{2\pi}\varphi], \quad \bar{z}_\sigma = (2\pi a_0)^{-1/4} \exp[-i\sigma\sqrt{2\pi}\bar{\varphi}], \quad \sigma = \pm 1.$$

$$\mathrm{z}_\sigma = z^+_{-\sigma}.$$

For instance, the WZNW matrix field for the Heisenberg model and the 1DEG fermions

$$\hat{G}(x) = (-1)^n \Big[A(\mathbf{S}_n \mathbf{S}_{n+1}) + iB(\mathbf{S}_n) \Big], \quad x = a_0 n,$$

can be written

$$G_{\sigma\sigma'} = \frac{1}{\sqrt{2}} e^{i\pi(1-\sigma\sigma')/4} z^H_\sigma [\bar{z}^H{}_{\sigma'}]^+.$$

$$R_\sigma = \xi_\sigma \left(z^c_- z^s_\sigma \right), \quad L_\sigma = \xi_\sigma \left(\bar{z}^c_- \bar{z}^s_\sigma \right)$$

From z-quanta of various WZNW one can construct **nonlocal OPs** of the spin liquid:

$$\langle \mathcal{O}_{rL} \rangle = \sum_\sigma \langle z^s_\sigma [\bar{z}^H{}_\sigma]^+ \rangle, \quad \langle \mathcal{O}_{lR} \rangle = \sum_\sigma \langle [\bar{z}^s_\sigma]^+ z^H_\sigma \rangle$$

Building D>1 model.

- Electrons tunnel between the chains. This tunneling will also generate an exchange between the Heisenberg chains.

$$H_{tunn} = t\sum_y \int dx(\psi_y^+(x)\psi_{y+1}(x) + H.c.),$$

$$H_{ex} = \sum_y \tilde{J}\int dx \mathbf{N}_y(x)\mathbf{N}_{y+1}(x), \qquad \tilde{J}_{ferro} \sim -J_K^2 t^2/W^3$$

In Random Phase approximation we have

$$G(\omega, \mathbf{k}) = [G_{1D}^{-1}(\omega, k_x) - t(\mathbf{k})]^{-1},$$

There are quasiparticle poles when $|t(k_y)| > 3.33\Delta(v_F/v_H)^{1/2}$

Electron and hole pockets appear.

The Green's functions

- The *single particle Green's function* is calculated from the symmetry considerations using a minimal information from the exact solution (Essler, Tsvelik, 2001):

$$G(\omega, k \pm k_F) = G_{RR,LL}(\omega, k), \quad G_{RR}(\omega, k) = G_{LL}(\omega, -k)$$

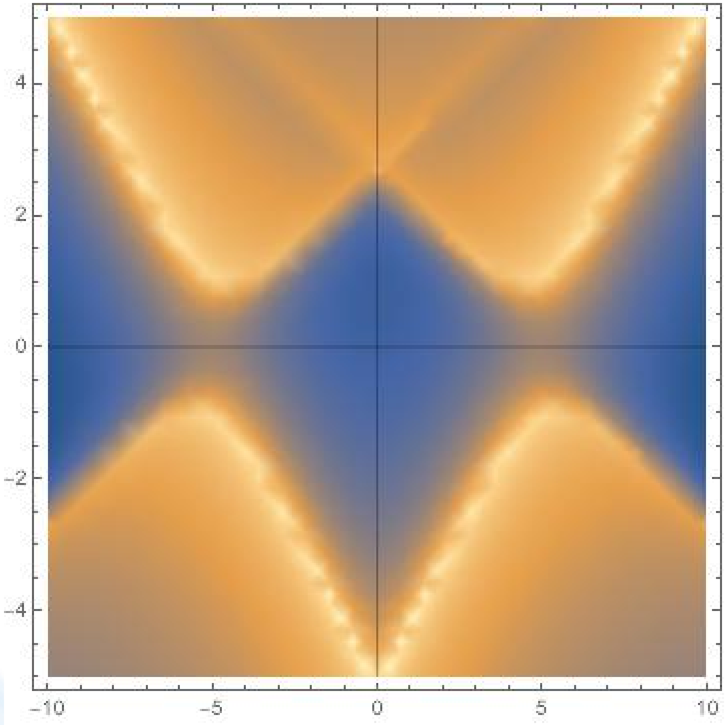
$$\mathrm{G}_{RR}(\omega, k) = \frac{Z_0}{\omega - v_F k}\left[\frac{\Delta}{\sqrt{-(\omega - v_F k)(\omega + v_H k) + \Delta^2}} - 1\right] + \ldots,$$

The Luttinger theorem is fulfilled through zeroes:

$$G(0, \pm k_F) = 0$$

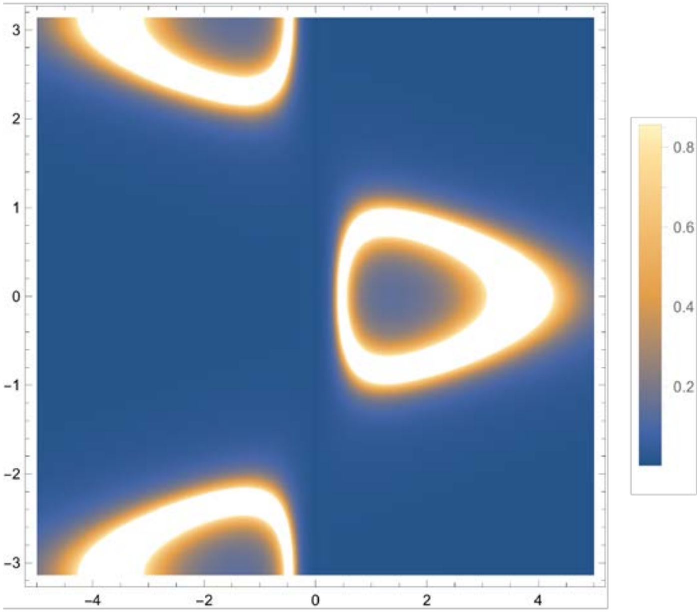
The *single particle Green's function* for a single chain calculated from the exact solution (Essler, Tsvelik, 2001):

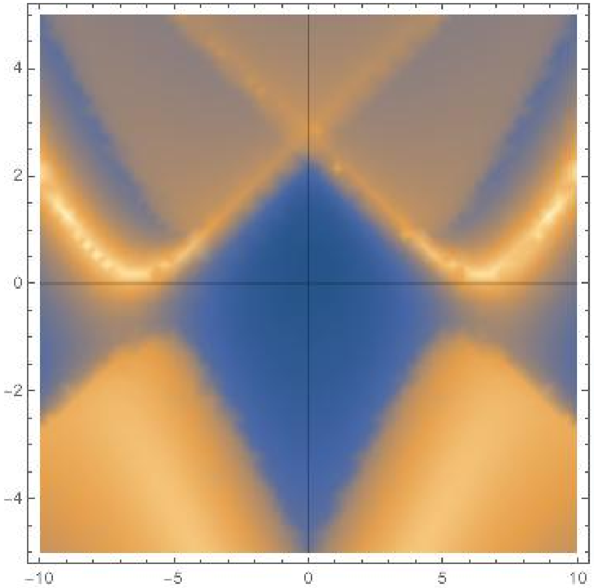
$$G(\omega, k \pm k_F) = G_{RR,LL}(\omega, k), \quad G_{RR}(\omega, k) = G_{LL}(\omega, -k)$$



$$\mathrm{G}_{RR}(\omega, k) = \frac{Z_0}{\omega - v_F k}\left[\frac{\Delta}{\sqrt{-(\omega - v_F k)(\omega + v_H k) + \Delta^2}} - 1\right] + \ldots,$$

When interchain tunneling is allowed, spinons and holons recombine into quasiparticles which propagate in D>1. The q.-p. dispersion is in the gap.





The plot of the quasiparticle weight near $k_x = k_F$ for $t_0 (v_H / v_F)^{1/2} / \Delta = 5$ and $v_F / v_H = 0.1$. The vertical axis is $k_y b$, the horizontal is $q = (k_x - k_F)(v_H v_F)^{1/2} / \Delta$.

In Random Phase approximation we have

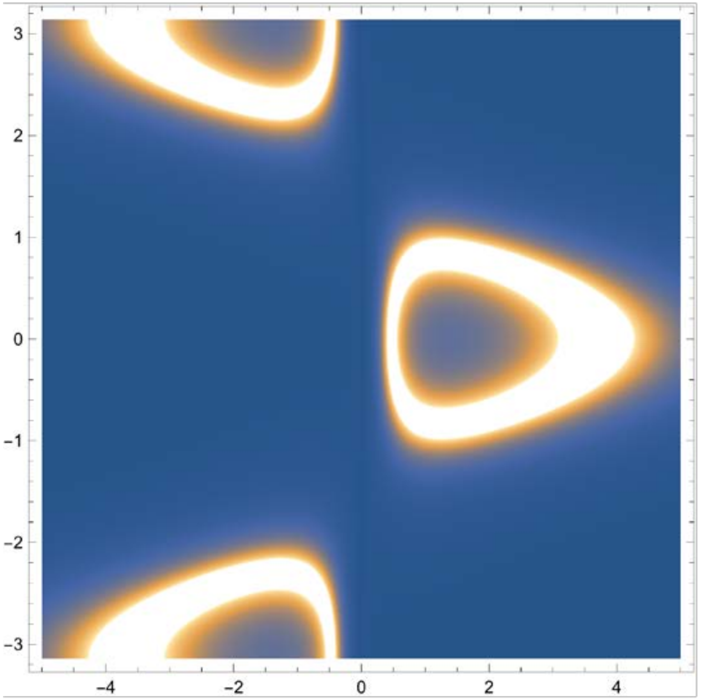
$$G(\omega, \mathbf{k}) = [G_{1D}^{-1}(\omega, k_x) - t(\mathbf{k})]^{-1},$$

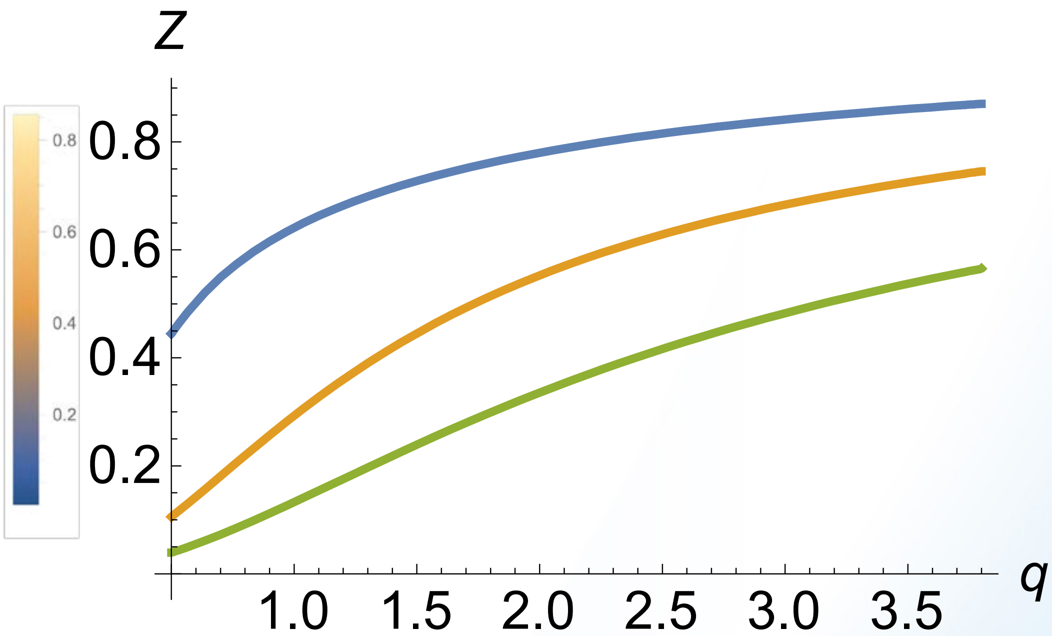
There are quasiparticle poles when

$$|t(k_y)| > 3.33\Delta(v_F/v_H)^{1/2}$$

That is how small Fermi surface is formed!

The fractionized particles still exist at finite energies.





The plot of the quasiparticle weight near $k_x = k_F$ for $t_0 (v_H / v_F)^{1/2} / \Delta = 5$ and $v_F / v_H = 0.1$. The vertical axis is $k_y b$, the horizontal is $q = (k_x - k_F)(v_H v_F)^{1/2} / \Delta$.

The *quasiparticle residue* *Z* as a function of $q = k_x (v_H v_F)^{1/2} / \Delta$ for (from top to bottom) $v_F / v_H = 3, 1, 0.1$.

Stability of the RPA solution

- The quasiparticle FS can be destroyed by two processes.

- **A.)** There is interaction between the gapless collective modes which leads to 3D order.
- - The coupling between the OPs from different chains is an independent parameter: **$T_c << E_F$** .
- **B.)** The QPs can couple to the collective modes:
- **- not possible,** the OPs wave vectors do not connect particle and hole FSs.

Is the ground state topological?

- Forget for a moment that the charge modes interact.
- Then the GS of spin sector of each chain is 4-times degenerate. Hence the GS of the array is 4^N – degenerate.
- This degeneracy cannot be probed by any local operator.
- In reality this picture holds only approximately, since the charge sector orders at some T.

Ginzburg-Landau functional

- Since OPs contain localized spins, to arrange the Josephson coupling *one needs spin exchange* besides the tunneling:

$$S = \sum_y \left[W[g_y] - \mathcal{J} \int_0^{1/T} d\tau \int dx \mathrm{Tr}(\sigma^z g_y \sigma^z g^+_{y+1} + H.c.) \right]$$

$$\mathcal{J} \sim \tilde{J}(t/\Delta)^2$$

To get the Fermi pockets one needs **t ~Δ**, but since the exchar
 is an independent
parameter, the transition temperature
may be **<<** than the Fermi energy of QPs.

Ginzburg-Landau theory – similar to He^3 -A

- The order parameter is SU(2) matrix and can be parametrized by three angles or by angle ϕ and a unit vector

$$\vec{n} = (\cos\theta, \sin\theta\cos\psi, \sin\theta\sin\psi)$$

$$\mathcal{F} = \frac{a_0}{8\pi}[\vec{\nabla}\times\vec{A}]^2 + \frac{\rho_\perp}{2}(\partial_\mu\vec{n})^2 + \frac{\lambda}{2}[\omega_\mu^3 - (2e/c)A_\mu]^2 - \frac{a_0}{4\pi}\mathbf{H}[\vec{\nabla}\times\vec{A}]$$

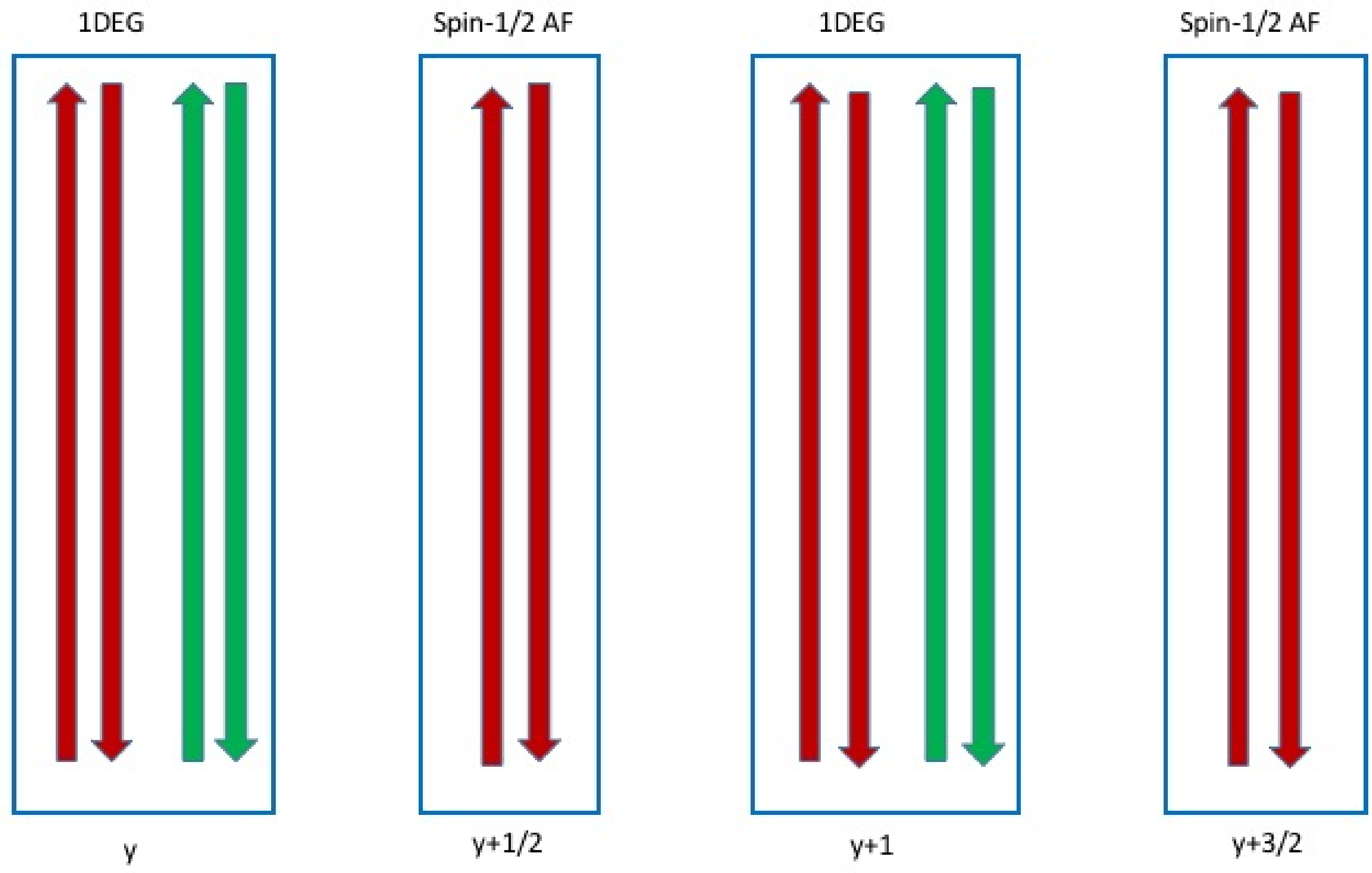
$$\omega_\mu^3 = \partial_\mu\phi + \cos\theta\partial_\mu\psi$$

Magnetic field will not destroy the OP, it will just rotate it from SC to CDW. At $H > H_{c1}$ the flux is equal to the *topological charge* of **n**-field.

Conclusions

- One may have a metallic state where the FS volume is not related to the electron density (in the given case $V_{FS} = 0$).
- The Luttinger theorem is fulfilled due to the *zeroes* of G(0,k).
- For the KH model it is shown that this state is topologically nontrivial, as was suggested by Senthil *et.al* (2005).

The schematic picture of the bosonized model



Red arrows - chiral