Numerical study and proposal for experiment of the Sachdev-Ye-Kitaev model

2016 Annual Meeting of NCTS in Condensed Matter Physics

"Quantum Simulations and Numerical Studies in Many-Body Physics"

11:30 – 12:20, 11 December 2016

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(Department of Physics, Kyoto University)

Collaborators

arXiv:1611.04650

Numerical study and relation to random matrix, black hole:

Jordan Saul Cotler, Guy Gur-Ari, Masanori Hanada, Joseph Polchinski,
Stanford Stanford YITP & Hakubi @ Kyoto; Stanford UCSB

Phil Saad, Stephen H. Shenker, Douglas Stanford, Alexandre Streicher
Stanford Stanford IAS Stanford

arXiv:1606.02454

Proposal for Experimental Quantum Gravity:

Ippei Danshita, Masanori Hanada
YITP YITP & Hakubi @ Kyoto; Stanford

Videos (on “videosfromIAS”)

Natifest – 17 September 2016 “Black Holes and Random Matrices” by Stephan Shenker

https://youtu.be/3qEoMBZLf10

26 October 2016 “The Sachdev-Ye-Kitaev quantum mechanics model, black holes, and random matrices” by Douglas Stanford

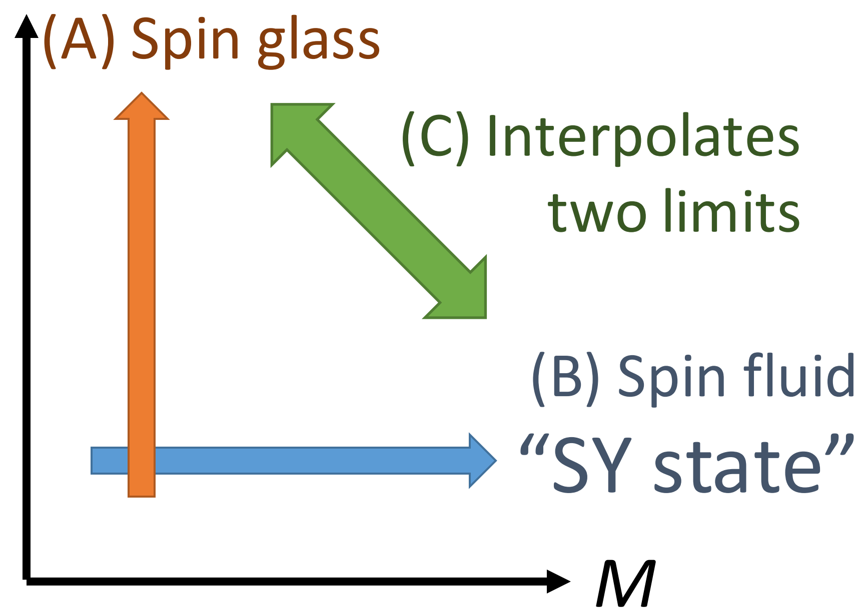
https://youtu.be/hK2S-pyAf0c

Introduction: Sachdev-Ye model

[S. Sachdev and J. Ye, PRL **70**, 3339 (1993)]
N SU(M) spins $\hat{\boldsymbol{S}}$ with all-to-all random coupling J_{ij}

$$\mathcal{H} = \frac{1}{\sqrt{NM}} \sum_{i>j} J_{ij} \hat{\mathcal{S}}_i \cdot \hat{\mathcal{S}}_j$$

n_{b}: # of columns in the Young tableau ~ "spin size"



Spin glass – **spin liquid** phase transition
"strange metals" ~ holographic metals
[S. Sachdev, PRL **105**, 151602 (2010)]

- (B): Non-Fermi liquid with nonzero entropy at $T \to 0$;

Local dynamic spin susceptibility

$$\bar{\chi}(\omega) = X \left[\ln\left(\frac{1}{|\omega|}\right) + i\frac{\pi}{2}\mathrm{sgn}(\omega) \right] + \cdots,$$

cf. Dynamic neutron scattering experiments on disordered antiferromagnets
[B. Keimer et al. PRL 1991 (LSCO); S.M. Hayden et al. PRL 1991 (LBCO);
C. Broholm et al. PRL 1990 (Kagome planes of Cr^{3+} ions in $Sr(Cr,Ga)_{12}O_{19}$)]

Sachdev-Ye-Kitaev model

N Majorana- or Dirac- fermions randomly coupled to each other, in groups of four

[Majorana version]

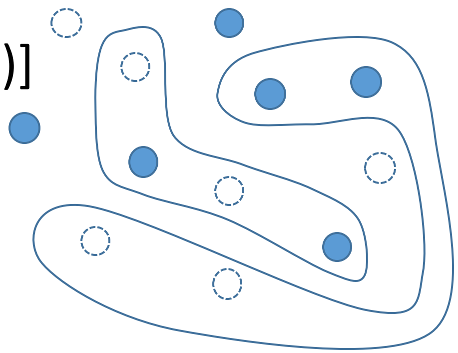
$$\hat{H} = \sum_{1\leq a<b<c<d\leq N_{\mathrm{M}}} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d,$$

[A. Kitaev: talks at KITP (2015)]

[Dirac version]

$$\hat{H} = \frac{1}{(2N)^{3/2}}\sum_{ijkl} J_{ij;kl}\hat{c}_i^\dagger\hat{c}_j^\dagger\hat{c}_k\hat{c}_l,$$

[S. Sachdev: PRX **5**, 041025 (2015)]



- Solvable at large-*N* (strong coupling when β*J*>>1),
  finite entropy / *N* at *T*→0
- Realizes SY state without SU(*M*>2) spin
- Satisfies the chaos bound
  (Lyapunov exponent $2\pi k_{\mathrm{B}}T$ as in black holes
  [Maldacena, Shenker, and Stanford: arXiv:1503.01409])

➔ Fast quantum information scrambler

- Holographic correspondence to 1+1D black holes

Some of earlier works in the literature

Papers citing the 1993 Sachdev-Ye paper: 1 in 2014, 4 in 2015, 46+ in 2016

Joseph Polchinski and Vladimir Rosenhaus,
arXiv:1601.06768 (JHEP04(2016)001)
Large-*N* results on two- and four-point functions; conformal symmetry breaking

Yi-Zhuang You, Andreas W. W. Ludwig, Cenke Xu: arXiv:1602.06964
BDI class, N_χ interacting Majorana fermions

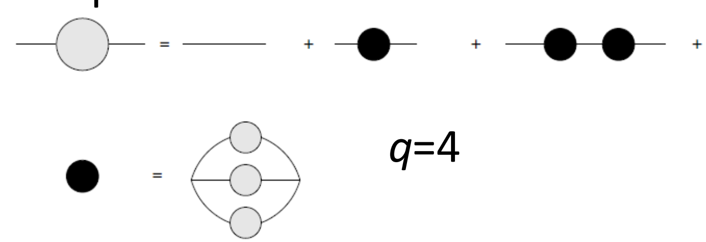
$N_\chi \pmod 8$	0	1	2	3	4	5	6	7
qdim	1	$\sqrt{2}$	2	$2\sqrt{2}$	2	$2\sqrt{2}$	2	$\sqrt{2}$
lev. stat.	GOE	GOE	GUE	GSE	GSE	GSE	GUE	GOE
$\mathcal{C}\ell_{0,N_\chi-1}$	$\mathbb{R}\oplus\mathbb{R}$	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{H}\oplus\mathbb{H}$	$\mathbb{H}$	$\mathbb{C}$	$\mathbb{R}$

Symmetry governs level statistics

Antal Jevicki, Kenta Suzuki, and Junggi Yoon,
arXiv:1603.06246 (JHEP07(2016)007)
Replica analysis for large-*N* results

Wenbo Wu and Subir Sachdev,
arXiv:1603.05246 (PRB **94**, 035135 (2016))
Large-*N* limit, saddle point solution for $G(i\omega n)$
Entropy compared with high-temperature expansion and finite-*N* exact diagonalization

Juan Maldacena and Douglas Stanford,
arXiv:1604.07818 (PRD **94**, 106002 (2016))
Large-*N* structure by diagrammatic analysis, four point function, *q* dependence, bulk interpretation



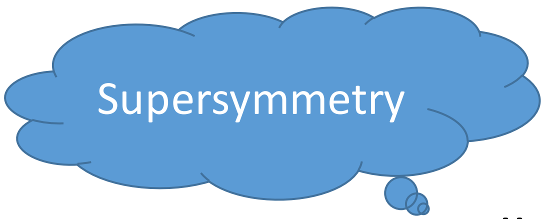
q=4

$$H = (i)^{\frac{q}{2}} \sum_{1\le i_1<i_2<\cdots<i_q\le N} j_{i_1 i_2 \cdots i_q} \psi_{i_1}\psi_{i_2}\cdots\psi_{i_q}$$

Holographic connection to black hole (BH) physics

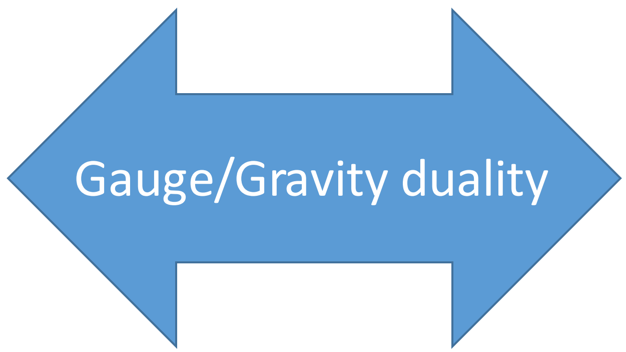
[Sachdev; Maldacena-Stanford, Hosur-Qi-Roberts-Yoshida; Polchinski-Rosenhaus, …]

Finite entropy of a BH from a bulk perspective; what is going on inside a BH?



Super Yang-Mills

IIA/IIB-string
M-theory



SYK model

0+1D

BH in AdS_2

1+1D

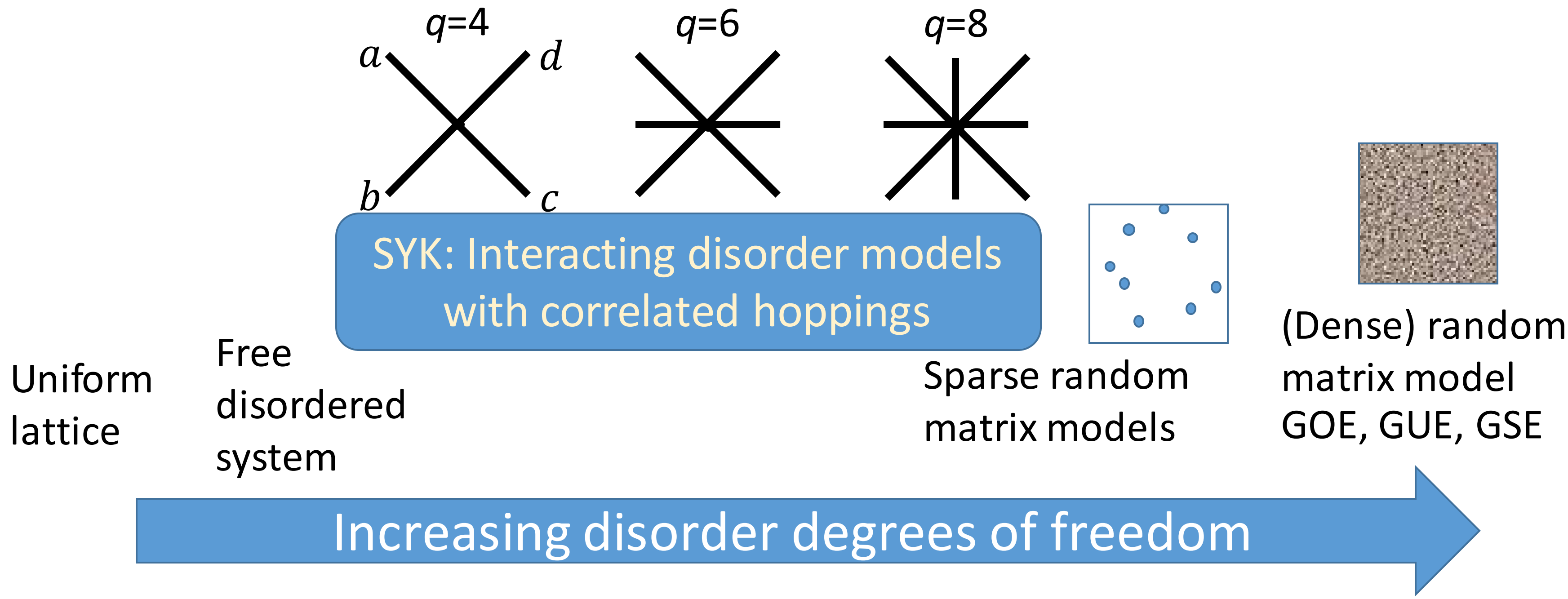
Correspondence in terms of thermodynamics, correlation functions

Finite entropy density at $T \rightarrow 0$ ⇔ Bekenstein-Hawking BH entropy

➔ SYK: simple model to understand holography and quantum features of BH

Our motivation in studying the SYK model

What characterizes gravity?



Numerical simulation for finite *N*: dynamics

Try to understand from analytical approach;

large-*N* perturbation theory, random matrix theory, …

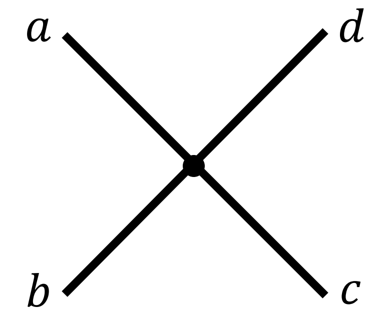
➔ Maximally chaotic model in 0+1D

➔ Near extremal black holes in nearly AdS_2 region

Real-time dynamics to study nonperturbative regime

After disorder sampling:

$0 < t << N/J$: perturbative ($1/N$) expansion good



$$\hat{H} = \sum_{1 \le a < b < c < d \le N_{\mathrm{M}}} J_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d,$$

How do we start to observe nonperturbative effects?

Time t

$t >> O(e^N)$: random matrix limit reached?

Diagonalization of the Hamiltonian → Eigenvalue spectrum

$$\hat{H} = \sum_{1 \le a < b < c < d \le N_{\mathrm{M}}} J_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d,$$

e.g. N = 34: $\binom{17}{4} = 46\,376$ independent random parameters J_{abcd}

Consider 17 complex fermions $\hat{c}_j = \frac{(\chi_{2j-1} + \mathrm{i}\chi_{2j})}{\sqrt{2}}, j = 1, 2, \ldots, 17$

$\chi\chi\chi\chi$ preserves parity of complex fermion number

$$\begin{pmatrix} H_{\mathrm{E}} & O \\ O & H_{\mathrm{O}} \end{pmatrix}$$

Each (even, odd) sector: $2^{16} = 65\,536$ states,

$\binom{17}{0} + \binom{17}{2} + \binom{17}{4} = 2517$ (~ 3.8 %) non-zero matrix elements on each row

~ 165 million non-zero matrix elements determined by J_{abcd}

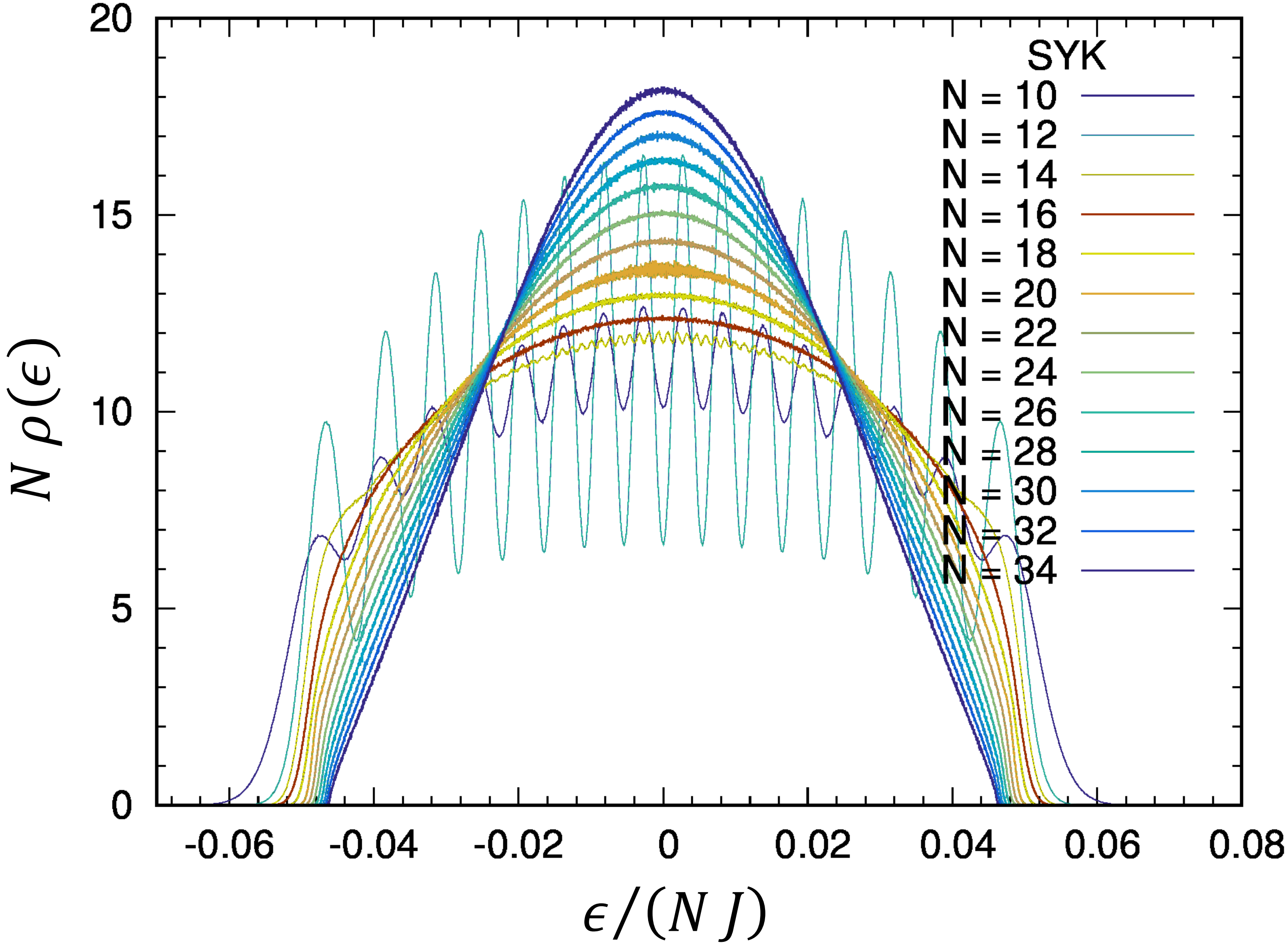
➔ Sparse and repetitive matrix

2^{32} complex matrix elements: 64 GB of memory

Diagonalization of the Hamiltonian → Eigenvalue spectrum

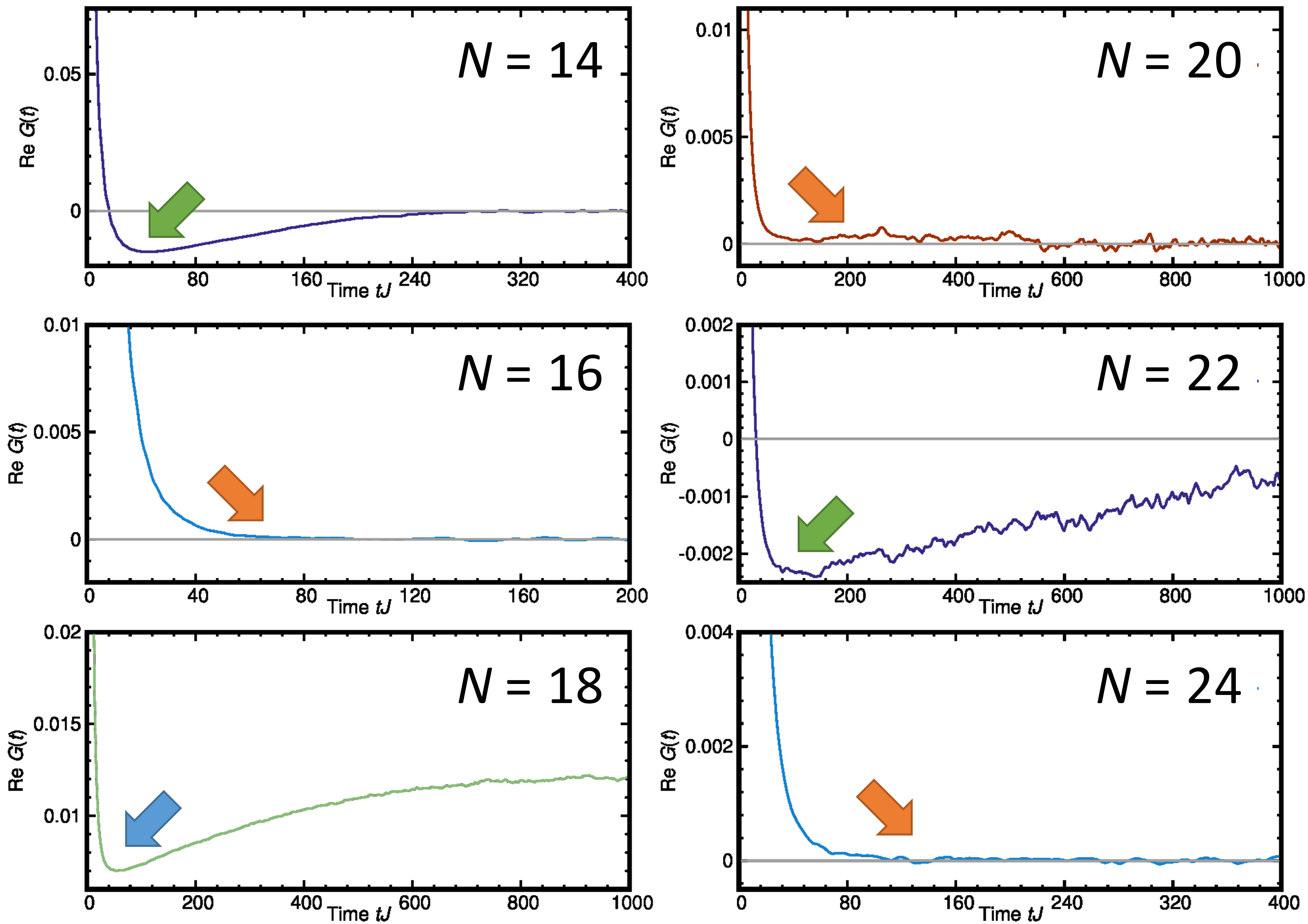
$$\hat{H} = \sum_{1\le a<b<c<d\le N_{\mathrm{M}}} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d,$$

J_{abcd} : Gaussian and variance $\sigma^2 = \frac{3!}{N^3}J$

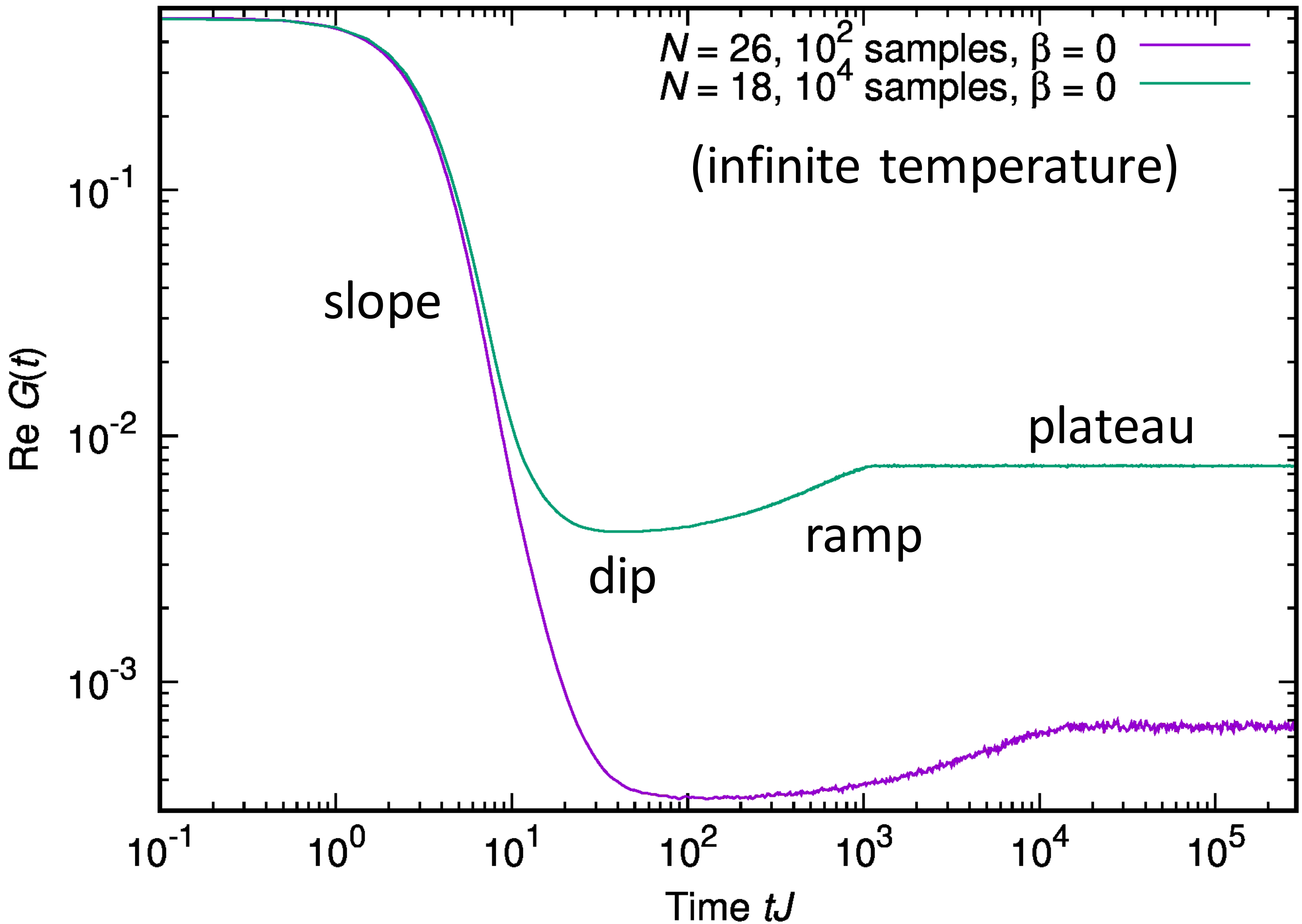


→ Approaches Gaussian for large *N* [A. M. García-García and J. J. M. Verbaarschot: 1610.03816]

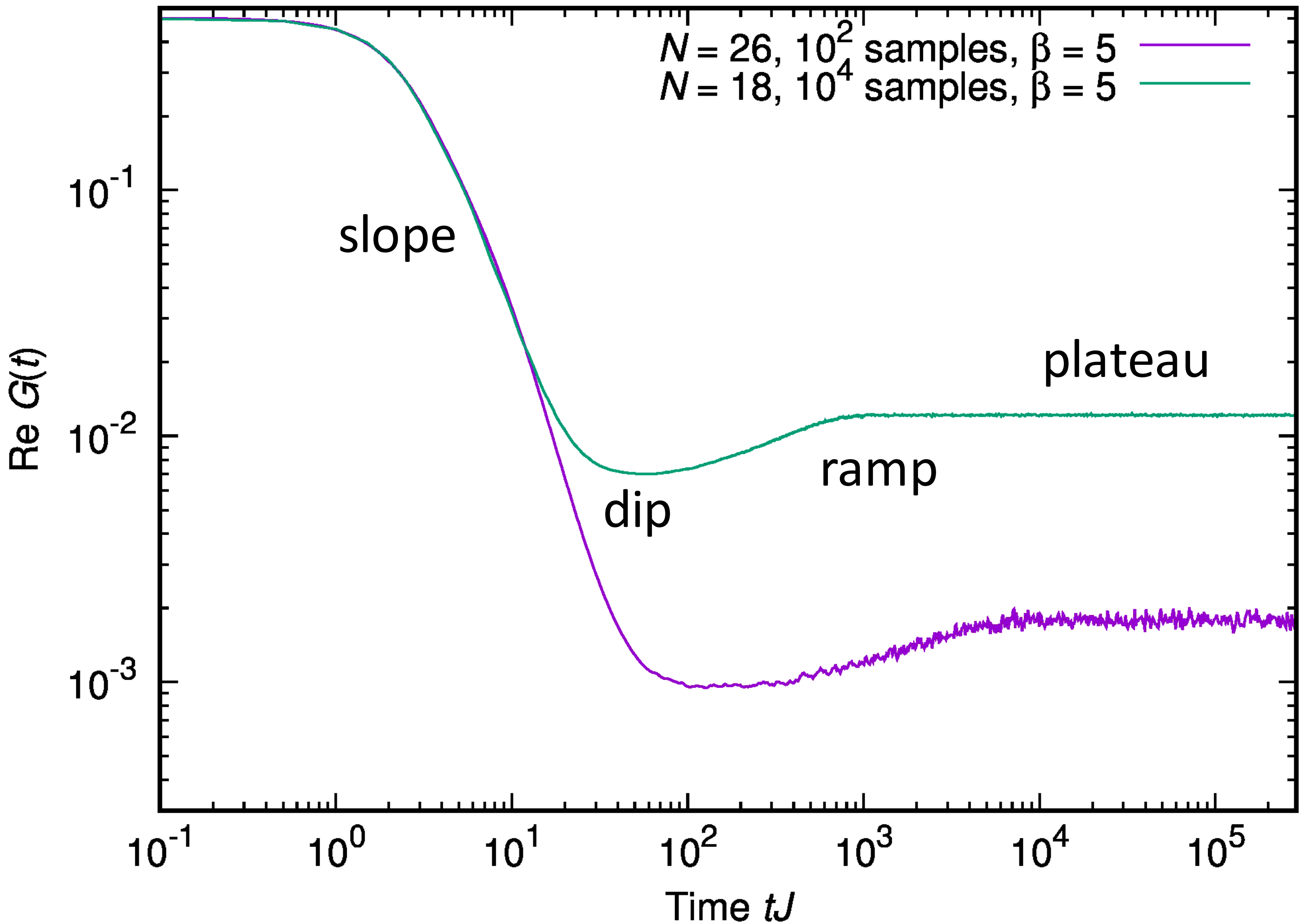
Correlation function $\left\langle \hat{\chi}_j(t)\hat{\chi}_j(0) \right\rangle_\beta$



Correlation function $\langle \hat{\chi}_j(t)\hat{\chi}_j(0) \rangle_\beta$ for $N \equiv 2 \pmod 8$



Correlation function $\left\langle \hat{\chi}_j(t)\hat{\chi}_j(0)\right\rangle_\beta$ for $N \equiv 2 \pmod 8$



Time-dependent partition function

$$Z(\beta,t)=Z(\beta+\mathrm{i}t)=\mathrm{Tr}\left(\mathrm{e}^{-\beta\hat{H}-\mathrm{i}\hat{H}t}\right)$$

For taking the disorder average, here we adopt "annealed"
(as opposed to "quenched") definition for simplicity in studying with replicas:

$$g(\beta,t)=\frac{\langle|Z(\beta,t)|^2\rangle_J}{\langle Z(\beta)\rangle_J^{\,2}}=g_{\mathrm{d}}(\beta,t)+g_{\mathrm{c}}(\beta,t)$$

but numerically this is typically within a percent of the disorder average of

$$\left|\frac{Z(\beta,t)}{Z(\beta,t=0)}\right|^2=\frac{1}{Z(\beta,t=0)^2}\sum_{m,n}\mathrm{e}^{-\beta(E_m+E_n)}\mathrm{e}^{\mathrm{i}(E_m-E_n)t}$$

Correlation: $\left\langle\hat{\chi}_j(t)\hat{\chi}_j(0)\right\rangle_\beta=\frac{1}{Z(\beta,t=0)}\sum_{m,n}\mathrm{e}^{-\beta E_m}\left|\langle m|\hat{\chi}_j|n\rangle\right|^2\mathrm{e}^{\mathrm{i}(E_m-E_n)t}$

Connected part

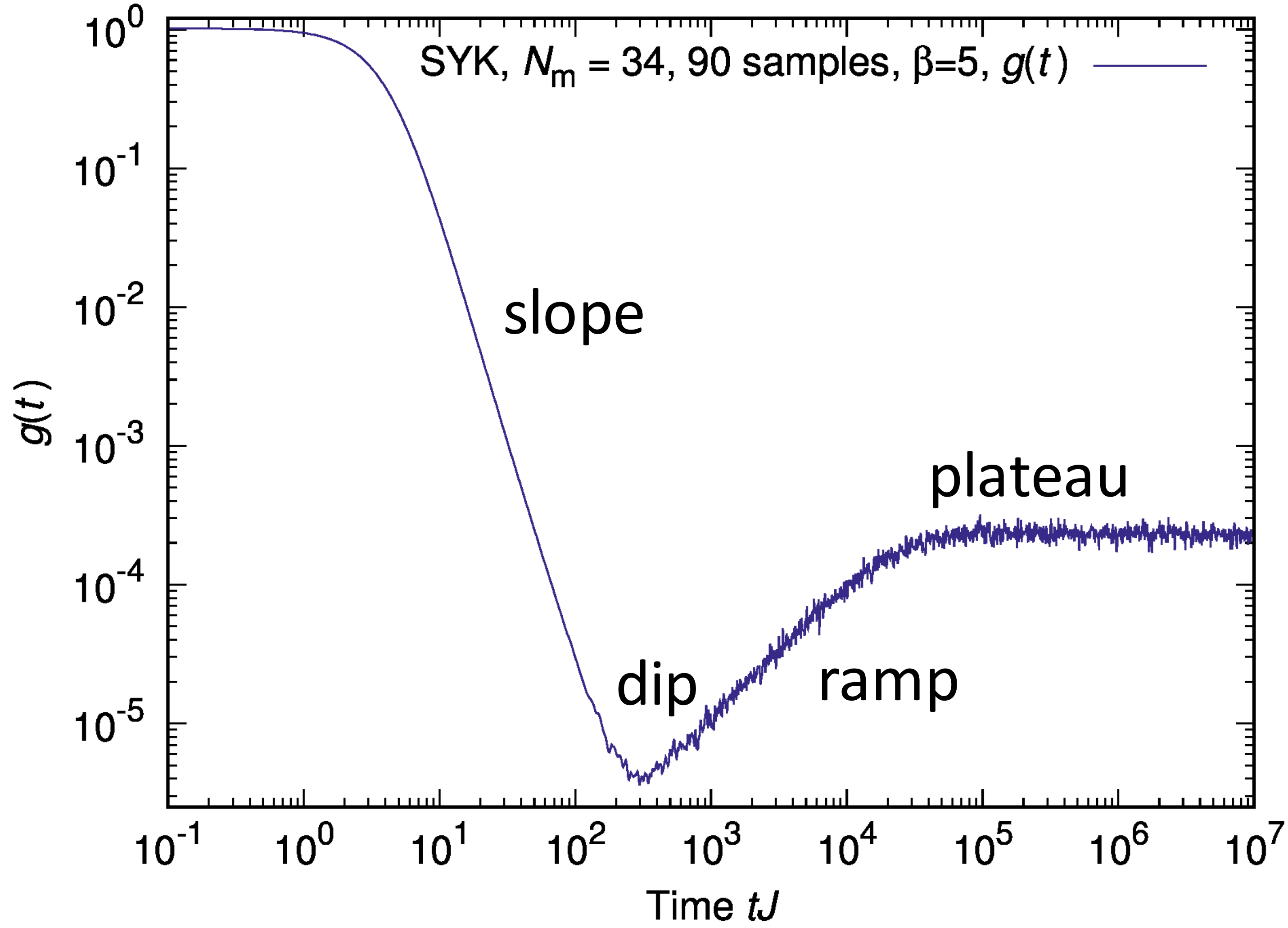
$$g_{\mathrm{c}}(\beta,t)=\frac{\langle|Z|^2\rangle_J-\left|\langle Z\rangle_J\right|^2}{\langle Z(\beta)\rangle_J^{\,2}}$$

Disconnected part

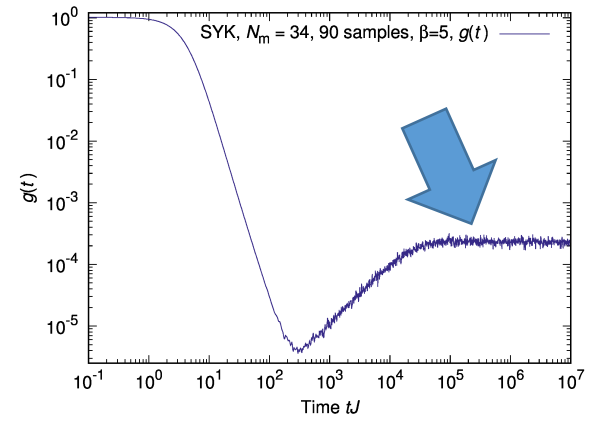
$$g_{\mathrm{d}}(\beta,t)=\frac{\left|\langle Z\rangle_J\right|^2}{\langle Z(\beta)\rangle_J^{\,2}}$$

Time-dependent partition function

$$g(\beta,t) = \frac{\langle |Z(\beta,t)|^2 \rangle_J}{\langle Z(\beta) \rangle_J^{\ 2}}$$



Plateau height determined by $Z(\beta)$



$$g(\beta,t) = \frac{\langle |Z(\beta,t)|^2 \rangle_J}{{\langle Z(\beta) \rangle_J}^2}$$

For each sample, consider the long time average of

$$\left| \frac{Z(\beta,t)}{Z(\beta,t=0)} \right|^2 = \sum_{m,n} \mathrm{e}^{-\beta(E_m+E_n)} \mathrm{e}^{\mathrm{i}(E_m-E_n)t}$$

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T dt \left| \frac{Z(\beta,t)}{Z(\beta,t=0)} \right|^2 = \frac{\sum_E {N_E}^2 \mathrm{e}^{-2\beta E}}{Z(\beta,t=0)^2} = N_E \frac{Z(2\beta)}{Z(\beta)}$$

(if degeneracy of E: N_E is independent of E)

Because $Z \sim \mathrm{e}^{aS}$ $(a>0)$, long-time average will be $\sim \mathrm{e}^{-aS}$ (non-perturbative in $1/N$)

Degeneracy

[L. Fidkowski and A. Kitaev: PRB **83**, 075103 (2011)]
[Y. Z. You, A. W. W. Ludwig, and C. Xu: 1602.06964]
[W. Fu and S. Sachdev: PRB **94**, 035135 (2016)]

$$P = K\prod_{j=1}^{N_\mathrm{D}}\left(\hat{c}_j^\dagger + \hat{c}_j\right), \hat{c}_j = \frac{\left(\chi_{2j-1} + \mathrm{i}\chi_{2j}\right)}{\sqrt{2}}, N_\mathrm{D} = N/2$$

K: complex conjugate

$$\left(\hat{c}_j^\dagger + \hat{c}_j\right)^2 = \left\{\hat{c}_j^\dagger, \hat{c}_j\right\} = 1, \qquad \left(\hat{c}_j^\dagger + \hat{c}_j\right)\left(\hat{c}_k^\dagger + \hat{c}_k\right) = -\left(\hat{c}_k^\dagger + \hat{c}_k\right)\left(\hat{c}_j^\dagger + \hat{c}_j\right)\ (j \neq k)$$

$$P^2 = (-1)^{N_\mathrm{D}(N_\mathrm{D}-1)/2} = \begin{cases} +1 & (N_\mathrm{D} \bmod 4 = 0, 1) \\ -1 & (N_\mathrm{D} \bmod 4 = 2, 3) \end{cases}$$

$$P\hat{c}_j^\dagger P = \eta\hat{c}_j, P\hat{c}_j P = \eta\hat{c}_j^\dagger, \eta = (-1)^{(N_\mathrm{D}-1)}P^2 = \begin{cases} +1 & (N_\mathrm{D} \bmod 4 = 1, 2) \\ -1 & (N_\mathrm{D} \bmod 4 = 0, 3) \end{cases}$$

$$\therefore P\chi P = \eta\chi, PHP = \eta^4 H = H$$

$$[H, P] = 0$$

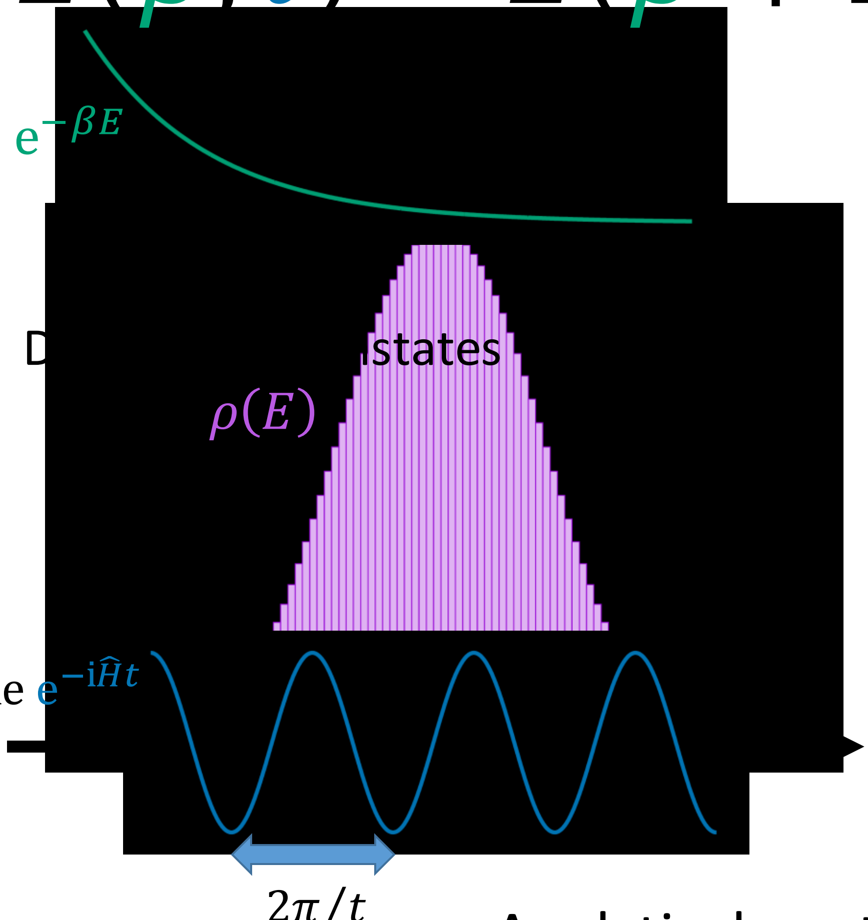
$\chi\chi\chi\chi$ preserves parity of complex fermion number

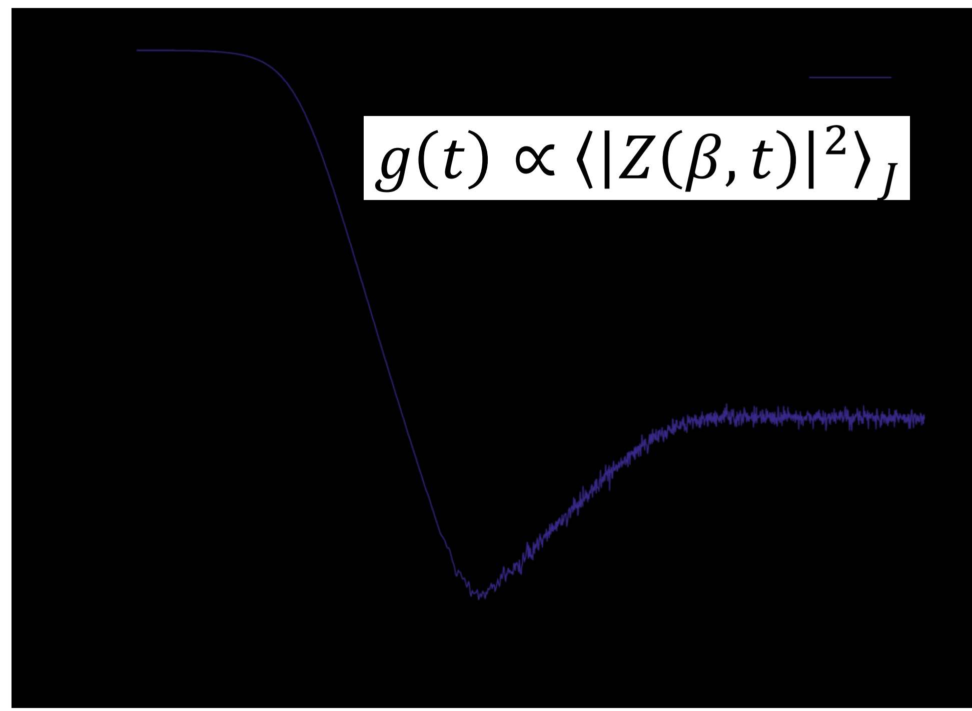
$N \equiv 0 \pmod 8$: P maps each charge parity sector to itself and $P^2 = 1$ (no degeneracy)
$N \equiv 2 \pmod 8$: P maps each sector to the other and $\langle\mathrm{even}|\chi|\mathrm{odd}\rangle$ finite
$N \equiv 4 \pmod 8$: P maps each charge parity sector to itself and $P^2 = -1$ (degeneracy)
$N \equiv 6 \pmod 8$: P maps each sector to the other but $\langle\mathrm{even}|\chi|\mathrm{odd}\rangle = 0$

Time-dependent partition function and energy scale

$$\mathcal{H}_{\mathrm{M}} = \sum_{a,b,c,d}^{N} J_{abcd}\widehat{\chi_a}\widehat{\chi_b}\widehat{\chi_c}\widehat{\chi_d}$$

$$Z(\beta, t) = Z(\beta + \mathrm{i}t) = \mathrm{Tr}\left(\mathrm{e}^{-\beta\hat{H} - \mathrm{i}\hat{H}t}\right)$$





- Analytical continuation of partition function $Z(\beta)$
- Fourier transform of $\rho(E)$ modified by temperature

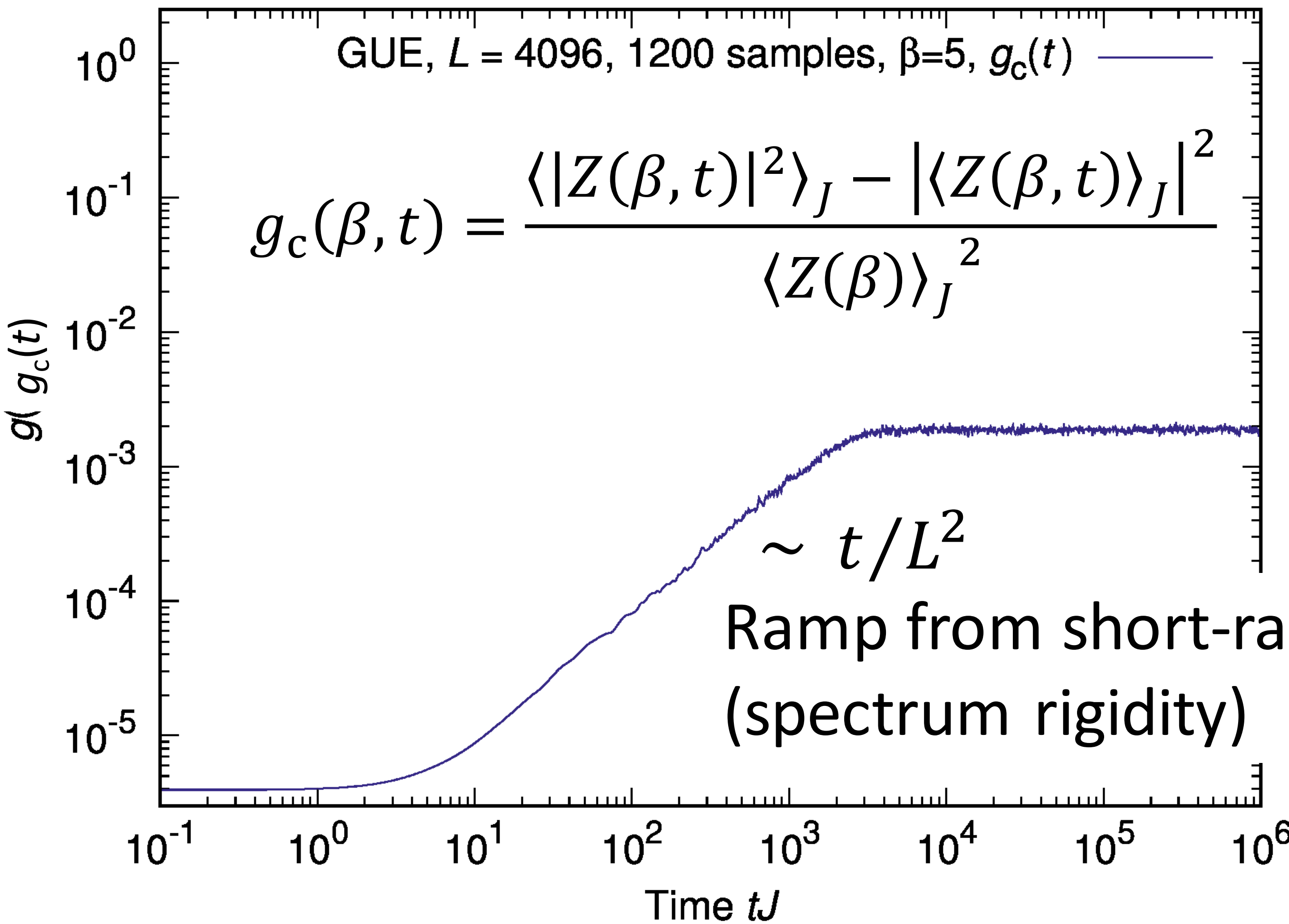
Late time: governed by $g_c(t)$

[You, Ludwig, Xu: arXiv:1602.06964]

BDI class, N_χ Majorana fermions

$N_\chi \pmod 8$	0	1	2	3	4	5	6	7
qdim	1	$\sqrt{2}$	2	$2\sqrt{2}$	2	$2\sqrt{2}$	2	$\sqrt{2}$
lev. stat.	GOE	GOE	GUE	GSE	GSE	GSE	GUE	GOE
$\mathcal{C}\ell_{0,N_\chi-1}$	$\mathbb{R}\oplus\mathbb{R}$	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{H}\oplus\mathbb{H}$	$\mathbb{H}$	$\mathbb{C}$	$\mathbb{R}$

Dense random matrix reproduces
the late-time ramp & plateau behavior



$$g_c(\beta,t) = \frac{\langle |Z(\beta,t)|^2\rangle_J - |\langle Z(\beta,t)\rangle_J|^2}{\langle Z(\beta)\rangle_J^{\ 2}}$$

L: matrix dimension

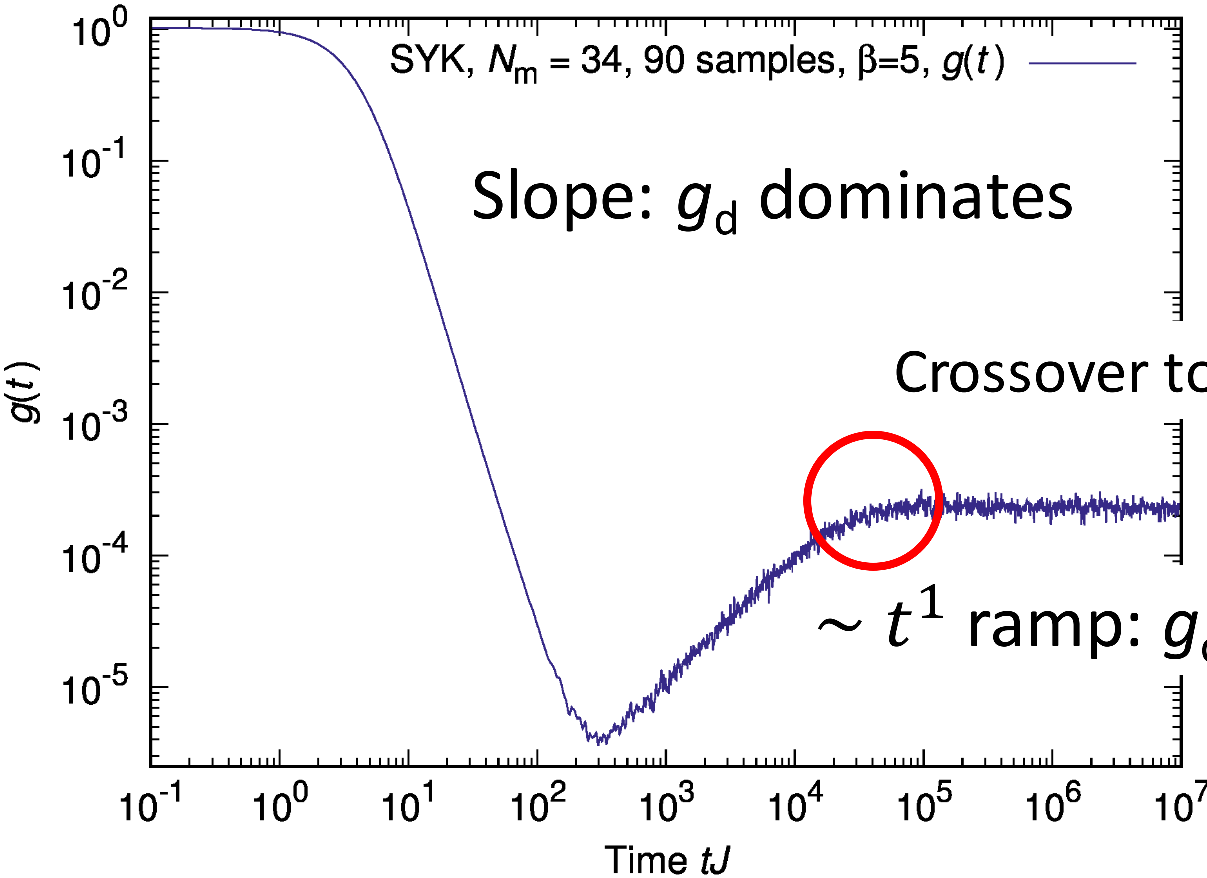
Ramp from short-range correlation
(spectrum rigidity)

Slope-dip-ramp-plateau structure of $g(\beta, t)$

Early time: $\sim t^{-3}$

$$Z(\beta, t) = \mathrm{Tr}\left(\mathrm{e}^{-\beta\hat{H} - \mathrm{i}\hat{H}t}\right)$$

$$g_{\mathrm{d}}(\beta, t) = \frac{\left|\langle Z(\beta, t)\rangle_J\right|^2}{\langle Z(\beta)\rangle_J^{\;2}}$$



Slope: g_{d} dominates

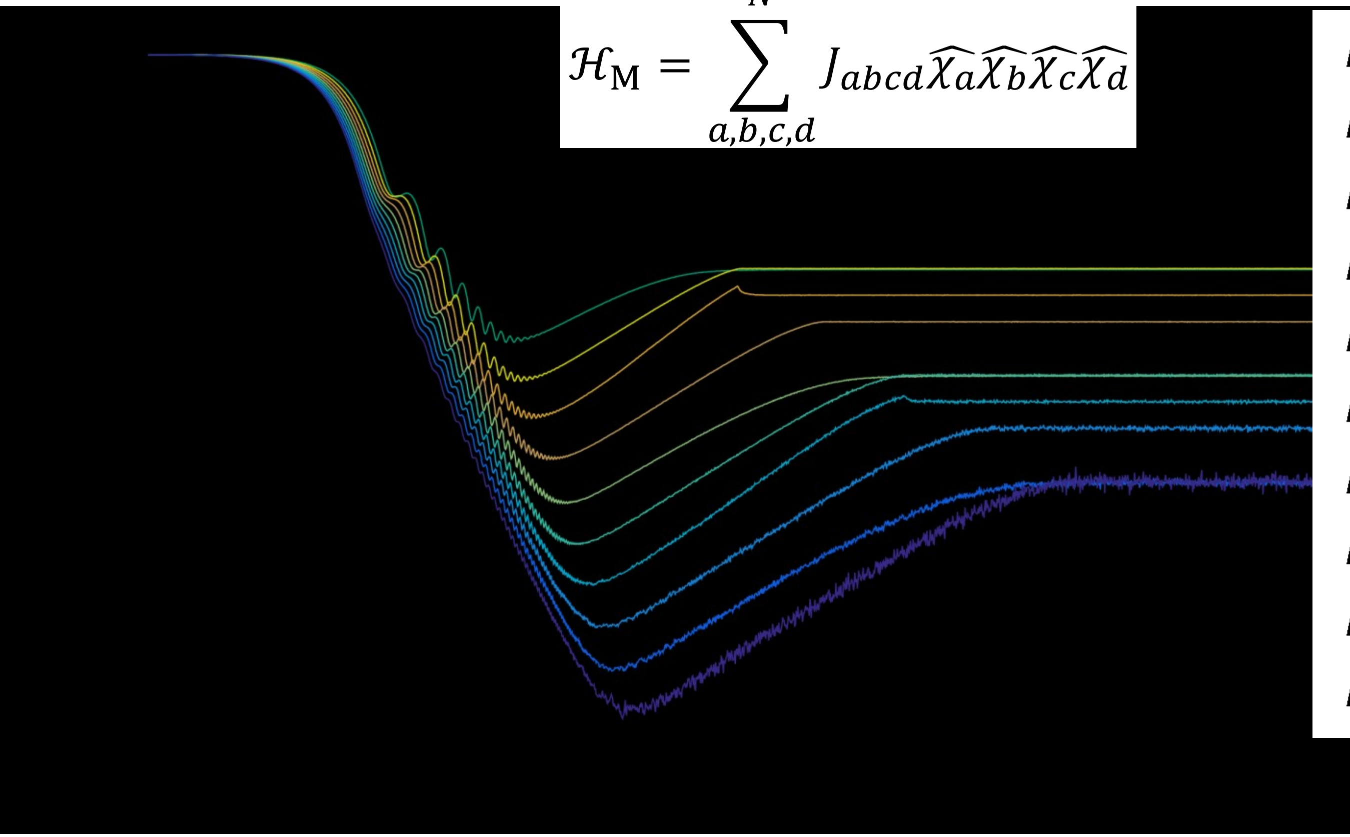
Crossover to plateau: symmetry

$\sim t^1$ ramp: g_{c} dominates

Long smooth ramp $\sim t^1$

➔ Spectrum rigidity in Sachdev-Ye-Kitaev model

$g(t)$: Dependence on N

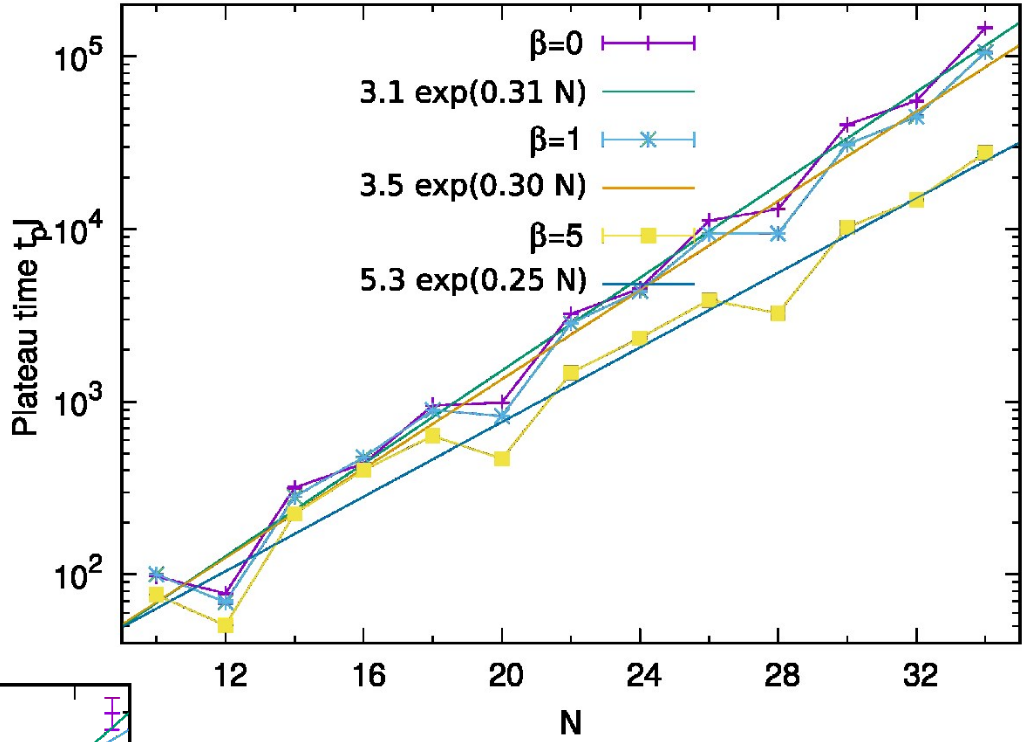


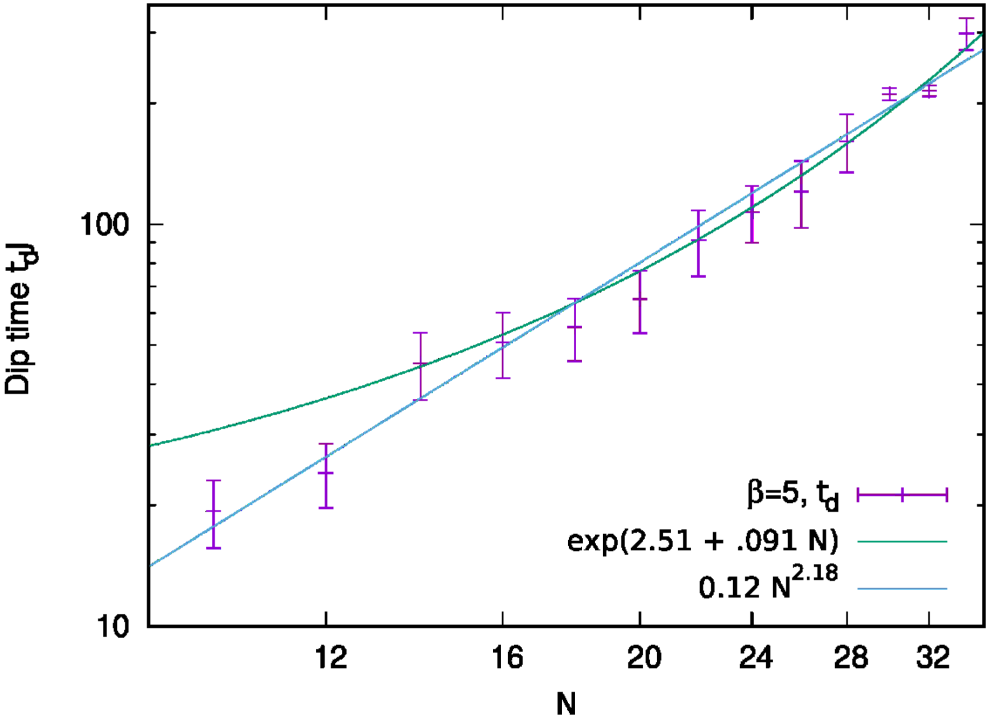
Classification of SPT order in class BDI: reduced from Z to Z_8 by interaction
[L. Fidkowski and A. Kitaev: PRB **81**, 134509 (2010); PRB **83**, 075103 (2011)]

Many-body level statistics ← corresponding (dense) random matrix ensemble
[Y.-Z. You, A. W. W. Ludwig, and Cenke Xu, 1602.06964]

N_χ (mod 8)	0	1	2	3	4	5	6	7
qdim	1	$\sqrt{2}$	2	$2\sqrt{2}$	2	$2\sqrt{2}$	2	$\sqrt{2}$
lev. stat.	GOE	GOE	GUE	GSE	GSE	GSE	GUE	GOE

Plateau time and dip time

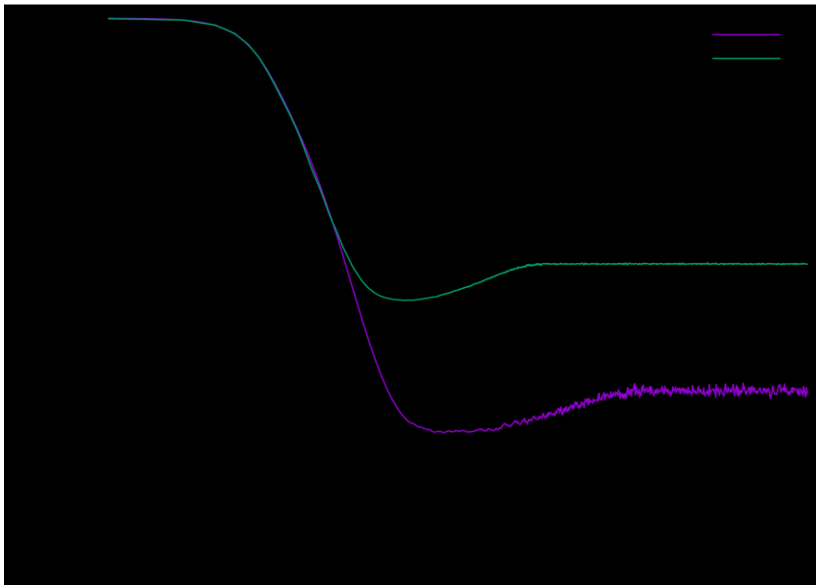




Correlation function

$$G(t) = \langle \chi_a(t)\chi_a(0) \rangle$$

Dip-ramp-plateau structure similar to $g(\beta, t)$ for $N \equiv 2 \pmod 8$



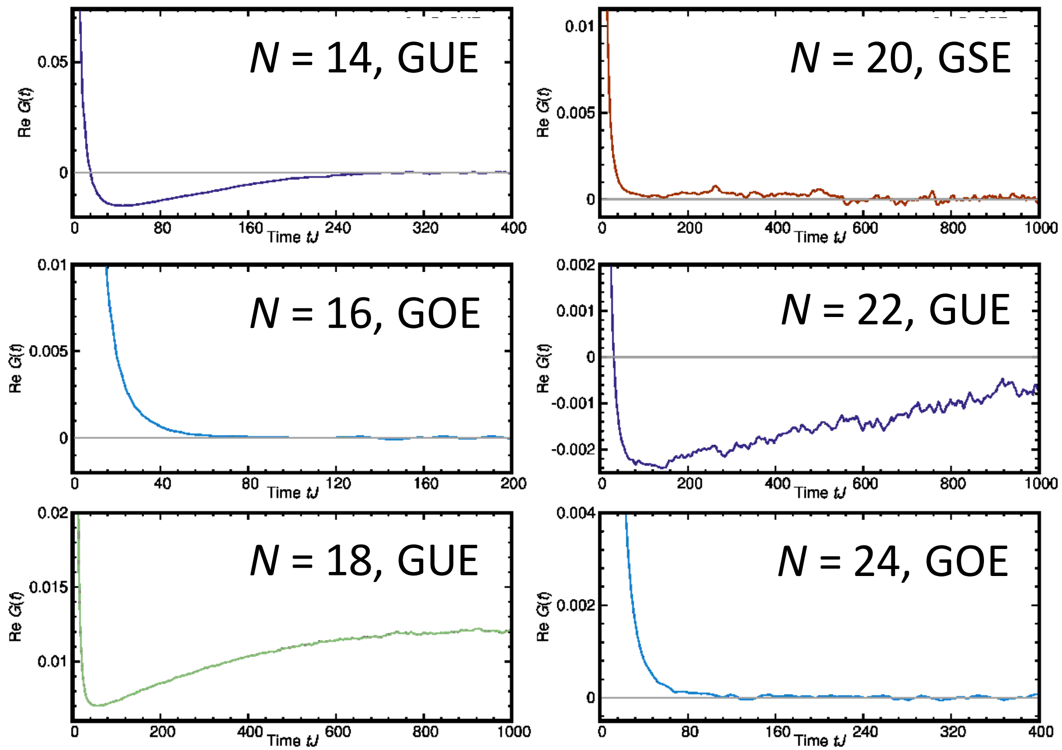
$$P = K \prod_{j=1}^{N/2} (\hat{c}_j^\dagger + \hat{c}_j), \hat{c}_j = \frac{(\chi_{2j-1} + \mathrm{i}\chi_{2j})}{\sqrt{2}}$$

$N \equiv 0 \pmod 8$: P maps each charge parity sector to itself and $P^2 = 1$ (no protected degeneracy)

$N \equiv 2 \pmod 8$: P maps each sector to the other and $\langle \mathrm{even}|\chi|\mathrm{odd}\rangle$ finite

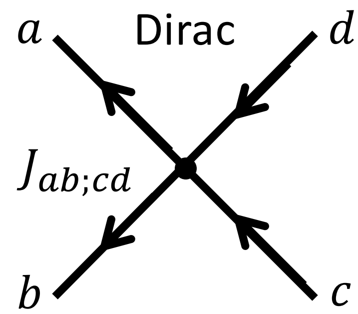
$N \equiv 4 \pmod 8$: P maps each charge parity sector to itself and $P^2 = -1$ (only internal degeneracy)

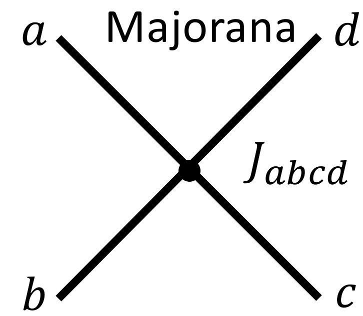
$N \equiv 6 \pmod 8$: P maps each sector to the other but $\langle \mathrm{even}|\chi|\mathrm{odd}\rangle = 0$



Summary of the first part [1611.04650]

[J. S. Cotler, G. Gur-Ari, M. Hanada, J. Polchinski, P. Saad, J. Saul, S. H. Shenker, D. Stanford, and MT]





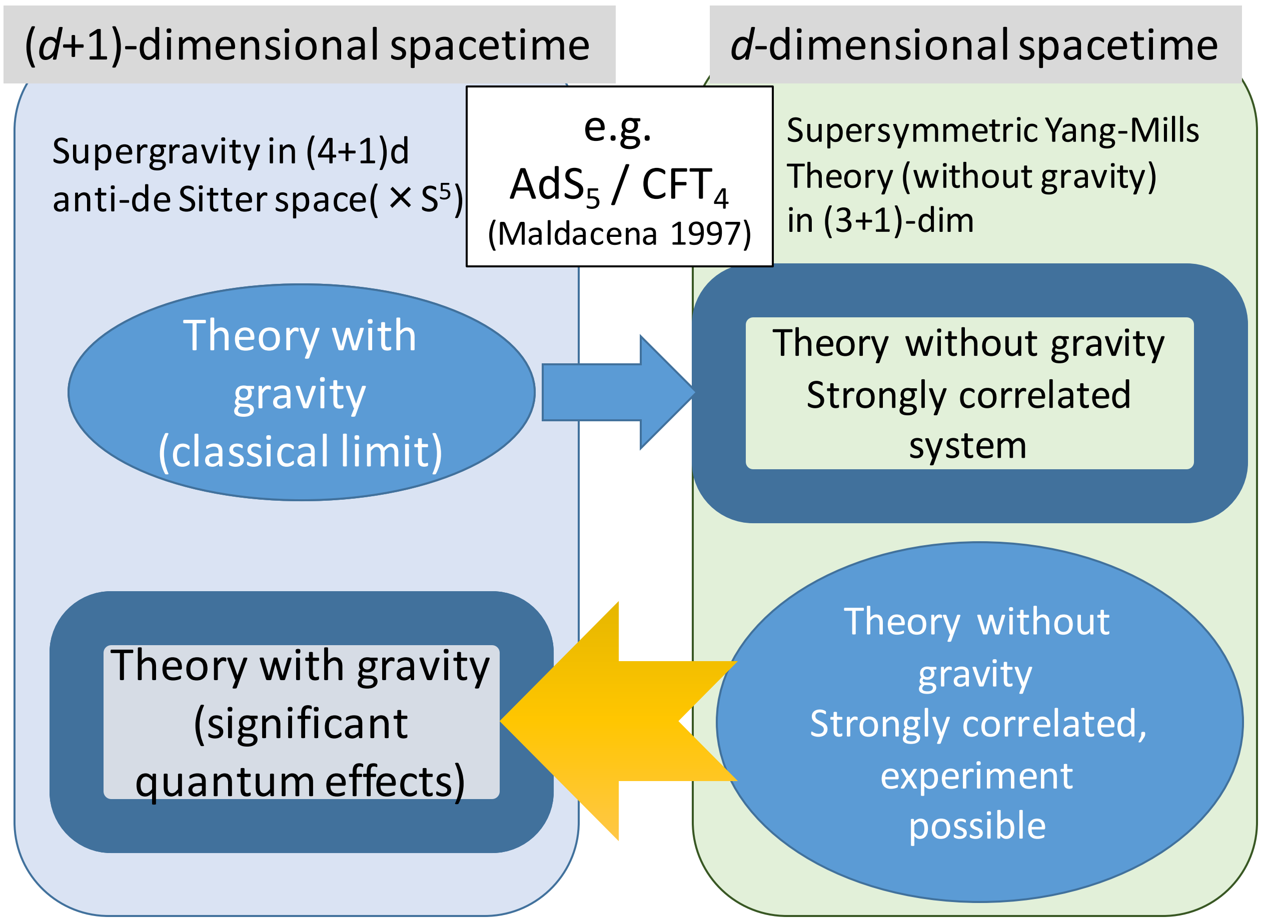
- Sachdev-Ye-Kitaev (SYK) model as a route to approach
  - Black hole entropy from bulk
  - Quantum information scrambling
  - Long-time behavior of inhomogeneous correlated systems

- Numerical results for finite degrees of freedom *N*
  - $g(\beta, t) \propto \langle |Z|^2 \rangle_J$, $g_c(\beta, t) \propto \langle |Z|^2 \rangle_J - \left|\langle Z \rangle_J\right|^2$ based on t-dep. partition func. $Z(\beta, t)$
    - Early-time dip: $\left|\langle Z \rangle_J\right|^2$ rapidly decays
    - Ramp + plateau: crossover to random matrix; 8-fold way (GOE, GUE, GSE)
  - Correlation $\langle \chi(t) \chi(0) \rangle$: have similar slope-dip-ramp-plateau for *N* mod 8 = 2
  - ➔ Conjectures about AdS black holes (see our paper)

➔ Ultracold gas experiment proposal

[I. Danshita, M. Hanada, MT: arXiv:1606.02454]

The holographic principle and quantum gravity



◎ Not limited to classical limit

➔ Several supporting evidences

e.g. check of the leading gravity correction for the black hole mass [M. Hanada, Y. Hyakutake, G. ishiki, and J. Nishimura, Science **344**, 882 (2013)]

Many “AdS/CMT” applications

This work:
approach quantum gravity by realizing corresponding non-gravity models in cold gases

Complex Dirac-SYK model

$$\hat{H} = \frac{1}{(2N)^{3/2}} \sum_{ijkl} J_{ij;kl} \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_k \hat{c}_l,$$

$<|J_{ij;kl}{}^2|> = J^2$; single parameter T/J
Consider large-N limit:
0+1d, conformal symmetry
Corresponding 1+1d gravity model in AdS_2
[A. Kitaev, talks at KITP (2015)]
[S. Sachdev, PRX **5**, 041025 (2015)]

Different from
other approaches to gravity in cold gases

Hawking radiation in sonic analogue of BEC
[J. Steinhauer, Nature Phys. **10**, 864 (2014); nphys3863 (2016)]
cf. theory [W.G. Unruh, PRL **46**, 1351 (1981)]

Sakharov Oscillations in quenched BEC
[C.-L. Hung, V. Gurarie, and C. Chin, Science **341**, 1213 (2013)]

- No supersymmetry (bosons ⇔ fermions) needed
- Not relativistic, no antiparticles
- Spinless fermions can be used

→ **Experimental realization?**

Out-of-time-order correlation (OTOC) functions

$$F(t) = \langle \hat{W}^\dagger(t)\hat{V}^\dagger(0)\hat{W}(t)\hat{V}(0)\rangle \qquad W(t) = \mathrm{e}^{iHt}W\mathrm{e}^{-iHt}$$

Butterfly effect: quantum version

$\partial x(t)/\partial x(t{=}0) = \{x(t), p\} = x(t)p + px(t)$

[N. Wiener 1938][Larkin & Ovchinnikov 1969]

Consider commuting operators V and W,

$C(t) = \langle |[W(t), V(t=0)]|^2\rangle = 2\big(1 - \mathrm{Re}\, F(t)\big)$

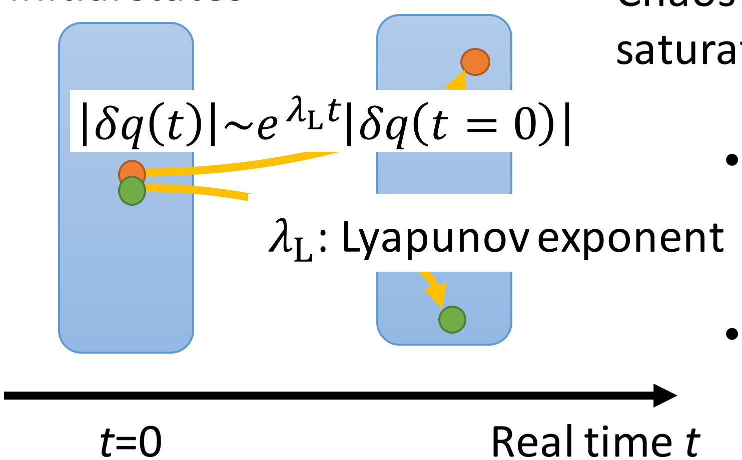
quantifies strength of quantum scrambling;

"BHs are fastest quantum scramblers"

[P. Hayden and J. Preskill 2007] [Y. Sekino and L. Susskind 2008]

[Shenker and Stanford 1306.0622]

Slightly different initial states



$\lambda_{\rm L} = 2\pi k_{\rm B}T/\hbar$

Chaos bound [J. Maldacena, S. H. Shenker, and D. Stanford, 1503.01409]

saturated by large-*N* SYK model [Maldacena and Stanford, 1604.07818]

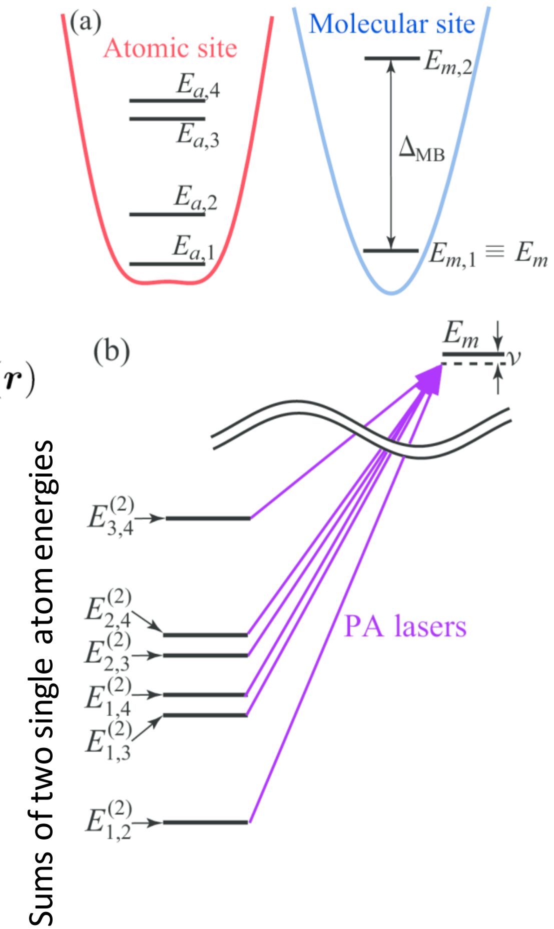
- Measurement protocol using a quantum dot
  [B. Swingle, G. Bentsen, M. Schleier-Smith, P. Hayden, PRB **94**, 040302 (2016)]
- Measurement protocols for SYK
  [I. Danshita, M. Hanada, and M. Tezuka, 1606.02454]
  [L. García-Álvarez, I. L. Egusquiza, L. Lamata, A. del Campo, J. Sonner, and E. Solano, 1607.08560]

Our proposal: coupled atom-molecule model

Consider atomic levels $i,j,\ldots = 1, 2, \ldots, N$ coupled to a molecule state m_1

$$\hat{H}_{\mathrm{m1}} = \nu \hat{m}^\dagger \hat{m} + \sum_{i,j} g_{ij} (\hat{m}^\dagger \hat{c}_j \hat{c}_i + h.c.)$$

$$g_{ij} = \frac{1}{2}\mathrm{sgn}(j-i) \int d\boldsymbol{r}\, \Omega_{i,j}(\mathbf{r}) w_m(\boldsymbol{r}) w_{a,i}(\boldsymbol{r})\, w_{a,j}(\boldsymbol{r})$$



Detuning ν: controlled by laser energy

$\Omega_{i,j}$: space-dependent photoassociation laser
w_m: molecular site wavefunction
$w_{a,i(j)}$: atomic site wavefunction

$s = 1, 2, \ldots, n_s$

Consider multiple molecular states; assume they are short-lived
➔ integrate them out and obtain the effective model for atoms

$$\hat{H}_{\mathrm{eff}} = \sum_{s,i,j,k,l} \frac{g_{s,ij} g_{s,kl}}{\nu_s} \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_k \hat{c}_l$$

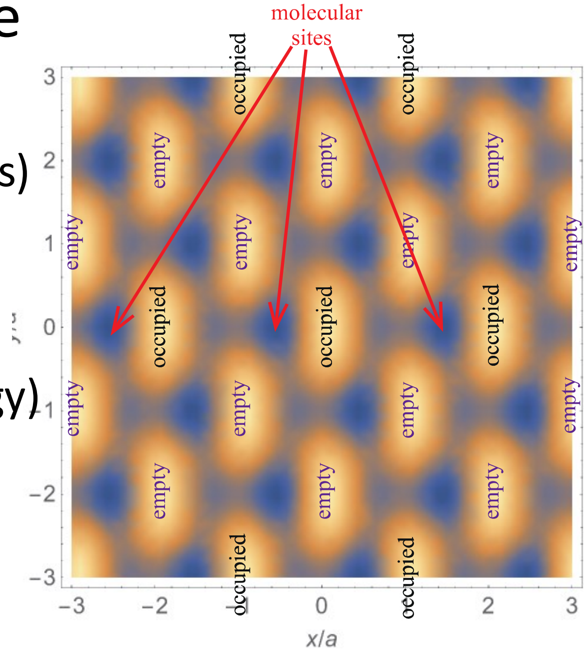
Optical lattice setup in our proposal

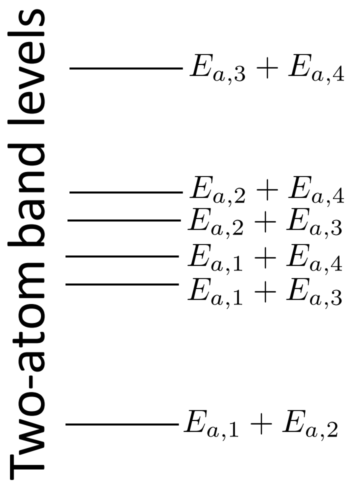
A double-well optical lattice

(no degeneracy in the band levels)

with ^{6}Li

(large recoil energy)





Possible to satisfy required conditions

$$\max(t_i) \lesssim \hbar/\tau_{\text{exp}} \ll J,$$

$$\max(\hbar\Gamma_{\text{PA}}, \hbar\Gamma_{\text{ms},s}) \ll |\nu_s| \ll \Delta_{\text{min}}, \text{ for all } s,$$

$$\Delta_{\text{max}} < \Delta_{\text{MB}} < \tilde{\Delta},$$

$$|\nu_s| \ll |U_{s,s'}|, \text{ for all } s \text{ and } s',$$

$$|U_{s,s'}| < \Delta_{\text{min}} \text{ or } \Delta_{\text{max}} < |U_{s,s'}|, \text{ for all } s \text{ and } s'.$$

Realizing real Dirac SYK model

$$\hat{H}_{\text{eff}} = \sum_{s,i,j,k,l} \frac{g_{s,ij} g_{s,kl}}{\nu_s} \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_k \hat{c}_l$$

$s = 1, 2, \ldots, n_s$

(For simplicity we take $\nu_s = (-1)^s \sqrt{n_s}\sigma_s$)

Can be shown to approach the real Dirac version of the SYK model as $n_s \to \infty$.

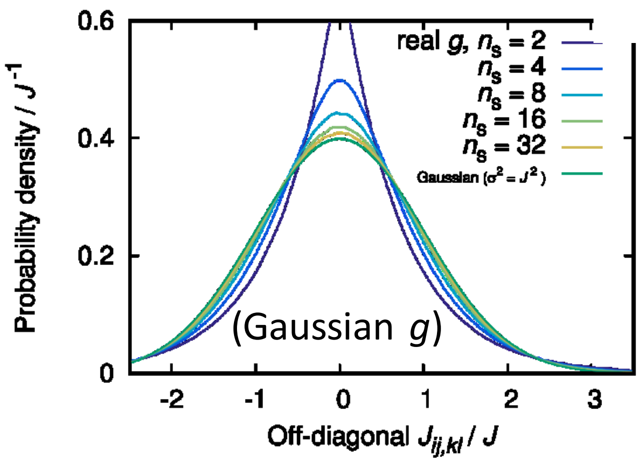
$$\hat{H} = \frac{1}{(2N)^{3/2}} \sum_{ijkl} J_{ij;kl} \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_k \hat{c}_l,$$

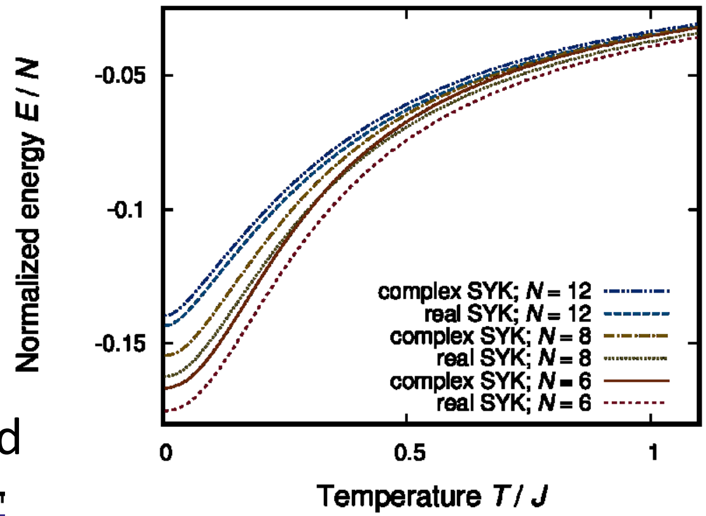
$$J_{ij;kl} = -J_{ji;kl} = -J_{ij;lk},$$

$$J_{ij;kl} = J_{kl,ij}$$

$$\overline{|J_{ij;kl}|^2} = \begin{cases} J^2 & (\{i,j\} \neq \{k,l\}) \\ 2J^2 & (\{i,j\} = \{k,l\}) \end{cases}$$

Gaussian J reproduced



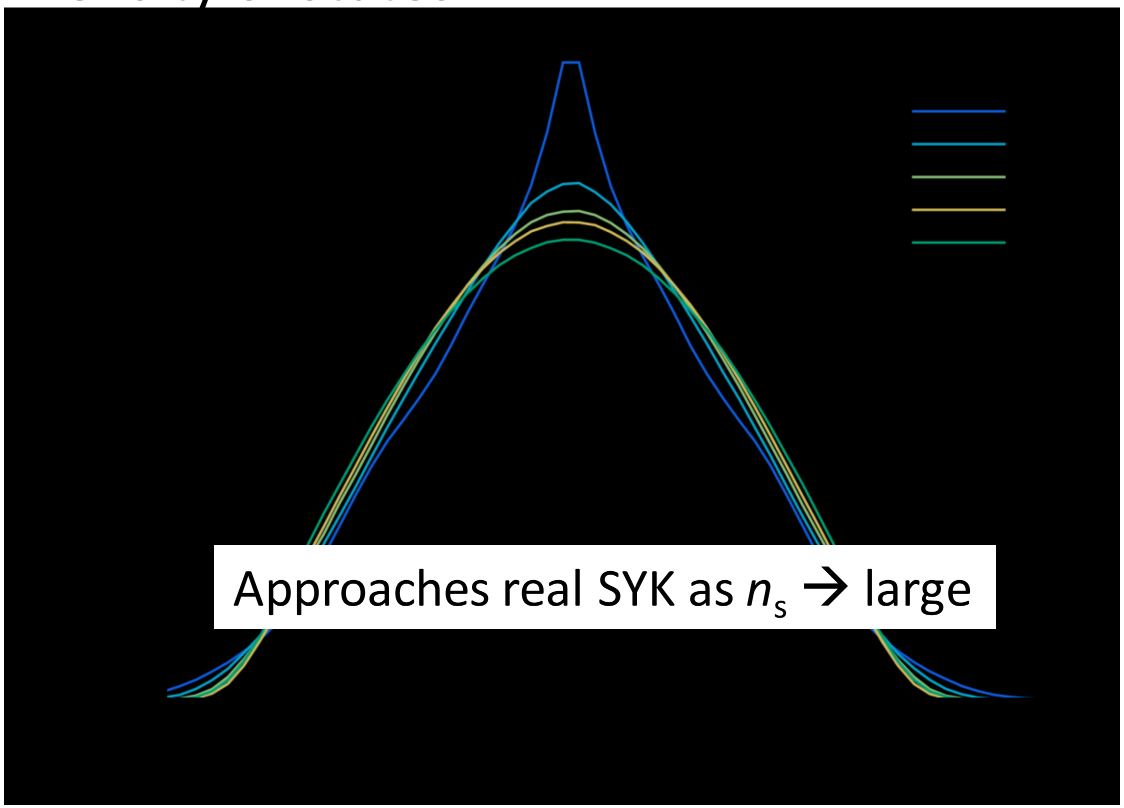


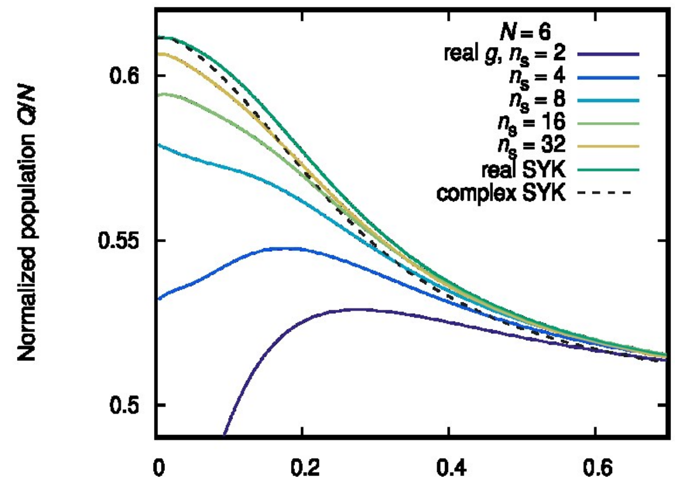
Real SYK:
Physical quantities coincide with those for complex SYK in $N \to \infty$ limit

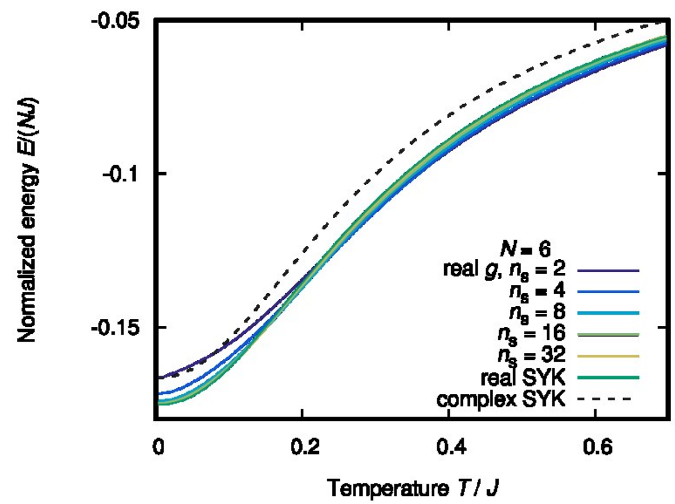
Many molecule states → real SYK realized

→ Thermodynamics also agree well

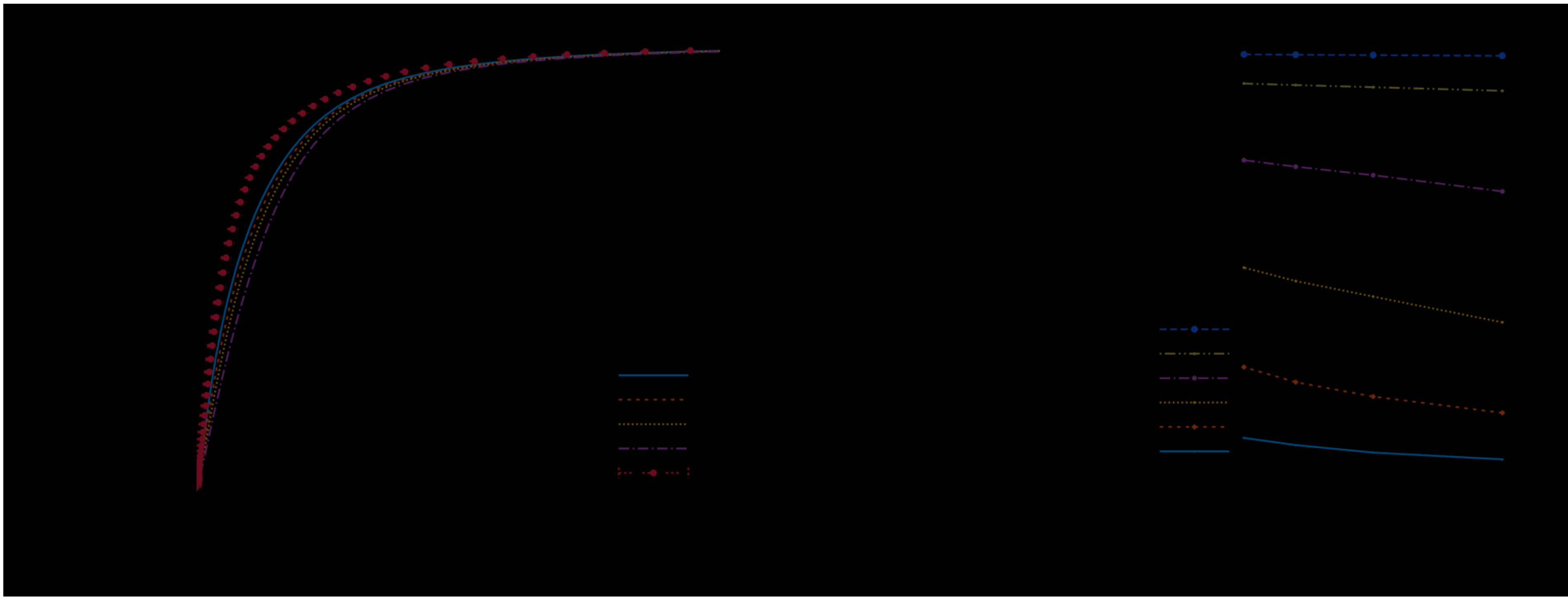
Density of states







Entropy at low temperatures



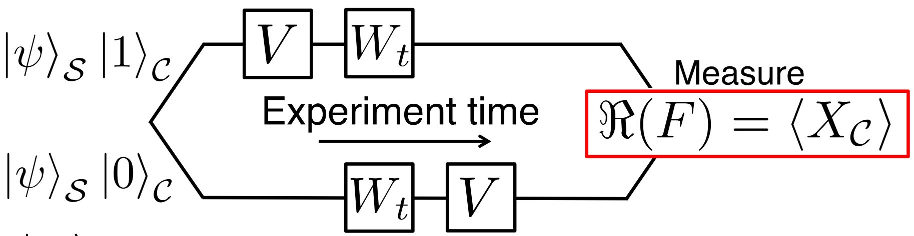
➔ Suggests non-zero entropy at the large-*N*, low *T* limit

Out-of-time-order correlation measurement

B. Swingle *et al.*: PRB **94**, 040302 (2016)

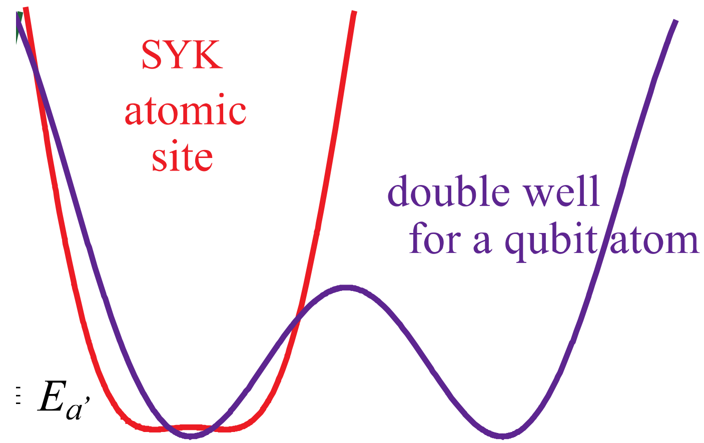
$$F(t) = \langle \hat{W}^\dagger(t)\hat{V}^\dagger(0)\hat{W}(t)\hat{V}(0)\rangle.$$

Interferometric protocol:



$|\psi\rangle_S$: Initial state of the probed system

$|0\rangle_C, |1\rangle_C$: states of the control qubit



$$\widehat{W}(t) = \boxed{e^{iHt}}\,\widehat{W}\,e^{-iHt}$$

Time evolution with
$H' = -H \;(\nu' = -\nu)$

Create the cat state

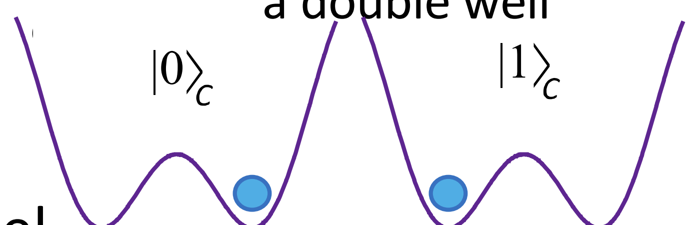
$$|\Psi\rangle = \widehat{W}(t)\hat{V}|\psi\rangle_S|1\rangle_C + \hat{V}\widehat{W}(t)|\psi\rangle_S|0\rangle_C$$

by applying

$\widehat{I_S} \otimes |0\rangle\langle 0|_C + \hat{V} \otimes |1\rangle\langle 1|_C$, $\widehat{W}(t) \otimes \widehat{I_C}$,

and $\hat{V} \otimes |0\rangle\langle 0|_C + \widehat{I_S} \otimes |1\rangle\langle 1|_C$ in this order, then measure the qubit to find $\mathrm{Re}\,F(t)$ and $\mathrm{Im}\,F(t)$.

Our qubit *C*:
A single particle in a double well



➔Implementation of this protocol in our model using a qubit on additional optical double well [1606.02454]

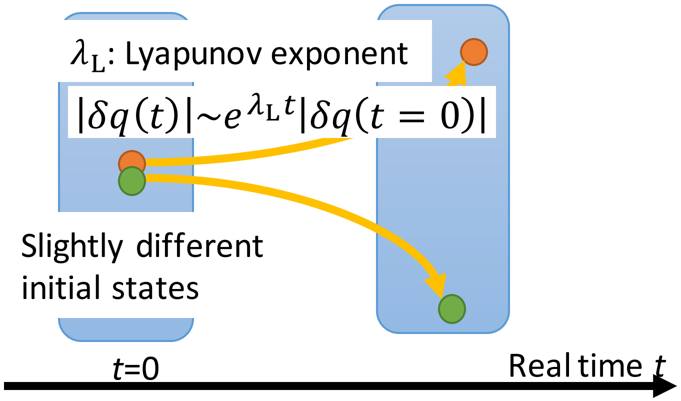
Calculation of OTOC for our model

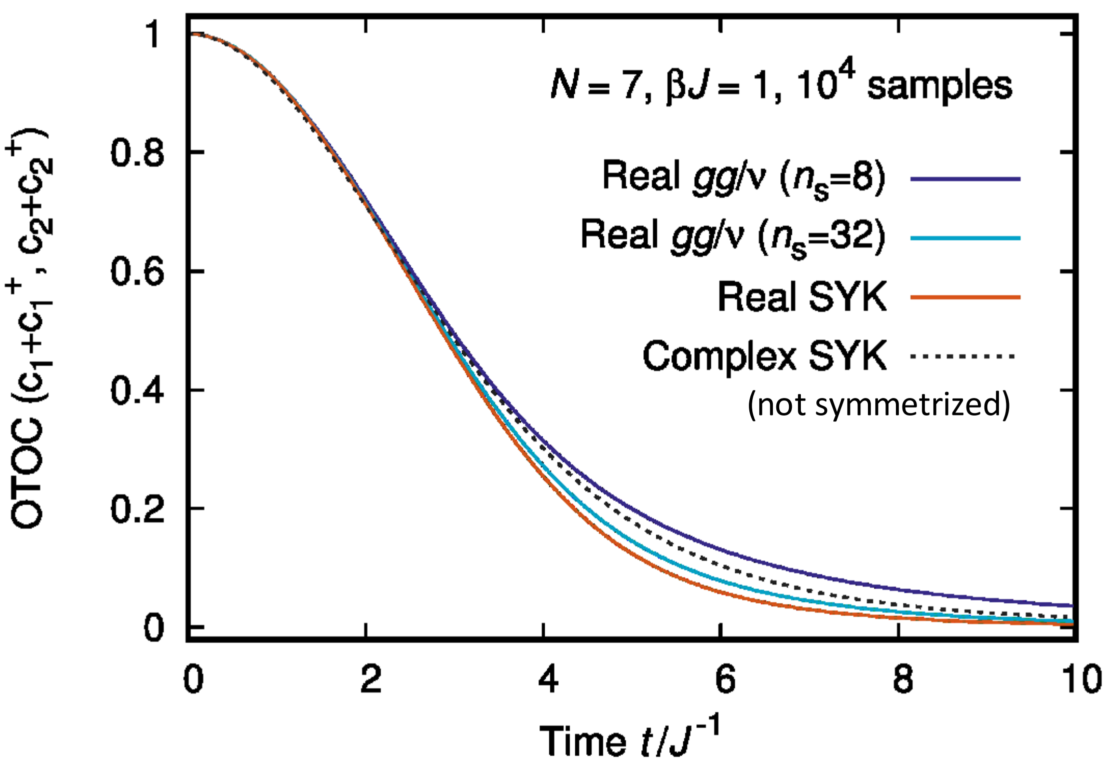
Out-of-time-order correlator (OTOC)
of initially commuting operators ($[V, W]=0$)

$$F(t) = \langle \hat{W}^\dagger(t)\hat{V}^\dagger(0)\hat{W}(t)\hat{V}(0)\rangle \sim 1 - Ce^{\lambda_{\mathrm{L}} t}\ ?$$

$(V, W) = (\hat{c_1} + \hat{c_1}^\dagger, \hat{c_2} + \hat{c_2}^\dagger)$: first calculated in [Fu and Sachdev, PRB 2016 (1603.05246)]

$\lambda_{\mathrm{L}} = 2\pi k_{\mathrm{B}} T/\hbar$: Chaos bound
[Maldacena et al., 1503.01409]
saturated by large-N SYK
[Maldacena and Stanford, 1604.07818]





Summary of the second part [1606.02454]

Ippei Danshita, Masanori Hanada, and Masaki Tezuka: arXiv:1606.02454

Realize strongly correlated model without gravity + holographic principle

➔ "Experimental quantum gravity"

Specific example here:
Sachdev-Ye-Kitaev (SYK) model
(dual to charged black hole in AdS_2)

- A coupled atom-molecule model approaches SYK as the number of molecule levels is increased
- Finite low-temperature entropy
- Eigenvalue spectrum, out-of-time-order correlation (OTOC) also reproduced

● How to realize coupled atom-molecule model?

$$\hat{H}_{\mathrm{m}} = \sum_{s=1}^{n_s} \left\{ \nu_s \hat{m}_s^\dagger \hat{m}_s + \sum_{i,j} g_{s,ij} \left(\hat{m}_s^\dagger \hat{c}_i \hat{c}_j - \hat{m}_s \hat{c}_i^\dagger \hat{c}_j^\dagger \right) \right\}$$

➔ Single sites of designed optical lattice + photo-association lasers

● How to measure OTOC?

$$C_{i,j}(t) = \langle \hat{c}_j^\dagger(t) \hat{c}_i^\dagger(0) \hat{c}_j(t) \hat{c}_i(0) \rangle$$

➔ Use interference by coupling to a controlling qubit (cat state)

Grant-in-Aid for Scientific Research (KAKENHI) on Innovative Areas
"Topological Materials Science"

Duration

July 2015 – March 2020 (5 Years Project)

Leader Norio Kawakami (Kyoto)

Topological Materials Science

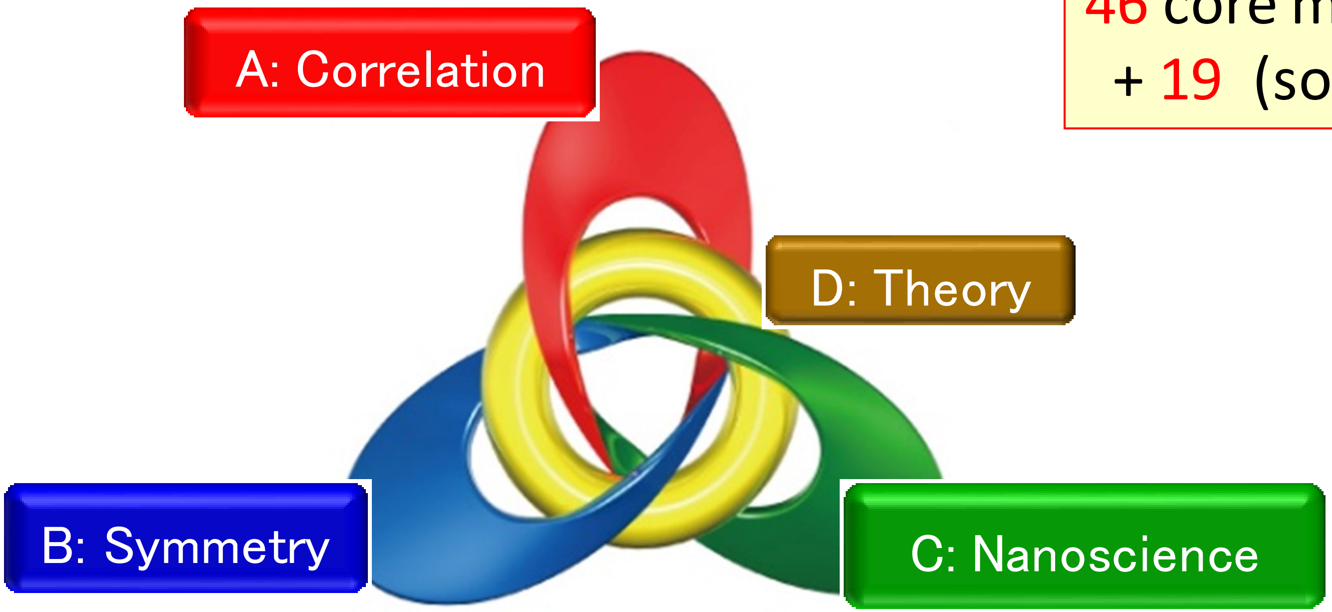
4 Teams

Team Leaders

A: Yoshi Maeno (Kyoto)
B: Takafumi Sato (Sendai)
C: Toshimasa Fujisawa (Tokyo Tech.)
D: Norio Kawakami (Kyoto)

【Sub-Projects】

A: Topology and Correlation (Yoshi Maeno)
B: Topology and Symmetry (Takafumi Sato)
C: Topology and Nanoscience (Toshimasa Fujisawa)
D: Topology and New Concepts (Norio Kawakami)



46 core members
+ 19 (solicitations)

"Topo-Q" Proposal

International Networking on Topological Quantum Matter

● What is Topo-Q?

"Topo-Q" is a global network that aims to promote the research progress on topological quantum matter by strengthening international collaborations through various programs such as exchange of researchers and hiring postdocs.

● Who conduct Topo-Q? (See the world map on next page.)

It is an equal-partnership network jointly conducted by research projects such as

"Topological Materials Science (TMS, Japan)",
"CIFAR Quantum Materials (Canada)",
"Emergent Phenomena in Quantum Systems (EPiQS, Moore Foundation, USA)",

as well as by institutes such as

Max Planck Institutes, Stuttgart and Dresden (Germany)
Institute-Spin Salerno (Italy),
Shanghai Center for Complex Physics (SCCP, China).

Topo-Q International Network

Russia
[Superconducting TQP]
(Golubov)
2014-

Canada
[CIFAR-QM]
2013 - 2018
(Taillefer)

CIFAR
CANADIAN INSTITUTE FOR ADVANCED RESEARCH

Germany
[Koeln]
(Ando)

Germany
[MPIs]
(Takagi,
Mackenzie)

Topological Materials Science

Japan[TMS]
2015.7 - 2020.3

GORDON AND BETTY MOORE FOUNDATION

USA
[Moore foundation, EPiQS]
2014.11 - 2019.10

Italy
[CNR-SPIN]
(Vecchione,
Cuoco)
2011 -

China
[SCCP]
(Leggett, Liu)
2013 -

"Topo-Q"
Research Collaborations,
Alliance Workshops, etc.

Programs provided by TMS

Topo-Q Programs	Contents	Duration	Range of support	How to apply
JREP Junior Researcher Exchange Program	Invitation and dispatch of junior researchers (PDs and graduate students)	A few weeks to a few months	Travel and lodging	Submit application from a host or PI researcher who is one of the core or open-solicitation project members of TMS.
REP Researcher Exchange Program	Invitation and dispatch of staff researchers	Up to three months	Travel, lodging, and per diem	
PD	Hiring of PDs for international collaboration	Up to 24 months	Salary	

http://topo-mat-sci.jp/en/

- Alliance workshop (Topo-Q program)

  2016 (Sep.) MPI (Germany)

  2016 (Dec.) Tahiti (EPiQS, USA)

  2017 Feb. 13-18, Kyoto (Peking Univ., China)

  2017 Apr. 25-28, Dresden SPIN (Italy)

  etc.

- International conference

  2017 May 9-13, Tokyo

- Second annual meeting
  2016 Dec. 16-18, Sendai

- Interactive meeting
  2017 Jan. 6-7, Nagoya

(Second annual meeting and Interactive meeting: Domestic)

International school + workshop, 13-18 Feb 2017 at Yukawa Inst. for Theoretical Physics, Kyoto University

Invited Lecturers

- Haruki Watanabe (University)
- Xiong-Jun Liu (Peking University)
- Ryuichi Shindou (Peking University)
- Keisuke Totsuka (YITP, Kyoto University)

Organiziners

- Guo-qing Zheng (Okayama University)
- **Ippei Danshita** (YITP)
- Keisuke Totsuka (YITP)
- Ryuichi Shindou (Peking University)
- Masatoshi Sato (YITP)
- Jian Wei (Peking University)
- Yuan Li (Peking University)
- Chi Zhang (Peking University)

“Quantum Gravity, String Theory and Holography”

(Not TMS Project workshop)

Invited Speakers (tentative)
David Berenstein (University of California, Santa Barbara)
Pavel Buividovich (University of Regensburg)
Simon Catterall (Syracuse University)
Ippei Danshita (Kyoto University)
Jan de Boer (University of Amsterdam)
Frank Ferrari (Université libre de Bruxelles)*
Koji Hashimoto (Osaka University)
Michal Heller (Perimeter Institute)*
Shinji Hirano (University of the Witwatersrand)*
Dan Kabat (Lehman College, CUNY)
Yuji Okawa (University of Tokyo)
Hirosi Ooguri (Caltech/IPMU)
Kyriakos Papadodimas (CERN)*
Joao Penedones (École polytechnique fédérale de Lausanne)
Enrico Rinaldi (Brookhaven/RIKEN)
Dan Roberts (Institute for Advanced Study, Princeton)*
Paul Romatschke (University of Colorado, Boulder)
Andreas Schaefer (University of Regensburg)
Kostas Skenderis (University of Southampton)
Toby Wiseman (Imperial College)

Dates
3 April 2017 - 7 April 2017

Place
Panasonic Auditorium,
Yukawa Institute for Theoretical Physics

Organizers
Adam Brown (Stanford), Valentina (Humboldt U., Berlin), **Guy Gur-Ari** (Stanford), **Masanori Hanada** (YITP/Hakubi Center), Yasuaki Hikida (YITP), Goro Ishiki (Tsukuba), Jun Nishimura (KEK), Masaki Shigemori (QMUL/YITP), Tadashi Takayanagi (YITP)

Cherry Blossoms
In Kyoto, average first bloom is on March 28, average full bloom is on April 5.

Summary

- Real-time behavior of Majorana SYK (1611.04650)

  with Jordan Saul Cotler, Guy Gur-Ari, Masanori Hanada, Joseph Polchinski, Phil Saad, Stephen H. Shenker, Douglas Stanford (IAS), Alexandre Streicher

  - Partition function $Z(\beta, t)$: dip-ramp-plateau structure
  - Correlation function: similar structure for $N \equiv 2 \pmod 8$

- Proposal for experiment of Dirac SYK (1606.02454)

  with Ippei Danshita (YITP), Masanori Hanada (YITP & Stanford)

  - Real-coupling Dirac SYK
  - Designed optical lattice + photoassociation laser coupling two fermions to molecular states
  - Finite low-temperature entropy
  - Out-of-time order correlation function (OTOC) can be measured using a controlling qubit

  ➔ "Experimental quantum gravity"