Interesting Dynamics Interplay with Symmetry, Topology and Entropy

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Complex Dynamics

Simple Dynamics

Simple Dynamics

Fundamental and Universal

Simple Dynamics

Fundamental and Universal

Symmetry, Topology and Entropy

Symmetry

Scale Invariance

$$i\hbar\frac{\partial}{\partial t}\Psi = -\sum_i \frac{\hbar^2}{2m}\nabla_i^2\Psi$$

Scale Transformation

$$\mathbf{r}_i \longrightarrow \Lambda\mathbf{r}_i$$
$$t \longrightarrow \Lambda^2 t$$

$$\frac{1}{\Lambda^2} \qquad\qquad \frac{1}{\Lambda^2}$$

No other energy scale except for the Fermi energy

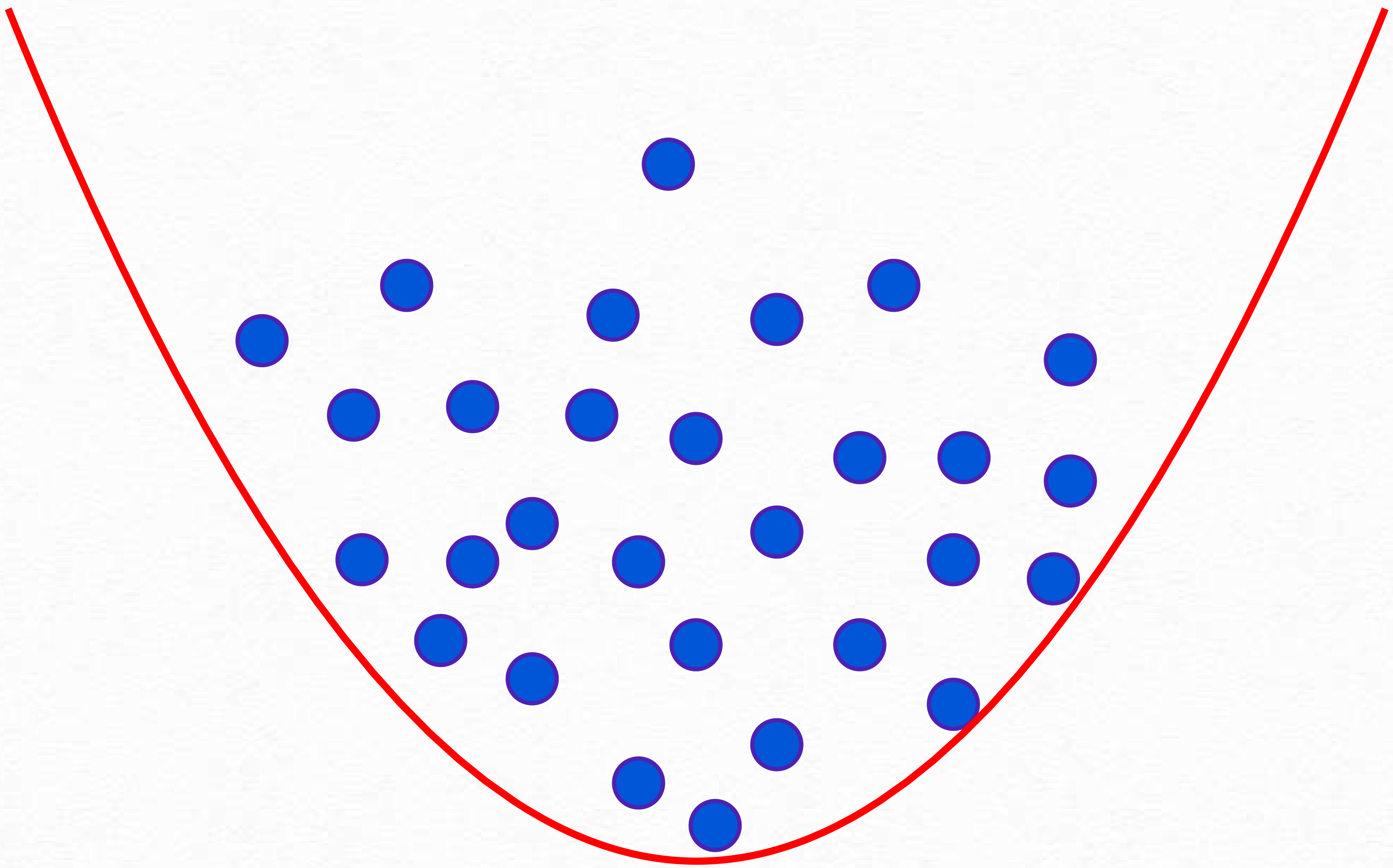
Zoo of Scale Invariant Quantum Gases

Non-interacting bosons/ fermions at any dimension	No other length scale except for density
Unitary Fermi gas at three dimension	Density and a_s $a_s = \infty$
Tonks gas of bosons/ fermions at one dimension	Density and g_{1D} $g_{1D} = \infty$

Universal behavior:

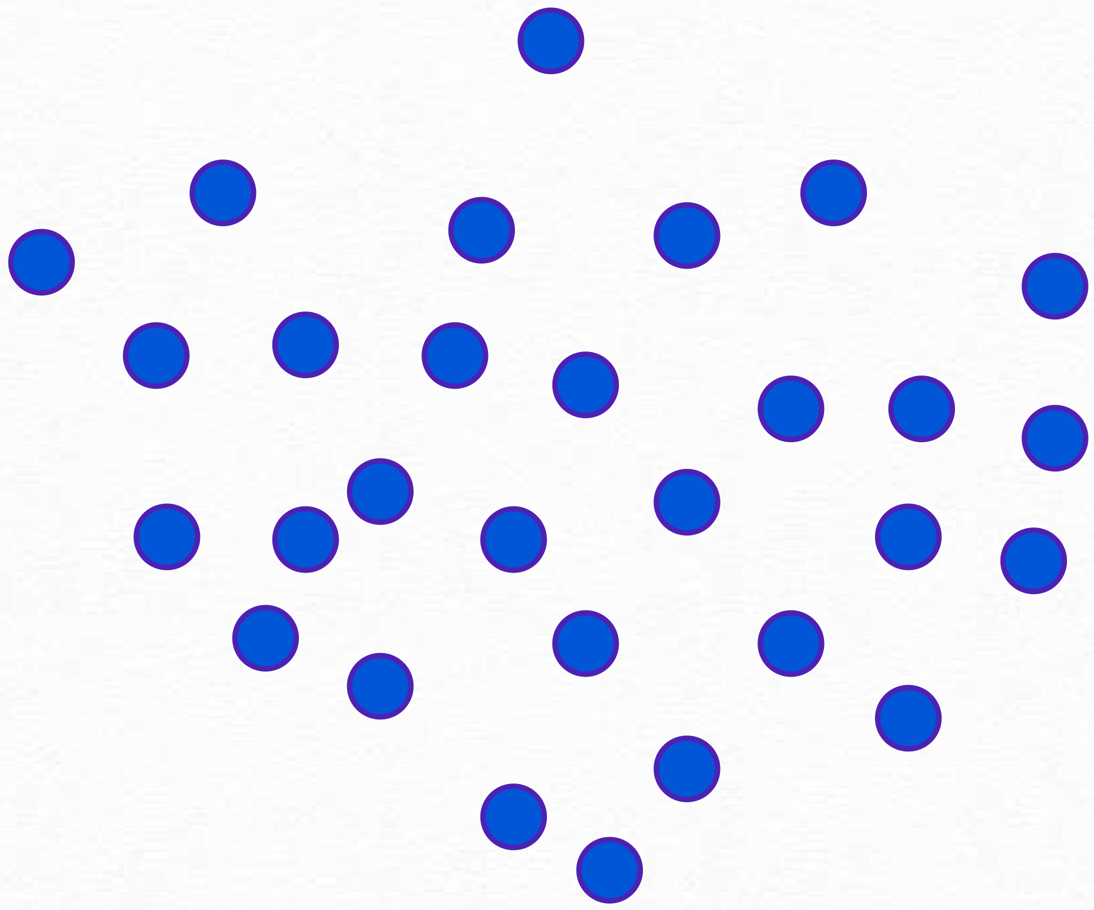
$$\langle V \rangle = \alpha \langle T \rangle$$

Expansion Experiment with Cold Atoms



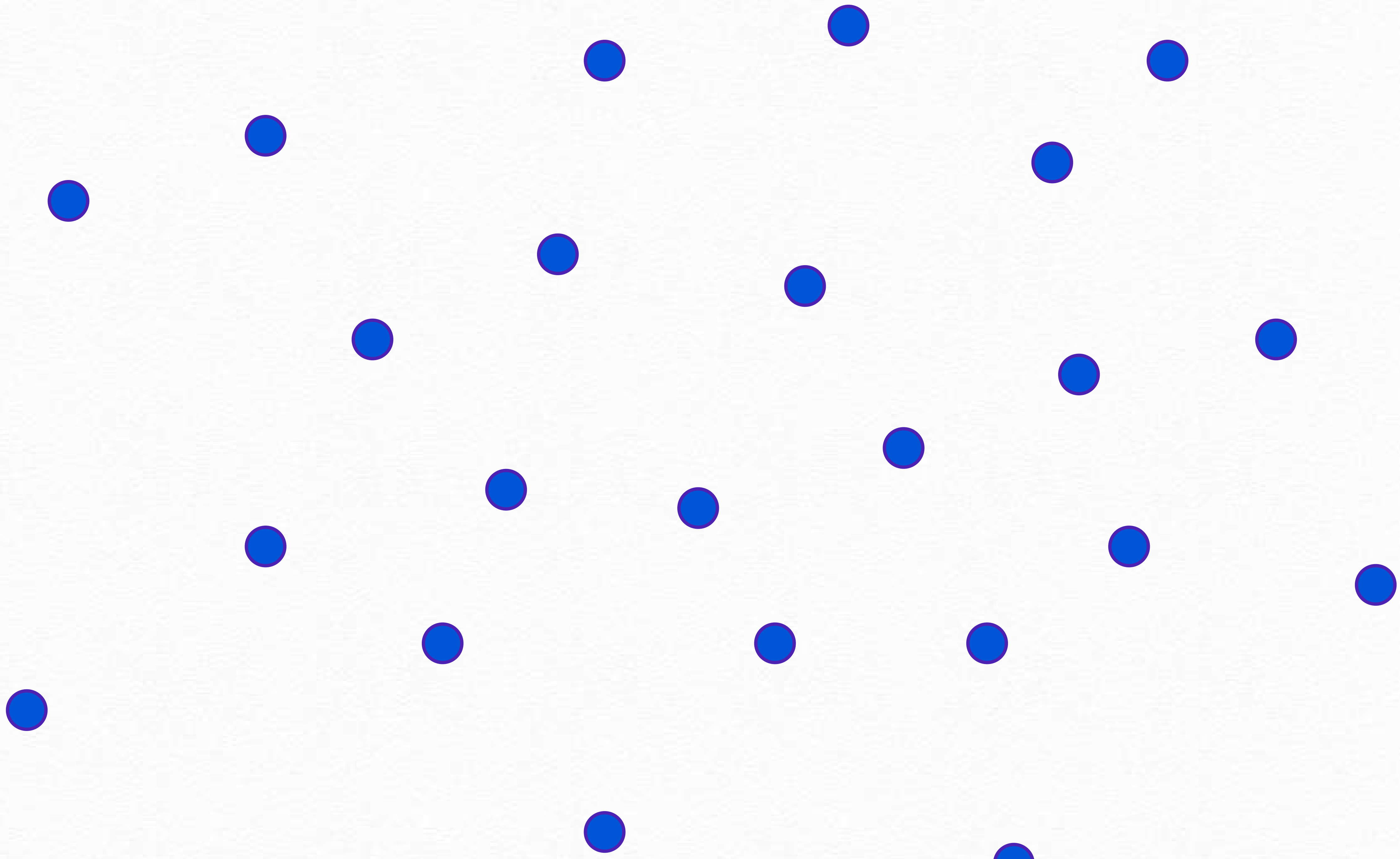
Gas hold in a trap

Expansion Experiment with Cold Atoms



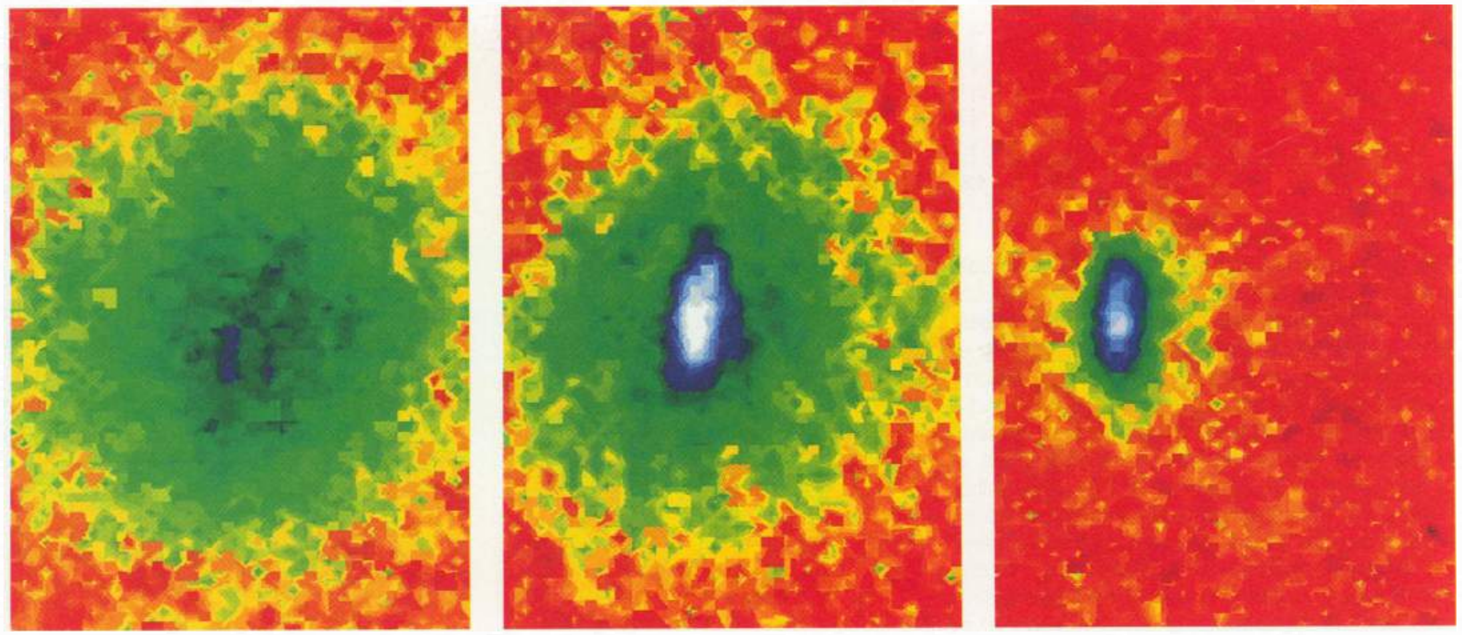
Turn off the trap

Expansion Experiment with Cold Atoms

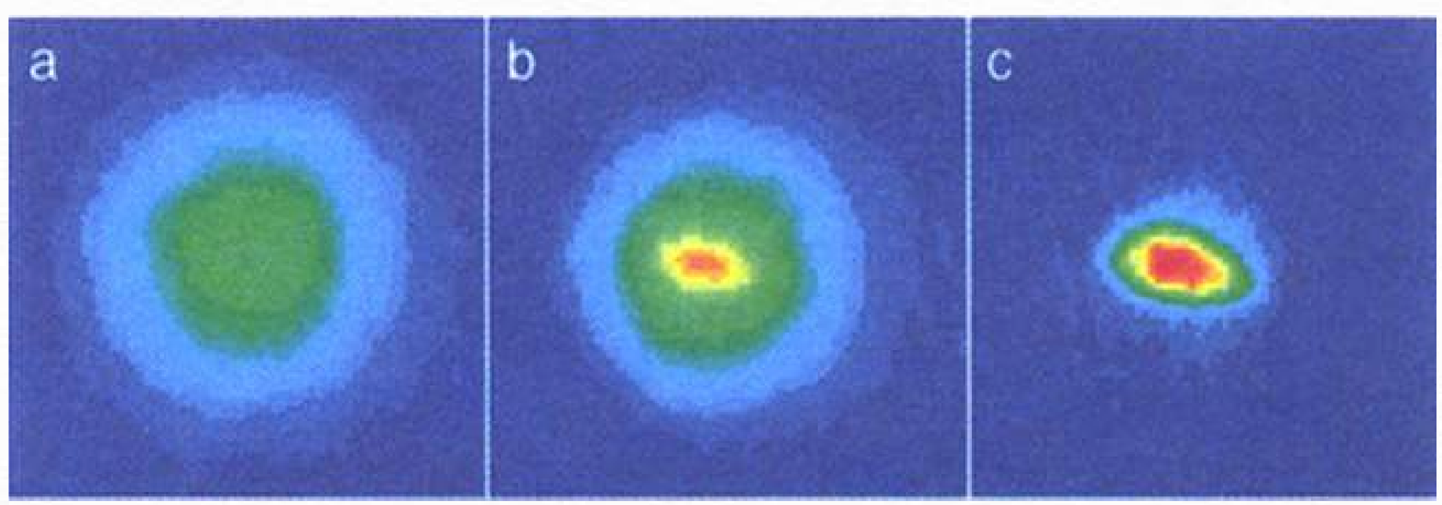


Expansion

The First BEC Experiments

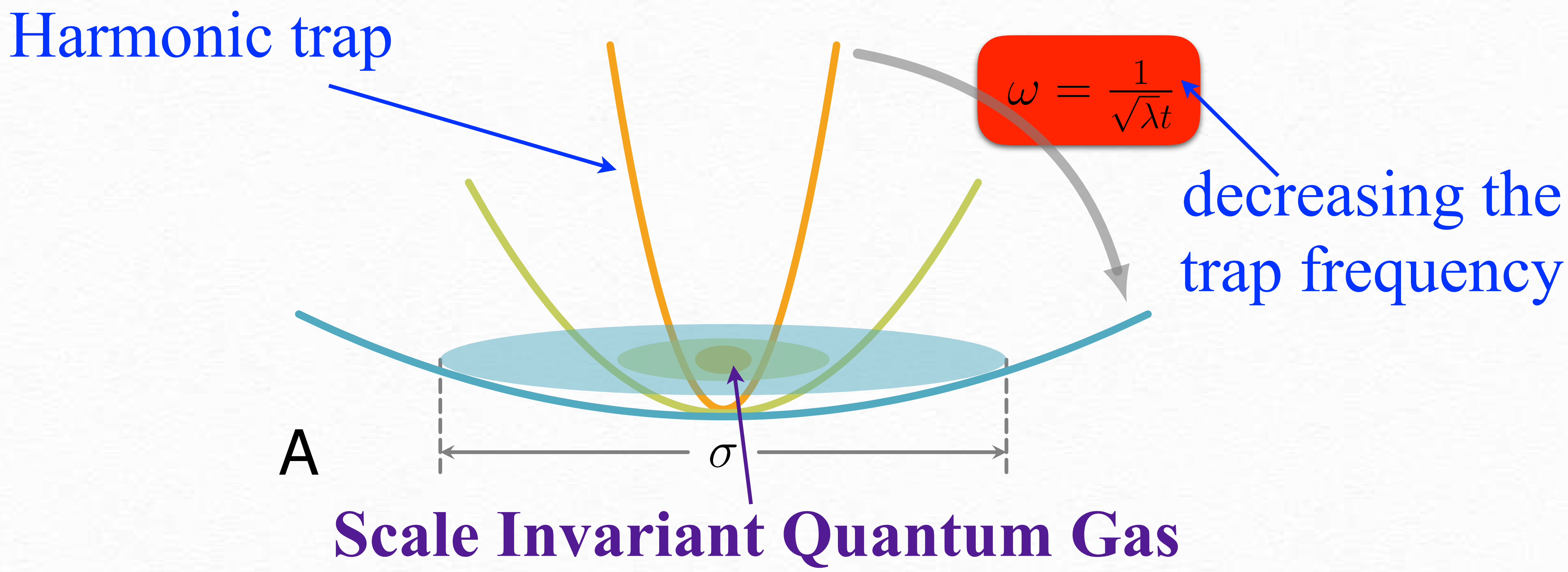


JILA Group

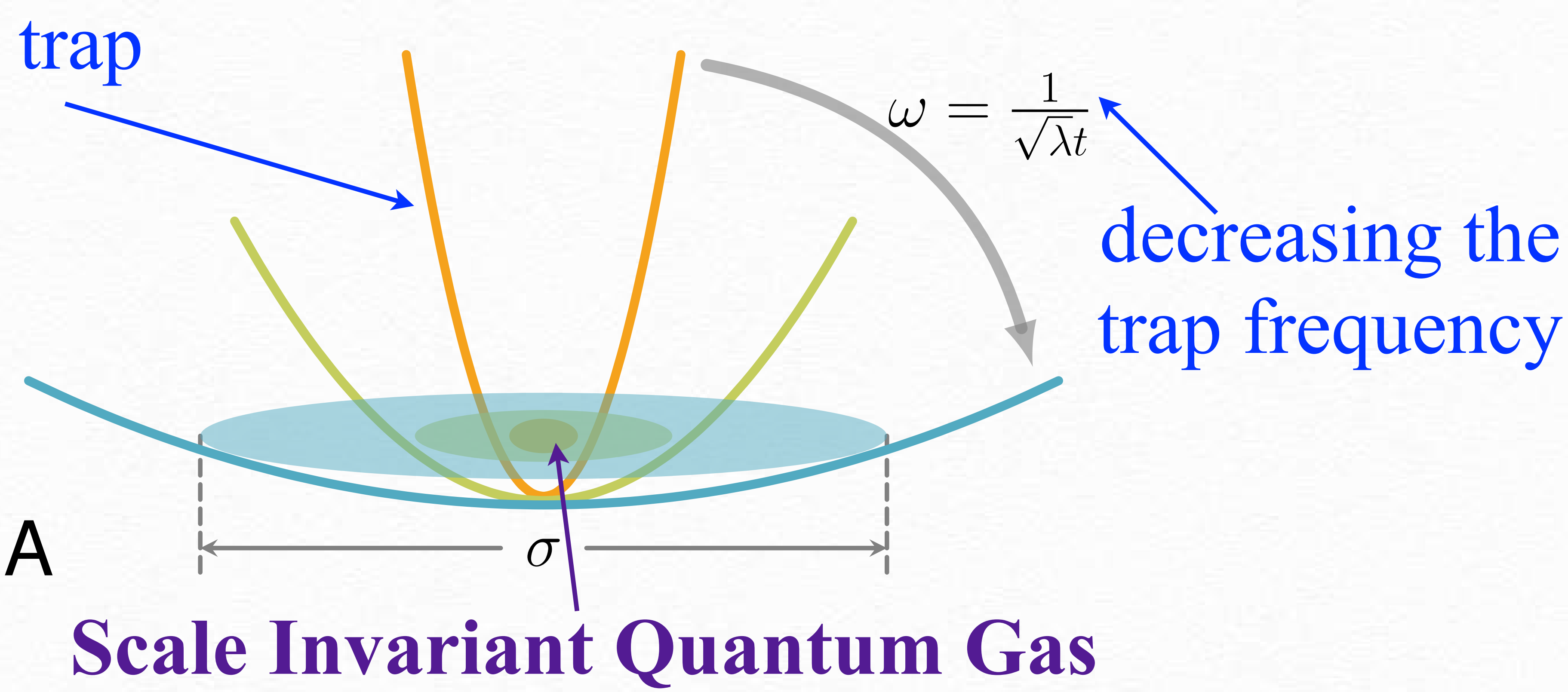


MIT Group

Our Proposal



Our Proposal

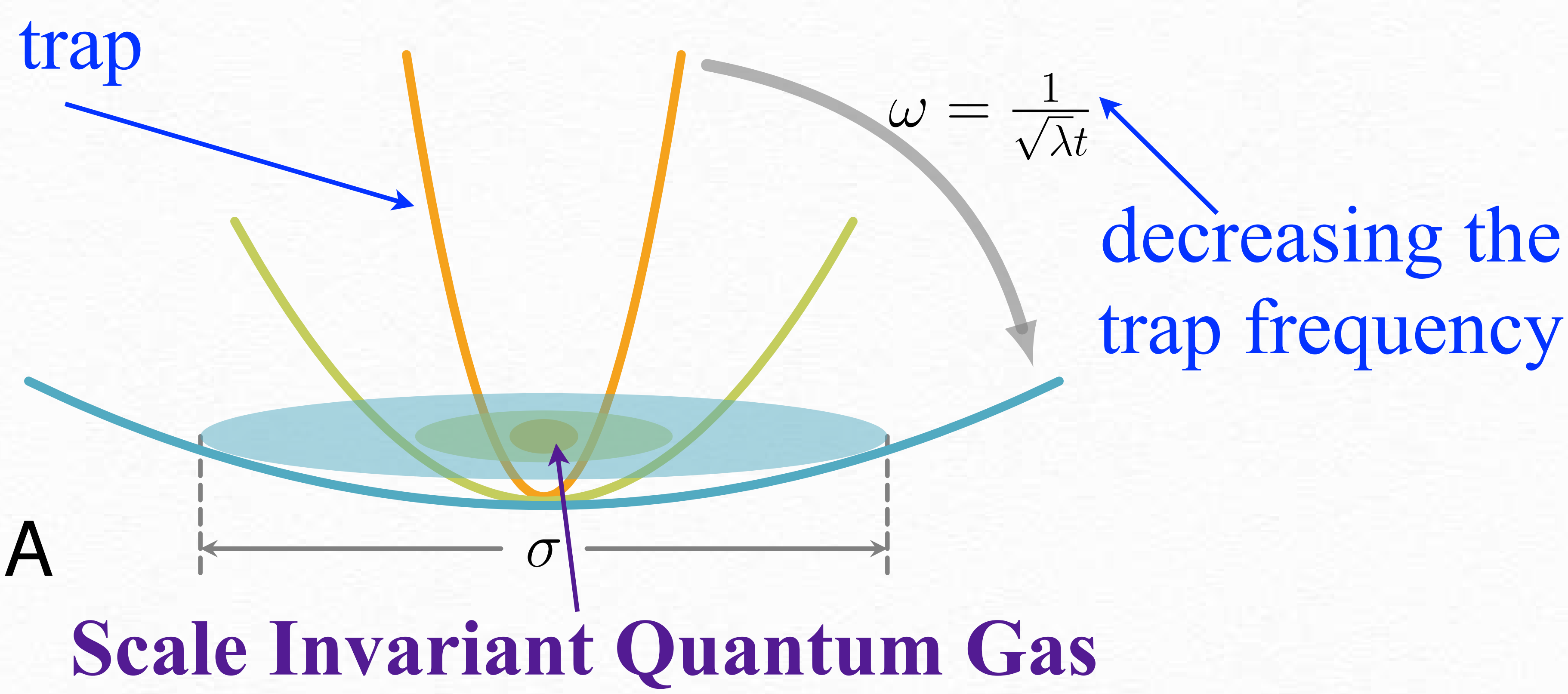


$$R = \sqrt{\langle \sum r_i^2 \rangle}$$ A

Harmonic length: $$a = \sqrt{\frac{\hbar}{m\omega}}$$

By dimension analysis: $$\mathcal{R} \sim \sqrt{t}$$

Our Proposal



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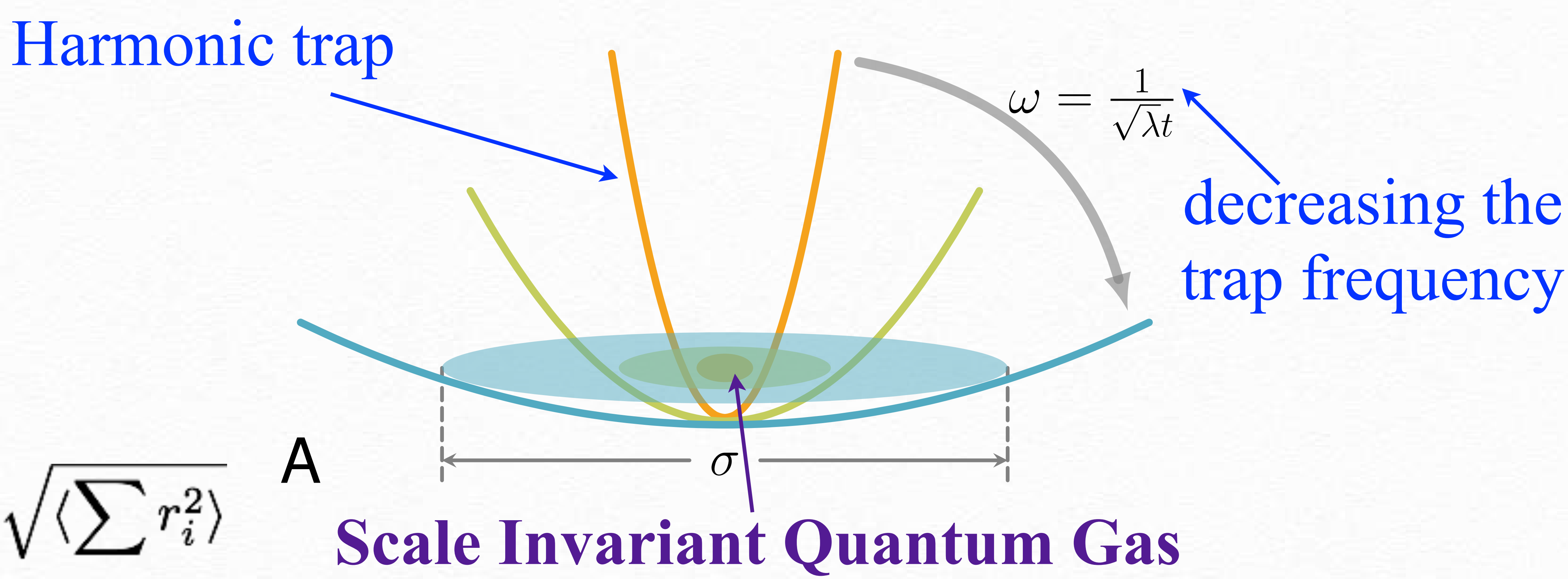
Our Proposal



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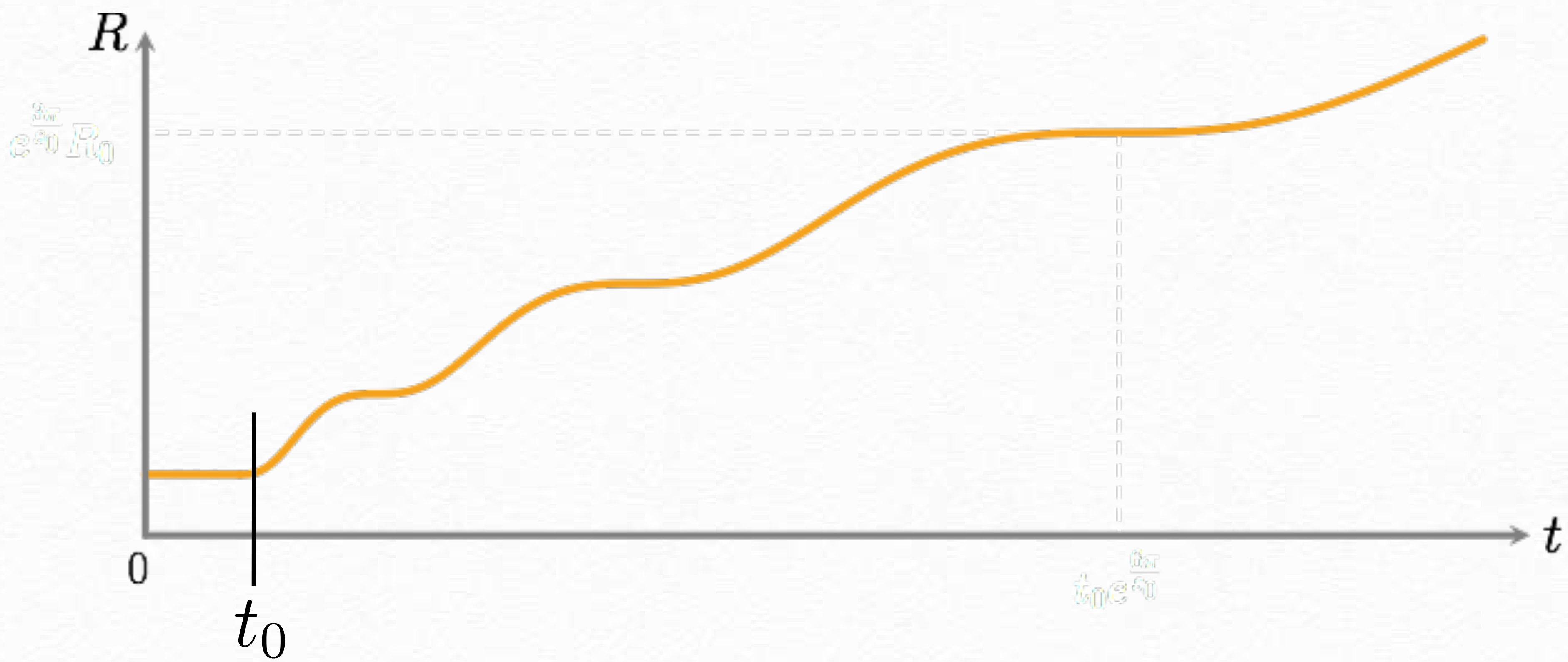
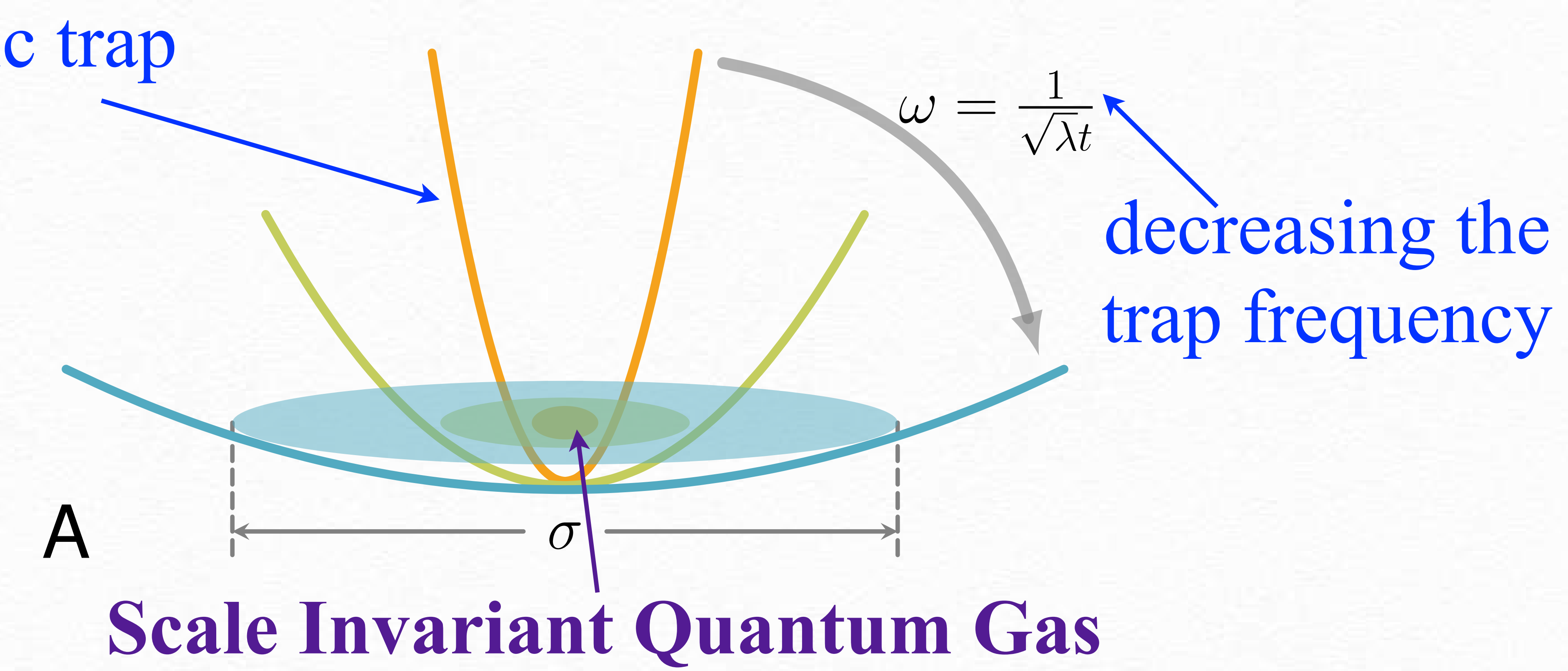


Our Proposal



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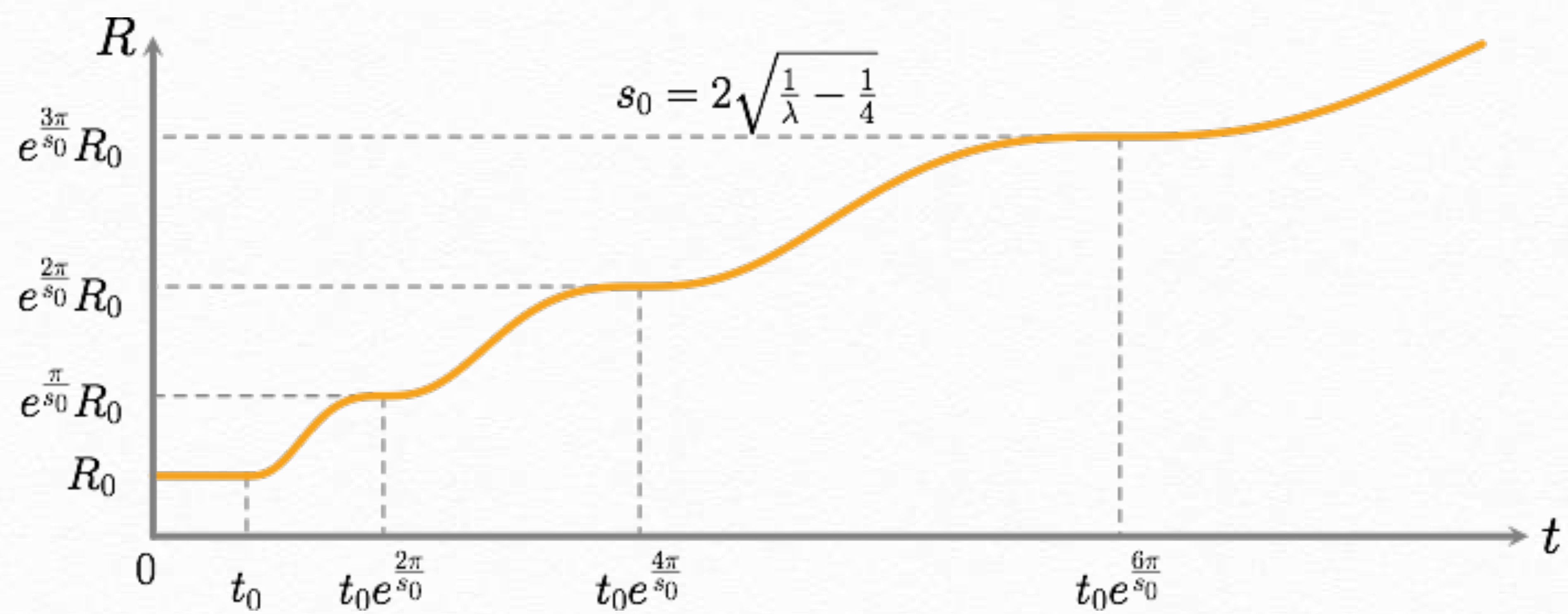

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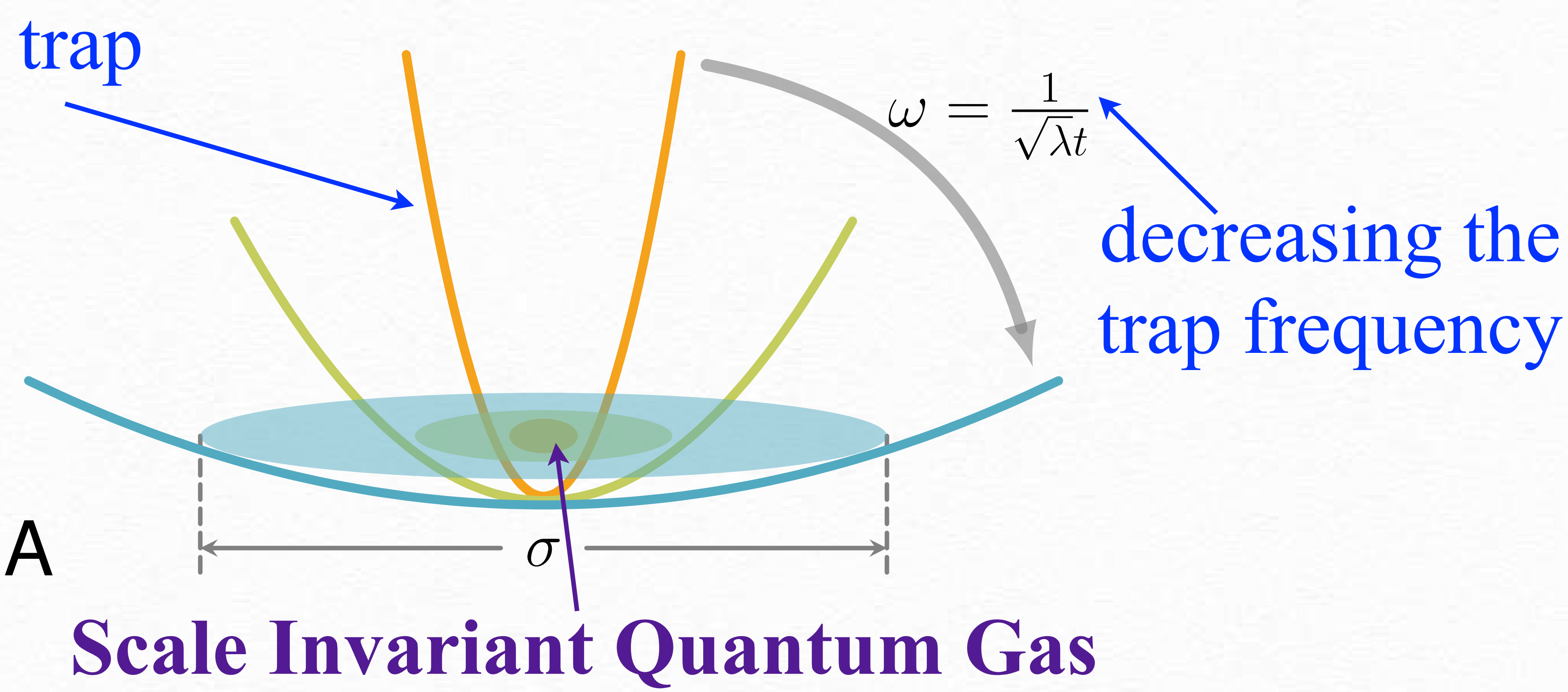

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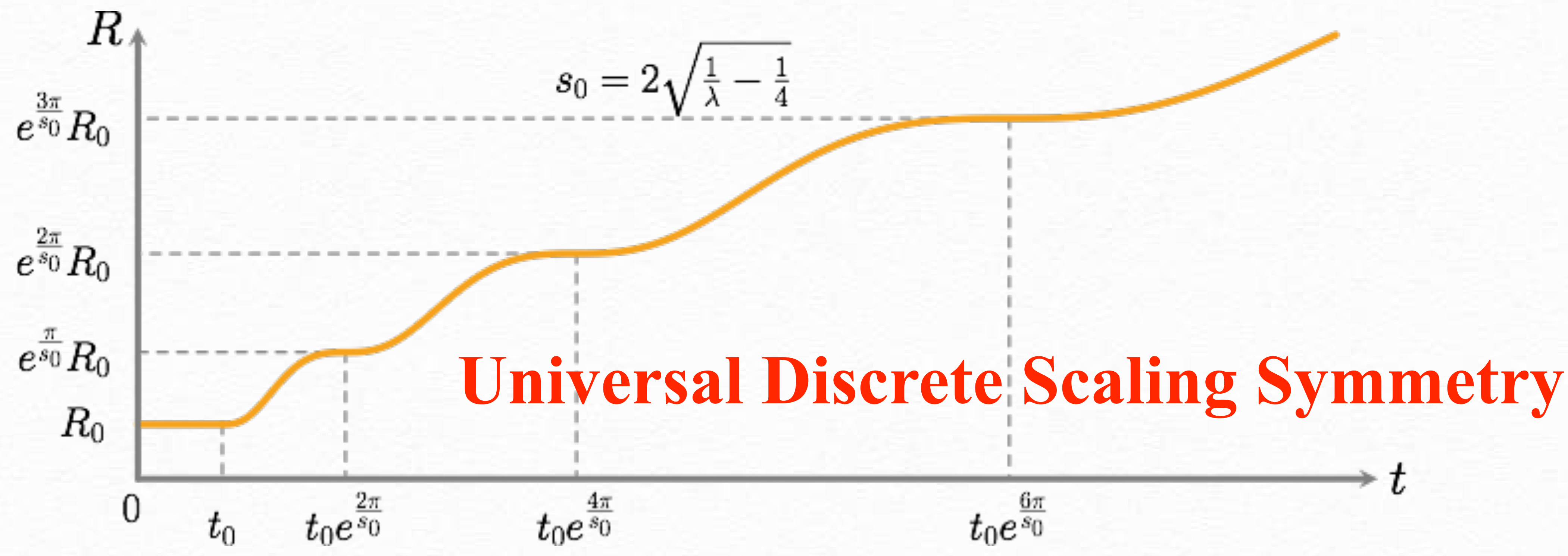
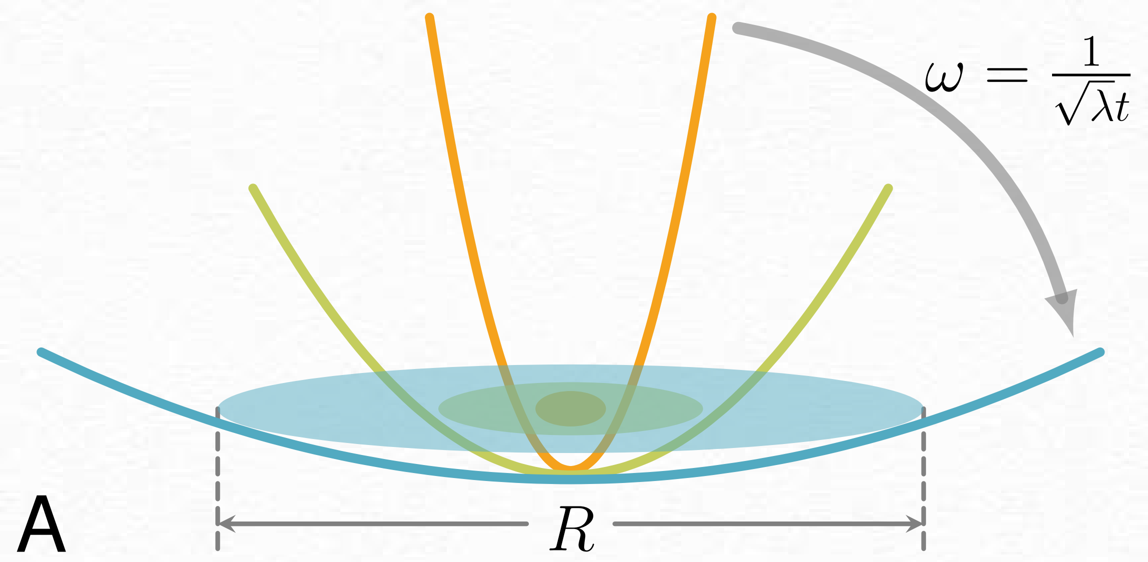


Significance

- Universal
    - Independent of Temperature
    - Independent of State of Matter
    - Independent Dimension
- " Hidden Symmetry "

Scaling Symmetry in a Harmonic Trap



$$i\hbar\frac{\partial}{\partial t}\Psi = \left[H + \sum_i \frac{1}{2} m\omega^2 r_i^2\right]\Psi$$

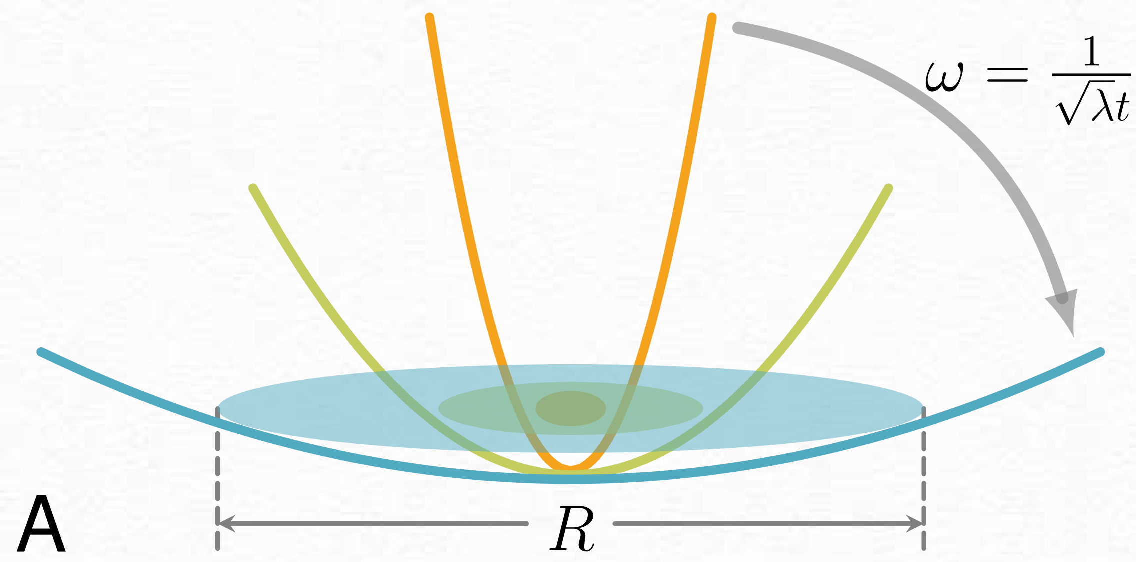
Scale Transformation

$$\mathbf{r}_i \to \Lambda \mathbf{r}_i$$

$$t \to \Lambda^2 t$$

$$\frac{1}{\Lambda^2} \qquad \frac{1}{\Lambda^2} \qquad \frac{1}{\Lambda^2} \ ?$$

Scaling Symmetry in a Harmonic Trap



$$i\hbar\frac{\partial}{\partial t}\Psi = \left[H + \sum_i \frac{1}{2}m\omega^2 r_i^2\right]\Psi$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$\frac{1}{\Lambda^2} \qquad \frac{1}{\Lambda^2} \qquad \frac{1}{\Lambda^2}$$

Scale Transformation

$$\mathbf{r}_i \longrightarrow \Lambda\mathbf{r}_i$$
$$t \longrightarrow \Lambda^2 t$$

This scaling symmetry exists only if

$$\omega = \frac{1}{\sqrt{\lambda}t}$$

Expansion Dynamics

$$i\frac{d}{dt}R^2 = \sum_i \langle [r_i^2, H] \rangle = 2i\langle \hat{D} \rangle$$

$\frac{1}{2}\sum_i (\mathbf{r}_i \cdot \mathbf{p}_i + \mathbf{p}_i \cdot \mathbf{r}_i)$

Generator of spatial scaling transformation

Expansion Dynamics

$$i\frac{d}{dt}R^2 = \sum_i \langle [r_i^2, H] \rangle = 2i\langle \hat{D} \rangle$$

$$i\frac{d}{dt}\langle \hat{D} \rangle = \langle [\hat{D}, H] \rangle = 2i\Big(\langle H \rangle - \omega^2 R^2 \Big)$$

$$\frac{d}{dt}\langle H \rangle = \langle \frac{\partial}{\partial t} H \rangle = \omega\dot{\omega}R^2$$

Expansion Dynamics

$$i\frac{d}{dt}R^2 = \sum_i \langle [r_i^2, H] \rangle = 2i\langle \hat{D} \rangle$$

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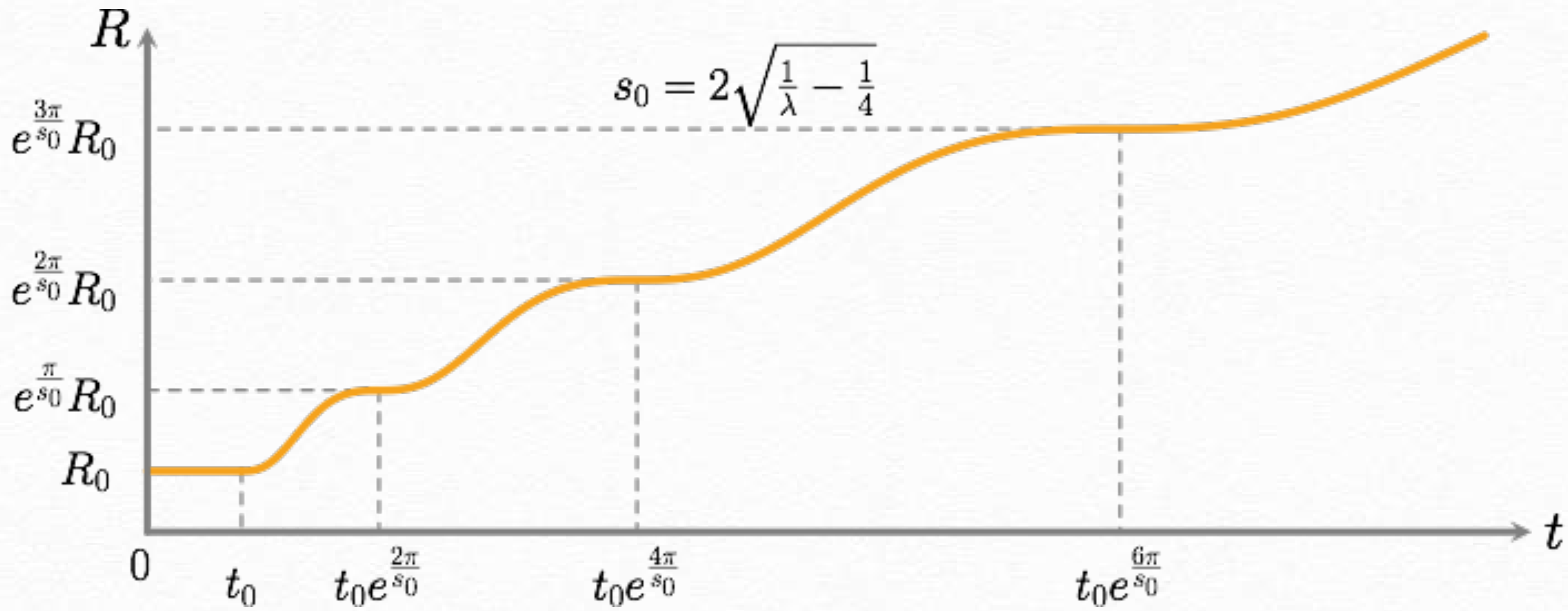
$$\frac{d}{dt}\langle H \rangle = \langle \frac{\partial}{\partial t} H \rangle = \omega\dot{\omega}R^2$$

$$\frac{d^3}{dt^3}R^2 + 4\omega^2\frac{d}{dt}R^2 + 4\omega\dot{\omega}R^2 = 0$$

Expansion Dynamics

$$\lambda < 4$$

$$\frac{\langle \hat{R}^2 \rangle (t)}{R_0^2} = \frac{t}{t_0} \frac{1}{\sin^2 \varphi} \left[1 - \cos\varphi \cdot \cos\left(s_0 \ln \frac{t}{t_0} + \varphi \right) \right]$$



Why plateaus ?

$$\frac{d^n}{dt^n} \langle \hat{R}^2 \rangle |_{t=t_0} = 0$$

Expansion Dynamics

$$i\frac{d}{dt}R^2 = \sum_i \langle [r_i^2, H] \rangle = 2i\langle \hat{D} \rangle$$

$$i\frac{d}{dt}\langle \hat{D} \rangle = \langle [\hat{D}, H] \rangle = 2i\Big(\langle H \rangle - \omega^2 R^2 \Big)$$

$$\frac{d}{dt}\langle H \rangle = \langle \frac{\partial}{\partial t} H \rangle = \omega\dot{\omega}R^2$$

$$\frac{d^3}{dt^3}R^2 + 4\omega^2\frac{d}{dt}R^2 + 4\omega\dot{\omega}R^2 = 0$$

Why the equation-of-motion closes ? An incident ?

Remark I: Emergent Conformal Symmetry

The Schrodinger Group Symmetry

temporal translation: $H = -i\partial_t$

spatial translation: $P^i = -i\partial_i$

spatial rotation: $M^{ij} = ix_i\partial_j - ix_j\partial_i,$

Galilean boost: $K^i = -it\partial_i - mx_i$

dilation: $D = -2it\partial_t - ix_i\partial_i - i\frac{d}{2}$

special Schrodinger transformation: $C = -it^2\partial_t - itx_i\partial_i - \frac{1}{2}m\vec{x}^2 - i\frac{d}{2}t.$

Generalization to relativistic case to probe conformal symmetry

Remark II: Connection to the Efimov Effect

$$\frac{d^3}{dt^3}R^2 + 4\omega^2\frac{d}{dt}R^2 + 4\omega\dot{\omega}R^2 = 0$$

Theorem: **The general solution can be constructed as**

$$R^2(t) = C_1 x_1^2 + C_2 x_1 x_2 + C_3 x_2^2$$

$$\ddot{x} + \omega^2(t)x = 0$$

Remark II: Connection to the Efimov Effect

$$\frac{d^3}{dt^3}R^2 + 4\omega^2\frac{d}{dt}R^2 + 4\omega\dot{\omega}R^2 = 0$$

Theorem: **The general solution can be constructed as**

$$R^2(t) = C_1 x_1^2 + C_2 x_1 x_2 + C_3 x_2^2$$

$$\omega = \frac{1}{\sqrt{\lambda} t}$$

$$\ddot{x} + \frac{1}{\lambda t^2}x = 0$$

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Equivalence:

$$x \leftrightarrow \psi \qquad t \leftrightarrow \rho$$

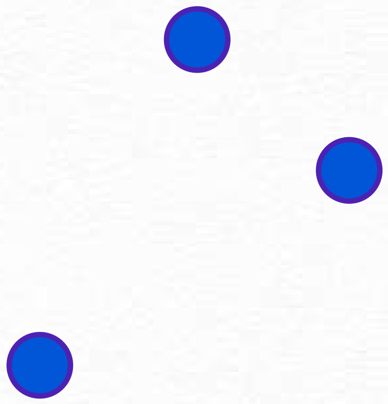
Efimov effect:

$$\left[-\frac{\hbar^2 \mathrm{d}^2}{2m\mathrm{d}\rho^2} - \frac{s_0^2 + 1/4}{m\rho^2}\right]\psi = E\psi$$

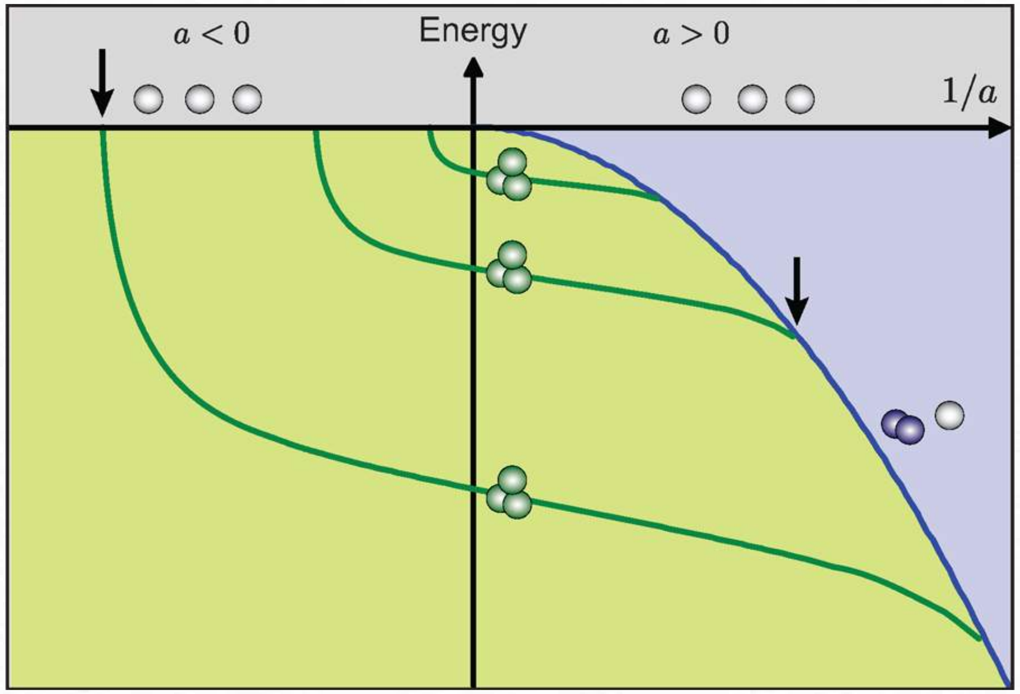
$$\psi = \sqrt{\rho}\cos[s_0 \log(\rho/\rho_0)]$$

The Efimov Effect

Problem: Three bosons interacting through a short-range interaction



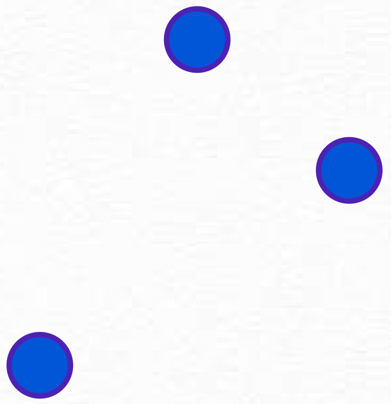
1970



Universal Discrete Scaling Symmetry

The Efimov Effect

Problem: Three bosons interacting through a short-range interaction



1970

$$\left[-\frac{\hbar^2 \mathrm{d}^2}{2m\mathrm{d}\rho^2} - \frac{s_0^2 + 1/4}{m\rho^2}\right]\psi = E\psi$$

$$\psi = \sqrt{\rho}\cos[s_0 \log(\rho/\rho_0)]$$

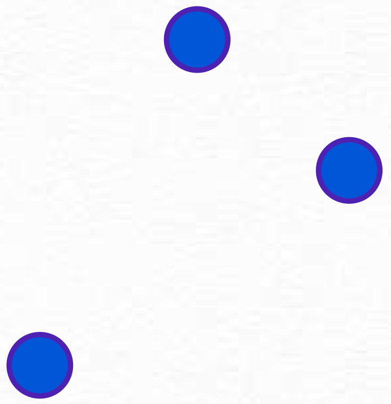
$$\rho \longrightarrow e^{2\pi/s_0}\rho$$

$$E_{\mathrm{T}}^{(n+1)}/E_{\mathrm{T}}^{(n)} \simeq e^{-2\pi/s_0}$$

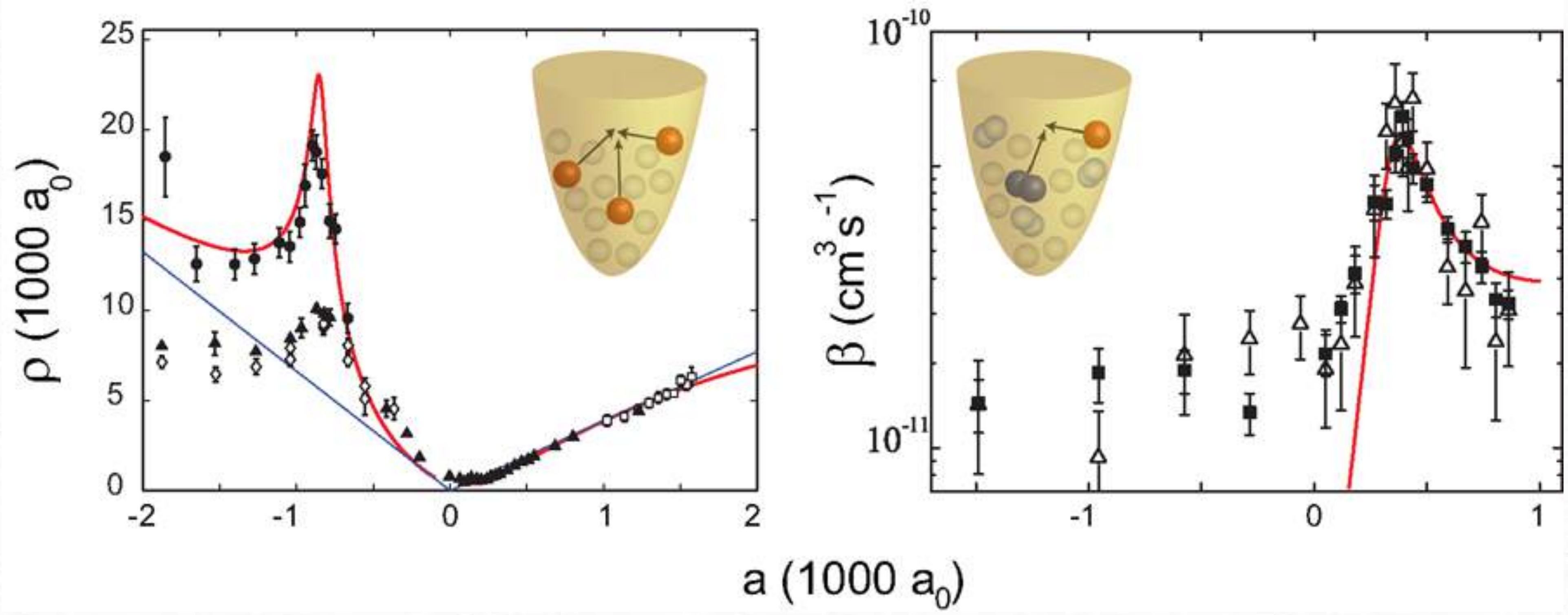
Discrete Scaling Symmetry

The Efimov Effect

Problem: Three bosons interacting through a short-range interaction



1970



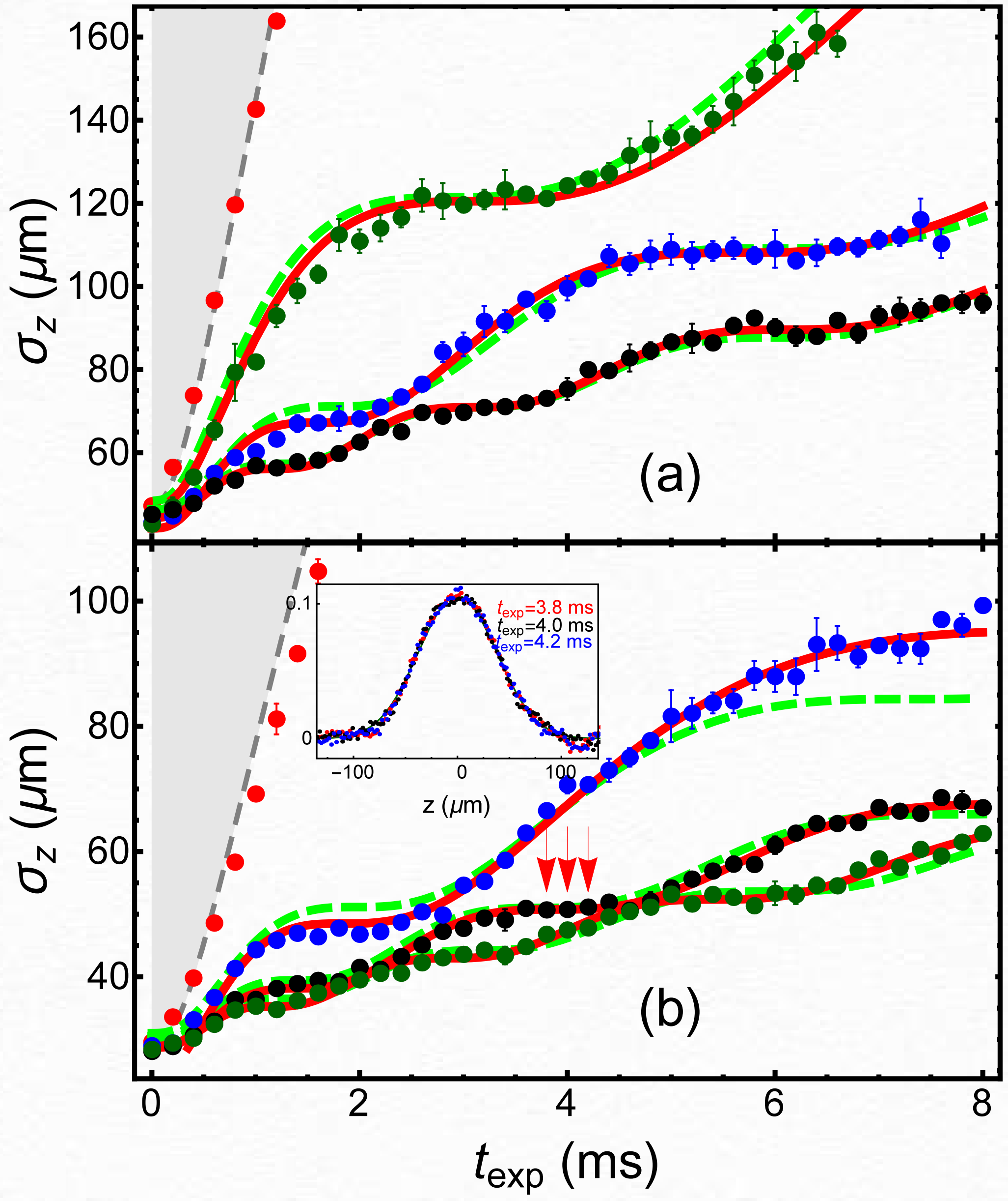
Innsbruck 2005, and many later

Experimental Observation

The Efimov Effect	The "Efimovian" Expansion
$-\frac{\hbar^2 d^2}{2md^2\rho}\psi - \frac{\lambda}{\rho^2}\psi = E\psi$	$\frac{d^3}{dt^3}\langle\hat{R}^2\rangle + \frac{4}{\lambda t^2}\frac{d}{dt}\langle\hat{R}^2\rangle - \frac{4}{\lambda t^3}\langle\hat{R}^2\rangle = 0.$
Spatial continuous scaling symmetry	Temporal continuous scaling symmetry
Short-range boundary condition	Initial time
$\psi = \sqrt{\rho}\cos[s_0\log(\rho/\rho_0)]$	$\frac{\langle\hat{R}^2\rangle(t)}{R_0^2} = \frac{t}{t_0}\frac{1}{\sin^2\varphi}\left[1-\cos\varphi\cdot\cos\left(s_0\ln\frac{t}{t_0}+\varphi\right)\right]$
Spatial discrete scaling symmetry $\rho \to e^{2\pi/s_0}\rho$	Temporal discrete scaling symmetry $t \to e^{2\pi/s_0}t$

Experimental Observation

by Haibin Wu in East China Normal University



Non-interacting

Unitary Fermions

$$\frac{\langle \hat{R}^2 \rangle(t)}{R_0^2} = \frac{t}{t_0} \frac{1}{\sin^2 \varphi} \left[1 - \cos\varphi \cdot \cos\left(s_0 \ln \frac{t}{t_0} + \varphi \right) \right]$$

Experimental Observation

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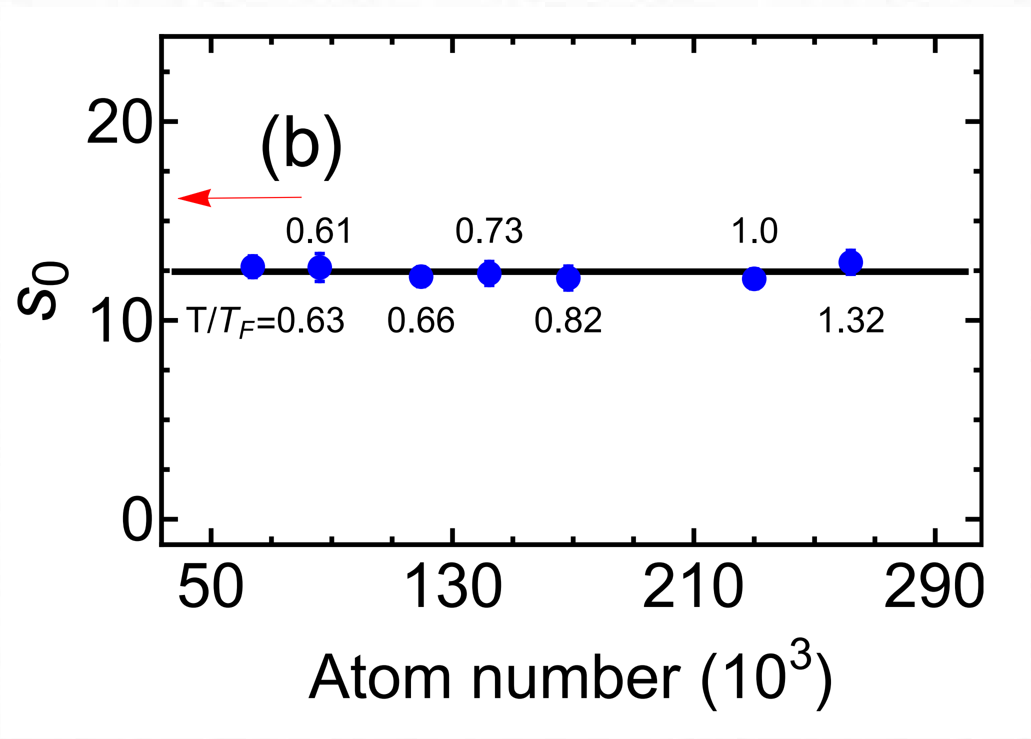

Independent of Temperature

Independent of State of Matter

Experimental Observation

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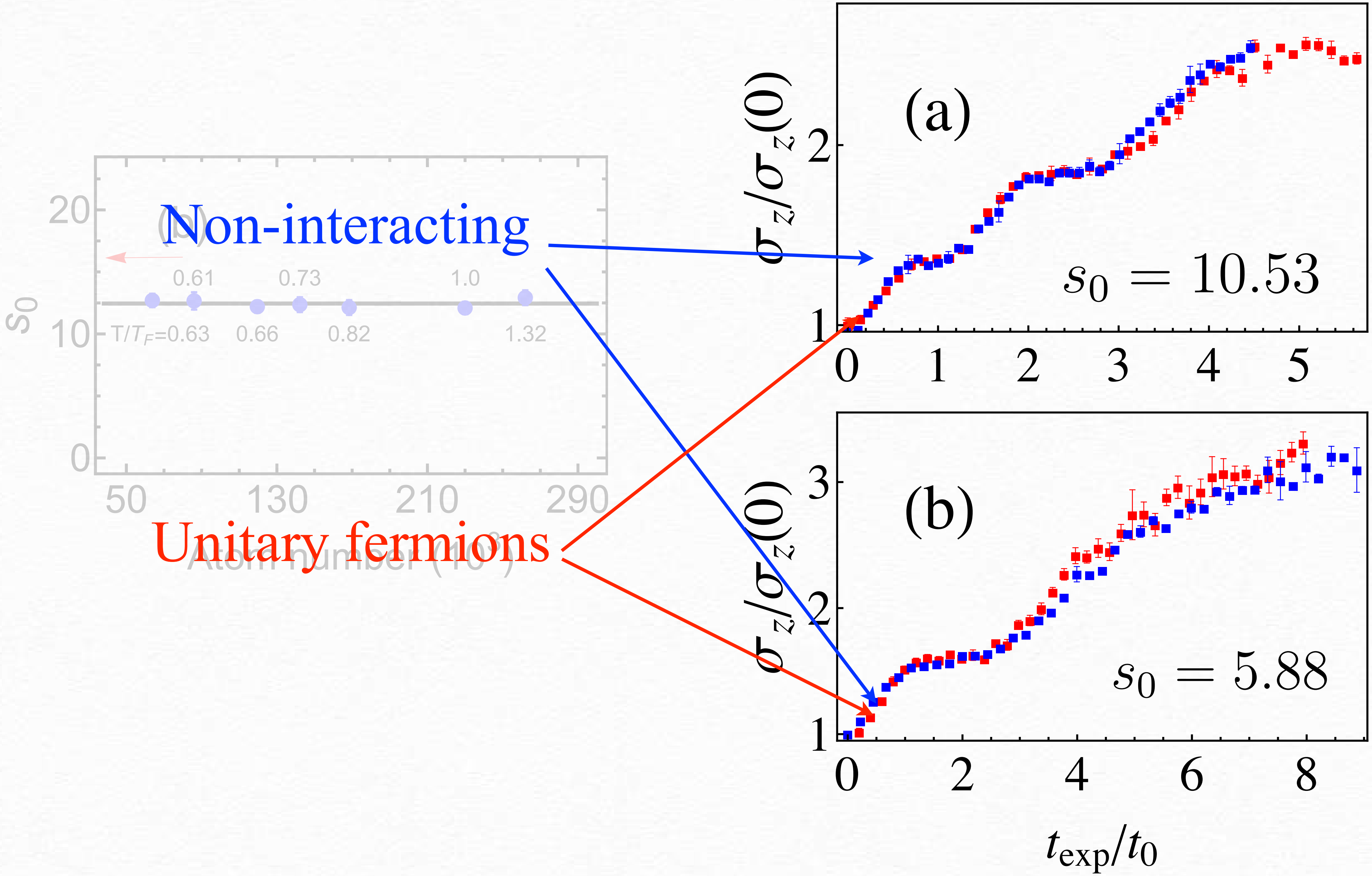
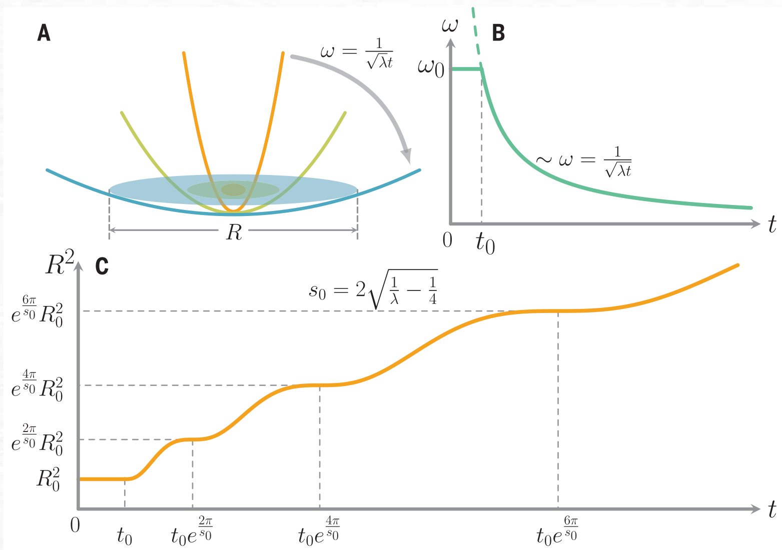


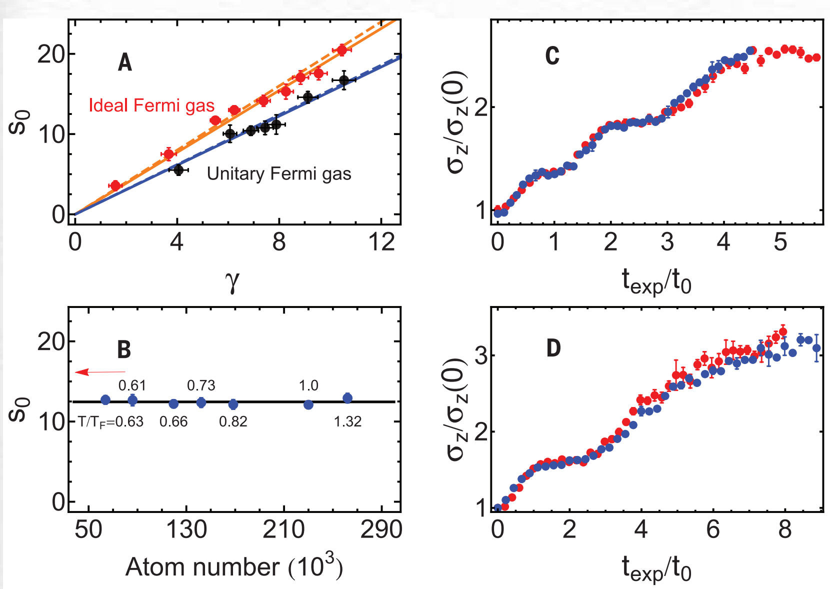
Independent of Temperature **Independent of State of Matter**

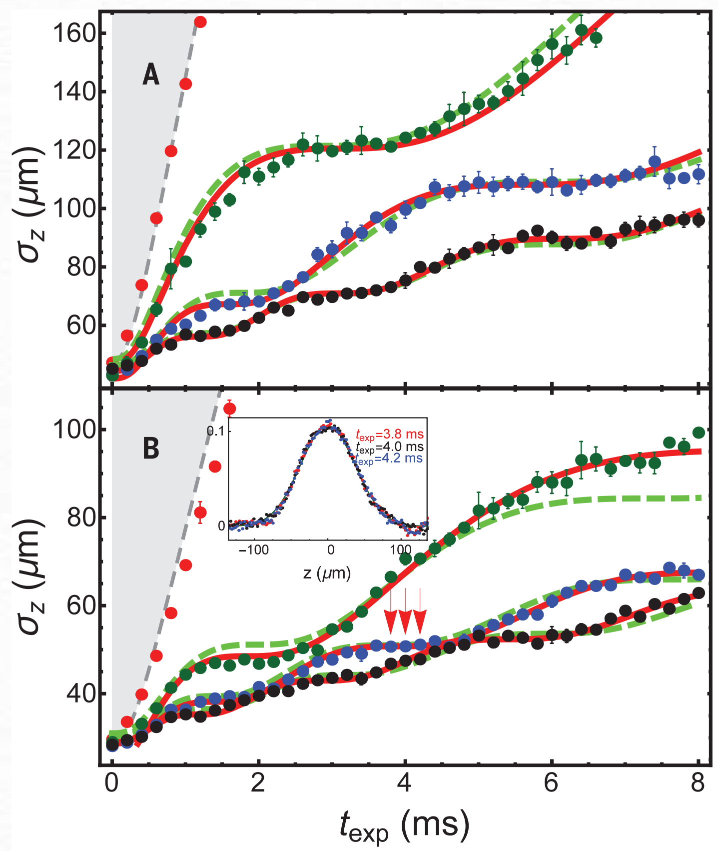
QUANTUM GASES

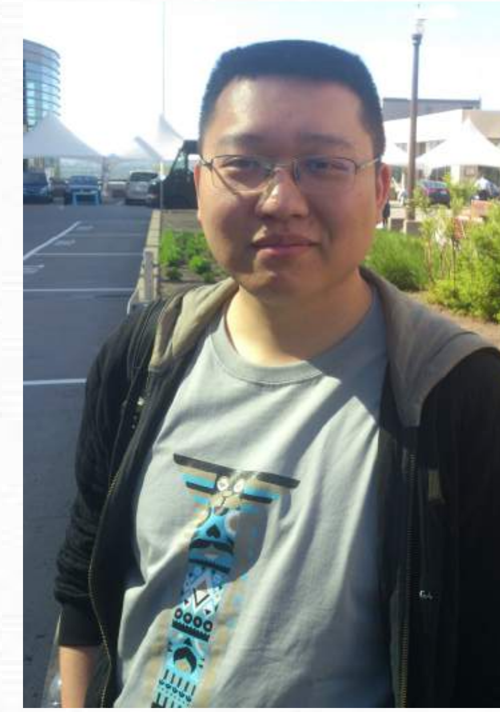
Observation of the Efimovian expansion in scale-invariant Fermi gases

Shujin Deng,[1*] Zhe-Yu Shi,[2*] Pengpeng Diao,[1] Qianli Yu,[1] Hui Zhai,[2] Ran Qi,[3†] Haibin Wu[1,4†]









Dr. Zheyu Shi

Prof. Ran Qi at Renmin University

Science, 371, 353 (2016)

Haibin Wu'g group in ECNU

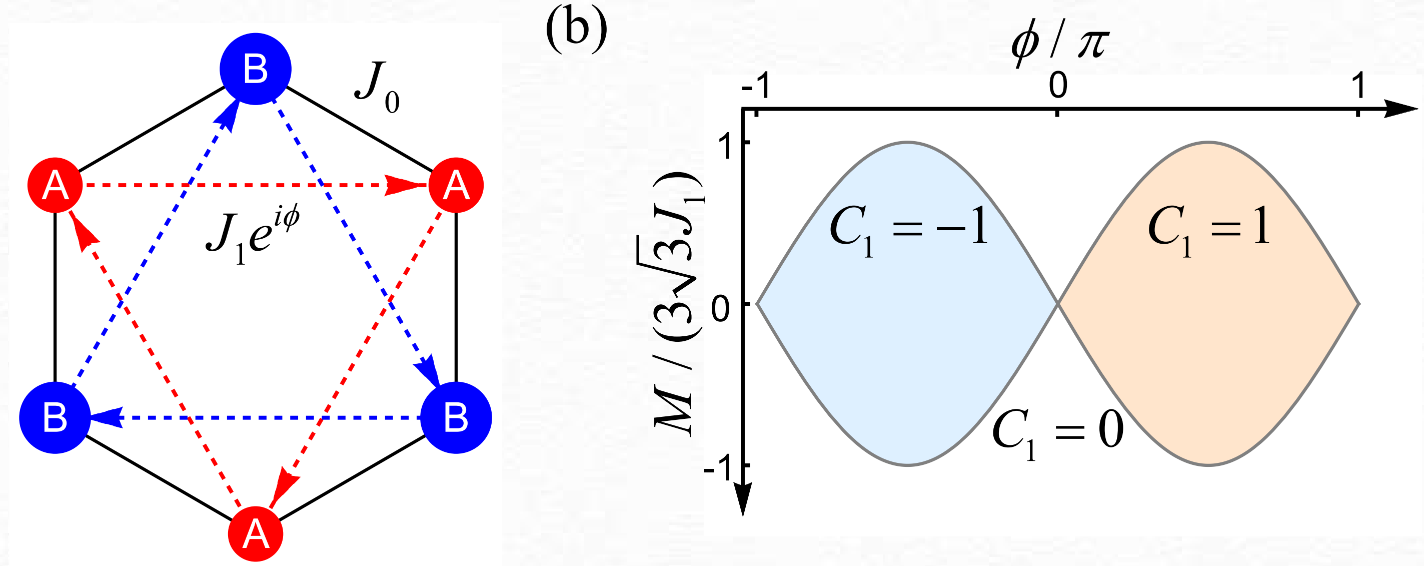
Topology

The Haldane Model

A two-band Chern Insulator $\mathcal{H}(\mathbf{k}) = \frac{1}{2}\mathbf{h}(\mathbf{k}) \cdot \boldsymbol{\sigma}$

For instance,
the Haldane model:

(a) (b)



How do I know the Chern Number of the Hamiltonian ?
At equilibrium:

The Haldane Model

A two-band Chern Insulator $$\mathcal{H}(\mathbf{k}) = \frac{1}{2}\mathbf{h}(\mathbf{k}) \cdot \boldsymbol{\sigma}$$

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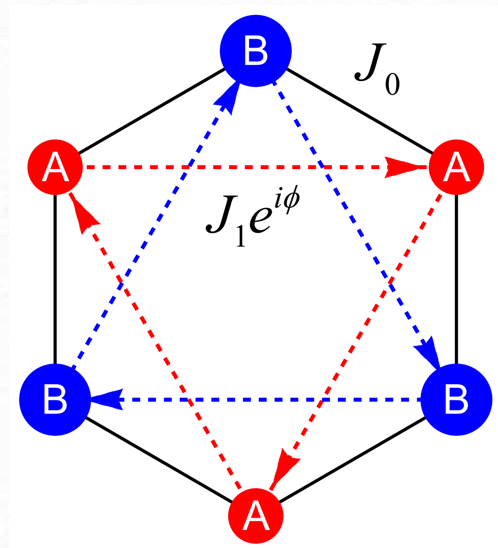
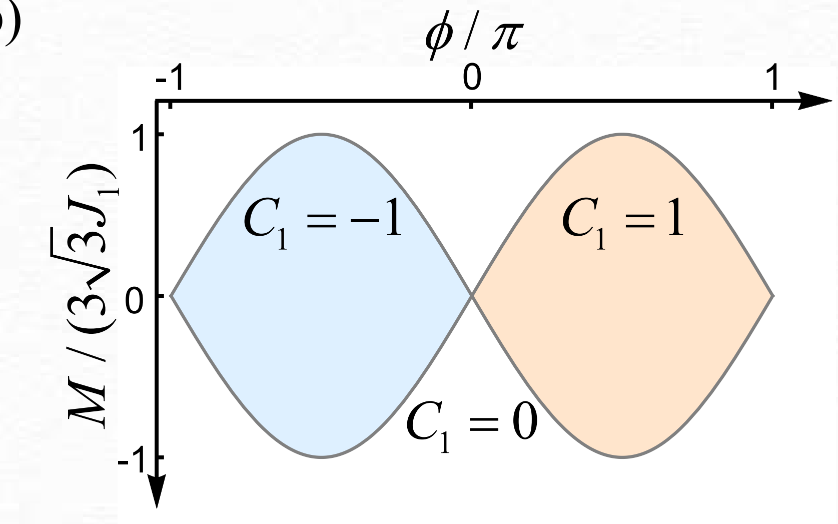
(a)



(b)



How do I know the Chern Number of the Hamiltonian ?

At equilibrium:

Number of Edge States **=** **Bulk Chern Number** **=** **Quantized Hall Conductance**

The Haldane Model

A two-band Chern Insulator

$$\mathcal{H}(\mathbf{k}) = \frac{1}{2}\mathbf{h}(\mathbf{k}) \cdot \boldsymbol{\sigma}$$

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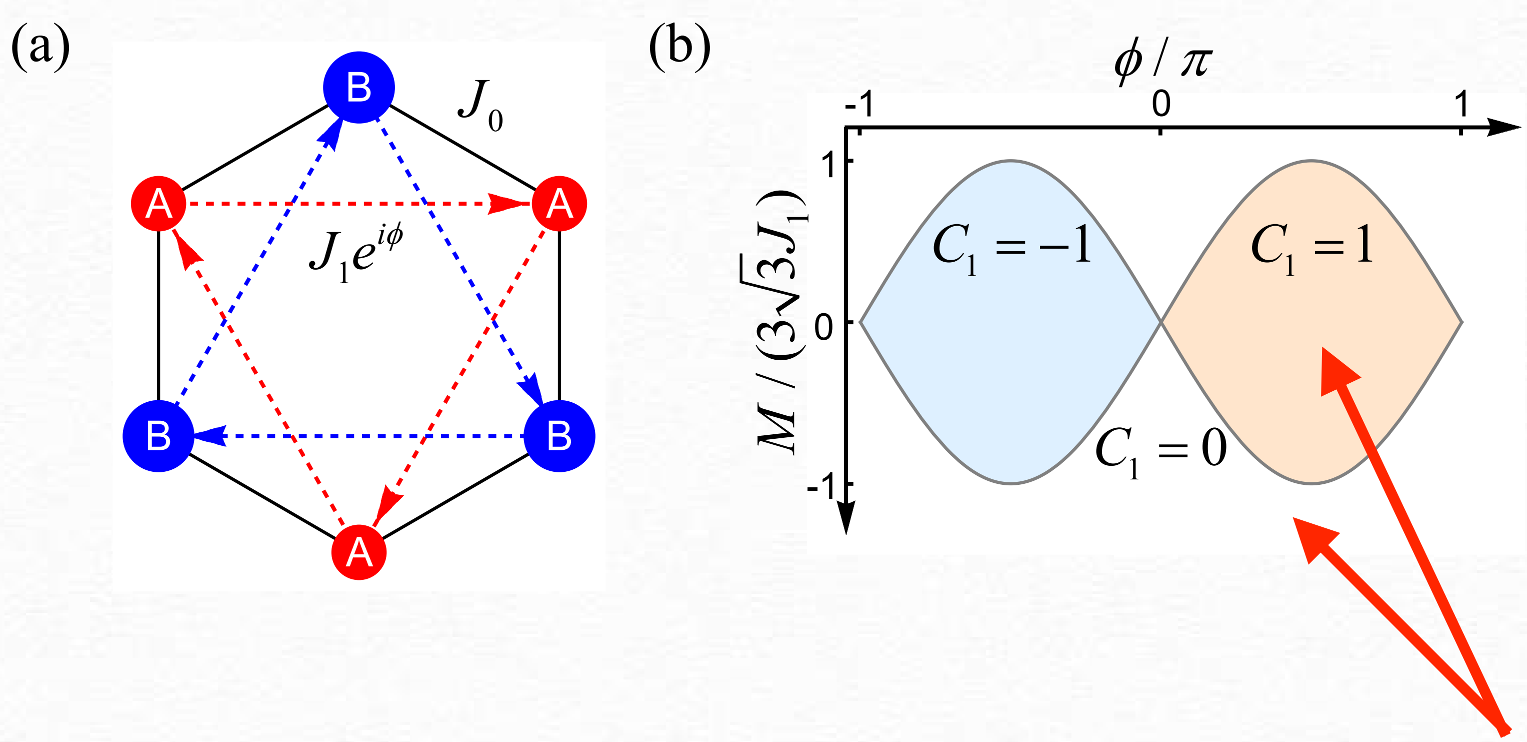
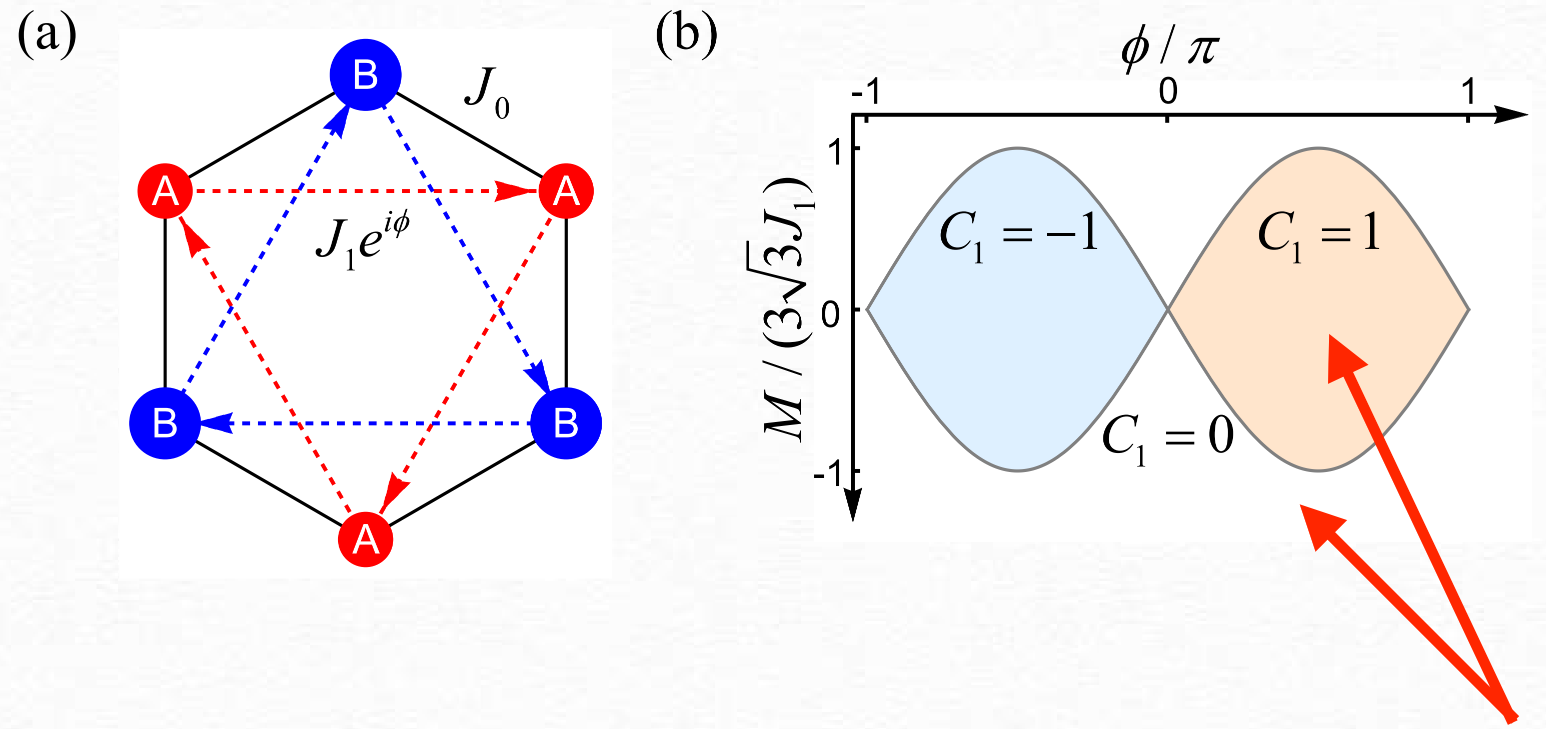


How do I know the Chern Number of the Hamiltonian ?
Quench:

The Haldane Model

A two-band Chern Insulator $\mathcal{H}(\mathbf{k}) = \frac{1}{2}\mathbf{h}(\mathbf{k}) \cdot \boldsymbol{\sigma}$

For instance, the Haldane model:



How do I know the Chern Number of the Hamiltonian ?

Quench:

Number of Edge States

Chern Number of Hamiltonian

≠

Chern Number of Wave Function

Quantized Hall Conductance

D'Alessio and Rigol (2015)

The Haldane Model

A two-band Chern Insulator $\mathcal{H}(\mathbf{k}) = \frac{1}{2}\mathbf{h}(\mathbf{k}) \cdot \boldsymbol{\sigma}$

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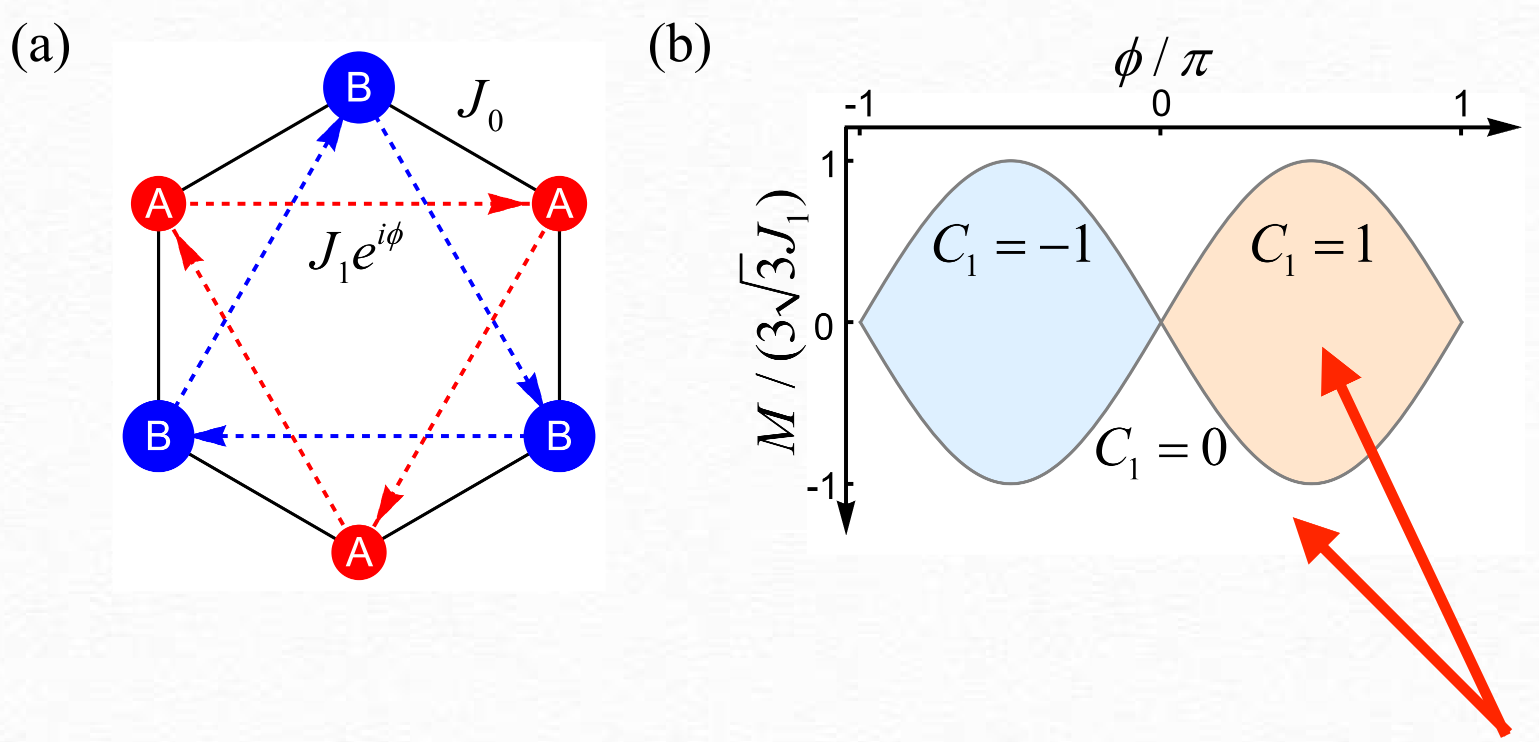

How do I know the Chern Number of the Hamiltonian ?

Quench:

Number of Edge States ≠ **Chern Number of Hamiltonian** ≠ **Chern Number of Wave Function**

Quantized Hall Conductance

D'Alessio and Rigol (2015)
Caio, Cooper, Bhassen (2015)

The Haldane Model

A two-band Chern Insulator $$\mathcal{H}(\mathbf{k}) = \frac{1}{2}\mathbf{h}(\mathbf{k}) \cdot \boldsymbol{\sigma}$$

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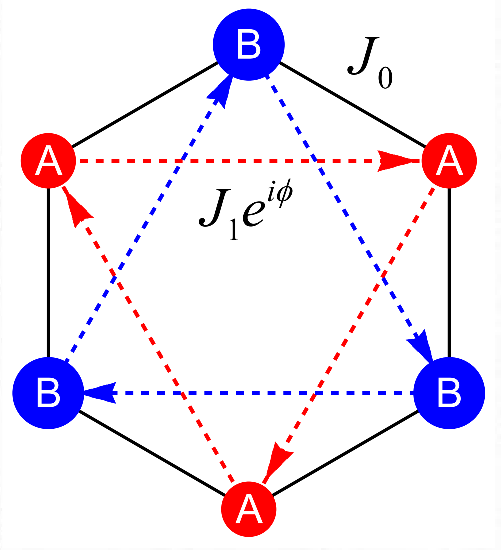
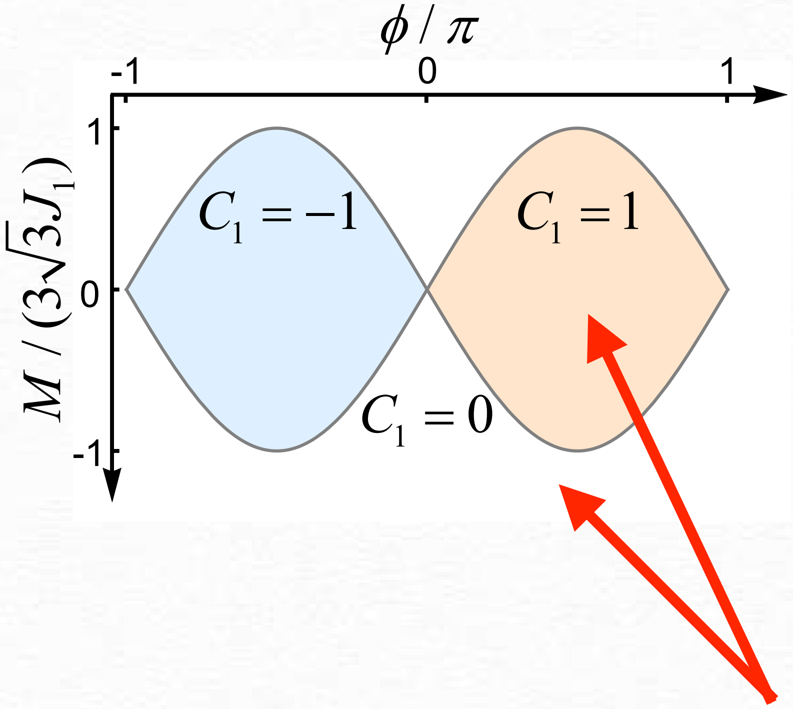
(a)



(b)



How do I know the Chern Number of the Hamiltonian ?

Quench:

Number of Edge States ≠ **Chern Number of Hamiltonian** ≠ **Quantized Hall Conductance**

≠

Chern Number of Wave Function

D'Alessio and Rigol (2015)
Caio, Cooper, Bhassen (2015)

Hu, Zoller, Budike (2016),
Wilson, Song, Refael (2016)
Unal, Mueller, Oktel (2016)

The Haldane Model

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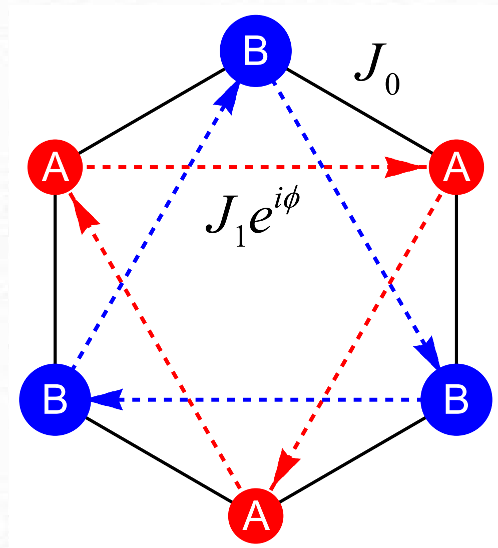

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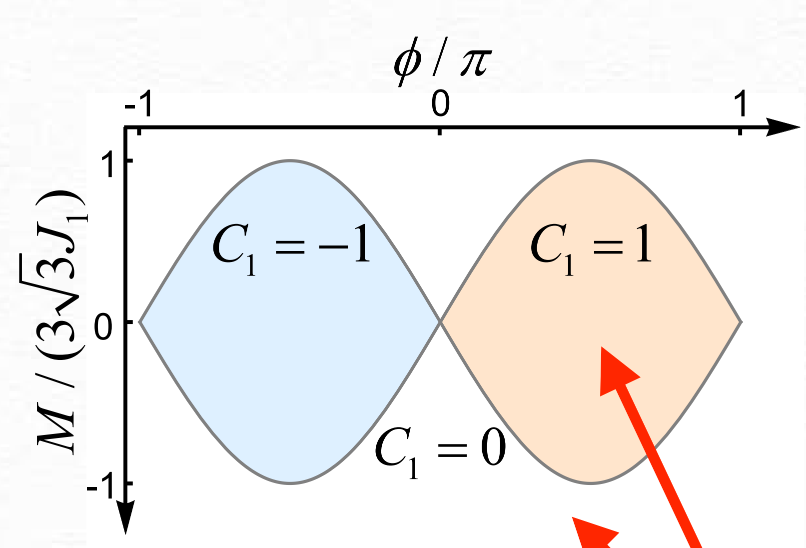

How do I know the Chern Number of the Hamiltonian ?

Quench:

Is there a Quantized Value one can extract from the quench dynamics, and this value equals to the bulk Chern number.

Our Proposal

A two-band Chern Insulator $\mathcal{H}(\mathbf{k}) = \frac{1}{2}\mathbf{h}(\mathbf{k}) \cdot \boldsymbol{\sigma}$

Quench from $\mathbf{h}^{\mathrm{i}}(\mathbf{k}) \rightarrow \mathbf{h}^{\mathrm{f}}(\mathbf{k})$

$$\zeta(\mathbf{k}, t) = \exp\left\{-\frac{i}{2}\mathbf{h}^{\mathrm{f}}(\mathbf{k}) \cdot \boldsymbol{\sigma} t\right\} \zeta^{\mathrm{i}}(\mathbf{k}),$$

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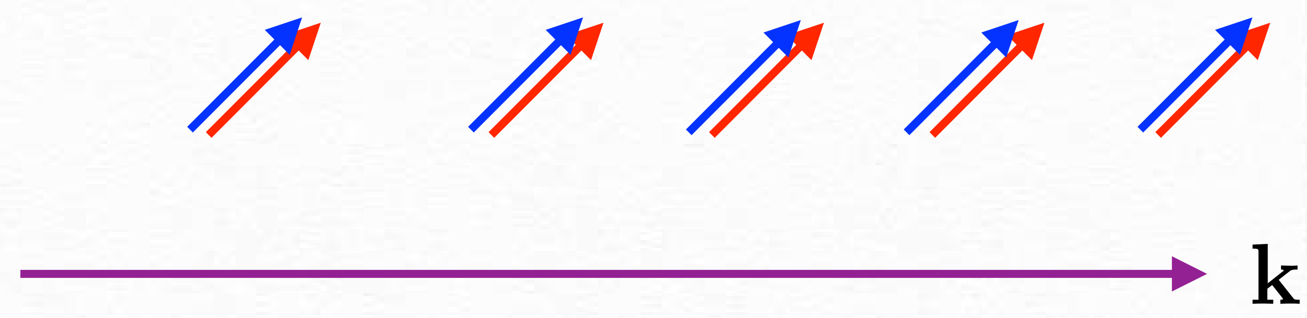
$$[k_x, k_y, t] \rightarrow \mathbf{n}$$



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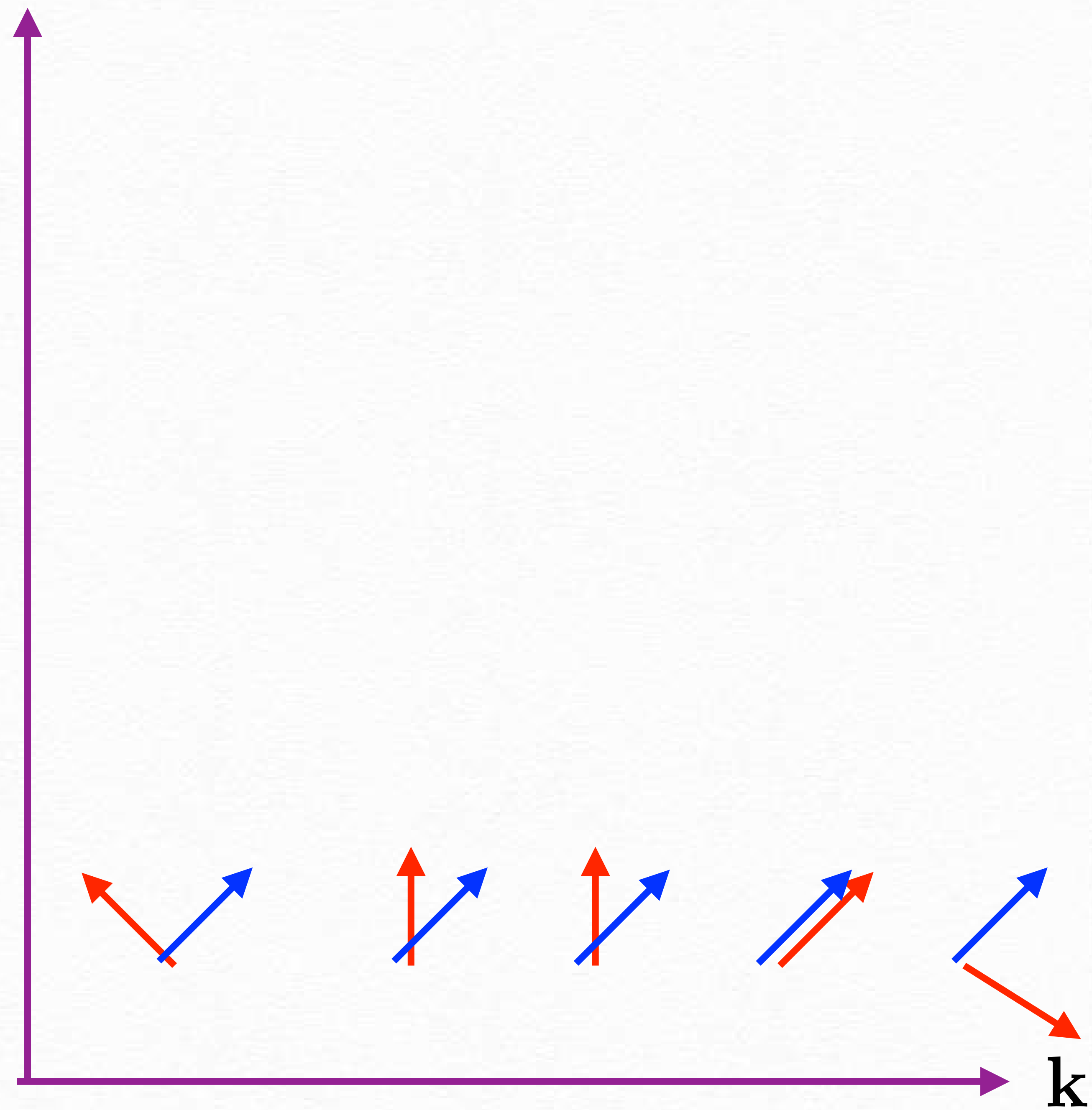

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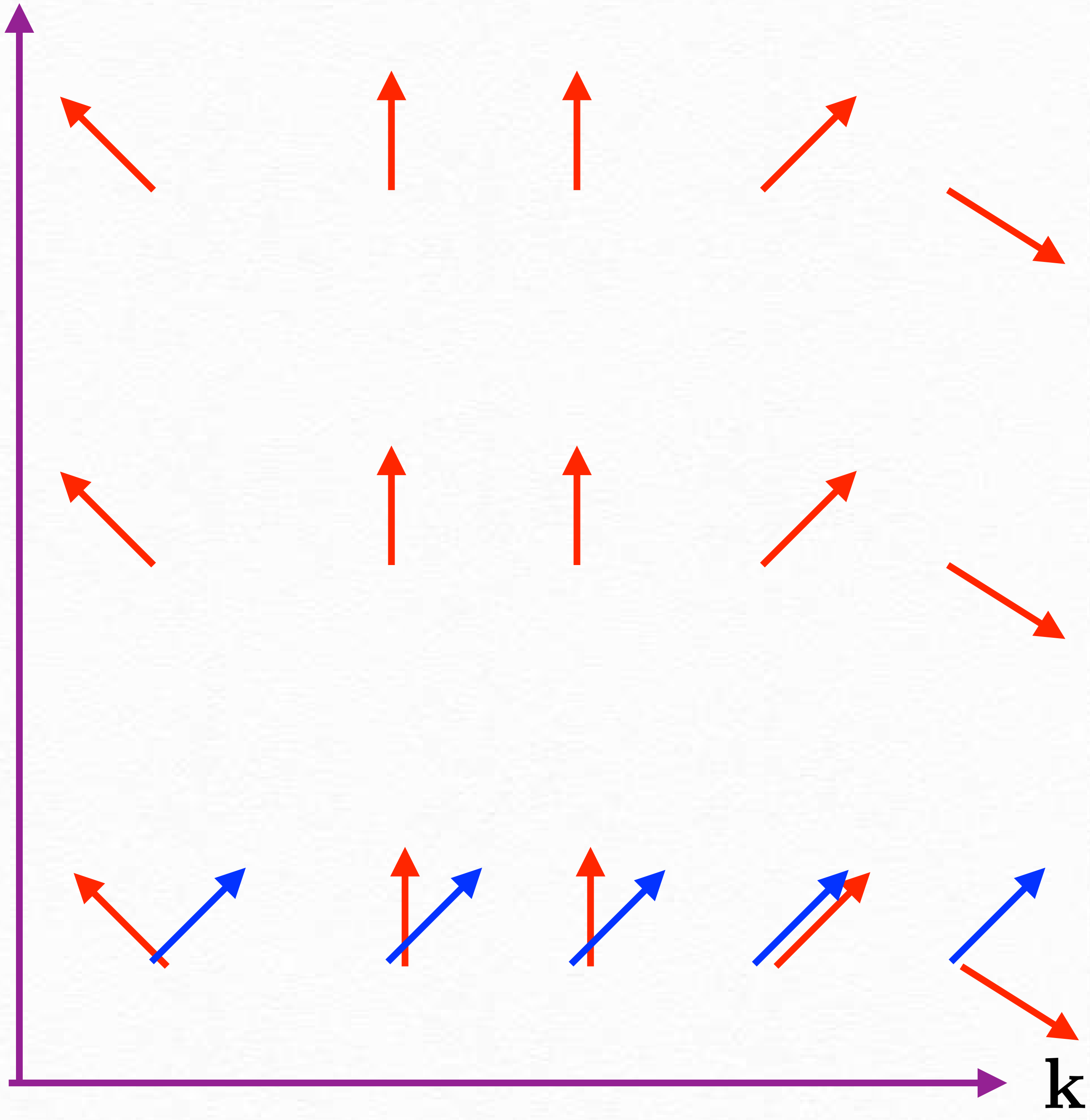

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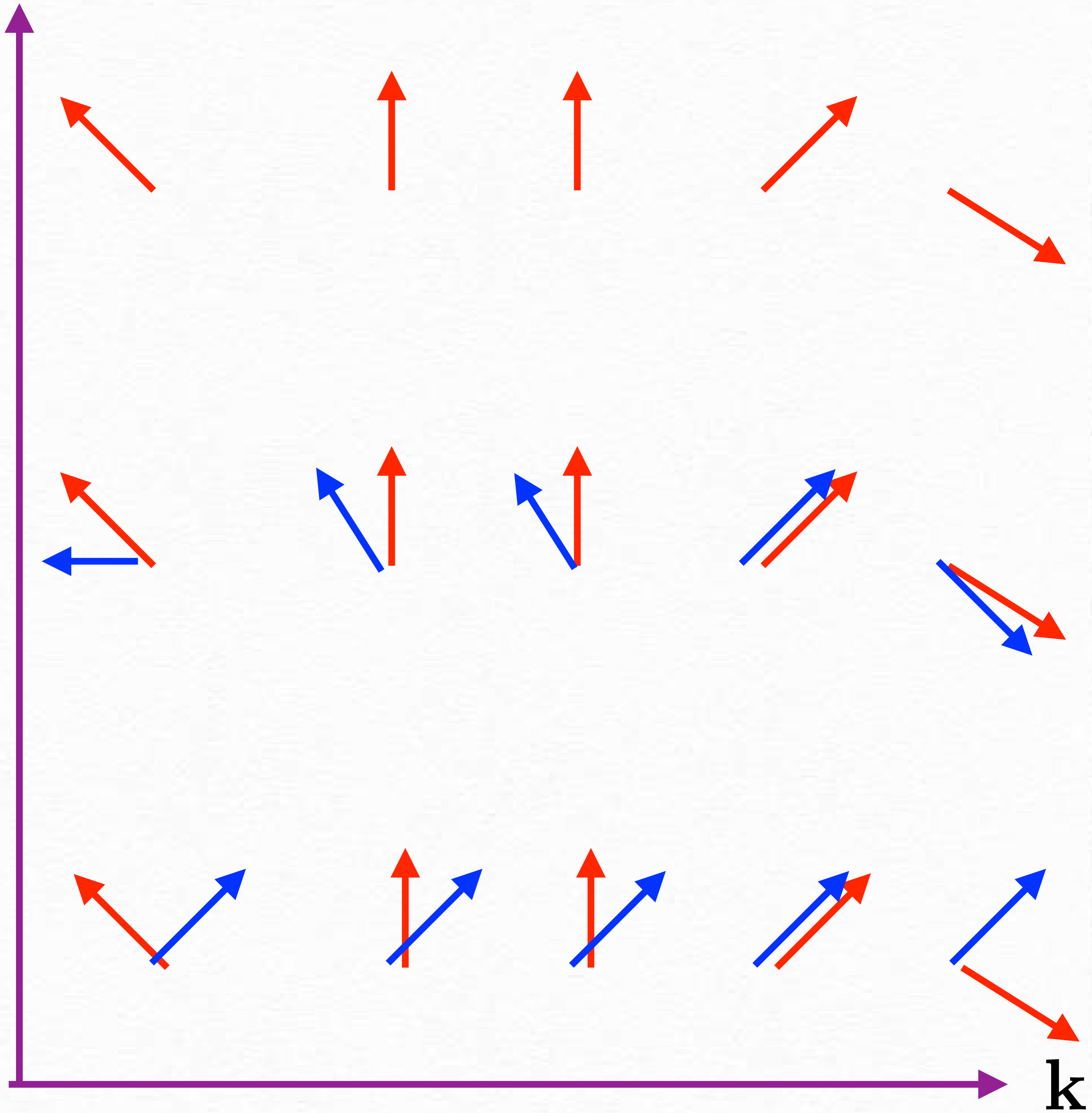

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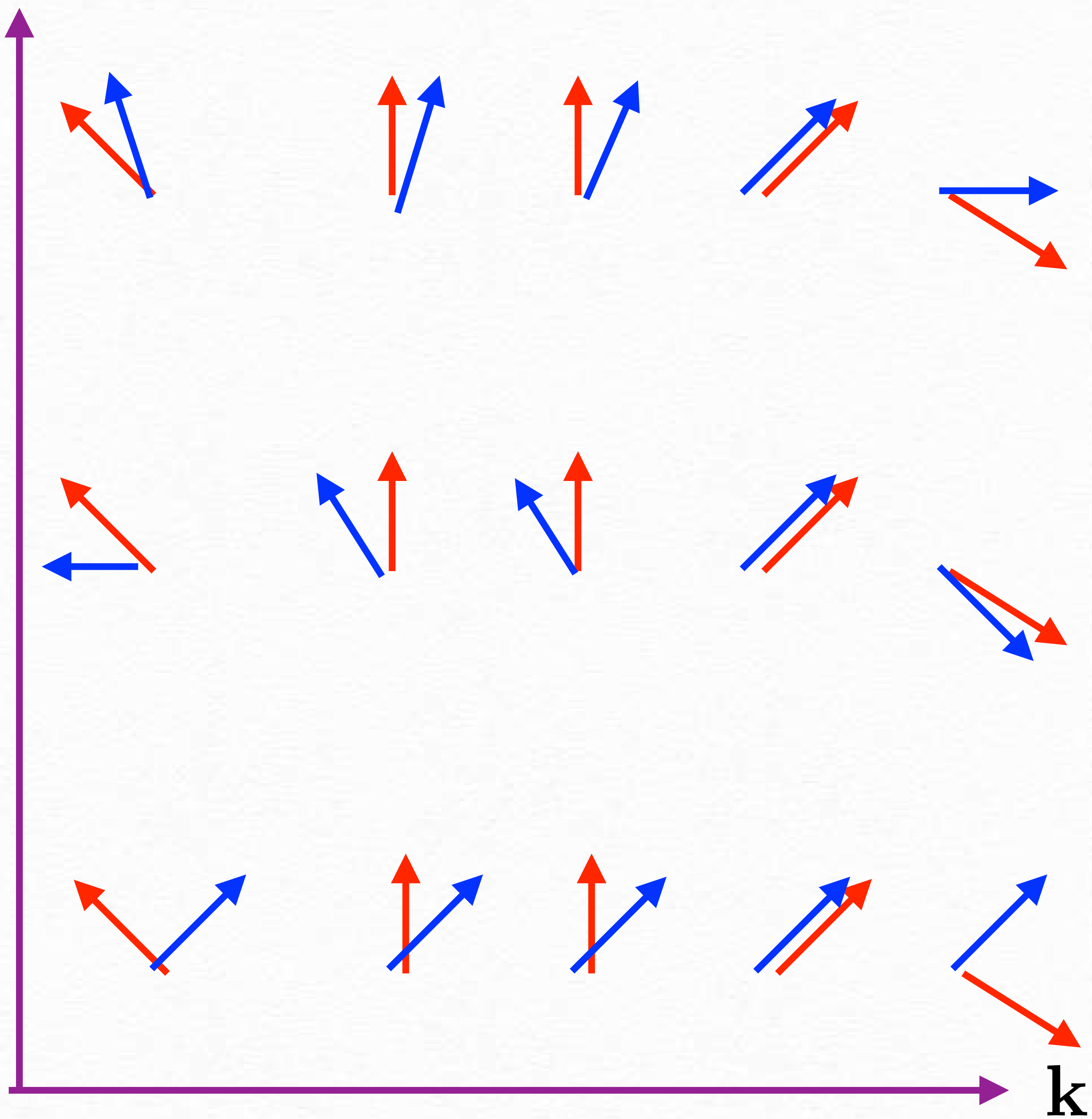

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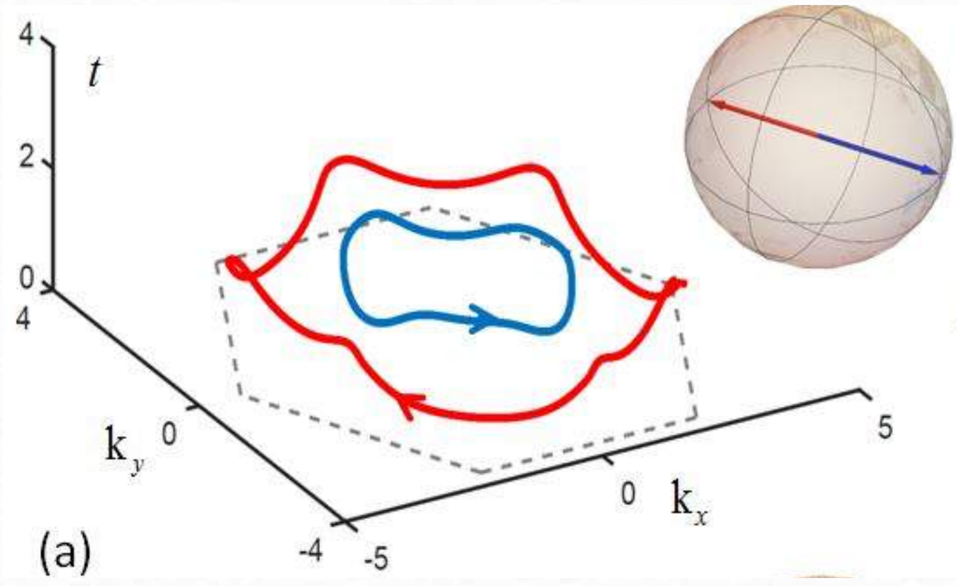

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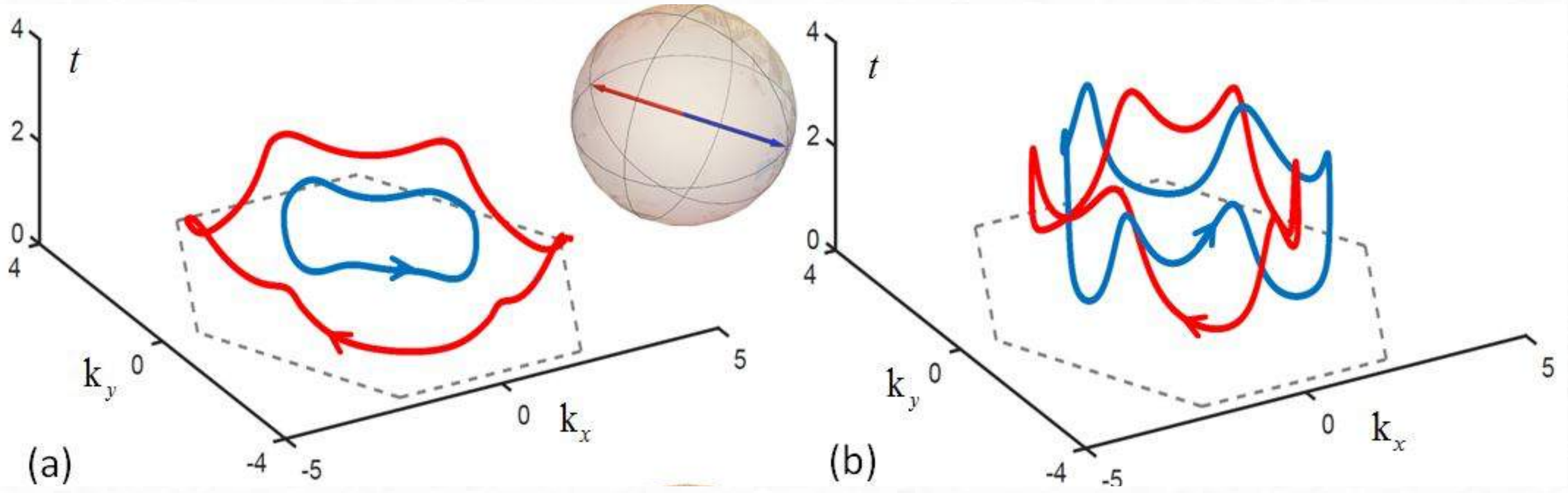

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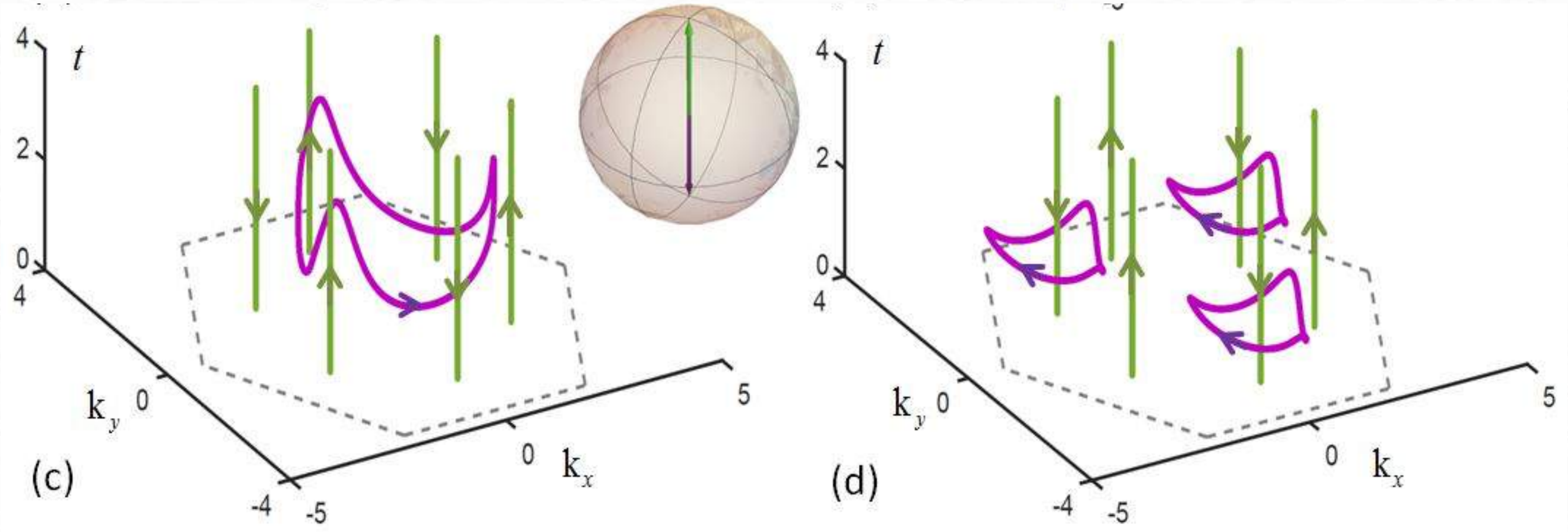
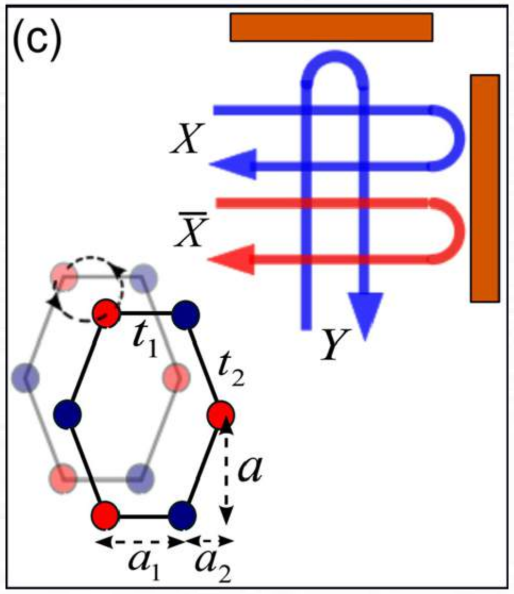


Application to Cold Atom Experiments



Wei Zheng and Hui Zhai, PRA 2014

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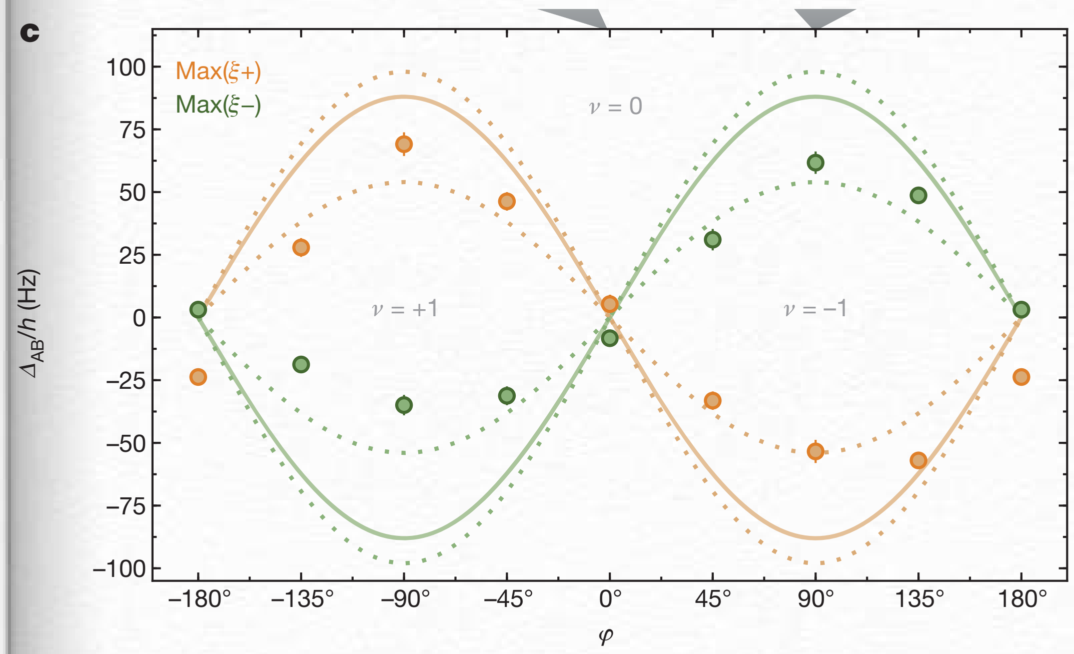

ETH group, Nature, 2014

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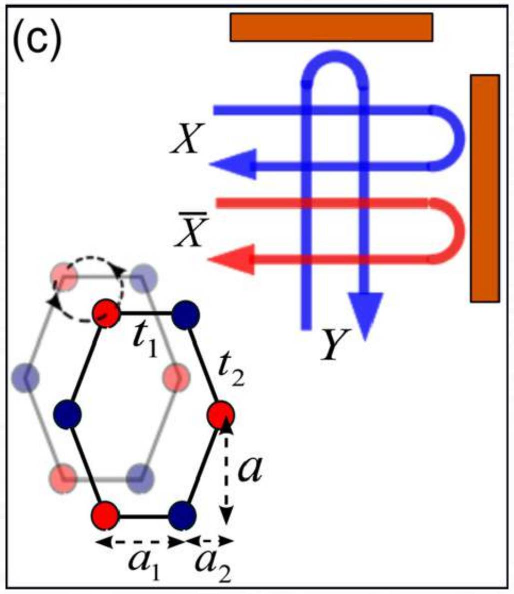

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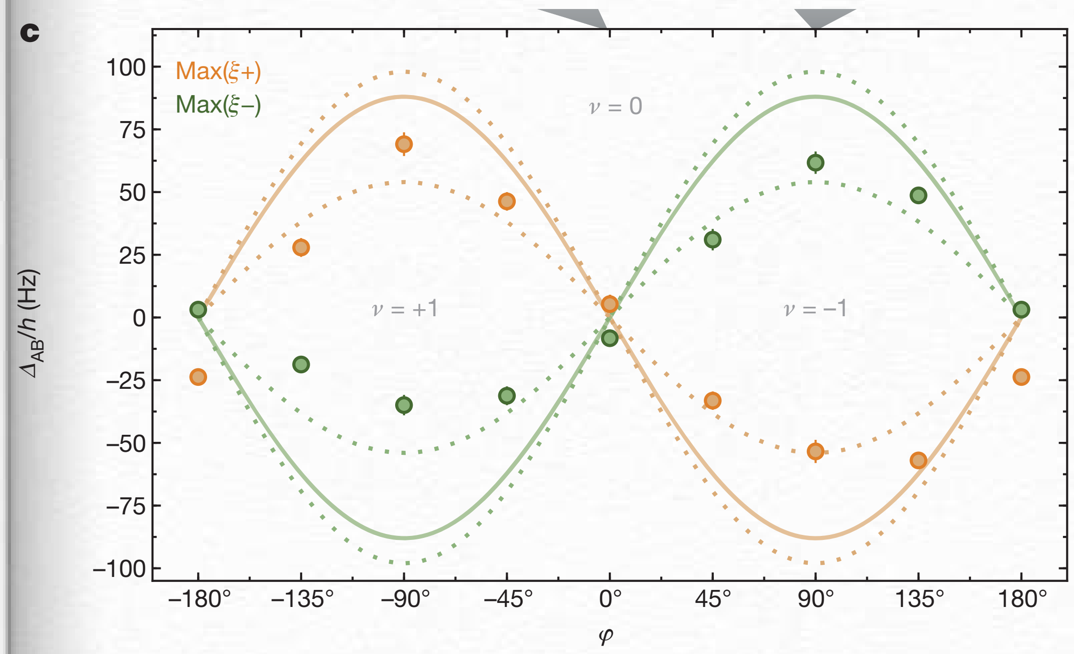

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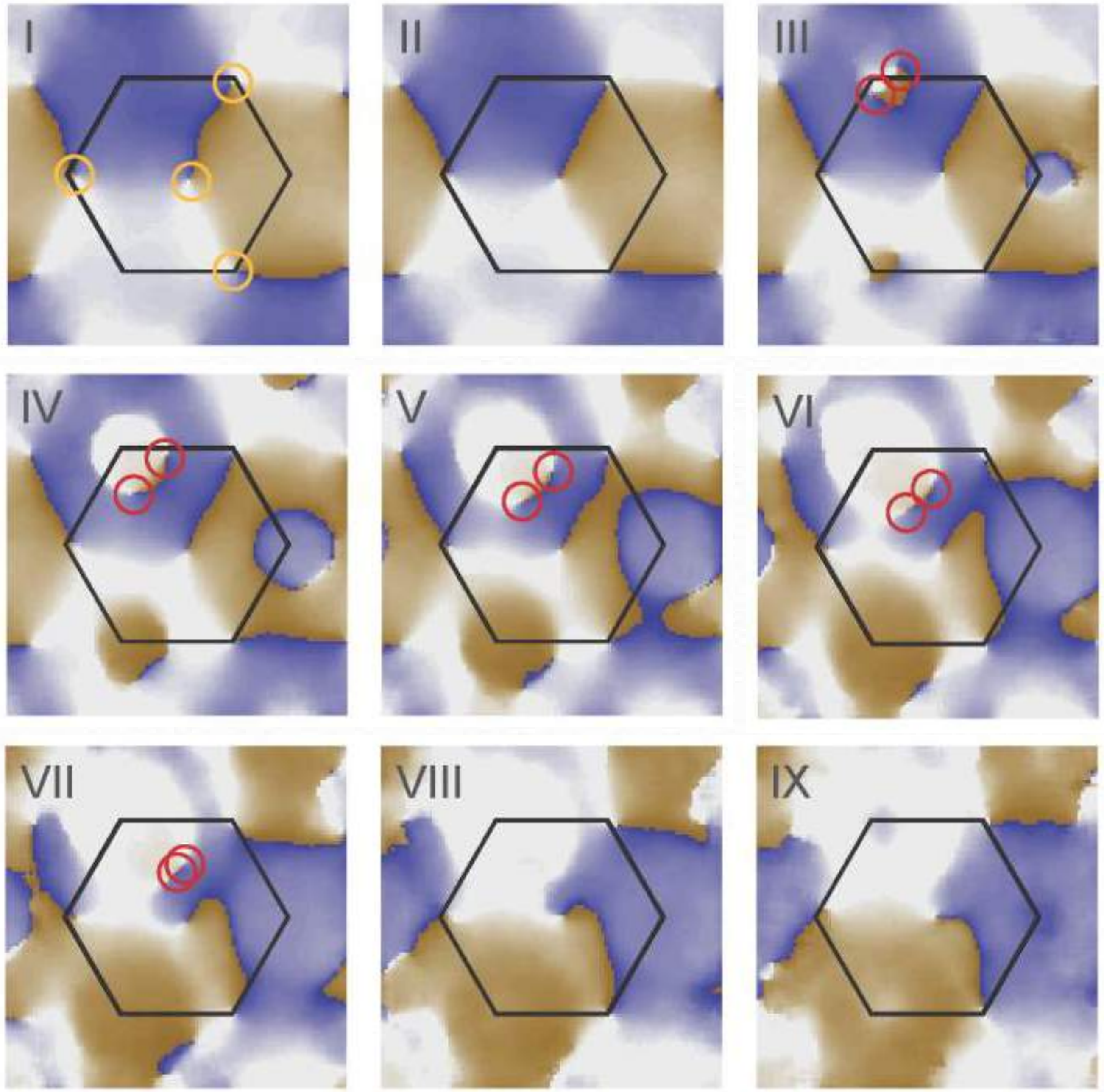

Hamburg group arXiv: 1608.05616

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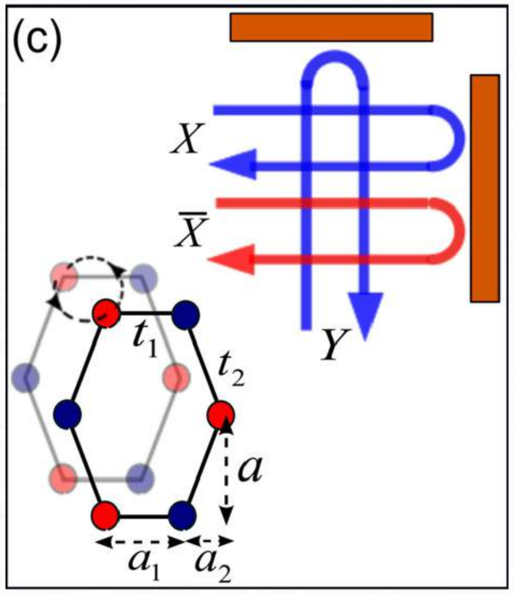

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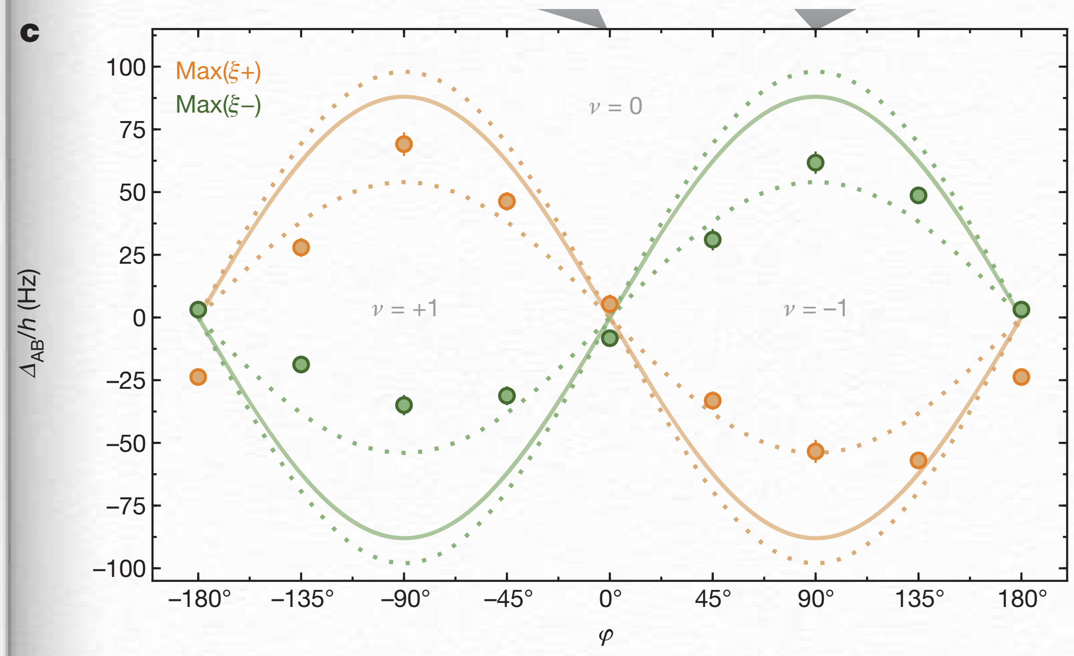

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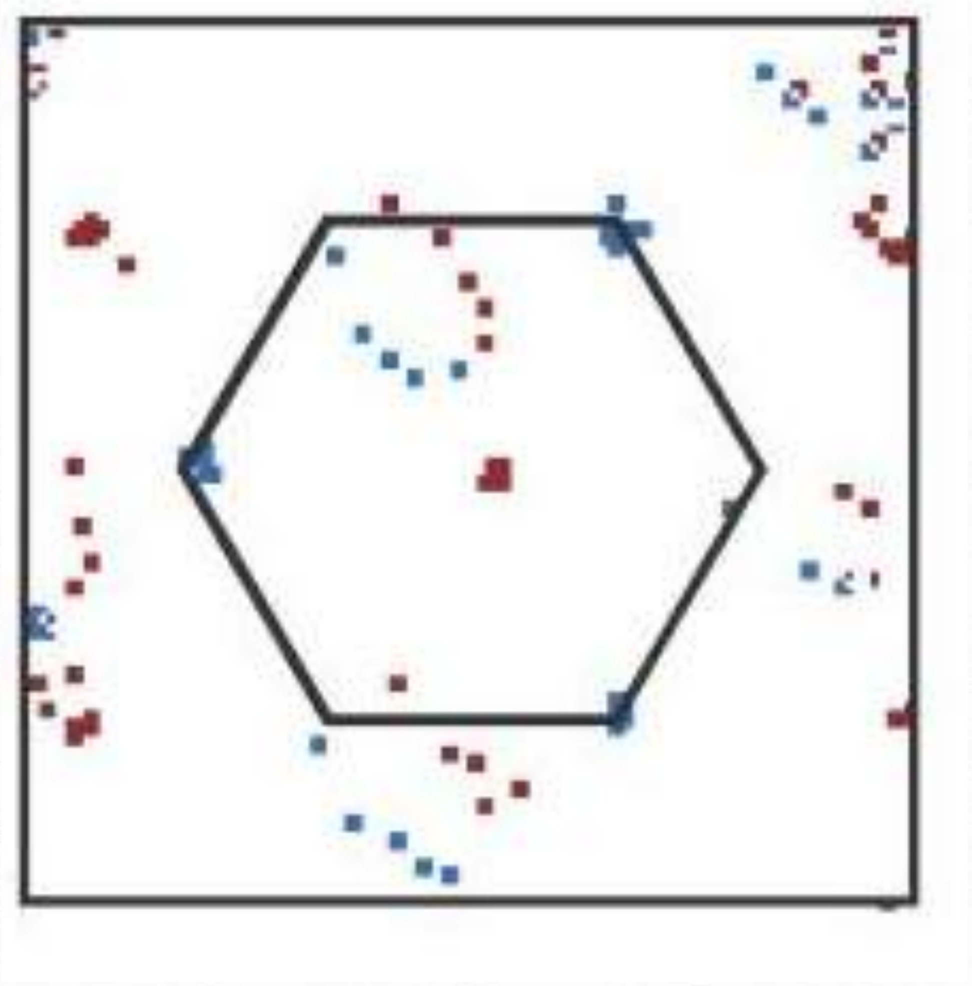

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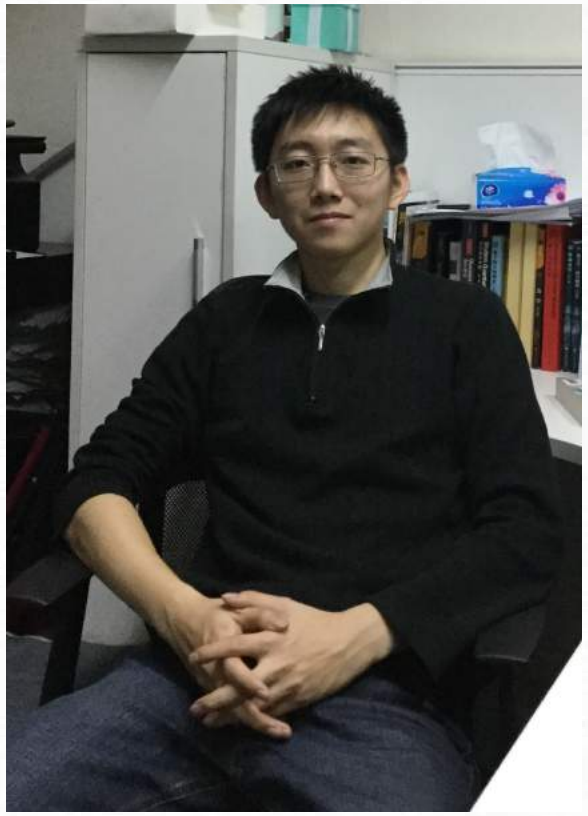
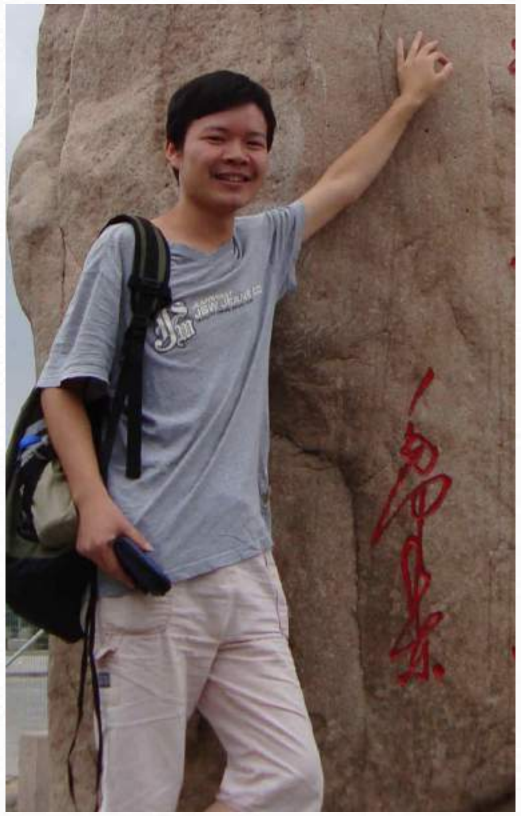
Measuring Topological Number of a Chern-Insulator from Quench Dynamics

Ce Wang, Pengfei Zhang,* Xin Chen, Jinlong Yu, and Hui Zhai†
Institute for Advanced Study, Tsinghua University, Beijing, 100084, China
(Dated: November 28, 2016)

arXiv:1611.03304





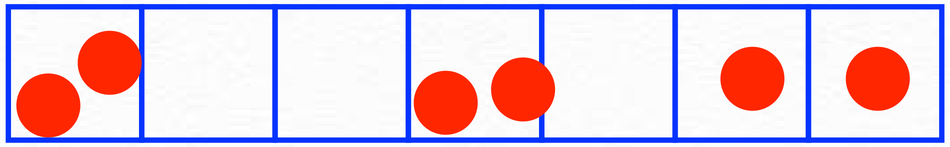
Ce Wang **Pengfei Zhang** **Xin Chen** **Dr. Jinlong Yu**

Entropy

Quench Experiment

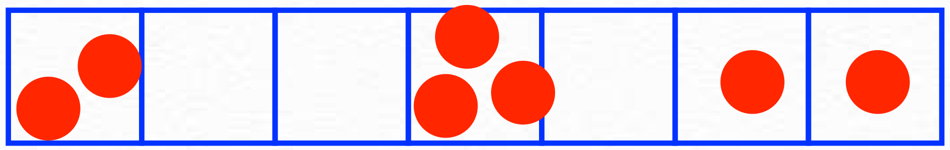
Quench Experiment

Local quench



Quench Experiment

Local quench $\hat{b}_i^\dagger|\Psi\rangle$



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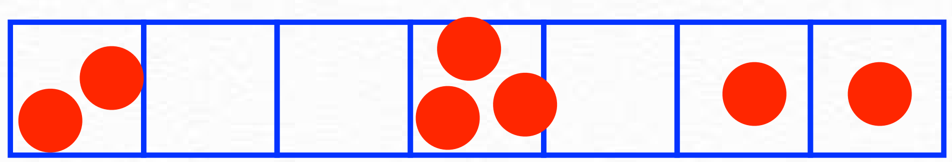
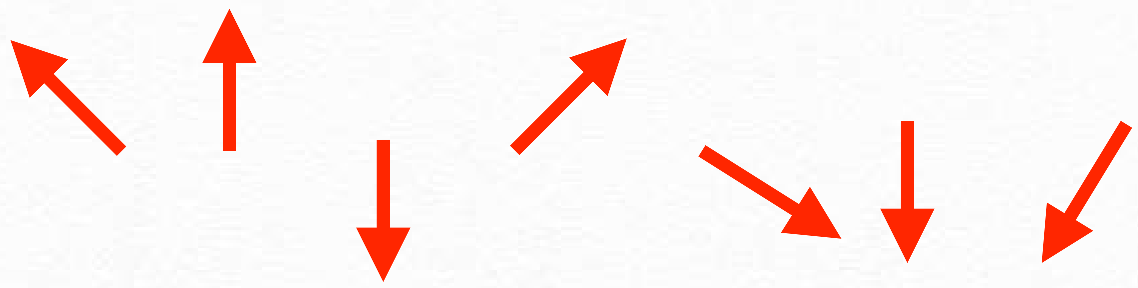
Local quench $\hat{b}_i^\dagger|\Psi\rangle$

Quench Experiment

Local quench $\hat{b}_i^\dagger|\Psi\rangle$ $S_i^-|\Psi\rangle$





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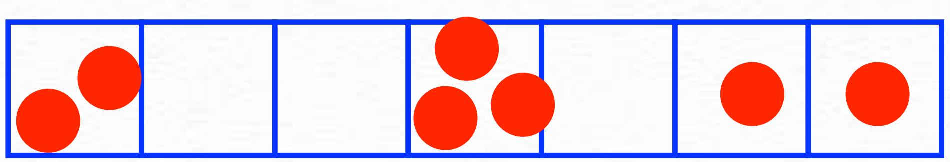

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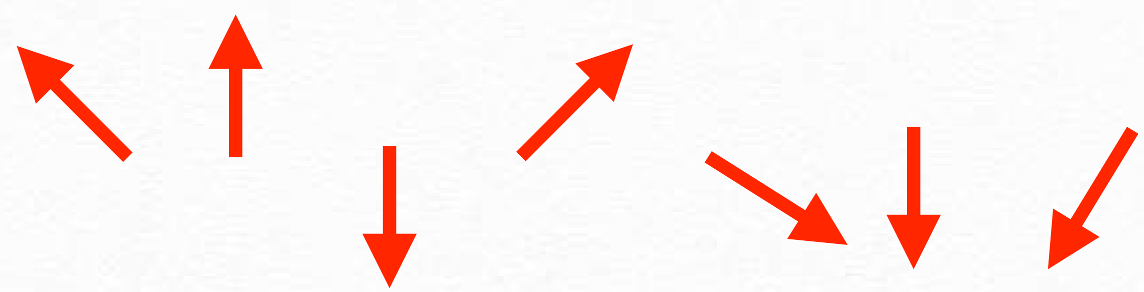
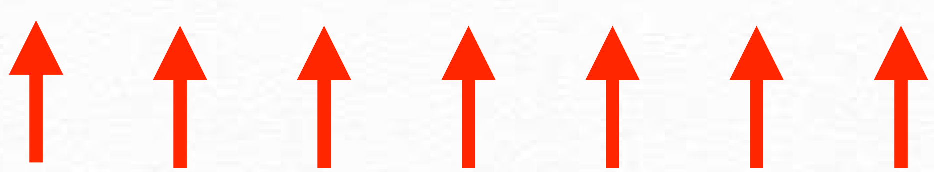


Global quench $\mathcal{P}|\Psi\rangle$



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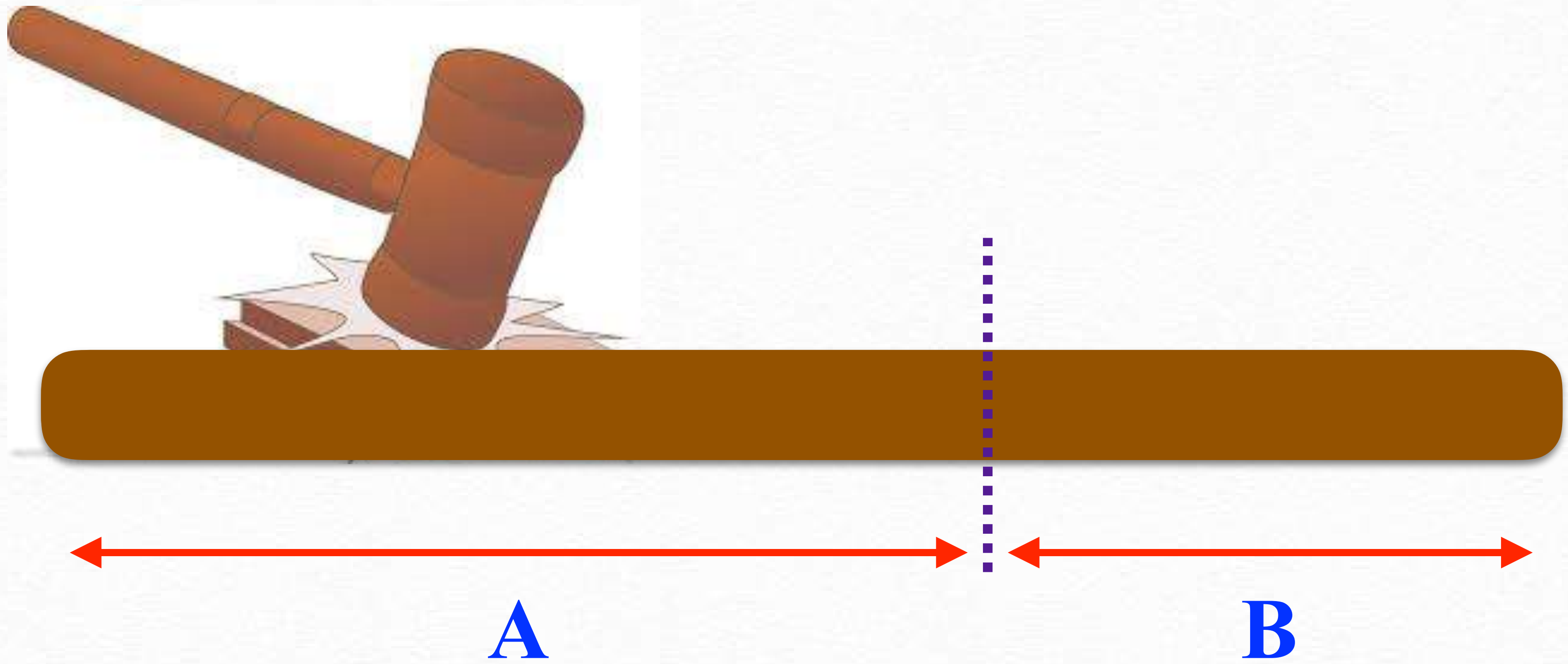
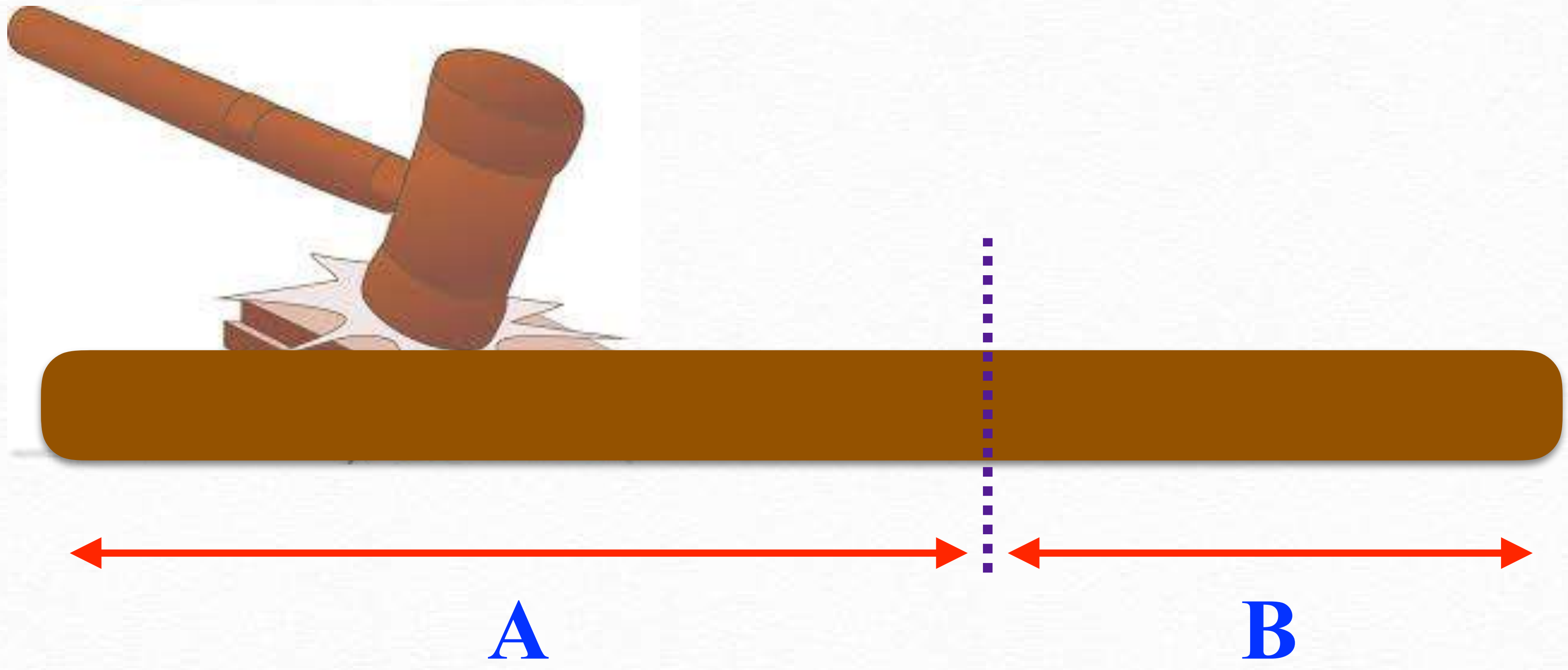


$$\rho_A = \mathrm{Tr}_B \rho$$

The Second Renyi Entropy $$S_A^{(2)} = -\log \mathrm{Tr}_A \hat{\rho}_A^2$$

Quench Experiment



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The Second Renyi Entropy

$$S_A^{(2)} = -\log \mathrm{Tr}_A \hat{\rho}_A^2$$

$$\exp(-S_A^{(2)}) = \sum_{M \in B} \mathrm{Tr}[\hat{M}(t)\hat{V}(0)\hat{M}(t)\hat{V}(0)]$$

Out-of-Time-Ordered Correlation (OTOC)

$$\langle \hat{W}^\dagger(t)\hat{V}^\dagger(0)\hat{W}(t)\hat{V}(0)\rangle_\beta$$

$$\hat{W}(t) = e^{i\hat{H}t}\hat{W}e^{-i\hat{H}t}$$

Normal correlation you can find in any textbook:

$$\langle \hat{W}^\dagger(t)\hat{W}(t)\hat{V}^\dagger(0)\hat{V}(0)\rangle_\beta$$

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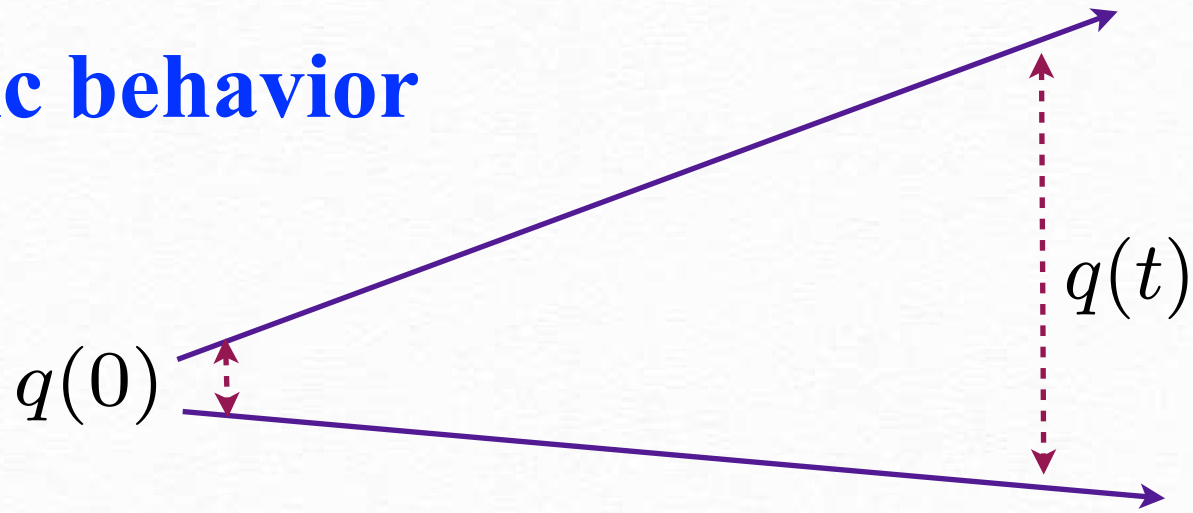
- **OTOC diagnoses chaotic behavior**

$$\frac{\partial q(t)}{\partial q(0)} \sim e^{\lambda_L t}$$



λ_L **Lyapunov exponent**

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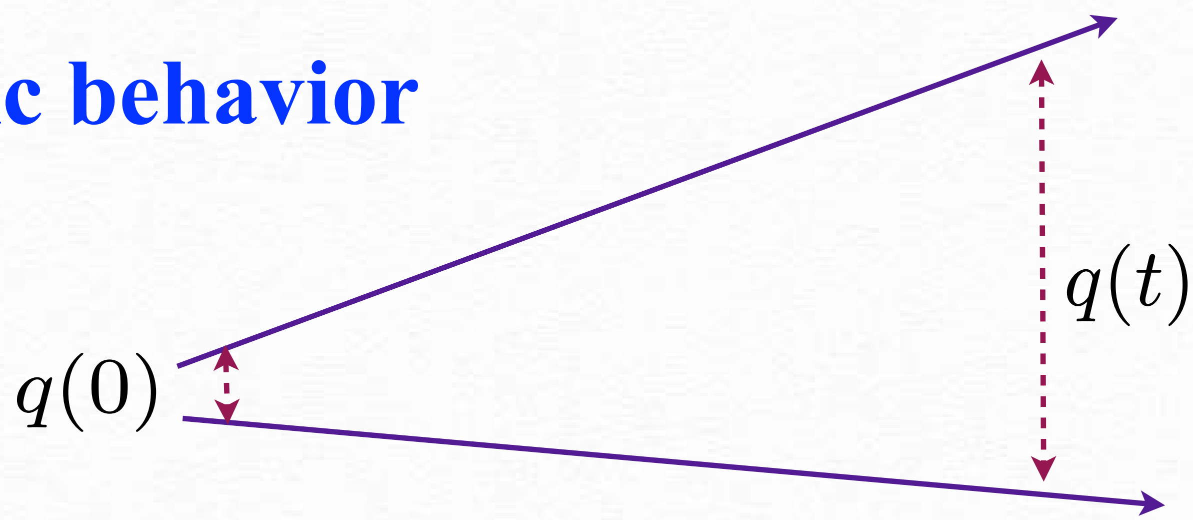

λ_L **Lyapunov exponent**

$$C(t) = \langle |[W(t), V(0)]|^2 \rangle_\beta$$

Larkin, Ovchinnikov 1969

Out-of-Time-Ordered Correlation (OTOC)

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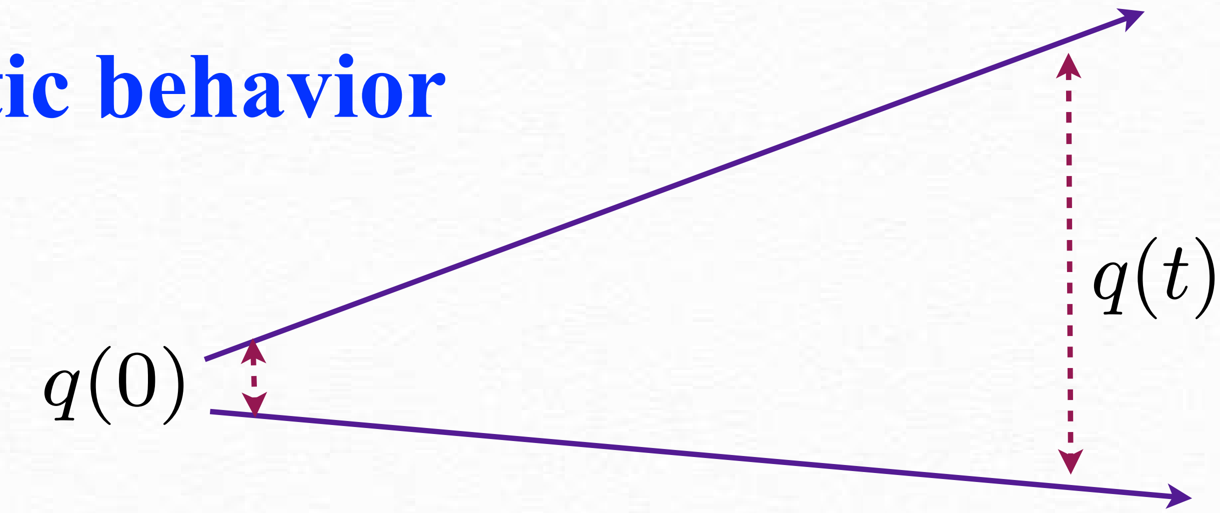


λ_L **Lyapunov exponent**

$$C(t) = \underbrace{\langle V^\dagger(0)W^\dagger(t)W(t)V(0) + W^\dagger(t)V^\dagger(0)V(0)W(t)}_{\text{“Accessible correlators”}}$$

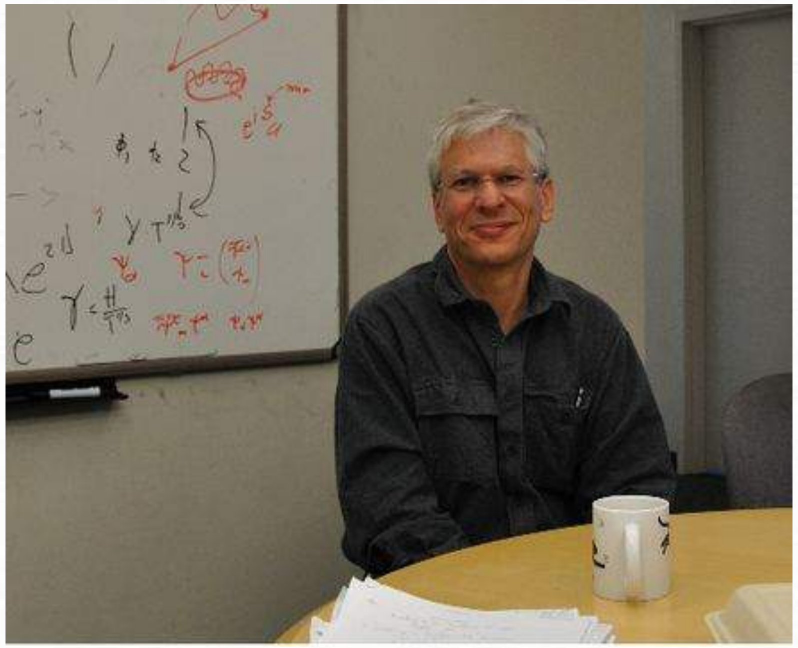
$$\underbrace{-W^\dagger(t)V^\dagger(0)W(t)V(0) - V^\dagger(0)W^\dagger(t)V(0)W(t)\rangle_\beta}_{\text{“Out-of-time-ordered correlators”}}$$

Larkin, Ovchinnikov 1969 Kitaev, KITP, 2014

Out-of-Time-Ordered Correlation (OTOC)

- **OTOC has also emerged in studying gravity models. The calculation with a black hole shows that**

$$\lambda_L = \frac{2\pi}{\beta}$$



Kitaev, KITP, 2014; Shenker and Stanford, JHEP, 2014, 2015

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An example is the SYK model

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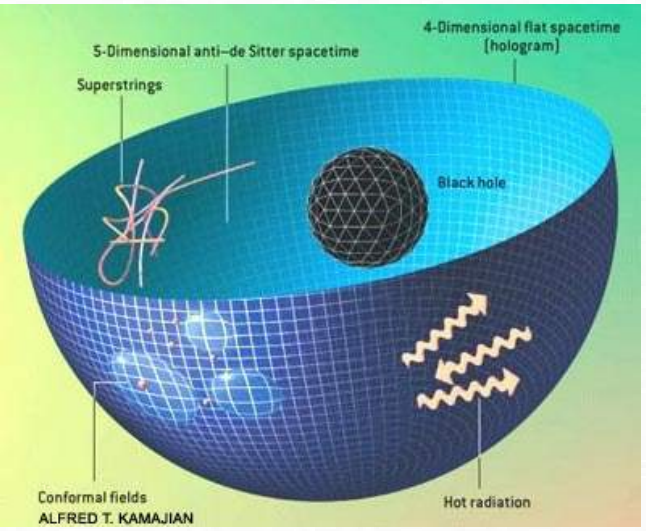

Holographic duality: A quantum many body system (strongly interacting, emergent conformal field symmetry) in D-dimension can be "mapped" to an Einstein gravity theory in D+1-dimension

Kitaev, KITP, 2015; Maldacena, Shenker and Stanford, 2015

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A quantum system with holographic duality saturates the bound
An example is the SYK model

- **Delocalization of information is closely related to the decay of the OTOC, and the butterfly effect in quantum system implies the information-theoretic definition of scrambling.**

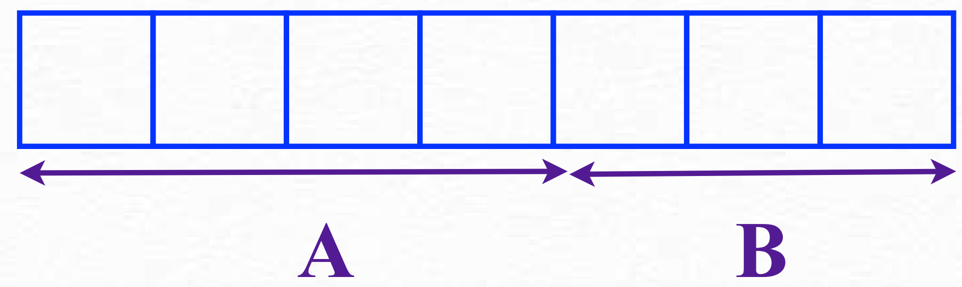
Hosur, Qi, Roberts and Yoshida, 2015

Out-of-Time-Ordered Correlation (OTOC)

Out-of-Time-Ordered Correlator

$$\langle W^\dagger(t) V^\dagger(0) W(t) V(0) \rangle$$

OTOC v.s. Entanglement Entropy



$$\exp(-S_A^{(2)}) = \sum_{M \in B} \mathrm{Tr}[\hat{M}(t)\hat{V}(0)\hat{M}(t)\hat{V}(0)]$$

Non-Equilibrium Properties

Quench the system by arbitrary operator $\mathcal{O}$

Entanglement Entropy

=

Equilibrium Properties

$$\hat{V} = \hat{O}\hat{O}^\dagger$$

$\hat{M}$ **is a complete set of operators in B**

OTOC

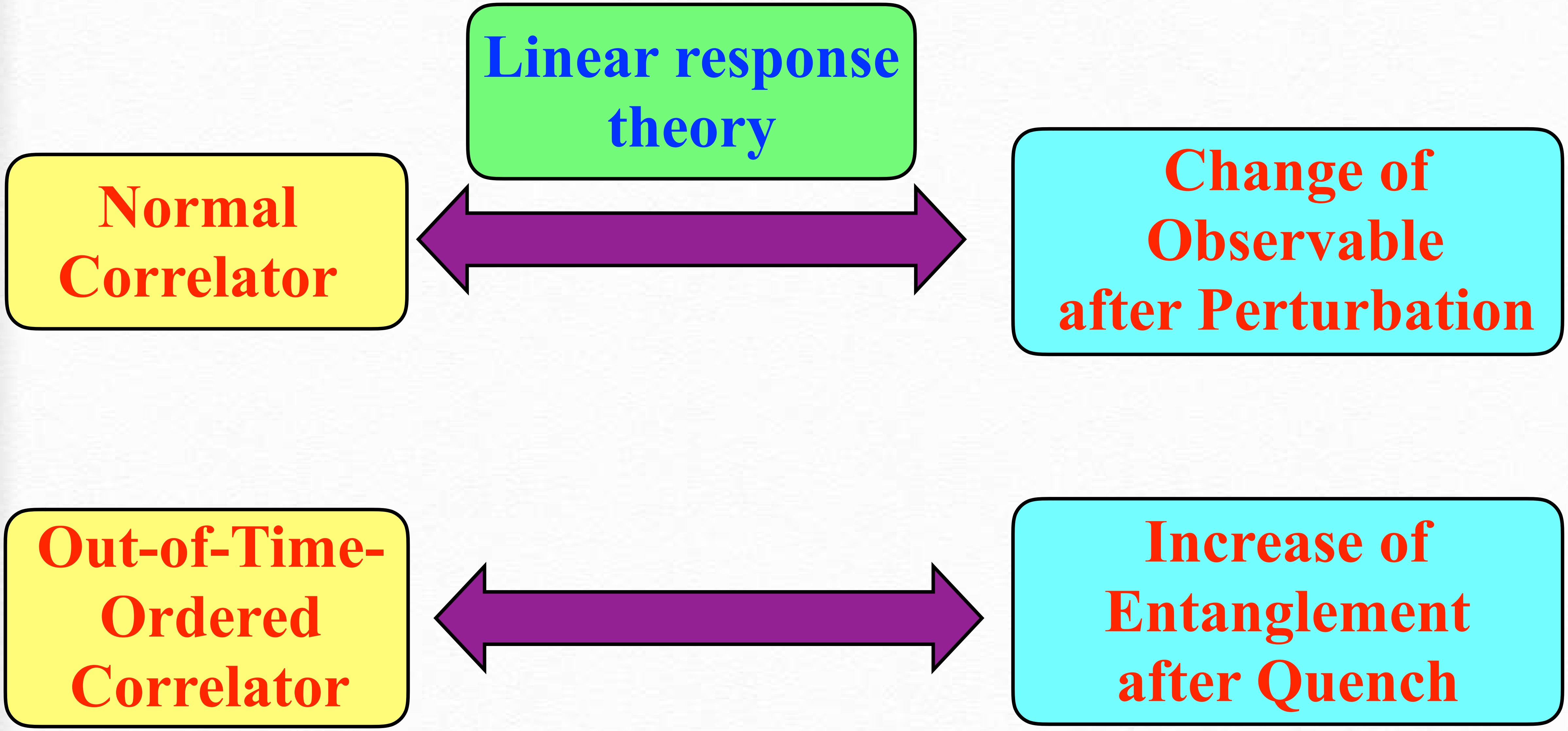
OTOC v.s. Entanglement Entropy

Linear response theory

Normal Correlator ⟷ Change of Observable after Perturbation

Out-of-Time-Ordered Correlator ⟷ ?

OTOC v.s. Entanglement Entropy



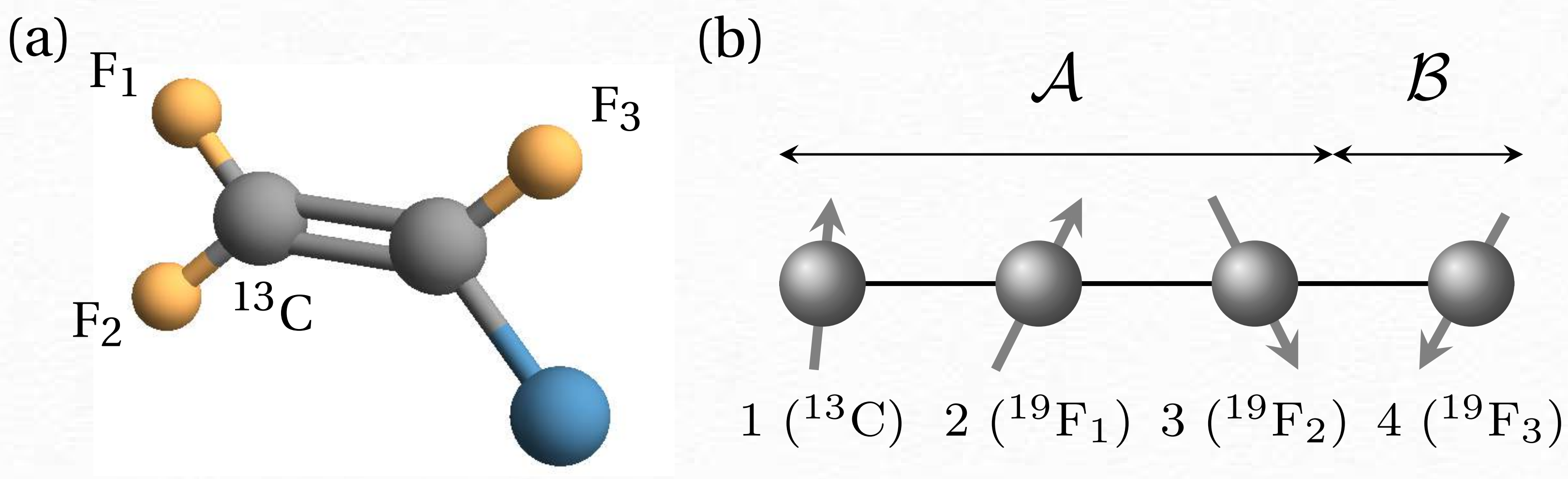
OTOC v.s. Entanglement Entropy

$$\exp(-S_A^{(2)}) = \sum_{M \in B} \mathrm{Tr}[\hat{M}(t)\hat{V}(0)\hat{M}(t)\hat{V}(0)]$$

Thermal Phase	Single-Particle Localized	Many-Body Localized
Power-law spreading of entanglement	No spreading of entanglement	Logarithmic spreading of entanglement
OTOC exponential decay	OTOC remains constant	OTOC power-law decay

Our Results

NMR Quantum Simulation Measuring OTOC



$$F(t) = \langle \hat{B}^\dagger(t)\hat{A}^\dagger(0)\hat{B}(t)\hat{A}(0)\rangle_\beta$$

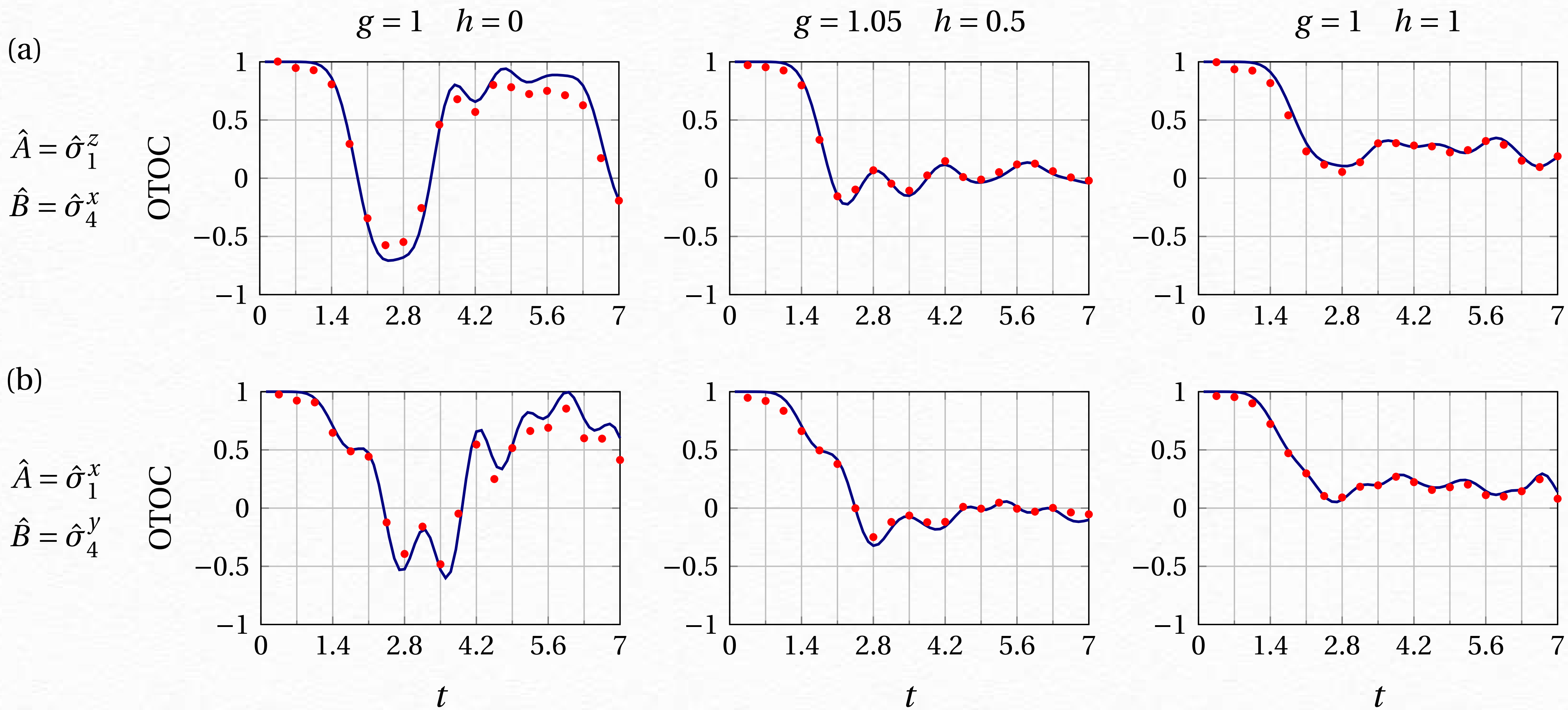
$$\hat{H} = \sum_i \left(-\hat{\sigma}_i^z\hat{\sigma}_{i+1}^z + g\hat{\sigma}_i^x + h\hat{\sigma}_i^z\right)$$

- **Zero h: Integrable case**
- **Non-Zero h: Non-Integrable case**

Measurements of OTOC for Ising Chain

$$\hat{H} = \sum_i \left(-\hat{\sigma}_i^z \hat{\sigma}_{i+1}^z + g\hat{\sigma}_i^x + h\hat{\sigma}_i^z\right)$$

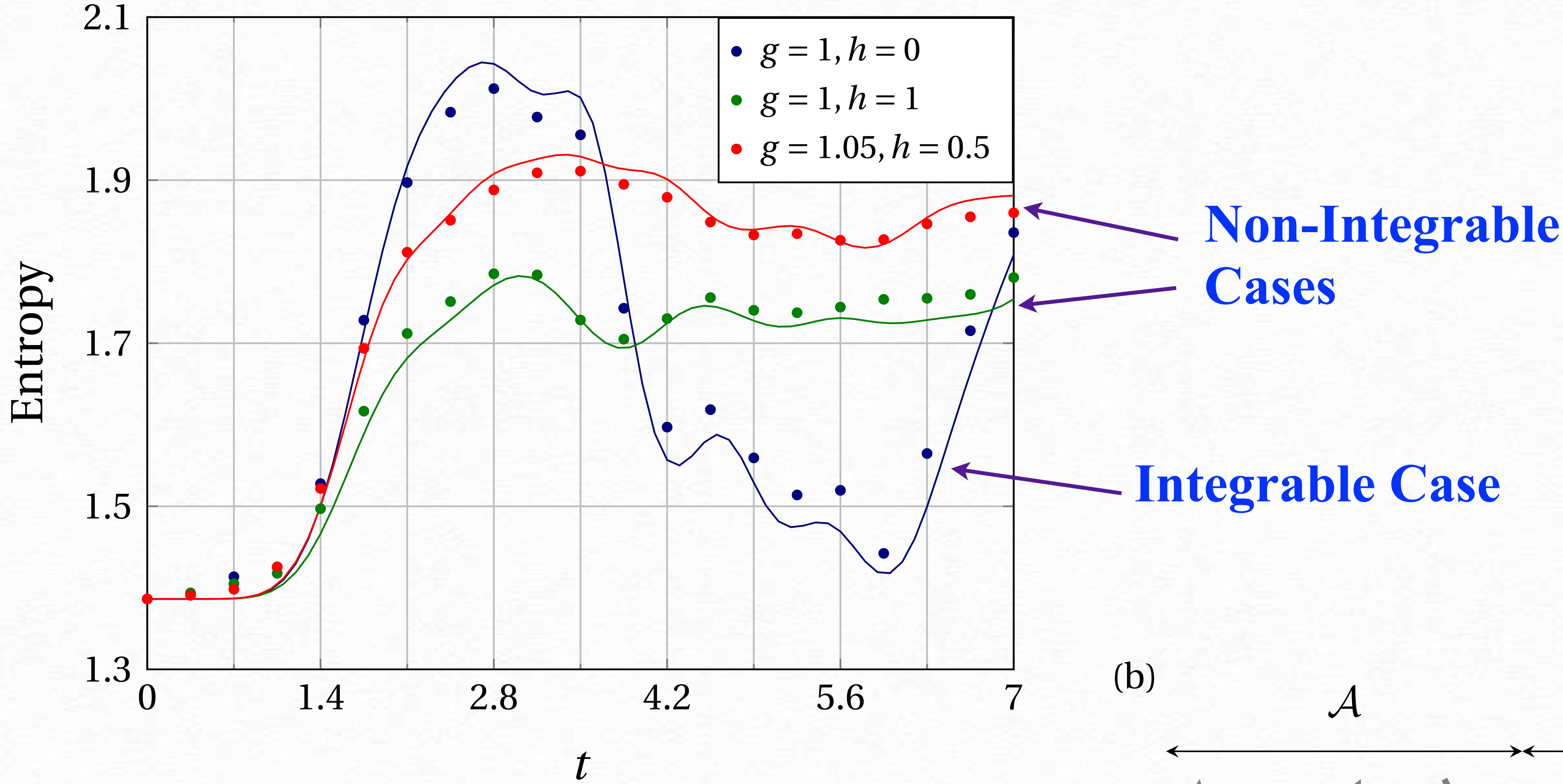
Integrable Case **Non-Integrable Cases**

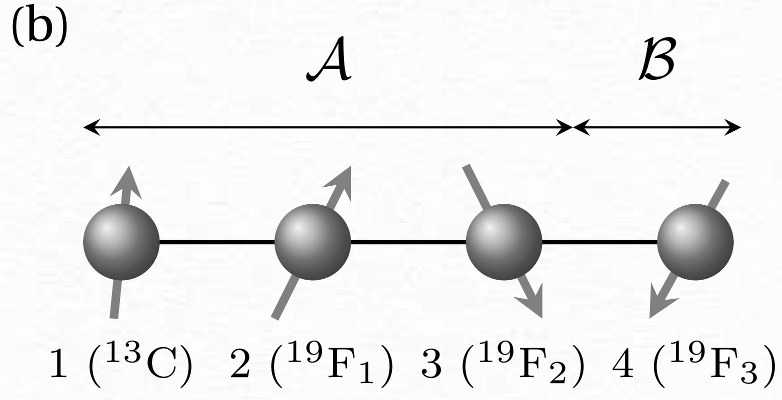


Hosur, Qi, Roberts and Yoshida, 2015

Measurements of OTOC for Ising Chain

$$\exp(-S_A^{(2)}) = \sum_{M \in B} \mathrm{Tr}[\hat{M}(t)\hat{V}(0)\hat{M}(t)\hat{V}(0)]$$





Integrable Case: Information Oscillates

Non-Integrable Case: Information Scrambles

Out-of-Time-Order Correlation for Many-Body Localization

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(Dated: August 16, 2016)

arXiv:1608.01914

Measuring out-of-time-order correlators on a nuclear magnetic resonance quantum simulator

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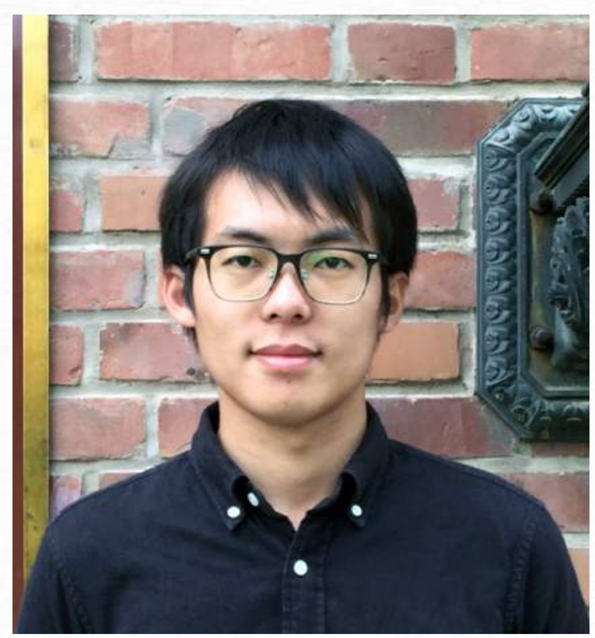
arXiv:1609.01246





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Thank You Very Much for Attention !