Quantum Quench of P-wave Superfluid Fermi Gases

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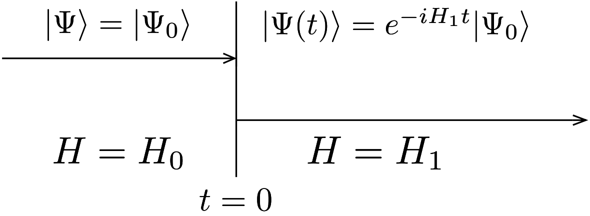
Outline

Non-Equilibrium Dynamics and Cold Atomic Gases

Cold Atomic Gases are good playgrounds for the experimental observation and control of the dynamics

- Intrinsic time scale is large compared with the conventional solid/condensed-matter systems
- Large characteristic length scales
- Can be controlled to be well isolated from the environment for the unitary evolution (just to see the effect of quench alone) after quench

Quench Dynamics

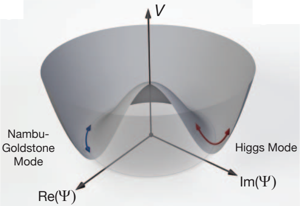


At $t = 0$, a sudden quench (change of system parameter, e.g. coupling constant) is made faster than any time scale of the system.

Dynamics of Order Parameter

Dynamics of Pairing Field in Superfluid

Spontaneous Symmetry Breaking of $U(1)$ symmetry
$\rightarrow$ dynamics of a complex order parameter $\Psi = |\Psi|e^{i\phi}$



-Bogoliubov-Anderson (Nambu-Goldstone) mode : phase dynamics of $\Delta(t)$
-Higgs mode : amplitude dynamics of $\Delta(t)$

- 'Higgs' mode in S-wave Fermi Gas (next section)
- 'Higgs' mode at the 2d Superfluid/Mott insulator transition : M. Endres *et al.* , Nature (2012).

Dynamics of Order Parameter

Caveat

- Gas sample with size L smaller than correlation length.
  $\rightarrow$ Inhomogeneous phase fluctuation and vortices are ignored. (Kibble-Zurek mechanism (KZM), a theory of defect formation, wiil NOT be discussed here.)
- Non-dissipative limit

S-wave Fermi Gas

Pairing Dynamics in the BCS regime

$$\mathcal{H} = \sum_{p,\sigma} \xi_p a^\dagger_{p,\sigma} a_{p,\sigma} - \frac{\lambda(t)}{2} \sum_{p,q} a^\dagger_{p,\uparrow} a^\dagger_{-p,\downarrow} a_{-q,\downarrow} a_{q,\uparrow} \tag{1}$$

Time-dependent many-body BCS state is represented by

$$|\Psi(t)\rangle = \prod_k [u_k(t) + v_k(t) a^\dagger_{p,\uparrow} a^\dagger_{-p,\downarrow}]|0\rangle \tag{2}$$

Time-dependent mean-field pairing function

$$\Delta(t) = \lambda(t) \sum_k u_k(t) v_k^*(t) \tag{3}$$

Bogoliubov-de Gennes equation

$$i\partial_t \begin{pmatrix} u_k \\ v_k \end{pmatrix} = \begin{pmatrix} \xi_k & \Delta \\ \Delta^* & -\xi_k \end{pmatrix} \begin{pmatrix} u_k \\ v_k \end{pmatrix} \tag{4}$$

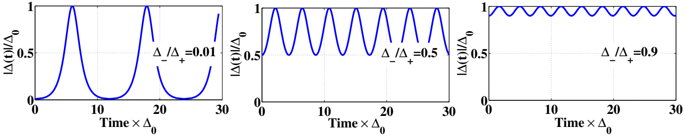
S-wave Fermi Gas

BCS regime

Barankov *et al.* PRL (2004)

$$\lambda(t) = \begin{cases} \lambda_s & \text{at } t < 0, \\ \lambda & \text{at } t > 0. \end{cases}$$

(Δ_0 : Equilibrium value of gap at the final coupling λ)

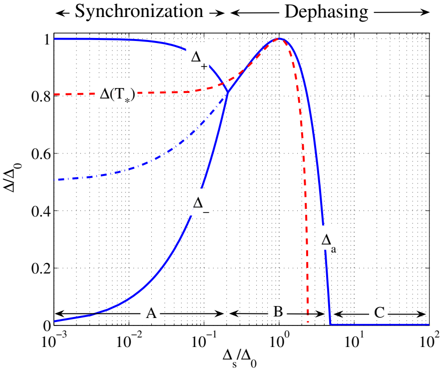


- The system oscillates between the normal/superfluid and superfluid state.
- Integrable : mapped to Bloch precession of Anderson pseudospins
- detectable with the rf-absorption spectroscopy technique : M. Dzero *et al.* PRL (2007)

S-wave Fermi Gas

BCS regime

Barankov *et al.* PRL (2006), Yuzbashyan *et al.* PRL (2006)



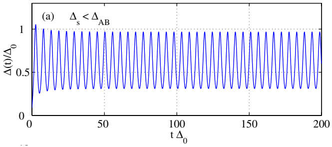
(A) $\omega < 2\Delta$:
no damping (no coupling to quasiparticles)

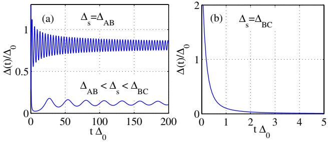
(B) $\omega = 2\Delta$:
nondissipative damping (Landau damping)

$$\Delta(t) = \Delta_a + \frac{A}{\sqrt{t}}\cos(2\Delta_a t + \alpha)$$

Volkov and Kogan, JETP (1974)

(C) overdamped





S-wave Fermi Gas

Unitary Fermi Gas

Gap in a unitary regime is large enough to be measurable.
- Density functional approach : A. Bulgac, PRA (2007)
- Dynamics of the Pairing Correlations in a Unitary Fermi Gas : A. Bulgac and S. Yoon, PRL (2009)

Only one scale ($n^{-1/3}$) exists at the unitary and the simplest energy density functional is, by dimensional analysis,

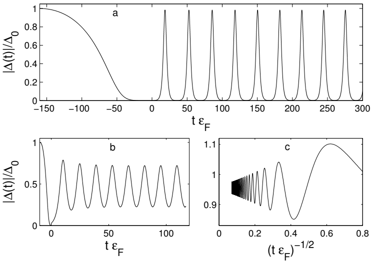
$$\mathcal{E} = \alpha\frac{\tau_c}{2} + \beta\frac{3(3\pi^2)^{2/3}n^{5/3}}{10} + g_{eff}|\nu_c|^2 \ , \frac{1}{g_{eff}} = \frac{n^{1/3}}{\gamma} + \Lambda_c$$

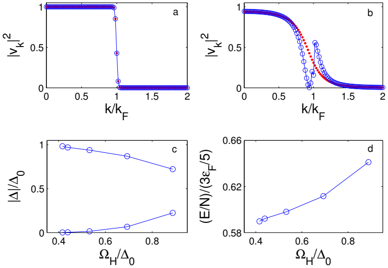
$$i\partial_t \begin{pmatrix} u_k \\ v_k \end{pmatrix} = \begin{pmatrix} h-\mu & \Delta \\ \Delta^* & -(h-\mu) \end{pmatrix} \begin{pmatrix} u_k \\ v_k \end{pmatrix} \quad (5)$$

$$h = -\frac{\alpha\nabla^2}{2} + \frac{\delta\mathcal{E}}{\delta n} \quad , \quad \Delta = -g_{eff}\nu_c \quad (6)$$

S-wave Fermi Gas

Unitary Fermi Gas





Higgs modes (excitations of $|\Delta(t)|$) exist also in a unitary regime by superfluid local density approximation (SLDA) formulation.

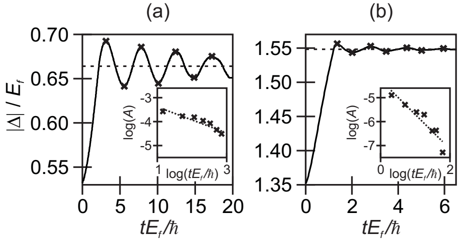
S-wave Fermi Gas

- BEC regime : V. Gurarie, PRL (2009)

$$|\Delta(t)| = \Delta_a + \frac{A}{t^{3/2}} \cos(2\sqrt{\mu^2 + \Delta_a}\ t + \alpha)$$

$t^{-3/2}$: probability of the molecular decay as a function of time

- Crossover regime : R. Scott *et al.* PRA (2012) - BdG eqs.



(a) $1/k_f a_0 = 0.2$ to 0 (b) $1/k_f a_0 = 0.8$ to 1

Single-species P-wave Fermi Gas

- Superfluids paired at a finite angular momentum :
  richer order parameters and phase transitions within the superfluid phase
- Single-species Fermi gas :
  p-wave scattering dominates due to Pauli exclusion principle

P-wave Fermi Gas

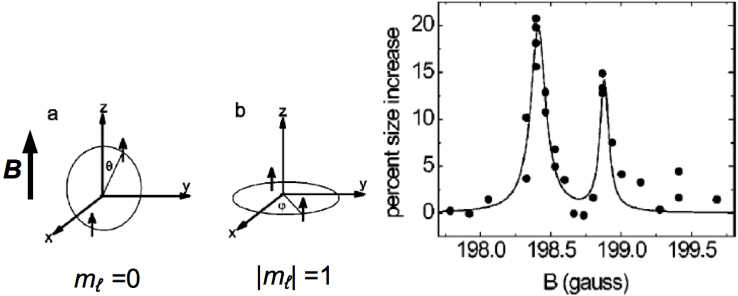
Splitting of p-wave FRs by dipolar interaction

Ticknor et al. PRA (2004) :

-Valence electron spins are polarized along B.

-Magnetic dipole-dipole interaction splits FRs with $|m_l| = 0$ and 1.

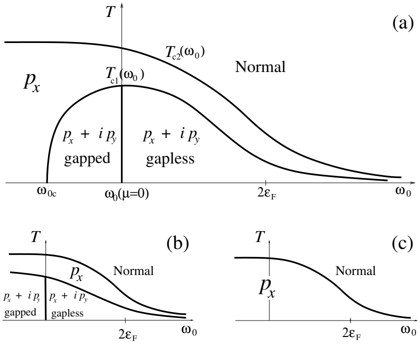
^{40}K: large splitting



Single-species P-wave Fermi Gas

Quantum Phase Transitions across a P -Wave Feshbach Resonance

V. Gurarie *et al.* PRL (2005), V. Gurarie *et al.* Ann. Phys. (2007)



(c) Large FR splitting :
- Pairing occurs in $m = 0$ state
- Polar state undergoes QPT (at $\mu = 0$) from 'Gapless' phase to 'Gapped' phase

T. Ho and R. Diener, PRL (2005)

$$H = \sum_{\mathbf{k}} \xi_{\mathbf{k}} \hat{a}^{\dagger}_{\mathbf{k}} \hat{a}_{\mathbf{k}} + \frac{1}{2} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} V_{l=1}(\mathbf{k},\mathbf{k}') \hat{a}^{\dagger}_{\mathbf{k}+\frac{\mathbf{q}}{2}} \hat{a}^{\dagger}_{-\mathbf{k}+\frac{\mathbf{q}}{2}} \hat{a}_{-\mathbf{k}'+\frac{\mathbf{q}}{2}} \hat{a}_{\mathbf{k}'+\frac{\mathbf{q}}{2}}$$

$$V_1(\mathbf{k},\mathbf{k}') = -4\pi g \Gamma^*(\mathbf{k}) \Gamma(\mathbf{k}')$$

$$\Gamma(\mathbf{k}) = \frac{k k_0}{k^2 + k_0^2} Y_{1,m}(\hat{\mathbf{k}})$$

- Determine g and k_0 (momentum cutoff) matching the low-energy scattering amplitude for the p-wave channel (a_1: scattering length, r_1: effective range, b: range of potential)

$$f_{l=1}(k) \overset{kb \ll 1}{=} \frac{(kb)^2}{-\frac{1}{a_1} + \frac{r_1 k^2}{2} - i(kb)^2 k} = \frac{k^2}{-\frac{1}{a_p} + \frac{r_p k^2}{2} - i(k)^2 k} \qquad (7)$$

P-wave Fermi Gas II

Formulation

$$\frac{1}{4\pi g} = -\frac{MV}{16\pi^2 a_p k_0^2} + \sum_k \frac{|\Gamma(k)|^2}{2\epsilon(k)}, \;\; r_p = -\left(k_0 + \frac{4}{k_0^2 a_p}\right)$$

($a_p \equiv a_1 b^2$ and $r_p = r_1/b^2$ have the dimensions of volume and inverse length.)

- Time evolution of an initial state $|\Psi(t=0)\rangle$

$$|\Psi(t)\rangle = e^{-iHt}|\Psi(t=0)\rangle = \prod_k [u_k(t) + v_k(t)\hat{a}_k^\dagger \hat{a}_{-k}^\dagger]|0\rangle$$

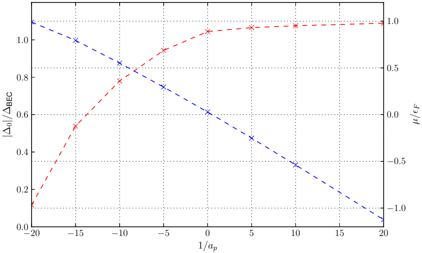
$$i\frac{\partial}{\partial t}\begin{pmatrix} u_{\mathbf{k}}(t) \\ v_{\mathbf{k}}(t) \end{pmatrix} = \begin{pmatrix} h_{\mathbf{k}} & \Delta_{\mathbf{k}}(t) \\ \Delta_{\mathbf{k}}^*(t) & -h_{\mathbf{k}} \end{pmatrix}\begin{pmatrix} u_{\mathbf{k}}(t) \\ v_{\mathbf{k}}(t) \end{pmatrix} \tag{8}$$

$$\Delta_{\mathbf{k}}(t) = \sum_{\mathbf{k}'} V_1(\mathbf{k}, \mathbf{k}') u_{\mathbf{k}'}^*(t) v_{\mathbf{k}'}(t) \tag{9}$$

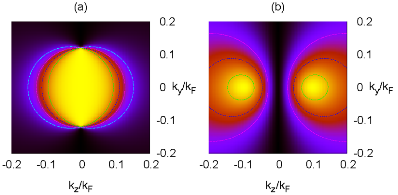
P-wave Fermi Gas

Equilibrium Properties of the Polar State

Iskin and Sá de Melo, PRL (2006)



momentum distribution

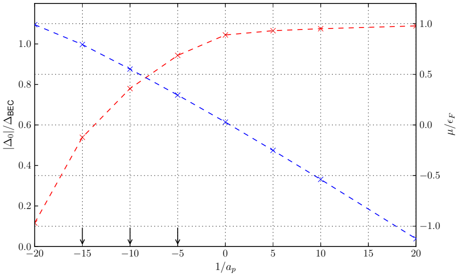


(a) BCS ($\mu > 0$) (b) BEC ($\mu < 0$)

- Near $1/k_F^3 a_p = 0$, μ changes sign.
- Quasiparticle spectrum : $E = \sqrt{(\epsilon_k - \mu)^2 + |\Delta_0 \cos\theta|^2}$
  gapless for $\mu > 0$ to gapped for $\mu < 0$: QPT at $\mu = 0$

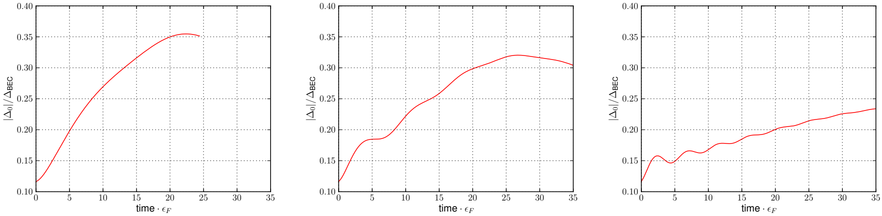
P-wave Fermi Gas

Quenching Polar State ($m = 0$)



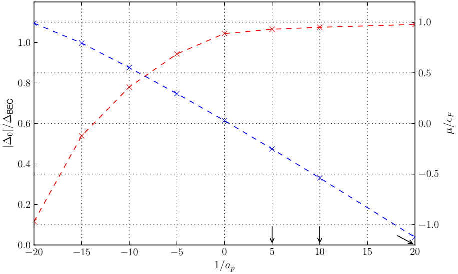
Within BCS side :

- Two time-scales appear
- Large time-scale is connected with a large depletion of momentum occupation inside the Fermi sea (will be shown later)

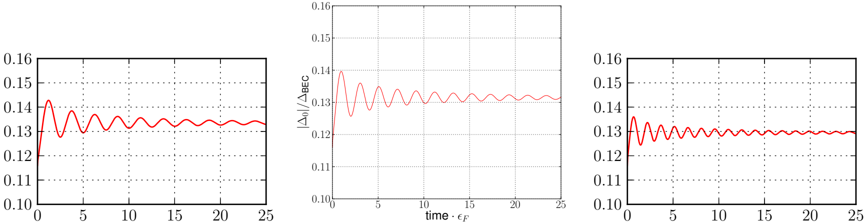


P-wave Fermi Gas

Quenching Polar State ($m = 0$)

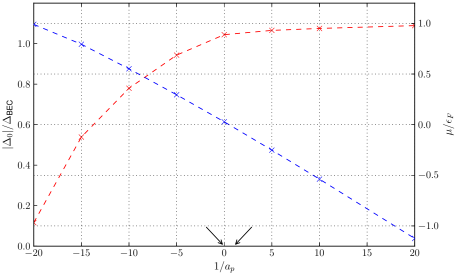


To BEC side :
Decaying oscillation with one time-scale.



P-wave Fermi Gas

Quenching Polar State ($m = 0$)

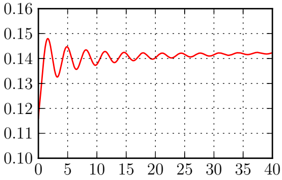


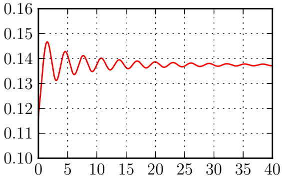
Close to QPT point :

$\mu_{eq}/\epsilon_F = +0.03$ at $1/a_p = 0$

$\mu_{eq}/\epsilon_F = -0.03$ at $1/a_p = 1$

long time-scale disappears after quenching across QPT.





P-wave Fermi Gas

Quenching Polar State ($m = 0$)

Quasiparticle momentum distribution (Quench within BCS side)

$1/a_p = -20 \to -15$

from -20 to -15

from -20 to -15

$1/a_p = -20 \to -5$

from -20 to -5

from -20 to -5

P-wave Fermi Gas

Quenching Polar State ($m = 0$)

Quasiparticle momentum distribution

$1/a_p = -20 \to 0$

from -20 to 0

from -20 to 0

$1/a_p = -20 \to +10$

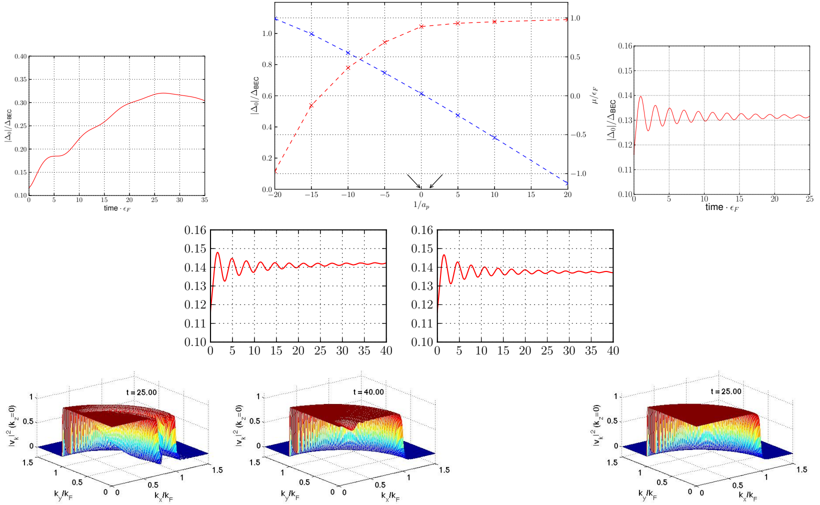
from -20 to +10

from -20 to +10

Summary

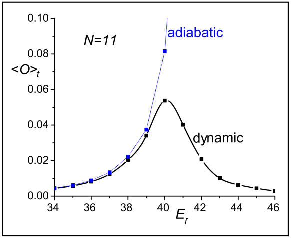
Quenching Polar State ($m = 0$)

Quench Dynamics of $|\Delta_0|$ (solid red line).



Discussion

Quantum dynamics of the 1D dipole model of the Mott insulator in a potential gradient : K. Sengupta *et al.* PRA (2004)



$$H_{1\mathrm{D}}[E] = -w\sqrt{n_0(n_0+1)}\sum_\ell (d_\ell^\dagger + d_\ell) + (U-E)\sum_\ell d_\ell^\dagger d_\ell$$

Study the dynamics of the Ising density wave order parameter $O = \frac{1}{N}\langle\Psi|\sum_\ell(-1)^\ell d_\ell^\dagger d_\ell|\Psi\rangle$ as E is changed rapidly across the QCP ($E_c = 41.85$)

- $\langle O\rangle_t$ stays close to O_{ad} as long as there is a large overlap between the initial and the new ground states.

In the case of the P-wave polar mode, the qualitative behavior is similar to the case studied by K. Sengupta *et al.* PRA (2004) :

- The change of the magnitude of the order parameter is very small when the final couplings are at the BEC side while the initial coupling is at the BCS side.
- When the coupling is changed within BCS side, longer time-evolution is need for the clarification.

Conclusion

Quench Dynamics of $|\Delta_0|$ (Pairing field of a polar state is expressed as $\Delta(\mathbf{k}) \sim \Delta_0 f(k) Y_{1,0}(\hat{\mathbf{k}})$).

- Two time-scales appear in the dynamics of a p-wave Fermi gas after a sudden quench within BCS side ($\mu > 0$).
- Large time-scale oscillation disappears after a sudden quench across the QPT point.
- Large depletion of momentum occupation inside the Fermi sea approaches the center of the Fermi sea when the final coupling approaches the QPT point from BCS side and it disappears when the final coupling is at the BEC side. The time-scale of large depletion of momentum occupation corresponds to large time-scale of pairing field dynamics.