Extrinsic Spin Hall Effect in Graphene

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NCTS workshop QC13 2013/09/03

Acknowledgements

Collaborators

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Singapore

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Rio de Janeiro

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Singapore

Discussions

Barbaros Ozylmaz
Singapore

Jayalakumar
Singapore

“Spintronic Dreams”

Spin **+** *Electronics* **=** *Spintronics*

"Spintronic Dreams"

Spin + *Electronics* = *Spintronics*

Dissipationless Quantum Spin Current at Room Temperature

Shuichi Murakami,[1*] Naoto Nagaosa,[1,2,3] Shou-Cheng Zhang[4]

Although microscopic laws of physics are invariant under the reversal of the arrow of time, the transport of energy and information in most devices is an irreversible process. It is this irreversibility that leads to intrinsic dissipations in electronic devices and limits the possibility of quantum computation. We theoretically predict that the electric field can induce a substantial amount of dissipationless quantum spin current at room temperature, in hole-doped semiconductors such as Si, Ge, and GaAs. On the basis of a generalization of the quantum Hall effect, the predicted effect leads to efficient spin injection without the need for metallic ferromagnets. Principles found here could enable quantum spintronic devices with integrated information processing and storage units, operating with low power consumption and performing reversible quantum computation.

Murakami, Nagaosa, Zhang, Science (2003)

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Spin + *Electronics* = *Spintronics*

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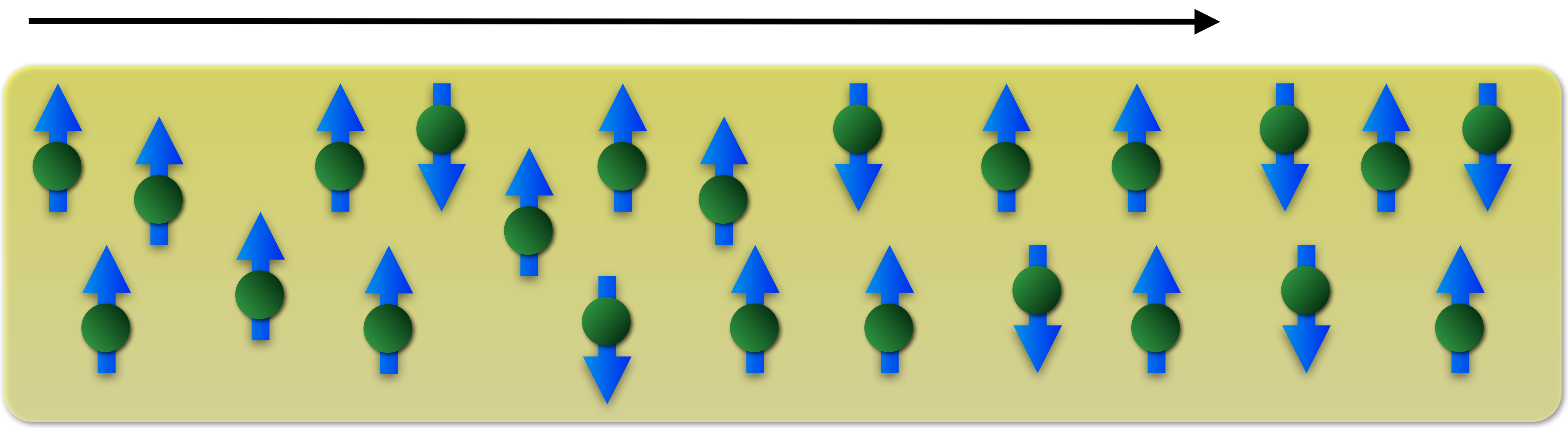
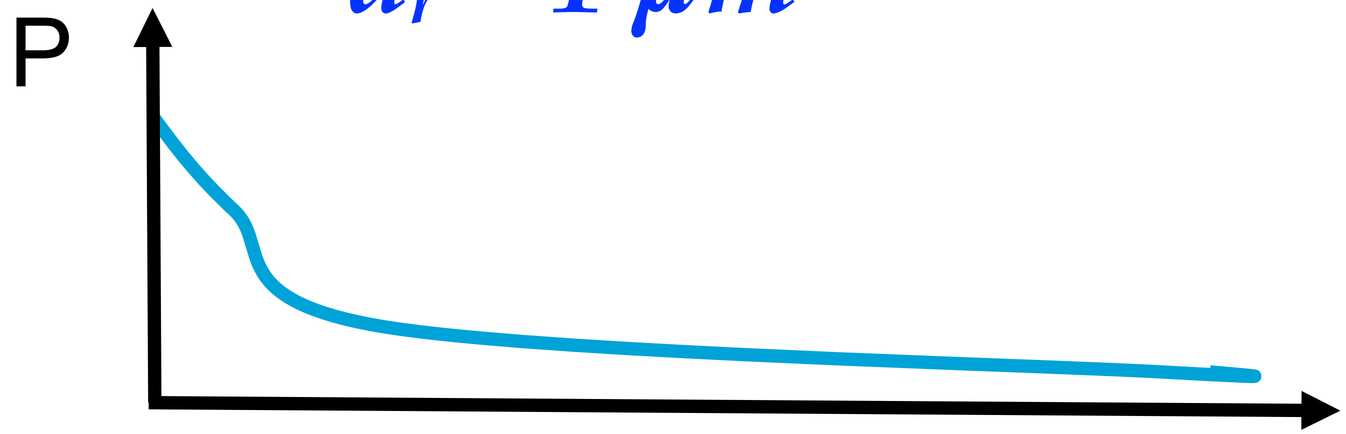
Murakami, Nagaosa, Zhang, Science (2003)

Spin Relaxation in Solids

$\tau_s \sim 1\ ns$

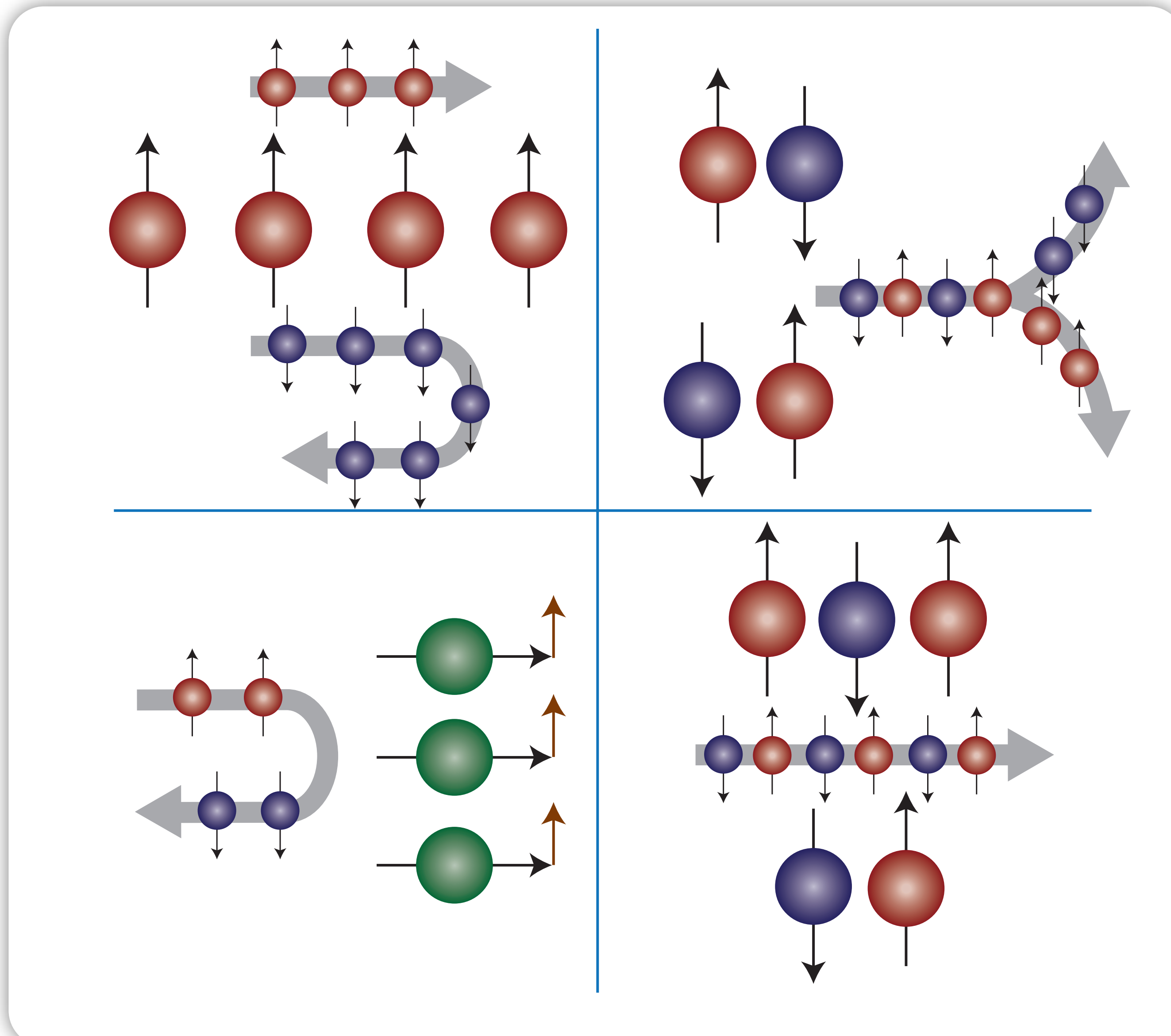


$d_r \sim 1\ \mu m$

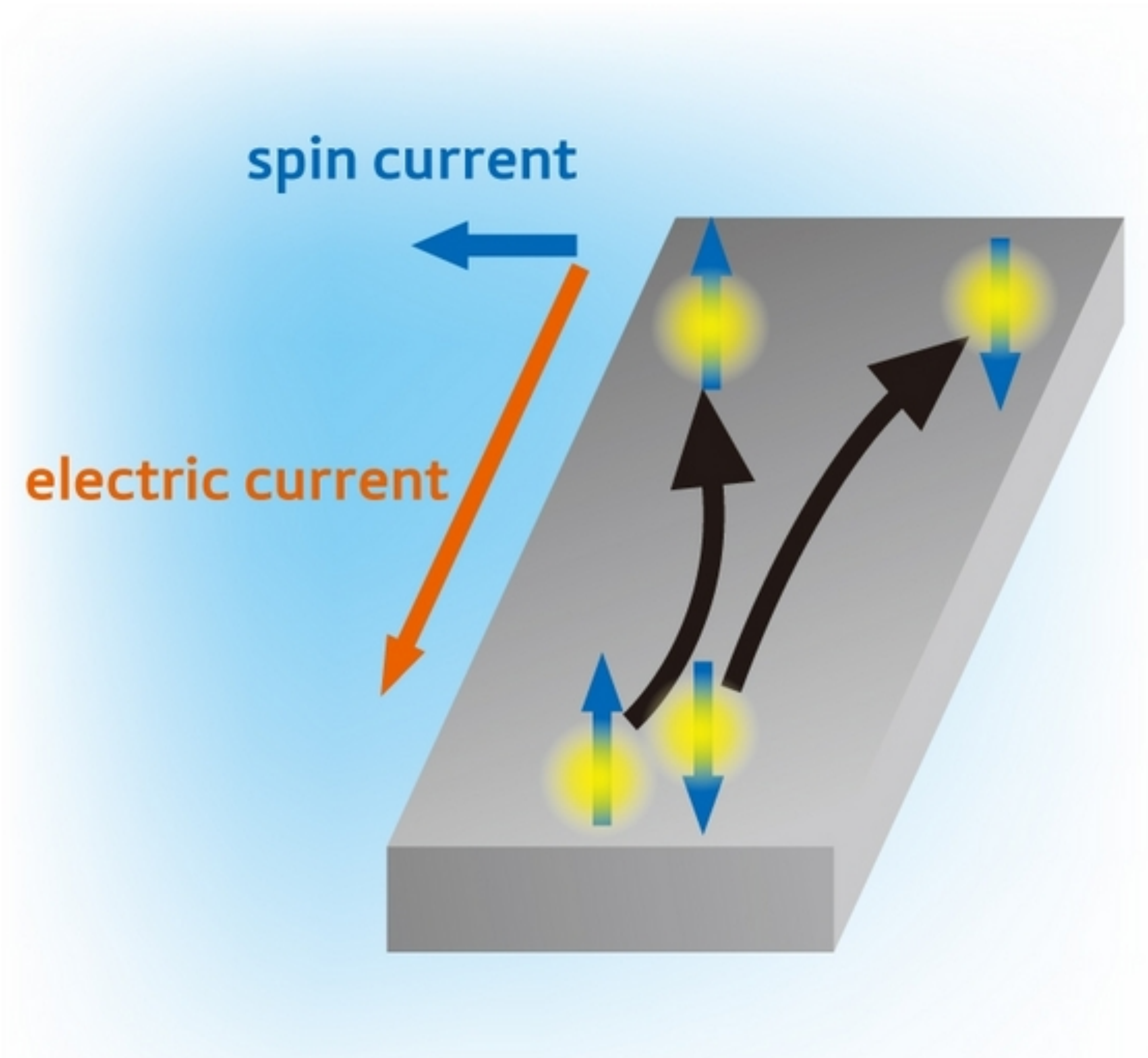


Generation of Spin Currents

Magnetic *Non-magnetic*



The Spin Hall Effect



Dyakonov & Perel (1971)
J Hirsch (1999)

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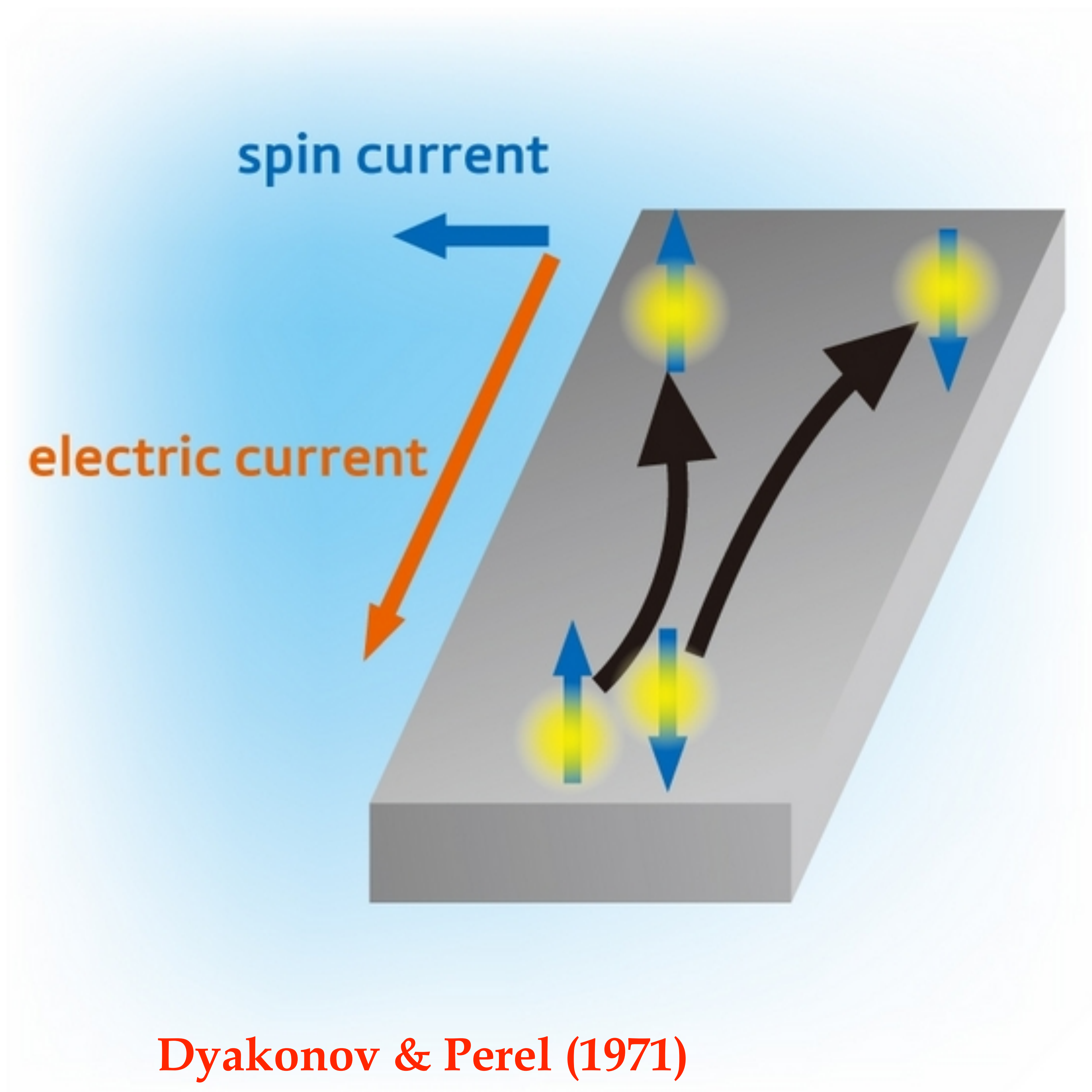

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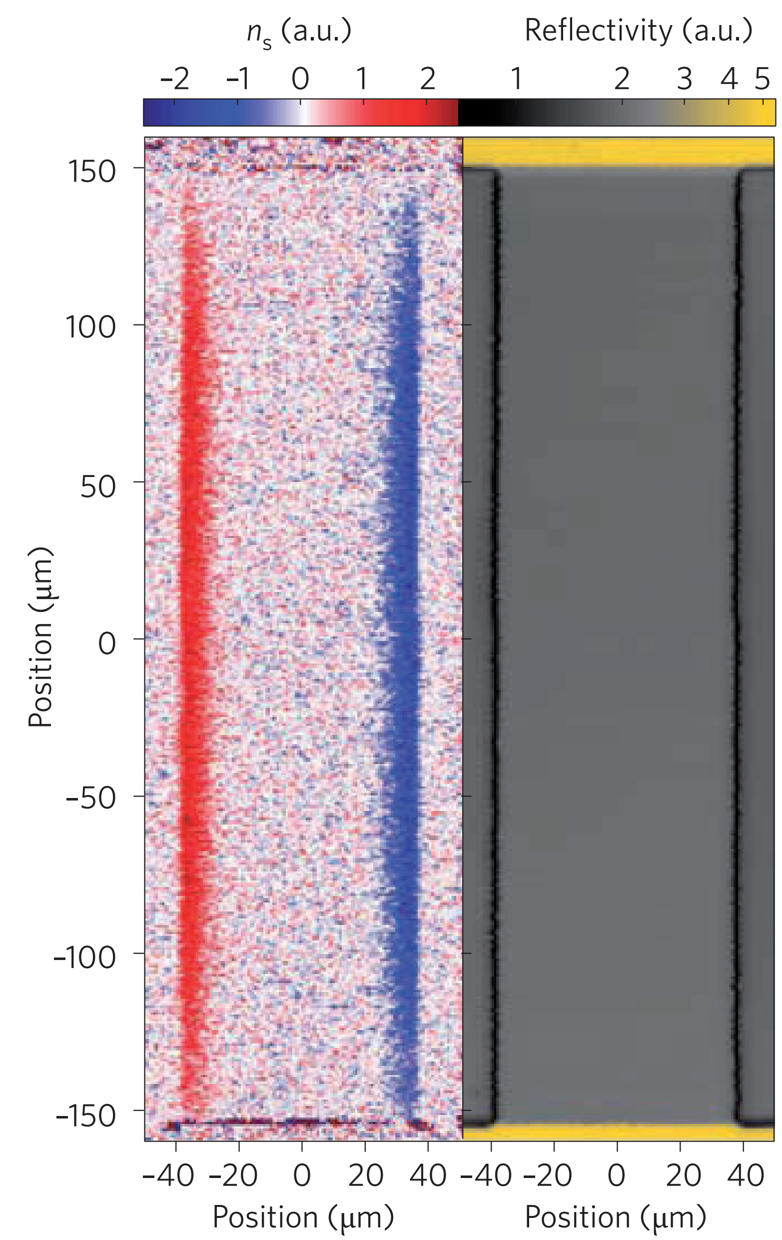
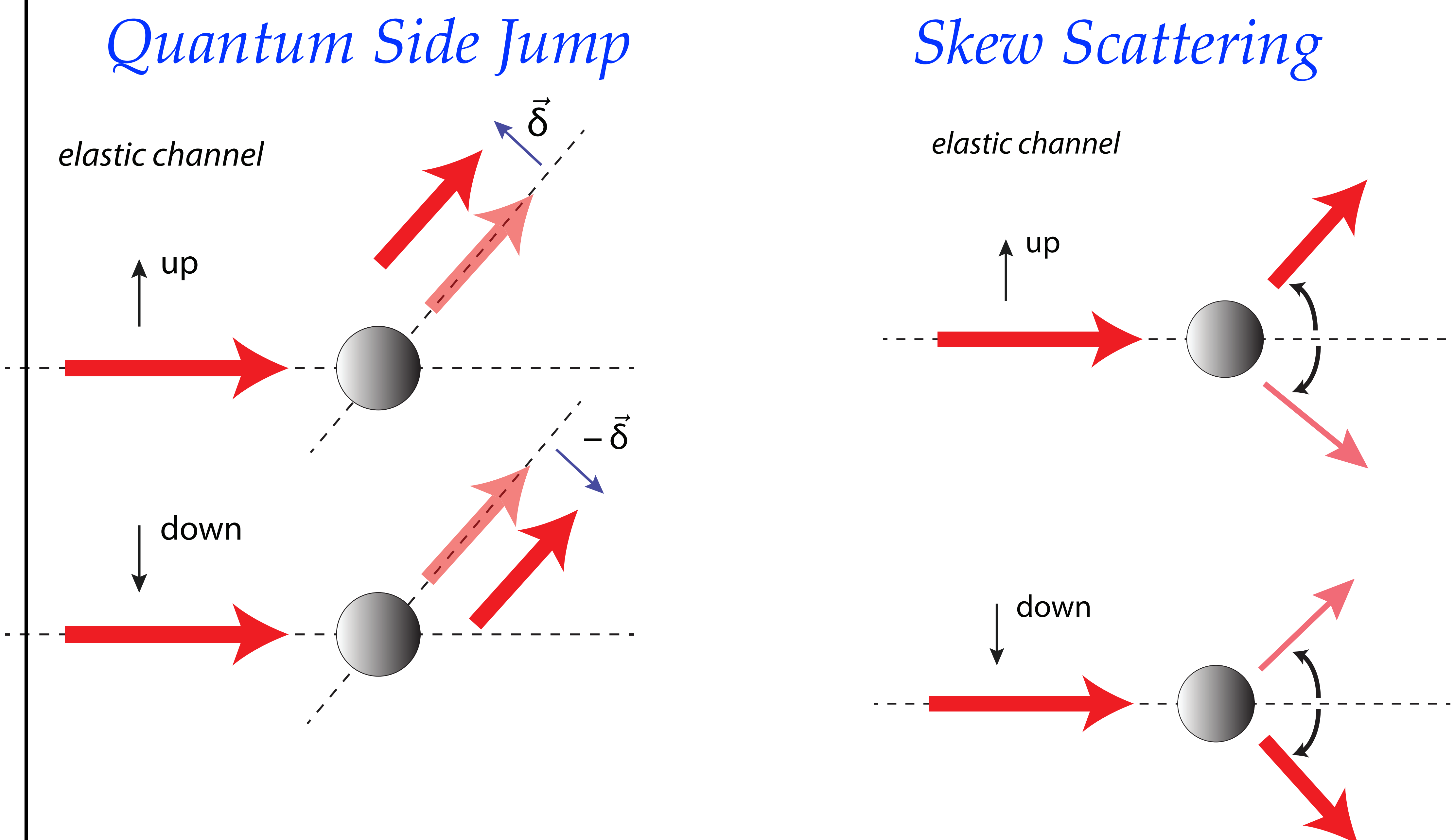
GaAs



Magneto-optical Kerr Microscope
Kato *et al* Science (2004)

Extrinsic Mechanisms for SHE



Nagaosa, Sinova, Onoda, MacDonald, Ong, Rev. Mod. Phys. (2010)

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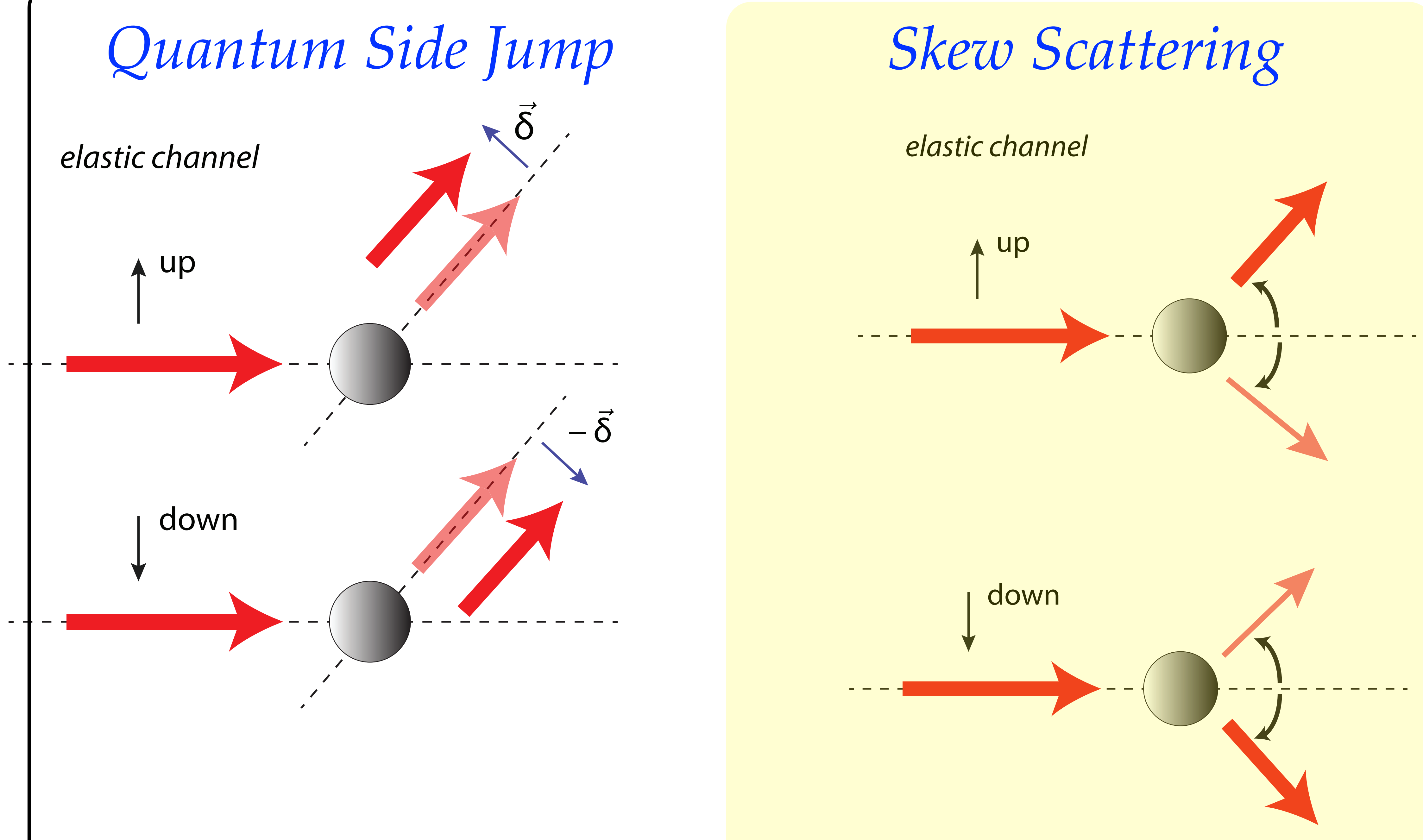
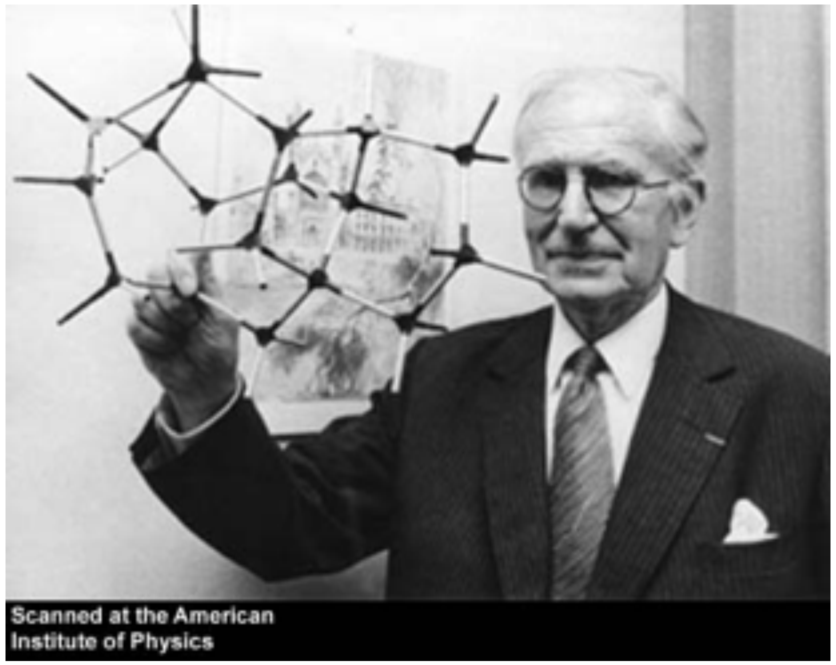
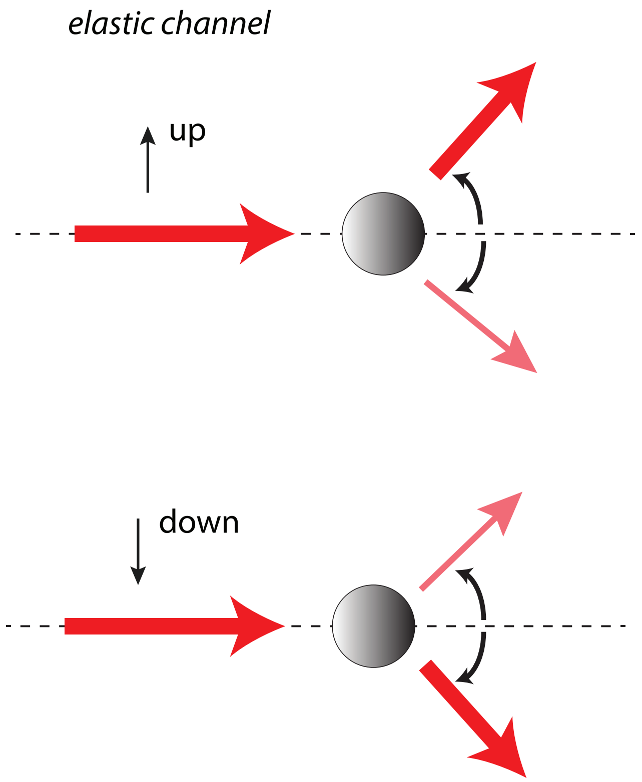


Nagaosa, Sinova, Onoda, MacDonald, Ong, Rev. Mod. Phys. (2010)

Skew Scattering Mechanism



Sir Nevil F. Mott



$$W_{\alpha\beta}(\mathbf{k},\mathbf{p}) = |\langle \mathbf{k}\alpha|T|\mathbf{p}\beta\rangle|^2; T = V + V\frac{1}{\epsilon - H_0 + i\epsilon^+}V$$

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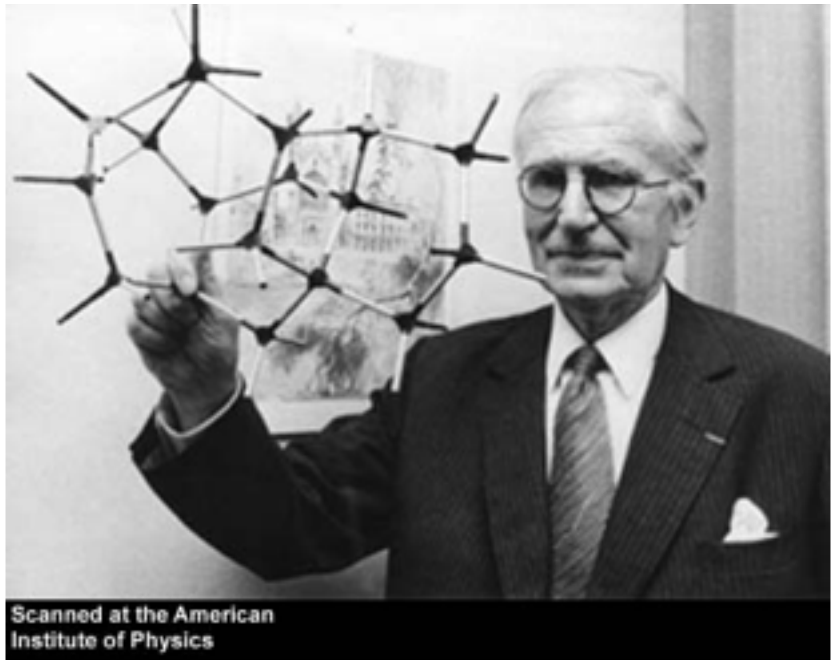

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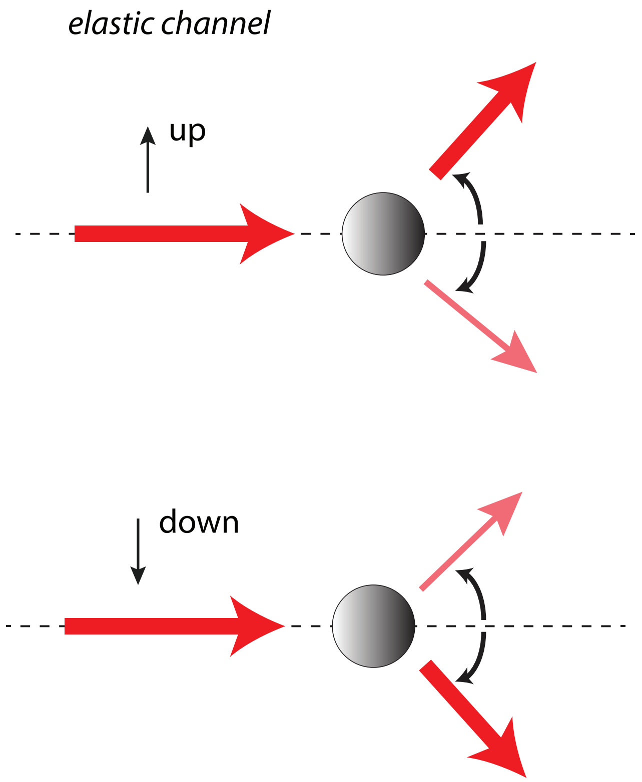

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Spin-Orbit coupling $\Rightarrow$ Breaking of micro-reversivility

$$W_{\alpha\beta}(\mathbf{k},\mathbf{p}) \neq W_{\alpha\beta}(\mathbf{p},\mathbf{k})$$

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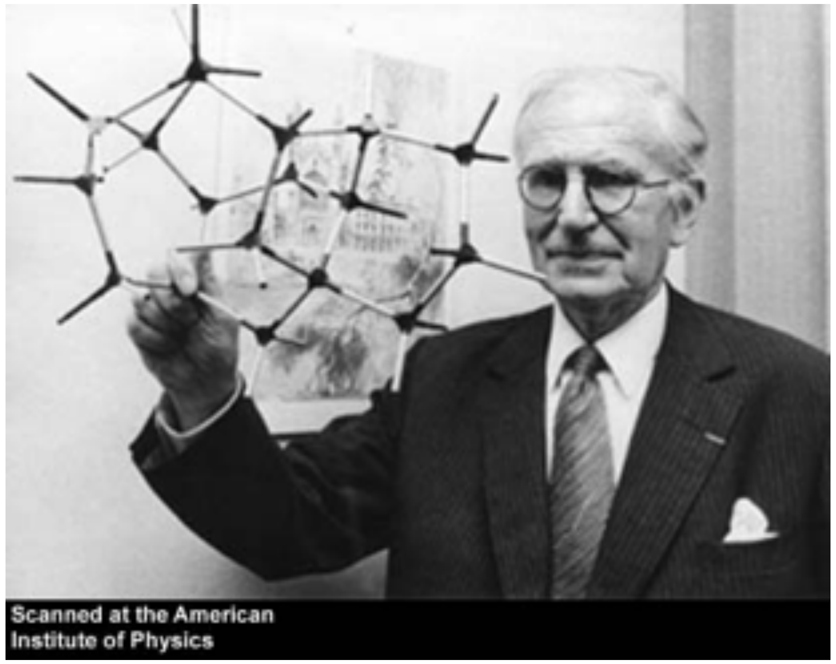

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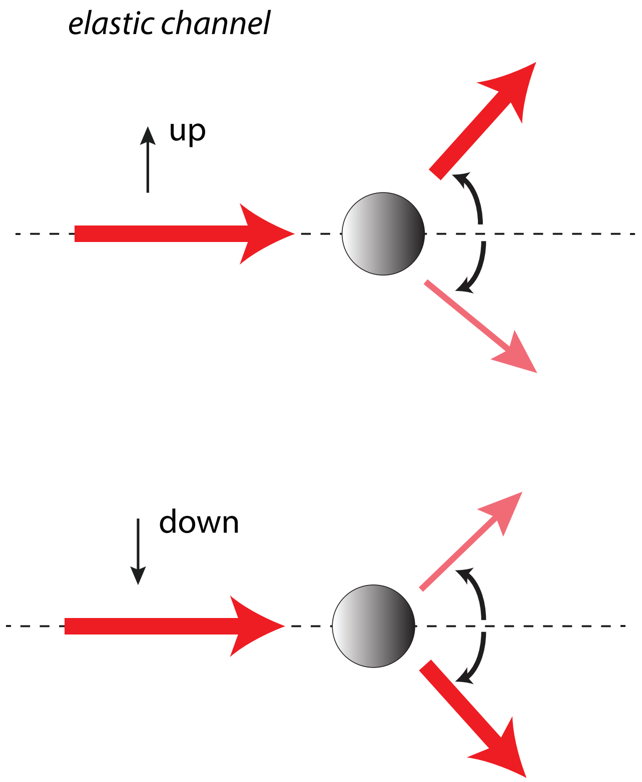

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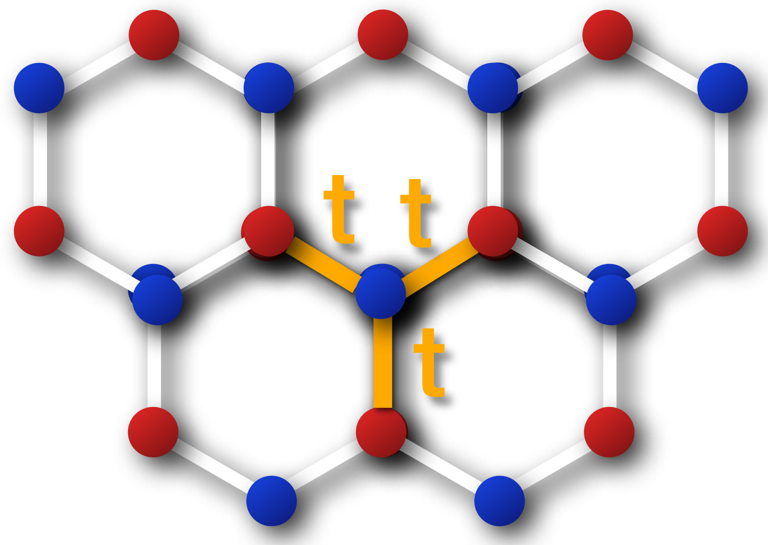
Spin-Orbit coupling ⇒ Breaking of micro-reversivility

$$W_{\alpha\beta}(\mathbf{k},\mathbf{p}) \neq W_{\alpha\beta}(\mathbf{p},\mathbf{k})$$

It cannot be captured by the 1st Born approximation $T \simeq V$

Graphene

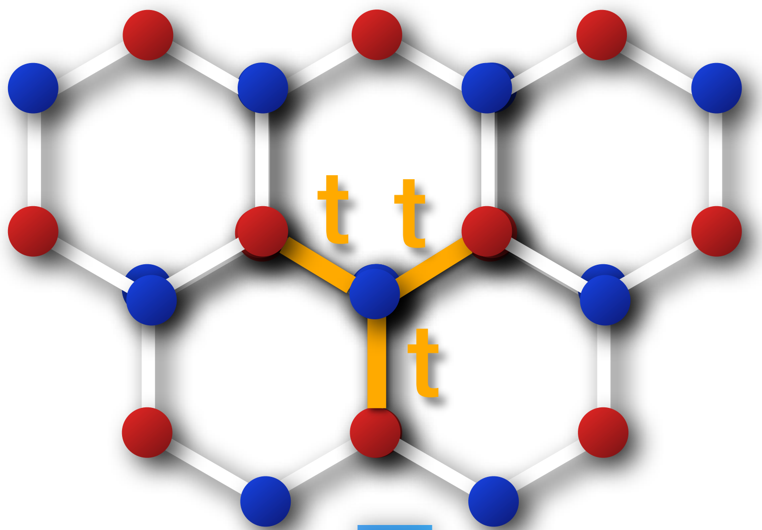


$t \simeq 2.7$ eV

Carbon π band: tight binding

$$\mathcal{H}_g = -t \sum_{\sigma,\langle ij\rangle} a_\sigma^\dagger(\mathbf{R}_i) b_\sigma(\mathbf{R}_j) + \text{h.c.}$$

Graphene



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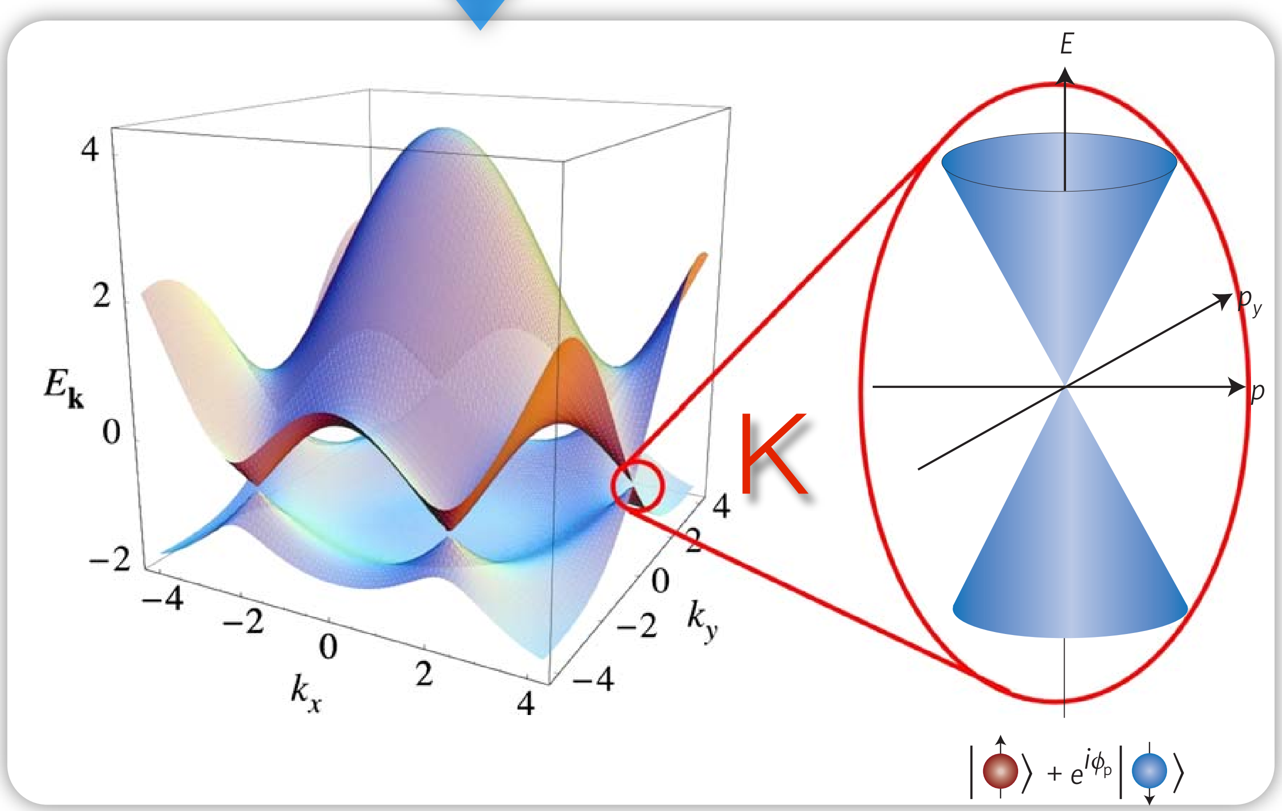

Low-energy description

$$H = v_f \sigma \cdot \mathbf{p}$$

$$v_f \sim c/300$$

Graphene Spintronics?

Spin-Orbit Coupling in Grahene

Project $\vec{L} \cdot \vec{S}$ Interaction onto $\boldsymbol{\pi}$ band

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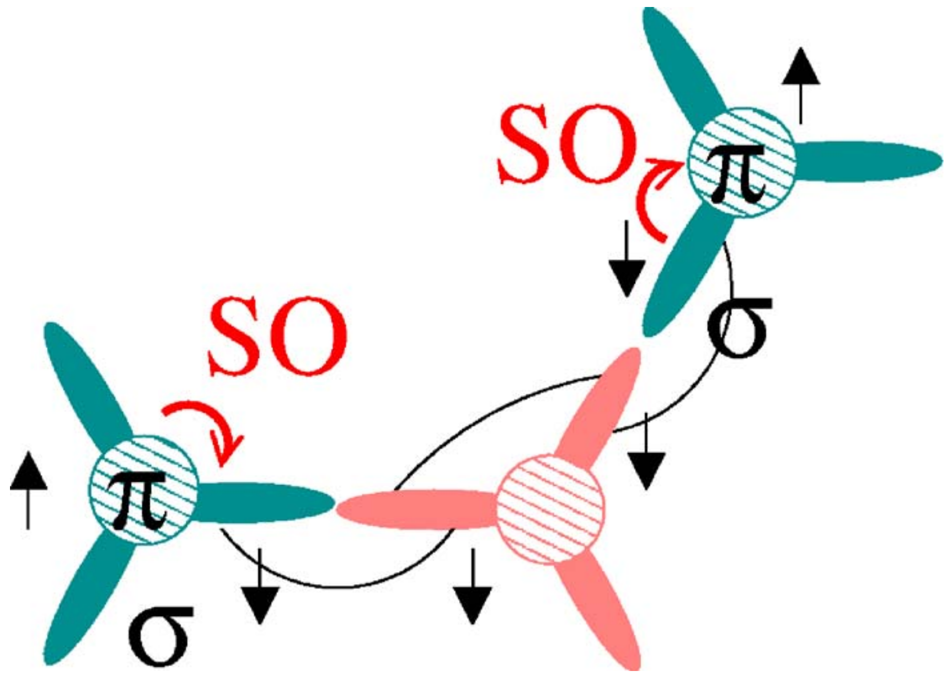
$$\Delta_{\mathrm{SO}} \sim 50\mu\mathrm{eV}$$

(Extremely weak)

s-p mixing in flat graphene

D Huertas-Hernando *et al*
Phys Rev B (2006)



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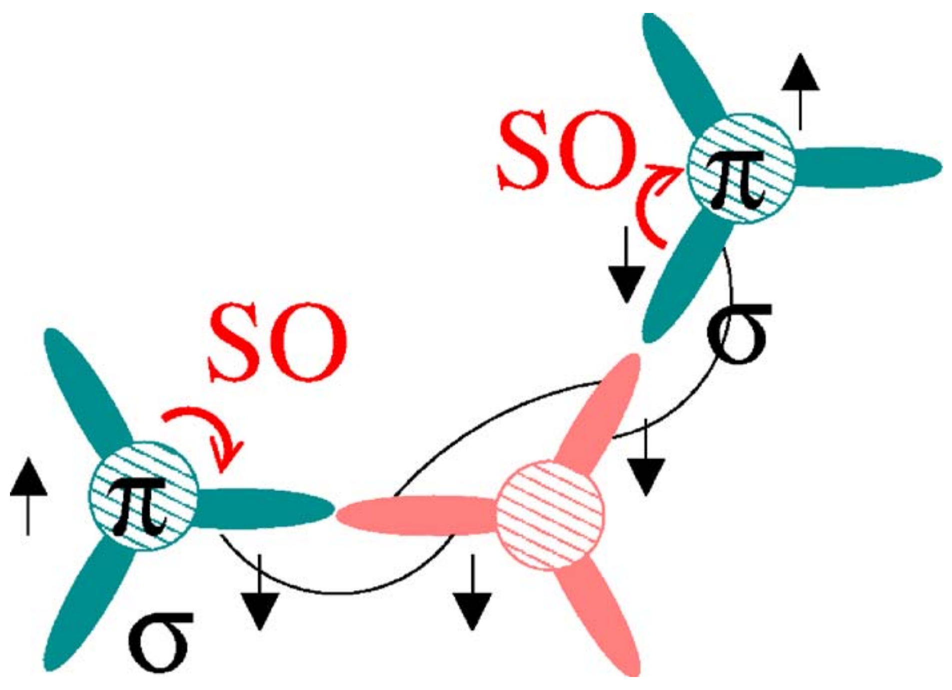
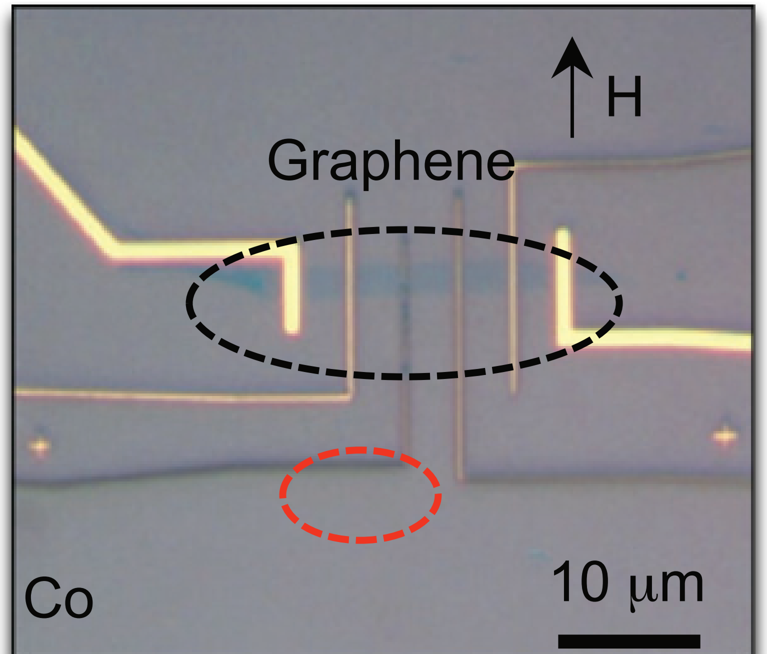
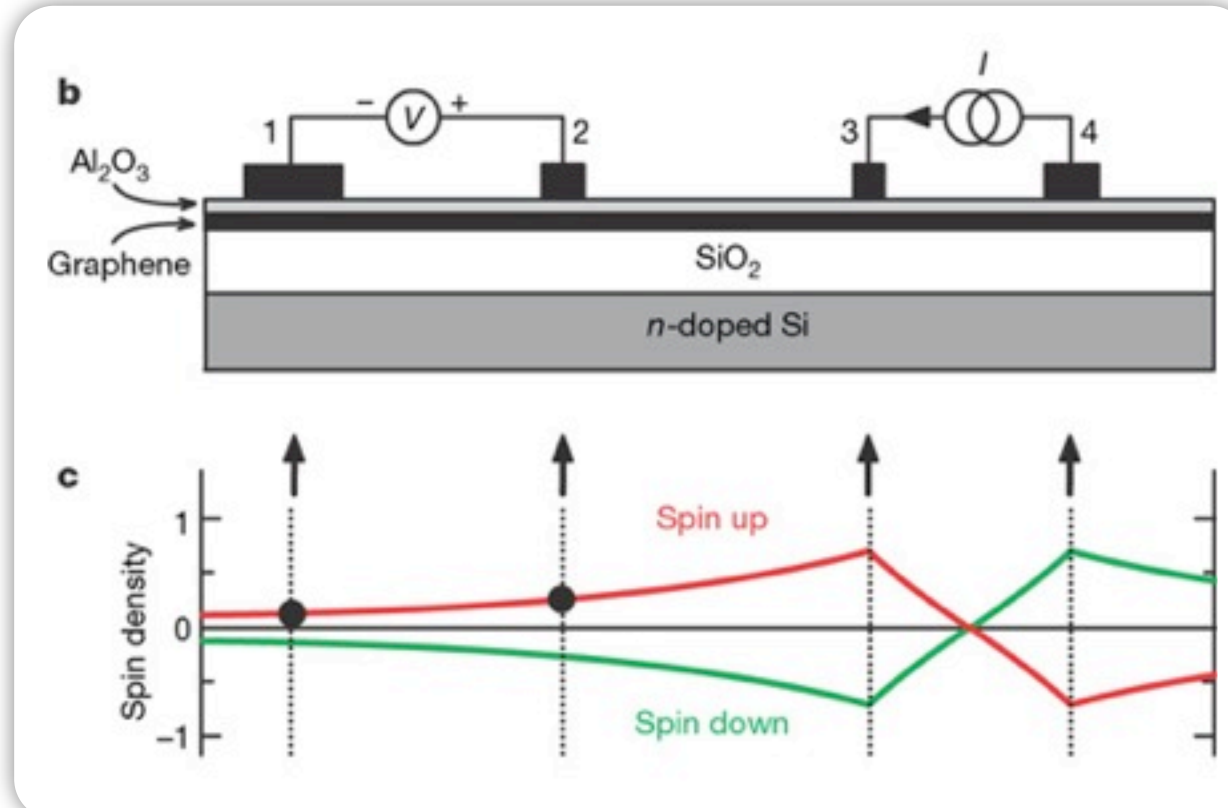


s-p and d-p mixing
are comparable in magnitude

S Konchuh et al, PRB (2010)

Passive Spintronics in Graphene





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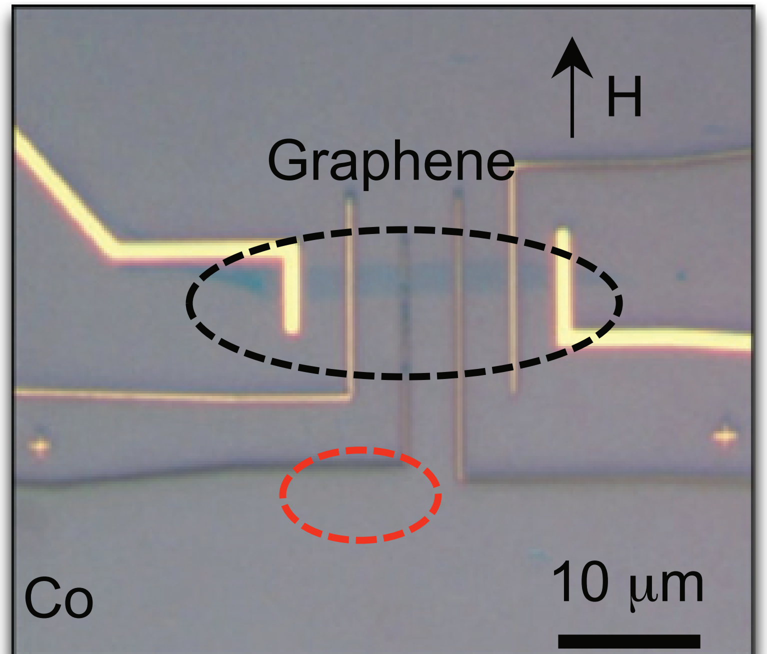

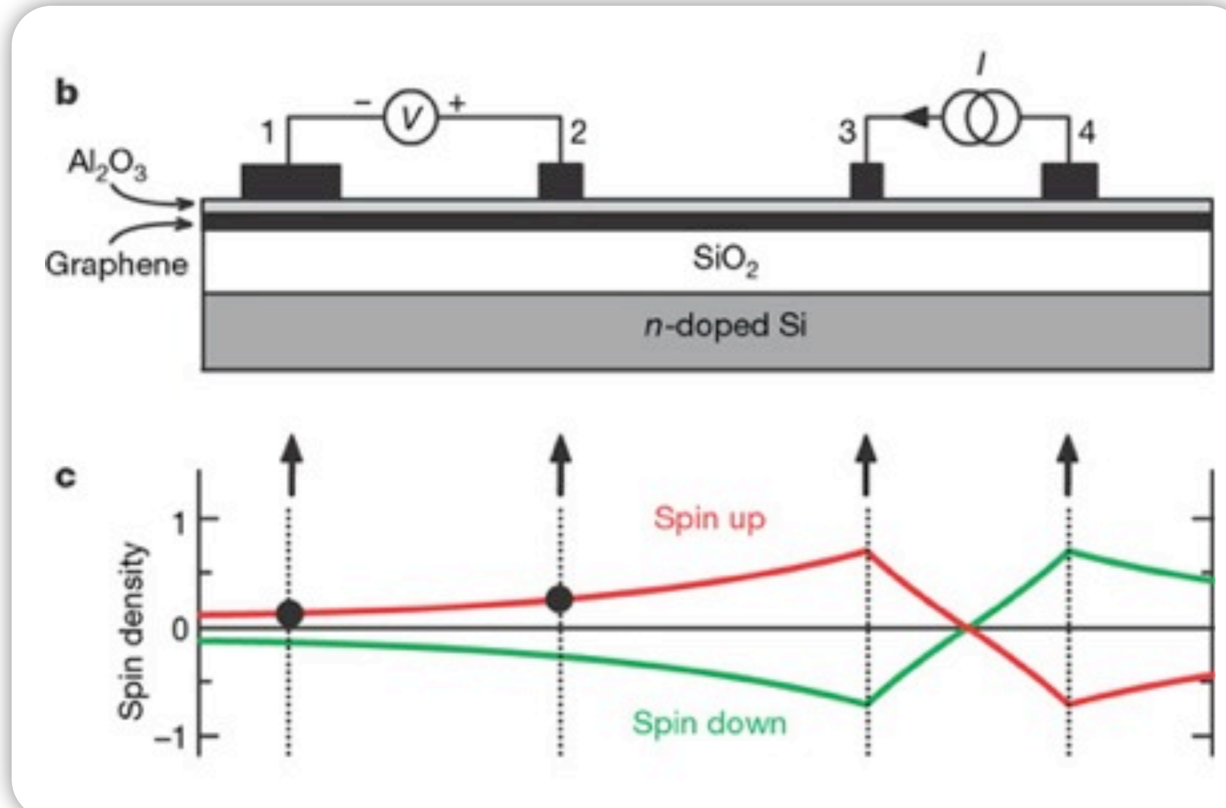

Long Spin Diffusion

λ_s ***up to*** $20\mu\mathrm{m}$ **Zomer *et al*, PRB(R) (2012)**
(graphene on BN)

λ_s ***up to*** $250\mu\mathrm{m}$ **Dlubak *et al*, Nat. Phys. (2012)**
(epitaxial graphene)

Active Spintronics in Graphene

Quantum Spin Hall Effect in Graphene

C. L. Kane and E. J. Mele
Dept. of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA
(Received 29 November 2004; published 23 November 2005)

$$H_{SO} = \Delta_I \, \tau^z \sigma^z s^z$$

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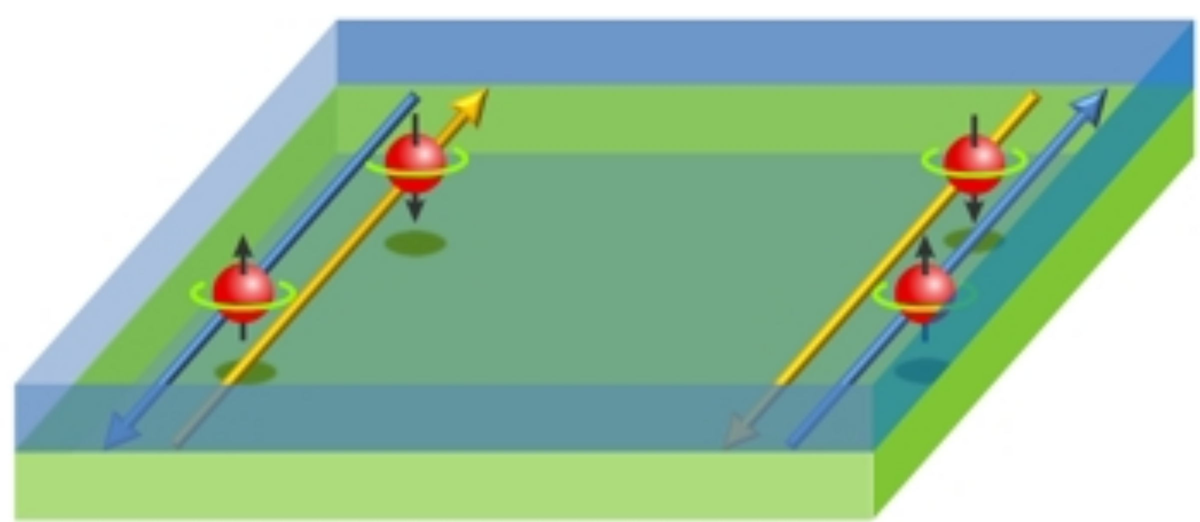
Symmetry Protected Topological Phase



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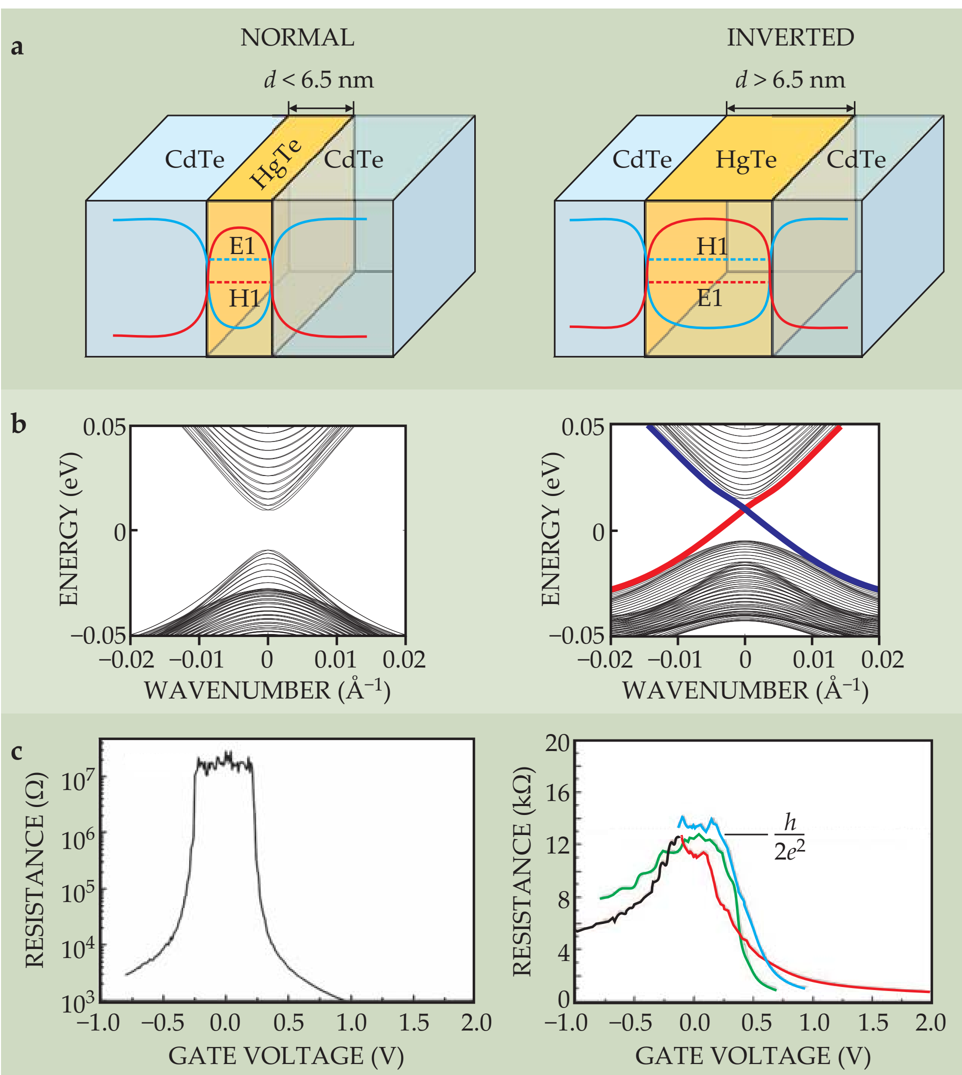

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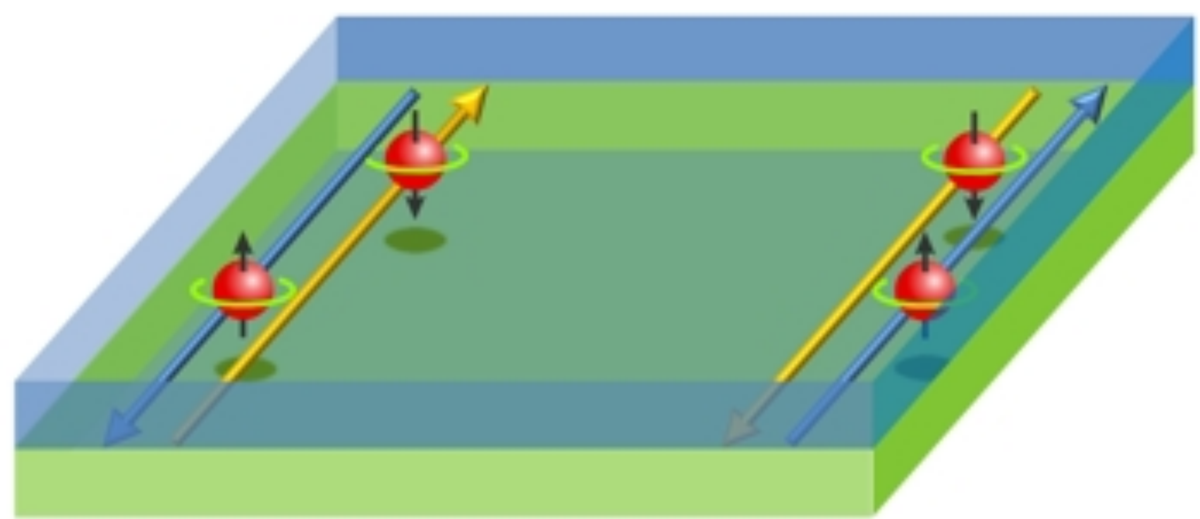

HgTe Quantum Wells

Engineering SOC and TI's in Graphene

PHYSICAL REVIEW X **1**, 021001 (2011)

Engineering a Robust Quantum Spin Hall State in Graphene via Adatom Deposition

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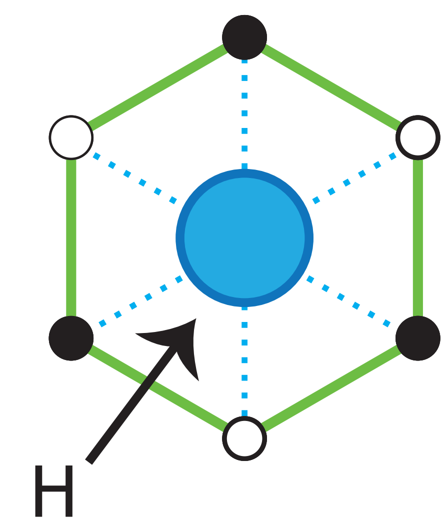

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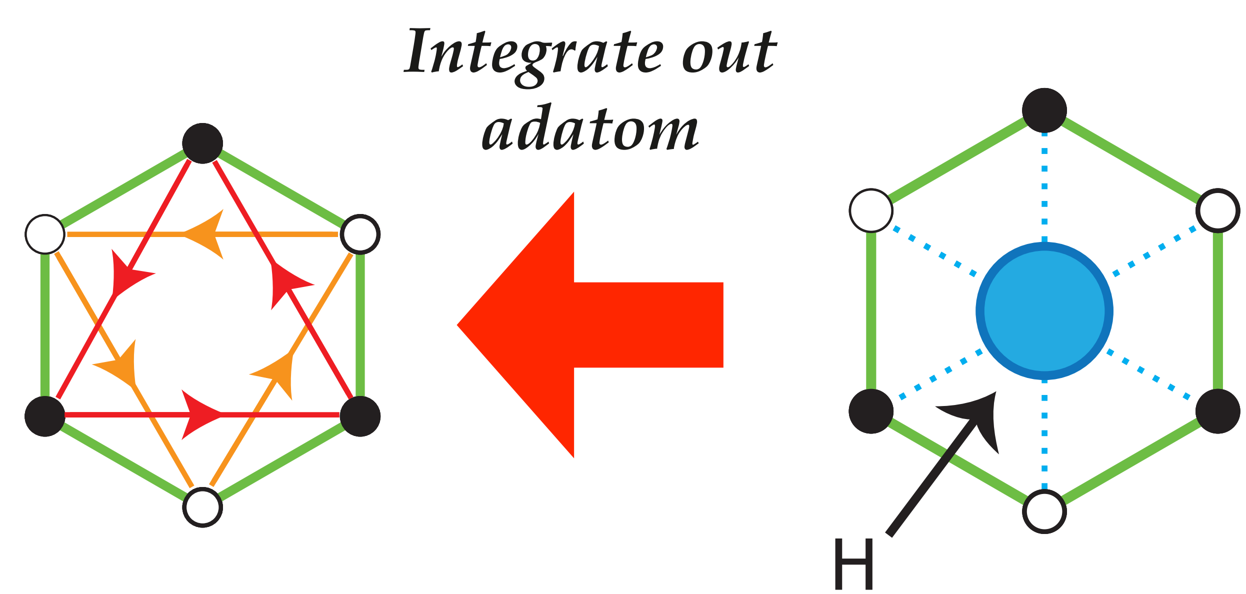
Adatom-mediated chiral hopping

(Kane-Mele Model) $$H_{\mathrm{SO}} = \lambda_{\mathrm{SO}} \sum_{\langle\langle \boldsymbol{r}\boldsymbol{r}'\rangle\rangle} \left(i\nu_{\boldsymbol{r}\boldsymbol{r}'} c^\dagger_{\boldsymbol{r}} \hat{s}_z c_{\boldsymbol{r}'} + \mathrm{H.c.} \right)$$



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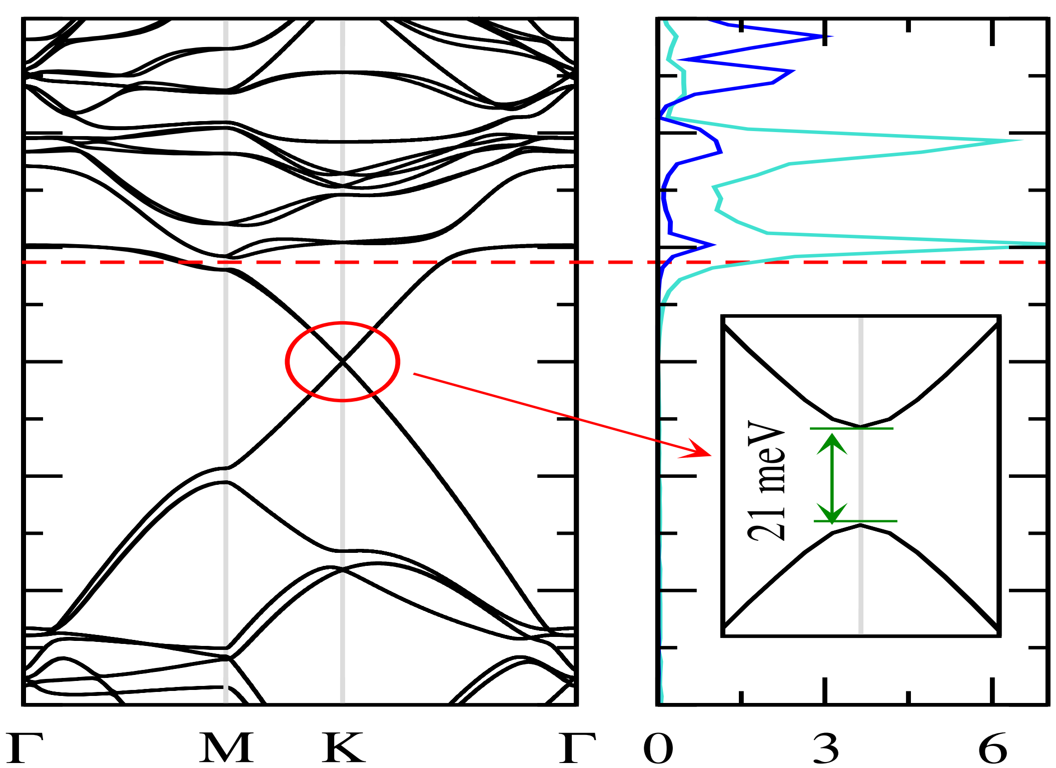
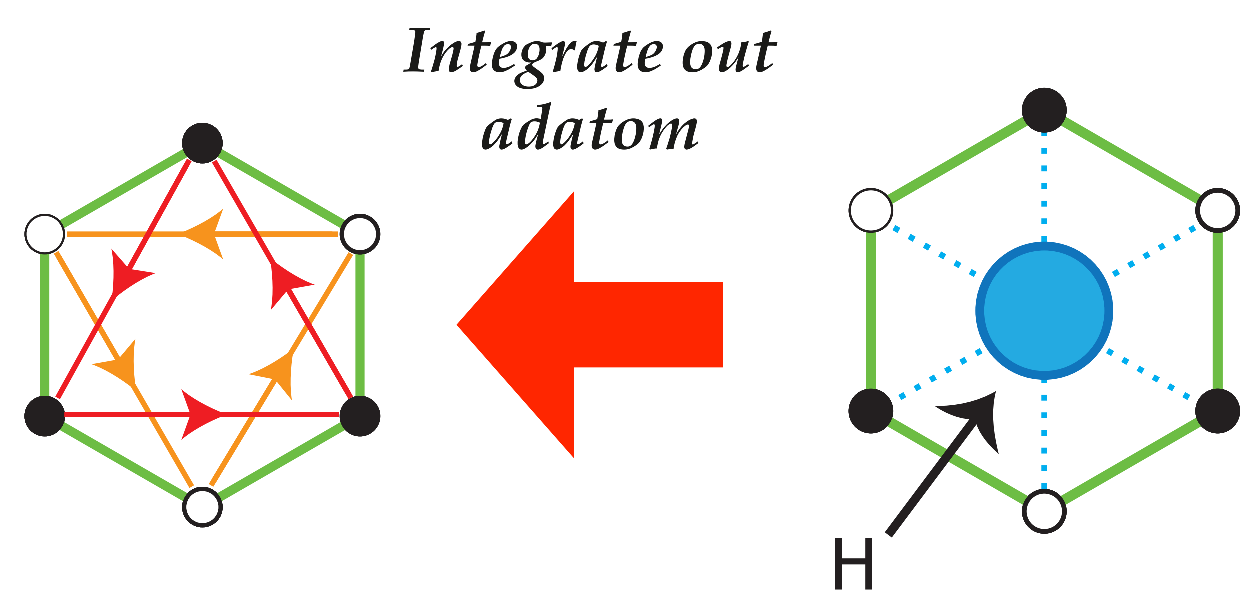
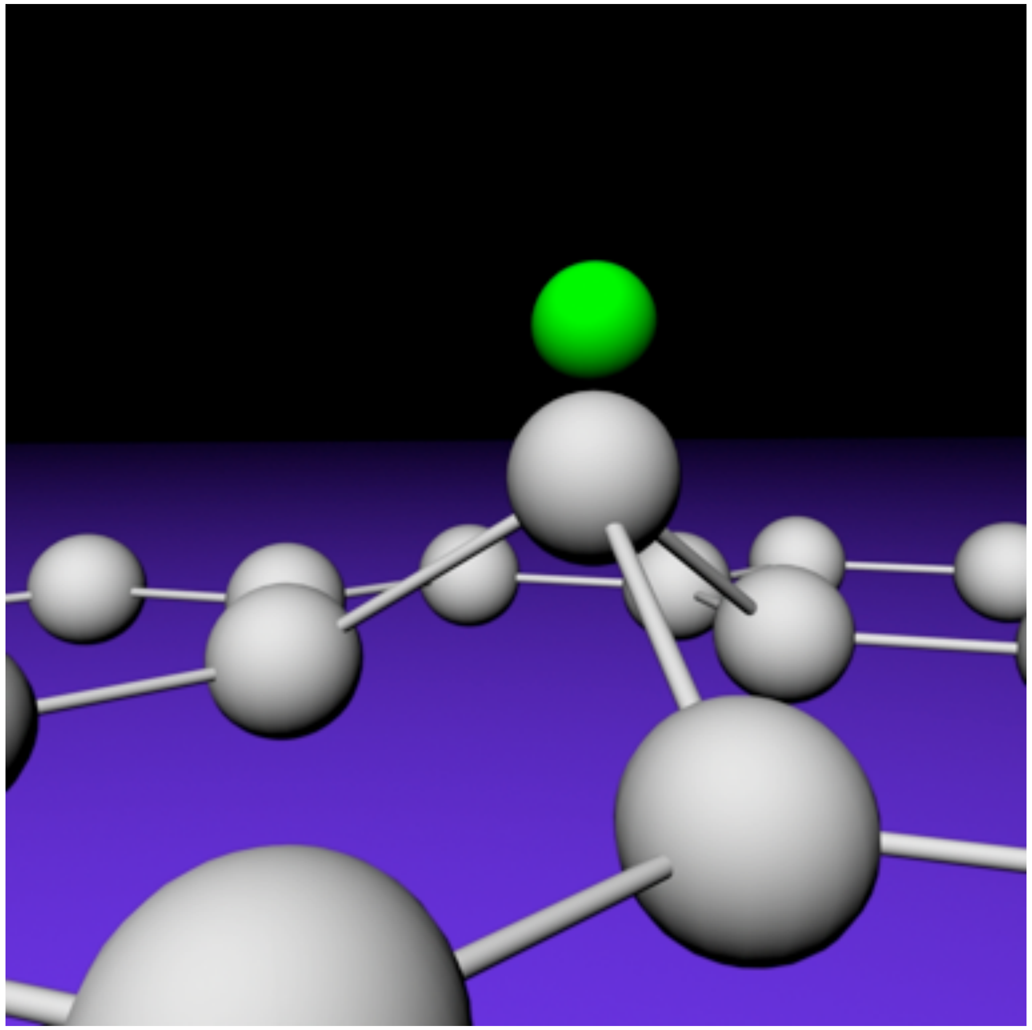




Engineering SOC in Graphene

Local enhancement of SOC (SOC active scatterers)



A.H. Castro Neto & F. Guinea
Phys. Rev. Lett. (2009)

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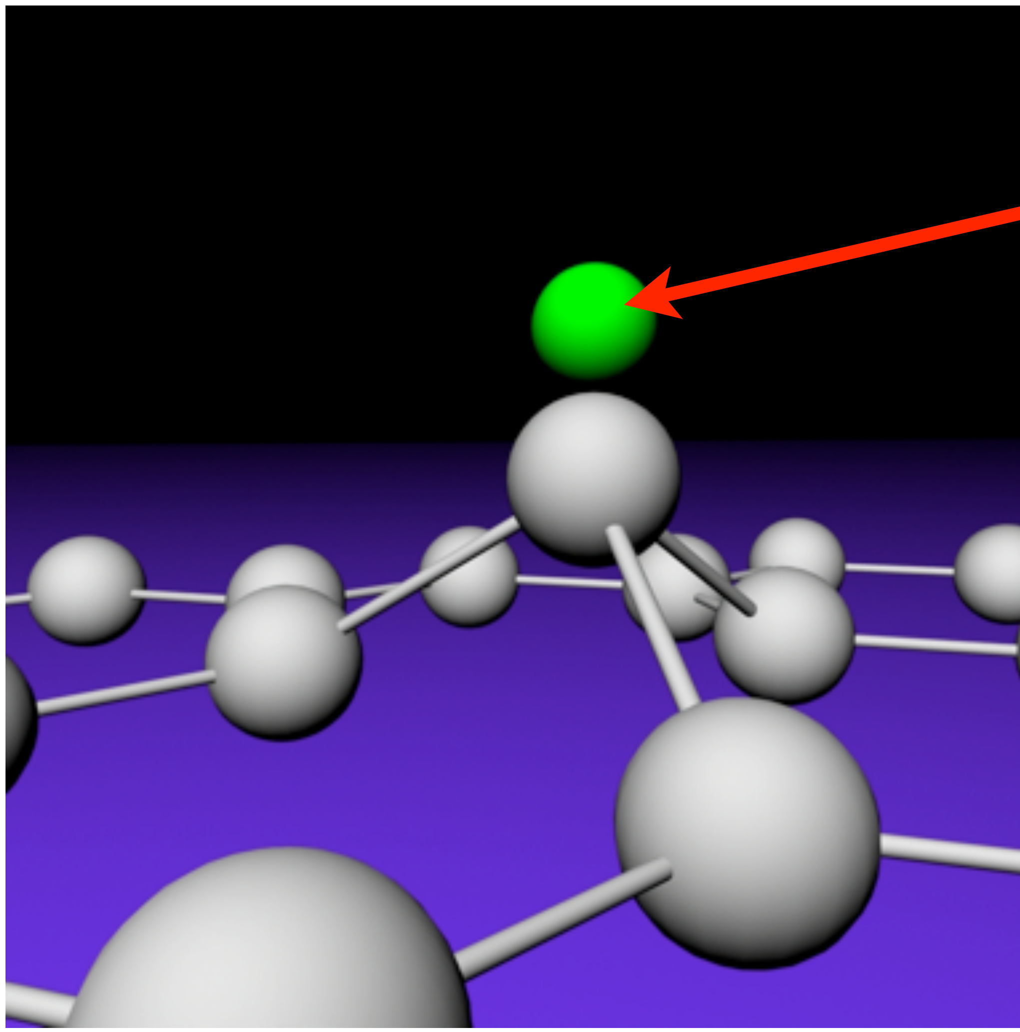

Chemisorbed Adatom
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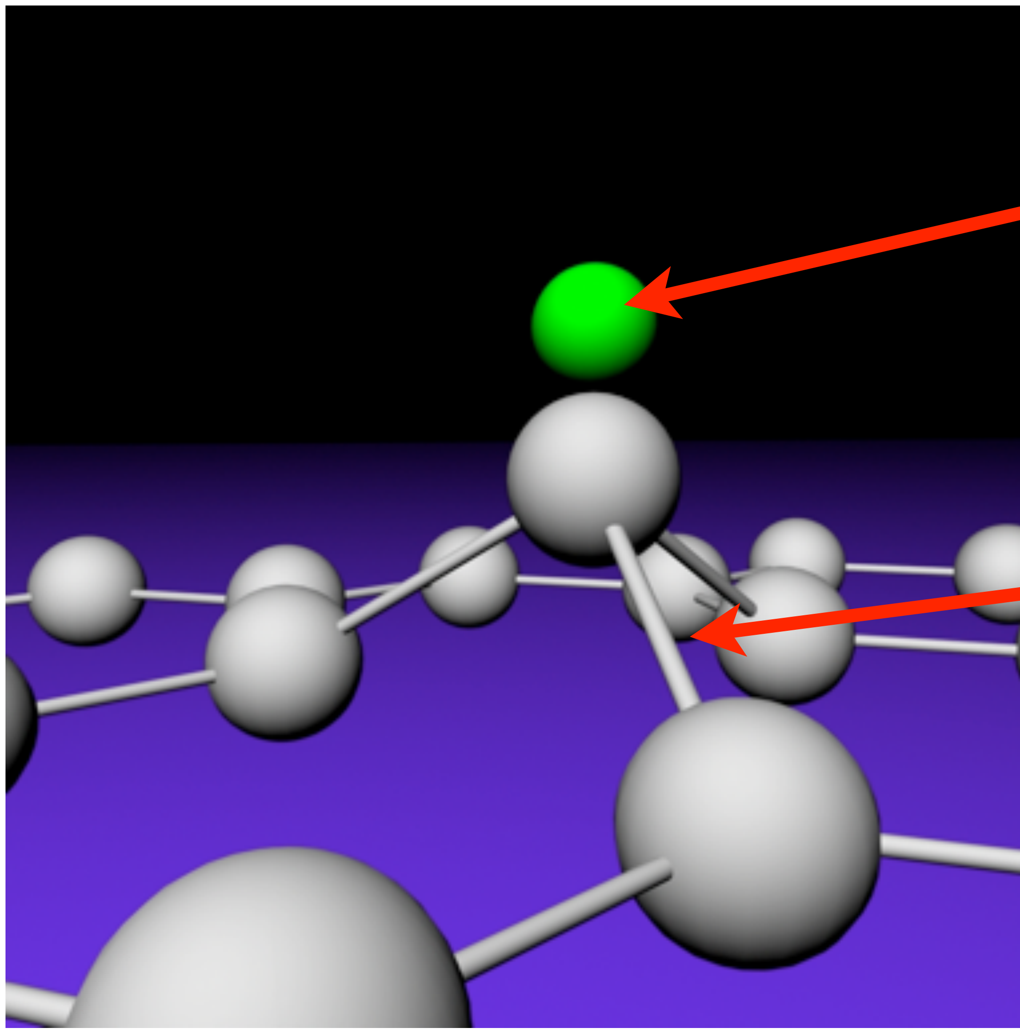

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Local Lattice distortion
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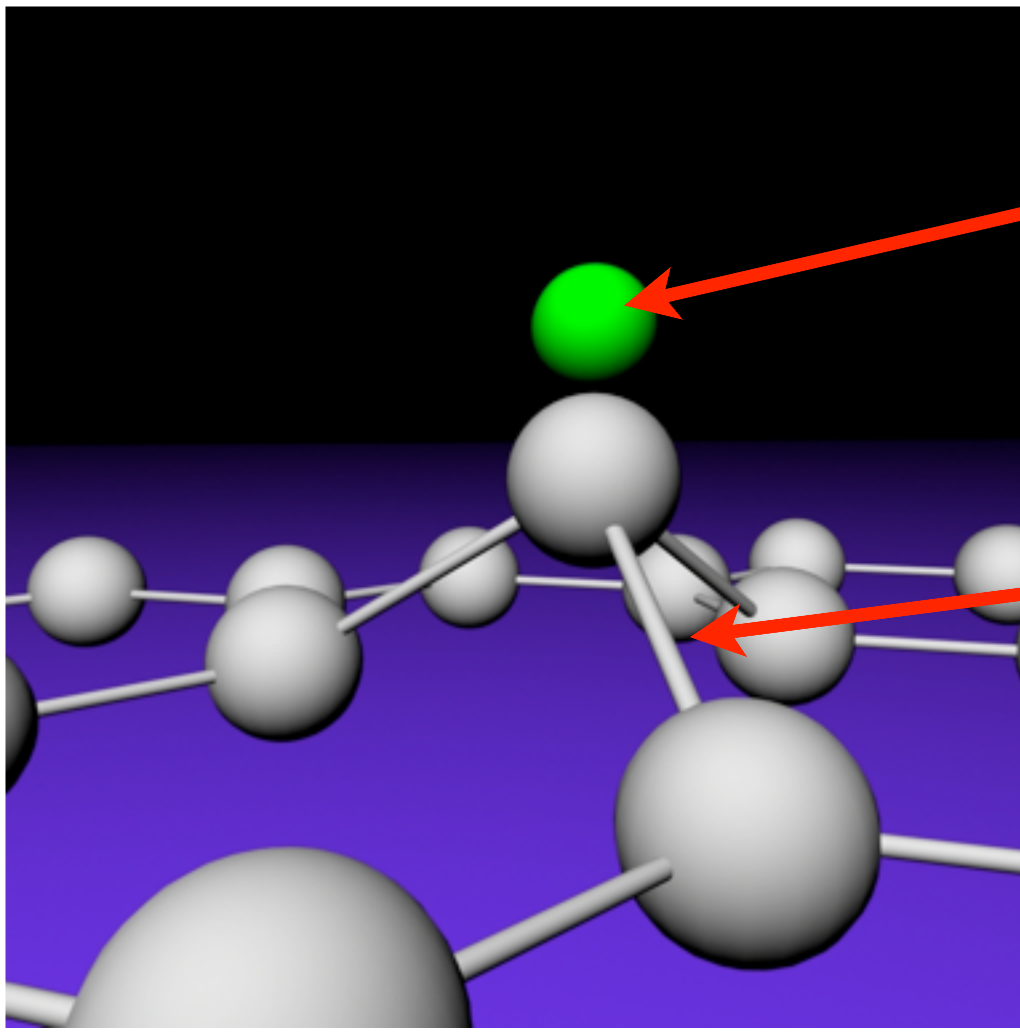


Chemisorbed Adatom (H, F, ...)

Local Lattice distortion (sp^2-sp^3 hybridization)

$$\Delta_{SO} \sim \lambda_{\mathrm{at}}^{SO} \sim 1\text{-}10 \text{ meV}$$

Comparable to diamond, Silicene, semiconductors

A.H. Castro Neto & F. Guinea
Phys. Rev. Lett. (2009)

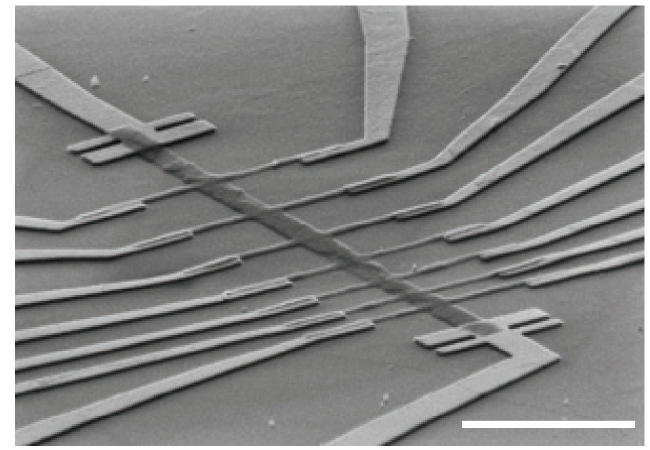
SHE in "decorated" Graphene

LETTERS
PUBLISHED ONLINE: 17 MARCH 2013 | DOI: 10.1038/NPHYS2576
nature physics

Colossal enhancement of spin–orbit coupling in weakly hydrogenated graphene

Jayakumar Balakrishnan[1,2†], Gavin Kok Wai Koon[1,2,3†], Manu Jaiswal[1,2‡], A. H. Castro Neto[1,2,4] and Barbaros Özyilmaz[1,2,3,4★]

a



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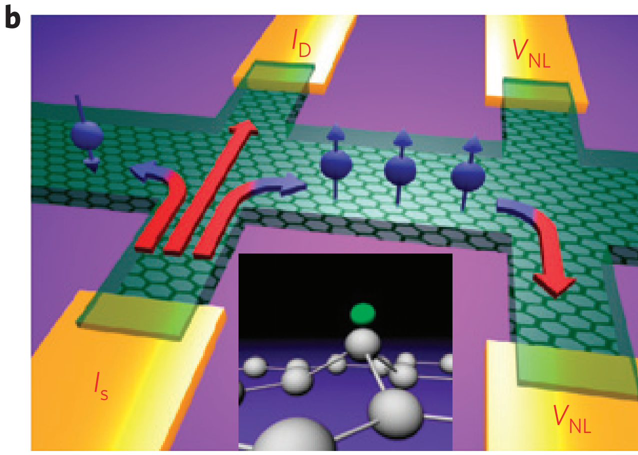

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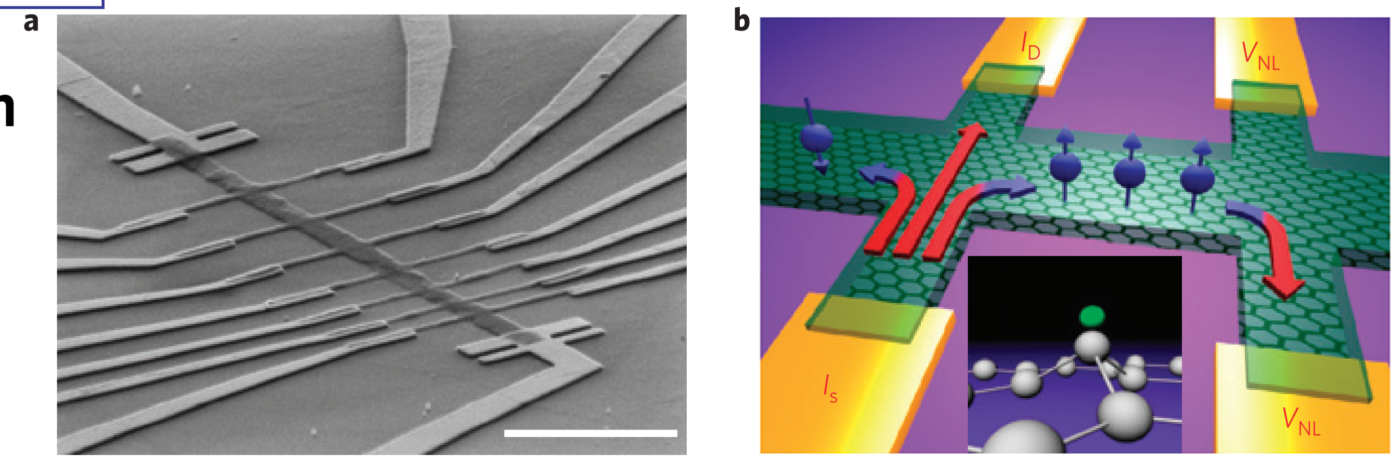

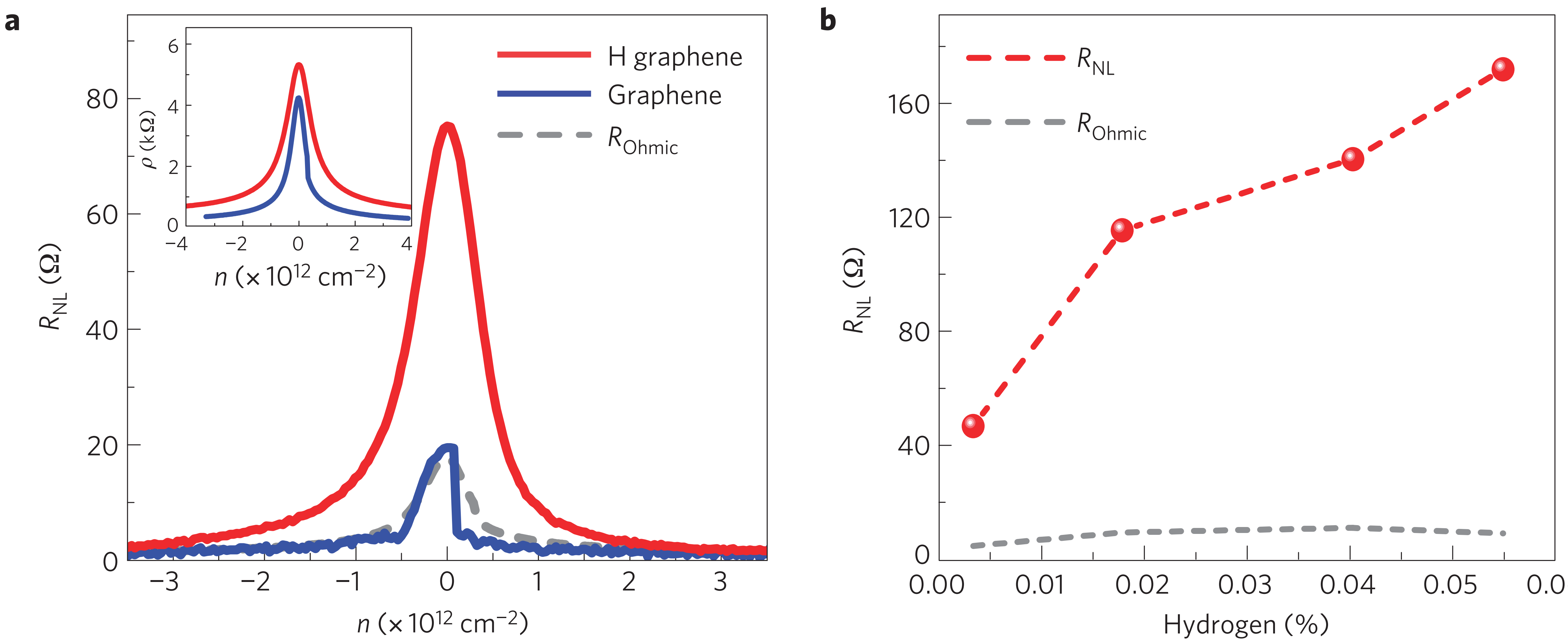

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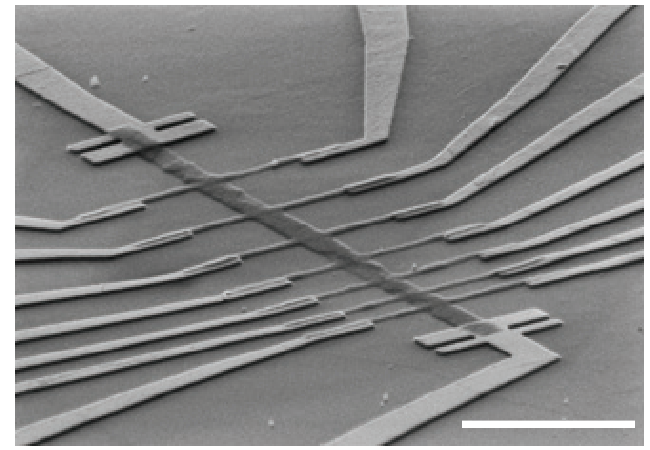

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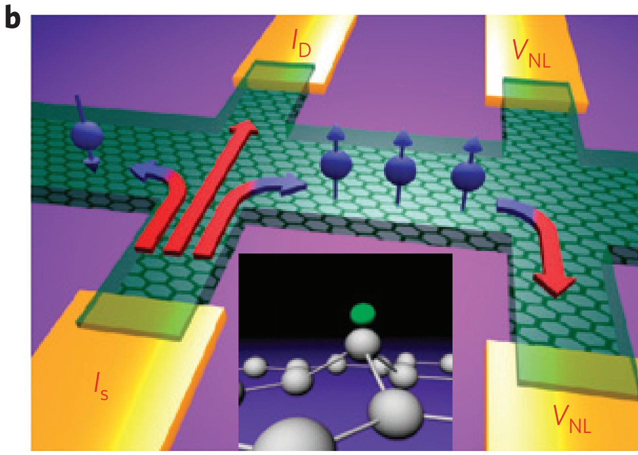
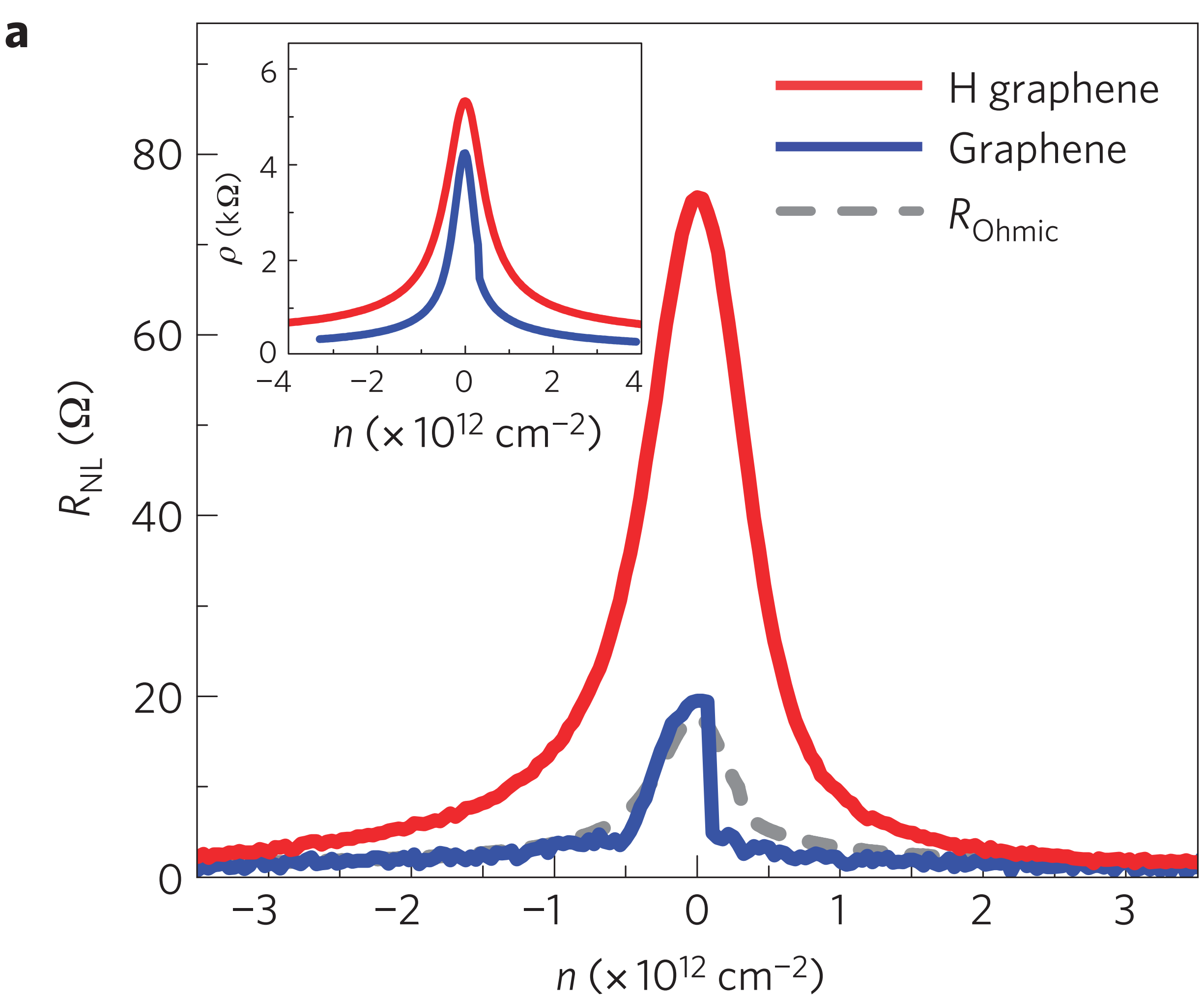
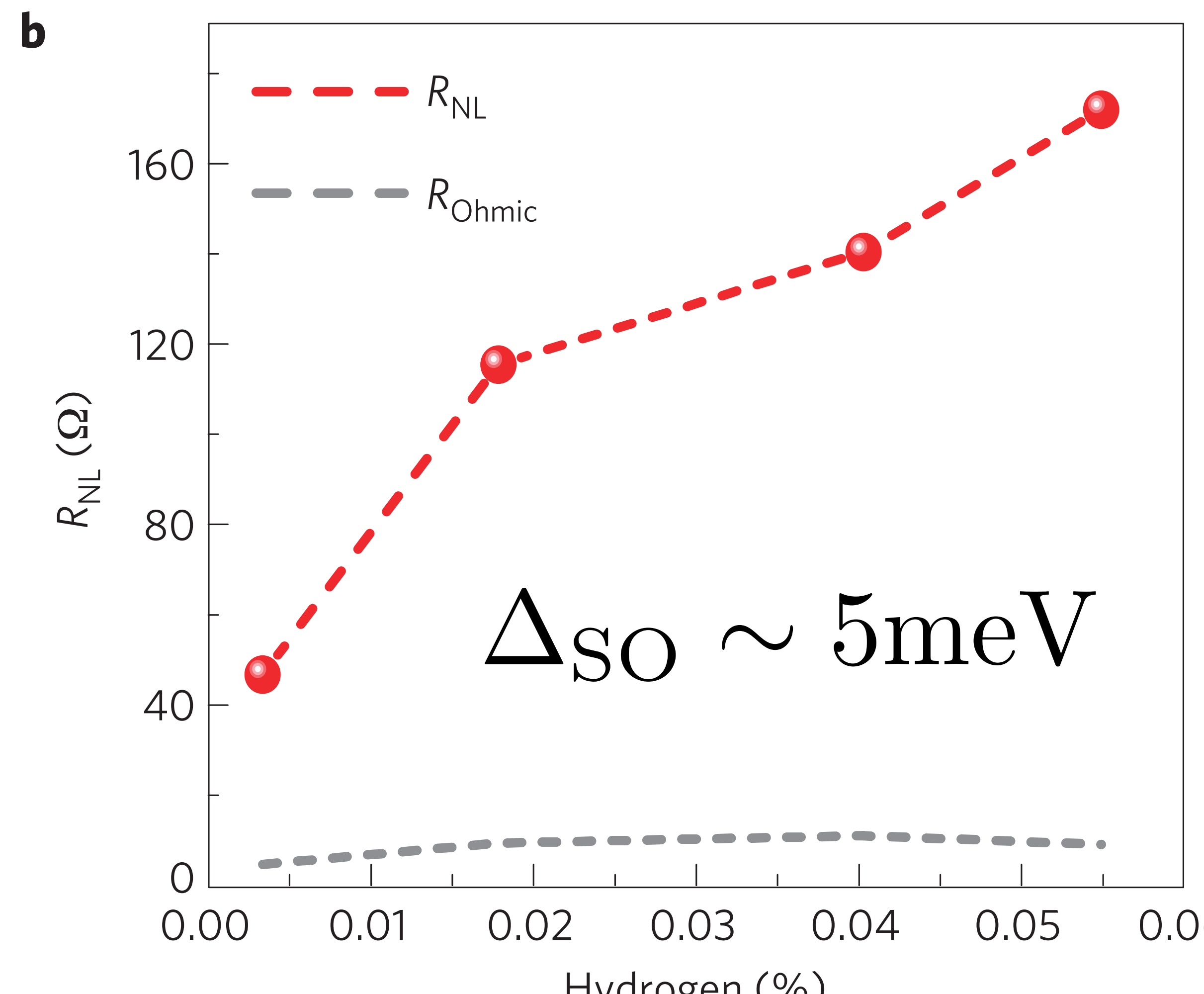


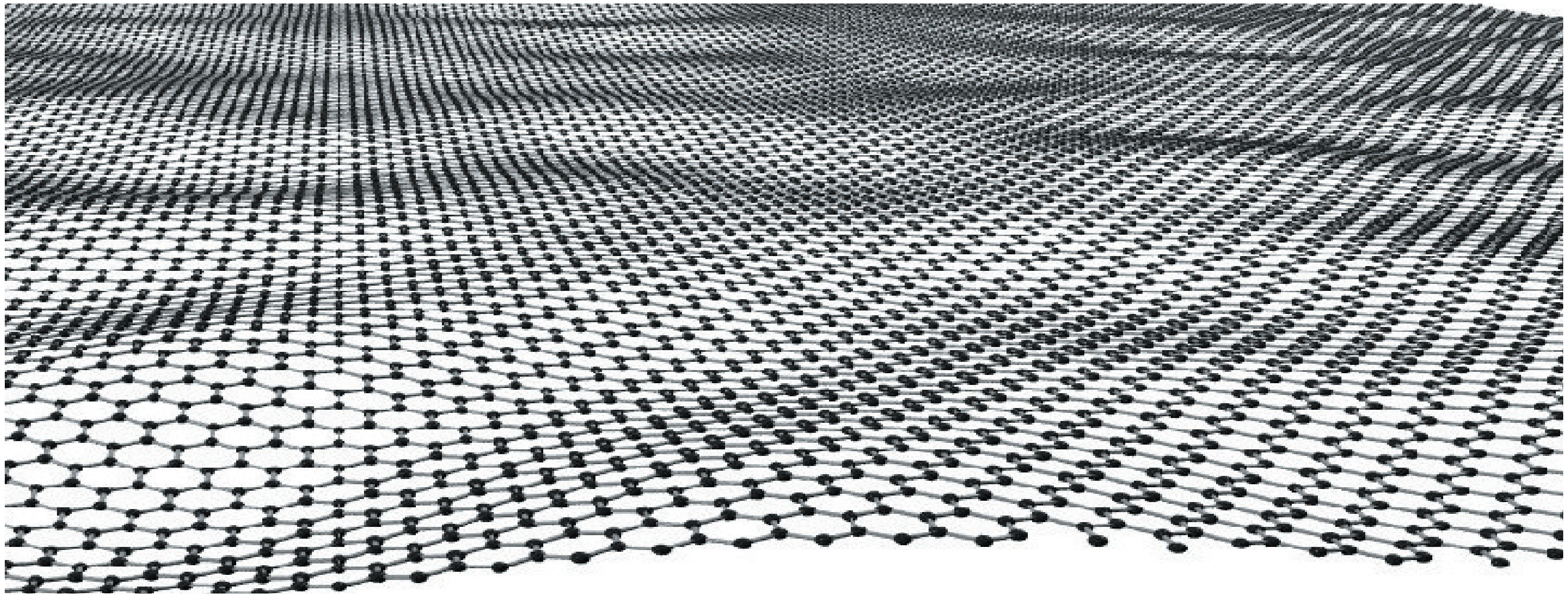




But... Clustering happens!

Graphene does not show structural long-range order in its free form

Frozen ripples



Ripples in graphene

Meyer et al. Nature 2007

O. Gülseren *et al*, PRL (2001)

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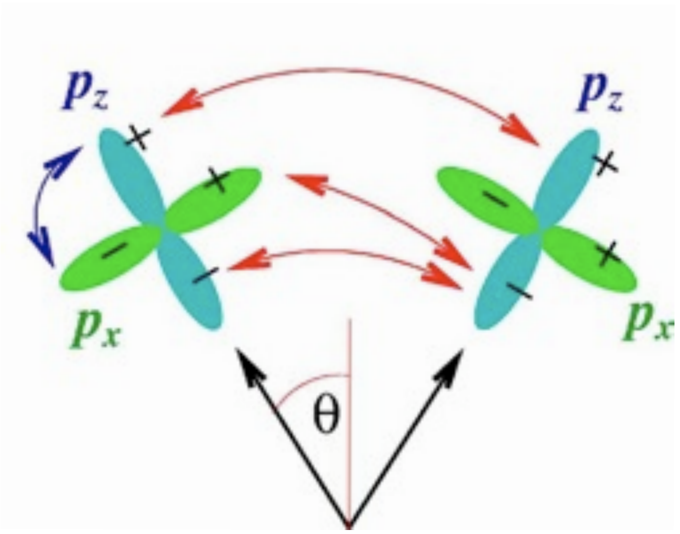
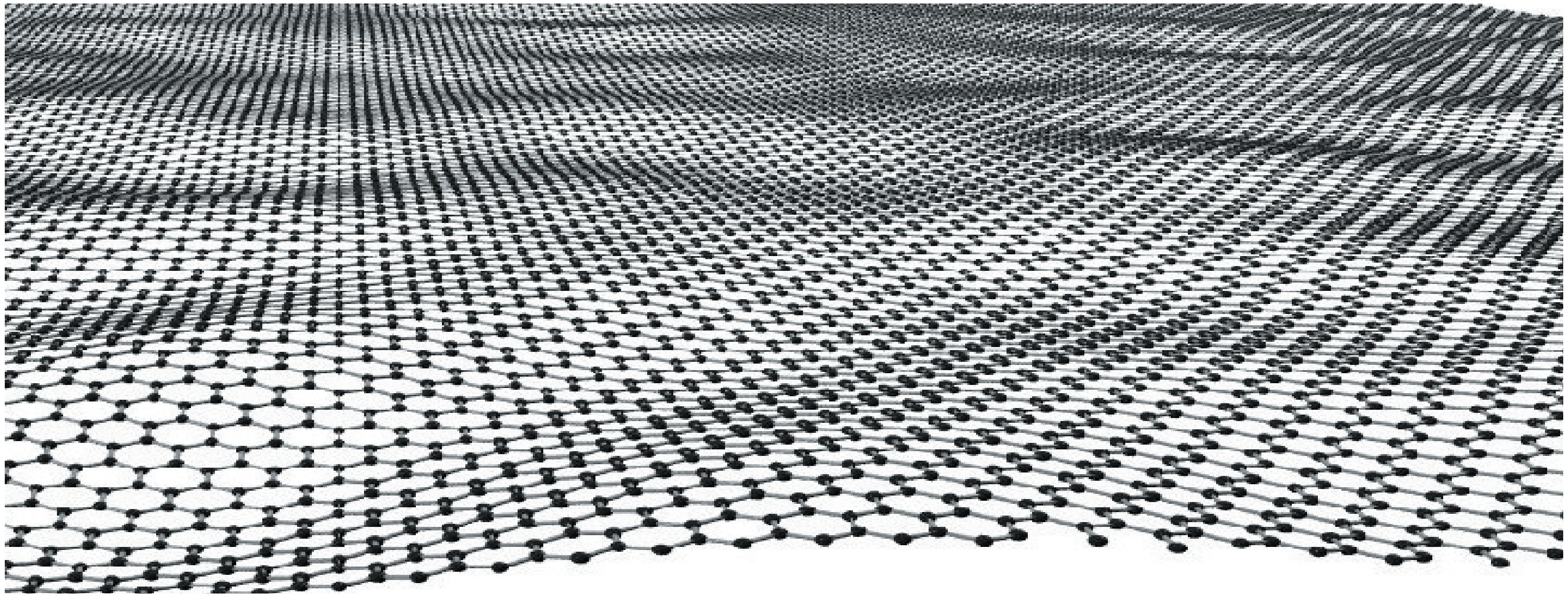




Ripples in graphene

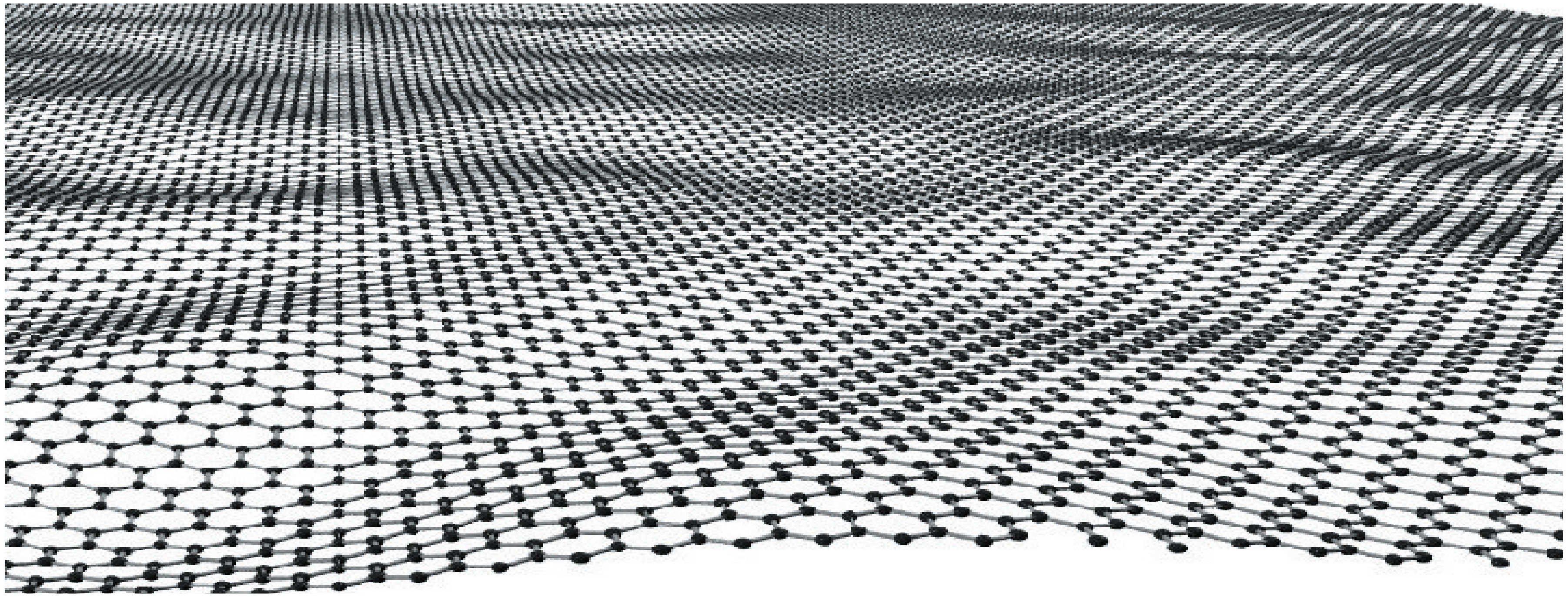
Meyer et al. Nature 2007

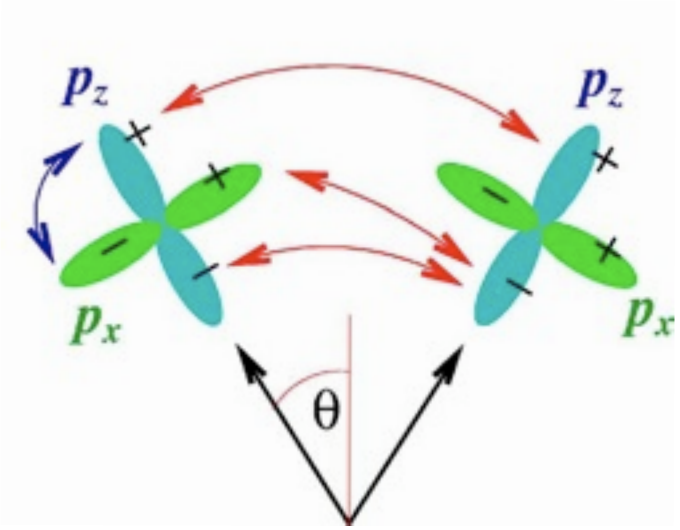
O. Gülseren *et al*, PRL (2001)

But... Clustering happens!

Graphene does not show structural long-range order in its free form

Frozen ripples





Ripples in graphene

Meyer et al. Nature 2007

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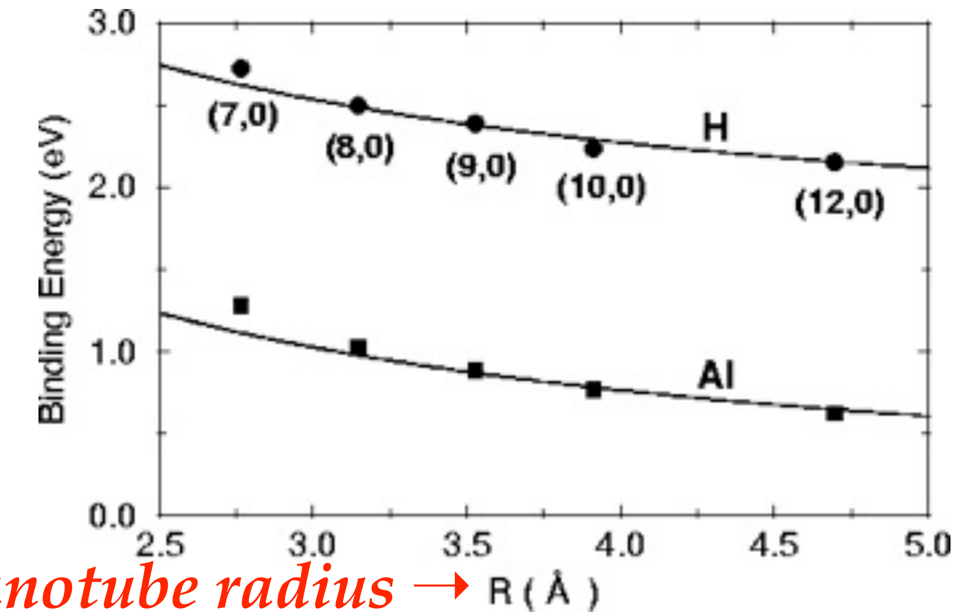
The p_z orbitals approach each other in the valleys but distance themselves in the hills

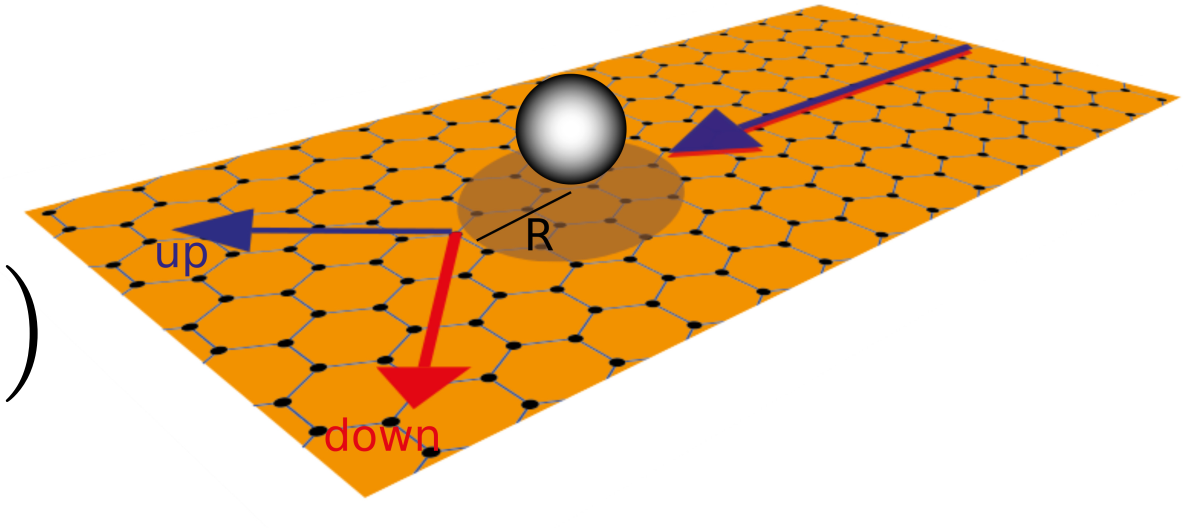
Probability that an adatom hybridizes with a hill-C is larger than a valley-C atom



Nanotube radius →

Skew Scattering in Graphene

Model for a single SO scatterer



$$V(r) = V_0(r) + H_{SO}(r)$$

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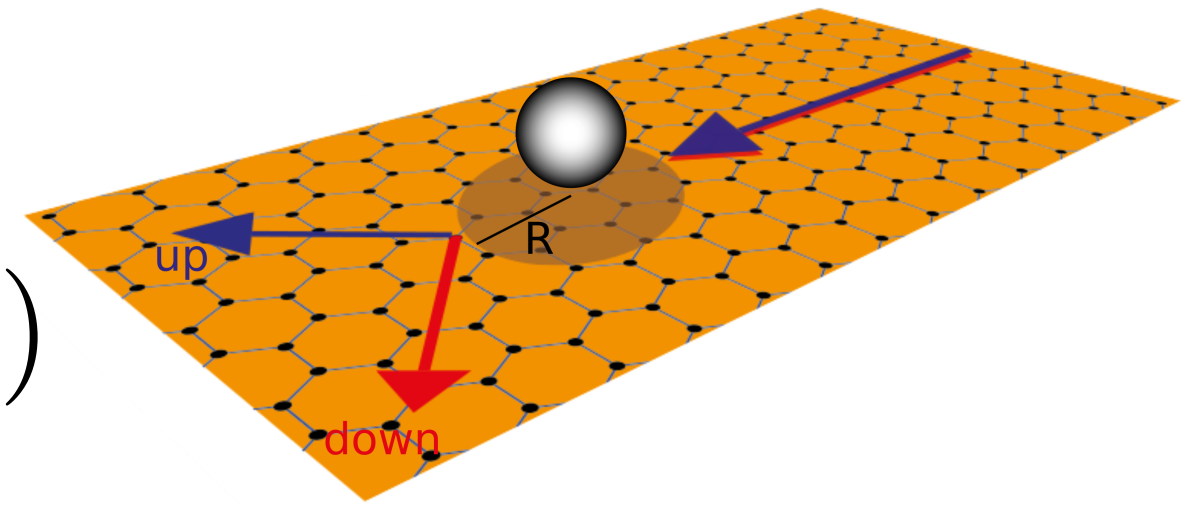

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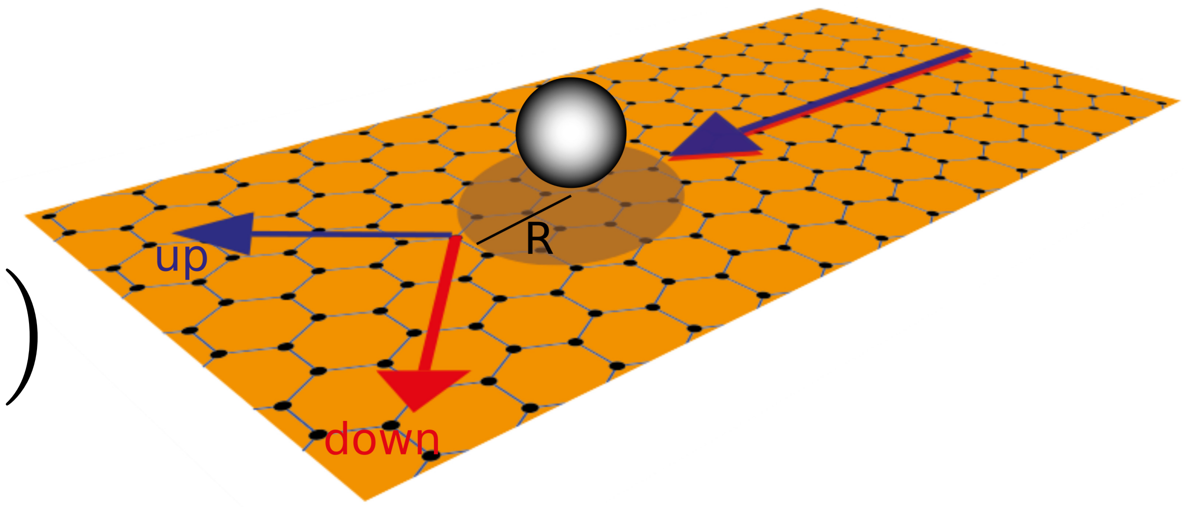
Types of SOC in Graphene

$$H_{SO} = \Delta_I \, \tau^z \sigma^z s^z$$ *("Intrinsic or Kane-Mele type")*

$$H_{SO} = \Delta_R(\tau^z \sigma^x s^y - \sigma^y s^x)$$ *("Rashba")* *[Broken mirror symmetry]*

Skew Scattering in Graphene



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Non-perturbative solution of the Scattering problem

$$\psi_{\lambda,\mathbf{k},s}(\mathbf{r}) = \begin{pmatrix} 1 \\ \lambda \end{pmatrix} e^{ikr\cos\theta}|s\rangle + \frac{f^{ss}(\theta)}{\sqrt{-ir}} \begin{pmatrix} 1 \\ \lambda e^{i\theta} \end{pmatrix} e^{ikr}|s\rangle + \frac{f^{s\bar{s}}(\theta)}{\sqrt{-ir}} \begin{pmatrix} 1 \\ \lambda e^{i\theta} \end{pmatrix} e^{ikr}|\bar{s}\rangle$$

Skew Scattering in Graphene: Symmetries

$$H = H_0 + [V_0 + \Delta_I \tau_z \sigma_z s_z + \Delta_R (\tau_z \sigma_x s_y - \sigma_y s_x)] \theta(R - r)$$

Skew Scattering in Graphene: Symmetries

$$H = H_0 + [V_0 + \Delta_I \tau_z \sigma_z s_z + \Delta_R(\tau_z \sigma_x s_y - \sigma_y s_x)]\theta(R - r)$$

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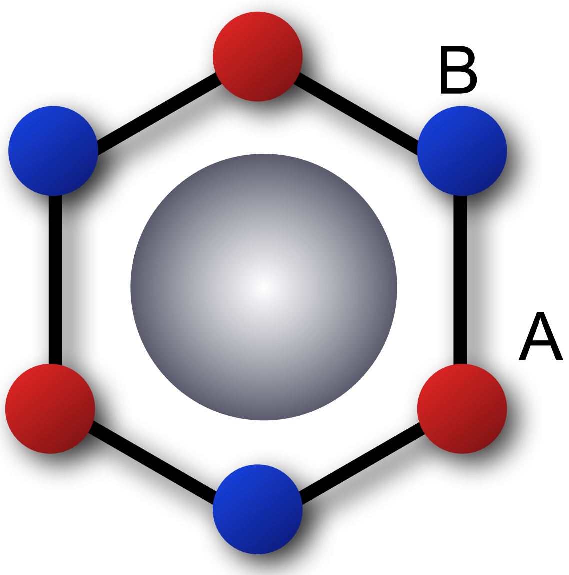

C_{6v} (Continuum limit) $\Rightarrow$ ***$C_{\infty v}$ + π rotations***

No inter-valley scattering $[T, \tau^z] = 0$

Time-reversal invariance (no magnetic moments)

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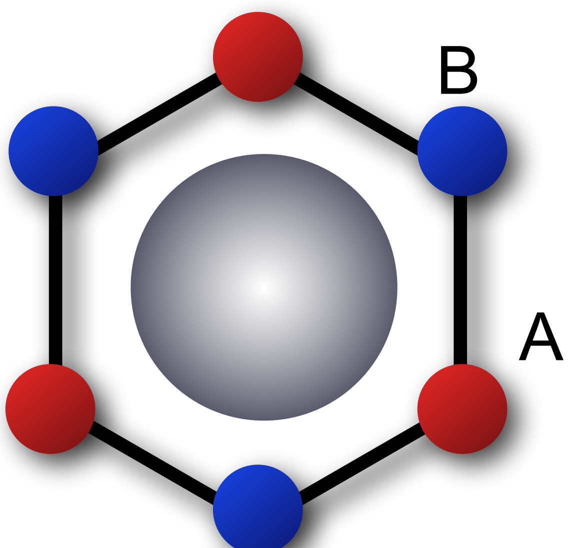

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Time-reversal invariance (no magnetic moments)

$$\mathcal{T}^s_{\alpha\beta;uv} = \langle \mathbf{k}\alpha u | T(\epsilon = \epsilon_{s\mathbf{k}} = \epsilon_{s\mathbf{p}}) | \mathbf{p}\beta v \rangle$$ ***On-shell 4 x 4 T-matrix***

$$\boxed{\mathcal{T}^s(\mathbf{k},\mathbf{p}) = a^s(\mathbf{k}\cdot\mathbf{p})s^0\tau^0 + [b^s(\mathbf{k}\cdot\mathbf{p})s^z + c^s(\mathbf{k}\cdot\mathbf{p})(\mathbf{k}-\mathbf{p})\cdot\mathbf{s}]\,(\mathbf{k}\wedge\mathbf{p})\,\tau^0}$$

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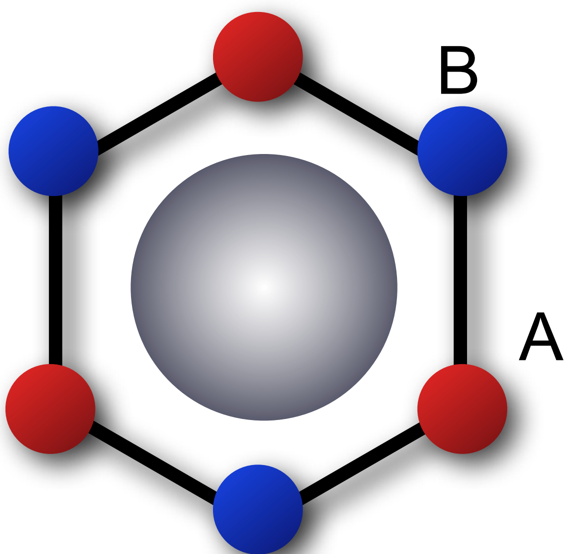

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Potential Scattering

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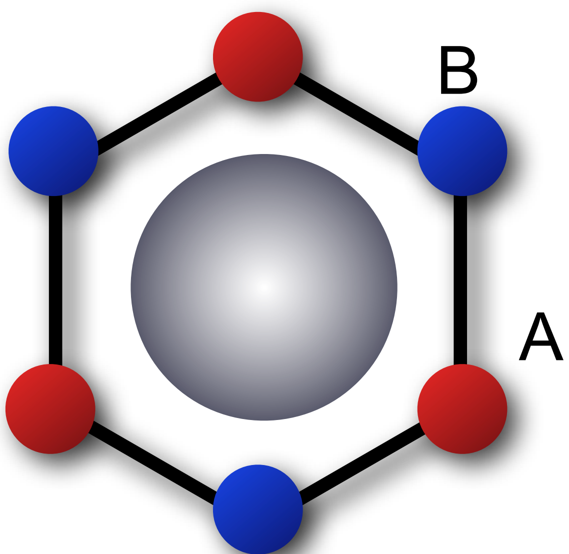

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Potential Scattering *Skew Scattering (s^z basis)*

Skew Scattering in Graphene: Symmetries

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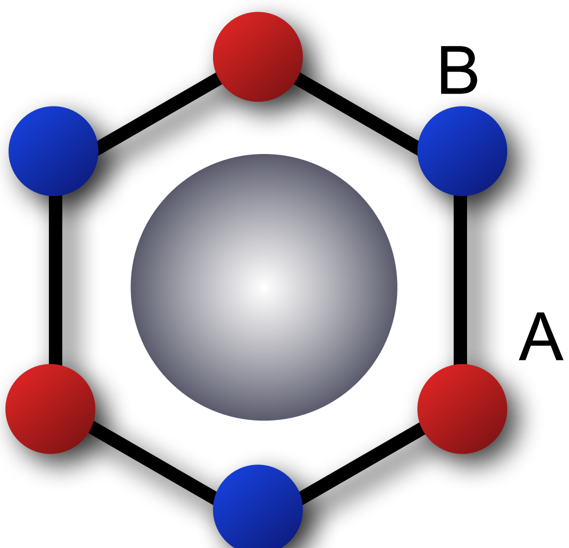

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Potential Scattering *Skew Scattering (s^z basis)* *spin-flip (s^z basis)*

Skew scattering in Graphene

Total Scattering Cross Section

$$\frac{d\sigma(\phi)}{d\phi} \sim \mathrm{Tr}\left[\mathcal{T}^s \mathcal{T}^{s\dagger}\right] = |a^s(\cos\phi)|^2 + \left[|b^s(\cos\phi)|^2 + |c^s(\cos\phi)|^2(1-\cos\phi)\right]\sin^2\phi$$

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Skew Scattering Cross Section (unpolarized electrons)

$$\frac{d\sigma_{\mathrm{skew}}(\phi)}{d\phi} \sim \mathrm{Tr}\left[\mathcal{T}^s \mathcal{T}^{s\dagger} s^z\right] k_y \sim \mathrm{Re}\left[a^{s*}(\cos\phi) b^s(\cos\phi)\right]\sin^2\phi$$

Skew scattering in Graphene

Total Scattering Cross Section

$$a^s \sim \frac{1}{E - E_R + i\Gamma_R}$$

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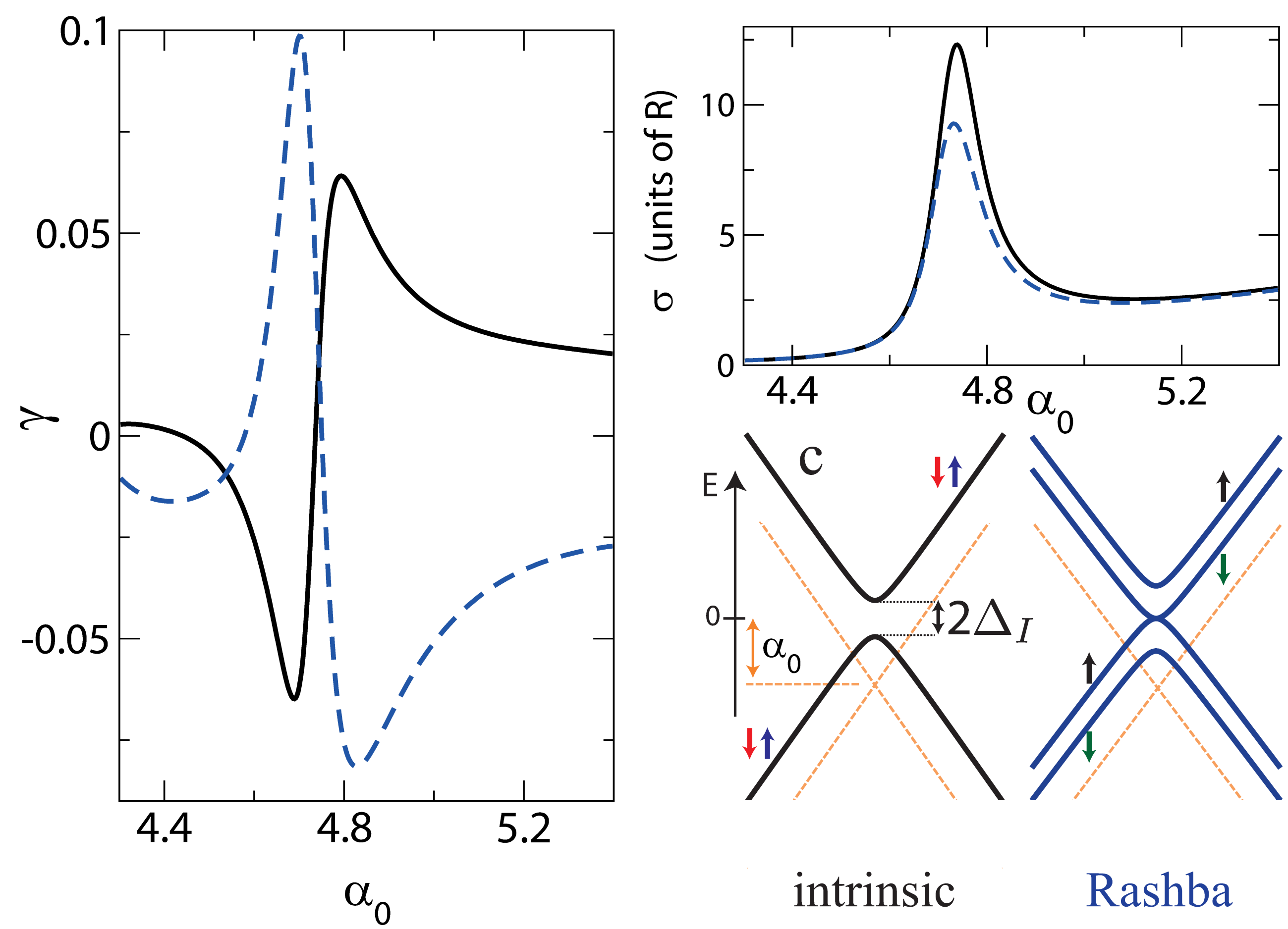
Scattering skewness

$$\gamma \sim \frac{\sigma_{\mathrm{skew}}}{\sigma}$$

$$\alpha_0 = \frac{V_0 R}{\hbar k}$$

$$\epsilon_{s\mathbf{k}} = \hbar v_F k = 100\,\mathrm{meV}$$

$$R = 4\,\mathrm{nm}.$$



Resonant Scattering in Graphene

LS Equation (short rage potential)

$$T(E) = U_0 + U_0 G_0^+(E) T(E)$$

$$T(\epsilon) = \frac{1}{\frac{1}{U_0} - G_0^+(E)}$$

$$G_0^+(E) = F_0(E) + i\pi\rho_0(E)$$

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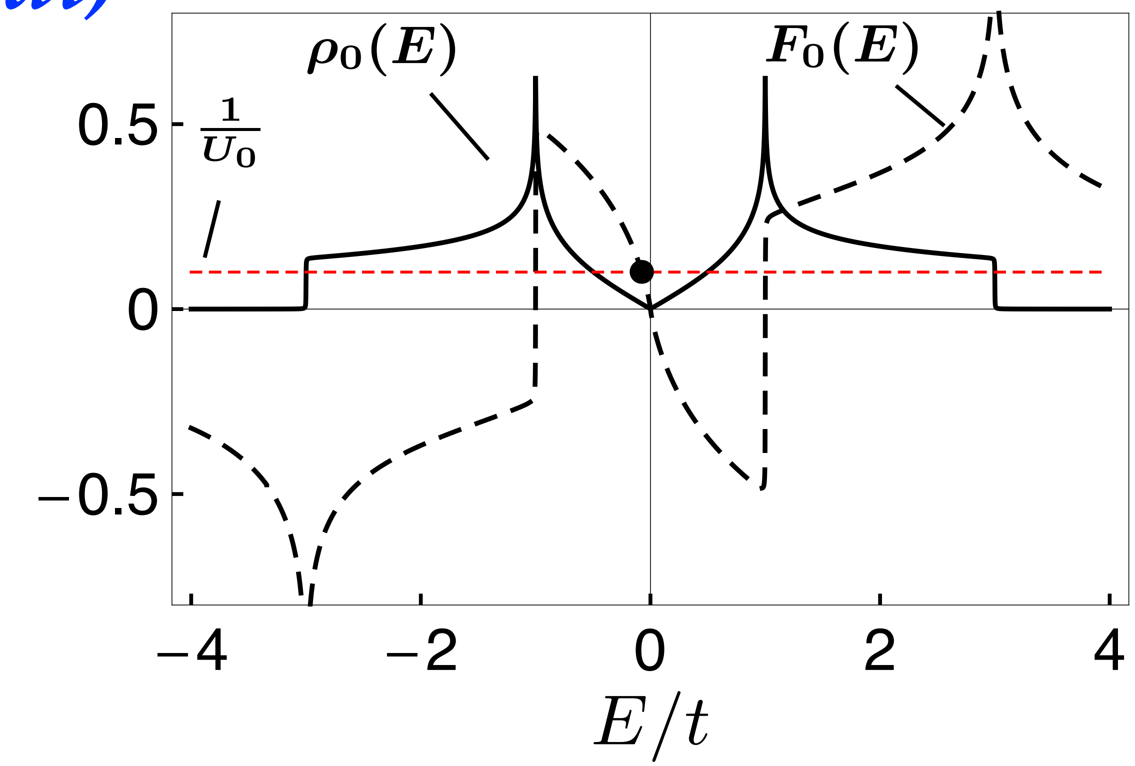
Poles of T(E): Graphic solution



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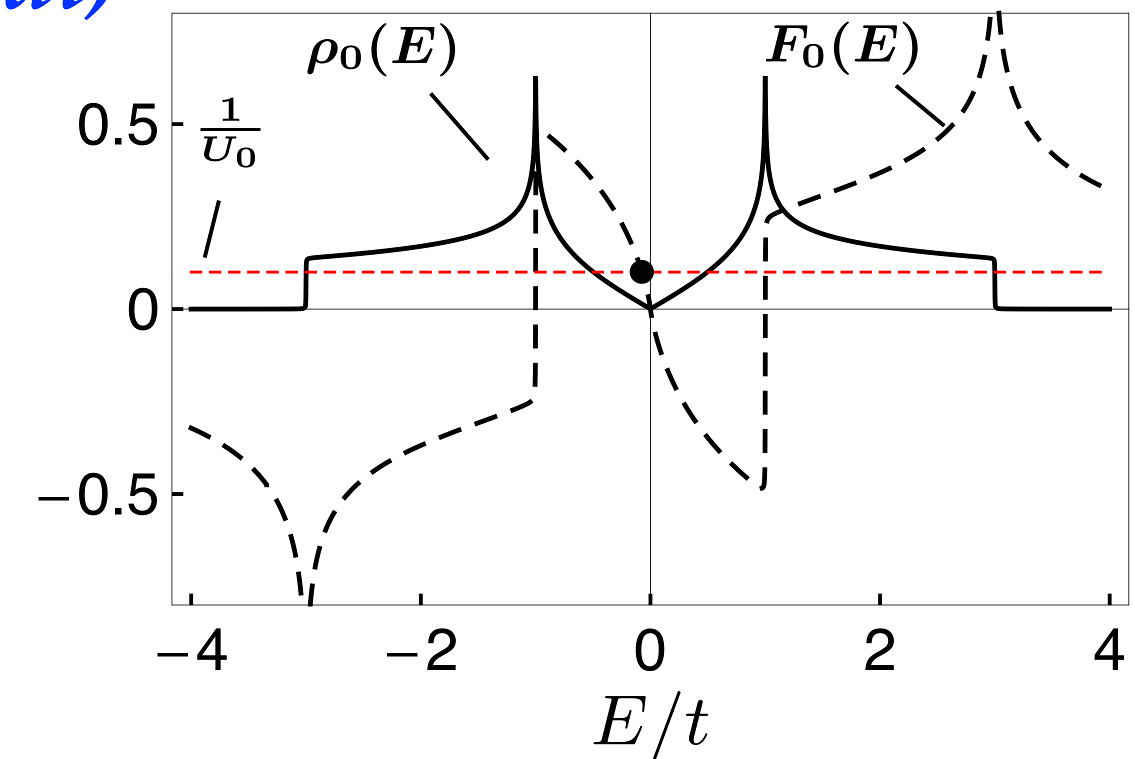

$$\rho_0(E) \sim |E|$$

Relatively easy to induce

narrow resonances around the Dirac point!

Resonant Scattering in Graphene

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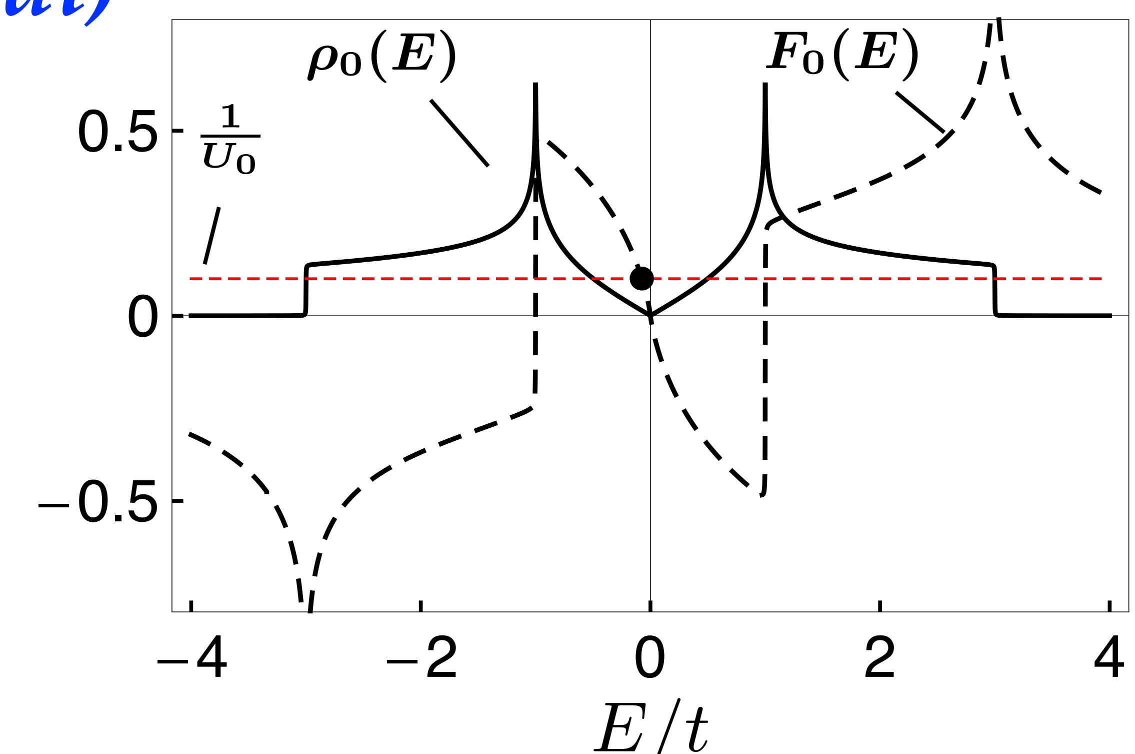

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PHYSICAL REVIEW B **79**, 195426 (2009)

Scattering of electrons in graphene by clusters of impurities

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(Received 28 April 2009; published 20 May 2009)

Theory

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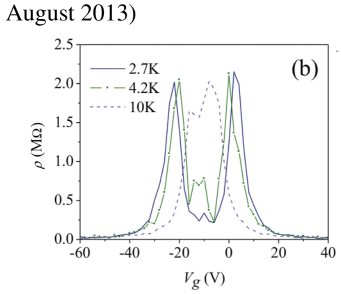
Low-energy resonant scattering from hydrogen impurities in graphene

Bernard R. Matis,[1] Brian H. Houston,[2] and Jeffrey W. Baldwin[2,*]
[1]*NRC Postdoctoral Associate, Naval Research Laboratory, Washington, DC 20375, United States*
[2]*Naval Research Laboratory, Code 7130, Washington, DC 20375, United States*
(Received 25 April 2013; published 30 August 2013)

Experiment



Skew scattering in Graphene

Boltzmann Equation ***(dilute random ensemble of scatterers)***

$$\nabla_{\mathbf{k}} n_\alpha(\mathbf{k}) \cdot (e\mathbf{E}) = \sum_{\mathbf{p},\beta=\uparrow,\downarrow} W_{\alpha\beta}(\mathbf{k},\mathbf{p})\left[n_\beta(\mathbf{p}) - n_\alpha(\mathbf{k})\right]$$

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Exact solution using $\delta n_\alpha(\mathbf{k}) = \nabla_{\mathbf{k}} n^0(\mathbf{k}) \left[A_\alpha(\mathbf{k}) e\mathbf{E} + B_s(\mathbf{k}) (\hat{\mathbf{z}} \times e\mathbf{E}) \right]$

$$\theta_{SH} = \frac{\sigma_{SH}}{\sigma_{xx}}$$

$$\theta_{\mathrm{sH}}\big|_{T=0} = \frac{\tau_{\parallel}}{\tau_{\perp}}$$

$$\tau_{\parallel}^{-1} = \sum_{\mathbf{p},\beta} (1 - s_\alpha s_\beta \cos\phi)\, W_{\alpha\beta}(\mathbf{k},\mathbf{p})$$

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Robust vs. T and impurity average

$$\theta_{SH} = \frac{\sigma_{SH}}{\sigma_{xx}}$$

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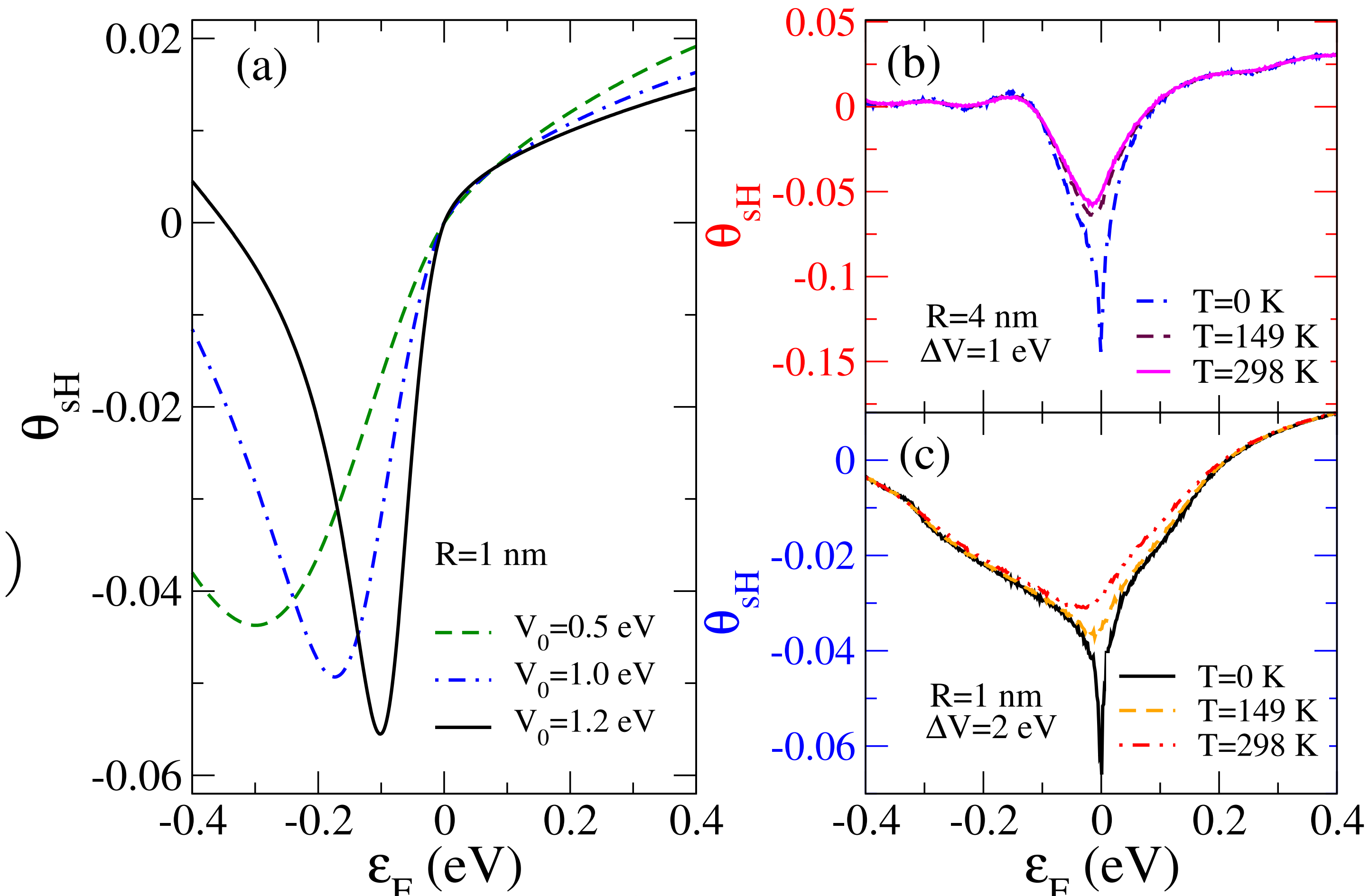

$$\Delta_I = 25\mathrm{meV}$$

Conclusions

- *It is possible to engineer SHE in Graphene by decoration with a dilute random array of adatom clusters.*
- *Resonant Scattering dramatically enhances the SHE angle.*
- *The effect is robust against temperature and disorder average.*