Correlated Bubble Crystals in Rapidly Rotating Dipole-blockaded Fermi Gases

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Collaborators:

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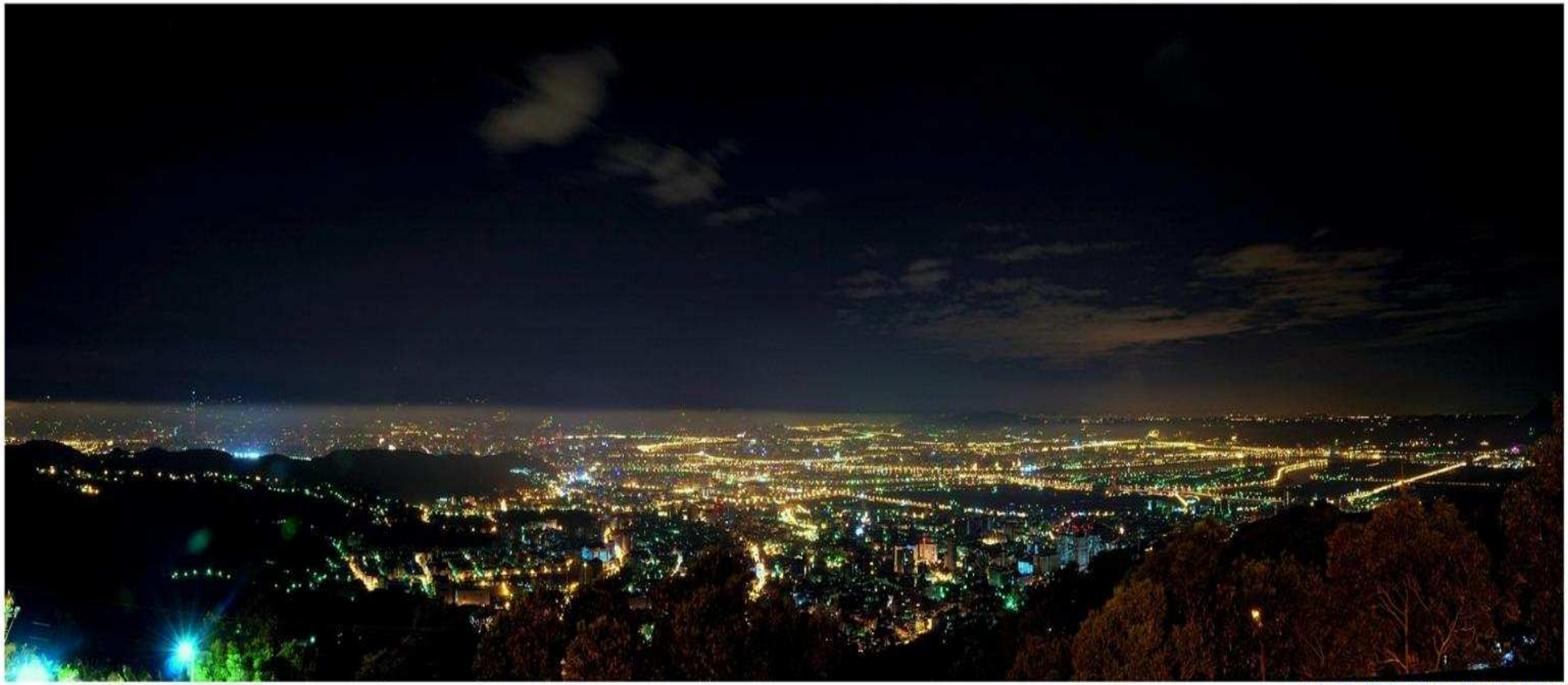
Institute of Physics, National Chiao Tung University, Taiwan

Supported by NSC of Taiwan: NSC 99-2112-M-034-002-MY3



Mark-san Image photography studio

July 25, 2009, released by Mark Hsu
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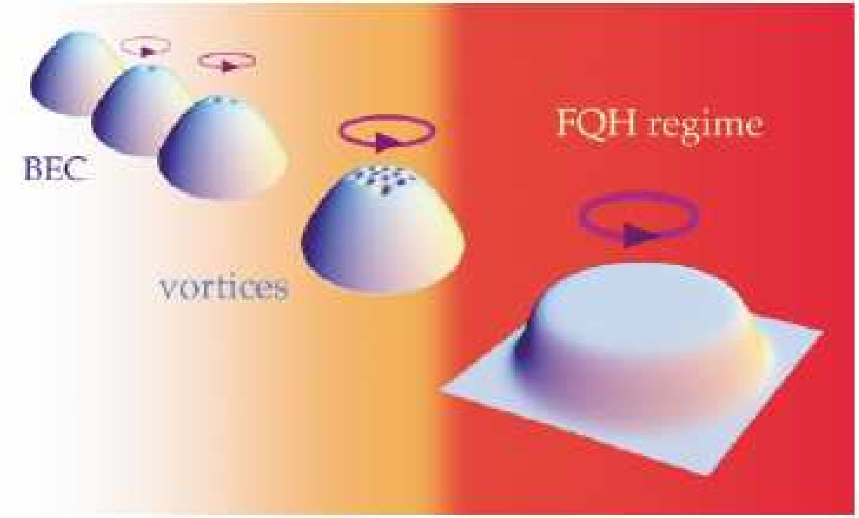
I. Introduction:



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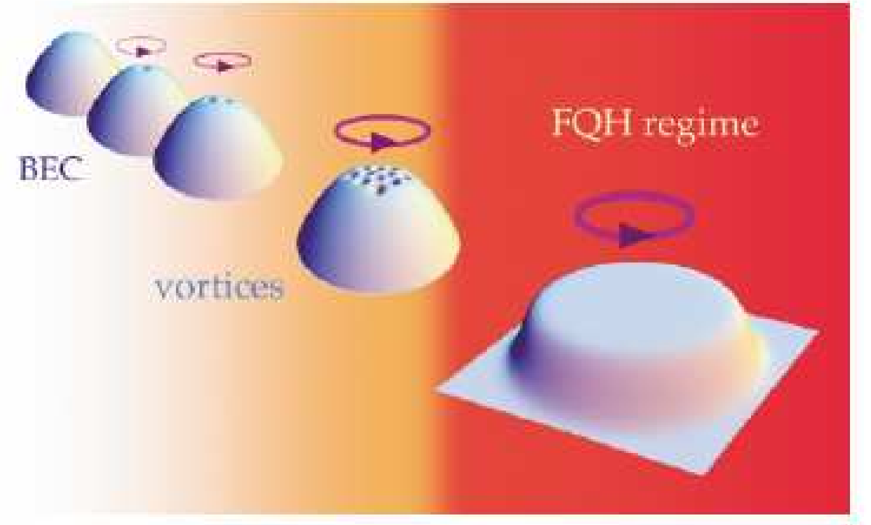

$$\mathcal{H} = \sum_{i=1}^{N} \frac{1}{2m}(\mathbf{p}_i - m\omega\hat{\mathbf{z}} \times \mathbf{r}_i)^2 + \frac{1}{2}m(\omega_0^2 - \omega^2)(x_i^2 + y_i^2) + \frac{1}{2}m\omega_z^2 z_i^2 + \sum_{i<j}^{N} V(\mathbf{r}_i - \mathbf{r}_j),$$

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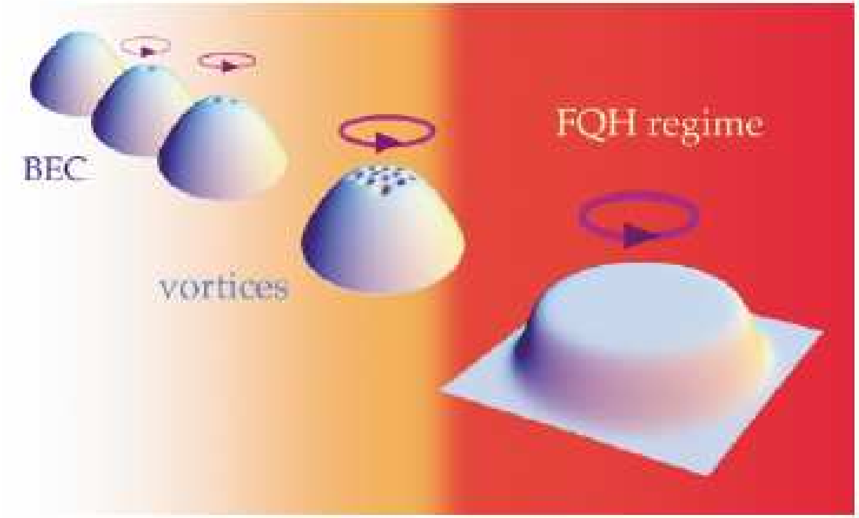

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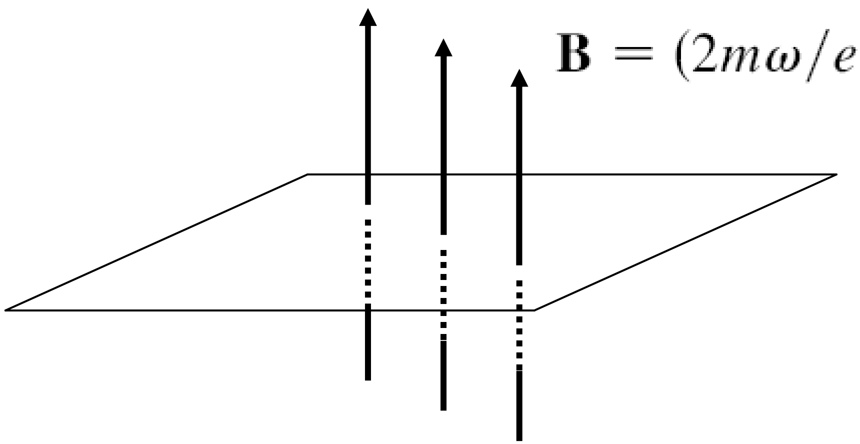
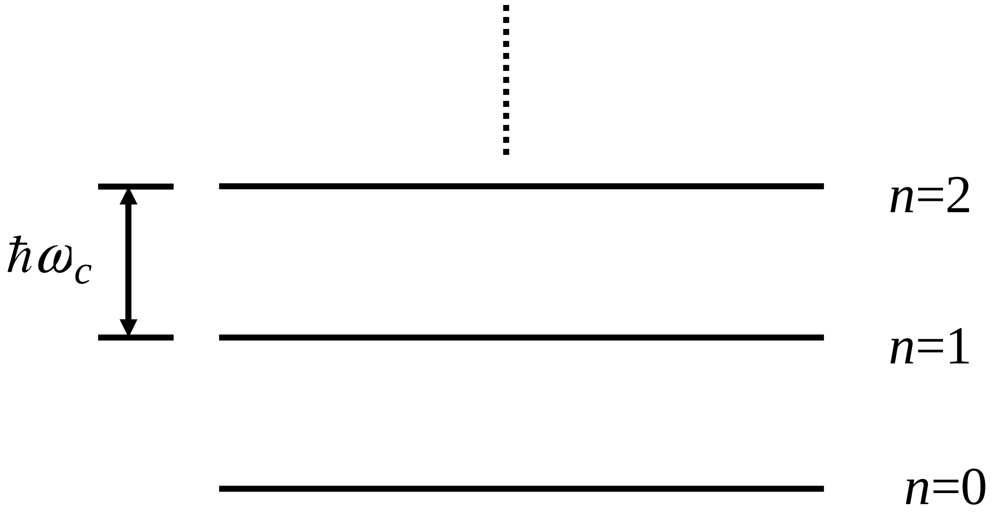
$$\mathbf{B} = (2m\omega/e)\hat{\mathbf{z}}.$$

If $\omega \approx \omega_0$



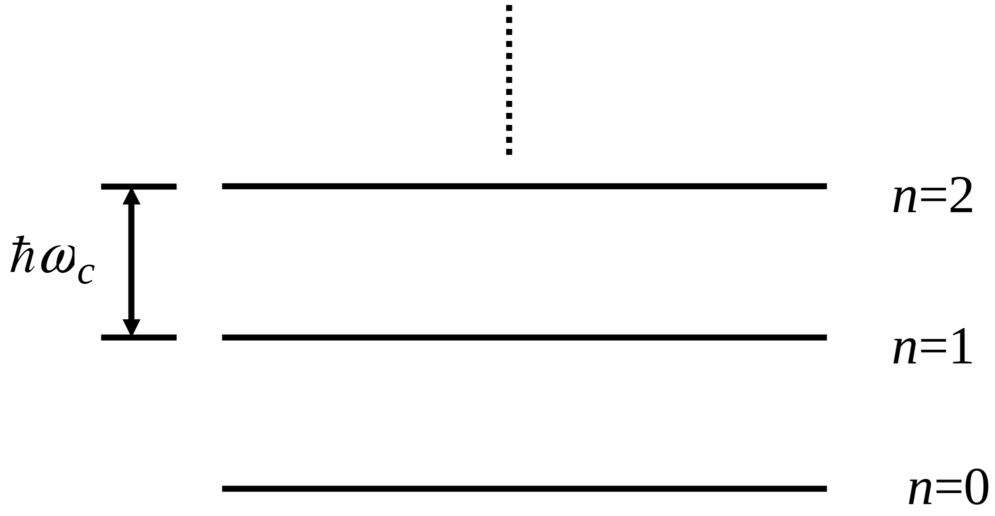
Landau levels



Cyclotron frequency: $\omega_c = 2\omega$

Magnetic length: $a = \sqrt{\hbar / 2M\omega} = 1$

Landau levels



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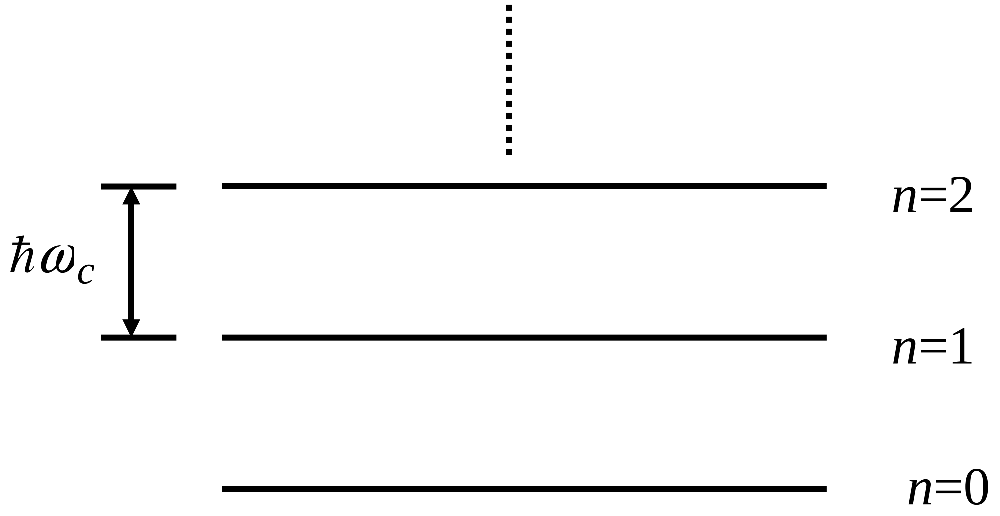
Landau levels



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$$\Gamma = \frac{< V >}{< K >} >> 1,$$ A crystal state is favored.

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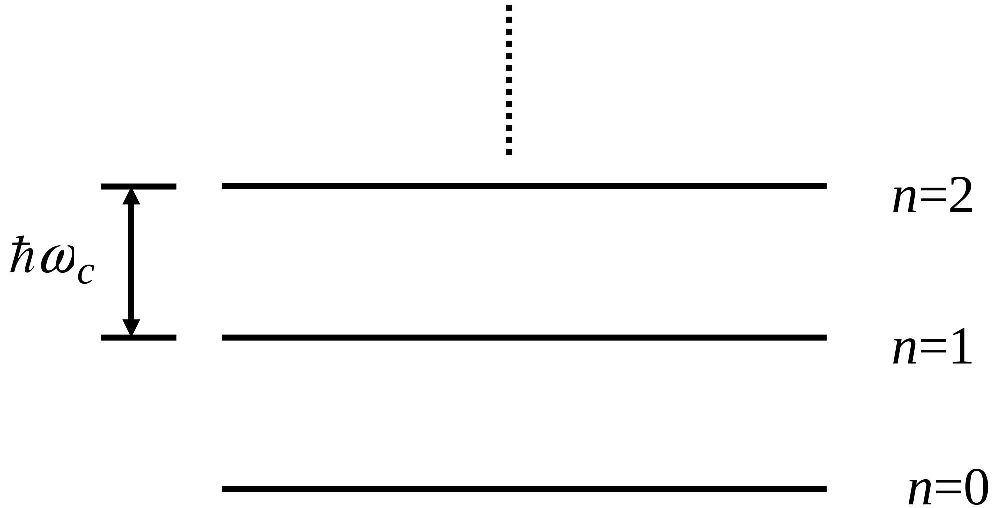

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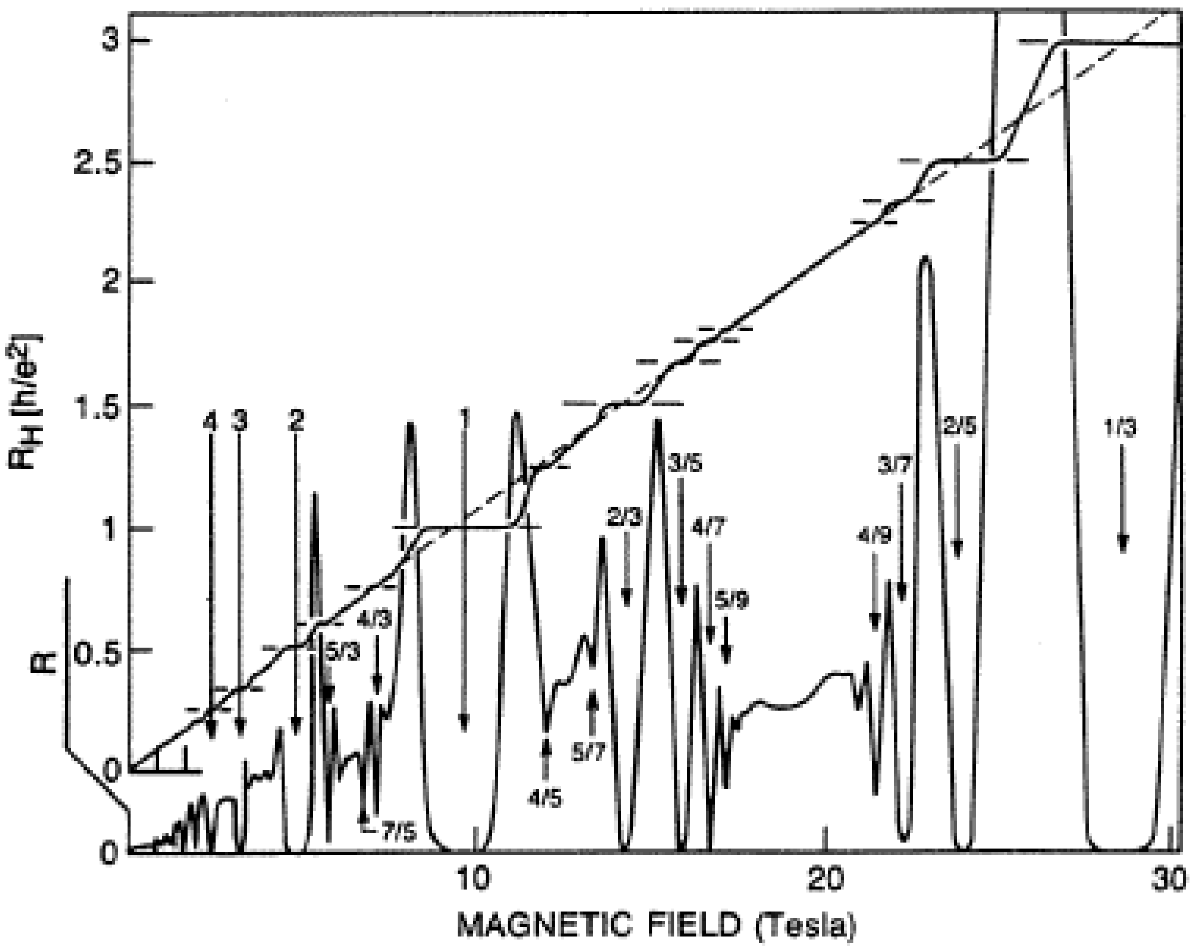
$$\Gamma = \frac{< V >}{< \mathrm{K} >} >> 1,$$ A crystal state is favored.

The kinetic energy is frozen and a crystal state is favorable in the lowest Landau level.
(Wigner Crystal or WC)

But

The Laughlin liquids are favorable for ν=1/3, 1/5, 1/7, 1/9.
(Fractional Quantum Hall Effect)



Eisenstein and Stormer, 1990.

(Laughlin liquid)

$$\Psi_m = \prod_{i<j}^{N} \left(z_i - z_j\right)^m \prod_{\ell} e^{-|z_\ell|^2/4}, \qquad m=1, 3, 5, 7, 9 \ldots. \qquad z=x+iy:$$

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For electrons:

Wigner crystals for ν<1/7 and Laughlin liquids for ν ≧1/7.

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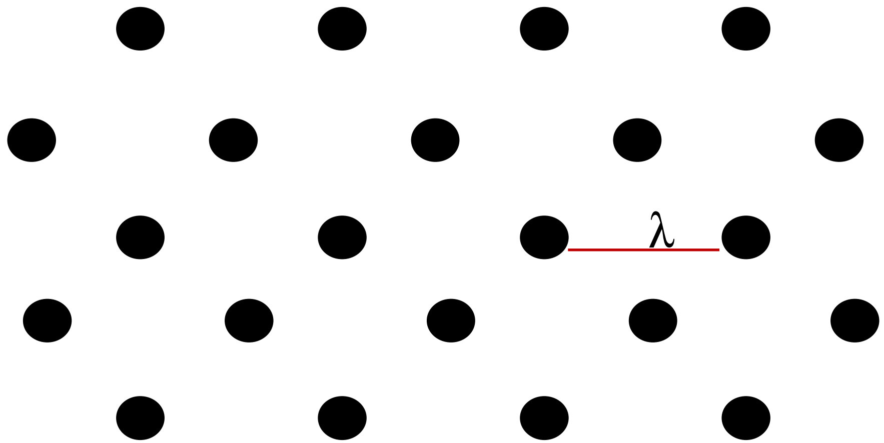
For electrons:

Wigner crystals for $\nu<1/7$ and Laughlin liquids for $\nu \geqq 1/7$.

(Wigner crysal)

$$\Psi_{WC} = A \prod_i \varphi_{\mathbf{R}_i}\left(\mathbf{r}_i\right), \qquad \varphi_{\mathbf{R}}\left(\mathbf{r}\right) = \sqrt{\frac{1}{2\pi}} \exp\left[-\frac{1}{4}(\mathbf{r}-\mathbf{R})^2 + \frac{1}{2} i (\mathbf{r}\times\mathbf{R}) \bullet \mathbf{e}_z\right]$$



(Triangular structure)

$$\psi_{\vec{k}} = \bar{\rho}_{\vec{k}}\ \psi_m$$

Girvin, S. M., and A. H. MacDonald, and P.
M. Platzman, Phys. Rev. Lett. **54,** 581 (1985).

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$$\bar{\rho}_{\mathbf{k}} = \sum_{j=1}^{N} \exp(-ik\partial/\partial z_j) \; \exp(-ik^* z_j/2)$$

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$$\Delta(k) = \frac{\bar{f}(k)}{\bar{s}(k)}$$

$$\bar{s}(k) \quad \equiv \quad \frac{1}{N}\left\langle \psi_m | \bar{\rho}^{\dagger}_{\vec{k}}\ \bar{\rho}_{\vec{k}} | \psi_m \right\rangle \qquad \bar{f}(k) \equiv \langle \psi_{\vec{k}} | (H - E_0) | \psi_{\vec{k}} \rangle$$

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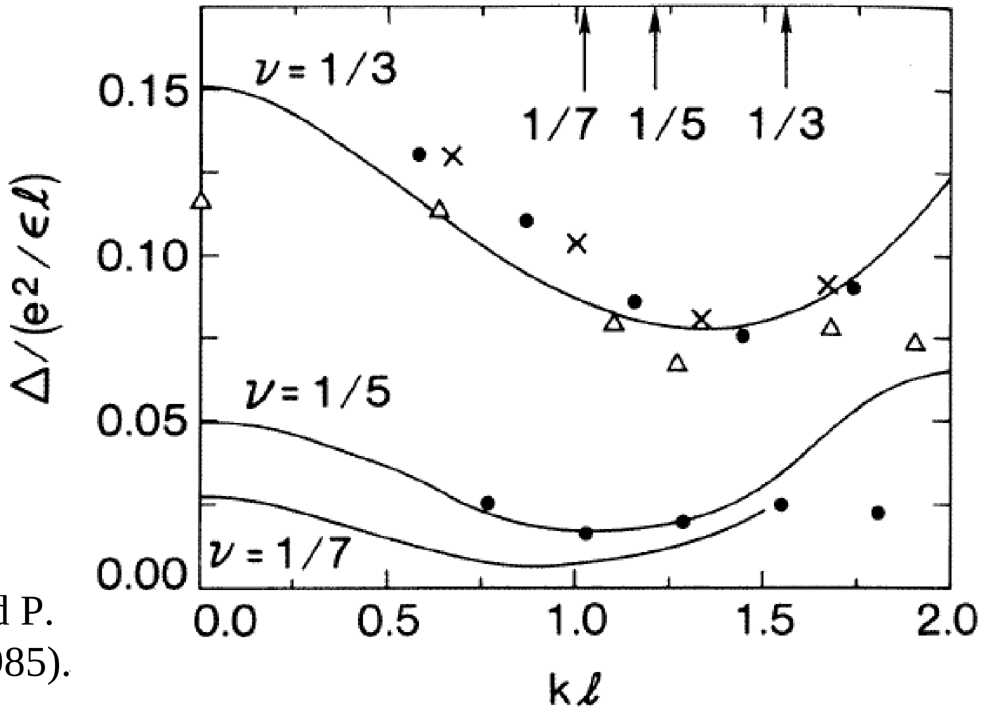

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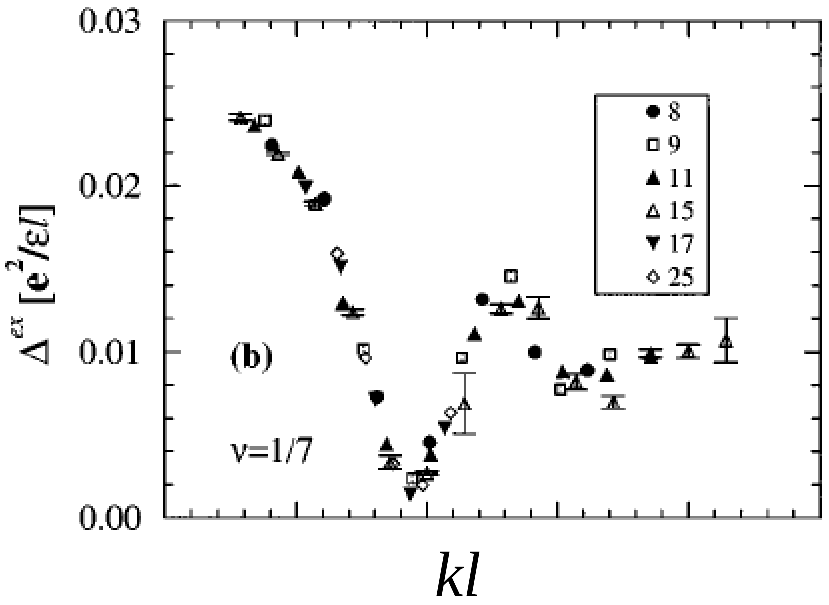
Exciton excitation from the composite-fermion gas



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Phys. Rev. B 55, R13417 (1997)

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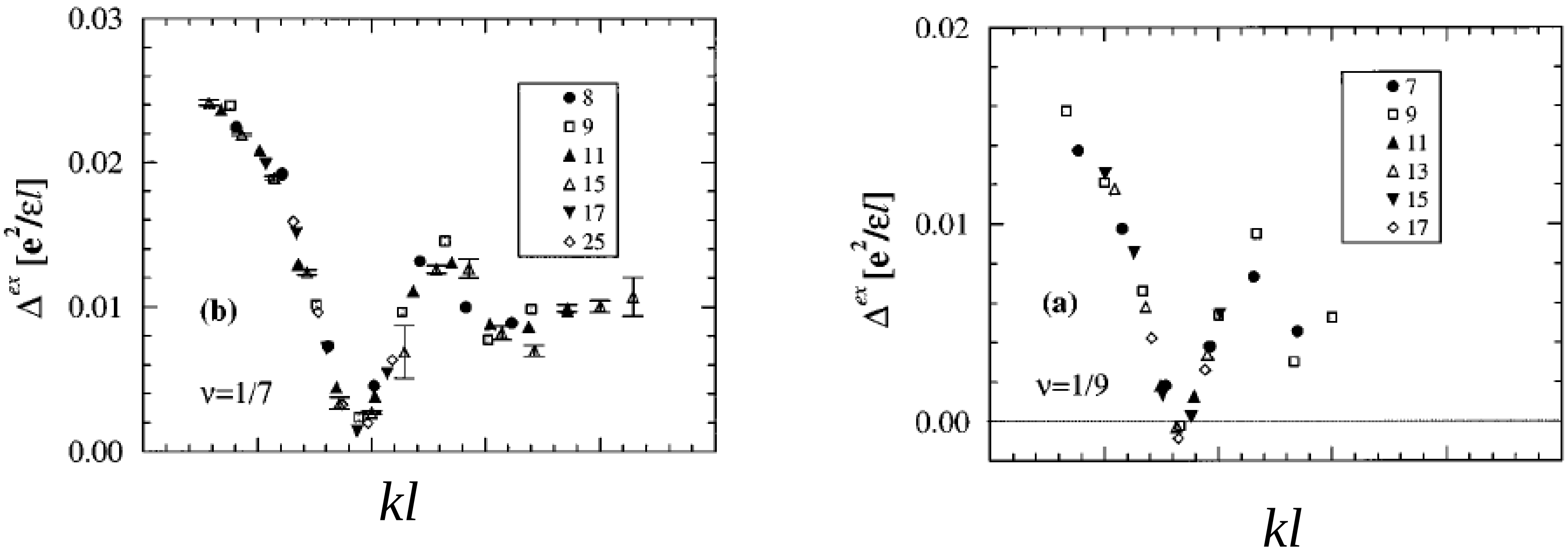

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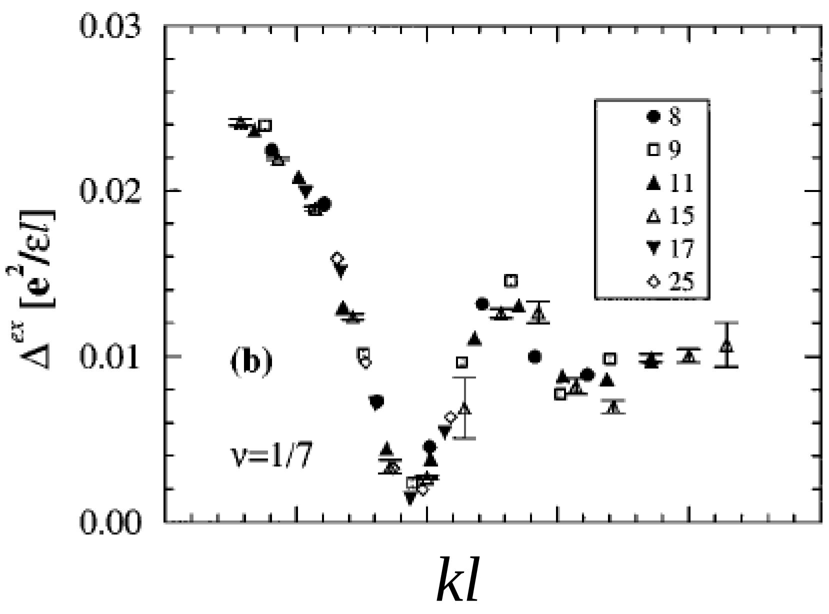

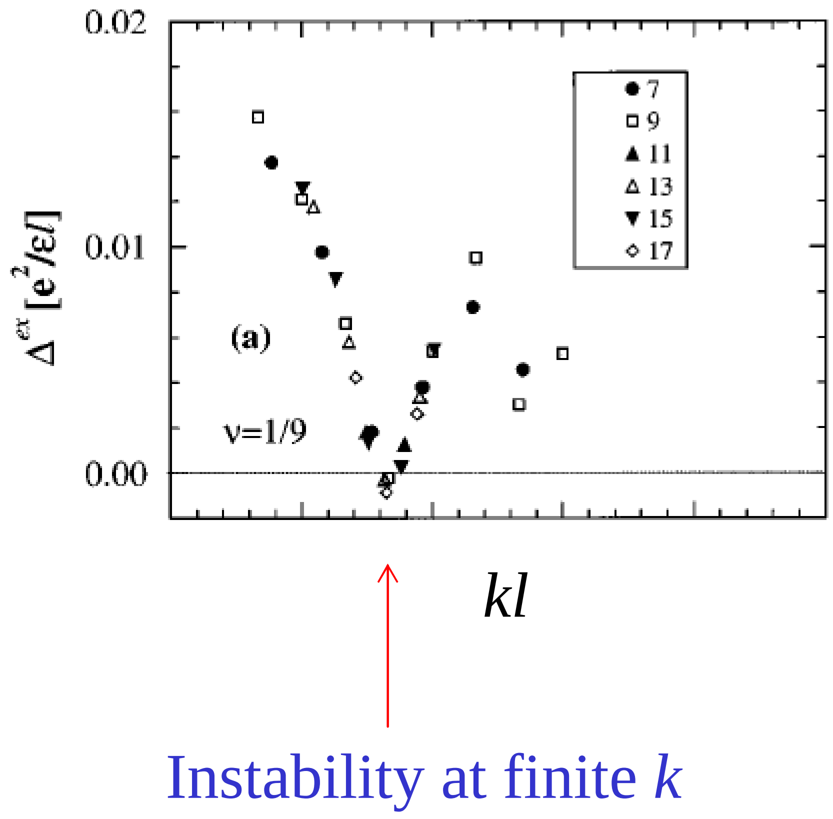

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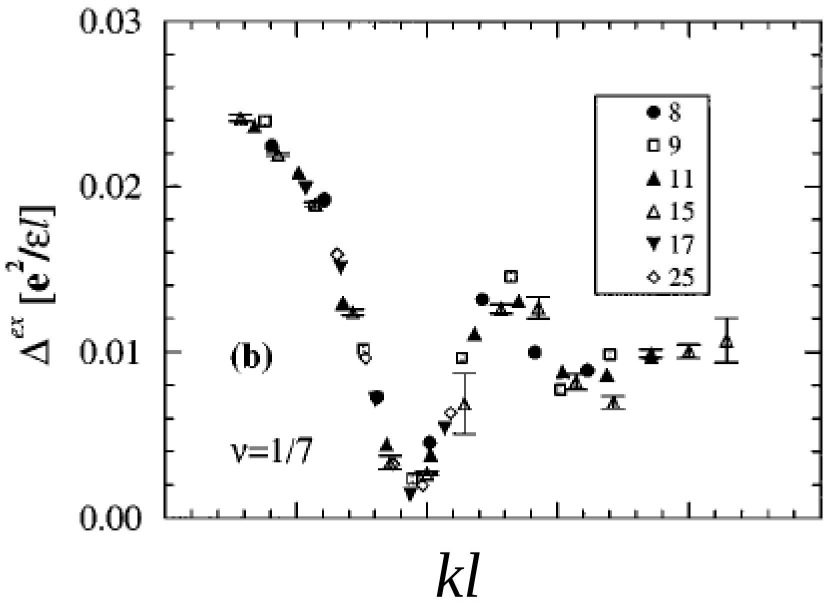

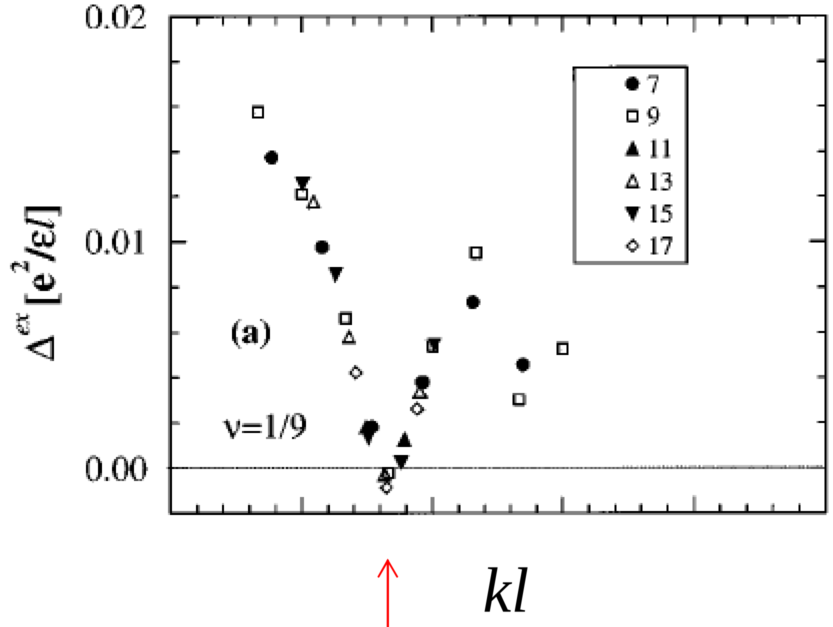

Instability at finite *k*

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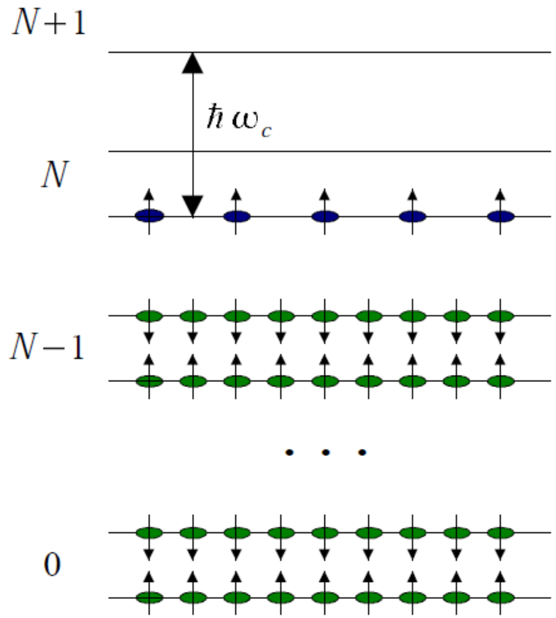
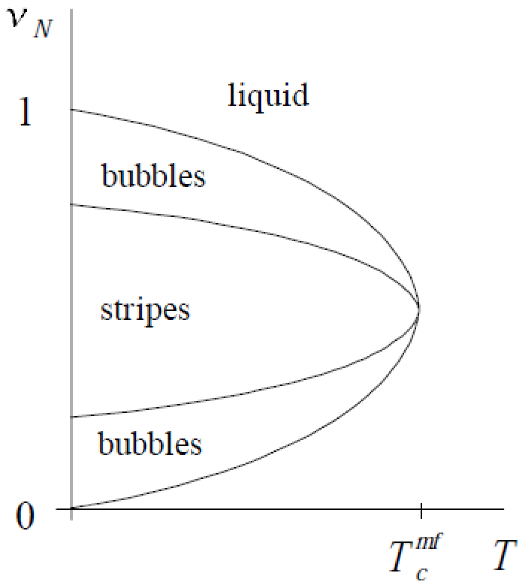
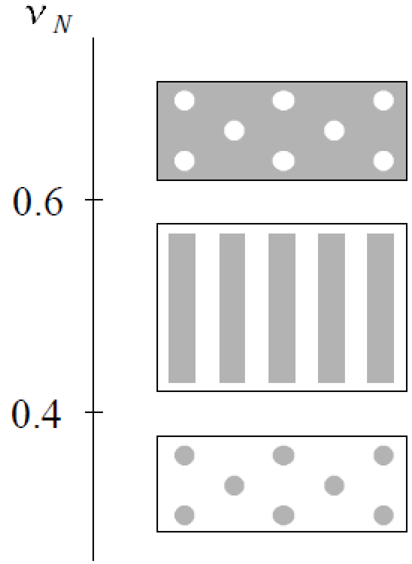
Wigner Crystallization

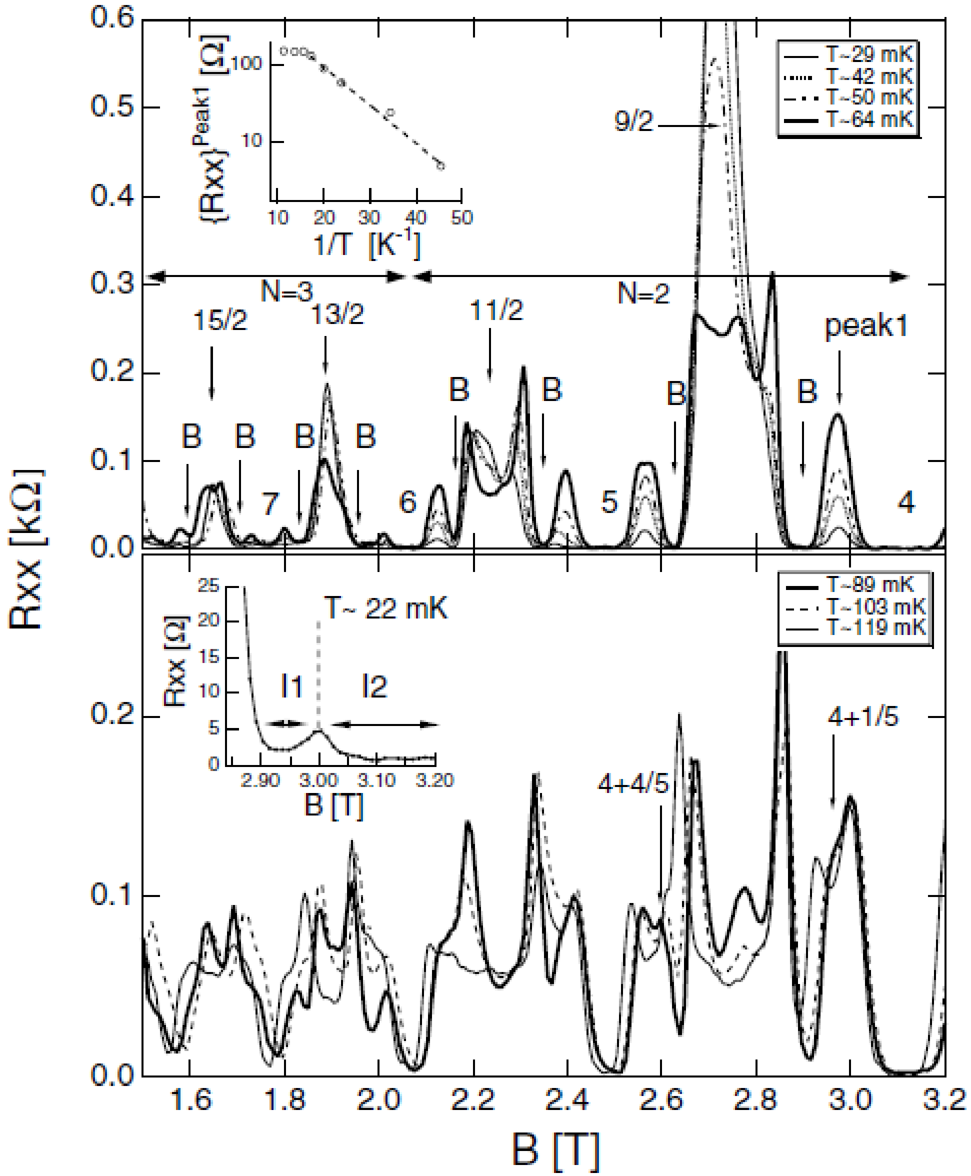
R. K. Kamilla and J. K. Jain, Phys. Rev. B 55, R13417 (1997)

Bubble crystals for a higher Landau level









G. Gevais *et a*l., Phys. Rev. Lett. **93,** 266804 (2004).

Any quantum Hall effect in the atomic Fermi gas?

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Atoms with long-range interactions.

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Rydberg atoms have long-range interactions.

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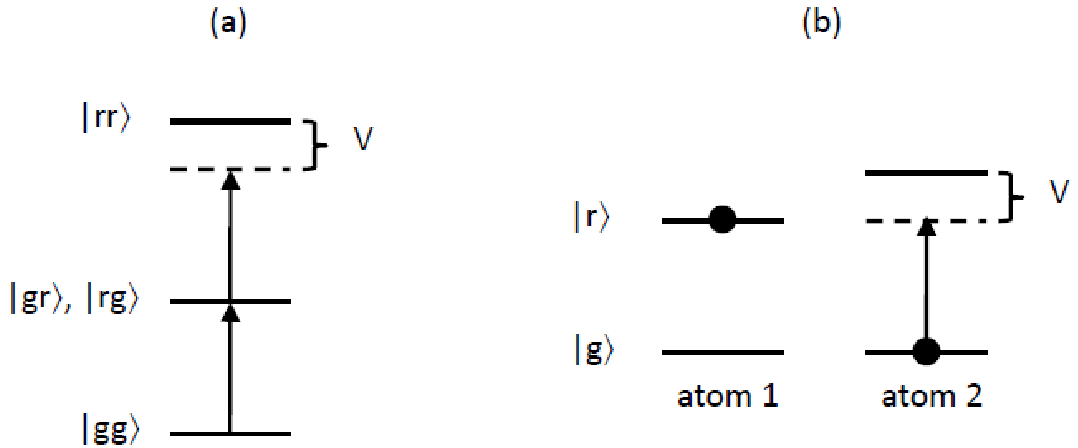
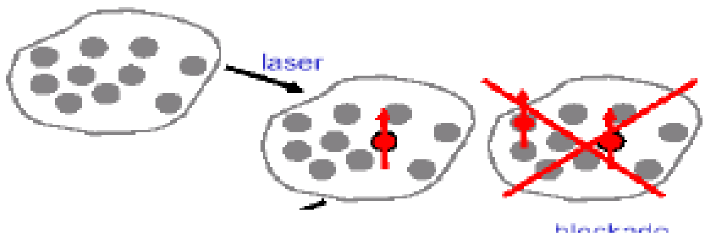
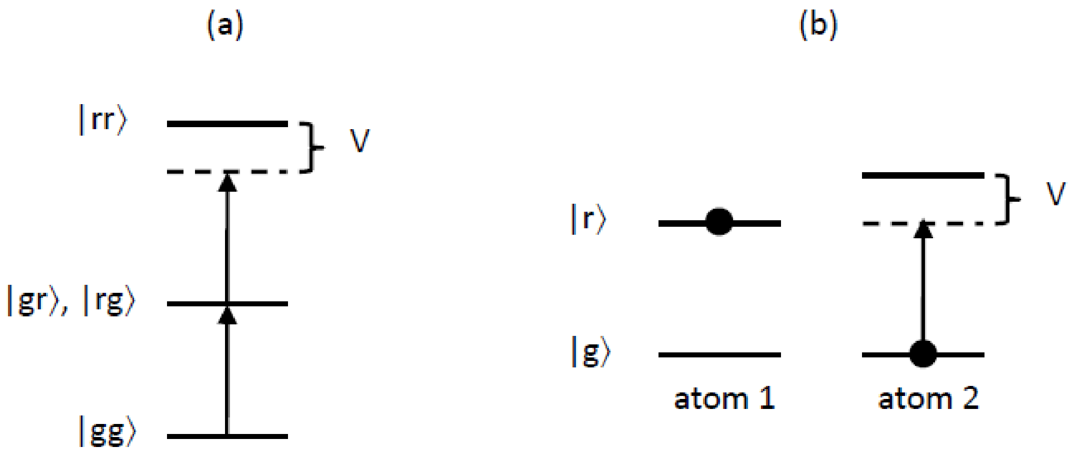
Dipole-blockaded Effect for Rydberg atoms



(Taken from the thesis of Tony E. Lee)



Dipole Blockaded Effect



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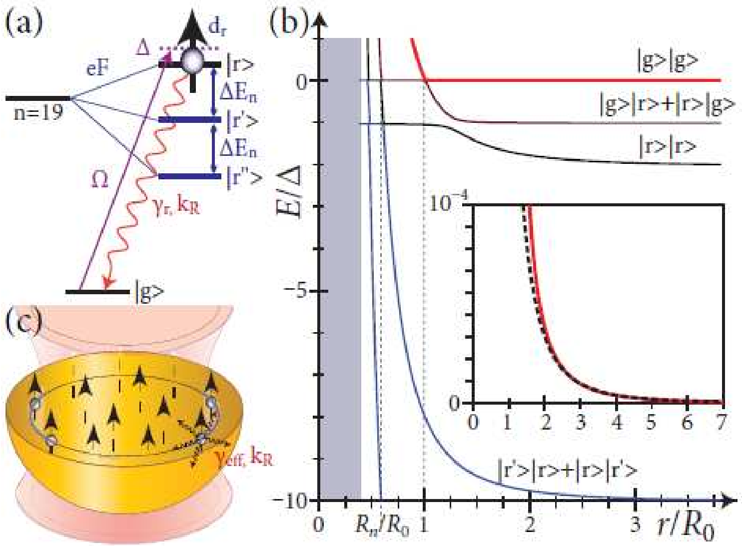
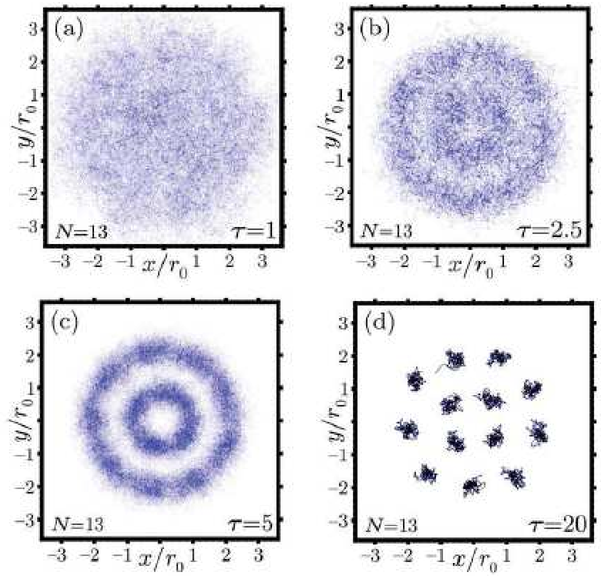


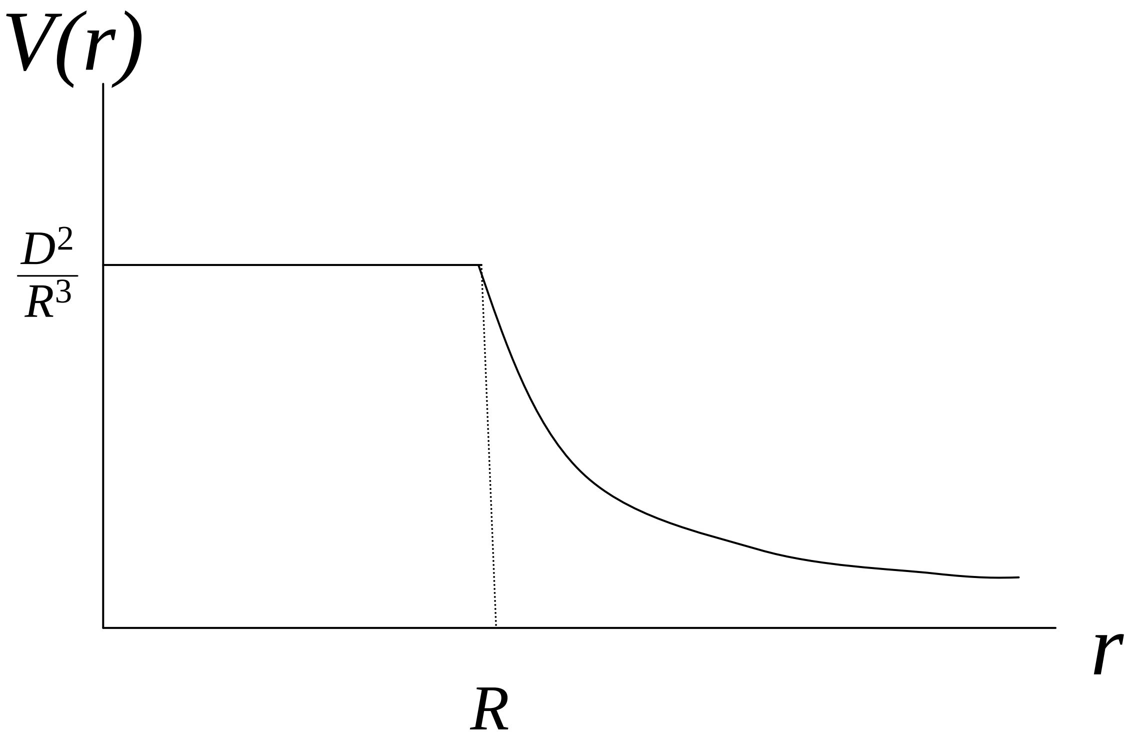


G. Pupillo *et al.*, PRL 104, 223002 (2010)

Dipole-blockaded Potential:

$$V(r)=\begin{cases} \frac{D^2}{R^3}, & r \le R \\ \frac{D^2}{r^3}, & r > R \end{cases}$$

R: blockade radius



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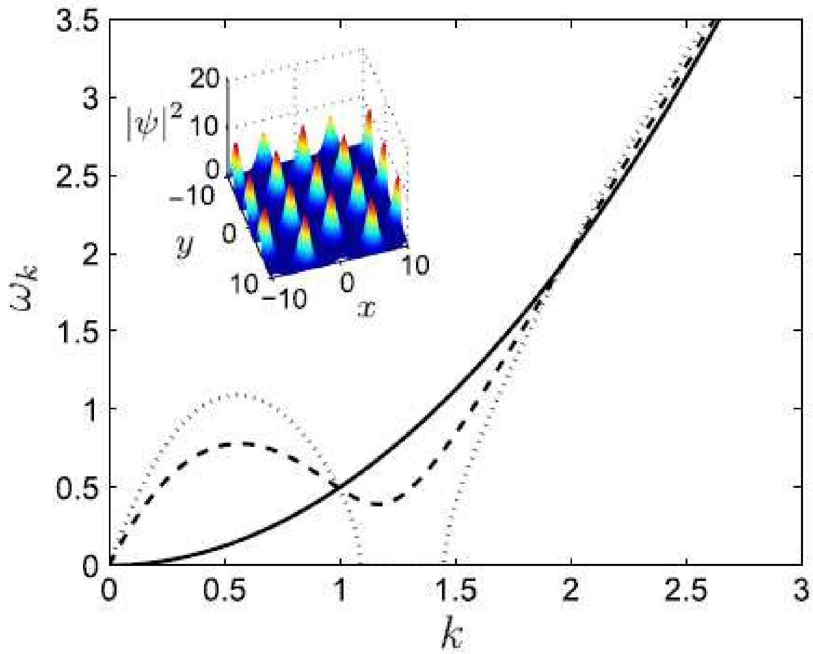
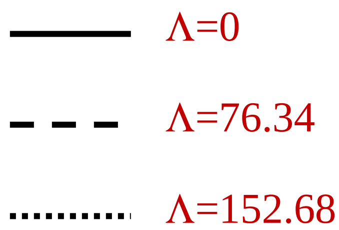
Excitation spectra of dipole-blockaded Bose gas:

Peter Mason *et al*., PRL 109, 045301 (2012)





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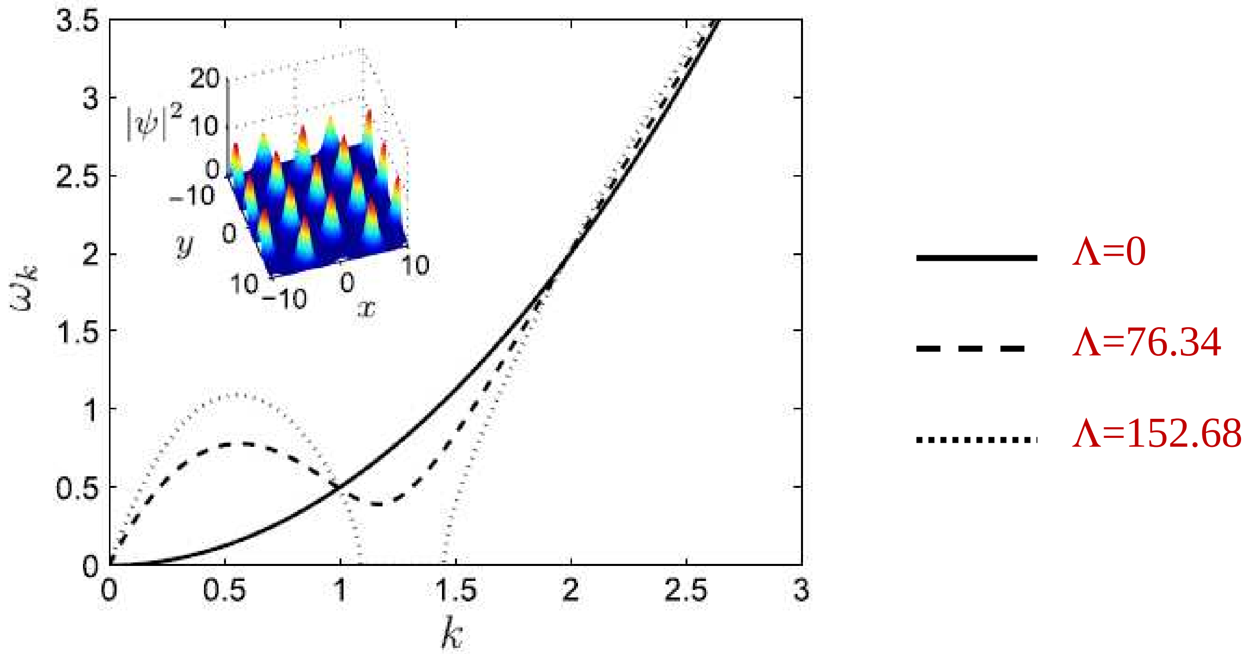

$\omega_k < 0$ at finite k → A crystal state

Monte Carlo simulations: F. Cinti *et al*., PRL 105, 135301 (2010)

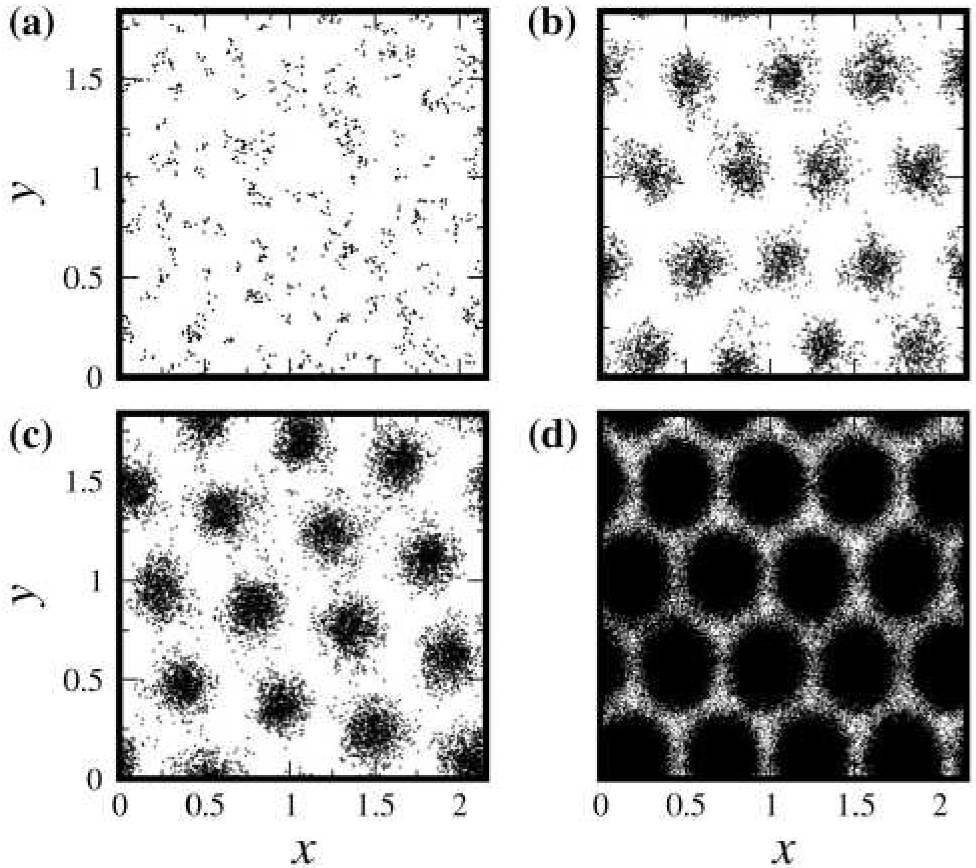

FIG. 1. Snapshots of a system of bosons interacting via potential (1), at the four different temperatures 200 (a), 20 (b), 1.0 (c) and 0.1 (d), expressed in units of ϵ_0. Points shown are taken along individual particle world lines. The nominal value of r_s in this case is 0.14, whereas the cutoff of the potential (1) is $R_c =$ 0.3.

F. Cinti *et al.*, PRL 105, 135301 (2010)

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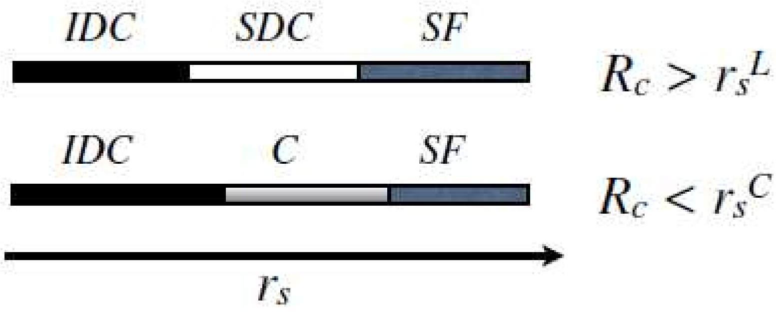

IDC: insulting droplet crystal

SDC: superfluid droplet crystal

SF: superfluid

C: a single-particle crystal

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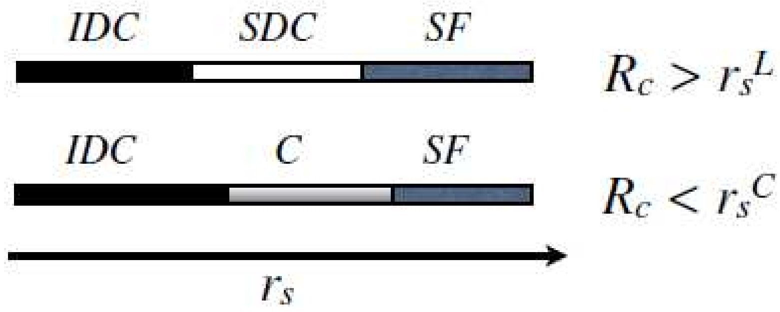

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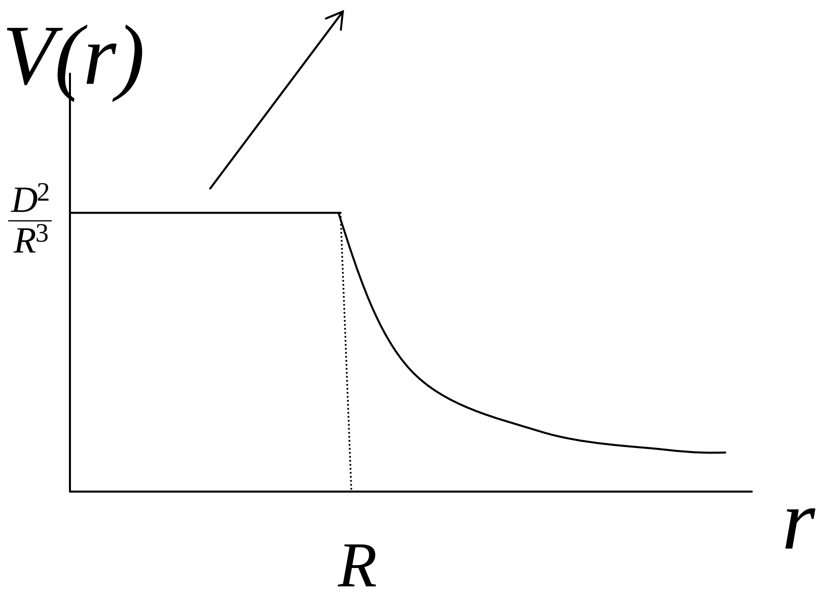
Rydberg atoms can easily form a bubble crystal.



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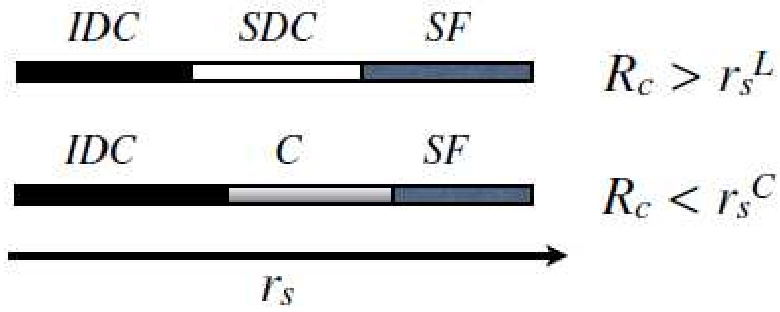

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Effect of gauge fields on the dipole-blockaded gas?

F. Cinti *et al.*, PRL 105, 135301 (2010)

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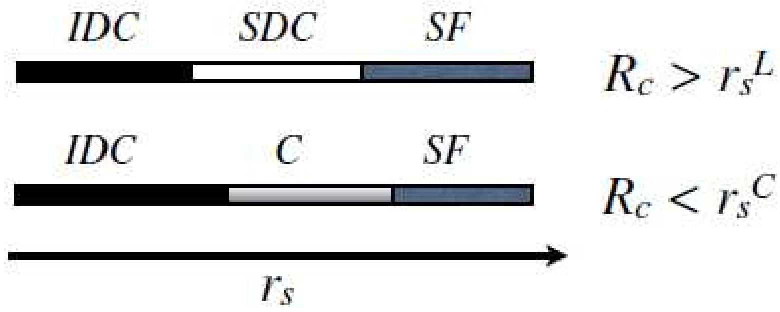

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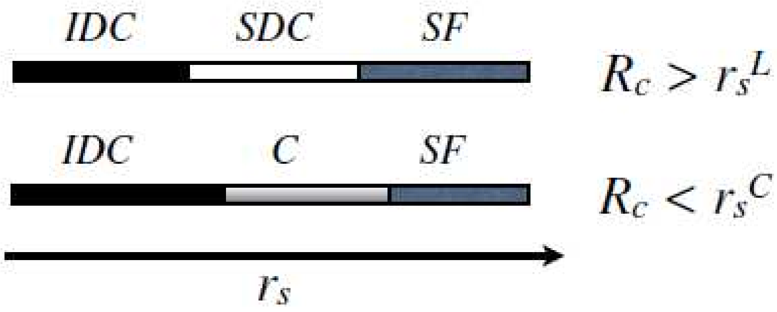

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How rotation affect crystallization?

II. Correlated Wigner Crystals for Rotating Dipolar Fermions:

Dipole Blockate Radius *R*=0

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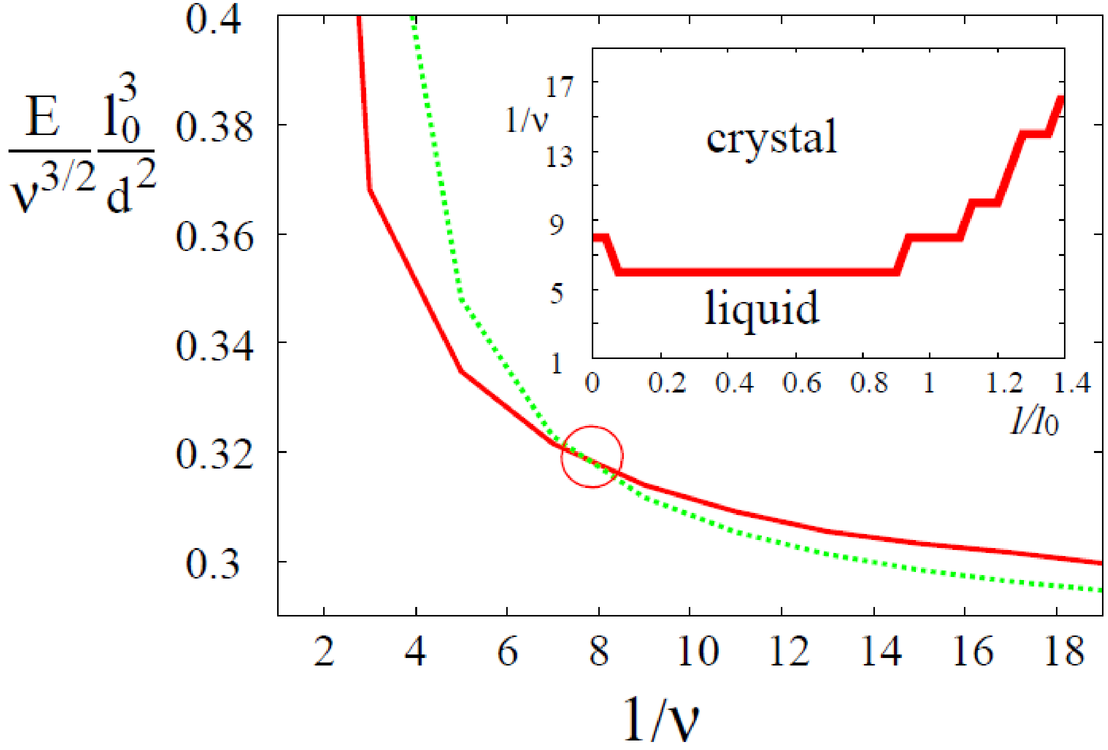

M. A. Baranov, *et. al*.
PRL **100**, 200402 (2008).

The Wigner-crystal states exist in the regime ν<1/7 for zero thickness.

Collective Excitation Spectra of the Laughlin Liquid:

(arXiv:cond-mat/1305.3494)

$$\bar{\rho}_{\mathbf{k}} = \sum_{j=1}^{N} \exp(-ik\partial/\partial z_j)\ \exp(-ik^* z_j/2)$$

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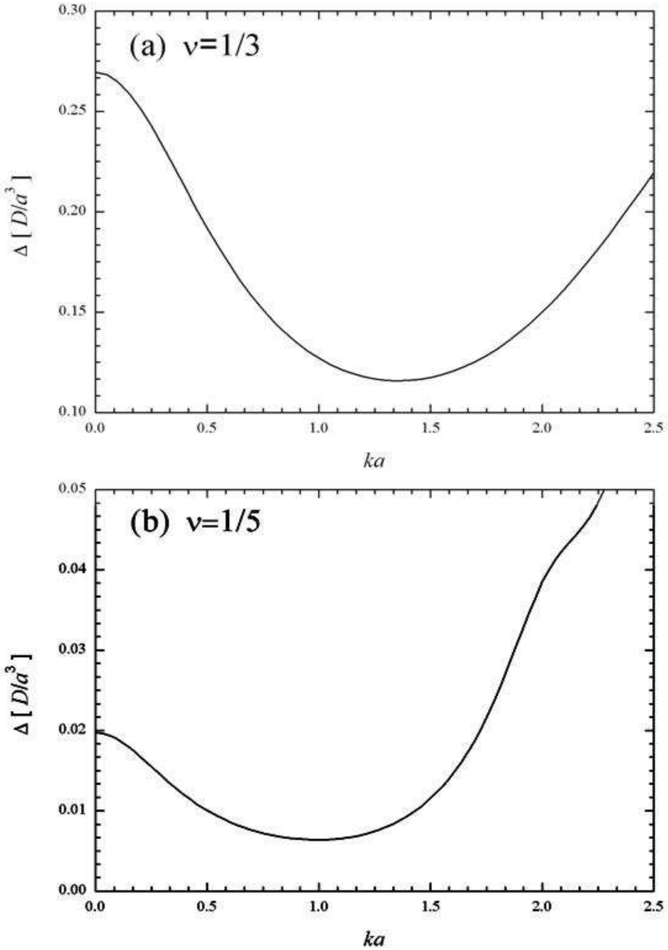

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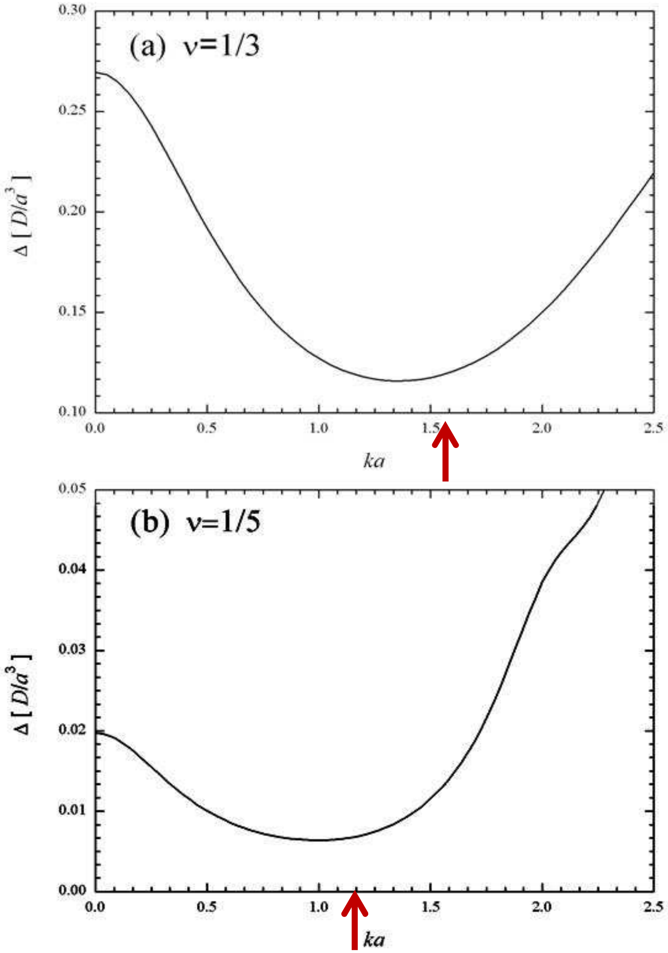

Reciprocal lattice vector of the Wigner crystal: $k_{WC}\, a = \sqrt{\frac{4\pi\nu}{\sqrt{3}}}$

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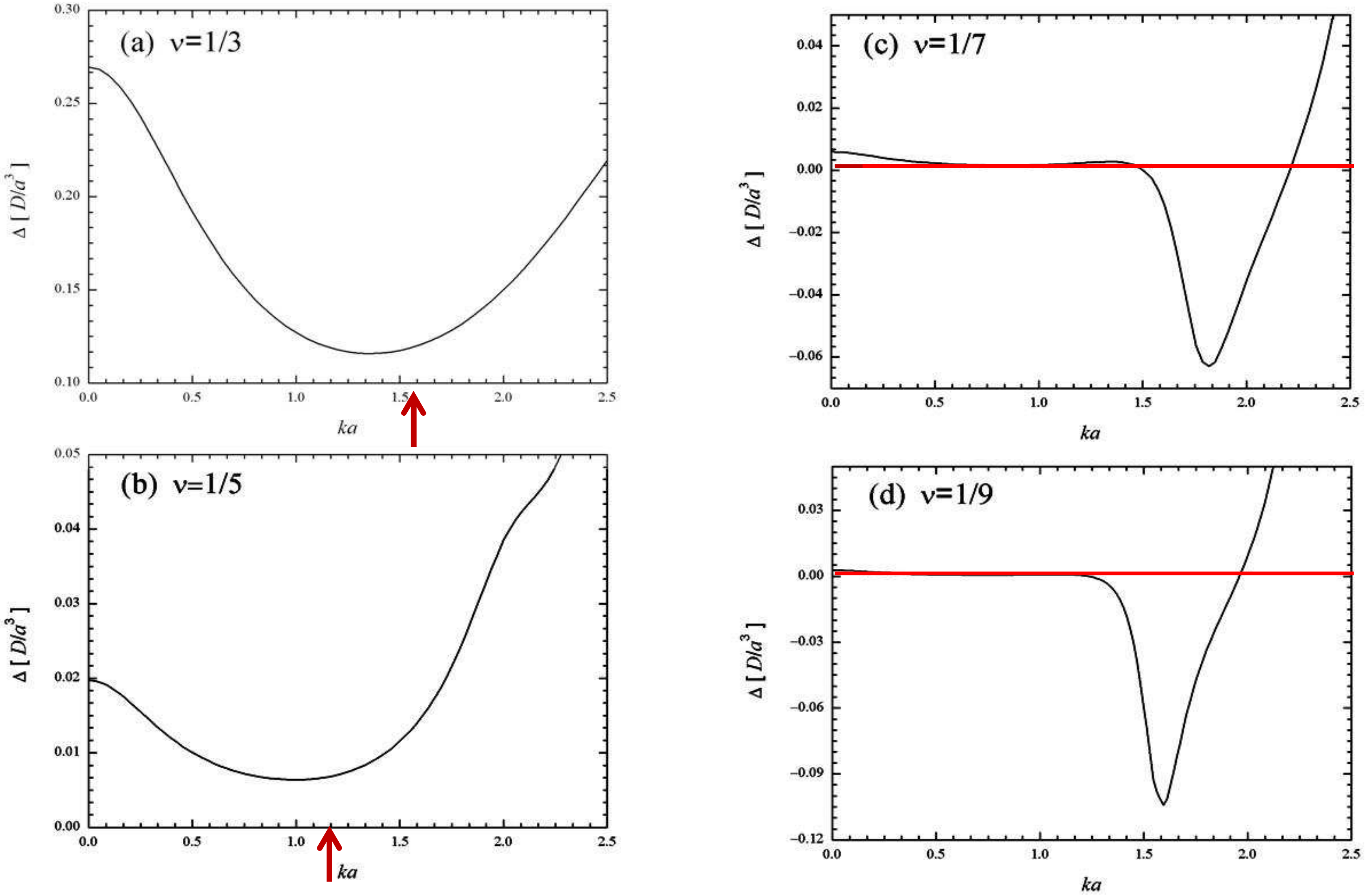

Reciprocal lattice vector of the Wigner crystal: $k_{WC}\, a = \sqrt{\dfrac{4\pi\nu}{\sqrt{3}}}$

Collective Excitation Spectra of the Laughlin Liquid:

(arXiv:cond-mat/1305.3494)

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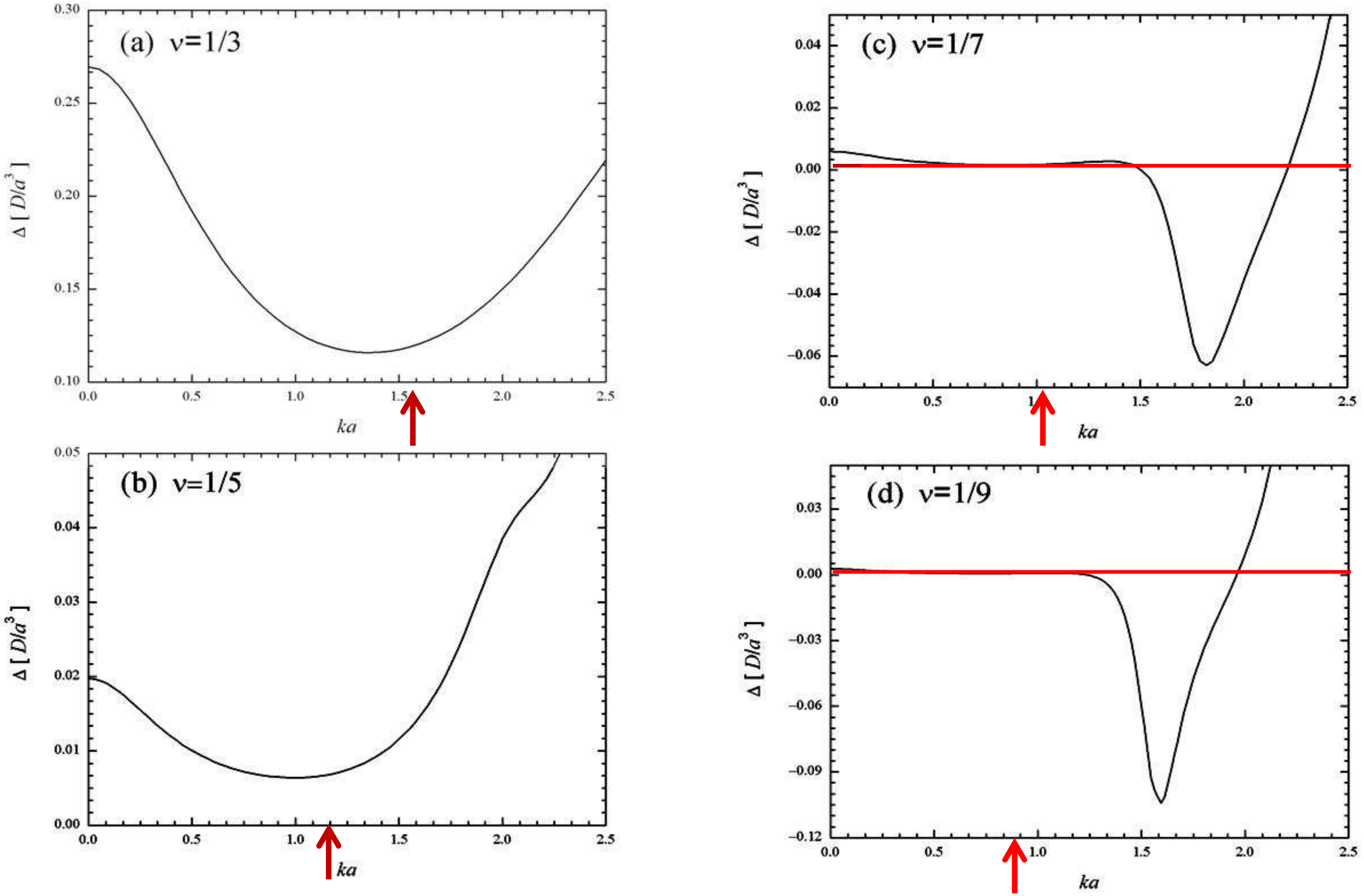
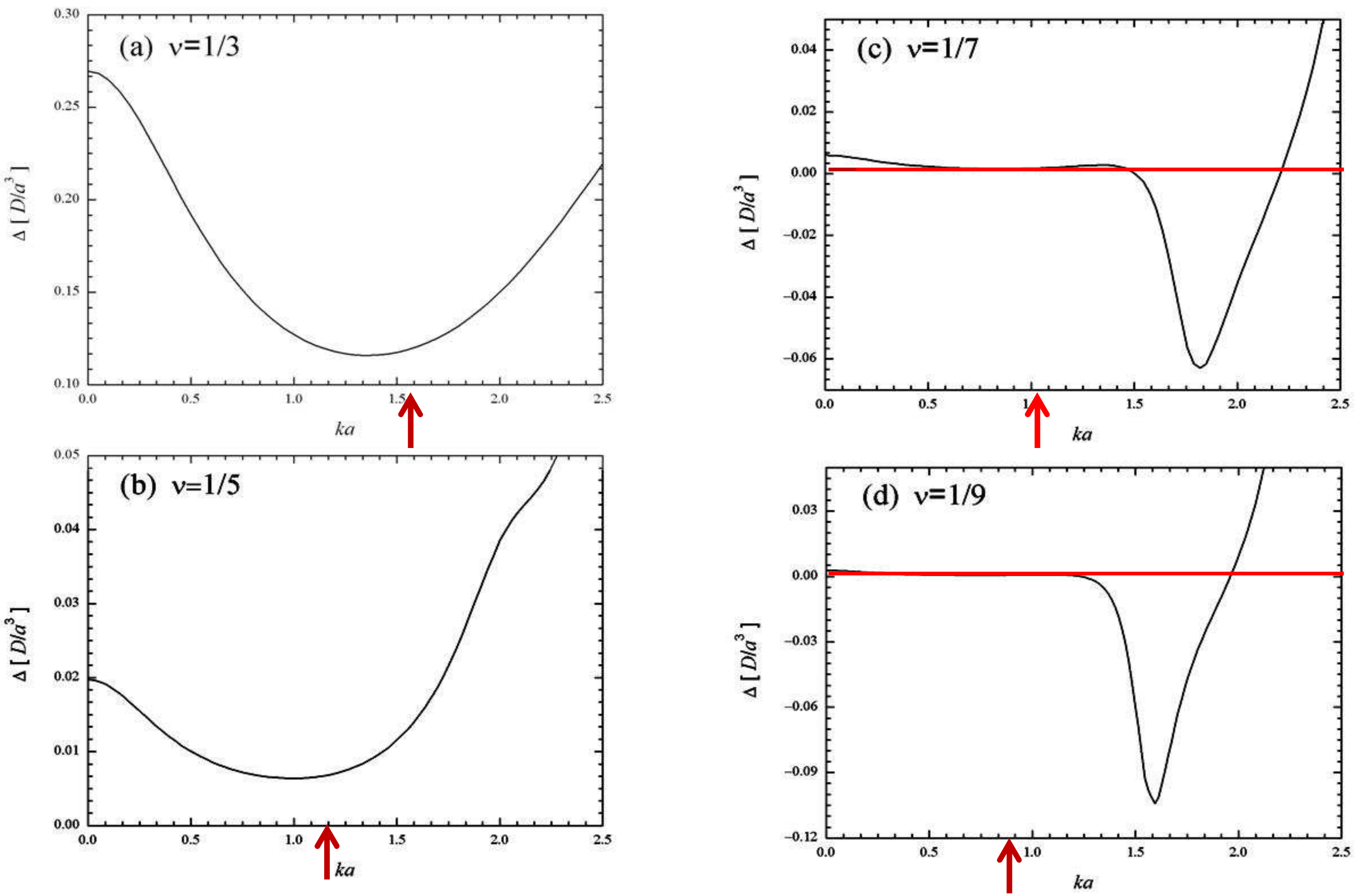


Reciprocal lattice vector of the Wigner crystal: $k_{WC}\, a = \sqrt{\frac{4\pi\nu}{\sqrt{3}}}$

The Laughlin liquid is unstable for $\nu < 1/5$!



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Correlated Wigner Crystal

Proposed by H. Yi and H. A. Fertig, Phys. Rev. B **58**, 4019 (1998)

$$\Psi = A \prod_{\ell<j} \left(Z_\ell - Z_j \right)^m \prod_k exp[-\frac{1}{4}(\vec{r}_k - \vec{R}_k)^2 + \frac{i}{2}\vec{r}_k \times \vec{R}_k \bullet \vec{z}]$$

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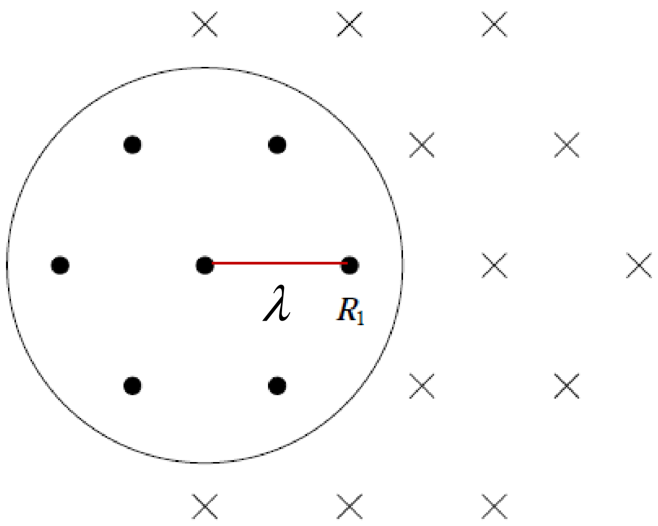
To obtain the energy: Monte Carlo Calculations.

N=20, 40, 60, 80, 100, 120, 140, 160 and 180 particles.



$\lambda \longrightarrow (1-\nu m)\,\lambda$

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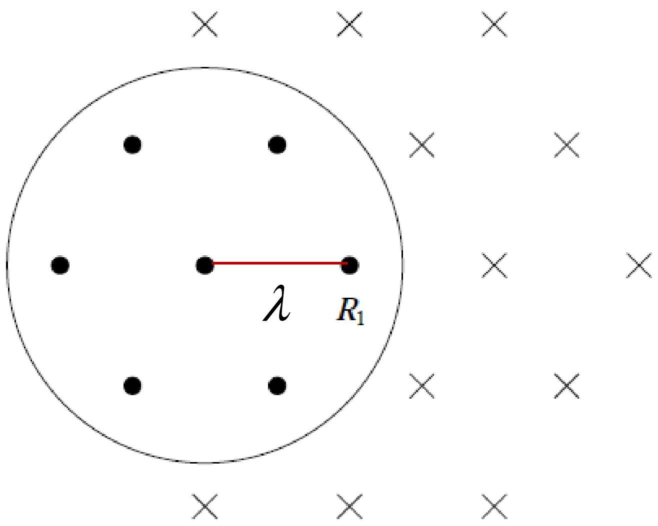

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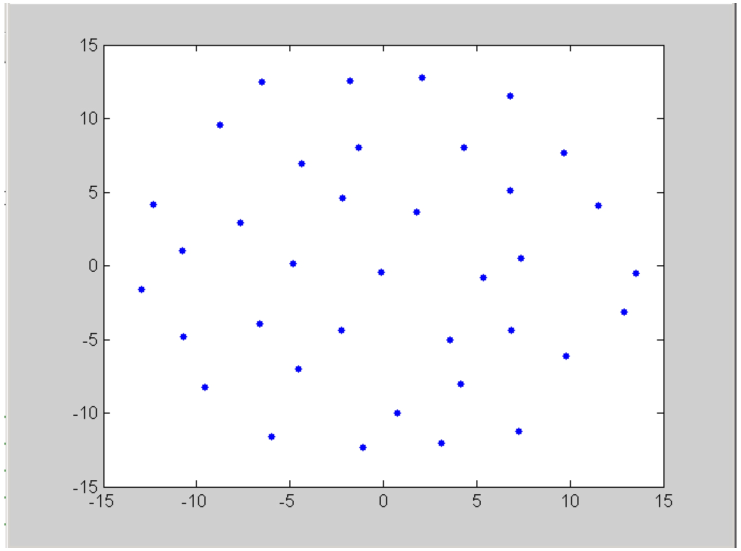
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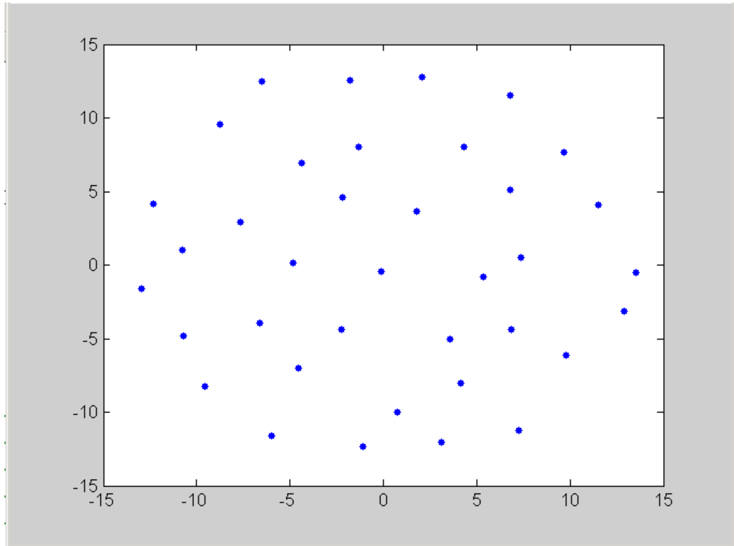
15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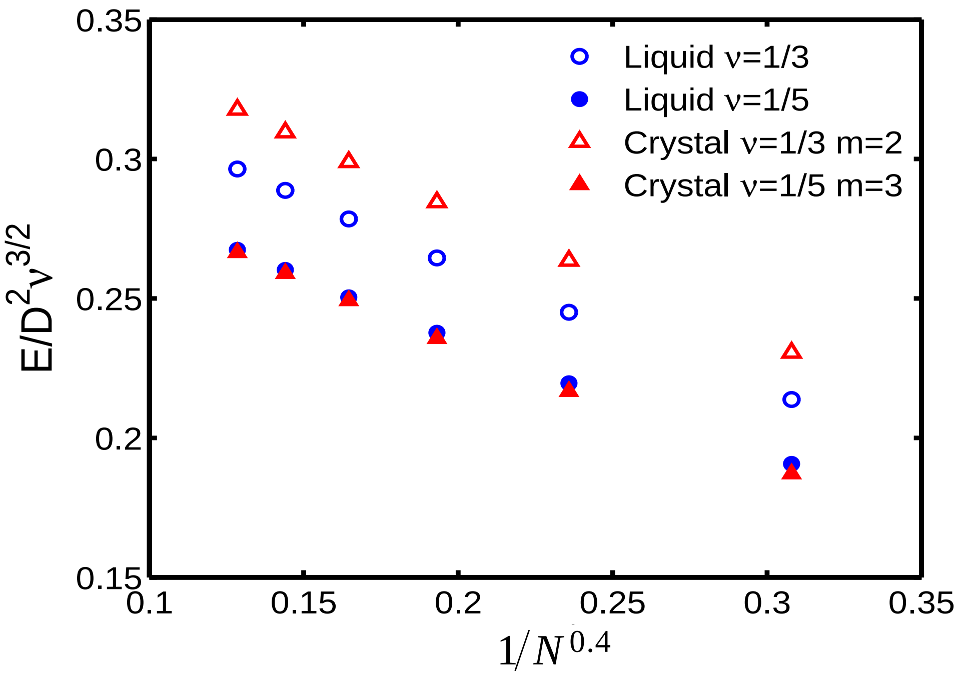
Liquid ν=1/3
Liquid ν=1/5
Crystal ν=1/3 m=2
Crystal ν=1/5 m=3
E/D²ν^{3/2}
1/N^{0.4}
0.35
0.3
0.25
0.2
0.15
0.1
0.15
0.2
0.25
0.3
0.35

ERROR: undefined
OFFENDING COMMAND: sg2@\etlinewidth

STACK:

6.25