Entanglement and its evolution across defects in critical chains

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Entanglement in pure quantum states

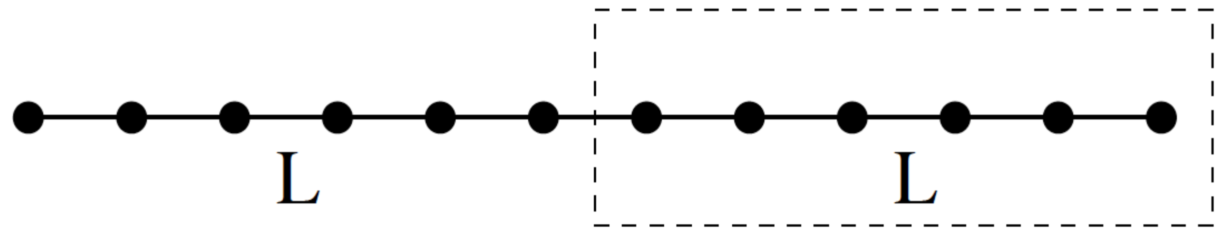
- Amount of entanglement is crucial for classical simulability (DMRG, MPS, etc.)
- Quantum ground states are typically not too much entangled: one has the “area law”
- However, time evolution induced e.g. by a quench leads to growth of entanglement
- How fast does it grow with time?
- Measured by von Neumann / Rényi entropies

Outline of talk

- Overview of results in homogeneous chains
- Introduce the defect problem
- Analyze entanglement in ground state
- Introduce the quench problem
- Point out connections between statics and dynamics

V. Eisler and I. Peschel, Ann. Phys. (Berlin) 522, 679 (2010)
I. Peschel and V. Eisler, J. Phys. A: Math. Theor. 45, 155301 (2012)
V. Eisler and I. Peschel, EPL 99, 20001 (2012)

Entanglement in homogeneous chains



- Main object: reduced density matrix

$$S = -\mathrm{tr}\,(\rho \ln \rho) \qquad\qquad S_n = \frac{1}{1-n} \ln \mathrm{tr}(\rho^n)$$

- Scaling with subsystem size:
  - c/6 ln L for critical (gapless) systems
  - c/6 ln ξ for gapped systems
- Fully understood by CFT methods
- Central charge appears in prefactor!

P. Calabrese and J. Cardy, J. Phys. A: Math. Theor. 42, 504005 (2009)

Free fermion/boson systems

- RDM can be written in a simple form: $\rho = \frac{1}{Z}\,\mathrm{e}^{-\mathcal{H}}$
- $\mathcal{H}$ is again a free-particle Hamiltonian
- Can be obtained through 2-point correlations (reduced correlation matrix)
- Eigenvalues ζ_l contain all the information about entanglement, e.g. for fermions

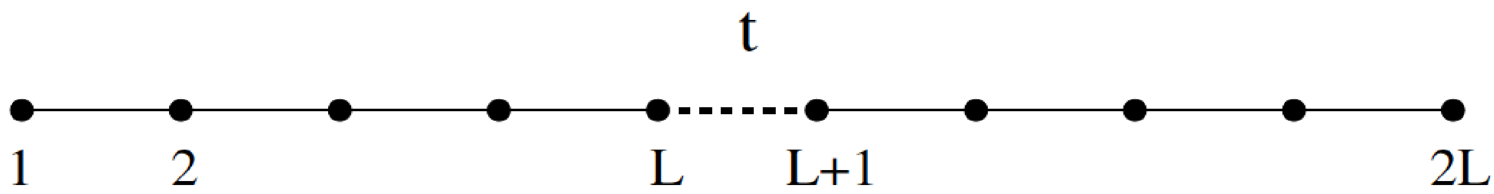
$$\zeta_l'(t) = \frac{1}{\mathrm{e}^{2\omega_l(t)} + 1}$$

- Simple formulas for entropies, e.g von Neumann:

$$S = \pm \sum_k \ln(1 \pm \mathrm{e}^{-2\omega_k}) + \sum_k \frac{2\omega_k}{\mathrm{e}^{2\omega_k} \pm 1}$$

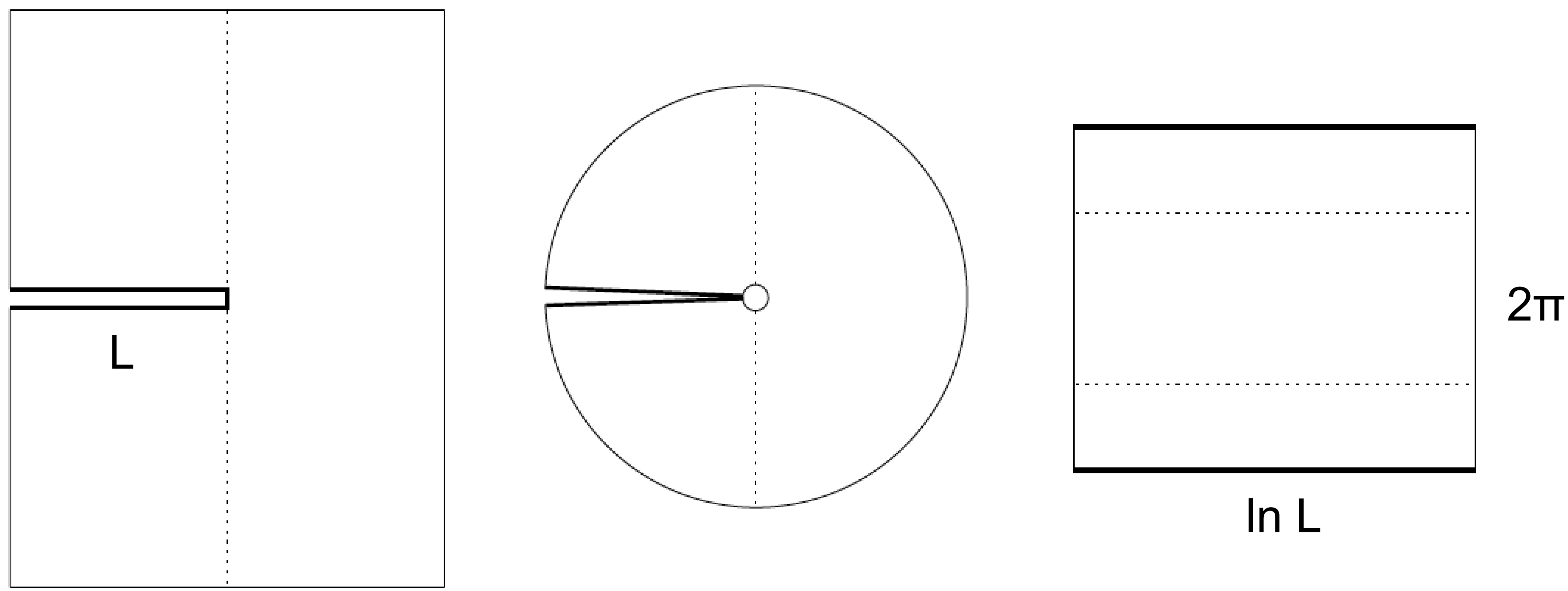
I. Peschel and V. Eisler J. Phys. A: Math. Theor. 42, 504003 (2009)

Chains with defects



- Defect breaks conformal invariance locally
- Marginal perturbation in non-interacting case
- How does the spectrum & entropies change?
- Numerics shows: $S \sim c_{eff}/6 \ln L$
- Models considered:
  - Transverse Ising chain
  - XX chain
  - Coupled oscillators

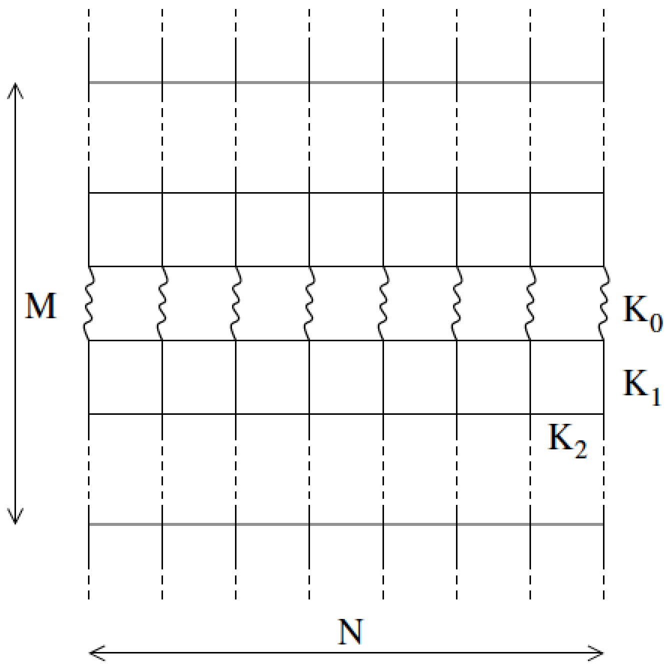
Representation of RDM



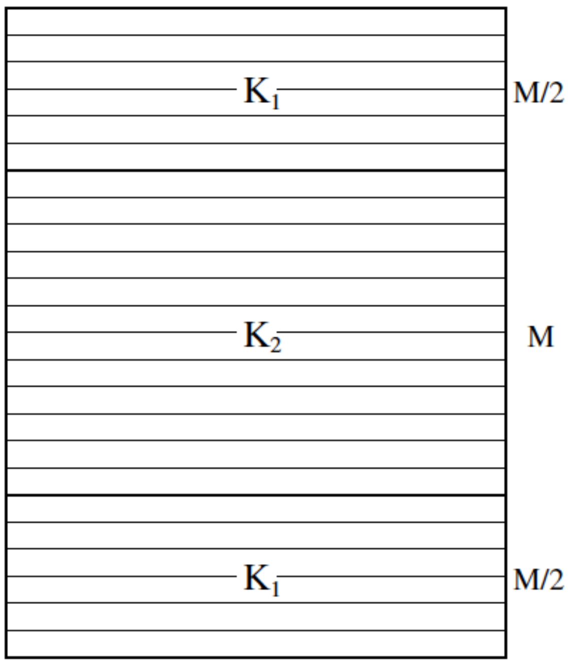
- Simpler geometry through conformal mapping
- Result: finite strip geometry
- To calculate: transfer matrix of strip with defect lines

Transfer matrices

TI chain



Oscillator chain



- 2D Ising model with ladder defect
- Defect parameter has to be renormalized

$$t = \frac{\operatorname{th} K_0}{\operatorname{th} K_1}$$

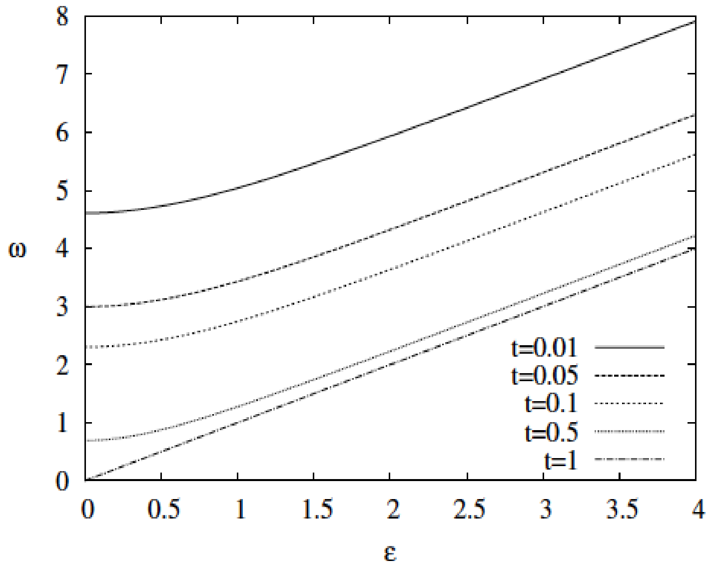
- Gaussian model on square lattice

$$V_1 = \exp\left(\frac{1}{2}K^* \sum_q \frac{\partial^2}{\partial \phi_q^2}\right), \qquad V_2 = \exp\left(-\frac{1}{2}K \sum_q \Omega_q^2 \phi_q^2\right)$$

$$W = V_1^{1/2} V_2 V_1^{1/2} \qquad W_{\text{tot}} = W_1^{M/2} W_2^{M} W_1^{M/2}$$

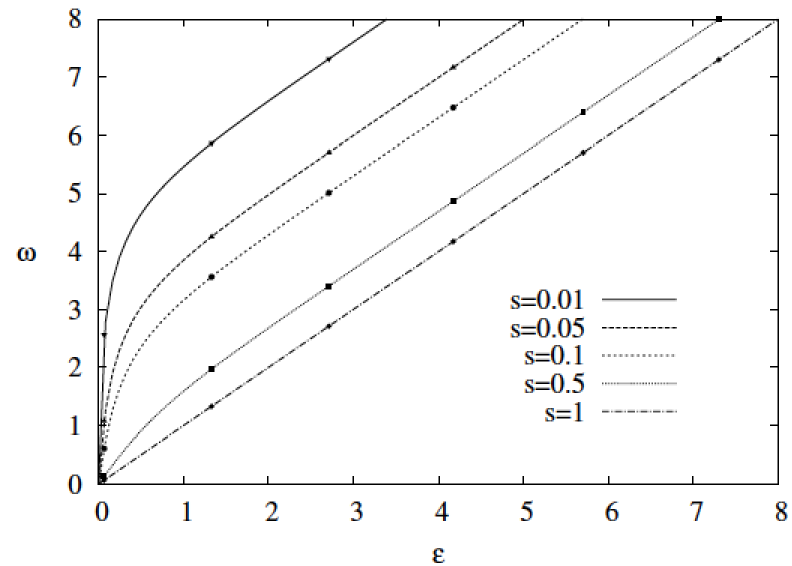
Dispersions

Fermions



$$\mathrm{ch}\,\omega_k = \frac{1}{s}\mathrm{ch}\,\varepsilon_k$$

Bosons



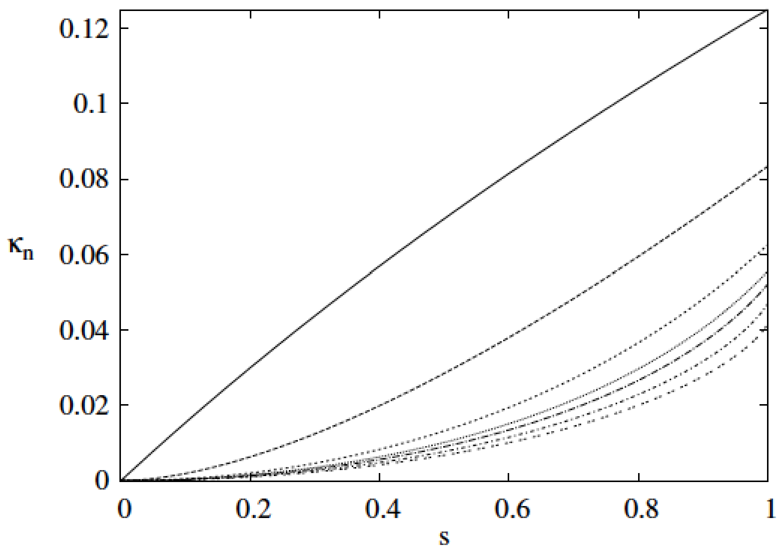
$$\mathrm{sh}\,\omega_k = \frac{1}{s}\mathrm{sh}\,\varepsilon_k$$

- Parameter s corresponds to the transmission amplitude of the defect!
- Spacing of ε_k is ~ 1/ln L (I. Peschel, JSTAT P06004 (2004))

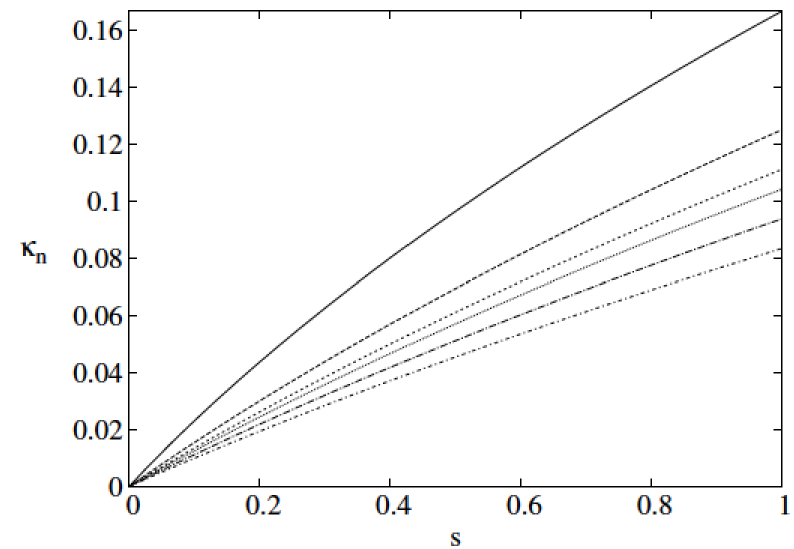
Entropies

$$S_n = \kappa_n \ln L$$

Fermions



Bosons



All the prefactors are available analytically!

$$\kappa_F(s) = -\frac{1}{2\pi^2}\{[(1+s)\ln(1+s) + (1-s)\ln(1-s)]\ln s + [(1+s)\mathrm{Li}_2(-s) + (1-s)\mathrm{Li}_2(s)]\}$$

$$\kappa(s) = \tfrac{1}{4}s - \kappa_F(s)$$

Time evolution after local quench

- What happens if we connect the chains through a defect? (Ising, XX)
- In *homogeneous* case CFT result shows:

  $S \sim c/3 \ln t$ Calabrese & Cardy JSTAT P01023 (2008)
- Central charge appears also in time evolution!
- Numerical results show for quench through defect:

  $S \sim \hat{c}_{eff}/3 \ln t$
- Is there any connection between static and dynamic effective central charges?

Conformal defect

$$H' = \frac{1}{2}\sum_{m,n=1}^{2L} H'_{m,n} c_m^\dagger c_n$$

$$H'_{m,m+1} = H'_{m+1,m} = \begin{cases} -1 & m \neq L \\ -\lambda & m = L \end{cases}$$

$$H'_{L,L} = -H'_{L+1,L+1} = \sqrt{1-\lambda^2}$$

Solved by a simple rescaling of the homogeneous eigenvectors:

$$\phi'_k(m) = \begin{cases} \alpha_k \phi_k(m) & 1 \leq m \leq L \\ \beta_k \phi_k(m) & L < m \leq 2L \end{cases}, \quad \Omega'_k = \Omega_k$$

Calculate time dependent reduced correlation matrix $\langle c_m^\dagger(t) c_n(t) \rangle$

$$\bar{\mathbf{C}}'(t) = \mathrm{e}^{i\bar{\mathbf{H}}'t}\bar{\mathbf{C}}(0)\mathrm{e}^{-i\bar{\mathbf{H}}'t}$$

$$\zeta'_l(t) = \frac{1}{\mathrm{e}^{2\omega_l(t)}+1}$$

$$S(t) = \sum_l \ln(1+\mathrm{e}^{-2\omega_l(t)}) + \sum_l \frac{2\omega_l(t)}{\mathrm{e}^{2\omega_l(t)}+1}$$

$$= \sum_l H(\zeta'_l(t))$$

Start with equal fillings

- Initial correlation matrix: $$\bar{\mathbf{C}}(0) = \begin{pmatrix} \mathbf{C}^0 & 0 \\ 0 & \mathbf{C}^0 \end{pmatrix}$$
- After time evolution one has the relation

$$(2\mathbf{C}'(t) - 1)^2_{mn} = \lambda^2 (2\mathbf{C}(t) - 1)^2_{mn} + (1 - \lambda^2)\delta_{mn}$$

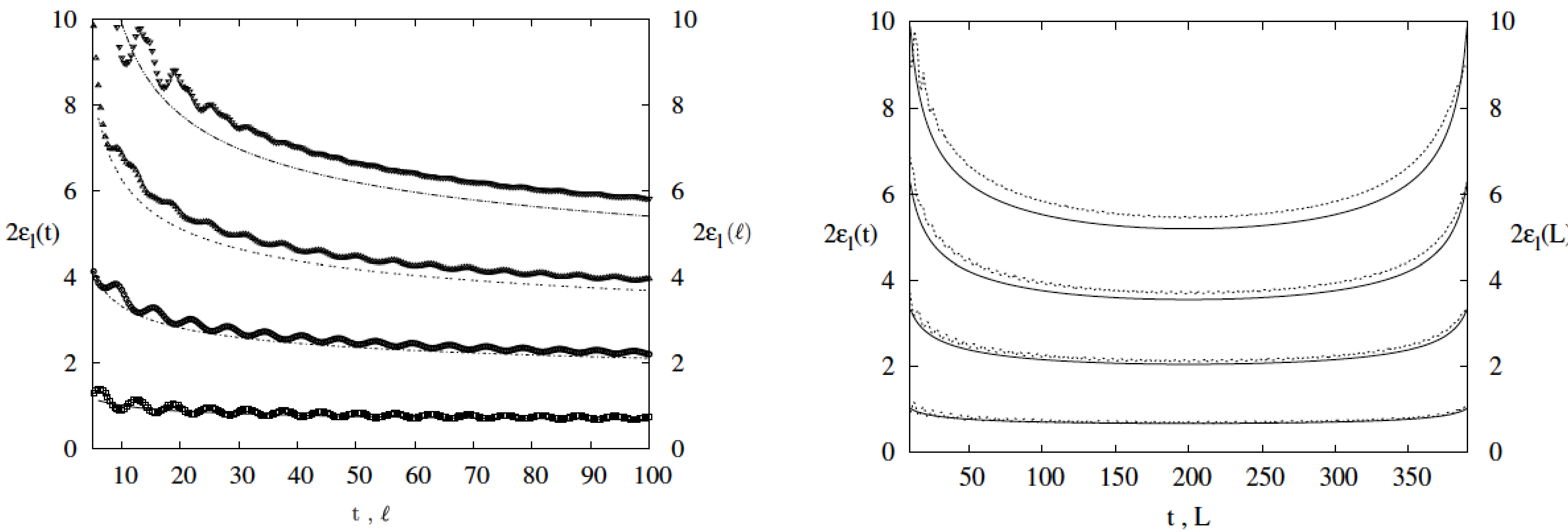
which can be rewritten using s = λ

$$\operatorname{ch} \omega_l(t) = \frac{1}{s} \operatorname{ch} \varepsilon_l(t)$$

and leads to

$$\hat{c}_{\text{eff}} = c_{\text{eff}}$$

Spectra and finite size effects



- Homogeneous spectrum is not known analytically
- Entropy formula (t<<L) analogous to equilibrium result for the segment in an infinite chain if one substitutes t with ℓ
- For finite L one has an analogy with a block in a ring (PBC!)

$$S(t) = \frac{c}{3} \ln \left| \frac{2L}{\pi} \sin \frac{\pi v_F t}{2L} \right| + \text{const} \qquad c \to c_{\text{eff}}$$

J.-M. Stéphan and J. Dubail, JSTAT P08019 (2011)

Biased case

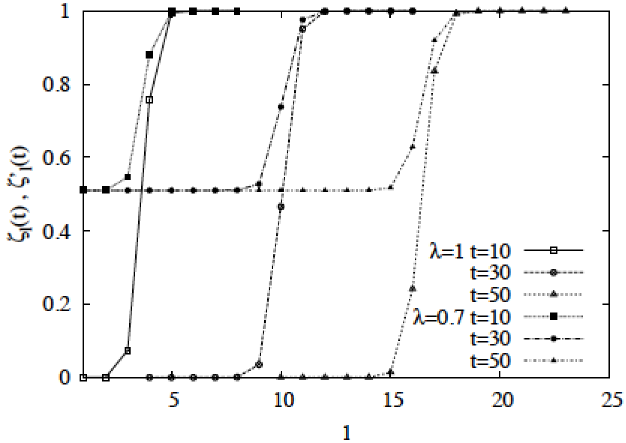
- Initial correlation matrix:

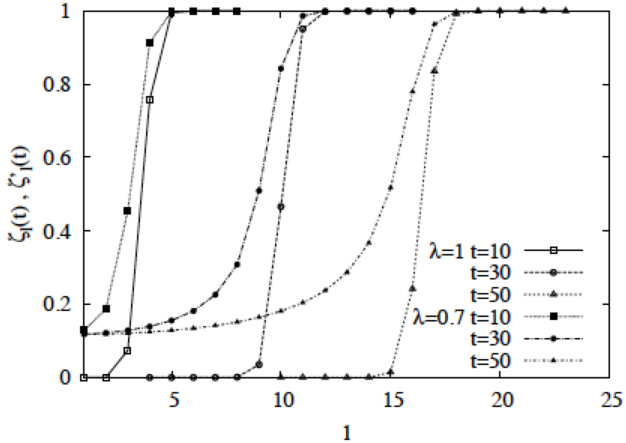
$$\bar{\mathbf{C}}(0) = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & 0 \end{pmatrix}$$

- After time evolution:

$$C'_{mn}(t) = \lambda^2 C_{mn}(t) + (1-\lambda^2)\delta_{mn}$$

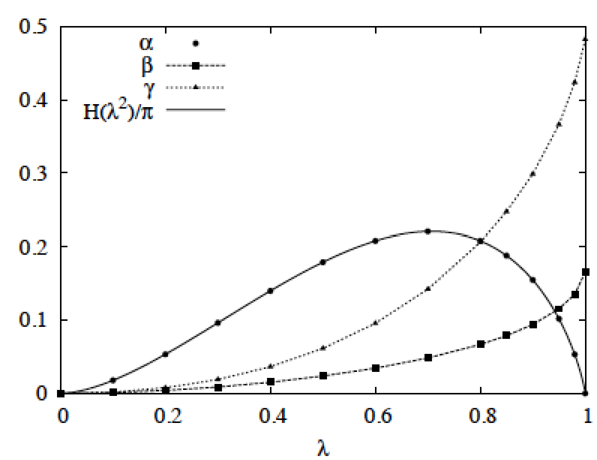
which can be rewritten as $\zeta'_l(t) = \lambda^2 \zeta_l(t) + 1 - \lambda^2$

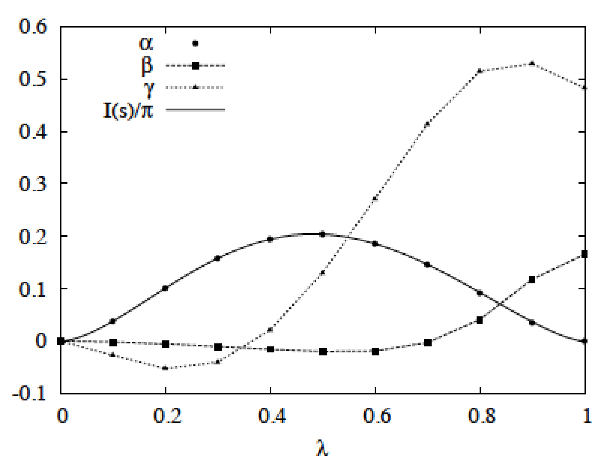




Quasi-classical picture

- Incoming particles partially transmitted / reflected
- Steady flow of particles and backscattering
- Entanglement is created steadily at the defect
- Entropy growth will be linear: $S(t) = \alpha t + \beta \ln t + \gamma$
- Ansatz for slope: $\alpha = \int_0^\pi \frac{\mathrm{d}q}{2\pi} v_q \, H(T_q)$
- Agrees perfectly with numerics:





Conclusions

- Statical defect problem solved exactly, entropy growth is logarithmic in block size
- Quench can be solved exactly for conformal defect, entropy grows logarithmically in time for equal fillings
- Biased case leads to a linear entropy growth!
- Entropy is generated locally but steadily
- Quasi-classical description á la

  Calabrese & Cardy / Rieger & Iglói