Quench dynamics of interacting bosons in 1-dimension

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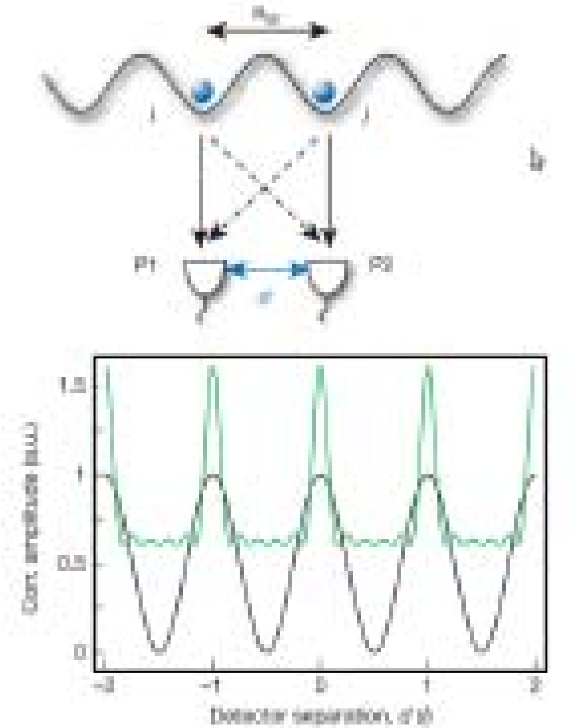


Hanbury-Brown Twiss effect

I Bloch et al.

with:

Deepak Iyer – Rutgers U

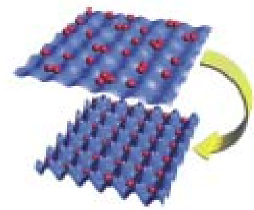
Correlations and Entanglement in Many-Body Systems Out-of-Equilibrium
NCTS, Hsinchu, Taiwan – Sept 2012

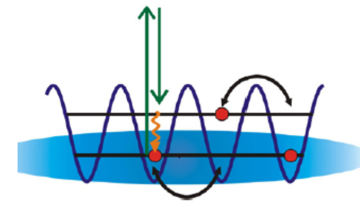
Quenching and Time Evolution

- **Prepare an isolated quantum many-body system in a state** $|\Phi_0\rangle$, typically eigenstate of H_0
- **At** $t=0$ **turn on interaction** H_1**, and evolve system with** $H = H_0 + H_1$ **:**

$$|\Phi_0, t\rangle = e^{-iHt}|\Phi_0\rangle$$

- **Many experiments: cold atom systems, nano-devices, molecular electronics, photonics**
- **New technologies, old questions**



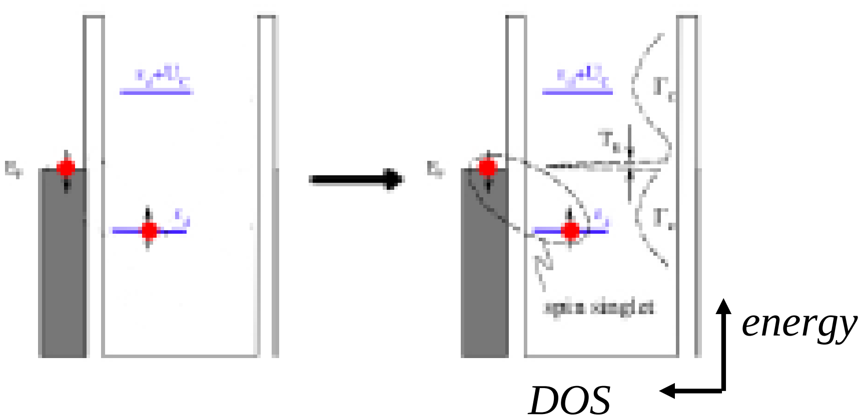


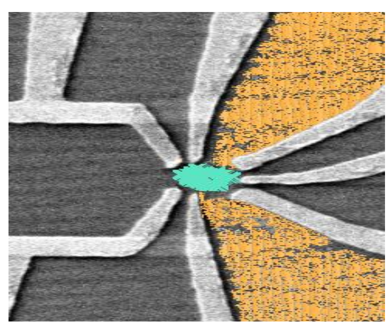
Questions: (as an introduction)

- **Time evolution of observables** $\langle O(t)\rangle = \langle \Phi_0, t|O|\Phi_0, t\rangle$
- **Evolution of correlation functions in quenched systems** $\langle\Phi_0|A(t+\tau)B(t)|\Phi_0\rangle = \langle\Phi_0,t)|A(\tau)B|\Phi_0,t\rangle$

  - **Dynamics of evolution of the Kondo resonance in a quantum dot: Anderson model**

    Quench at t = 0 : couple dot to leads





Measure time evolution of the Kondo peak.

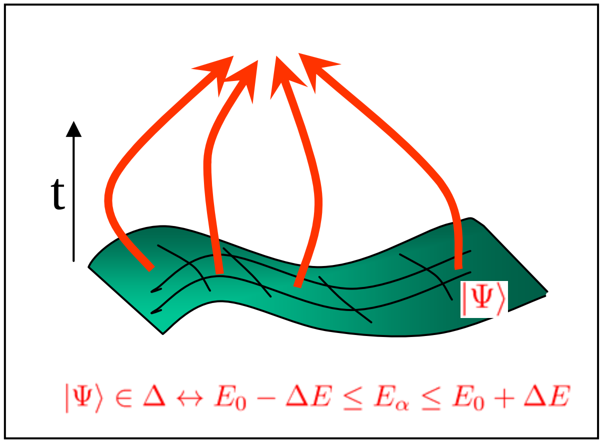
- Time resolved photo emission spectroscopy
- Time dependent current

Closed systems: quenching – long time limit, thermalization

- **Manifestation of interactions in time evolution dynamics**

The subject of this talk: bosons in 1-d

Time evolution and statistical mechanics:



- **Long time limit and thermalization:**

- is there a limit $\bar{O} = \lim_{t\to\infty} \langle O(t) \rangle$?
- is there a density operator ρ such that $\bar{O} = tr(\rho O)$?

Does it depend on $E_0 = \langle \psi_0 | H | \psi_0 \rangle$ not on $|\psi_0\rangle$

- **Scenarios of thermalization (**Rigol et al)

- Diagonal matrix elements of physical operators $A_{\alpha\alpha}$ do not fluctuate much around constant energy surface (**ETH-*eigenstate thermalization hypothesis***, Deutsch 92, Srednicki 94)

- Occupation numbers $|C_\alpha|^2$ do not fluctuate on the energy surface for reasonable IC

- Both fluctuate but are uncorrelated

- **Thermalization, Integrability, Non-Boltzmannian ensembles,** Rigol, Cardy, Cazalilla, Kollath

If conservation laws are present – how do they affect dynamics of thermalization?

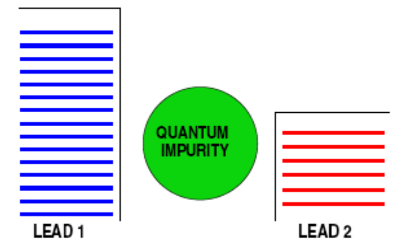
Open systems: quenching and non-thermalization, transport

Nonequilibrium currents

Goldhaber-Gordon *et al*, Conenwett *et al*, Schmid *et al*

• **Two baths or more**

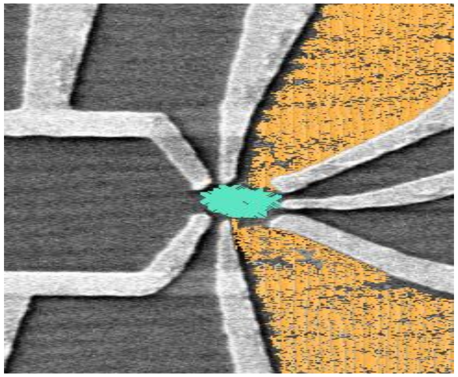
time evolution in a nonequilibrium set up



$H = H_{leads} + H_{imp} + H_{leads-imp}$

Interplay - strong correlations and nonequilibrium

Quench
or
Keldysh



- $t \leq 0$, leads decoupled, system described by: ρ_o
- $t = 0$, couple leads to impurity
- $t \geq 0$, evolve with $H(t) = H_0 + H_1$

• **What is the time evolution of the current** $\langle I(t) \rangle$ **?**

• **Asymptotic limit?**

• **Under what conditions is there a steady state? Dissipation?**

• **Steady state – is there a non thermal** ρ_s **?**

• **New effects out of equilibrium? New scales? Phase transitions, universality?**

Quenching in 1-d systems

Physical Motivation:

- **Natural dimensionality of many systems:**

  - wires, waveguides, optical traps, edges

- **Impurities: Dynamics dominated by s-waves, reduces to 1D system**
- **Many experimental realizations: Cold atom traps, nano-systems..**

Special features of 1- d : theoretical

- **Strong quantum fluctuations for any coupling strength**
- **Powerful mathematical methods:**

  - RG methods, Bosonization, CFT methods, Bethe Ansatz approach

- Bethe Ansatz approach: allows complete diagonalization of *H*

- Experimentally realizable: *Hubbard model, Kondo model, Anderson model, Lieb-Linniger model, Sine-Gordon model, Heisenberg model, Richardson model..*

- BA $\longrightarrow$ Quench dynamics of many body systems? Exact!

Others approaches: Keldysh, t-DMRG, t-NRG, t-RG

Much work in context of Luttinger Liquid: Cazalilla et al, Mitra et al

Time Evolution and the Bethe Ansatz

- A given state $|\Phi_0\rangle$ can be formally time evolved in terms of a complete set of energy eigenstates $|F^\lambda\rangle$

$$|\Phi_0\rangle = \sum_\lambda |F^\lambda\rangle\langle F^\lambda|\Phi_0\rangle \quad \longrightarrow \quad |\Phi_0, t\rangle = e^{-iHt}|\Phi_0\rangle = \sum_\lambda e^{-i\epsilon_\lambda t}|F^\lambda\rangle\langle F^\lambda|\Phi_0\rangle$$

If H integrable $\longrightarrow$ eigenstates $|F^\lambda\rangle$ are known via the Bethe-Ansatz

- Use the Bethe Ansatz to study quenching and evolution

- New technology is necessary:

- **Standard approach**: **impose PBC $\longrightarrow$ Bethe Ansatz eqns $\longrightarrow$ spectrum $\longrightarrow$ thermodynamics**
- **Non equilibrium entails more difficulties:**

i. Compute overlaps **ii. Sum over complete basis** **iii. Take limits**

Some progress was made - J. S. Caux et al

The Bethe Ansatz - Review

- **General N - particle state**

$$|F^\lambda\rangle_N = \int d^N x \, F^\lambda(\vec{x}) \prod_{j=1}^{N} \psi^\dagger(x_j)|0\rangle$$

- **Wave function very complicated in general**
- **The BA -wave function much simpler -**

Product of single particles wave functions $f_\lambda(x)$ and S-matrices S_{ij},

Example:

$$H = -\sum_{j=1}^{N} \partial^2_{x_j} + c \sum_{i<j} \delta(x_i - x_j)$$

$$f_\lambda(x) \sim e^{i\lambda x}$$

$$S_{ij}(\lambda_i - \lambda_j) = \frac{\lambda_i - \lambda_j + ic}{\lambda_i - \lambda_j - ic}$$

i. divide configuration space into N! regions $Q, \quad \{x_{Q1} \leq \cdots, \leq x_{QN}\}$

ii particles interact only when they cross: inside a region product of single particle wave funct.

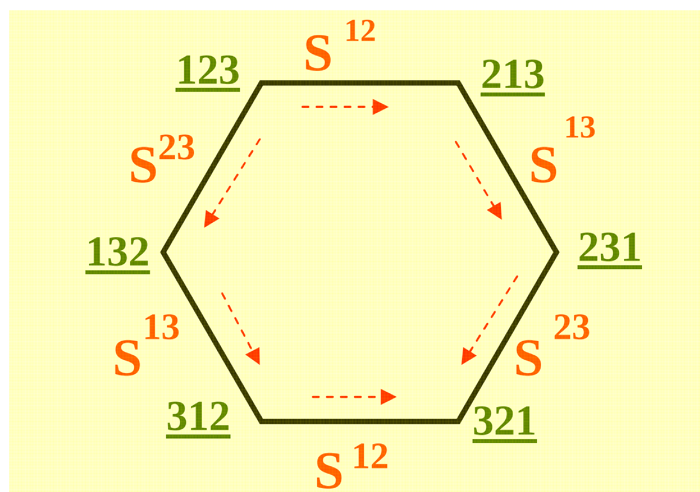
iii. assign amplitude A^Q to region Q

iv. amplitudes related by S-matrices S_{ij} (*e.g.* $A^{132} = S^{23} \, A^{123}$)

$$\rightarrow \boxed{F^\lambda(\vec{x}) = \sum_{Q \in S_N} A^Q \prod_j f_{\lambda_{Qj}}(x_j)}$$

v. do it consistently: ***Yang-Baxter relation***

$$S^{12} S^{13} S^{23} = S^{23} S^{13} S^{12}$$



The contour representation

Instead of $|\Phi_0\rangle = \sum_\lambda |F^\lambda\rangle\langle F^\lambda|\Phi_0\rangle$ introduce (directly in infinite volume):

Contour representation of $|\Phi_0\rangle$

$$|\Phi_0\rangle = \int_\gamma d^N\lambda \, |F^\lambda\rangle(F^\lambda|\Phi_0\rangle$$

V. Yudson, sov. phys. *JETP* (1985)

Computed S-matrix of Dicke model

with: $|F^\lambda\rangle$ Bethe eigenstate

$|F^\lambda)$ obtained from Bethe eigenstate by setting $S = I$ - easier to calculate

γ contour in momentum space $\{\lambda\}$ chosen according to **pole structure** of $S(\lambda_i - \lambda_j)$

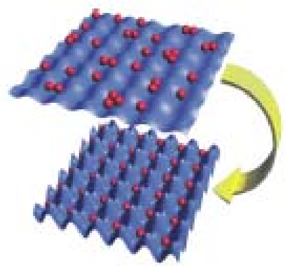
Note: in the infinite volume limit momenta $\{\lambda\}$ are not quantized
- no Bethe Ansatz equations, $\{\lambda\}$ free parameters

then:

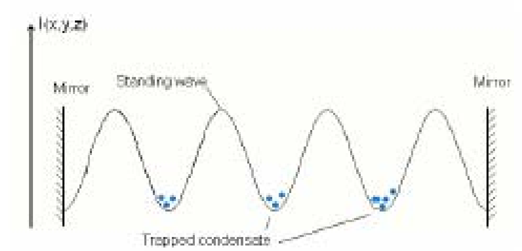
$$|\Phi_0, t\rangle = \int_\gamma d^N\lambda \, e^{-iE(\lambda)t}|F^\lambda\rangle(F^\lambda|\Phi_0\rangle$$

Boson Systems - experiments

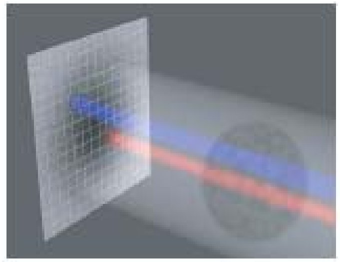
Bosons in optical traps



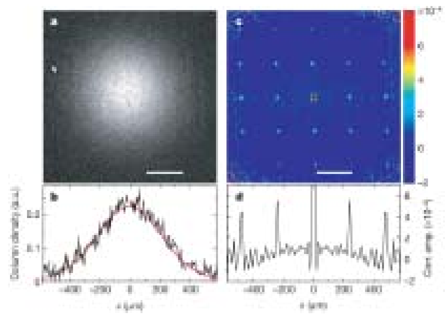
Superfluid Mott insulator transition



Mott insulator – initial condition



Imaging of density cloud using a CCD



Density and noise correlation functions

Bloch et al (Nature 2005, Rev Mod Phys 2008)

Interacting bosonic system

Bosons in a 1-d with short range interactions

$$H = -\int dx b^\dagger(x)\partial^2 b(x) + c\int dx\, b^\dagger(x)b(x)b^\dagger(x)b(x)$$

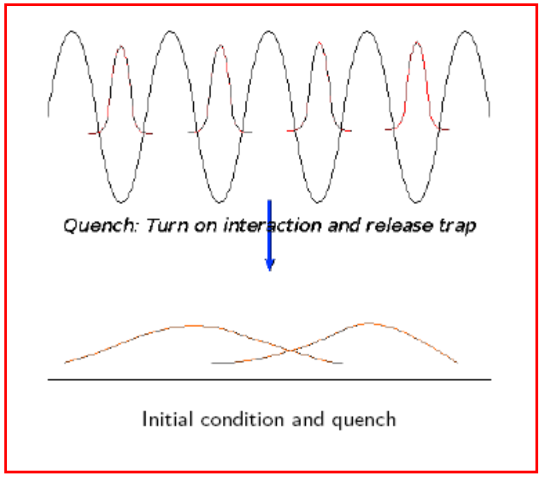
c **- coupling constant**

$c > 0$ *repulsive*

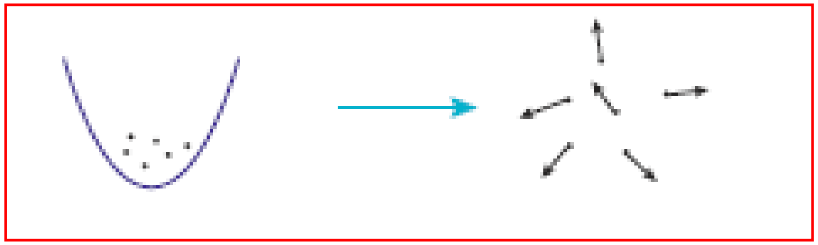
$c < 0$ *attractive*

Equivalently:

$$H = -\sum_{j=1}^{N}\partial_{x_j}^2 + c\sum_{i<j}\delta(x_i - x_j)$$



- **Initial condition I : bosons in a periodic optical lattice**



- **Initial condition II : bosons in a trap - condensate**

Bosonic system – BA solution

The N-boson eigenstatestate (Lieb-Linniger '67)

$$|\lambda_1, \cdots, \lambda_N\rangle = \int_y \prod_{i<j} Z^y_{ij}(\lambda_i - \lambda_j) \prod_j e^{i\lambda_j y_j} \; b^\dagger(y_j)|0\rangle$$

Eigenstates labeled by Momenta $\lambda_1, \cdots, \lambda_N$

- **i.** - ***satisfy BA eqns*** **if PBC imposed,**
  - ***unconstrained*** **in open space**
- **ii. real,** - **for** $c > 0$
  - **complex "strings"** - **for** $c < 0$

n-string

$$\lambda_j^{(n)} = \lambda_0 + (n/2 - j)ic$$

$$j = 0, \cdots, n-1$$

The 2-particle S-matrix

$$S_{ij}(\lambda_i - \lambda_j) = \frac{\lambda_i - \lambda_j + ic}{\lambda_i - \lambda_j - ic} \longrightarrow$$

Dynamic factor:

$$Z^y_{ij}(\lambda_i - \lambda_j) = \frac{\lambda_i - \lambda_j - ic\, sgn(y_i - y_j)}{\lambda_i - \lambda_j - ic}$$

The energy eigenvalues

$$H|\lambda_1, \cdots, \lambda_N\rangle = \sum_j \lambda_j^2 \, |\lambda_1, \cdots, \lambda_N\rangle$$

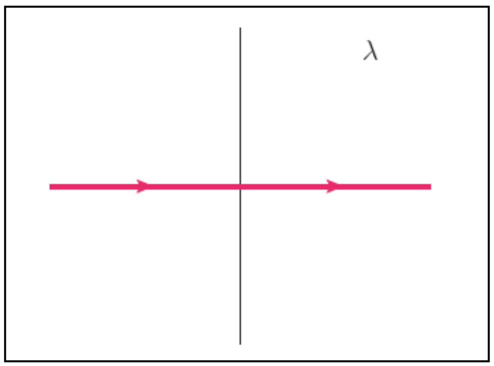
bosonic system: contour representation

"Central theorem"

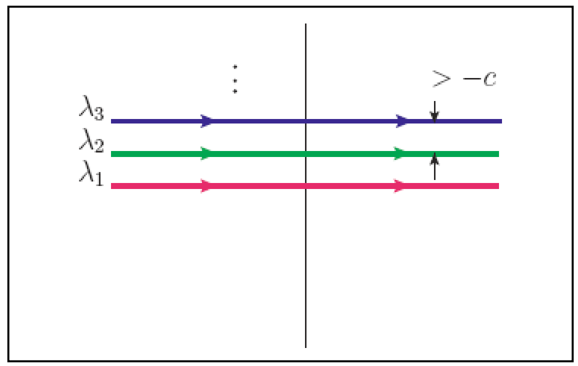
denote: $\theta(\vec{x}) = \theta(x_1 > x_2 > \cdots . x_N)$

$$|\Phi_0\rangle = \int_x \Phi_0(\vec{x}) b^\dagger(x_N)\cdots b^\dagger(x_1)|0\rangle =$$

$$= \int_{x,y}\int_\lambda \theta(\vec{x})\Phi_0(\vec{x}) \prod_{i<j} \frac{\lambda_i - \lambda_j - ic\, sgn(y_i - y_j)}{\lambda_i - \lambda_j - ic} \prod_j e^{i\lambda_j(y_j - x_j)} b^\dagger(y_j)|0\rangle$$



Repulsive c > 0



Attractive c < 0,

contour accounts for strings, bound states

It time evolves to:

$$|\Phi_0, t\rangle = \int_x \int_y \int_\lambda \theta(\vec{x})\Phi_0(\vec{x}) \prod_{i<j} \frac{\lambda_i - \lambda_j - ic\, sgn(y_i - y_j)}{\lambda_i - \lambda_j - ic} \prod_j e^{-i\lambda_j^2 t} e^{i\lambda_j(y_j - x_j)} b^\dagger(y_j)|0\rangle$$

- **Expression contains full information about the dynamics of the system**

Keldysh

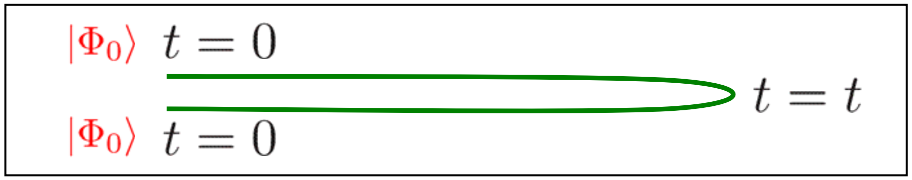
- **Time evolution of expectation values:**

$$O_{\Phi_0}(t) = \langle \Phi_0 | e^{iHt} \, \hat{O} \, e^{-iHt} | \Phi_0 \rangle = \langle \Phi_0, t | \, \hat{O} \, | \Phi_0, t \rangle$$

Non-perturbative Keldysh:

$$= \int \mathcal{D}\, b^* \mathcal{D}\, b \, \hat{O} \; e^{-i \int_C [S_0(b,b^*) + S_I(b,b^*)] \, dt}$$

carried out on the Keldysh contour C *, with separate fields for the top and bottom lines:*



What to calculate?

- **We shall study:**

1. Evolution of the density

$C_1(x,t) = \langle \rho(x,t) \rangle$ Time Of Flight experiment

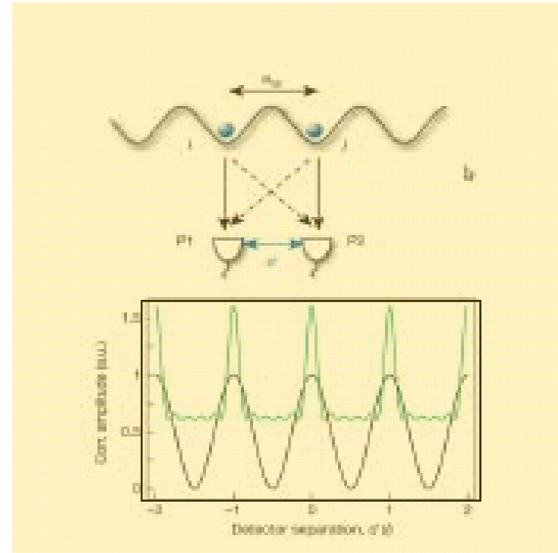
competition between quantum broadening and attraction

2. Evolution of noise correlation

$$C_2(x_1, x_2; t) = \frac{\langle \rho(x_1,t)\rho(x_2,t)\rangle}{\langle \rho(x_1,t)\rangle\langle \rho(x_2,t)\rangle} - 1$$

time dependent Hanbury-Brown Twiss effect

- repulsive bosons evolve into fermions
- attractive bosons evolve to a condensate



Hanbury-Brown Twiss effect

Measure: $C_2(x_1, x_2, t)$

- two sources: originally stars

Free bosons $C_2(x,-x) \sim \cos x$

Free Fermions $C_2(x,-x) \sim -\cos x$

- two free particles:

Similar, but time dependent

- many free particles:

More structure: main peaks, sub peaks

Effects of interactions?

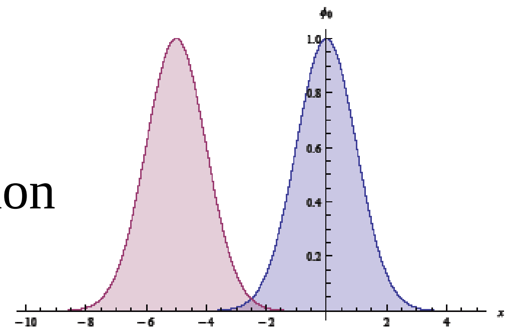
Evolution of a bosonic system: density

Density evolution:
(Time of flight experiment)

$$\langle \rho(x_0,t)\rangle = \langle \Phi_0(t)|b^\dagger(x_0)b(x_0)|\Phi_0(t)\rangle$$

Two bosons

Initial condition



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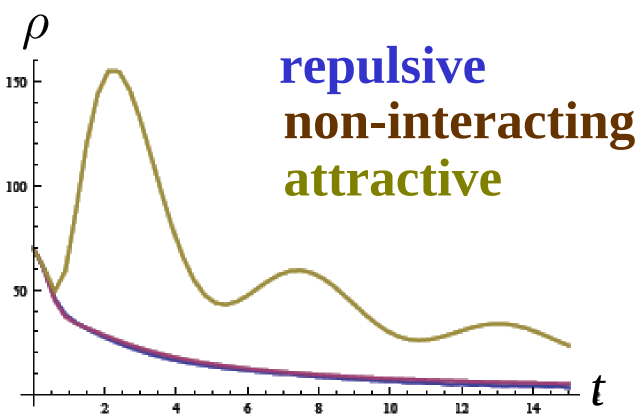


$$|\Phi_0\rangle = \frac{1}{(\pi\sigma^2)^{\frac{1}{2}}}\int_x e^{-\frac{(x_1)^2}{2\sigma^2}} e^{-\frac{(x_2+a)^2}{2\sigma^2}} b^\dagger(x_1)b^\dagger(x_2)|0\rangle$$

Attractive: Time period of oscillations $T = \frac{4\pi}{c^2}$
- independent of IC

Rep: $$|\Phi_0(t)\rangle_2 = \int_y \frac{e^{i\frac{(y_1-x_1)^2}{4t}+i\frac{(y_2-x_2)^2}{4t}}}{4\pi i t}\left(1 - c\sqrt{\pi i t}\,\theta(y_2-y_1)e^{\frac{i}{8t}\alpha^2(t)}\mathrm{erf}\left(\frac{i-1}{4}\frac{i\alpha(t)}{\sqrt{t}}\right)\right)$$

Att: $$|\Phi_0(t)\rangle_2 = \int_y \frac{e^{i\frac{(y_1-x_1)^2}{4t}+i\frac{(y_2-x_2)^2}{4t}}}{4\pi i t}\left(1 - c\sqrt{\pi i t}\,\theta(y_2-y_1)e^{\frac{i}{8t}\alpha^2(t)}\mathrm{erfc}\left(\frac{i-1}{4}\frac{i\alpha(t)}{\sqrt{t}}\right)\right)$$

with $\alpha(t) = 2ct - i(y_1-x_1) - i(y_2-x_2)$

Repulsive: almost coincides with free boson **Broadening**

Attractive: competition between **attraction** and **Broadening**

Emergence of an asymptotic Hamiltonian

Long time asymptotics - repulsive:

(cf. Buljan et al. '08

- **Bosons turn into fermions as time evolves (for any $c > 0$)**

$$|\Phi_0, t\rangle = \int_x \int_y \int_\lambda \theta(\vec{x})\Phi_0(\vec{x}) \prod_{i<j} \frac{\lambda_i - \lambda_j - ic\; sgn(y_i - y_j)}{\lambda_i - \lambda_j - ic} \prod_j e^{-i\Sigma_j \lambda_j^2 t - \lambda_j (y_j - x_j)} \prod_j b^\dagger(y_j)|0\rangle$$

$$= \int_x \int_y \int_\lambda \theta(\vec{x})\Phi_0(\vec{x}) \prod_{i<j} \frac{\lambda_i - \lambda_j - ic\sqrt{t}\; sgn(y_i - y_j)}{\lambda_i - \lambda_j - ic\sqrt{t}} e^{-i\Sigma_j \lambda_j^2 - \lambda_j (y_j - x_j)/\sqrt{t}} \prod_j b^\dagger(y_j)|0\rangle$$

$$\rightarrow \int_x \int_y \int_\lambda \theta(\vec{x})\Phi_0(\vec{x}) e^{-i\Sigma_j \lambda_j^2 - \lambda_j (y_j - x_j)/\sqrt{t}} \prod_{i<j} sgn(y_i - y_j) \prod_j b^\dagger(y_j)|0\rangle$$

$$= e^{-iH_0^f t} \int_{x,k} \mathcal{A}_x\, \theta(\vec{x})\Phi_0(\vec{x}) \prod_j c^\dagger(x_j)|0\rangle.$$

$\mathcal{A}_x$ *antisymmetrizer*

where

$$H_0^f = -\int_x c^\dagger(x)\partial^2 c(c)$$

- In the long time limit repulsive bosons for any $c > 0$ ***propagate under the influence of a*** **Tonks – Girardeau** ***Hamiltonian (hard core bosons=free fermions)***

- The state equilibrates, does not thermalize

- valid independently of Φ_0

- **Scaling argument fails for attractive bosons** (instead, they condense to a bound state)

Evolution of a bosonic system: saddle point app

Corrections to long time asymptotics -

Stationary phase approx at large times (carry out λ - integration)

- **Repulsive –** only stationary phase contributions (on real line); (cf. Lamacraft 2011)

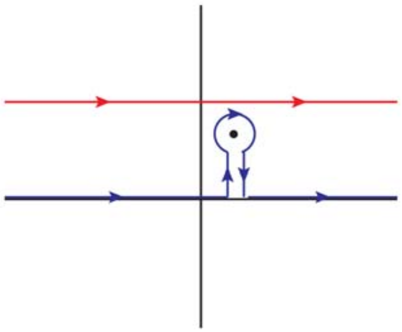
$$\phi\left(\xi\equiv\frac{y}{2t},x,t\right)=S_\xi\frac{1}{(4\pi i t)^{\frac{N}{2}}}\prod_{i<j}\frac{\xi_i-\xi_j-ic\;sgn(\xi_i-\xi_j)}{\xi_i-\xi_j-ic}e^{\sum_j i\xi_j^2 t-i\xi_j x_j}$$

- **Attractive –** contributions from stationary phases and poles.

For two particles:

Pole contributions from deformation of contours – formation of bound states

$$\phi(\xi,x,t)=S_\xi\Big[\frac{1}{4\pi i t}\frac{\xi_1-\xi_2-ic\;sgn(\xi_1-\xi_2)}{\xi_1-\xi_2-ic}e^{\sum_j i\xi_j^2 t-i\xi_j x_j}+$$
$$+\frac{2c\theta(\xi_2-\xi_1)}{\sqrt{4\pi i t}}e^{i\xi_1^2 t-i\xi_1 x_1-i(\xi_1-ic)^2 t+i(\xi_1-ic)(2t\xi_2-x_2)}\Big]$$



- repulsive correlations depend on $\xi=\frac{y}{2t}$ only (light cone propagation)
- attractive correlations maintain t dependence (bound states provide additional scales)

Evolution of a bosonic system

Long time asymptotics:

- **General expression – repulsive**

$$|\Phi_0, t\rangle = \int_x \int_y \theta(\vec{x})\Phi_0(\vec{x}) \prod_{i<j} \frac{\xi_i - \xi_j - ic\, sgn(\xi_i - \xi_j)}{\xi_i - \xi_j - ic} \prod_j \frac{1}{\sqrt{4\pi i t}} e^{i\xi_j^2 t - i\xi_j x_j} b^\dagger(y_j)|0\rangle$$

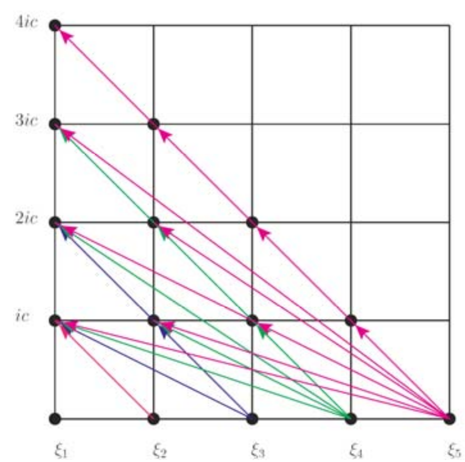
function of $\xi = y/2t$ **only, light-like propagation**

Exp: Bloch et al
Nature 2012

- **General expression – attractive (poles and bound states)**

$$|\Phi_0, t\rangle = \int_x \int_y \theta(\vec{x})\Phi_0(\vec{x}) \sum_{\xi_j^* = \xi_j, \xi_i^* + ic, i<j} \prod_{i<j} \frac{\xi_i^* - \xi_j^* + ic\, sgn(\xi_i - \xi_j)}{\xi_i^* - \xi_j^* + ic} \prod_j \frac{1}{\sqrt{4\pi i t}} e^{-i(\xi_j^*)^2 t + i\xi_j^*(2t\xi_j - x_j)} b^\dagger(y_j)|0\rangle$$

Pole contributions follow recursive pattern:



Pattern corresponds to successive formation and contributions of bound states

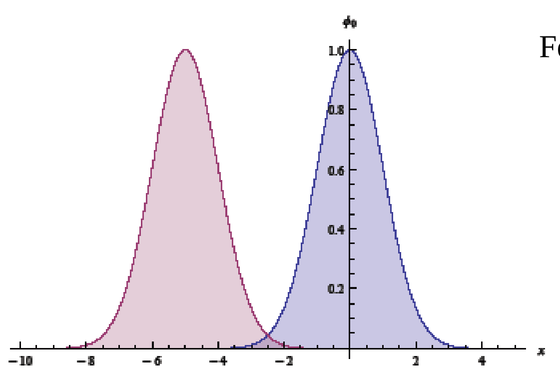
Evolution of repulsive bosons into fermions: HBT

Long time asymptotics - repulsive:

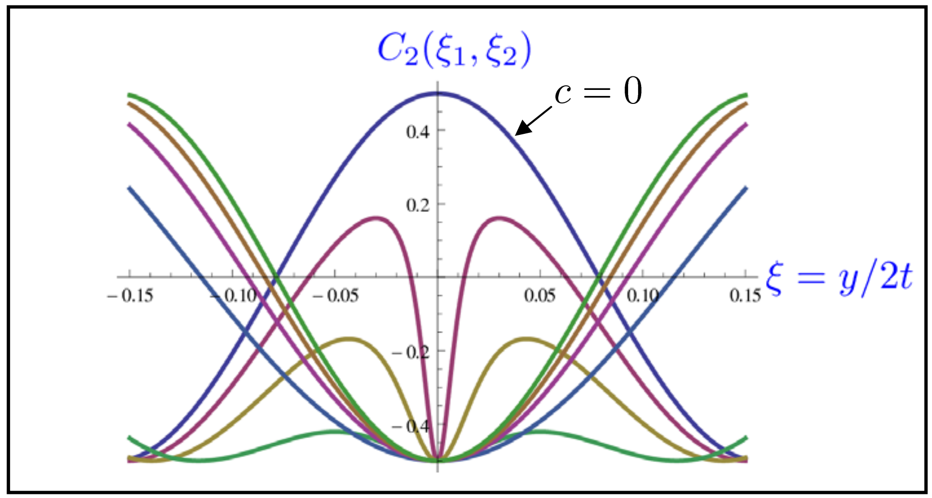
- **Bosons turn into fermions as time evolves (for any $c > 0$)**
- **Can be observed in the noise correlations: (dependence on t only via $\xi = x/2t$)**

$$C_2(x_1, x_2, t) \to C_2(\xi_1, \xi_2) = \frac{\langle \rho(\xi_1)\rho(\xi_2)\rangle}{\langle \rho(\xi_1)\rangle\langle \rho(\xi_2)\rangle} - 1,$$

$c/a = 0, .3, \cdots, 4$



Fermionic correlations evolve



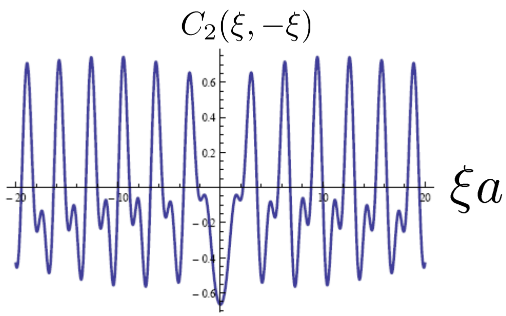
- **Fermionic dip develops for any repulsive interaction on time scale set by $1/c^2$**

Evolution of a bosonic system: noise correlations

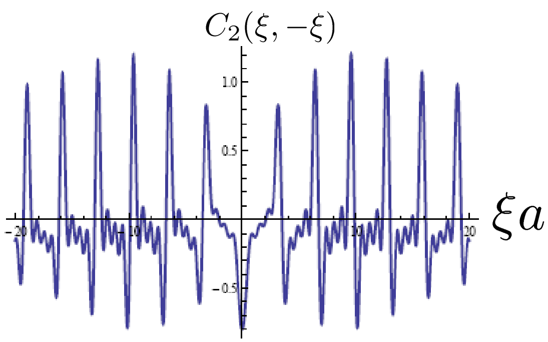
Noise correlations – many particles

Repulsive bosons

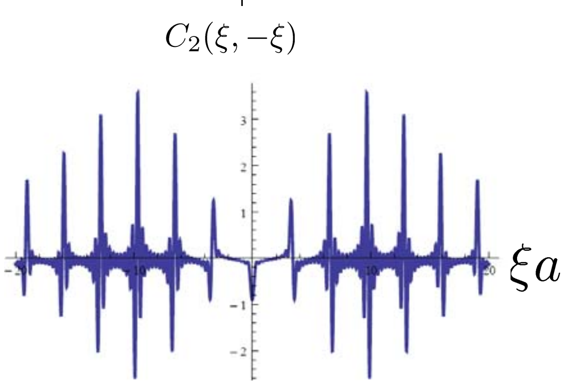
3 particles



5 particles



10 particles

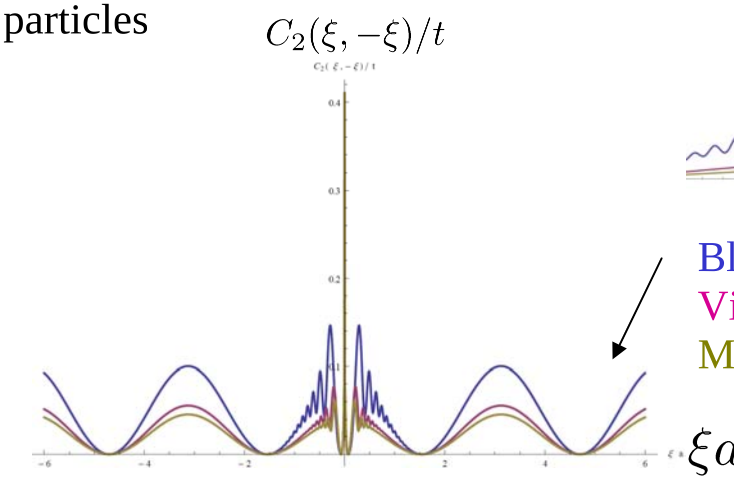


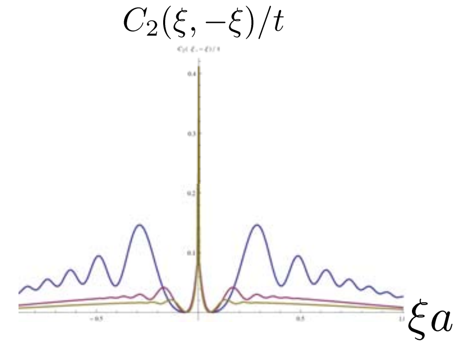
Fermionic dip as $\xi \to 0$

Structure emerges at $\xi a = \sigma$

Attractive bosons

2 particles

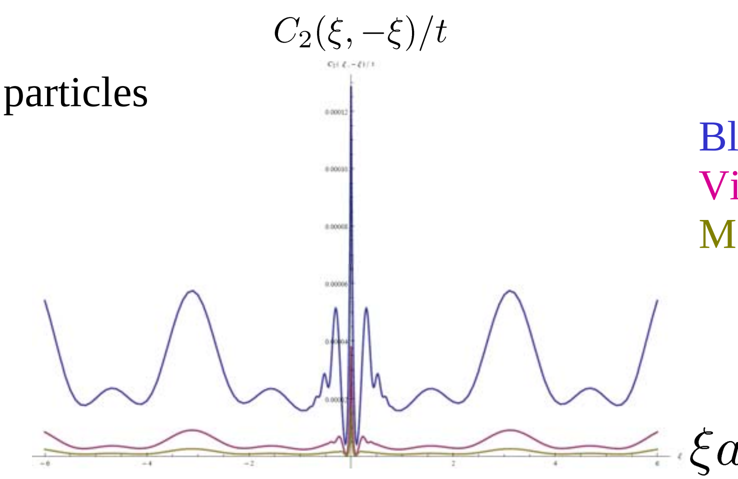




Blue -short times
Violet -longer
Magenta -longest

central peaks increase with time
- weight in the bound states increases

3 particles



Blue -short times
Violet -longer
Magenta -longest

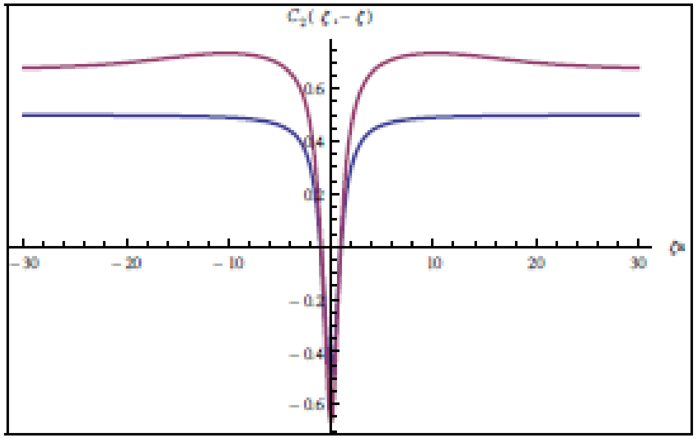
peaks diffuse – momenta redistribute

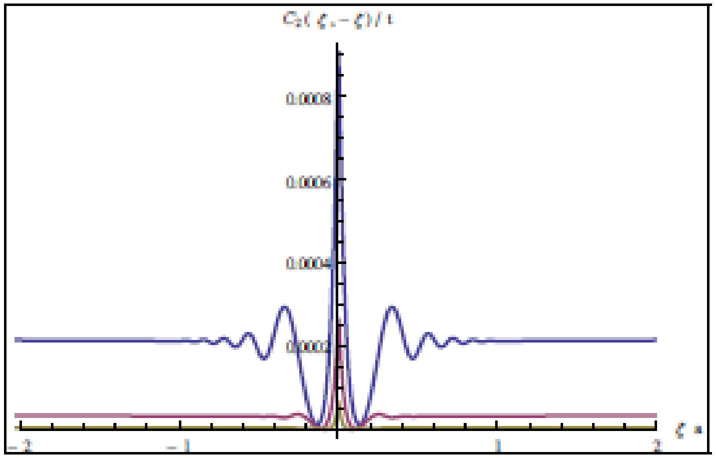
Evolution of a bosonic system: noise correlation

Noise correlations – starting from a condensate

Repulsive bosons

Attractive bosons





Two (blue) and three bosons,

Three bosons, at times: $tc^2 = 20, 40, 60$

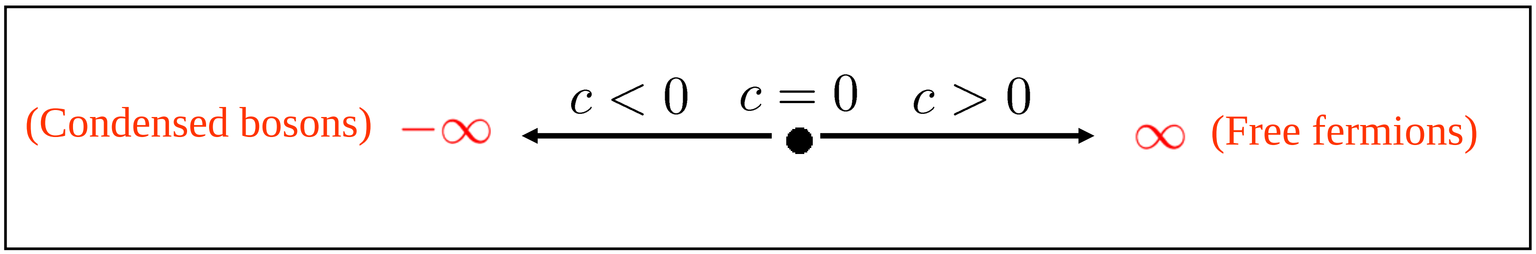
Time evolution "Renormalization Group"

"Dynamic" RG interpretation

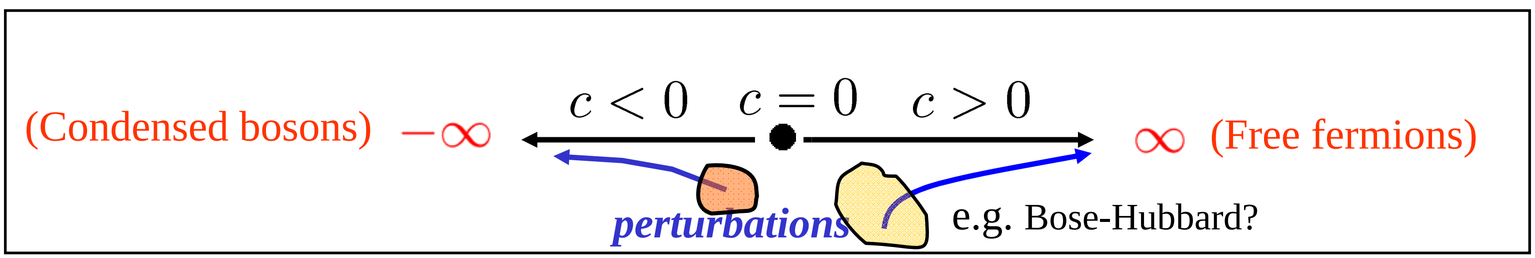
- **Universality out of equilibrium**

- **Can view time evolution as RG flow** $t \sim \ln(D_0/D)$

- As time evolves the weight of eigenstate contributions varies, time successively "integrates out" high energy states



- **Are there "basins of attraction" for perturbations flowing to dynamic fixed points**



- **Other fixed points?**

Evolution of a bosonic system

Conclusions:

- Does not need the spectrum of Hamiltonian or normalized eigenstates
- Takes into account existence of bound states without dealing with large sums over strings
- Asymptotics calculable for both repulsive and attractive interactions in the Lieb-Liniger model

To do list:

- Generalize to other integrable models: Heisenberg model (in progress, with Deepak Iyer), Anderson model (Deepak Iyer, Paata Kakashvili), Lieb-Linniger + impurity (Huijie Guan)
- Time evolution at finite volume, finite density (in progress, with Deepak Iyer)
- Time evolution at finite temperatures (under discussion)
- Study approach to nonequilibrium steady state (in progress, with P. Kakashvili)
- Numerical tests of *dynamic RG hypothesis* (in progress, with P. Schmitteckert, t-DMRG)
- Generalize to correlation functions (open)

Big Questions:

- What drives thermalization of pure states? Canonical typicality, entanglement entropy (Lebowitz, Tasaki, Short…)
- General principles, variational? F-D theorem out-of-equilibrium? Heating? Entanglement?
- What is universal? RG Classification?