Electromagnetic Induced Transparency and Slow Light in Strongly Correlated Atomic Gases

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Outline

- What is electromagnetic induced transparency (EIT) and basic theory?

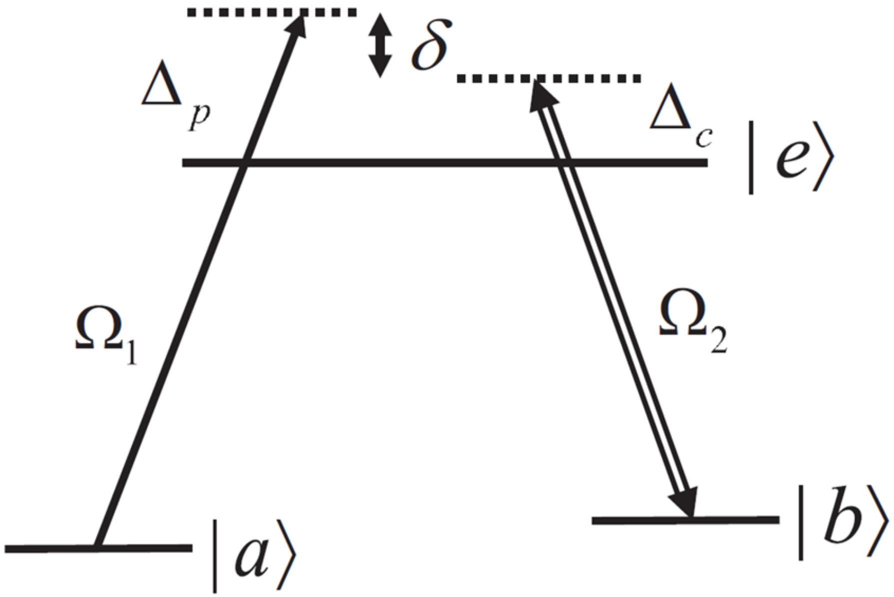
- EIT in non-interacting Bosons/Fermions and weakly interacting BEC

- A strong interacting 1-D Bosons - Luttinger liquids

Synthetic gauge fields v.s. EIT

EIT= Electromagnetic Induced Transparancy

Similarity: Λ-type atomic levels with two or more coupling laser fields.



Differences:

(1) Two dressing laser fields couple with large detunings

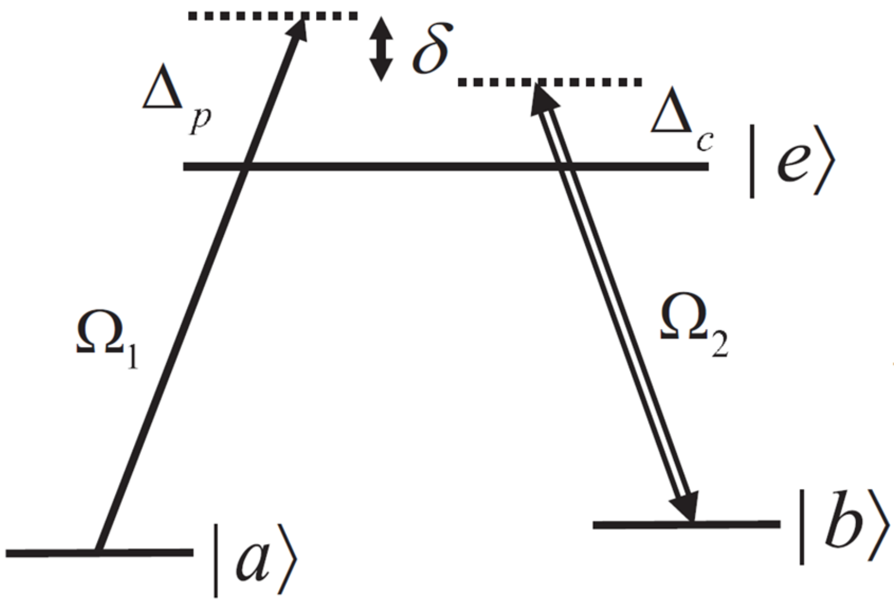
←→ **weak probe and strong control fields near resonance**.

(2) Inhomogeneous laser field Ω or detuning Δ

←→ **not necessarily inhomogeneous.**

(3) Recoil momentum matters ←→ **usually not relevant**

EIT – Maxwell Bloch equation



$$\frac{\partial E}{\partial z}+\frac{1}{c}\frac{\partial E}{\partial z}=\frac{ik}{2\varepsilon_0}P,$$

$$P(z,t) = \langle \rho d_{ae}\tilde{\sigma}_{ae}(z,t)\rangle$$

polarization

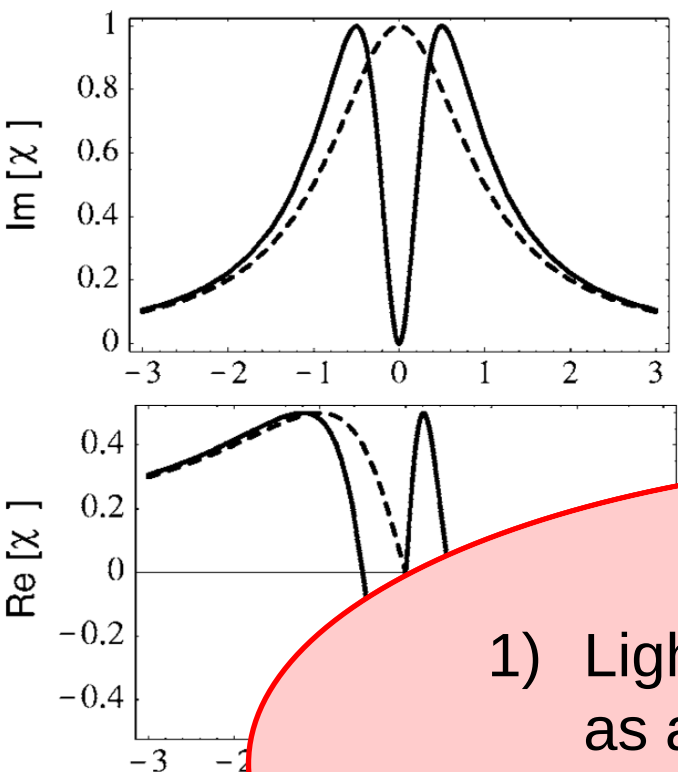
$$\tilde{\sigma}_{ae}(z,t) \equiv \frac{1}{N_z}e^{i\omega_1 t - ik_1 z}\sum_{\mu=1}^{N_z}|a\rangle_\mu\langle e|$$

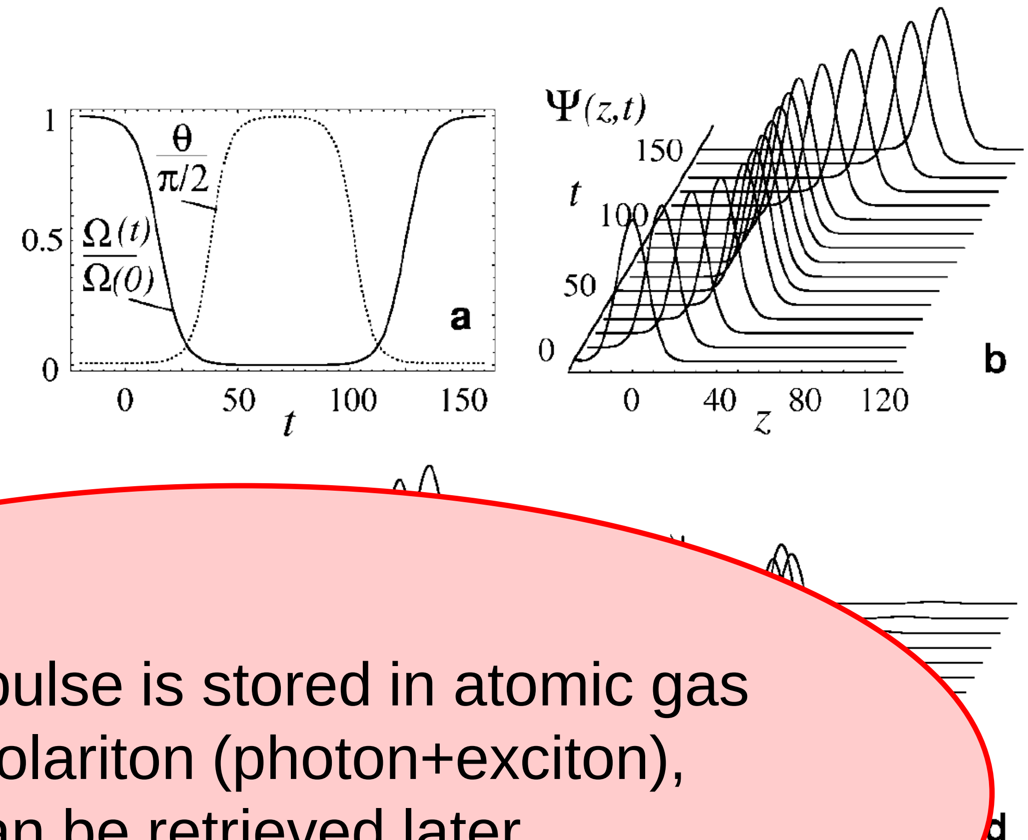
The polarization is typically calculated from the coherent state in the single-particle picture, independent of the dynamics and statistics.

Group velocity:

$$v_g = \frac{c}{1 + N|g|^2 \frac{d}{d\omega_1}\left[\mathrm{Im}\left(\frac{i\langle\tilde{\sigma}_{ae}\rangle}{\Omega_1}\right)\right]}$$

EIT – dispersion and dark state polariton

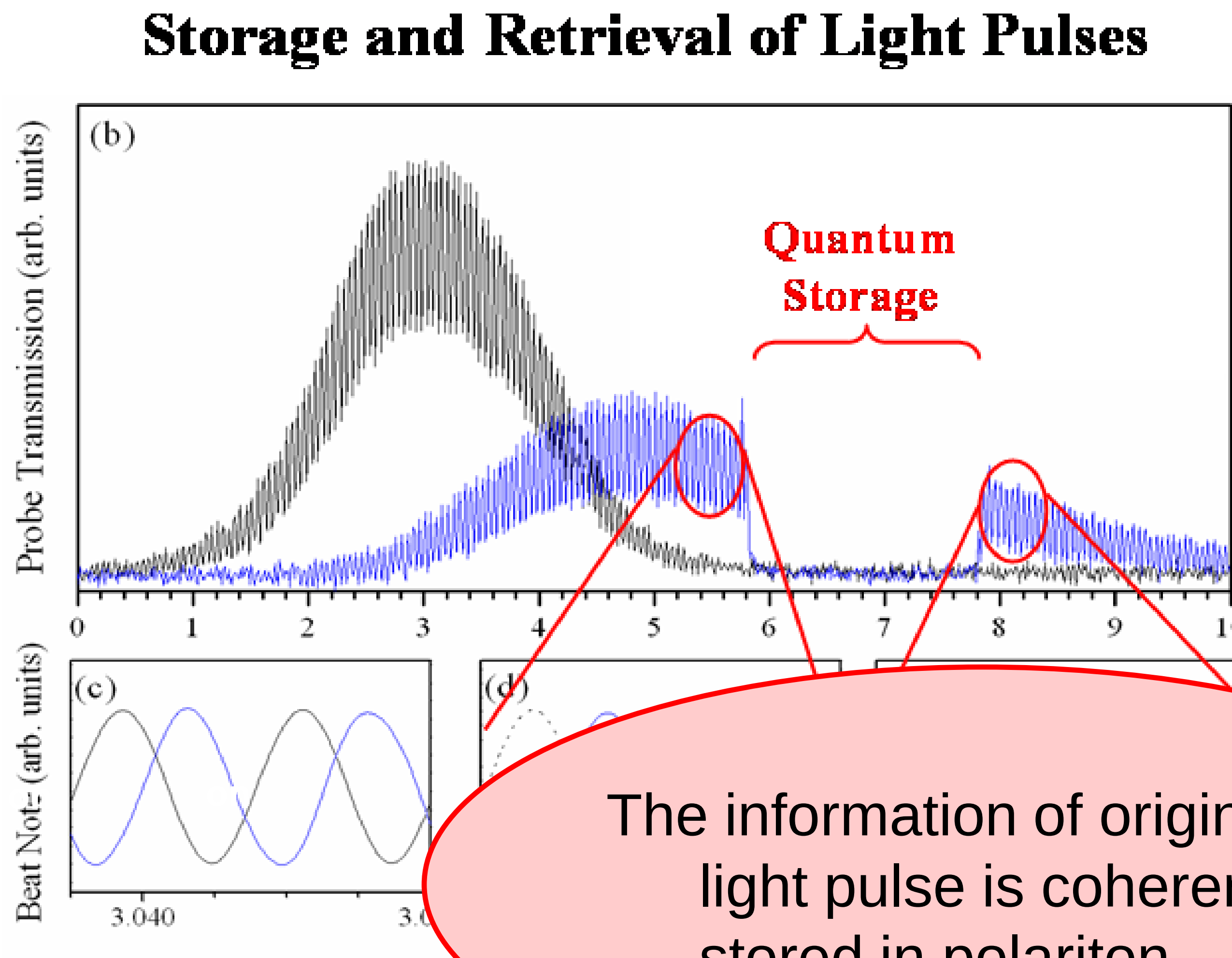




1) Light pulse is stored in atomic gas as a polariton (photon+exciton), and can be retrieved later.
2) Group velocity is strongly reduced.

M. Fleischhauer et al, RMP **77**, … **84**, 5094 (2000)

Storage and Retrieval of Light Pulses



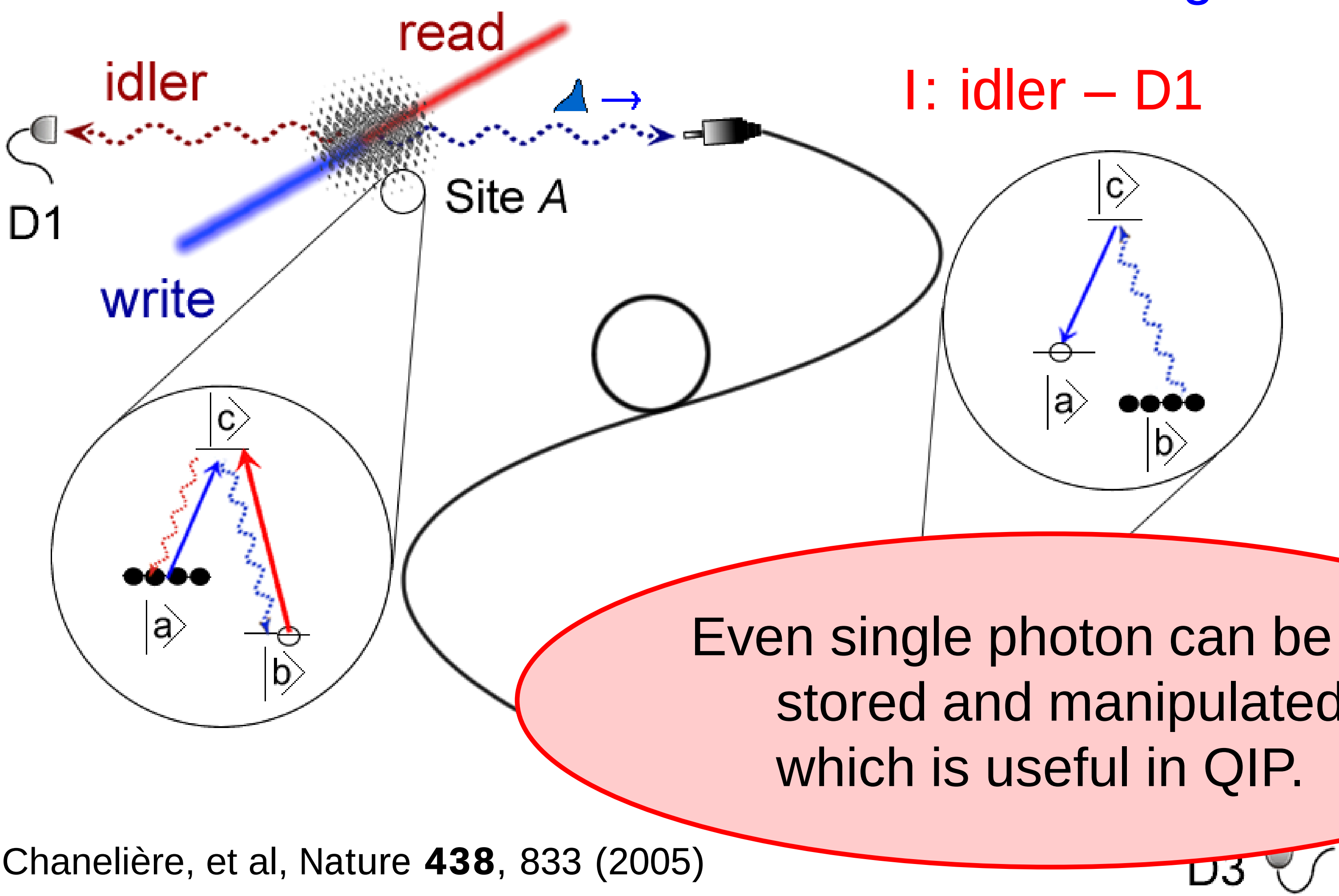
The information of original light pulse is coherently stored in polariton.

http://atomcool.phys.nthu.edu.tw/introduction.html

Quantum memory for single photons

S: stored signal – D2 or D3

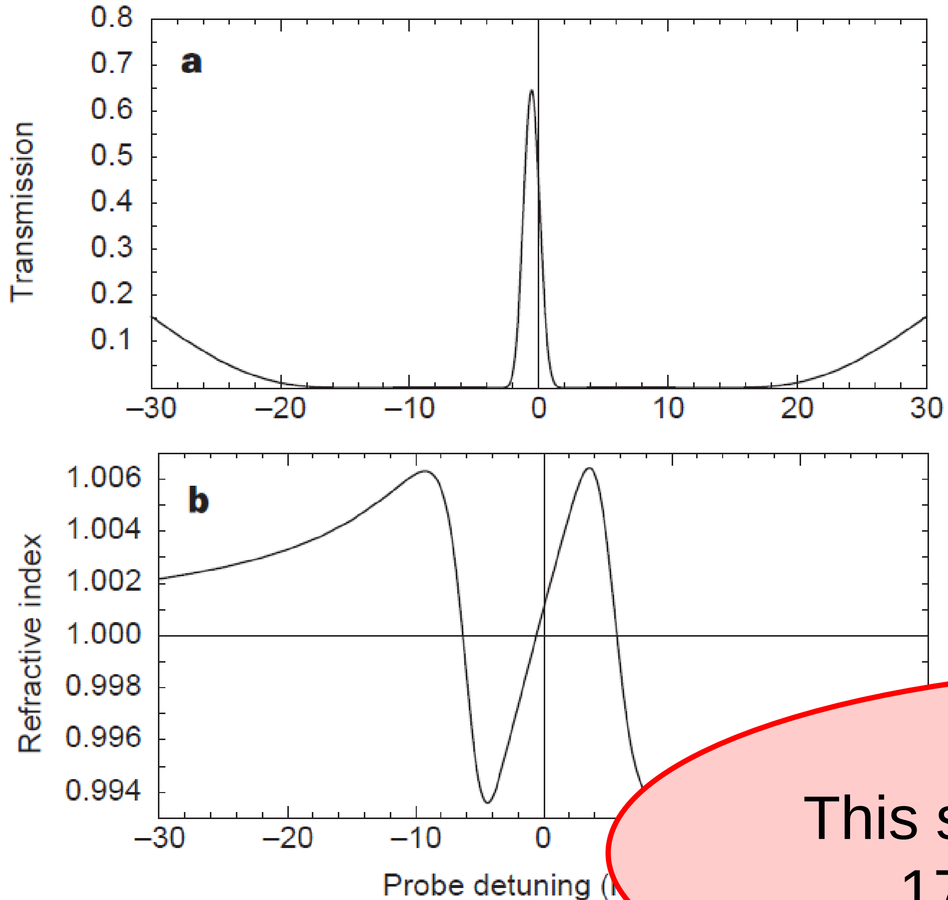
I: idler – D1

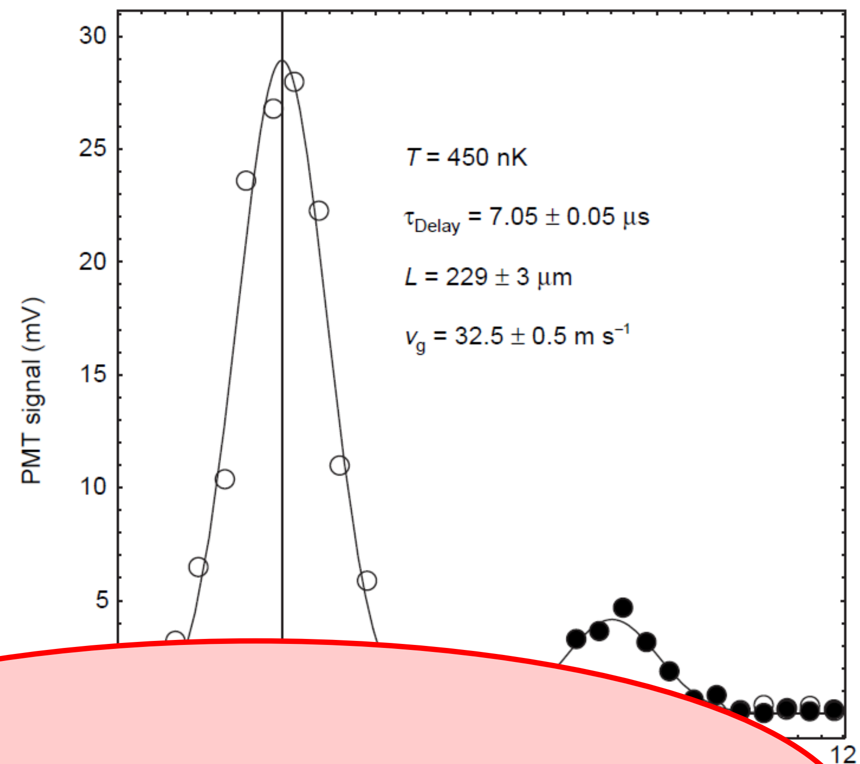


Even single photon can be stored and manipulated which is useful in QIP.

T. Chanelière, et al, Nature **438**, 833 (2005)

EIT in BEC





This sets up the record of 17m/s for slow light.

L. V. Hau, et al, Nature 397, 594 (1999)

What happens when EIT is realized in quantum degenerate atomic gases when atomic dynamics and statistics or even interaction matters?

Hamiltonian

$$H_A = \sum_p \frac{p^2}{2m}\left(\hat{a}_p^\dagger \hat{a}_p + \hat{b}_p^\dagger \hat{b}_p + \hat{e}_p^\dagger \hat{e}_p\right) + \hbar\Delta_1 \sum_p \hat{a}_p^\dagger \hat{a}_p + \hbar\Delta_2 \sum_p \hat{b}_p^\dagger \hat{b}_p,$$

$$H_{AL} = -\hbar\Omega_1 \sum_p \hat{e}_{p+p_1}^\dagger \hat{a}_p - \hbar\Omega_2 \sum_p \hat{e}_{p+p_2}^\dagger \hat{b}_p + h.c.,$$

$$U = \frac{1}{2V} \sum_{m,j=a,b,e} \sum_{p,p',q} U_{mj} \hat{m}_{p+q}^\dagger \hat{j}_{p'-q}^\dagger \hat{j}_{p'} \hat{m}_p, \qquad \text{(A2)}$$

Atom-light interaction

Atom-atom interaction

Dark state

H_0 → diagonalize → $\langle \sigma_{ae} \rangle$ → P → dispersion & v_g

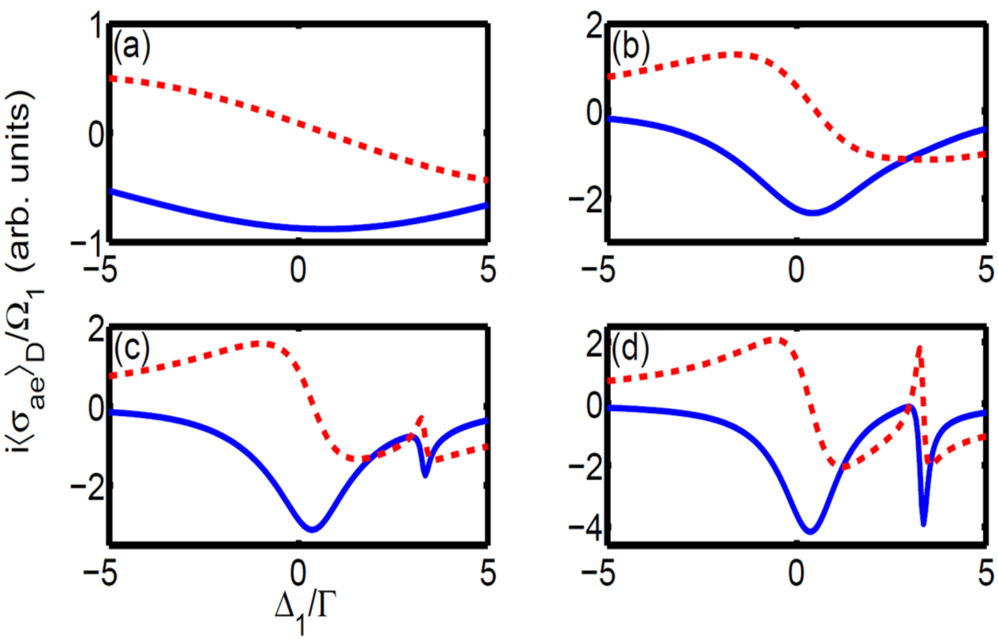
$$H_0 = \begin{bmatrix} \Delta_1 + \frac{p^2}{2m} & 0 & 0 \\ 0 & \Delta_2 + \frac{(p+p_1-p_2)^2}{2m} & -\Omega_2 \\ 0 & -\Omega_2 & \frac{(p+p_1)^2}{2m} \end{bmatrix}$$

Polarization:

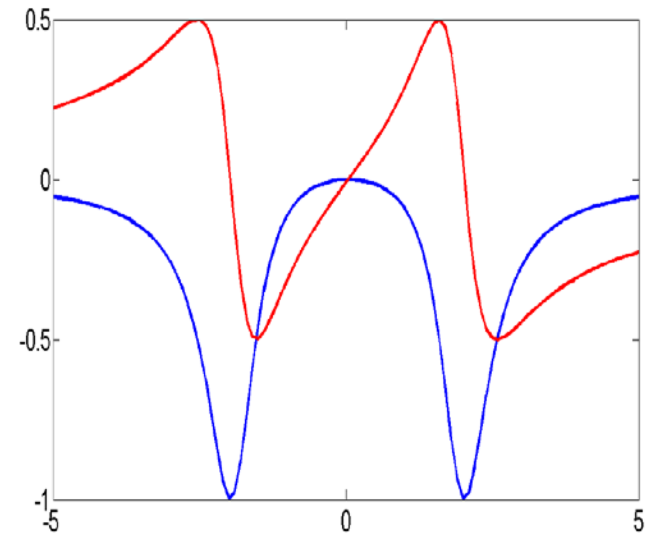
$$\frac{\langle \tilde{\sigma}_{ae}(T) \rangle_D}{\Omega_1} = \frac{\rho_c(T)}{\rho} F_{p=0} + \frac{1}{N} \sum_{p \neq 0} F_p n_{\beta_p}. \; F_p \equiv - \left(\frac{\sin^2 \phi}{E_0^{(0)} - E_+^{(0)}} + \frac{\cos^2 \phi}{E_0^{(0)} - E_-^{(0)}} \right)$$

Non-interacting quantum degenerate gases at finite temperature (1)

Counterpropagating bosons:



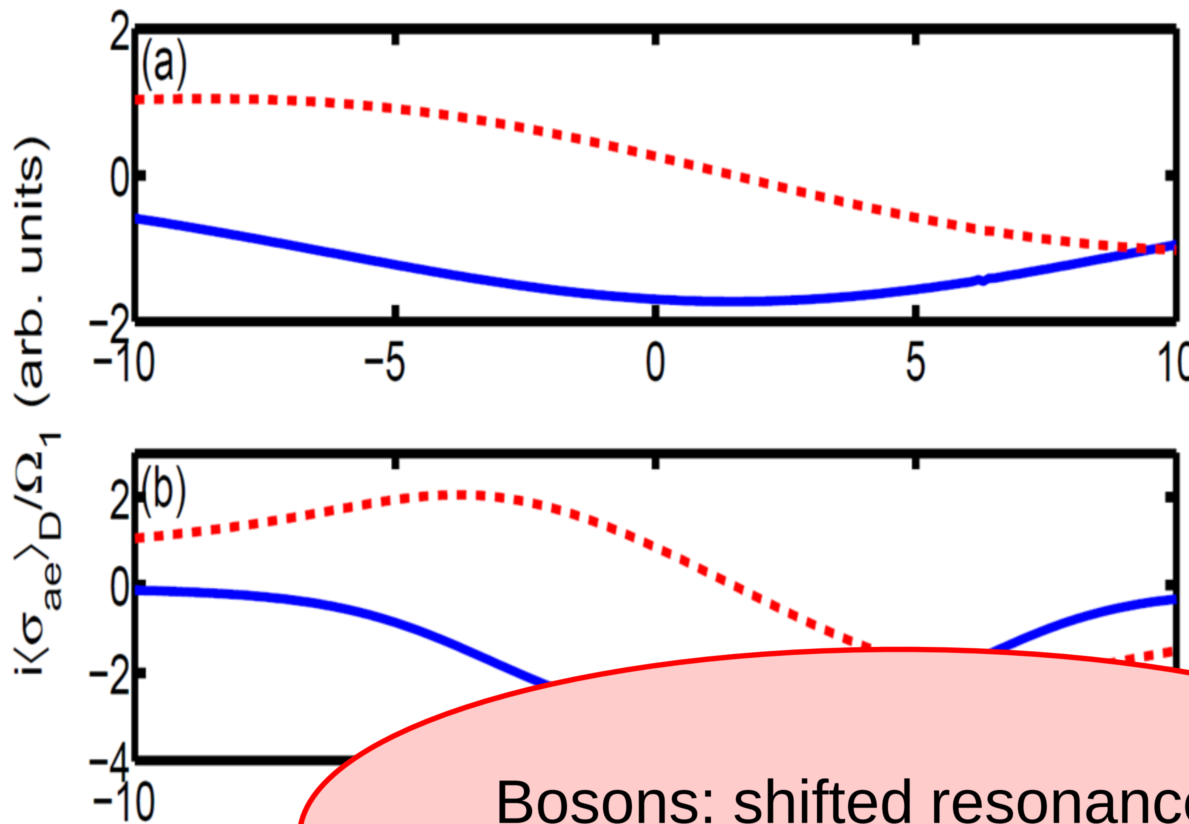
Typical atoms (frozen):



Tc ≈ 0.4 μK. . Γ^{-1} = 28.3 μs. (a) T=5 μK. (b) T=0.5 μK. (c) T=0.35 μK. (d) T=0.1 μK.

Non-interacting quantum degenerate gases at finite temperature (2)

Counterpropagating fermions:



Bosons: shifted resonance
Fermions: no EIT

$T_f = 1.98\ \mu K$ and Γ^{-1}

Weakly interacting Bose-Einstein condensate

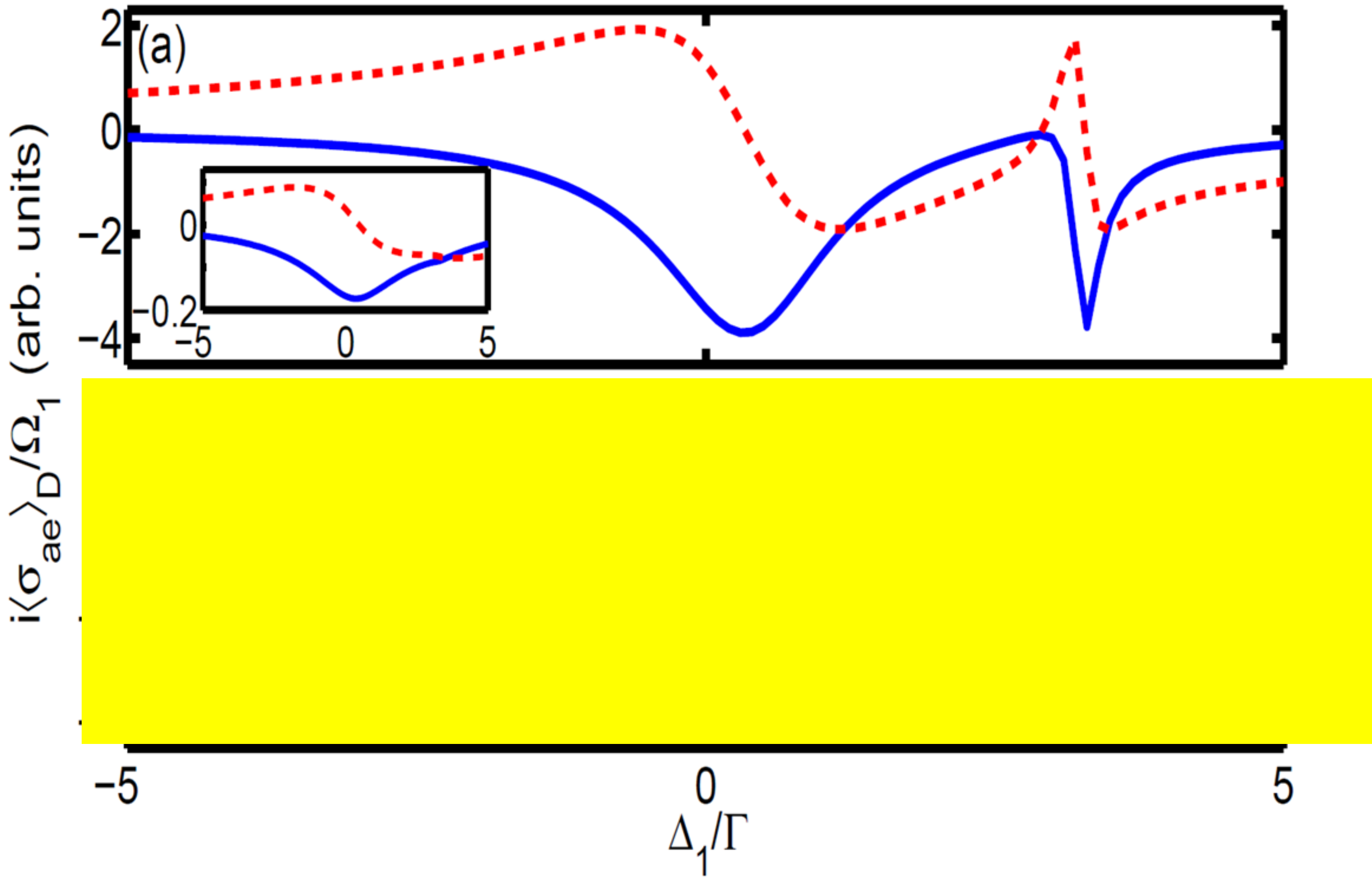
Bogoliubov excitations in dark state:

$$H_D - \mu \hat{N} = \epsilon_D(p=0) - \mu \langle \hat{\beta}_0 \rangle^2 + \frac{U_{aa}}{2V} \langle \hat{\beta}_0 \rangle^4$$
$$+ \sum_{q \neq 0} \left[\epsilon_1 \hat{\beta}_q^\dagger \hat{\beta}_q + \frac{\epsilon_2}{2} \hat{\beta}_q \hat{\beta}_{-q} + \frac{\epsilon_2}{2} \hat{\beta}_{-q}^\dagger \hat{\beta}_q^\dagger \right],$$

$$\epsilon_1(q) = \epsilon_D(q) - \mu + 2 n_0 U_{aa}, \; \epsilon_2 = n_0 U_{aa}$$

$$\frac{\langle \tilde{\sigma}_{ae} \rangle_B}{\Omega_1} = \frac{1}{N} \sum_{p \neq 0} F_p (\sinh^2 t + n_b \cosh 2t),$$

EIT profiles in weakly interacting BEC



(a) counter-propagating excitation fields. The scattering length $a_s = 7 \times 106a_0$ and temperature is chosen as T = 10nK. Insets are contributions from Bogoliubov particles and $n_{ex} \approx 0.1n$.

EIT in strong interacting systems

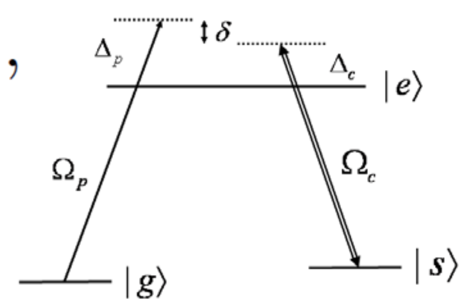
Hamiltonian

$$H = H^{(g)} + \sum_k \begin{bmatrix} \hat{a}_k^\dagger & \hat{b}_k^\dagger \end{bmatrix} \begin{bmatrix} E_a(k) & 0 \\ 0 & E_b(k) \end{bmatrix} \begin{bmatrix} \hat{a}_k \\ \hat{b}_k \end{bmatrix},$$

where $\hat{a}_k$ ($\hat{b}_k$) is combination of $\hat{s}'_{k+p_1-p_2}$ and $\hat{e}'_{k+p_1}$.

$$H_2 = -\frac{1}{V} \sum_{k,p} \left[\tilde{\Omega}_k(t) e^{-i\omega_p t} \hat{e}^\dagger_{k+p}(t) \hat{g}_p(t) + h.c. \right].$$

$$\delta \left\langle \hat{O}(t) \right\rangle = \frac{i}{\hbar} \int_{-\infty}^{t} dt' \left\langle \psi_0 \middle| \left[H_2(t'), \hat{O}(t) \right] \middle| \psi_0 \right\rangle,$$



Linear response of polarization

$$\delta\left\langle \hat{P}(q,\omega)\right\rangle$$

$$= \frac{id_0}{\hbar V}\sum_k \left\{\bar{\Omega}_q(\omega)\int_{-\infty}^{\infty}\theta(t)dt\int dx e^{ikx}\int_{-\infty}^{\infty}d\tilde{\omega}e^{i\tilde{\omega}t}\int d\tilde{k}e^{-i\tilde{k}x}\tilde{G}(\tilde{k},\tilde{\omega})\right.$$

$$\left.\left[\cos^2\phi_{k+q}e^{-i(\omega-(E_g-E_{a,k+q})/\hbar)t}+\sin^2\phi_{k+q}e^{-i(\omega-(E_g-E_{b,k+q})/\hbar)t}\right]-h.c.(q,\omega\to -q,-\omega)\right\},$$

Dynamical Green's

function:

$$\tilde{G}(k,\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}drdte^{ikr}e^{-i\omega t}\left\langle\hat{\psi}_g^{\dagger}(\vec{r},t)\hat{\psi}_g(0,0)\right\rangle$$

Correlation function for Luttinger liquid

The Luttinger ground state correlation can be calculated if recoil momentum is neglected for lower atomic transitions (not Rydberg levels) at low temperature.

$$\tilde{G}(k,\omega) = \frac{C_0(v,K)}{\left|(\omega - E_{g,k}/\hbar)^2 - k^2 v^2\right|^{1-1/(4K)}},$$

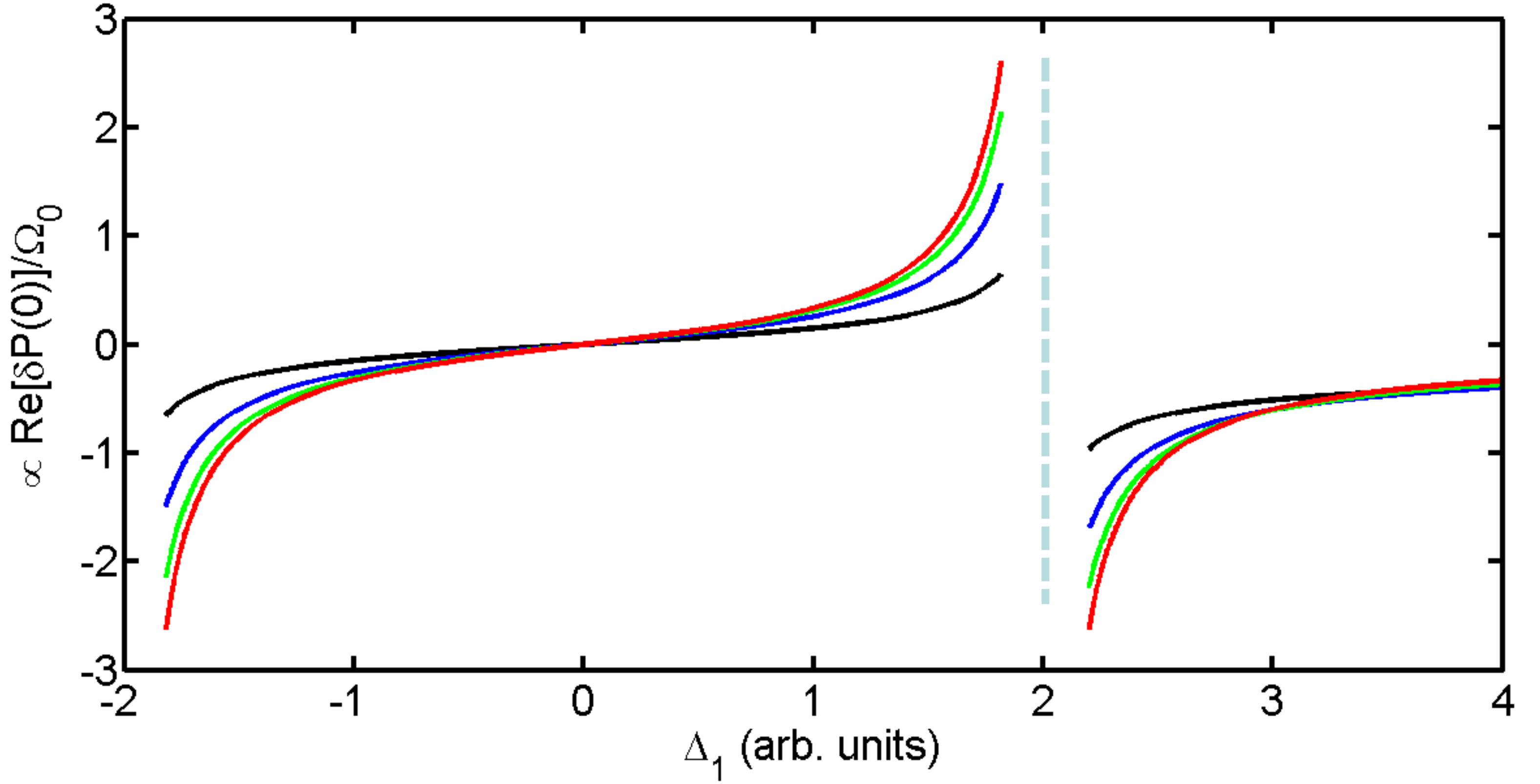
where C_0 is a constant that depends on interaction parameter K and phonon velocity v.

v, κ: Luttinger parameters. $1<\kappa<\infty$

Dispersion relation

$$\delta\left\langle \hat{P}(q,\omega)\right\rangle$$
$$= -i\bar{C}_0(v,K)\frac{d_0 v_s^{-1} e^{-i\pi/(4K)}\sec\frac{\pi}{4K}}{\hbar}\left\{\bar{\Omega}_q(\omega)\left[\frac{\cos^2\phi_0}{(\omega - E_{ga}/\hbar)^{1-1/(2K)}} + \frac{\sin^2\phi_0}{(\omega - E_{gb}/\hbar)^{1-1/(2K)}}\right]\right.$$
$$\left. - h.c.(q,\omega \to -q,-\omega)\right\},$$

Interaction strength
(a) κ=1,
(b) κ=2
(c) κ=5
(d) κ=1000



Outlook and conclusion

- Recoil momentum modify EIT profiles in non-interacting quantum degenerate atom gases.

- We develop a general theory of EIT for any quantum systems as long as a single particle Green's function is known.

- Another strong interacting quantum state - Mott state

Thank you for your attention!