Large-momentum effective theory for partons and light-cone physics

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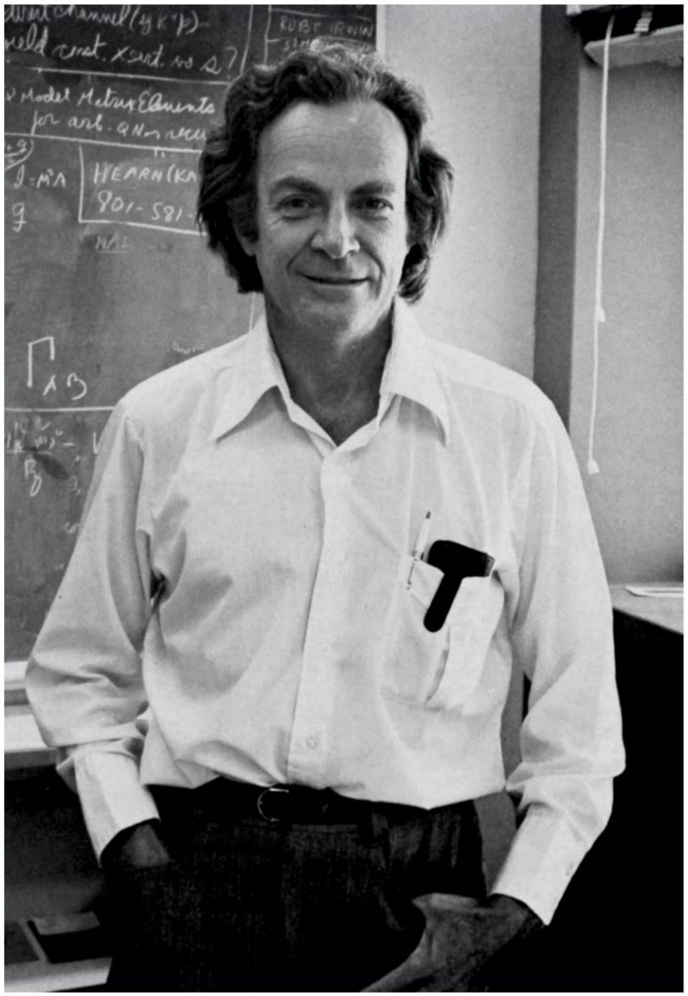
Outline

- Introduction to Partons
- Partons in Minkowskian formulation
    - Light-front quantization (LFQ)
    - Light-front (LF) correlations
- Back to Feynman: partons in infinite momentum frame
    - States, not operators!
- Large-momentum expansion and EFT (LaMET)
- Applications

Introduction to partons

Feynman language



- Feynman Diagrams
- Path Integrals
- Partons

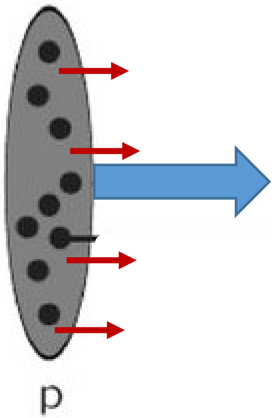
arXiv:2006.08594:
Feynman Lectures
On Strong Interactions

Feynman's parton model

- When proton travels at $v \sim c$, one can assume the proton travels exactly at v=c, or the proton momentum is

$$p=E=\infty$$

(Infinite momentum frame, IMF)



- Proton may be considered as a collection of interaction-free particles: partons

Parton distribution functions (PDF)

- Every parton has k=∞ (EFT), however,

  x=k/p = finite, ∈ [0,1]

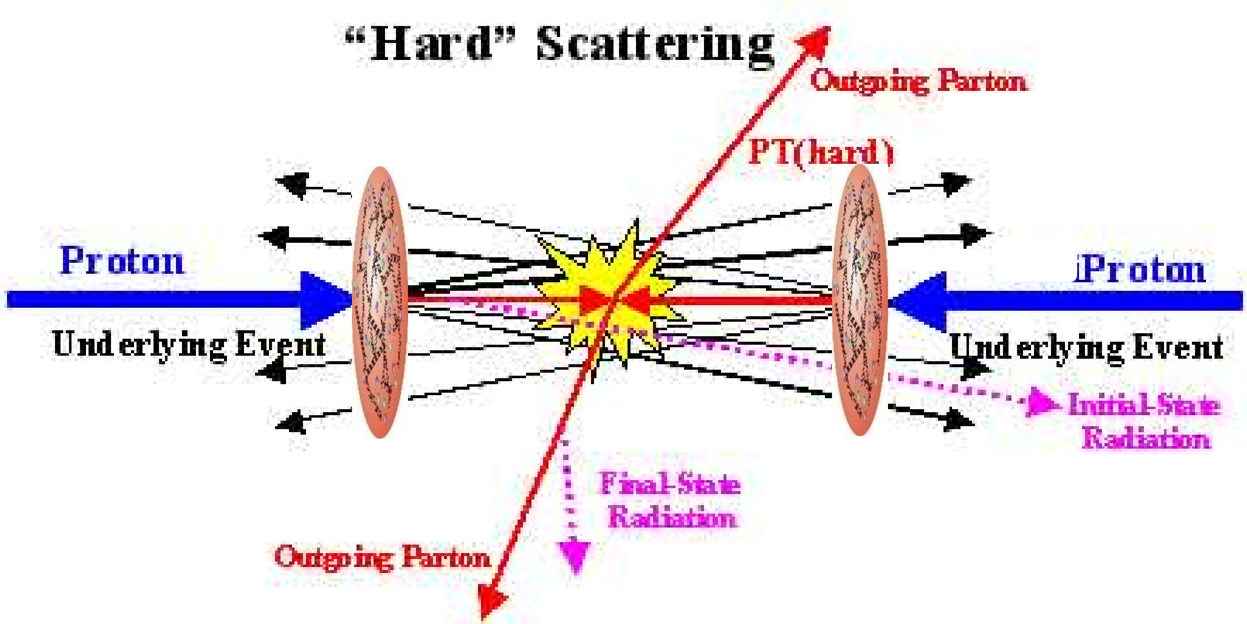
- Parton distribution function

  $$f(x)$$

is the probability of finding parton in a proton, carrying x fraction of the momentum of the parent.

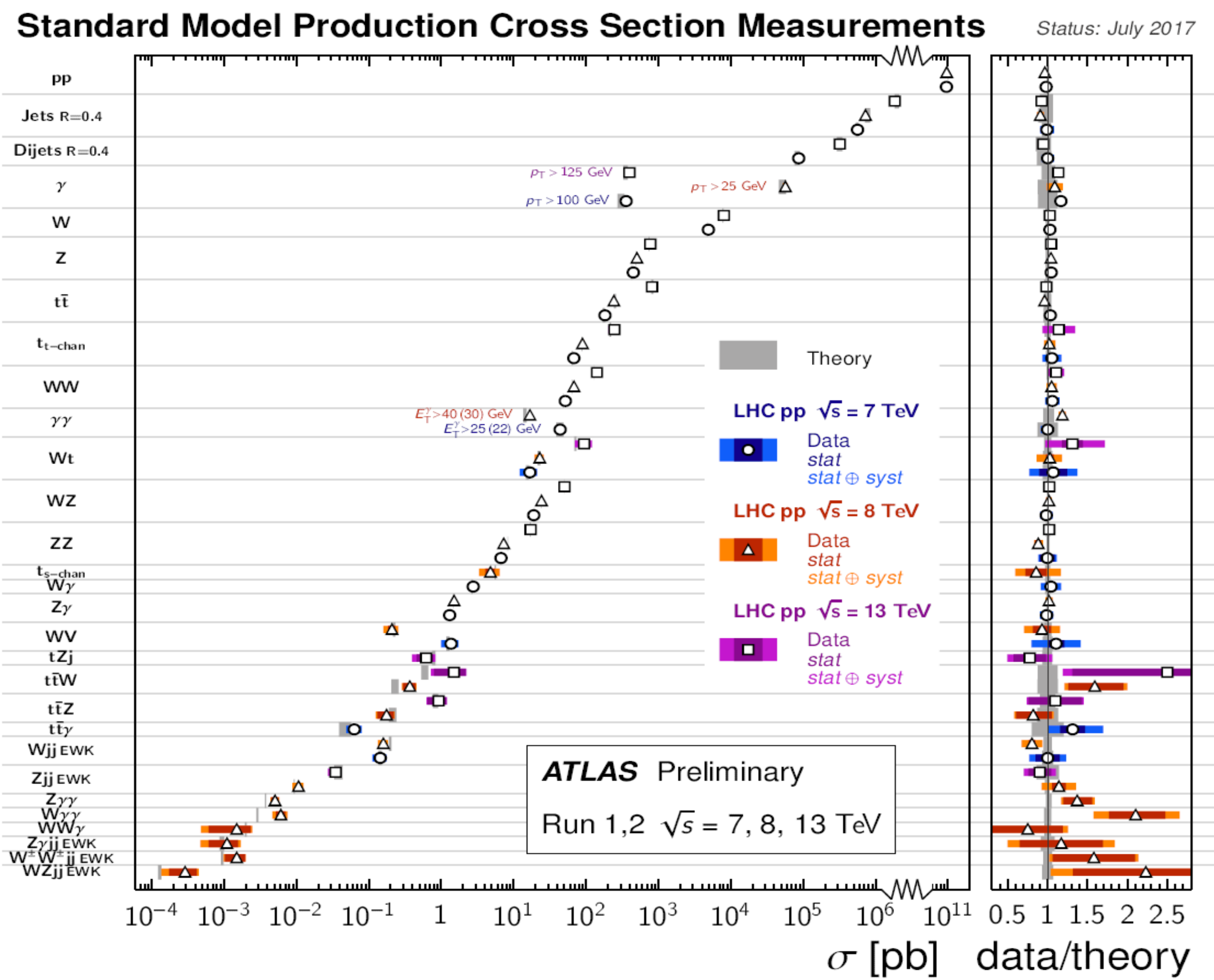
- PDF is a bound state property of the proton, essential to explain the results of high-energy collisions.

Hard scattering theory



- Factorization theorems: The scattering cross sections are factorized in terms of PDFs and parton x-section.

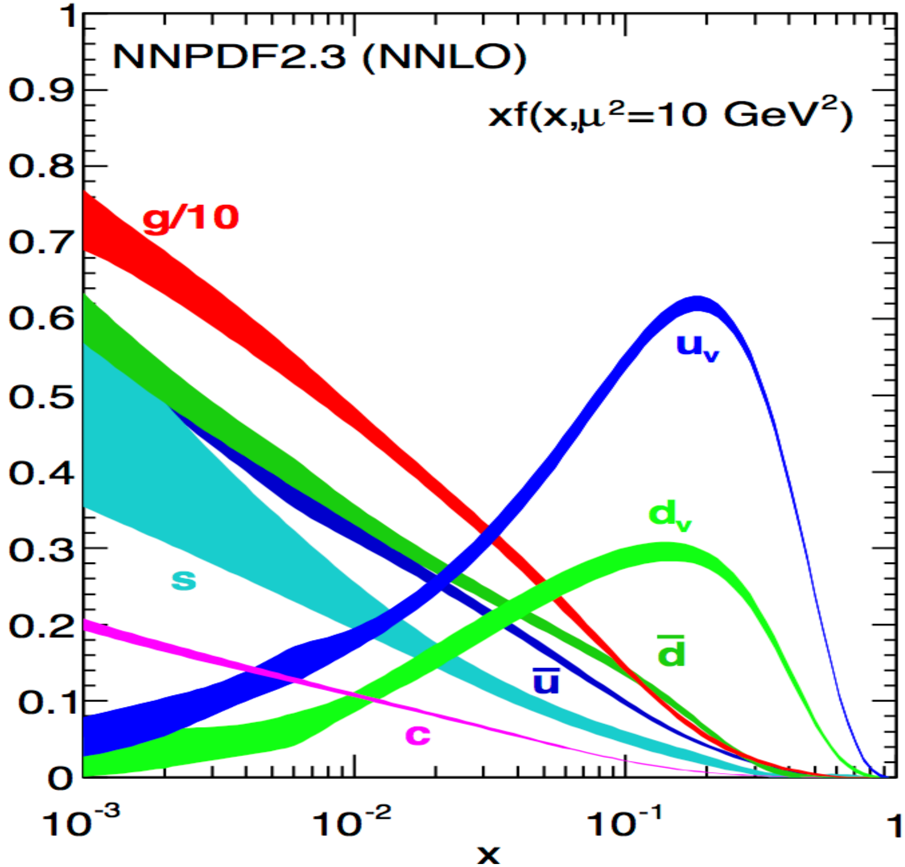
$$\sigma = \int dx_a dx_b f_{a/A}(x_a) f_{b/B}(x_b) \hat{\sigma}$$



Standard Model Production Cross Section Measurements
Status: July 2017
pp
Jets R=0.4
Dijets R=0.4
γ
W
Z
tt̄
t t−chan
WW
γγ
Wt
WZ
ZZ
t s−chan
Wγ
Zγ
WV
tZj
tt̄W
tt̄Z
tt̄γ
Wjj EWK
Zjj EWK
Zγγ
Wγγ
WWγ
Zγjj EWK
W±W±jj EWK
WZjj EWK
pT > 125 GeV
pT > 100 GeV
pT > 25 GeV
E_T^γ > 40 (30) GeV
E_T^γ > 25 (22) GeV
Theory
LHC pp √s = 7 TeV
Data
stat
stat ⊕ syst
LHC pp √s = 8 TeV
Data
stat
stat ⊕ syst
LHC pp √s = 13 TeV
Data
stat
stat ⊕ syst
ATLAS Preliminary
Run 1,2 √s = 7, 8, 13 TeV
10−4 10−3 10−2 10−1 1 10¹ 10² 10³ 10⁴ 10⁵ 10⁶ 10¹¹
σ [pb]
0.5 1 1.5 2 2.5
data/theory

Phenomenological PDFs

- Use experimental data (50 yrs) to extract PDFs



J. Gao, et al, Phys. Rept. 742 (2018) 1-121

Partons in Minkowskian formalism

Weinberg’s rules

- What does an object look like when travelling at infinite momentum or speed of light?
- S. Weinberg (scalar QFT)

  Dynamics at infinite momentum

  Phys. Rev. 150 (1966) 1313-1318

- All kinematic infinities can be removed from the calculations, resulting a set of rules for Hamiltonian perturbation theory (“old-fashioned p.t.”)
- The result is similar to a “non-relativistic” theory.

More Weinberg's rules… and a discovery

- L. Susskind, K. Bardakci, and M. B.Halpern,…
- S. J. Chang and S. K. Ma (1969)

Feynman rules and quantum electrodynamics at infinite momentum,

Phys. Rev. 180 (1969) 1506-1513

- J. Kogut and D. Soper

Quantum Electrodynamics in the Infinite Momentum Frame, Phys. Rev. D 1 (1970) 2901-2913

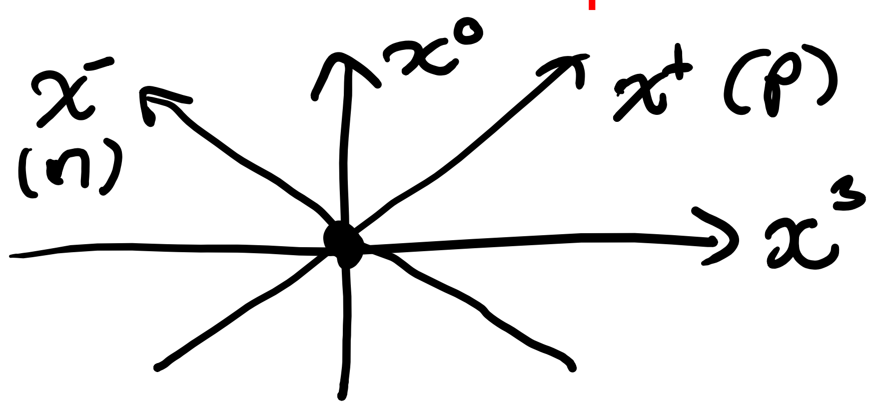
Chang and Ma's discovery

- All Weinberg's rules in the P=∞ limit can be obtained by quantizing the theory with "new coordinates"

$$x^+ = \frac{x^0 + x^3}{\sqrt{2}}, \;\; x^- = \frac{x^0 - x^3}{\sqrt{2}}$$

by treating x^+ as the new "time"

x^- as the new "space" dimension.



Dirac's form of dynamics

- The Weinberg's rules exactly correspond to what Dirac proposed in 1949.
- Paul A.M. Dirac,

Forms of Relativistic Dynamics,

Rev. Mod. Phys. 21 (1949) 392-399.

"Front form"

or Light-front quantization (LFQ)

Feynman’s parton model (1968)

- Formulated in IMF of Fubini and Furlan
- Not in a context of a QFT but with QM and relativity in mind.
- Connection with LF quantization and LF coordinates immediately recognized!
- Plus: a theory without kinematic infinity
- Minus: Theories in LFQ hard to solve!

Why it is hard to solve QCD in LFQ?

- All slow-moving stuff in zero-modes (vacuum).
- It is a strongly coupled problem!
- There is no demonstration that the weak coupling expansion actually works for QCD.

K. Wilson et. al. Phys. Rev. D49 (1994)

- There is no controlled approximation.

Light-front collinear modes

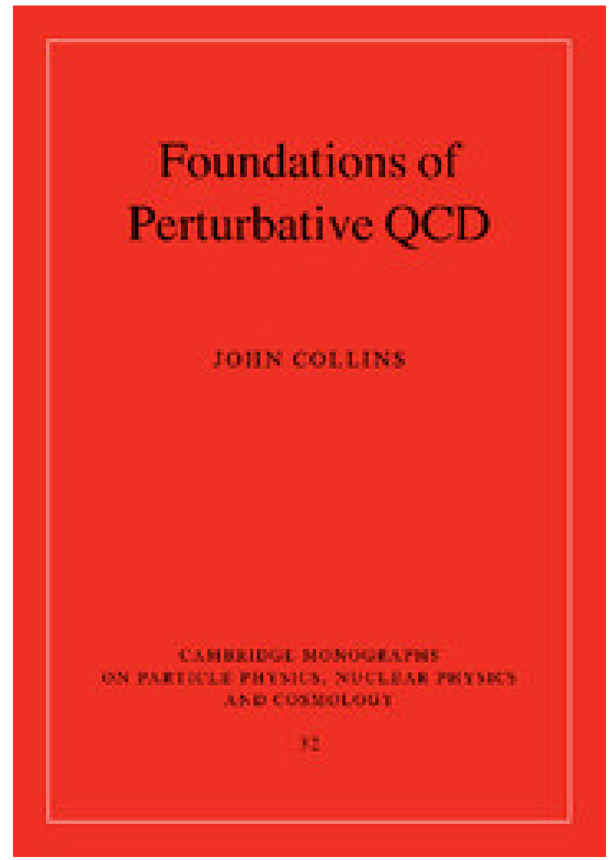
- In Lagrangian formulation of parton physics, the partons are represented by collinear modes in QCD

$$\psi(\lambda n^\mu)\,,\ n^2 = 0$$

λ is the distance along the LF

- Parton physics is related to correlations of these fields along n with distance λ.

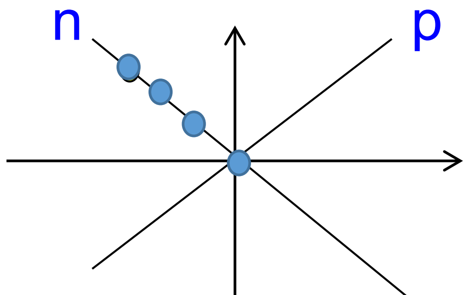
e.g. Soft-Collinear Effective Theory (SCET)



Partons as LF correlations

- Probes（operators) are light-cone correlations

$$\hat{O} = \phi_1(\lambda_1 \mathrm{n})\phi_2(\lambda_2 n) \ldots \phi_k(\lambda_k n)$$



- The matrix elements are independent of hadron momentum, and they can be calculated in the states in the rest frame.
- "Heisenberg picture"

Partons and critical phenomena

- Fourier trans. of PDFs gives a small-x behavior,

  f(x) → $x^{-\alpha}$

- When FT back to position space, one has

  $C(\lambda) \sim \lambda^{\alpha-1}$

  This corresponds to "infinite correlation" length

  $C(\lambda) \sim \exp(-\frac{\xi}{\lambda})$ with $\xi \to \infty$

No condensed matter theorists directly solve critical phenomena at T=Tc!

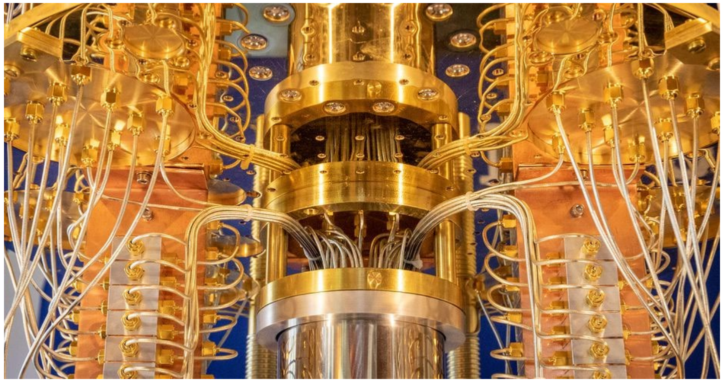
Real-time Monte Carlo in path integrals?

- Monte Carlo simulations have not been very successful with quantum real-time dynamics.

$$\exp(-iHt)$$

an oscillating phase factor!

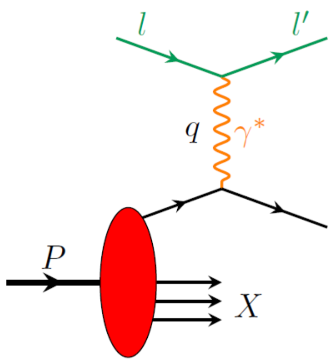
- "Sign problem": Hubbard model for high T_c.
- Signals are exponentially small!
- Quantum computer?



Back to Feynman: Partons in infinite-momentum frame

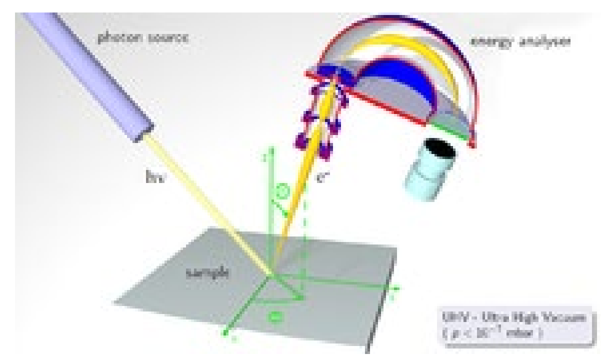
Origin of Parton Model

- Electron-proton deep-inelastic scattering (DIS)



- Knock out scattering in NR systems
  - e-scattering on atoms
  - ARPES in CM systems
  - Neutron scattering on liquid-^{4}He

  …



Momentum distribution in NR systems

- Knock-out reactions in NR systems probes momentum distribution

$$n\left(\vec{k}\right) = \left|\psi\left(\vec{k}\right)\right|^2$$

$$\sim \int \psi^*(\vec{r})\psi(0)e^{i\vec{k}\vec{r}}d^3r$$

$$\sim \int \langle\Omega\left|\hat{\psi}^+(\vec{r})\hat{\psi}(0)\right|\Omega\rangle e^{i\vec{k}\vec{r}}d^3r$$

- Mom.dis. are related to Euclidean correlations, generally amenable for Monte Carlo simulations.

Difference between relativistic and NR systems

- NR cases, the energy transfer is small.

$$q^0 \sim \frac{1}{M} \sim 0$$

- Relativistic systems:

  In DIS, if we choose a frame in which the virtual photon energy is zero

$$q^\mu = (0, 0, 0, -Q)\,,$$
$$P^\mu = \left(\frac{Q}{2x_B} + \frac{M^2 x_B}{Q}, 0, 0, \frac{Q}{2x_B}\right)\,,$$

In the Bjorken limit, $P^z \sim Q \to \infty$

Feynman's partons

- Consider the mom.dis. of constituents in a hadron

$$f(k^z, P^z) = \int d^2k_\perp\, f(k^z, k_\perp, P^z)$$

which depends on P^z because of relativity.

(H is not invariant under boost K)

- PDF is a result of the $P^z \rightarrow \infty$ limit,

$$f(k^z, P^z) \rightarrow_{P^z \rightarrow \infty} f(x) \quad with\; x = \frac{k^z}{P^z},$$

Euclidean formulation of partons

- Calculate the Euclidean correlation

$$C(\lambda) = \langle P^z = \infty | \overline{\psi}(z) \Gamma \psi(0) | P^z = \infty \rangle$$

$$\lambda = \lim_{P^z \to \infty, z \to 0} (zP^z).$$

- Parton distribution

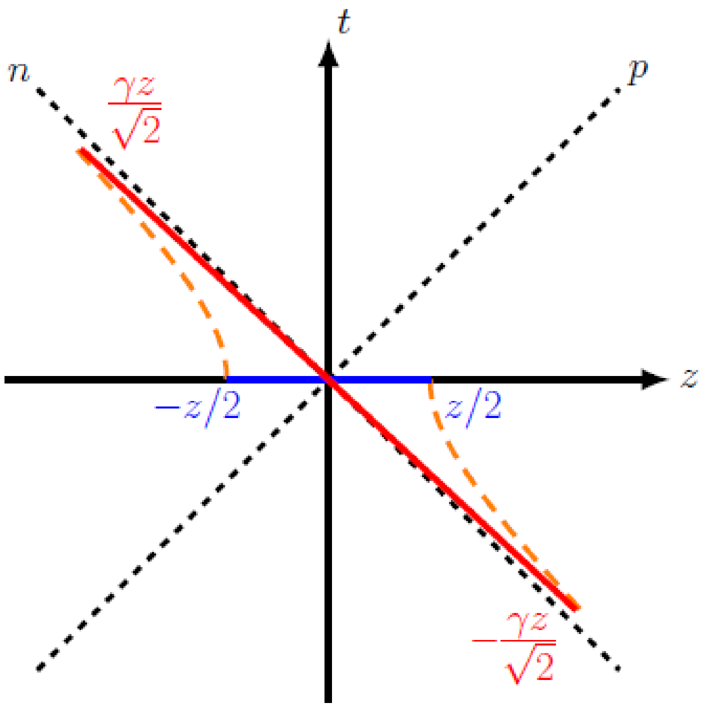
$$f(x) = \frac{1}{2P^+} \int \frac{d\lambda}{2\pi} e^{ix\lambda} C(\lambda) \ .$$

Relation between two parton formalisms

- Partons in LF correlation formamlism
  - Use LF collinear field operators
  - Parton physics in LF correlations (Heisenberg picture)
  - Independent of external state momentum
- Infinite-momentum-frame parton formalism
  - Use infinite-momentum states to select parton modes
  - Euclidean correlations (Schrodinger picture)
  - Can use different operators: universality class

Relation between Euclidean and Minkowskian parton formulations:

An infinite Lorentz boost



Large-momentum expansion and EFT

Take $P^z \to \infty$ limit

- Highly non-trivial. It must be studied in the context of a QFT (only a field theory can support ∞ momentum modes)
- Assuming the limit exist, the limiting process is controlled by expansion,

$$f(k^z, P^z) = f(x) + f_2(x)(M/P^z)^2 + ...$$

where M is a bound-state scale,

P^z is a large-momentum scale.

Naïve dimensional analysis

- $\epsilon = \left(\frac{M}{P^z}\right)^2$ is an expansion parameter

  M =1 GeV, P=2 GeV

  $\epsilon = 1/4$

  the expansion may already work.

Large momentum expansion
(X. Ji, 2013)

- Approximate p=∞ by a large P
- This is what we frequently do in QCD

Lattice QCD: approximate a continuum theory by a discrete one.

cut-off $\Lambda \to \infty$, on lattice $\Lambda = \pi/a$

0.1 fm ~ 2 GeV

HQET: $\epsilon = \Lambda_{QCD}/m_Q$

using $m_Q = \infty$ to approximate

m_c =1.5 GeV!

QFT subtleties

- There is a UV cut-off Λ_{UV}, $f(k^z, P^z)$ is not analytic at $P^z = \infty$!
- There are two possible $P^z \to \infty$ limits:
  1. $P^z \ll \Lambda_{UV} \to \infty$, IMF limit (lattice QCD)
  2. $P^z \gg \Lambda_{UV} \to \infty$ LFQ limit (HEP PDF)
- Calculating mom.dis. is done with the limit 1) and PDF is defined in limit 2).
- Solution: matching in EFT

  The difference is perturbative!

LaMET expansion

- One needs a proton fast enough

  the control parameter of expansion is

  $$(M/P^z)^2 \sim 1/\gamma^2$$

- Thus for finite momentum, one can have a factorization formula for large $\gamma \sim (2-5)$:

$$\tilde{f}(y, P^z) = \int Z(y/x, xP^z/\mu) f(x,\mu) dx$$

$$+ \mathcal{O}\Big(\frac{\Lambda^2_{\mathrm{QCD}}}{y^2(P^z)^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{(1-y)^2(P^z)^2}\Big),$$

$$\langle\gamma|\hat{O}(z_1, z_2, \ldots, z_k)|\gamma\rangle = Z(\alpha_s, \gamma, z_i, \lambda_i) \otimes \langle 1|\hat{O}(\lambda_1, \ldots, \lambda_k)|1\rangle +$$

$$+\, o(1/\gamma^2)$$

Momentum renormalization group equation

- Mom.dis. $f(k^z, P^z)$ has a non-trivial dependence on P^z (H is frame-dependent).
- At large P^z, this dependence shall be calculable in perturbation theory

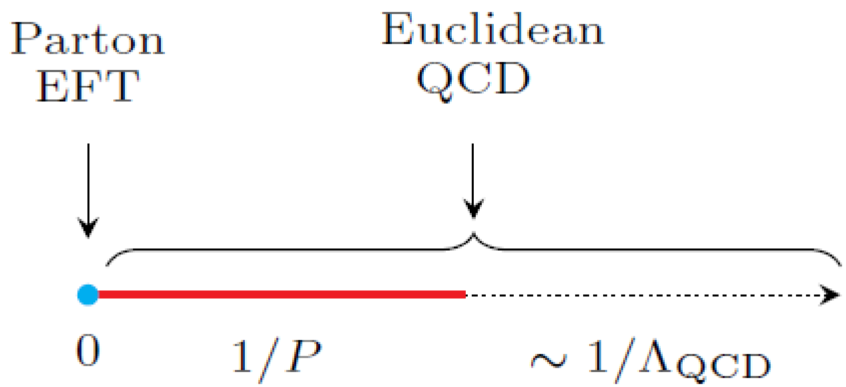
$$\frac{\partial O(P^z)}{\partial \ln P^z} = \gamma_P(\alpha_s) O(P^z)\,,$$

Momentum RGE

$$P^z \frac{\partial}{\partial P^z} \tilde{q}(y, P^z, \mu) = \int_0^1 \frac{dt}{|t|} P_{qq}(t) \times \tilde{q}\left(\frac{y}{t}, tP^z, \mu\right) - 2\gamma_F \tilde{q}(y, P^z, \mu)\ .$$

- DGLAP evolution is related to the change of mom.dis. with different CoM motion.

LaMET and critical phenomena



Universality

- One can practically choose ANY composite operator with arguments $z_1 \ldots z_n$, so long as γ large enough, they give the same collinear or soft physics.
- For different operators, flowing into the fixed point of large momentum will have different rates (which is faster?), but the limit is the same.

X. Ji, Sci. China (Phys. Mech.Astron.) 57, 1407-1412 (2014)

Y. Hatta, Ji, Zhao, Phys. Rev. D89 (2014).

Why is it an EFT?

- An EFT integrates out the DOF outside the model space

  P+Q=1

- LaMET P-space contains all modes with momentum between 0 and P^z , and cutoff $\Lambda_{UV} \gg P^z$
- Q-Space contains all modes between P^z and ∞
- Similar to lattice gauge theory approximating the continuum theory

Applications

Application 1: Gluon total helicity ΔG

- In QCD factorization, one can show that the gluon polarization is a matrix element of non-local light-cone correlation.

A. Manohar,Phys. Rev. Lett. 66 (1991) 2684

$$\Delta G = \int dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS|F_a^{+\alpha}(\xi^-) \times \mathcal{L}^{ab}(\xi^-, 0)\tilde{F}^+_{\alpha,b}(0)|PS\rangle,$$

- No one knows how to calculate this for nearly 30 years!

LaMET calculations

- In LaMET theory, one can start with the local operator $\vec{E} \times \vec{A}$, in any physical gauge

(gauge choices shall allow transverse polarized gluons):

  - Coulomb gauge $\nabla \cdot \vec{E} = 0$
  - Axial guage A_z=0
  - Temporal gauge A_0 =0

- Their matrix elements in the large momentum limit all go to ΔG (Weizsacker & Williams)

Ji, Zhang, Zhao, Phys. Rev. Lett.,111,112002(2013)

Y. Hatta, Ji, Zhao, Phys.Rev. D89 (2014).

First calculation (Y. Yang et al, PRL(2017))

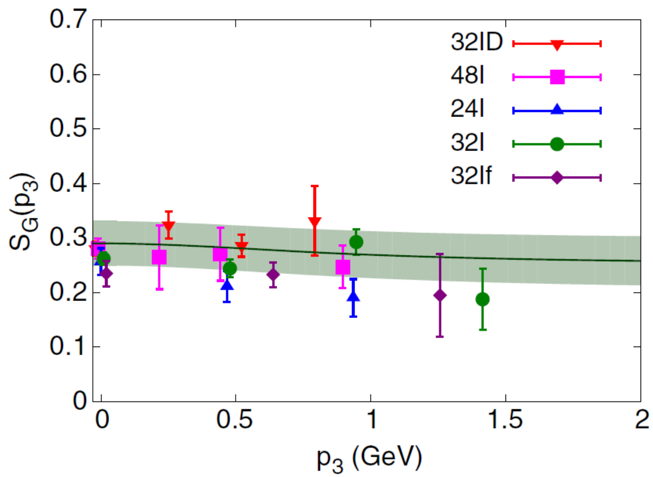
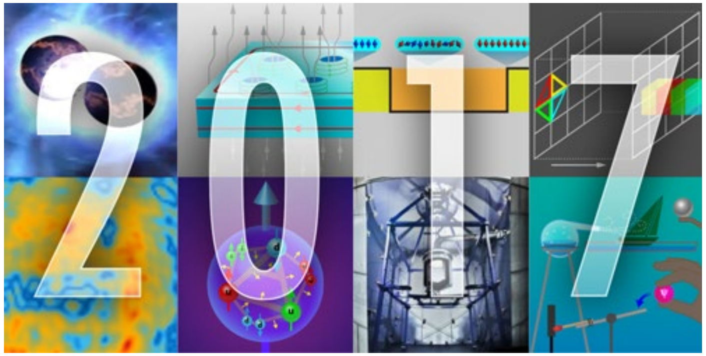


FIG. 4. The results extrapolated to the physical pion mass as a function of the absolute value of $\vec{p} = (0, 0, p_3)$, on all the five ensembles. All the results have been converted to $\overline{\text{MS}}$ at $\mu^2 = 10\ \text{GeV}^2$. The data on several ensembles are shifted horizontally to enhance the legibility. The green band shows the frame dependence of the global fit [with the empirical form in Eq. (11)] of the results.



Gluons Provide Half of the Proton's Spin

The gluons that bind quarks together in nucleons provide a considerable chunk of the proton's total spin. That was the conclusion reached by Yi-Bo Yang from the University of Kentucky, Lexington, and colleagues (see Viewpoint: **Spinning Gluons in the Proton**). By running state-of-the-art computer simulations of quark-gluon dynamics on a so-called spacetime lattice, the researchers found that 50% of the proton's spin comes from

Need more controlled calculations

Application 2: PDFs:

- PDF can be obtained from large momentum limit of a correlation

$$\langle P \left| \bar{\psi}(z) \Gamma W(z,0) \psi(0) \right| P \rangle$$

  - Γ can be γ^0or γ^3 or any combination.
  - W is a straight-line Wilson link

This is the starting point of quarsi-PDF,

X. Ji, Phys. Rev. Lett. 110 (2013)

Factorization was conjectured. The full-proved given by Ma and Qiu, PRD98 (2018) 074021

State-of-the-art calculations

- ETMC PRL121(2018) 112001; …
- LP3 (J.W. Chen et al.) PRL121(2018) 242003;…
- LPC PRD 101 (2020) 3, 034020;…
- Jlab & BNL groups:
- Some analyses now have controlled approximations
  - One-loop matching and scale setting, renormalization
  - Excited states, higher twist corrections

ETMC recent result

C. Alexandrou et al, Phys.Rev.Lett. 121 (2018) no.11, 112001

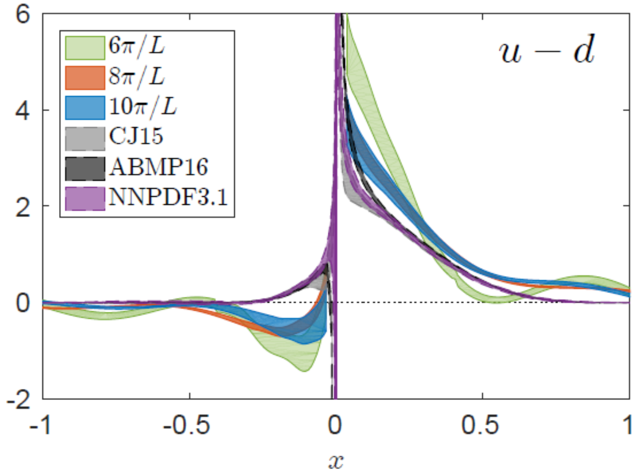


FIG. 4: Comparison of unpolarized PDF at momenta $\frac{6\pi}{L}$ (green band), $\frac{8\pi}{L}$ (orange band), $\frac{10\pi}{L}$ (blue band), and ABMP16 [39] (NNLO), NNPDF [40] (NNLO) and CJ15 [38] (NLO) phenomenological curves.

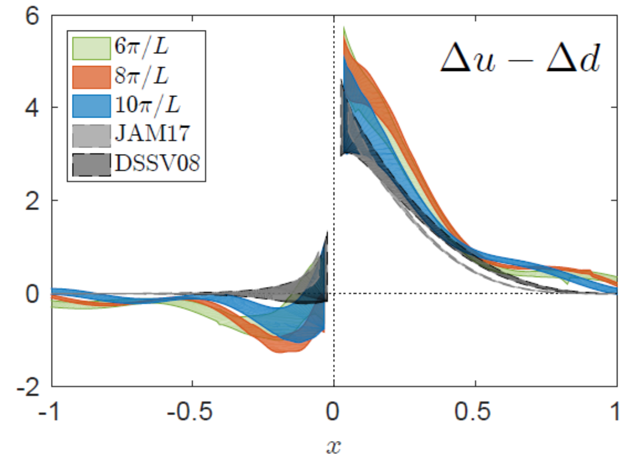


FIG. 5: Comparison of polarized PDF at momenta $\frac{6\pi}{L}$ (green band), $\frac{8\pi}{L}$ (orange band), $\frac{10\pi}{L}$ (blue band), DSSV08 [41] and JAM17 NLO phenomenological data [42].

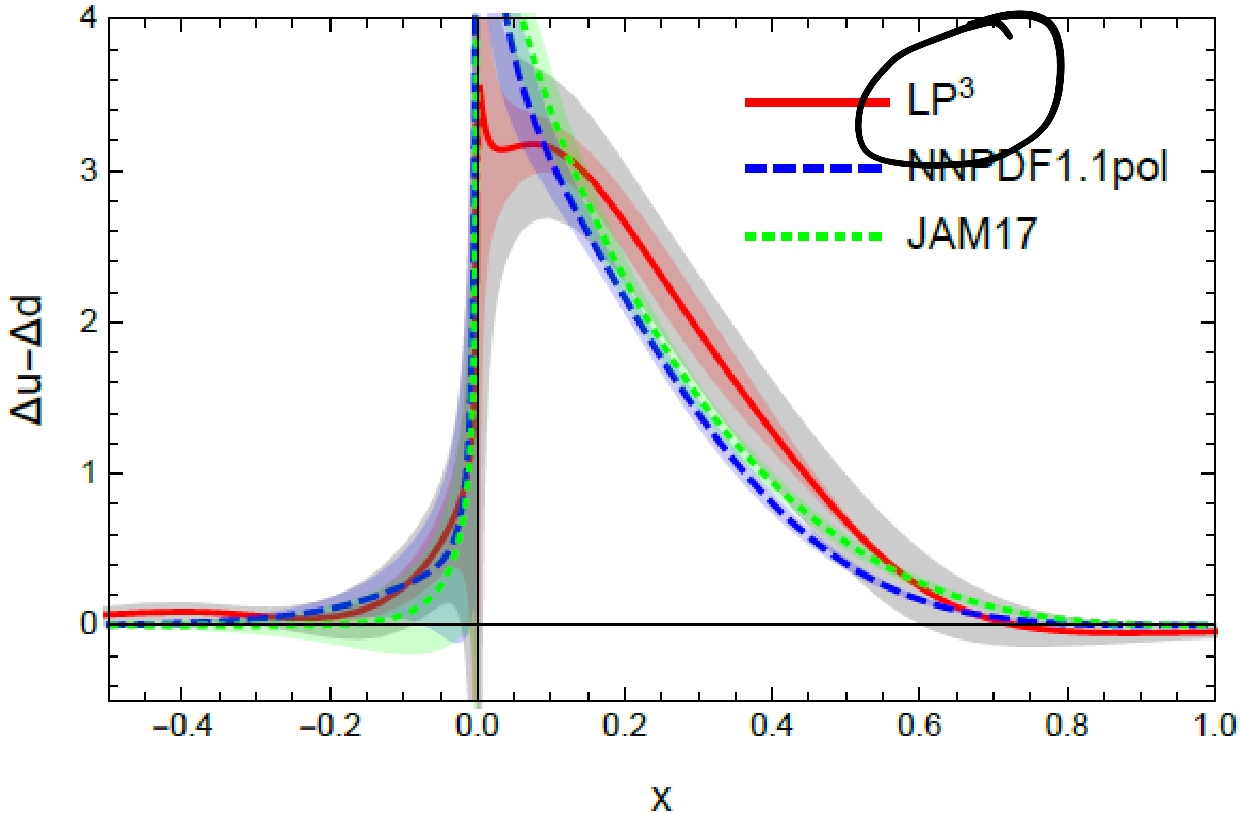


FIG. 21: Proton isovector quark PDFs [177]: The helicity PDF ($P^z = 3.0$ GeV) with red band contains statistic error and grey band further includes systematic error. NNPDF1.1pol [348] and JAM17 [350] are global fits.

LPC

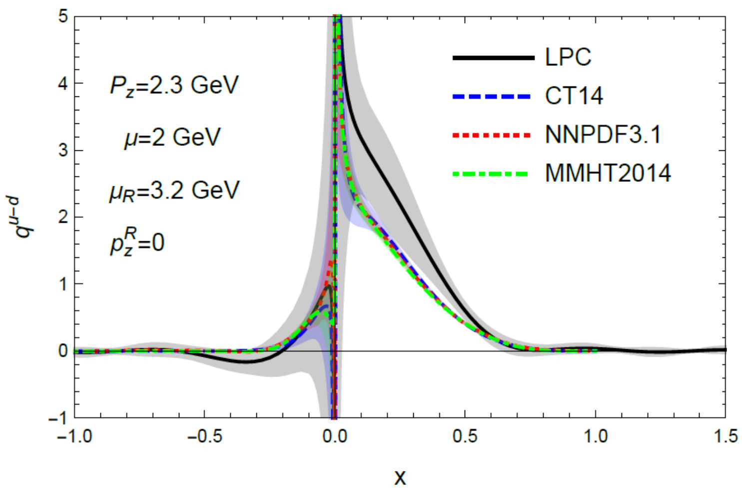
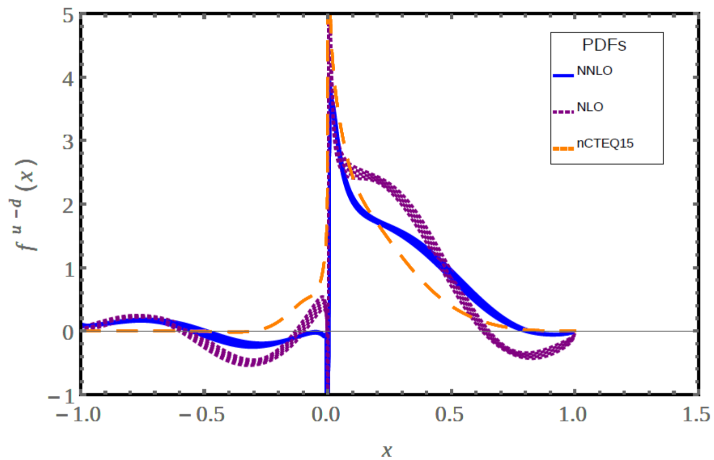


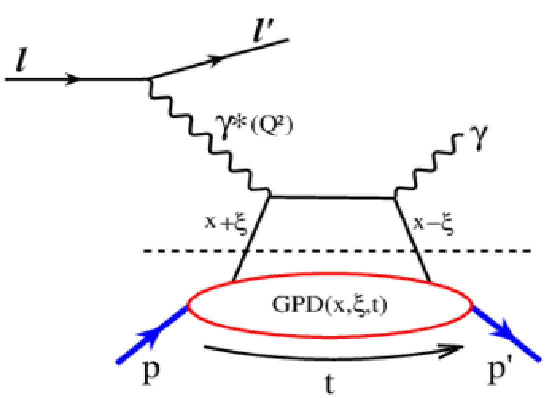
FIG. 10. Results for PDF at $\mu = 2$ GeV calculated from RI/MOM quasi-PDF at nucleon momentum $P_z = 2.3$ GeV: Comparing with CT14nnlo (90CL) [84], NNPDF3.1 (68CL) [85], and MMHT2014 (68CL) [86]. Our results agree with the global-analysis within uncertainties.



Chen, Wang, Zhu, 2020
arXiv:2006.14825

Application 3: Generalized parton distributions (GPD)

- GPD are form factors of parton distributions, discovered independently (Mueller et al.'94, Ji'96)
- An experimental process that can be used to measure GPD: Deeply virtual Compton scattering (Ji, PRL78, PRD55, 1997)



Kinematic variables: x, ξ, t

GPD on lattice

- GPD presents no additional difficulty compared with PDF
- One considers the off-forward matrix elements

$$\left\langle P' \middle| \bar{\psi}(z)\Gamma W(z,0)\psi(0) \middle| P \right\rangle$$

- Two more variables results
  - Momentum transfer, t
  - Skewness, ξ, need momentum transfer in z- direction.
- It has been very difficult to model these dependences in the literature. Great oppo to get guidance from lattice.
- Matching: Liu et al., PRD100 (2019) 034006

ETMC, Xiv:**1910.13229**

Nucleon unpolarized & helicity GPDs

★ N_f=2+1+1 twisted mass fermions

[C. Alexandrou et al., (ETMC), 2019]

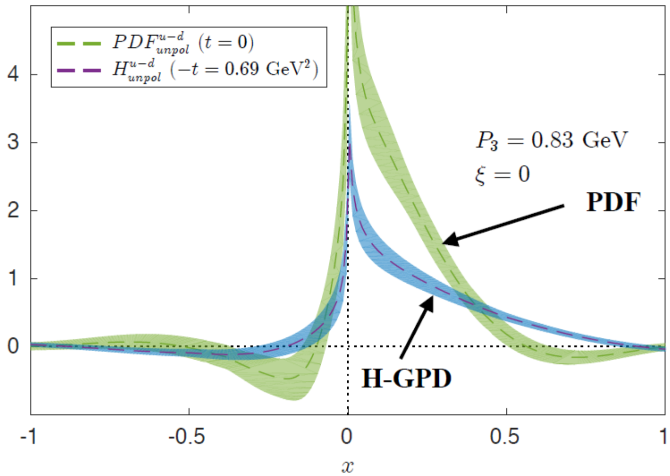
$m_\pi = 260\,\text{MeV},\ P = 0.83 - 1.66\,\text{GeV}$

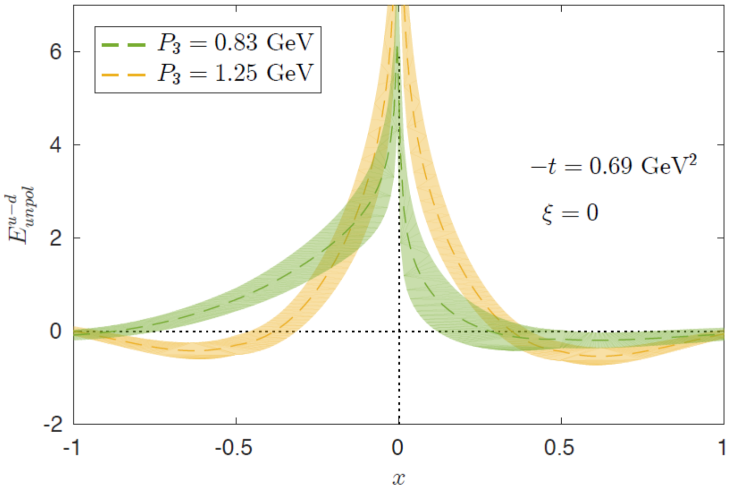
$|\xi| \geq 0,\ a = 0.09\,\text{fm},\ 32^3 \times 64$

★ GPDs decomposition (unpolarized)

$$\mathcal{M}_{\gamma_0}(\Gamma_0) = \tilde{H}_Q C_H(\Gamma_0, p_i, p_f) + \tilde{E}_Q C_E(\Gamma_0, p_i, p_f)$$

$$\mathcal{M}_{\gamma_0}(\Gamma_\kappa) = i\,\tilde{H}_Q\, C_H(\Gamma_\kappa, p_i, p_f) + i\,\tilde{E}_Q\, C_E(\Gamma_\kappa, p_i, p_f)$$



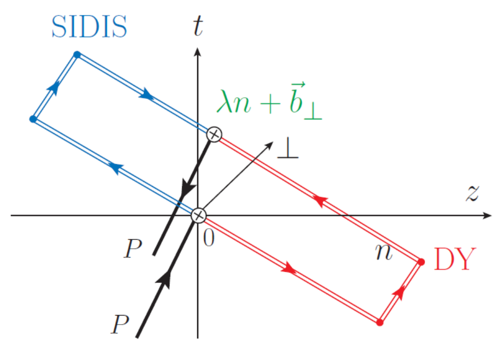


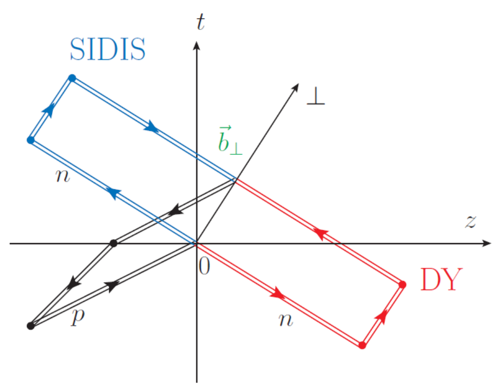
★ Comparison at same P_3 :
H suppressed compared to PDF

★ E becomes symmetric in x as P_3 increases

Application 4: TMD-PDFs

- A very important nucleon observable, many phenomenology related to spin physics (Sivers effect etc).
- It took sometime to figure out the correct definition





Echevarria, Idilbi, Scimemi (2013), Collins & Rogers (2013)

Lattice calculations

- Started from A. Schafer et al., much progress; no x dependence has yet been studied.

  Hagler et al, Much et al, Yoon et al. PRD96,094508 (2017)…

- A number of LaMET formulations:

  Ji et al., PRD91,074009 (2015); PRD99,114006(2019)

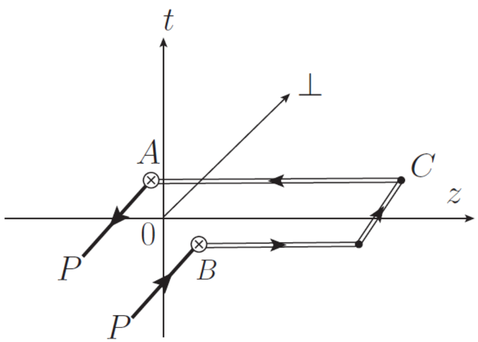
  Ebert, Stewart, Zhao, PRD99,034505 (2019), JHEP09,037(2019); arXiv:1910.08569

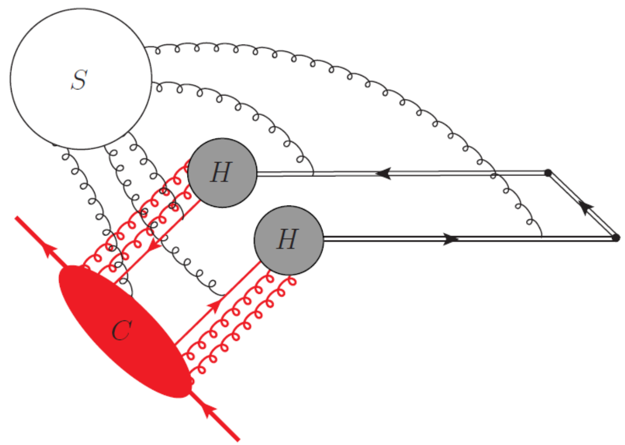
  Collins-Soper evolution kernel can be calculated.

  Soft function can be calculated Ji, Liu, Liu, NPB(2020)

Quasi-TMDPDF and factorization

Ji, Liu, Liu, 1911.03840, PLB



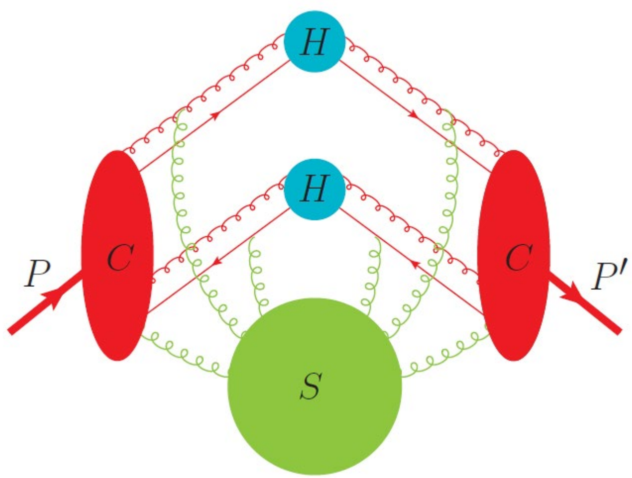


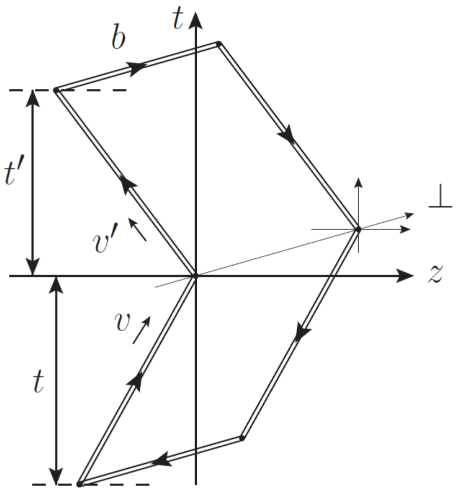
$$\tilde{f}(x, b_\perp, \mu, \zeta_z)\sqrt{S_r(b_\perp, \mu)}$$

$$= H\left(\frac{\zeta_z}{\mu^2}\right) e^{K(b_\perp,\mu)\ln(\frac{\zeta_z}{\zeta})} f^{\rm TMD}(x, b_\perp, \mu, \zeta) + \ldots$$

$$\mu \frac{d}{d\mu} \ln H\left(\frac{\zeta_z}{\mu^2}\right) = \Gamma_{\rm cusp} \ln \frac{\zeta_z}{\mu^2} + \gamma_C$$

Calculation of soft factor

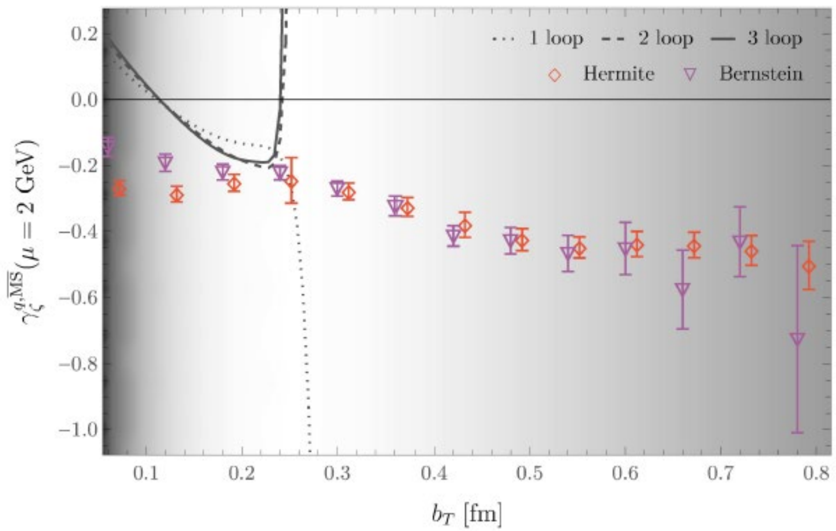




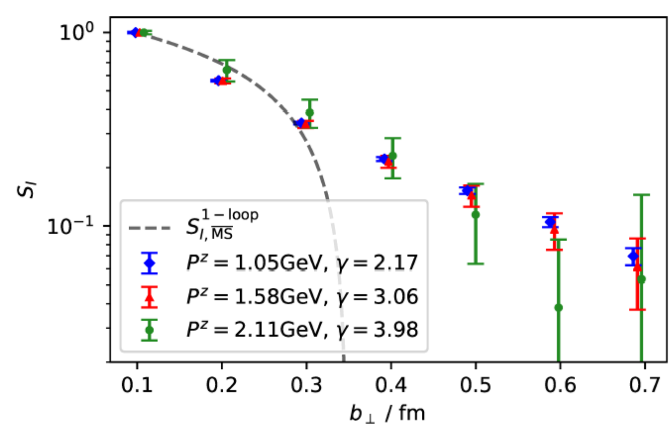
Factorization of form-factor of Light meson

Form-factors of heavy-quark pair

CS evolution and soft factor



P. Shanahan, Wagman, Zhao, 2003.06063 [hep-lat]



Q. A. Zhang et al, 2005.14572 [hep-lat]

App. 5： Light-Front Wave-Functions

- LF quantization focuses on the WFs, from which everything can be calculated: a very ambitious goal! Brodsky et al. Phys. Rept. 301 (1998)
- However, there are a number of reasons this approach has not been very successful.
- LaMET provides the practical way to calculate non-perturbative WF, at least for lowest few components. Ji, Liu, & Liu, to be published.
- All WF can be computed as gauge-invariant matrix elements

$$\left\langle 0 \left| \hat{O}\left(z_1, \vec{b}_1, z_2, \vec{b}_2 \ldots, z_k, \vec{b}_k\right) \right| P \right\rangle$$

Conclusions

- Partons can be calculated on lattice using a large-momentum hadron state.

$$\frac{\Lambda_{QCD}}{P} \ll 1$$

- LaMET3.0 (~ 5% error)

  two-loop matching

  P=3 GeV

  Improved non-pert renormalization

- 1% accuracy in 10-20 years