Topologically stable, finite-energy electroweak monopoles and the prediction of $\sin^2\theta_W$

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- TOPOLOGICALLY STABLE: The monopole is stable due to a topological conservation law characterized by an integer n. A monopole with $n = 1$ cannot change into a trivial configuration with $n = 0$ for example.
- FINITE ENERGY: A soliton with finite mass and finite size.
- ELECTROWEAK-SCALE MONOPOLE: Monopoles whose masses $\sim O(TeV) \rightarrow$ Accessible at the LHC and can be searched for at MoEDAL
- Who cares about monopoles? Many people: Dirac, Schwinger, 't Hooft, Polyakov,...
- For what reasons? Symmetry of Maxwell's equations, charge quantization,..., Consequences of Spontaneous Symmetry Breaking

. MAXWELL'S EQUATIONS (without monopoles)

$$\nabla.\vec{E} = 4\pi\rho_e$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla.\vec{B} = 0$$

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{J}_e$$

$$.\Downarrow$$

$$\boxed{\partial_\mu F^{\mu\nu} = j^\nu \; ; \qquad \partial_\mu \tilde{F}^{\mu\nu} = 0}$$

$$(\; \tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\sigma\rho}F^{\sigma\rho})$$

Symmetry for $j^\nu = 0$: $F^{\mu\nu} \to \tilde{F}^{\mu\nu}; \tilde{F}^{\mu\nu} \to -F^{\mu\nu}$.

$$\vec{E} \to \vec{B}; \vec{B} \to -\vec{E}$$

. MAXWELL'S EQUATIONS (with monopoles)

$$\nabla . \vec{E} = 4\pi \rho_e$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} + \frac{4\pi}{c}\vec{J}_m$$

$$\nabla . \vec{B} = 4\pi \rho_m$$

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{J}_e$$

$$.\Downarrow$$

$$\boxed{\partial_\mu F^{\mu\nu} = j^\nu \; ; \quad \partial_\mu \tilde{F}^{\mu\nu} = k^\nu}$$

Symmetry: $F^{\mu\nu} \to \tilde{F}^{\mu\nu}; \tilde{F}^{\mu\nu} \to -F^{\mu\nu}$.

$$j^\nu \to k^\nu; k^\nu \to -j^\nu$$

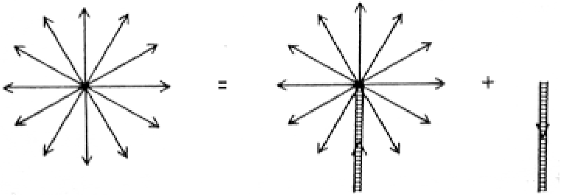
Coulomb-like magnetic field of a point-like magnetic monopole of magnetic charge g_M:

$$\vec{B} = g_M \frac{\vec{r}}{r^3}$$

$$g_M = \oint \vec{B}.d\vec{S}$$

But $\oint \vec{B}.d\vec{S} = 0$ if $\vec{B} = \nabla \times \vec{A}$. $\oint \vec{B}.d\vec{S} \neq 0$ if

$$g_M \frac{\vec{r}}{r^3} = \nabla \times \vec{A} + \underbrace{4\pi g_M \theta(-z)\delta(x)\delta(y)}_{\textit{Dirac string}}$$



- QM: we use $\vec{A}$. Northern patch: $\vec{A}^{(N)}$. Southern patch: $\vec{A}^{(S)}$. Related by gauge transformation: $\vec{A}^{(N)} - \vec{A}^{(S)} = \nabla(2g_M\phi)$. $\psi_{N,S}$: Solutions of the Schrödinger equation for each patch. Can show: $\psi_S(r,\theta,\phi) = e^{\imath 2eg_M\phi}\psi_N(r,\theta,\phi)$. $(\psi_S(r,\theta,0) = \psi_N(r,\theta,0))$

  Suppose $\psi_N(r,\theta,2\pi) = \psi_N(r,\theta,0)$ i.e. single-valued.

- $\psi_S(r,\theta,2\pi) = e^{\imath 2eg_M(2\pi)}\psi_N(r,\theta,0) = e^{\imath 2eg_M(2\pi)}\psi_S(r,\theta,0)$

  ψ_S is single-valued, i.e. $\psi_S(r,\theta,2\pi) = \psi_S(r,\theta,2=0)$ if and only if

$$\boxed{e g_M = \frac{m}{2}}$$

Dirac Quantization Condition (DQC)

MAGNETIC MONOPOLES with NO STRING ATTACHED

- Dirac monopole: Point-like object with a singular string attached. Just pure $U(1)_{em}$. No idea how heavy it could be.
- $U(1)_{em} \subset G$ could get rid of the Dirac string and predict the monopole mass.
- 't Hooft-Polyakov monopole: Topologically-stable, finite-energy solution to the field equations for the Georgi-Glashow model $SO(3) \to U(1)$: the monopole is a soliton with finite size and finite mass. The Dirac string is just a gauge artifact. Far away from the core of the monopole, it looks exactly like a Dirac monopole.
- Topologically stable? Finite energy?
- How does one find such a solution for a general class of models that contain $U(1)_{em}$ as a subgroup?
- What could the experimental and theoretical implications be?

TOPOLOGICAL STABILITY

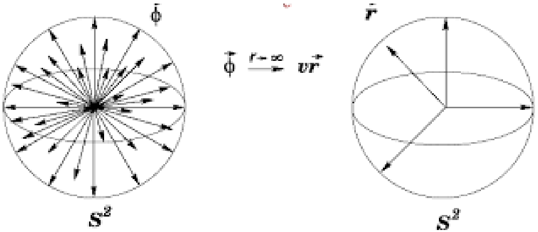
- A gauge group is spontaneously broken down to a subgroup by a Higgs multiplet ϕ^a: $\phi^a \phi^a = v^2$. This is a "sphere" in internal symmetry space: A vacuum manifold $\mathcal{M}$ (space of vacuum expectation values of the Higgs field).
- Higgs triplet of $SU(2)$ with 3 real components: $\phi_1^2 + \phi_2^2 + \phi_3^2 = v^2$. That's a 2-sphere S^2 (surface of a 3-dim internal symmetry sphere).
- Complex Higgs doublet with 4 real components: $\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = v^2$. A 3-sphere S^3.
- We are interested in how solutions to the classical field equations map the vacuum manifold $\mathcal{M}$ to the boundary of 3-dimensional space, also S^2.
- Trivial (perturbative) vacuum: $\langle \phi \rangle = (0, 0, ..., v)$, independent of spatial direction.
- Question: How many times one goes around in $\mathcal{M}$ when one goes around once in spatial S^2?

TOPOLOGICAL STABILITY

- The number of times is classified by $\Pi_2(\mathcal{M})$: The second Homotopy Group.
- For a real triplet, $\mathcal{M} = S^2$.
- Homotopy: $\Pi_2(S^2) = Z = n$. $n = 0, 1, 2, ...$
- Georgi-Glashow $SO(3)$ model with a real triplet: $\Pi_2(S^2) = Z$.
- 't Hooft-Polyakov monopole: Hedgehog änsatz (static):
  $\xi^a = \frac{r^a}{gr^2} H(v_M gr)$; $W_n^a = \epsilon_{aji} \frac{r^j}{gr^2}[1 - K(v_M gr)]$; $W_0^a = 0$

  This corresponds to $n = 1$.
- Non-trivial vacuum: $\xi^i \to v_M \frac{r^i}{r}$ as $r \to \infty$
- This monopole is topologically stable because it takes an infinite amount of energy to go from $n = 1$ to $n = 0$! It also has a finite energy!
- Far from the core (more on this later): $B_i \approx \frac{g_M}{r^2} \hat{r}_i$

TOPOLOGICAL STABILITY



TOPOLOGICAL STABILITY

- $\Pi_2(S^n) = 0$ for $n > 2$.
- SM with complex Higgs doublets: $\mathcal{M} = S^3 \Rightarrow \Pi_2(S^3) = 0$. No topologically stable monopole!
- Cho-Maison: SM with a Higgs doublet but now $\mathcal{M} = CP^1 \approx S^2 \Rightarrow \Pi_2(CP^1 \approx S^2) = Z$.
- Cho-Maison monopole is topologically stable but it has an infinite energy. To have a finite energy (more on this below), C-M modifies the kinetic term of the $U(1)_Y$ gauge field $\Rightarrow$ Unknown Physics BSM!
- Can one have a topologically stable (T-S), finite energy (F-E) monopole à la 't Hooft-Polyakov solely within the gauge group $SU(2) \times U(1)_Y$? Yes but one needs a real Higgs triplet of $SU(2)$!

EW-ν_R MODEL AND T-S, F-E MONOPOLE (PQH)

- What do neutrinos have to do with monopoles?
- The EW-ν_R model (PQH) has a real Higgs triplet. Why?
- The EW-ν_R model: A model of non-sterile ν_Rs with Majorana masses M_R proportional to the electroweak scale $\Lambda_{EW} \sim 246 GeV$. Gauge group: $SU(3) \times SU(2)_W \times U(1)_Y$.
- Non-sterile: $l_R^M = \begin{pmatrix} \nu_R^M \\ e_R^M \end{pmatrix}$ Mirror lepton doublet.
- Majorana mass term: $L_M = g_M \, l_R^{M,T} \sigma_2 \tau_2 \, \tilde{\chi} \, l_R^M$, where $\tilde{\chi} = (\chi^0, \chi^+, \chi^{++})$ is a complex triplet.
- $\langle \chi^0 \rangle = v_M \Rightarrow M_R = g_M v_M$.
- $M_R > M_Z/2$ (Z boson decay width constraint). $\Rightarrow$ $M_W \neq M_Z \cos\theta_W$ at tree level. VERY BADLY!
- CURE: Add a real triplet $\boxed{\xi(Y/2 = 0) = (\xi^+, \xi^0, \xi^-)}$ with $\langle \xi^0 \rangle = v_M \Rightarrow M_W = M_Z \cos\theta_W$!
- $\xi \Rightarrow$ Monopoles!

EW-ν_R MODEL AND T-S, F-E MONOPOLE

- The EW-ν_R model has a rich Higgs spectrum: Doublets $\Phi_i^{SM,MF}$, Complex triplet $\tilde{\chi}$, and ξ. (Also a singlet, irrelevant here.)
- Vacuum manifolds: $\Phi_i^{SM,MF} \to S^3$, $\tilde{\chi} \to S^5$, $\xi \to S^2$.
- Vacuum manifold of the EW-ν_R model: $\mathcal{M} = S^2 \times S^5 \times \prod S_i^3$.
- $\Pi_2(\mathcal{M}) = \Pi_2(S^2) \oplus \Pi_2(S^5) \oplus \Pi_2(\prod S_i^3) = \Pi_2(S^2) = Z$.
- The electroweak monopole is topologically stable.
- Hedgehog änsatz (static):
  $\xi^a = \frac{r^a}{gr^2} H(v_M gr)$; $W_n^a = \epsilon_{aji} \frac{r^j}{gr^2}[1 - K(v_M gr)]$; $W_0^a = 0$
- Classical static solution: Mass $=$ Energy $\Rightarrow$
  $\boxed{M = \frac{4\pi v_M}{g} f(\lambda/g^2) \sim 889\,GeV - 2.993\,TeV}$
  ($87\,GeV > v_M > 45.5\,GeV$, $f = 1 - 1.78$ and ($\sum_{i=1,2} v_i^2 + v_i^{M,2}) + 8v_M^2 = (246\,GeV)^2$). It is finite!
- The monopole is a finite-energy soliton with a core of radius $\boxed{R_c \sim (gv_M)^{-1} \sim 10^{-16}\,cm}$, with virtual $W^\pm$ and Z inside the core.

EW-ν_R MODEL AND T-S, F-E MONOPOLE

- Topological Quantization Condition: With $W_3^{\mu\nu} = \partial^\mu W_3^\nu - \partial^\nu W_3^\mu + \frac{1}{v_M^3 g}\varepsilon_{abc}\xi^a \partial^\mu \xi^b \partial^\nu \xi^c$, one constructs a topological current: $k_\mu = \frac{1}{2}\epsilon_{\mu\nu\sigma\rho}\partial^\nu W_3^{\sigma\rho}$. the topological magnetic charge is defined as $g_M = \int d^3 x k_0$. One obtains

$$\boxed{g\, g_M = n}$$

- $g_M = \frac{1}{g}$ for $n = 1$.
- $SU(2)_W \times U(1)_Y \rightarrow U(1)_W \times U(1)_Y \rightarrow U(1)_{em} \Rightarrow g = e/\sin\theta_W$

$$\boxed{g_M = \frac{\sin\theta_W}{e}}$$

- Magnetic field of the electroweak monopole far from the core:

$$\boxed{\vec{B} = \left(\frac{\sin\theta_W}{e}\right)\frac{\hat{r}}{r^2}}$$

. **Prediction of** $\sin^2\theta_W$

(John Ellis, Nick Mavromatos, PQH)

- DQC for an electron circling around the electroweak monopole: $eg_M = \frac{m}{2}$. Compare this DQC with the TQC: $g_M = \frac{\sin\theta_W}{e} \Rightarrow$ $\sin\theta_W = m/2$. Only $m = 1$ is allowed $\Rightarrow$

$$\boxed{\sin^2\theta_W = \frac{1}{4}}$$

. Prediction of $\sin^2\theta_W$

- Evolving $\sin^2\theta_W = 1/4$ from the monopole mass scale down to the Z-boson mass M_Z, gives $\sin^2\theta_W(M_Z) = x_W \approx 0.231$ compatible with experiment.

M_M (TeV)	F	n_H	n_3	$\bar{n}_3$	x_W
2.3	3	1	0	0	0.232
3	3	3	0	0	0.2314
3	3	1	1	1	0.2318
3	4	1	0	0	0.2328
3.5	4	1	0	0	0.232

- The compatibility with experiment appears to indicate that some (or all) mirror fermions are very heavy ($F \leq 4$). (The model contains 3 generations of SM fermions and 3 generations of mirror fermions.)
- The lightest mirror fermions are long-lived (details are in the EW-ν_R model-related papers). These are LLPs.

. Implications

- The existence in the EW-ν_R model of a real Higgs triplet ξ gives rise to topologically-stable, finite-energy electroweak monopole.
- The model predicts $\sin^2\theta_W = \frac{1}{4}$ which is evolved down to $\sin^2\theta_W \approx 0.231$ at the Z-mass.
- Monopole masses 2-3 TeV are accessible to the LHC and MoEDAL.
- The electroweak monopole mass $M = \frac{4\pi v_M}{g} f(\lambda/g^2)$ is related to the non-sterile right-handed neutrino mass $M_R = g_M v_M$!
- Apparently, the best production mechanism is to use heavy-ion collision because the production process is very different from that of a p-p collision (*exponentially-suppressed* as $\sigma \sim \exp(-4/\alpha = -548)$), coming mainly from a thermal Schwinger thermal pair production process.
- LHC signals in conjunction with those of the electroweak monopole signals?

. Implications

- The signals to look for are lepton-number violating signals at high energy: Like-sign dileptons from the decays $\nu_R \nu_R$ ($q\bar{q} \to Z \to \nu_R \nu_R$). Remember that ν_Rs are *non-sterile and Majorana*! One has $\nu_{Ri} \to e^M_{Rj} + W^+$ followed by $e^M_{Rj} \to e_{Lk} + \phi_S$ which occurs at *displaced vertices* due to the smallness of $g_{Sl} < 10^{-4}$. The signals at the LHC would be $q\bar{q} \to Z \to \nu_R i + \nu_{Ri} \to e_{Lk} + e_{Ll} + W^+ + W^+ + \phi_S + \phi_S$: Like-sign dileptons $e_{Lk} + e_{Ll}$ plus 2 jets (from 2 W) plus missing energies (from ϕ_S) $\Rightarrow$ Lepton-number violating signals! The appearance of like-sign dileptons $e^- e^-, \mu^- \mu^-, \tau^- \tau^-, e^- \mu^-, ...$ could occur at displaced vertices $> 1mm$ or even tens of centimeters depending on the size of g_{Sl}.

. EPILOGUE

Thank you and stay safe!