Implications of the NANOGrav pulsar-timing results on stochastic gravitational wave background

Kin-Wang Ng (吳建宏)

Institute of Physics &
Institute of Astronomy and Astrophysics
Academia Sinica

NCTS Annual Theory Meeting 2020
National Tsing Hua University
Hsinchu, Dec 9-11, 2020

Outline

- Stochastic gravitational wave background
- Pulsar timing arrays
- NANOGrav results
- Perspectives

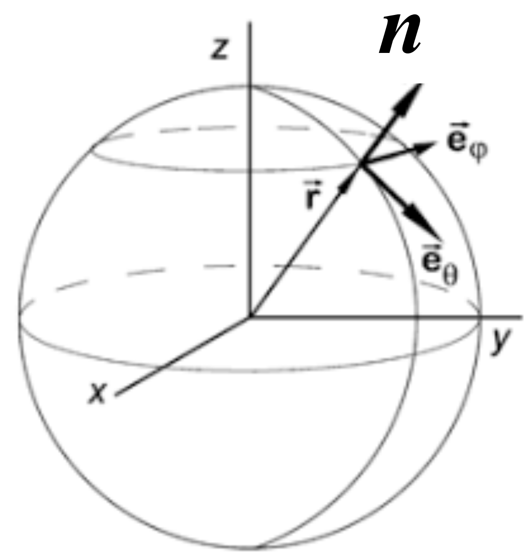
Gravitational Waves

$$h_{ij}(t,\boldsymbol{x}) = \sum_{P=+,\times} \int_{-\infty}^{\infty} df \int_{S^2} d\boldsymbol{n} h_P(f,\boldsymbol{n}) e^{2\pi i f(-t+\boldsymbol{n}\cdot\boldsymbol{x})} e_{ij}^P(\boldsymbol{n}). \tag{1}$$

Here, the bases for transverse-traceless tensor $\boldsymbol{e}^P$ ($P = +, \times$) are given as

$$\boldsymbol{e}^{+} = \hat{\boldsymbol{e}}_\theta \otimes \hat{\boldsymbol{e}}_\theta - \hat{\boldsymbol{e}}_\phi \otimes \hat{\boldsymbol{e}}_\phi, \qquad \boldsymbol{e}^{\times} = \hat{\boldsymbol{e}}_\theta \otimes \hat{\boldsymbol{e}}_\phi + \hat{\boldsymbol{e}}_\phi \otimes \hat{\boldsymbol{e}}_\theta,$$

$$\begin{pmatrix} \langle h_+(f,\boldsymbol{n}) h_+^*(f',\boldsymbol{n}')\rangle & \langle h_+(f,\boldsymbol{n}) h_\times^*(f',\boldsymbol{n}')\rangle \\ \langle h_\times(f,\boldsymbol{n}) h_+^*(f',\boldsymbol{n}')\rangle & \langle h_\times(f,\boldsymbol{n}) h_\times^*(f',\boldsymbol{n}')\rangle \end{pmatrix}$$
$$= \frac{1}{2}\delta_{\mathrm{D}}^2(\boldsymbol{n}-\boldsymbol{n}')\delta_{\mathrm{D}}(f-f')$$
$$\times \begin{pmatrix} I(f,\boldsymbol{n}) + Q(f,\boldsymbol{n}) & U(f,\boldsymbol{n}) - iV(f,\boldsymbol{n}) \\ U(f,\boldsymbol{n}) + iV(f,\boldsymbol{n}) & I(f,\boldsymbol{n}) - Q(f,\boldsymbol{n}) \end{pmatrix},$$



Q= <++> - <xx>
U is the same as in
a frame rotated by **π**/8

$$\boldsymbol{e}^R = \frac{(\boldsymbol{e}^+ + i\boldsymbol{e}^\times)}{\sqrt{2}}, \qquad \boldsymbol{e}^L = \frac{(\boldsymbol{e}^+ - i\boldsymbol{e}^\times)}{\sqrt{2}}$$

$$h_R = \frac{(h_+ - ih_\times)}{\sqrt{2}}, \qquad h_L = \frac{(h_+ + ih_\times)}{\sqrt{2}}$$

$$\begin{pmatrix} \langle h_R(f,\boldsymbol{n}) h_R(f',\boldsymbol{n}')^*\rangle & \langle h_L(f,\boldsymbol{n}) h_R(f',\boldsymbol{n}')^*\rangle \\ \langle h_R(f,\boldsymbol{n}) h_L(f',\boldsymbol{n}')^*\rangle & \langle h_L(f,\boldsymbol{n}) h_L(f',\boldsymbol{n}')^*\rangle \end{pmatrix}$$
$$= \frac{1}{2}\delta_{\mathrm{D}}(\boldsymbol{n}-\boldsymbol{n}')^2\delta_{\mathrm{D}}(f-f')$$
$$\times \begin{pmatrix} I(f,\boldsymbol{n}) + V(f,\boldsymbol{n}) & Q(f,\boldsymbol{n}) - iU(f,\boldsymbol{n}) \\ Q(f,\boldsymbol{n}) + iU(f,\boldsymbol{n}) & I(f,\boldsymbol{n}) - V(f,\boldsymbol{n}) \end{pmatrix}.$$

Cosmological GW spectral energy density

tion h_{ij} is gauged to be transverse-traceless. The latter can be decomposed into two polarization unit tensors as

$$h_{ij}(\eta,\vec{x}) = \sum_{\lambda=+,\times} \int \frac{d^3\vec{k}}{(2\pi)^{\frac{3}{2}}} h_\lambda(\eta,\vec{k})\epsilon^\lambda_{ij}(\hat{k})e^{i\vec{k}\cdot\vec{x}}, \quad (2)$$

where $h_\lambda(\eta,\vec{k})$ is a Gaussian random field that defines the power spectrum of tensor perturbation,

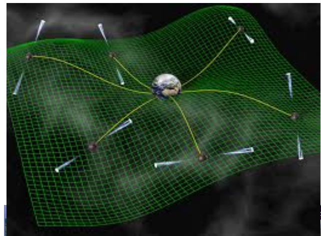
$$\langle h_\lambda(\eta,\vec{k})h^*_{\lambda'}(\eta,\vec{k}')\rangle = \delta(\vec{k}-\vec{k}')\frac{2\pi^2}{k^3}\mathcal{P}_h^{\lambda\lambda'}(\eta,k). \quad (3)$$

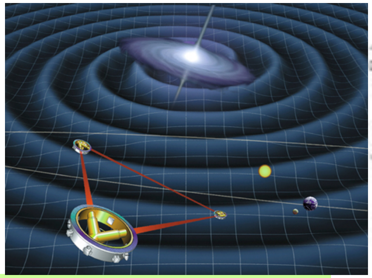
In the following, we will assume that $\mathcal{P}_h^{\lambda\lambda'}(\eta,k) = \delta_{\lambda\lambda'}\mathcal{P}_h(\eta,k)$. Then, the spectral energy density of the GWs relative to the critical density is given by

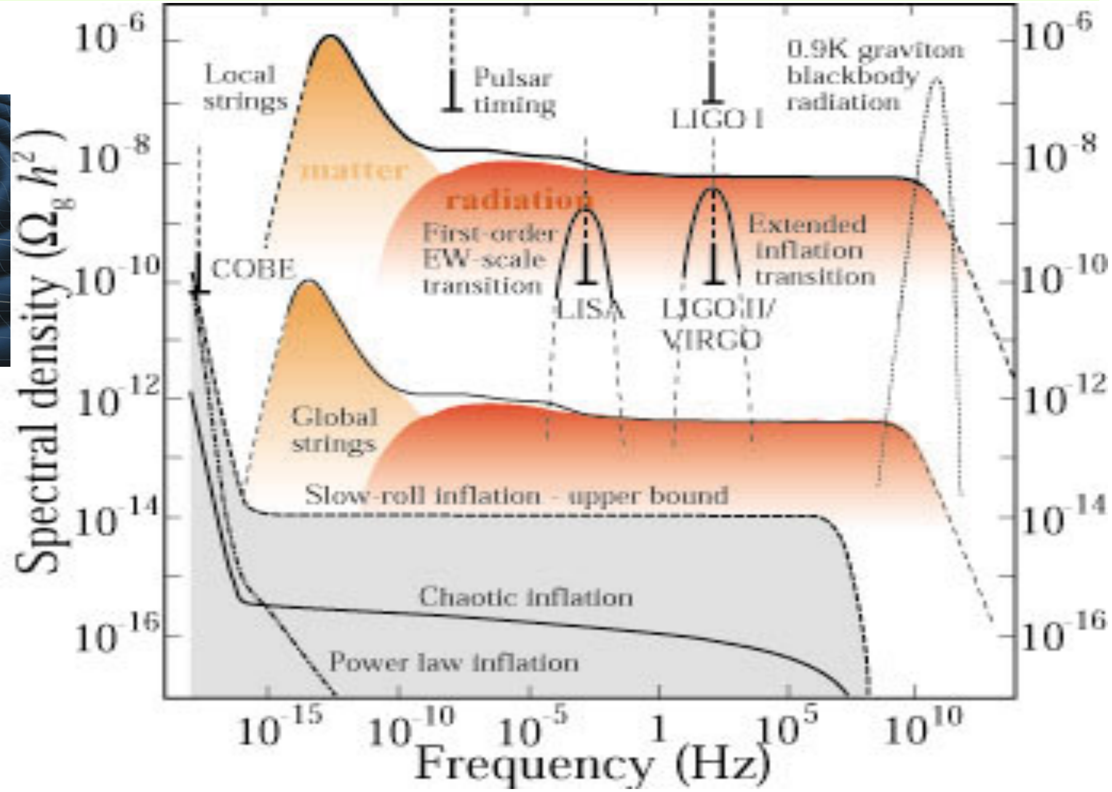
$$\Omega_{\rm GW}(\eta,k,\hat{k}) \equiv \frac{k}{\rho_c}\frac{d\rho_{\rm GW}}{dk d^2\hat{k}} = \frac{1}{96\pi}\left(\frac{k}{aH}\right)^2 \bar{\mathcal{P}}_h(\eta,k), \quad (4)$$

where $\rho_c = 3M_p^2H^2$ and the overbar denotes taking a time-average. For k-modes that re-enter the horizon

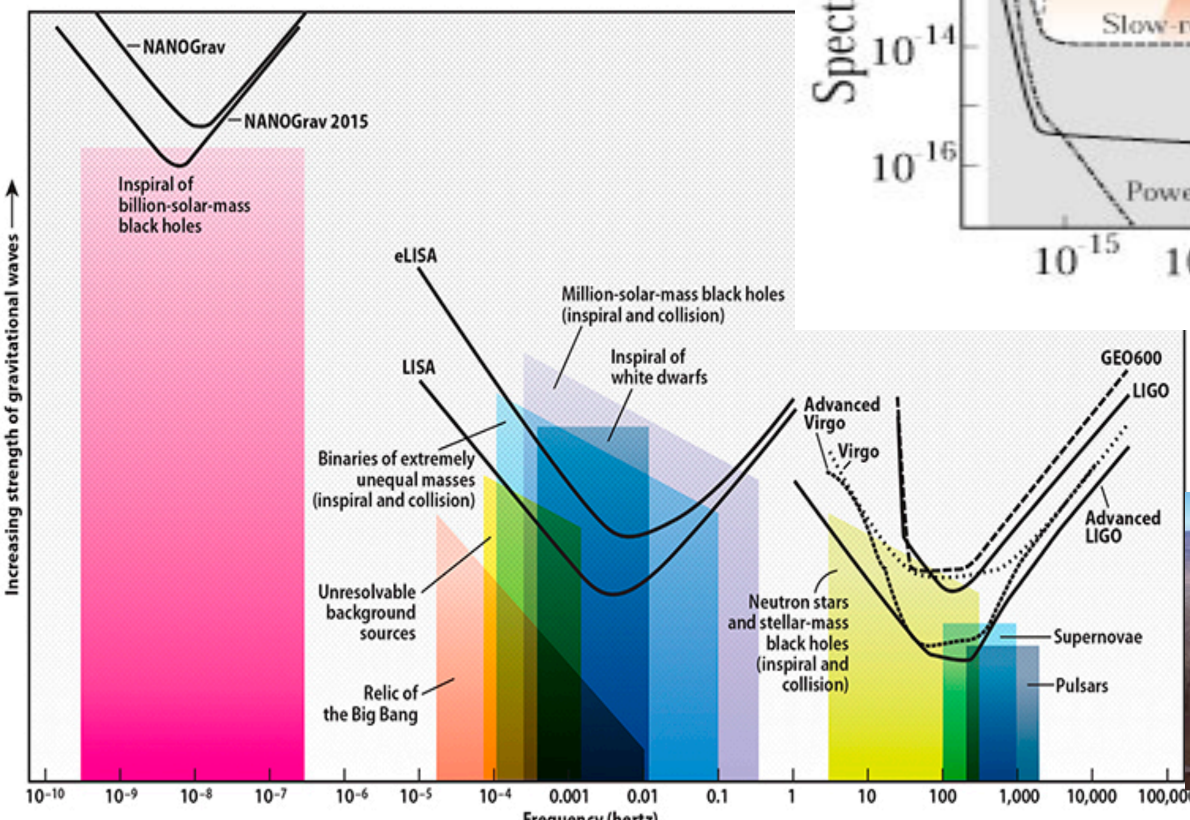
Cosmological sources for gravitational waves



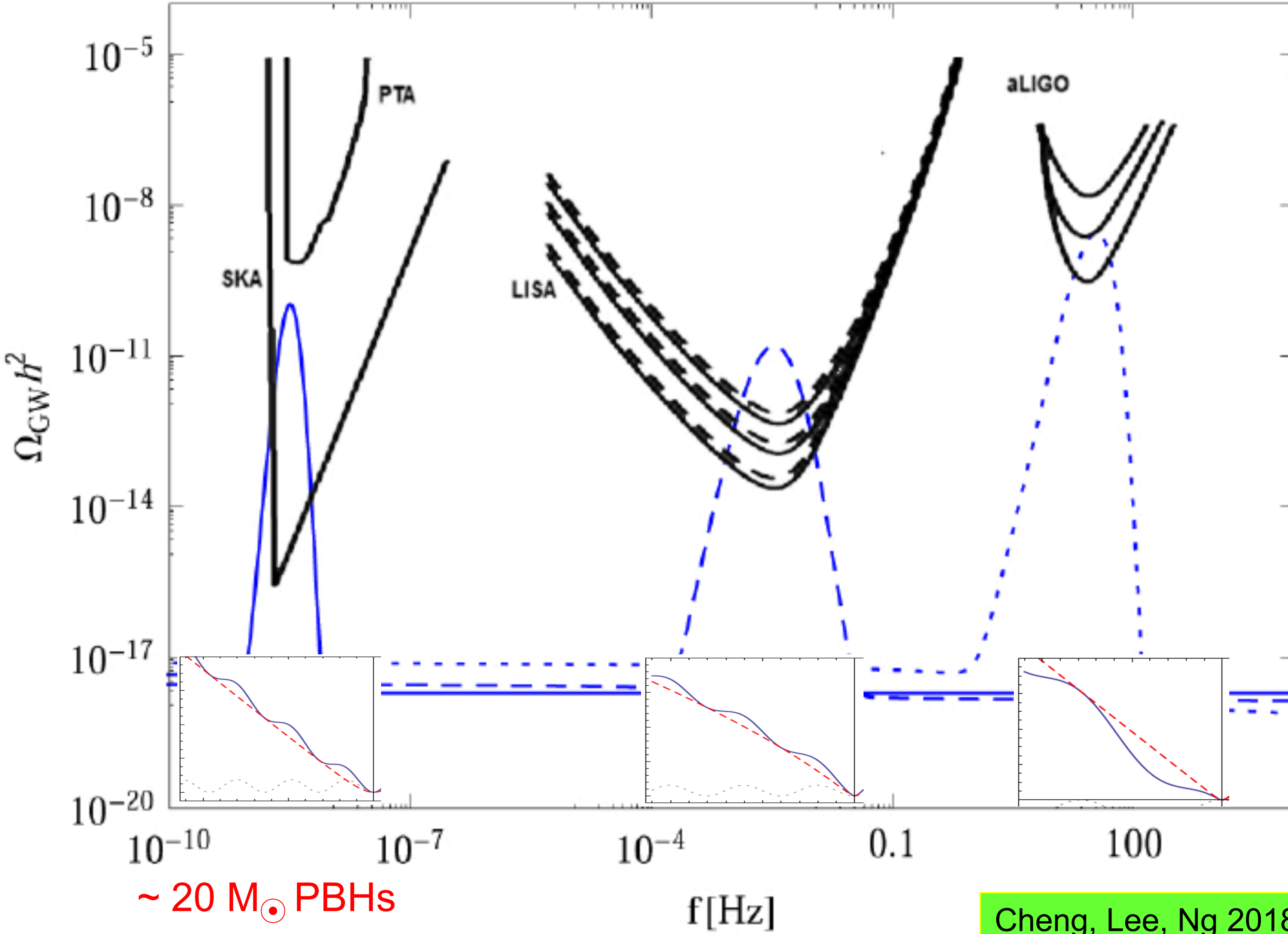




Astrophysical sources



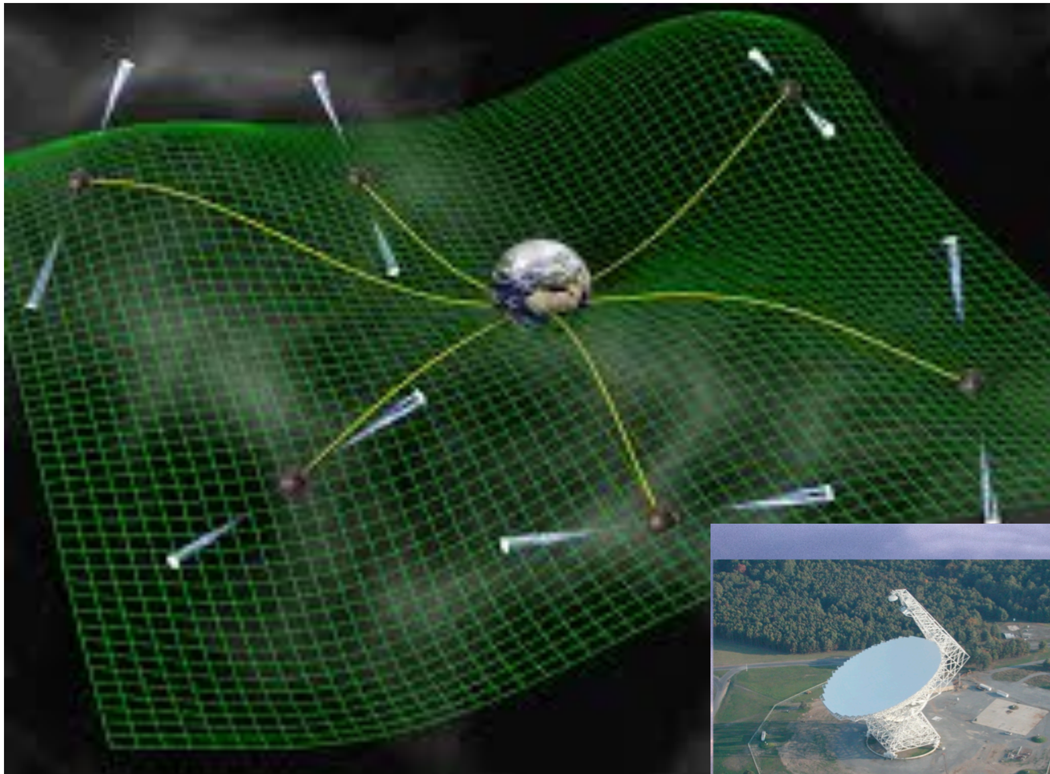
GWs associated with the formation of primordial black holes in axion inflation



~ 20 $M_\odot$ PBHs

Cheng, Lee, Ng 2018

Current Pulsar Timing Arrays



Nano-Hz GWs cause small correlated changes to the times of arrival of radio pulses from millisecond pulsars

International Pulsar Timing Arrays

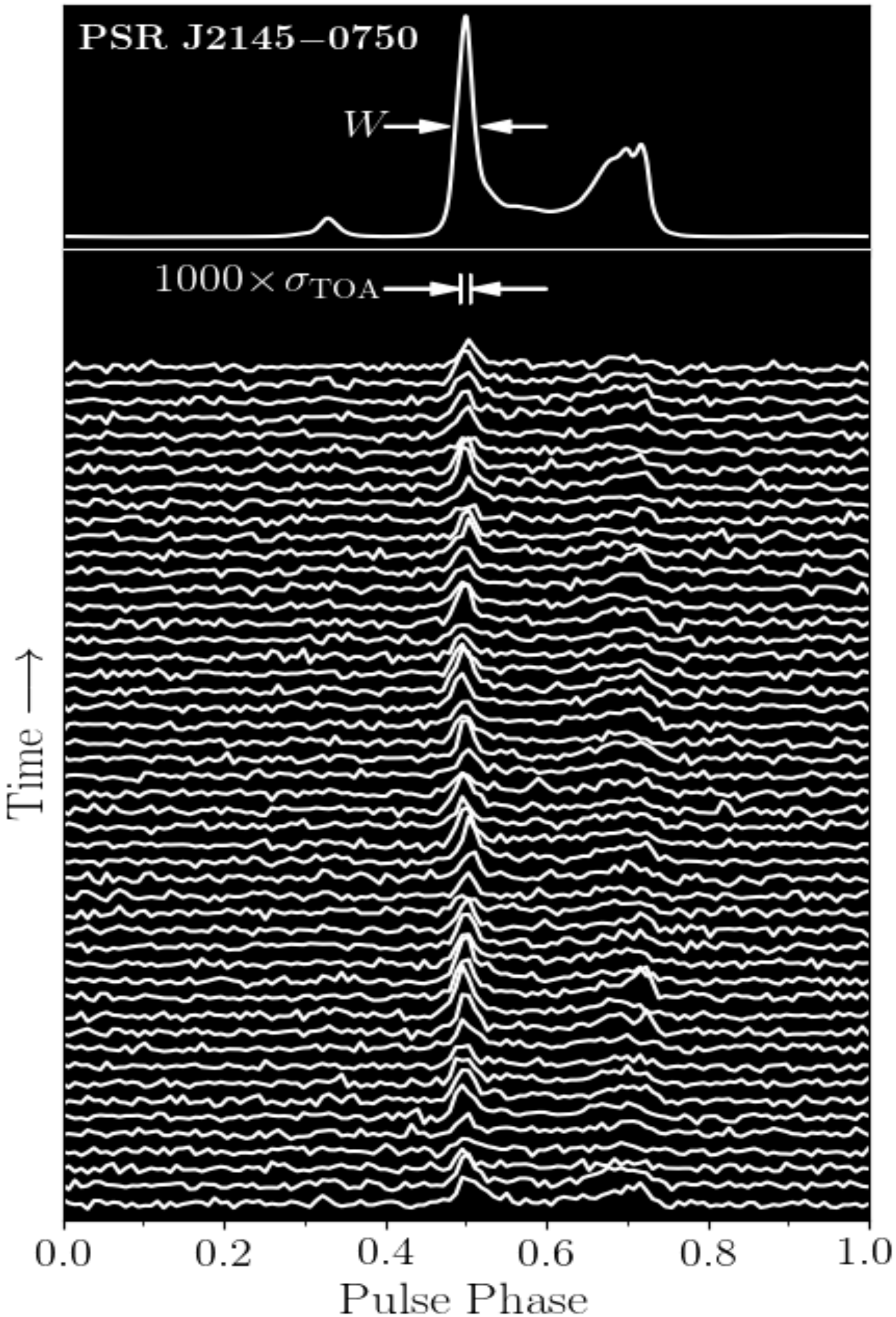


Image: Jitter in the NANOGrav pulsar J2145-0750. Each pulse is actually made of 10 individual pulses for clarity, but one can see that even these averages "jitter" around compared to the very stable average pulse at top. Reproduced from "Science with the Next-Generation VLA and Pulsar Timing Arrays"

The "miracle" of pulsar timing relies on the fact that the average pulse shape of pulsars is very stable. Over the course of decades, we can therefore determine the time of arrival (TOA) at our telescopes very precisely.

PSR J2145-0750
Period: 16.5ms
TOA Uncertainty: 0.229μs

North American Nanohertz Observatory for Gravitational Waves

North American Nanohertz Observatory for Gravitational Waves

A National Science Foundation Physics Frontiers Center

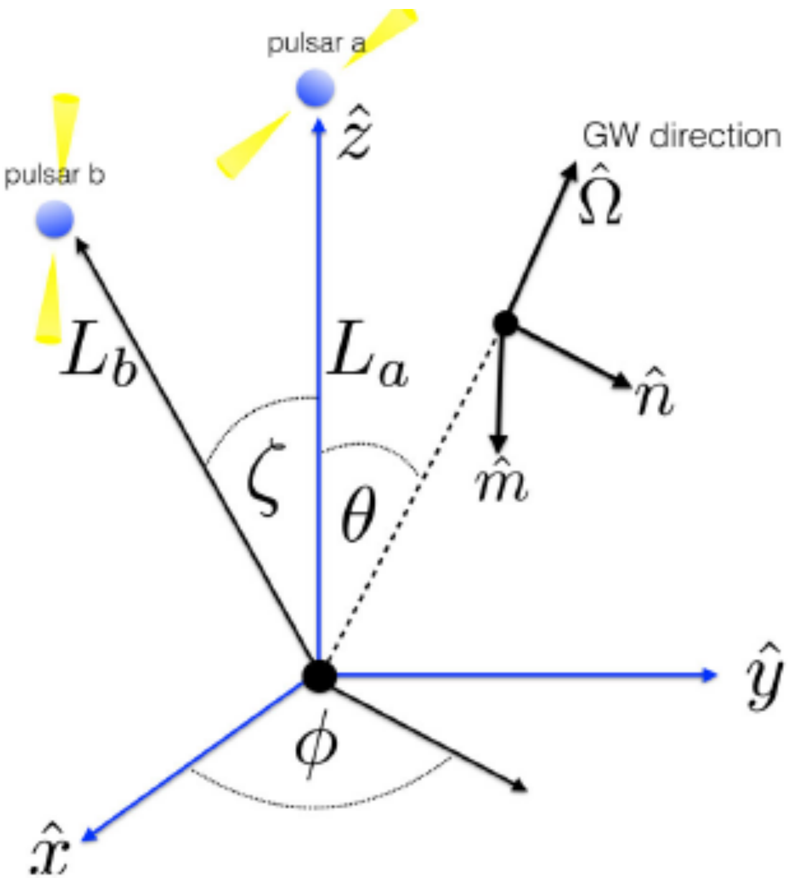
NANOGrav Physics Frontiers Center

The NANOGrav 12.5-year Data Set: Search For An Isotropic Stochastic Gravitational-Wave Background arXiv:2009.04496 9 Sep 2020

We search for an isotropic stochastic gravitational-wave background (GWB) in the 12.5-year pulsar timing data set collected by the North American Nanohertz Observatory for Gravitational Waves (NANOGrav). Our analysis finds strong evidence of a stochastic process, modeled as a power-law, with common amplitude and spectral slope across pulsars. The Bayesian posterior of the amplitude for a $f^{-2/3}$ power-law spectrum, expressed as characteristic GW strain, has median 1.92×10^{-15} and 5%–95% quantiles of 1.37–2.67×10^{-15} at a reference frequency of $f_{\rm yr} = 1\ \rm yr^{-1}$. The Bayes factor in favor of the common-spectrum process versus independent red-noise processes in each pulsar exceeds $10,000$. However, we find no statistically significant evidence that this process has quadrupolar spatial correlations, which we would consider necessary to claim a GWB detection consistent with General Relativity. We find that the process has neither monopolar nor dipolar correlations, which may arise from, for example, reference clock or solar-system ephemeris systematics, respectively. The amplitude posterior has significant support above previously reported upper limits; we explain this in terms of the Bayesian priors assumed for intrinsic pulsar red noise. We examine potential implications for the supermassive black hole binary population under the hypothesis that the signal is indeed astrophysical in nature.

$$\boxed{\Omega_{\rm GW} h^2 = \frac{2\pi^2}{3} h_c^2 \left(\frac{f_{\rm yr}}{100\,{\rm km\,s^{-1}Mpc^{-1}}}\right)^2 \simeq 2.3 \times 10^{-9}}$$

Pulsar Timing – MSPs are precise clocks



$$z(t,\hat{\Omega}) \equiv \frac{\nu(t)-\nu_0}{\nu_0} = \frac{1}{2}\frac{\hat{p}^i\hat{p}^j}{1+\hat{\Omega}\cdot\hat{p}}\Delta h_{ij}(t,\hat{\Omega}),$$

Shapiro time delay

$$\Delta h_{ij}(t,\hat{\Omega}) \equiv h_{ij}(t,\hat{\Omega}) - h_{ij}(t_{\mathrm{p}},\hat{\Omega})$$

$$z(t) = \int d\hat{\Omega} z(t,\hat{\Omega})$$

pulsar residual

$$r(t) = \int^t dt' z(t')$$

$$\langle r_a^*(t_j) r_b(t_k)\rangle = \left\langle \int^{t_j} dt' \int^{t_k} dt'' z_a^*(t') z_b(t'') \right\rangle$$

$$\langle r_a^*(t_j) r_b(t_k) \rangle = \int^{t_j} dt' \int^{t_k} dt'' \int_{-\infty}^{+\infty} df\, e^{-i2\pi f(t'-t'')} H(f)^{(ab)}\Gamma(f)$$

$$\langle h_A^*(f,\hat{\Omega}) h_{A'}(f',\hat{\Omega}') \rangle = \delta^2(\hat{\Omega},\hat{\Omega}')\delta_{AA'}\delta(f-f')H(f)P(\hat{\Omega})$$

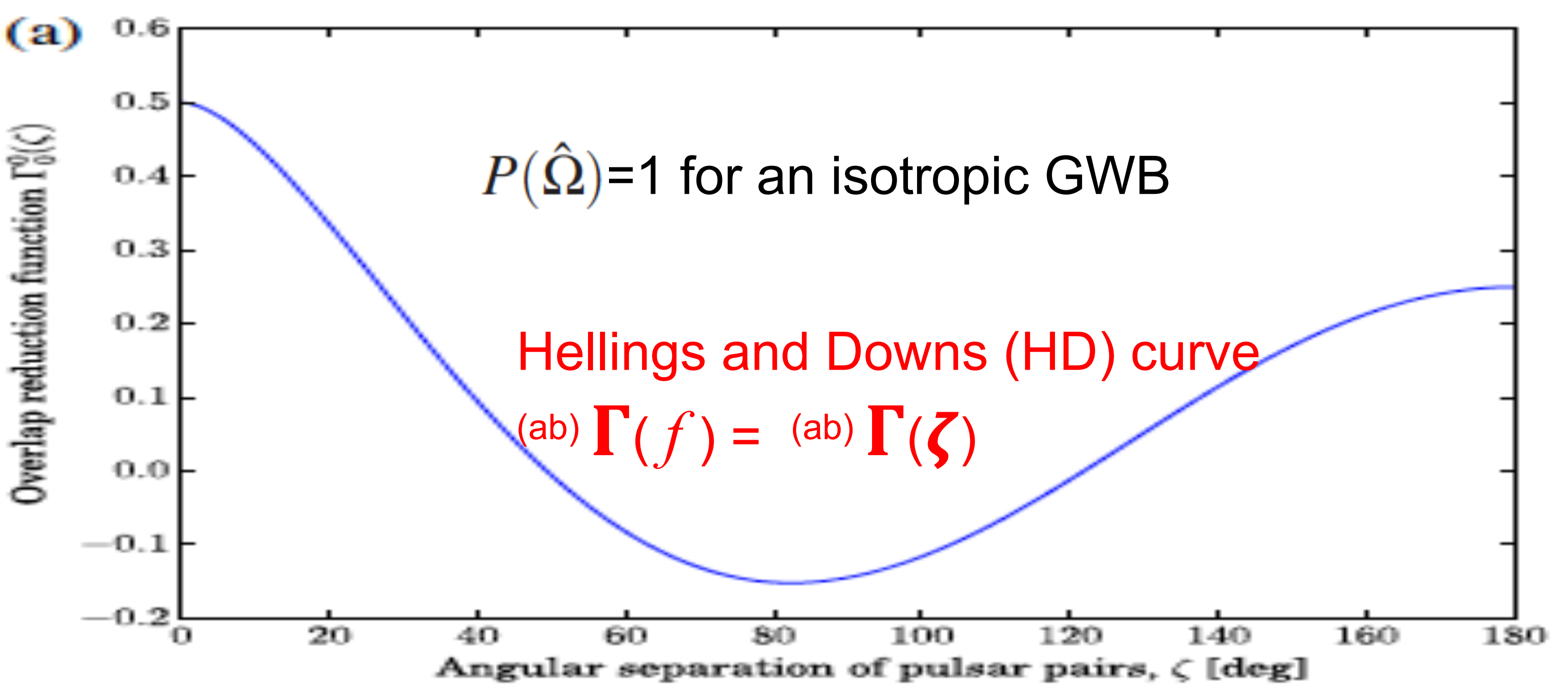
Overlap Reduction Function

$$^{(ab)}\Gamma(f) \equiv \int d\hat{\Omega}\, P(\hat{\Omega})\kappa_{ab}(f,\hat{\Omega})\left[\sum_A F_a^A(\hat{\Omega}) F_b^A(\hat{\Omega})\right],$$

$$\kappa_{ab}(f,\hat{\Omega}) \equiv [1 - e^{i2\pi f L_a(1+\hat{\Omega}\cdot\hat{p}_a)}][1 - e^{-i2\pi f L_b(1+\hat{\Omega}\cdot\hat{p}_b)}], \qquad F^A(\hat{\Omega}) = \left[\frac{1}{2}\frac{\hat{p}^i\hat{p}^j}{1+\hat{\Omega}\cdot\hat{p}} e_{ij}^A(\hat{\Omega})\right]$$

Earth term

pulsar term $fL >> 1$



The NANOGrav 12.5 year data set: 45 MSPs

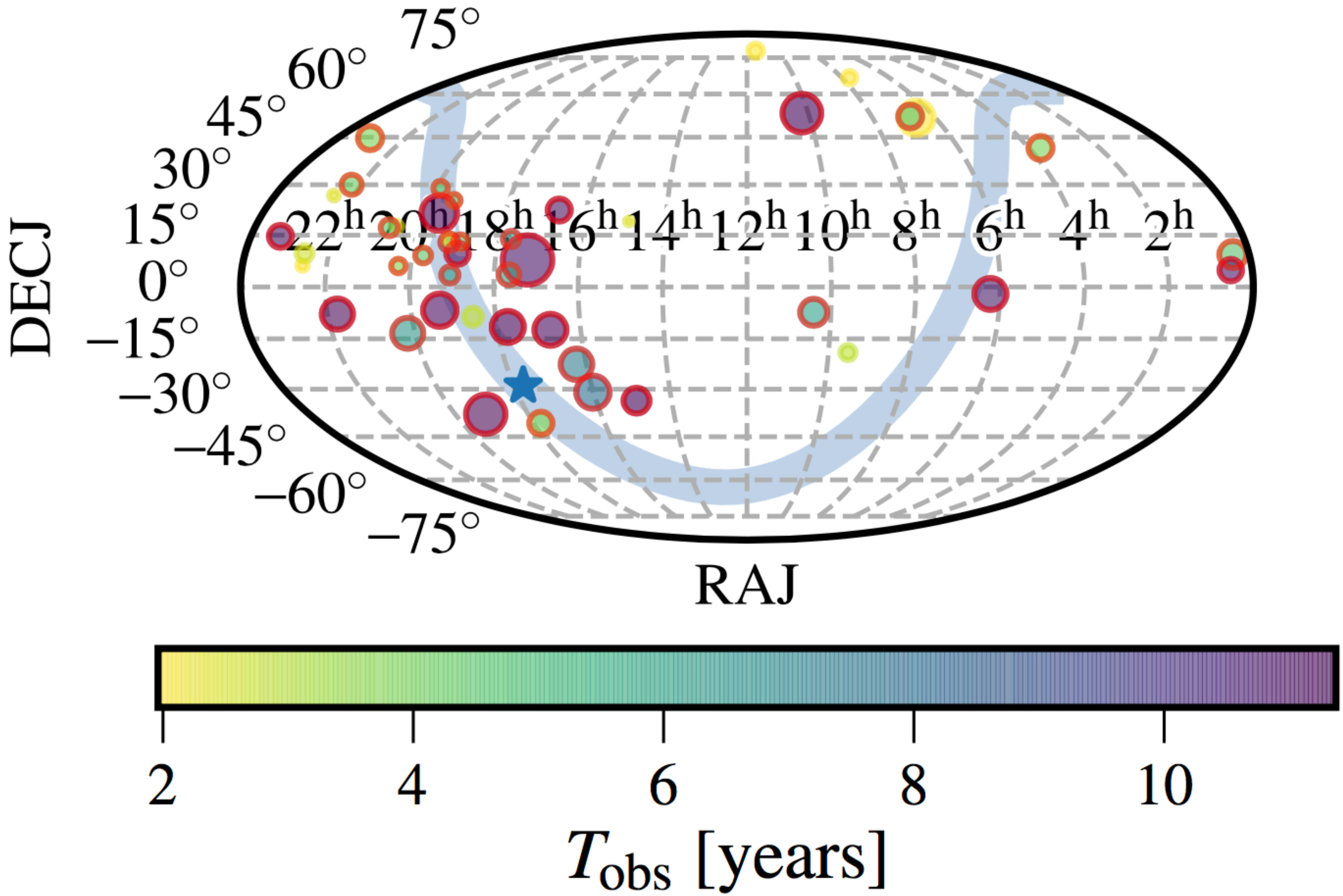


Image: sky map of NANOGrav pulsars in the 11-yr data set

Signal

$$h_c(f) = A_{\mathrm{GWB}} \left(\frac{f}{f_{\mathrm{yr}}}\right)^{\alpha} \qquad \alpha = -2/3 \text{ for a population of inspiraling SMBHBs}$$

Pulsar residual

$$\langle r_a^*(t_j) r_b(t_k) \rangle = S_{ab}(f) = \Gamma_{ab} \frac{{A_{\mathrm{GWB}}}^2}{12\pi^2} \left(\frac{f}{f_{\mathrm{yr}}}\right)^{-\gamma} f_{\mathrm{yr}}^{-3}. \qquad \boxed{\gamma = 3 - 2\alpha}$$

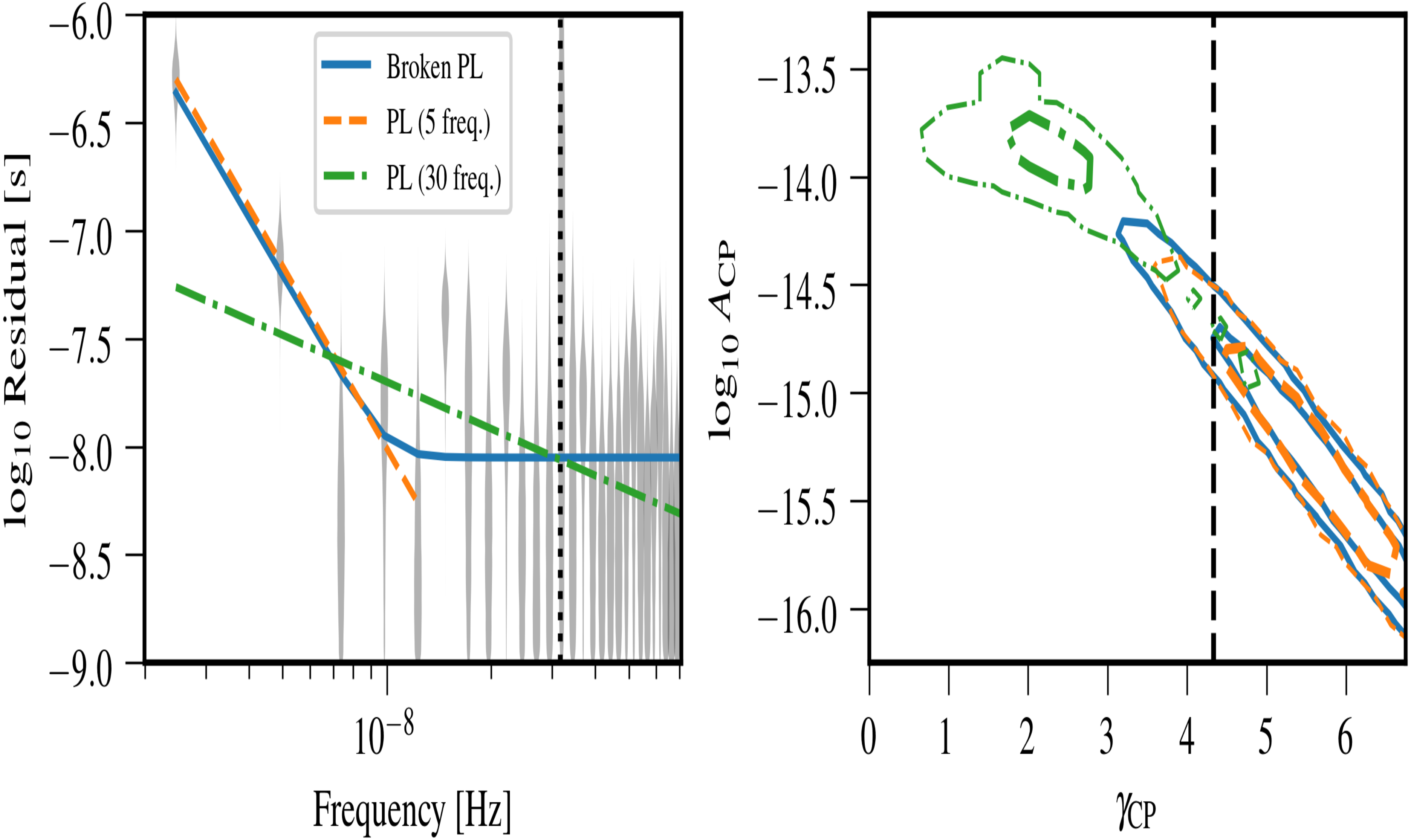
- Monopolar ORF $\Gamma_{ab}=1$ (*due to clock error*)
- Dipolar ORF $\Gamma_{ab}= \cos\zeta$ (*due to error in solar system ephemeris*)
- Quadupolar ORF Γ_{ab} = HD curve (*genuine GWB signal*)

Noise

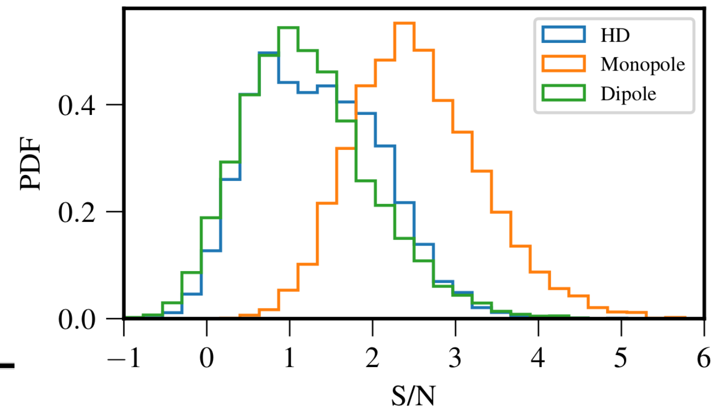
white noises (instrumental) + pulsar intrinsic red noise (including pulsar spin noise, pulsar profile changes, dispersion measure variations,…)

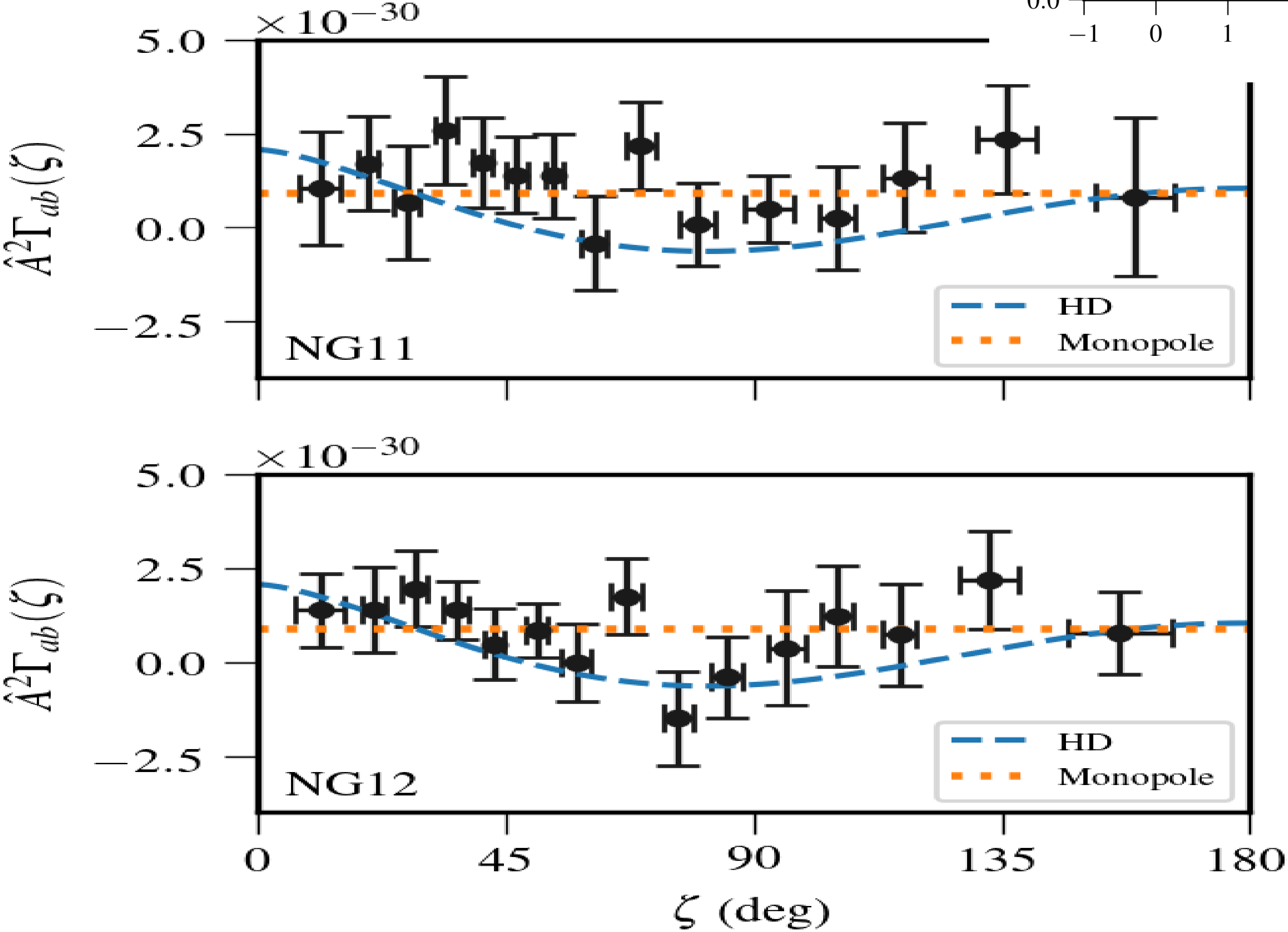
$$N_{aa}(f) = {A_{red}}^2 \,(\, f / f_{yr})^{-\gamma}\; {f_{yr}}^{-3}$$

common-spectrum process across MSPs $S_{aa}(f)$



Spatial correlation $S_{ab}(f)$

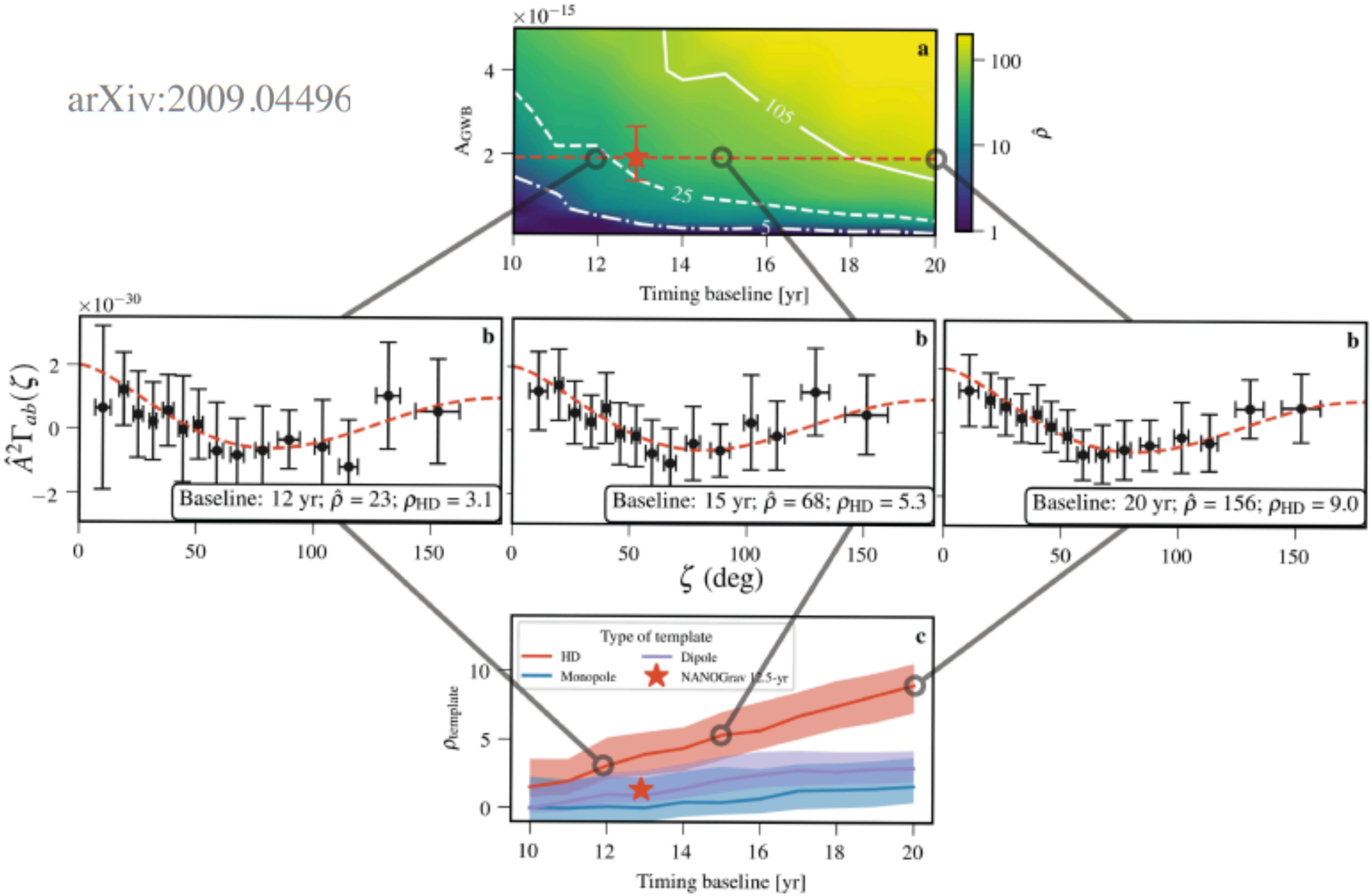




Perspectives

NANOGrav projection

arXiv:2009.04496



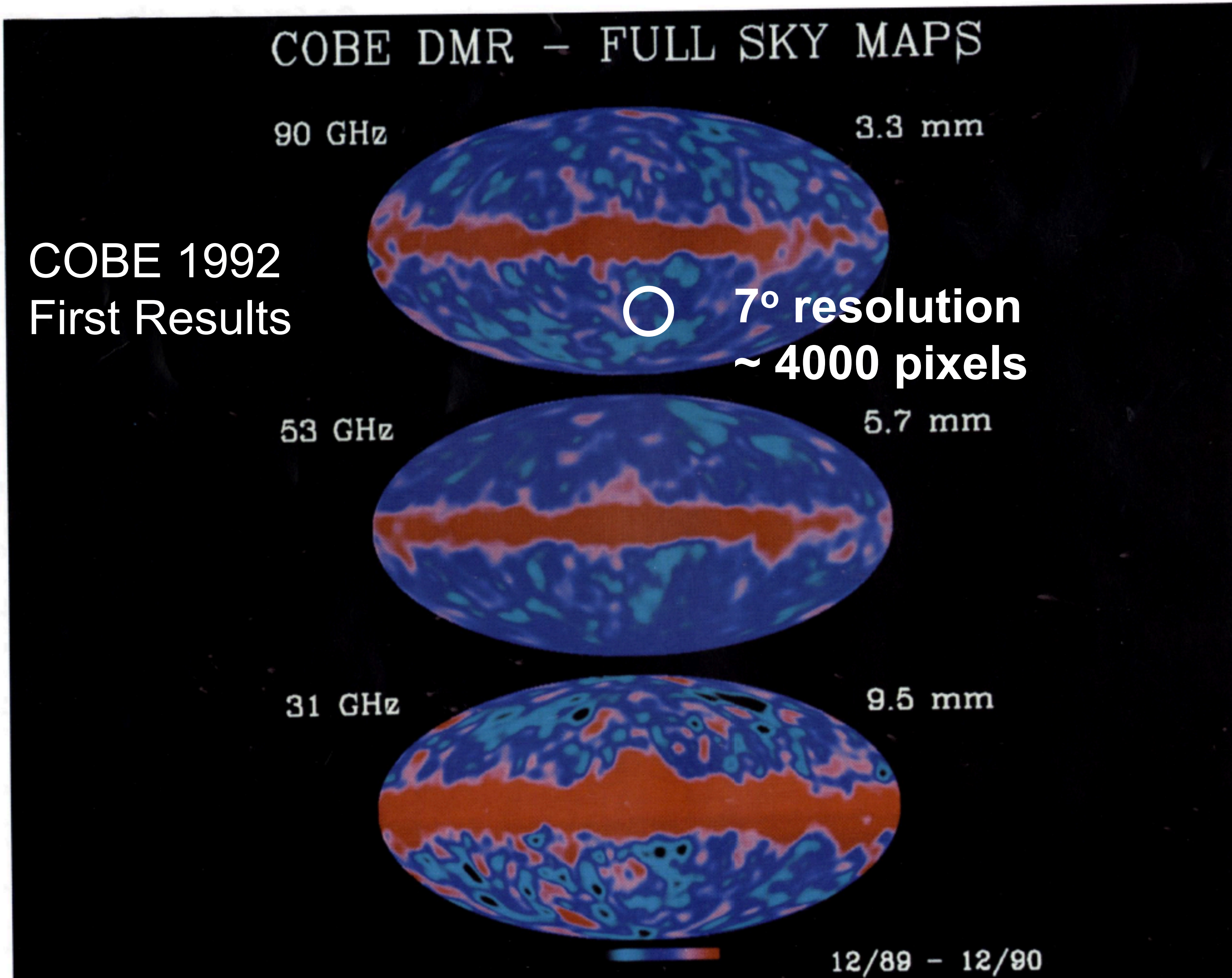


FIG. 1b

SMOOT et al. (see 396, L2)

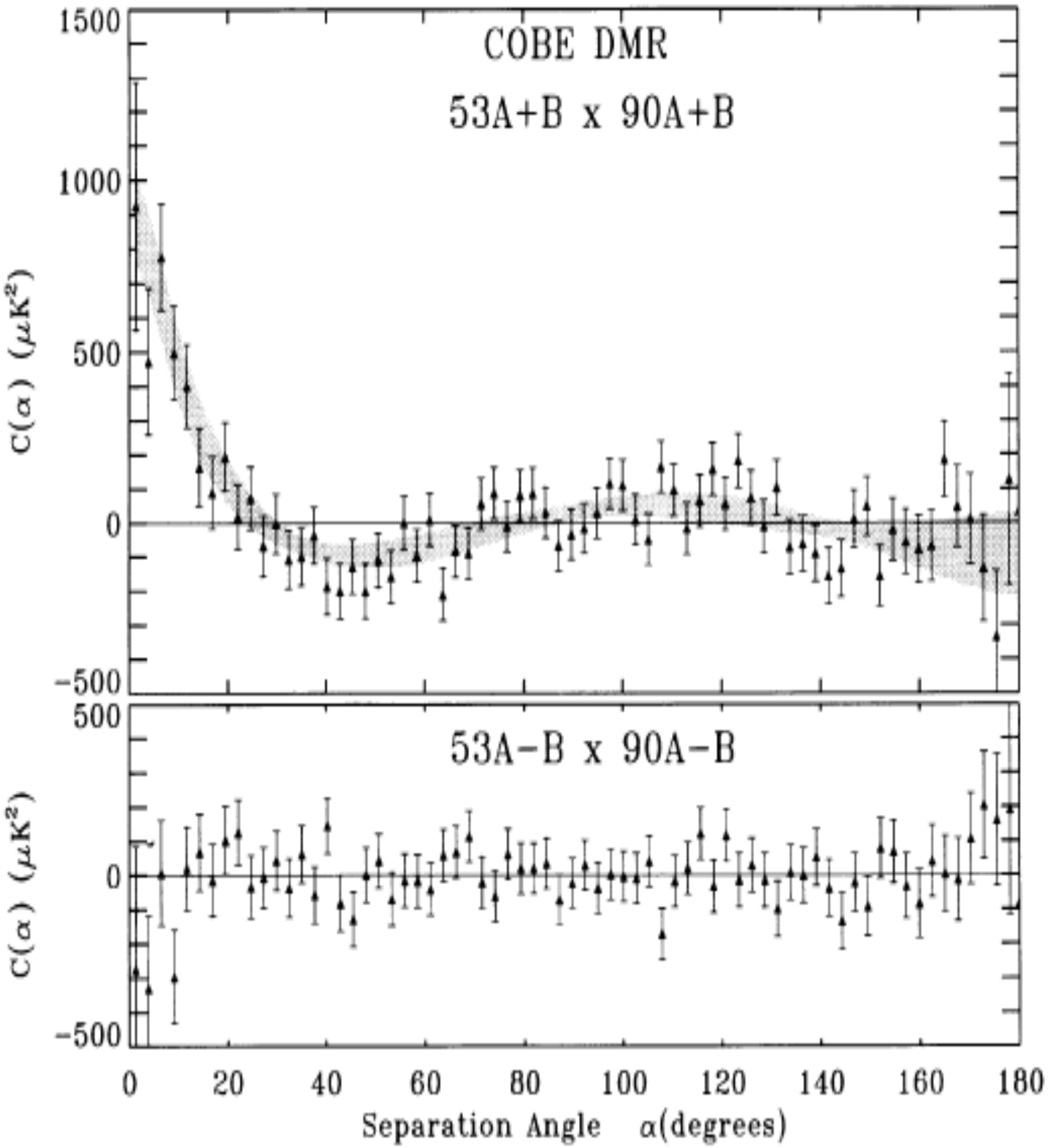
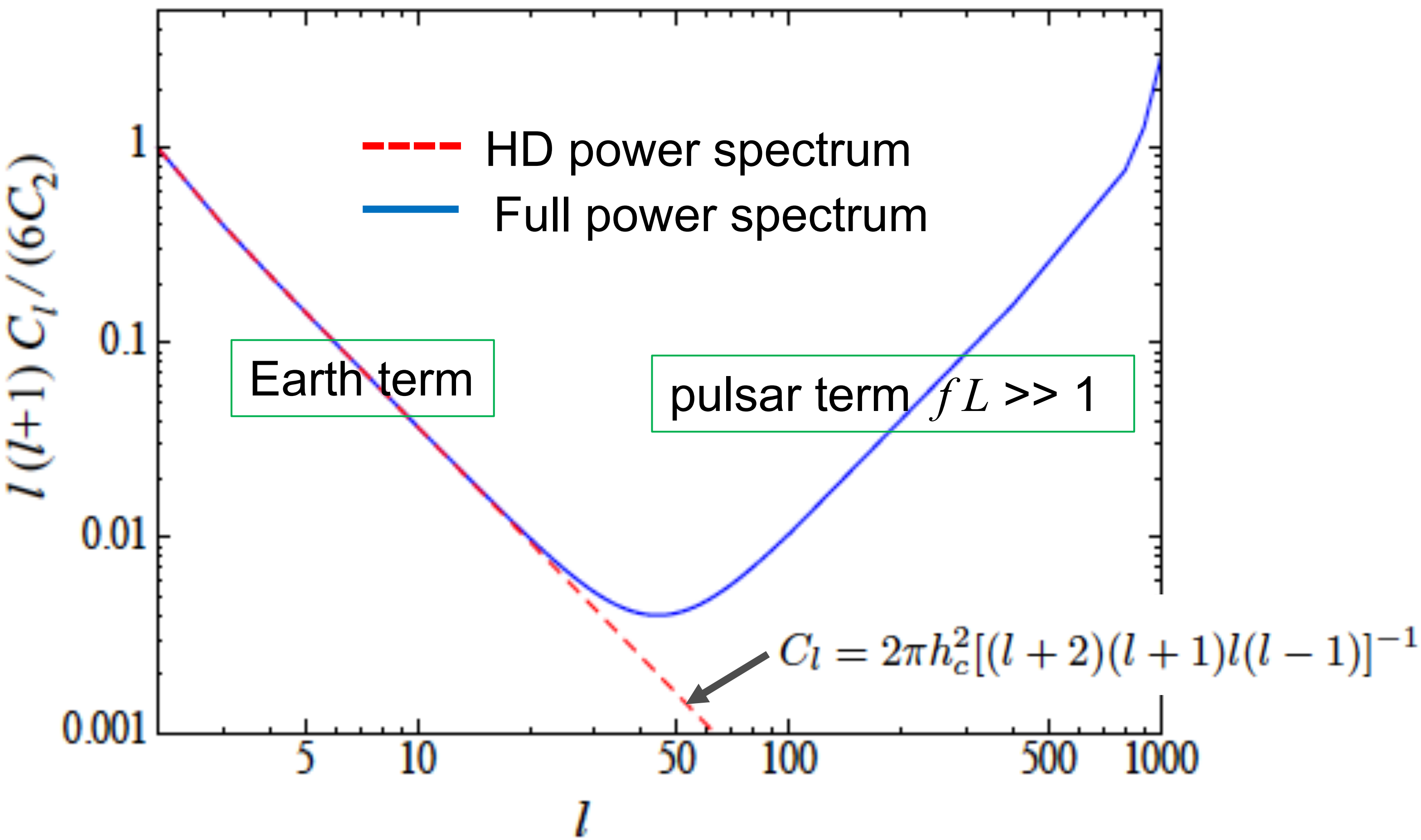


FIG. 3.—Cross-correlation of 53 GHz with 90 GHz for $|b| > 20^\circ$ plus the correlation function for a scale-invariant spectrum with an expected quadrupole amplitude of 15.4 μK: the gray band indicates 68% C.L. cosmic variations. *Top* is for the sum maps, and *bottom* is for the difference maps. The cross and autocorrelations for the various combination of maps all have consistent values.

Power spectrum of the correlation (l-space)

$$\langle r(t_1)r(t_2)\rangle = \int_0^{t_1} dt' \int_0^{t_2} dt'' \langle z(t')z(t'')\rangle$$
$$= \int_0^{t_1} dt' \int_0^{t_2} dt'' e^{-ik_*(t'-t'')} \langle z(\mathbf{e_1})z(\mathbf{e_2})\rangle$$

$$\langle z(\mathbf{e_1})z(\mathbf{e_2})\rangle = \sum_l \frac{2l+1}{4\pi} C_l P_l(\mathrm{e}_1 \cdot \mathrm{e}_2)$$



Extragalactic pulsars at redshift $\bar{z}$

$$\langle \delta z(\mathbf{e_1}) \delta z(\mathbf{e_2}) \rangle = (1+\bar{z})^2 \sum_l \frac{2l+1}{4\pi} C_l P_l(\mathbf{e}_1 \cdot \mathbf{e}_2)$$

Sachs-Wolfe effect

$$C_l = \frac{1}{2\pi}(l+2)(l+1)l(l-1) \times \int_0^\infty \frac{dk}{k} \left| \int_{\eta_e}^{\eta_r} d\eta \frac{dh(k\eta)}{d\eta} \frac{j_l[k(\eta_r - \eta)]}{k^2(\eta_r - \eta)^2} \right|^2$$

Assume GW of k_* have amplitude H_* at time η_*

$$C_l \simeq \frac{2H_*^2}{\pi M_p^2} \left(\frac{\eta_*}{\eta_0} \right)^4 \left[\frac{1}{(l+2)(l+1)l(l-1)} + \frac{1}{x_0^2} \frac{(l+2)(l-1)}{(l+1)l} \right]$$

$$x_0 = k_* \eta_0$$

η_0 is the present time

Anisotropic polarized SGWB

$$\langle r(t_a)r(t_b)\rangle = \int_{-\infty}^{\infty} \frac{\mathrm{d}f}{f^2}\,(1-e^{-2\pi i f t_a})(1-e^{2\pi i f t_b}) \sum_{L_1 M_1 L_2 M_2} i^{L_1}(-i)^{L_2} J_{L_1,L_2}(x_a,x_b) \times$$

$$Y_{L_1 M_1}(\hat{e}_a) Y^*_{L_2 M_2}(\hat{e}_b) \int_{S^2} \mathrm{d}\hat{k}\, R(f,\hat{k},\hat{e}_a,\hat{e}_b)\, Y^*_{L_1 M_1}(\hat{k}) Y_{L_2 M_2}(\hat{k})\,,$$

$$\boxed{x_a = 2\pi f L_a/c,\ x_b = 2\pi f L_b/c,}$$

$$\boxed{J_{L_1,L_2}(x_a,x_b) = \int_0^{x_a} \mathrm{d}x\, e^{ix} j_{L_1}(x) \int_0^{x_b} \mathrm{d}x\, e^{-ix} j_{L_2}(x)}$$

$$R(f,\hat{k},\hat{e}_a,\hat{e}_b) = \sum_{AA'} P_{AA'}(f,\hat{k})\, d_a^{ij} d_b^{kl}\, \mathrm{e}^{A}_{ij}(\hat{k}) \mathrm{e}^{*A'}_{kl}(\hat{k})$$

$$\boxed{\begin{aligned}
\mathbb{E}^{I}_{ijkl}(\hat{k}) &= \mathrm{e}^{R}_{ij}(\hat{k})\mathrm{e}^{*R}_{kl}(\hat{k}) + \mathrm{e}^{L}_{ij}(\hat{k})\mathrm{e}^{*L}_{kl}(\hat{k})\,,\\
\mathbb{E}^{V}_{ijkl}(\hat{k}) &= \mathrm{e}^{R}_{ij}(\hat{k})\mathrm{e}^{*R}_{kl}(\hat{k}) - \mathrm{e}^{L}_{ij}(\hat{k})\mathrm{e}^{*L}_{kl}(\hat{k})\,,\\
\mathbb{E}^{Q+iU}_{ijkl}(\hat{k}) &= \mathrm{e}^{L}_{ij}(\hat{k})\mathrm{e}^{*R}_{kl}(\hat{k})\,,\\
\mathbb{E}^{Q-iU}_{ijkl}(\hat{k}) &= \mathrm{e}^{R}_{ij}(\hat{k})\mathrm{e}^{*L}_{kl}(\hat{k})\,,
\end{aligned}}$$

$$\boxed{{}^{ijkl}\mathbb{D}_{ab} = d_a^{ij} d_b^{kl}\,,}$$

$$R(f,\hat{k},\hat{e}_a,\hat{e}_b) = \sum_{I=\{I,V,Q\pm iU\}} I(f,\hat{k})\, {}^{ijkl}\mathbb{D}_{ab}\, \mathbb{E}^{I}_{ijkl}(\hat{k})$$

$$\boxed{\begin{aligned} I(f,\hat{k}) &= \sum_{\ell m} I_{\ell m}(f)\, Y_{\ell m}(\hat{k})\,, \\ V(f,\hat{k}) &= \sum_{\ell m} V_{\ell m}(f)\, Y_{\ell m}(\hat{k})\,, \\ (Q+iU)(f,\hat{k}) &= \sum_{\ell m} (Q+iU)_{\ell m}(f)\ {}_{+4}Y_{\ell m}(\hat{k})\,, \\ (Q-iU)(f,\hat{k}) &= \sum_{\ell m} (Q-iU)_{\ell m}(f)\ {}_{-4}Y_{\ell m}(\hat{k})\,, \end{aligned}}$$

$$\boxed{\begin{aligned} \mathbb{E}^{I}_{ijkl}(\hat{k}) &= \sum_{\ell_e m_e} {}^{ijkl}\mathbb{E}^{I}_{\ell_e m_e} Y_{\ell_e m_e}(\hat{k})\,, \\ \mathbb{E}^{V}_{ijkl}(\hat{k}) &= \sum_{\ell_e m_e} {}^{ijkl}\mathbb{E}^{V}_{\ell_e m_e} Y_{\ell_e m_e}(\hat{k})\,, \\ \mathbb{E}^{Q+iU}_{ijkl}(\hat{k}) &= \sum_{\ell_e m_e} {}^{ijkl}\mathbb{E}^{Q+iU}_{\ell_e m_e}\ {}_{-4}Y_{\ell_e m_e}(\hat{k})\,, \\ \mathbb{E}^{Q-iU}_{ijkl}(\hat{k}) &= \sum_{\ell_e m_e} {}^{ijkl}\mathbb{E}^{Q-iU}_{\ell_e m_e}\ {}_{+4}Y_{\ell_e m_e}(\hat{k})\,, \end{aligned}}$$

$$\langle r(t_a) r(t_b)\rangle = \int_{-\infty}^{\infty} \frac{\mathrm{d}f}{f^2}\,(1-e^{-2\pi i f t_a})(1-e^{2\pi i f t_b}) \sum_{I=\{I,V,Q\pm iU\}} \sum_{\ell m} I_{\ell m}(f)\gamma^{I}_{\ell m}(x_a, x_b, \hat{e}_a, \hat{e}_b)$$

Overlap Reduction Functions

$$\gamma^{I,V}_{\ell m}(x_a, x_b, \hat{e}_a, \hat{e}_b) = \sum_{L_1 M_1 L_2 M_2} i^{L_1}(-i)^{L_2} J_{L_1 L_2} Y_{L_1 M_1}(\hat{e}_a) Y^*_{L_2 M_2}(\hat{e}_b) \sum_{\ell_e m_e} \mathbb{D}\cdot\mathbb{E}^{I,V}_{\ell_e m_e} \begin{pmatrix} L_1 & L_2 & \ell_e & \ell \\ M_1 & M_2 & 0\ m_e & 0\ m \end{pmatrix}$$

$$\gamma^{Q\pm iU}_{\ell m}(x_a, x_b, \hat{e}_a, \hat{e}_b) = \sum_{L_1 M_1 L_2 M_2} i^{L_1}(-i)^{L_2} J_{L_1 L_2} Y_{L_1 M_1}(\hat{e}_a) Y^*_{L_2 M_2}(\hat{e}_b) \sum_{\ell_e m_e} \mathbb{D}\cdot\mathbb{E}^{Q\pm iU}_{\ell_e m_e} \begin{pmatrix} L_1 & L_2 & \ell_e & \ell \\ M_1 & M_2 & \mp 4\ m_e & \pm 4\ m \end{pmatrix}$$

$$\boxed{\mathbb{D}\cdot\mathbb{E} = {}^{ijkl}\mathbb{D}_{ab}\mathbb{E}_{ijkl}}$$

$$\begin{pmatrix} L_1 & L_2 & l_1 & l_2 \\ M_1 & M_2 & s_1\ m_1 & s_2\ m_2 \end{pmatrix} \equiv \int \mathrm{d}\hat{k}\ Y^*_{L_1 M_1}(\hat{k}) Y_{L_2 M_2}(\hat{k})\ {}_{s_1}Y_{l_1 m_1}(\hat{k})\ {}_{s_2}Y_{l_2 m_2}(\hat{k})$$

Thank you for your attention!