Cosmic Relic Scattering at Gravitational Wave Detectors

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NCTS Annual Theory Meeting

Mostly based on collaborations with
V. Domcke [arXiv:1703.08629], C. Ting, R. Primulando [arXiv:1906.07356] and
C.-H. Lee, C. S. Nugroho [arXiv:2007.07908]

Outline

- Introduction
- Cosmic Neutrino Background
- Dark Matter
- Summary and Conclusions

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- **Introduction**
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Cosmic Relics

- Cosmic Relics are produced in the early universe and are still around today
- Examples:
  - Cosmic Microwave Background (not the focus here)
  - Cosmic Neutrino Background (no direct observation yet)
  - Dark Matter (only seen gravitationally)

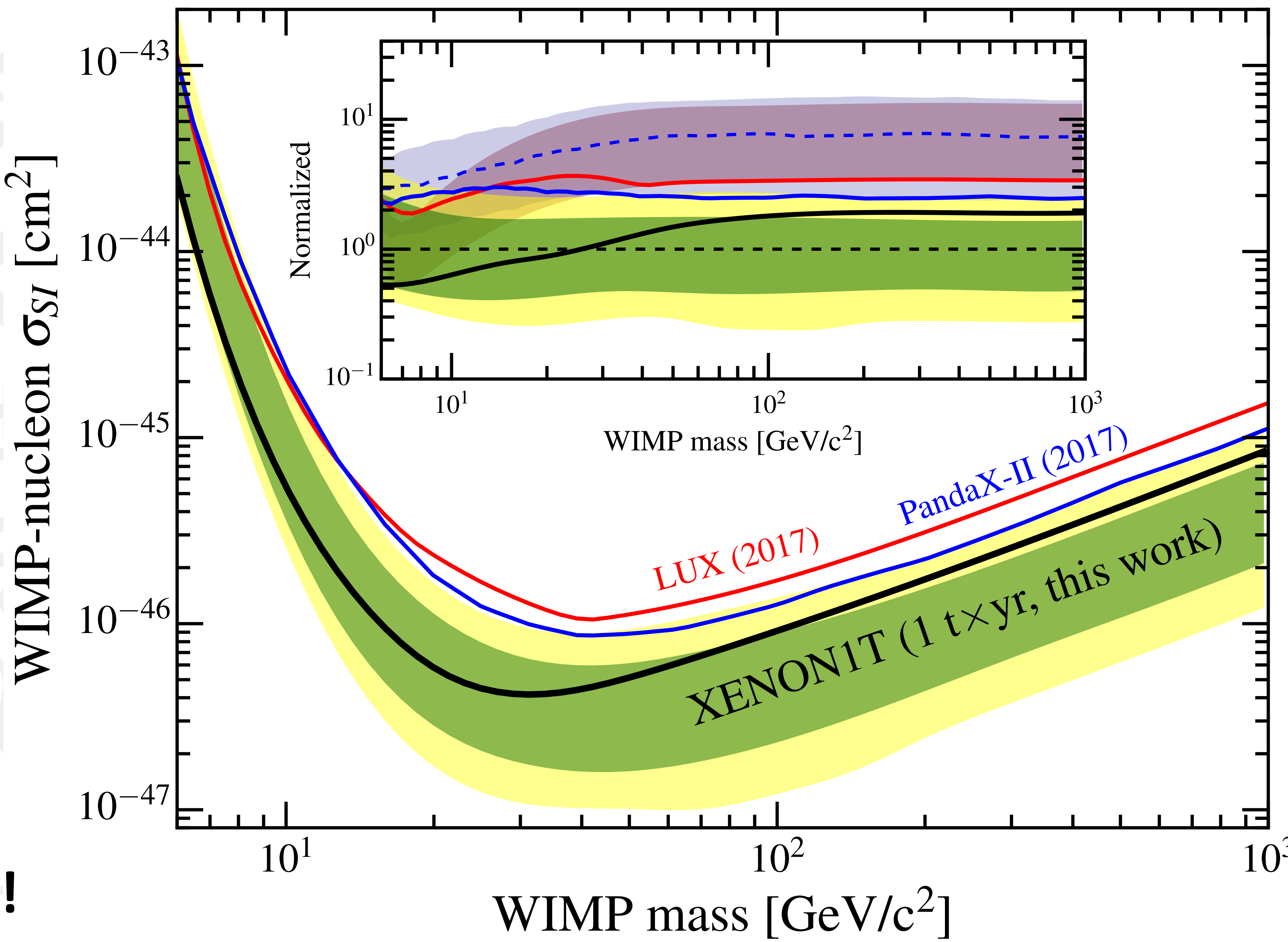
The Cosmic Neutrino Background

- Produced 1 s after Big Bang (CMB: 379k years)
- Number density: about 330 cm^{-3} = 6 n_0
- Temperature: 1.9 K
- Average kinetic energy: 0.5 meV
- Velocity: 10^{-3} - 1 c
- CNB neutron cross section: 10^{-27} pb (10^{-63} cm^2)

Cold Dark Matter

- Produced ? s after Big Bang
- Local energy density: 0.3-0.4 GeV/cm^3
- Mass: ??? (10^{-22} eV/c^2 - 10^5 $M_\odot$)
- Local peak velocity: $\sim$250 km/s
- Cross section to ordinary matter: ??? cm^2

WIMP Searches

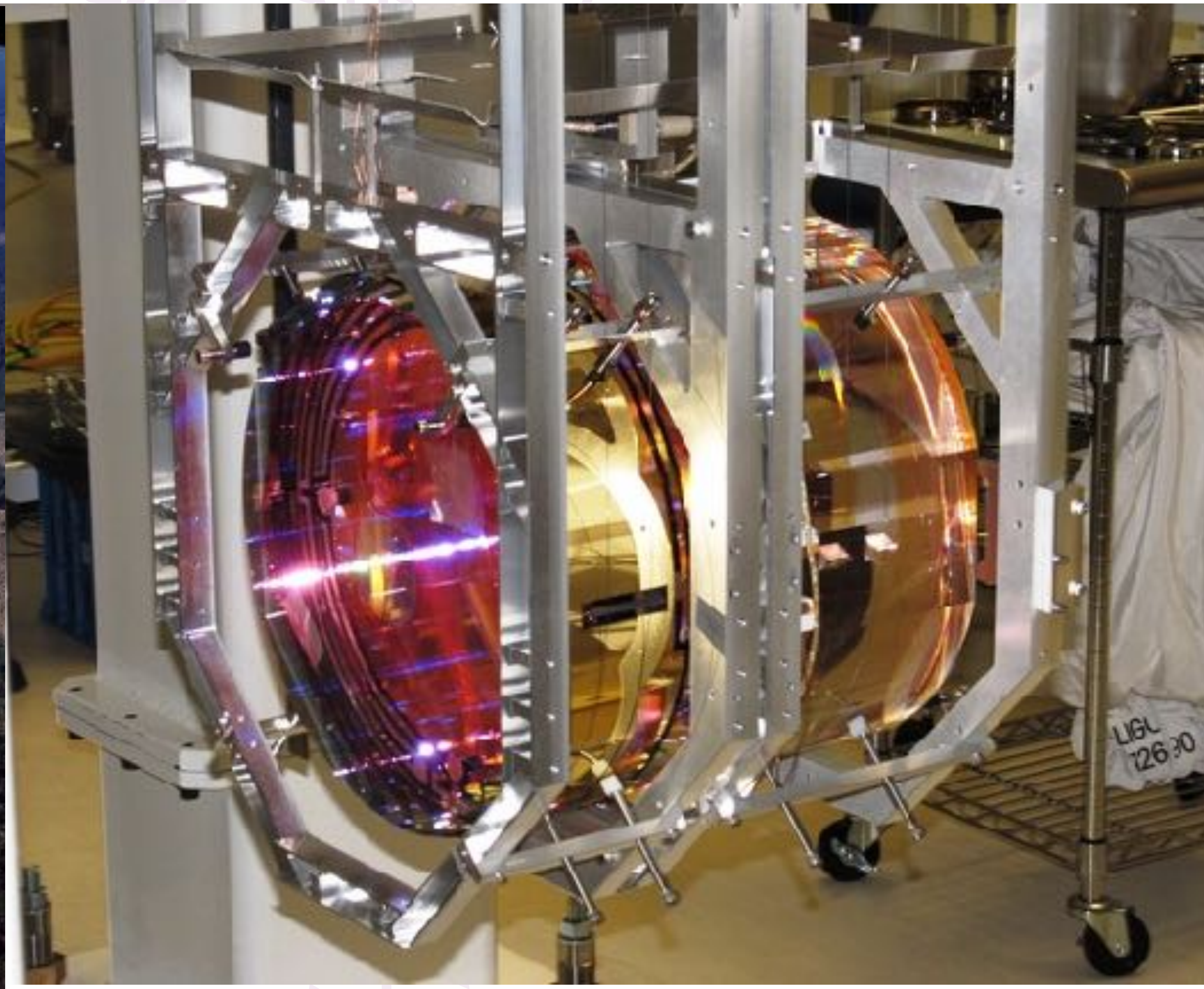


[XENON1T result '18]

Time to think again?!
Light WIMPs?

Gravitational Wave Detectors

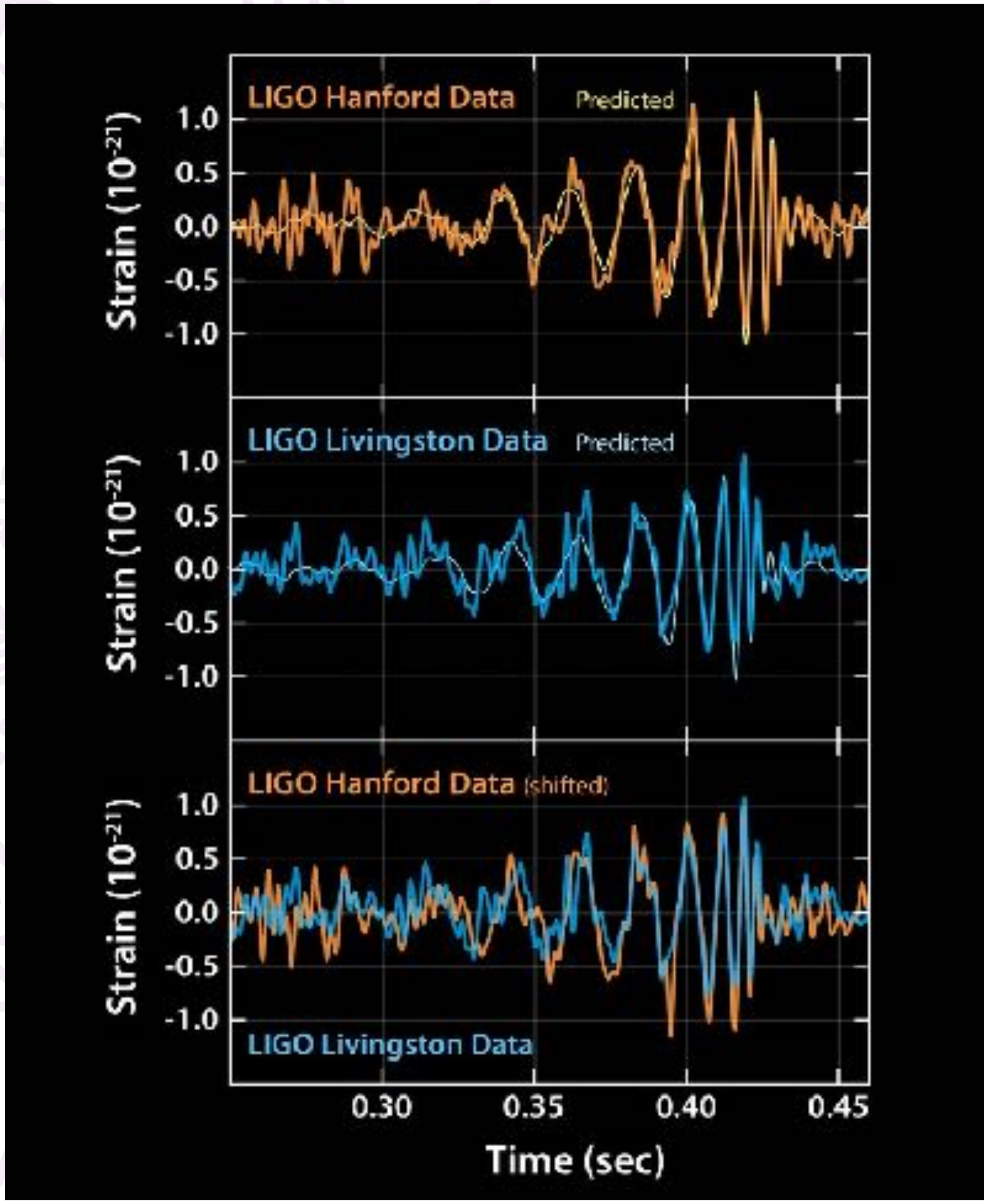
[LIGO Livingston, Courtesy Caltech/MIT/LIGO Laboratory 2016]



[LIGO test mass, Courtesy Caltech/MIT/LIGO Laboratory 2016]

Gravitational Wave Detectors

- Decade long R&D efforts
- Impressive sensitivities
- Impressive results
- Nobelprize 2017
- Other uses for this technology?



[Courtesy Caltech/MIT/LIGO Laboratory 2016]

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The Experiment

[Domcke, MS '17]

- Estimated sensitivity

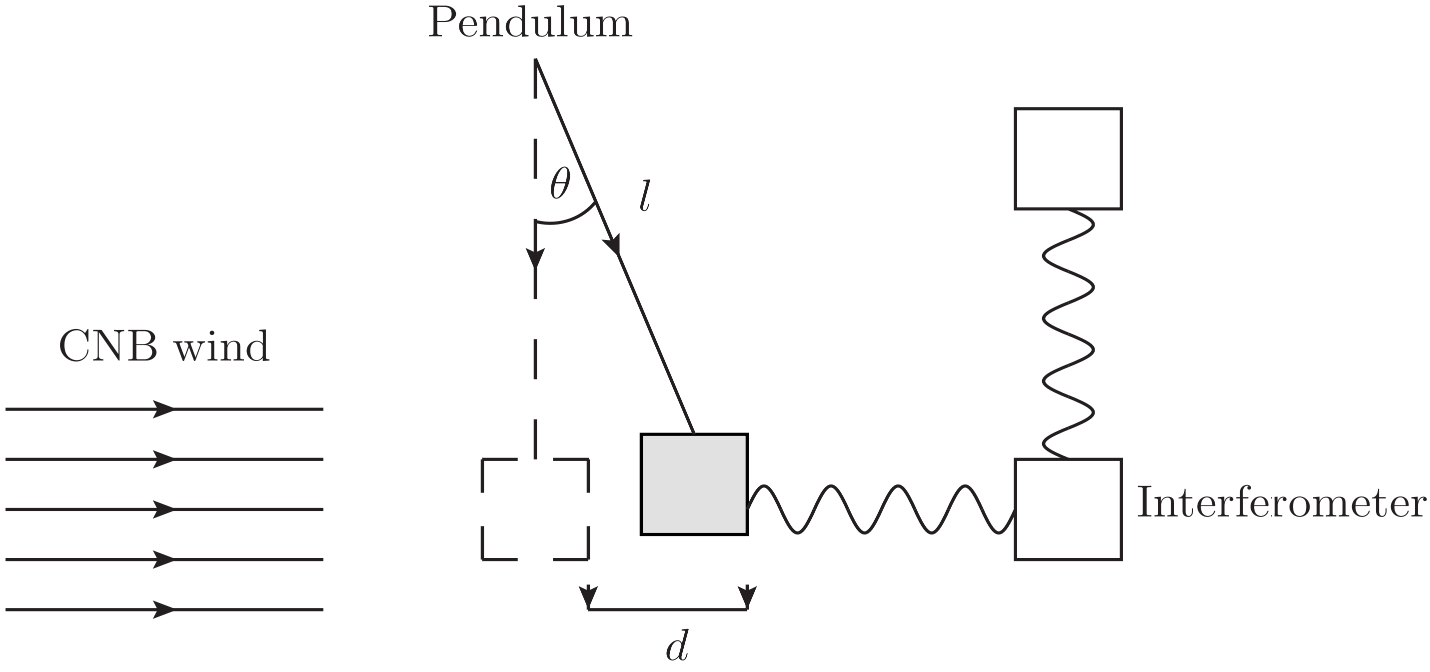
$$a \gtrsim \frac{g}{l} d_{\min}$$

- LIGO-like interferometers

$$a_{\mathrm{LIGO}} \gtrsim 10^{-16}\ \mathrm{cm/s}^2$$

- Einstein telescope maybe

$$a_{\mathrm{ET}} \gtrsim 3 \cdot 10^{-18}\ \mathrm{cm/s}^2$$



[For more general particle physics applications, see Englert, Hild, Spannowsky '17]

After some calculations...

[Domcke, MS '17; see also Duda *et al.* '01, ..., Opher '74]

- Estimate the average induced acceleration from the neutrino wind

$$a_{\mathrm{CNB}} \lesssim \mathcal{O}(10^{-27})\ \mathrm{cm/s}^2$$

- Compare to experimental sensitivity

$$a_{\mathrm{LIGO}} \gtrsim 10^{-16}\ \mathrm{cm/s}^2$$

Other "Winds"

[Domcke, MS ’17; see also Duda *et al.* '01]

- Solar neutrinos

$$a_{\mathrm{solar}-\nu} \approx 3 \cdot 10^{-26}\,\mathrm{cm/s}^2$$

- Cold WIMP Dark Matter (m_{DM} > 1 GeV/c^2)

$$a_{\mathrm{DM}} \approx 4 \cdot 10^{-30} \left(\frac{(A-Z)^2}{76\,A} \right) \left(\frac{\sigma_{X-N}}{10^{-46}\,\mathrm{cm}^2} \right) \left(\frac{\rho_{\mathrm{dark(local)}}}{10^{-24}\,\mathrm{g/cm}^3} \right) \left(\frac{\beta_X}{10^{-3}} \right)^2 \mathrm{cm/s}^2$$

- Light WIMP Dark Matter (m_{DM} = 3.3 keV/c^2)

$$a_{\mathrm{light\ DM}} \approx N_c\, a_{\mathrm{DM}} \approx 10^9\, a_{\mathrm{DM}}$$

[There is also plenty of works on ultralight bosonic DM not based on individual particle scattering, see, e.g., Arvanitaki *et al.* '15; Graham *et al.* '15; Aoki & Soda '16; Pierce *et al.* '18; Morisaki & Suyama '18; Fukuda, Matsumoto & Yanagida '18; ...]

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- **Dark Matter**
  - **Particle Physics Approach**
  - Gravitational Wave Astronomy Approach
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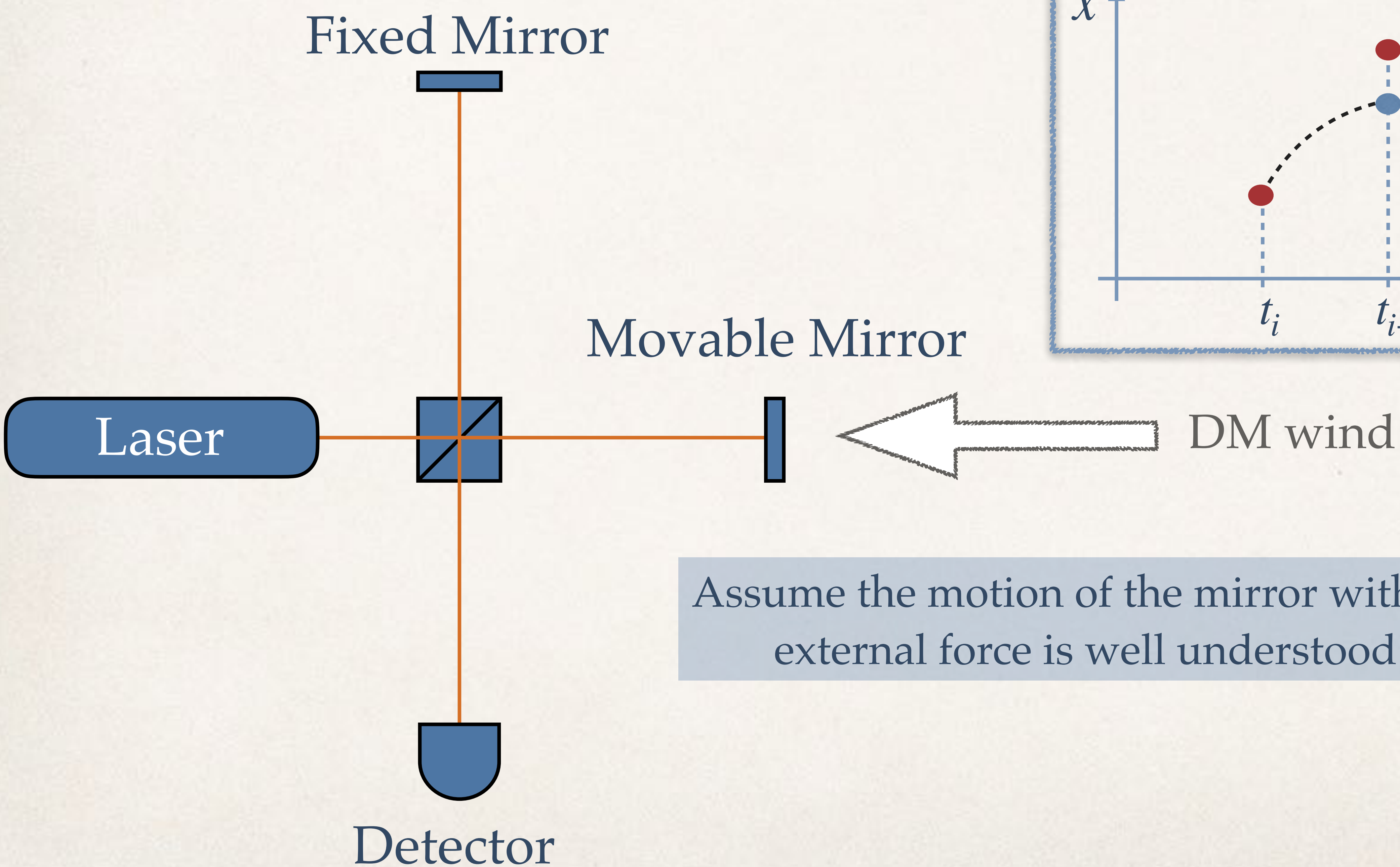
Dark Brownian Motion

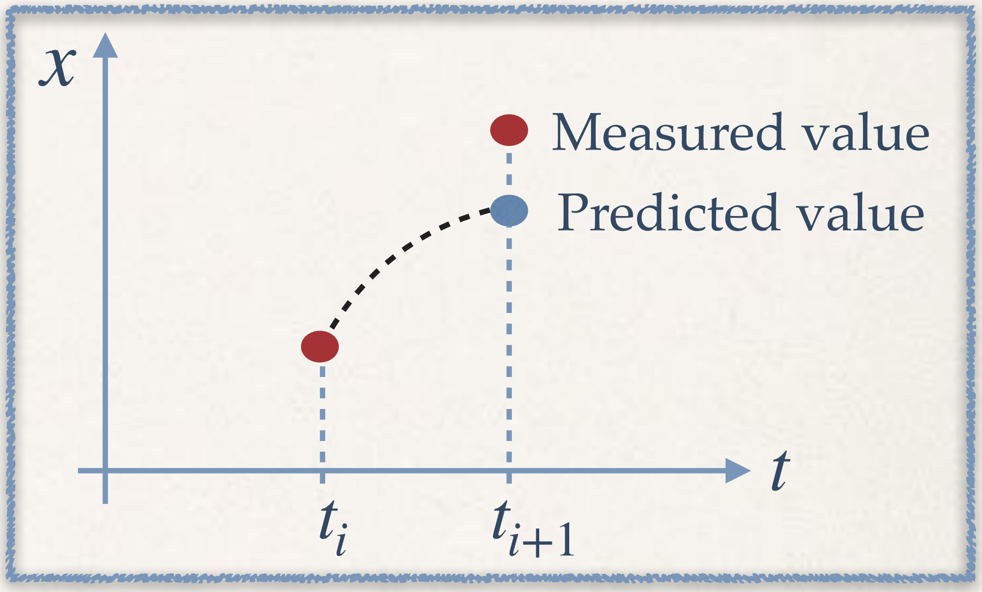
[Cheng, Primulando, MS '19]

- Any target mass in a bath of DM
- DM scatterings induce Brownian Motion
- Measure the position of a light target mass with high precision
- Look for time-dependent asymmetries

Potential Setup

Inspired by [Valerie Domcke and Martin Spinrath, 2017]





Assume the motion of the mirror without external force is well understood

The Asymmetry Factor

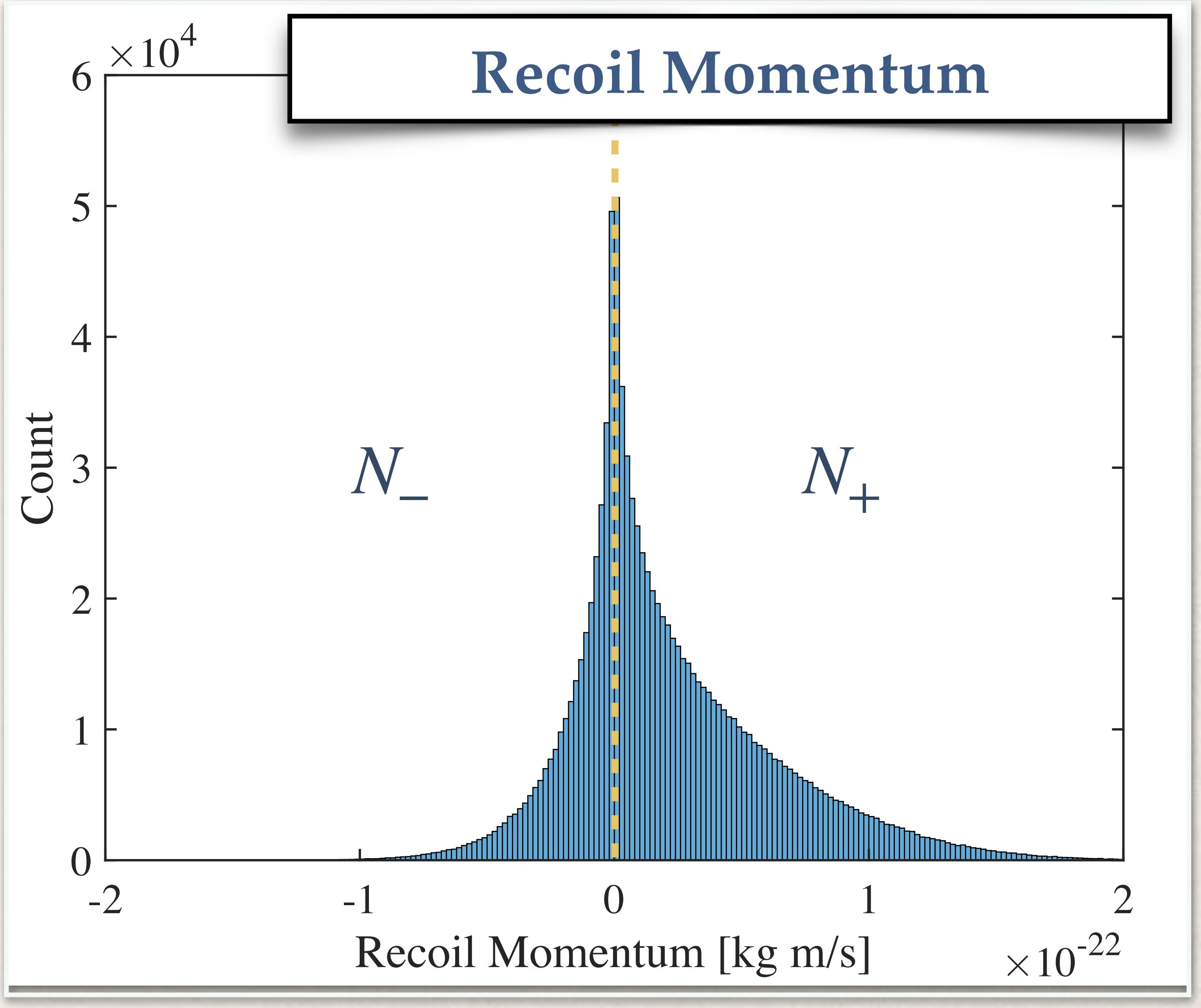
- The Asymmetry Factor :

$$A = \frac{N_+ - N_-}{N_+ + N_-} = p_+ - p_-$$

- Uncertainty of A :

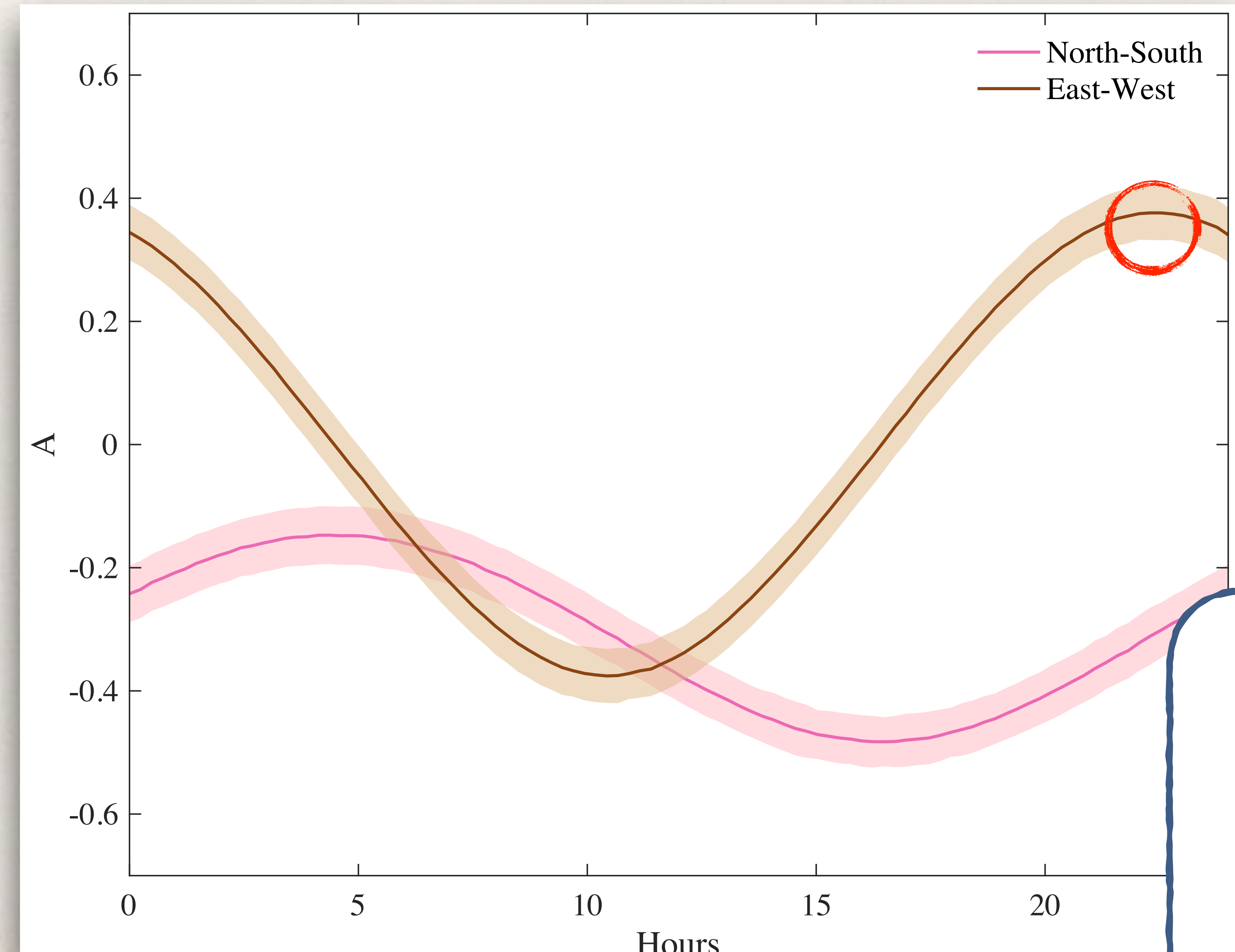
$$\sigma_A = \frac{2}{\sqrt{N}} \sqrt{p_+ p_-}$$

$A, p_\pm$ are independent of DM mass



[slide taken from Ting Cheng now at MPIK]

Daily Modulation of A



[T.C., M. Spinrath, R. Primulando 2019]

Error Bar:

$$\Delta t = 10 \quad \text{min}$$

$$\sigma_{DM-N} = 10^{-31} \quad \text{cm}^2$$

$$M_{DM} = 10 \quad \text{MeV}$$

Backgrounds

- Many potential backgrounds for our proposal
    - seismic noise, nearby traffic, radioactivity, etc.
- Two examples
    - Neutrinos
    - Hits from residual gas

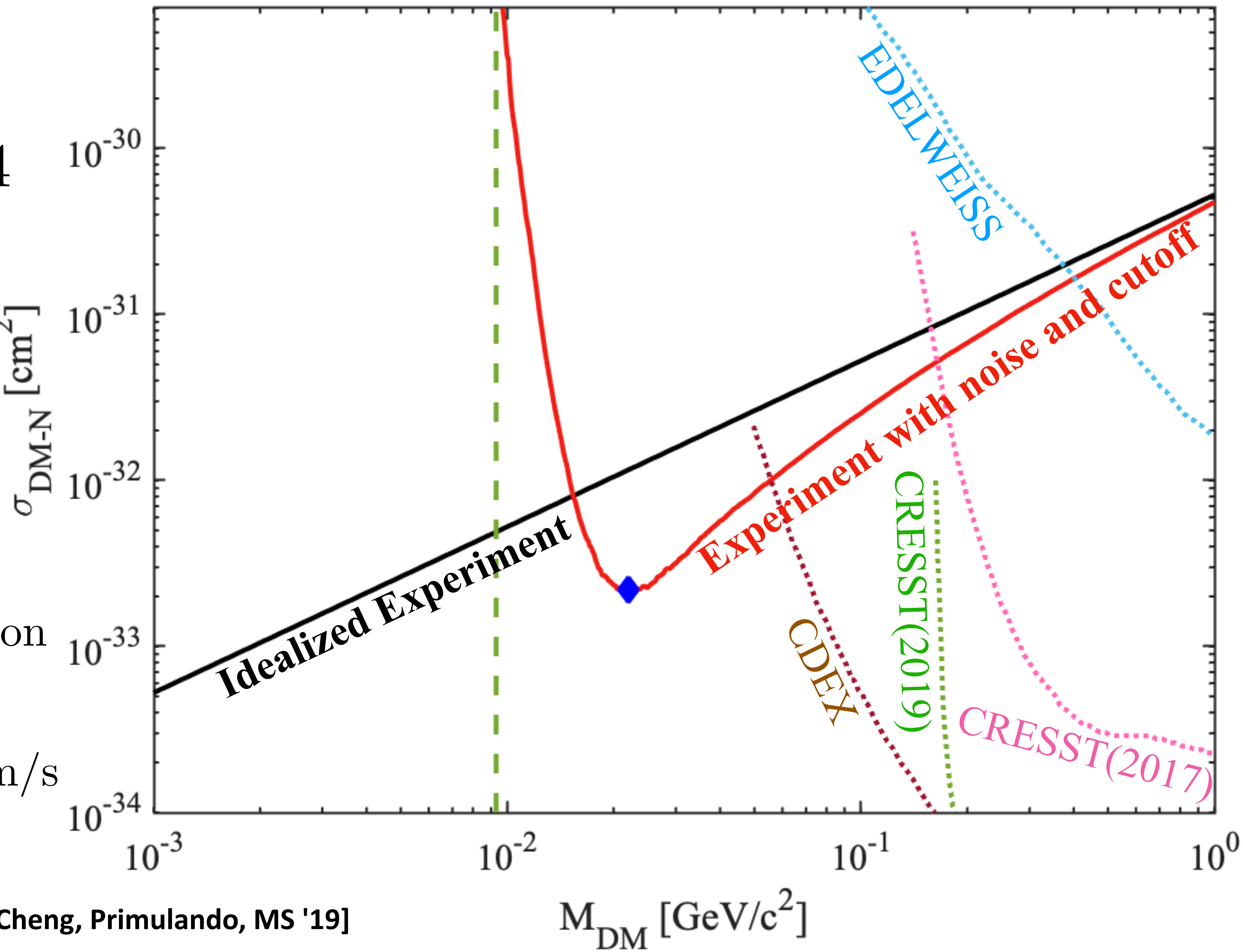
Backgrounds

- Small neutrino cross section and target mass
  - Negligible $O(10^{-14})$ neutrino events per sec
- Residual gas
  - Naively, many $O(10^9)$ events per sec
  - Not a relevant noise in LIGO/KAGRA
  - After momentum cutoff $O(10^{-9})$ events per sec

Sensitivity Estimate

$$\frac{\langle A \rangle^2}{\sigma_A^2 + \sigma_{\rm bkg}^2} = 4$$

$$M_T = 10^{-3} \text{ g of carbon}$$
$$\Delta t = 10 \text{ min}$$
$$q_{\rm min} = 2 \times 10^{-23} \text{ kg m/s}$$



[Cheng, Primulando, MS '19]

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Toy Model: Damped Harmonic Oscillator

[Lee, Nugroho, MS '20]

- We want to study a simple toy model first

$$m\,\ddot{x}_c + k_c\,(1 + \mathrm{i}\,\phi)\,x_c = \frac{F_{\mathrm{ext},c}}{L}$$

- The experimental output

[Moore, Cole, Berry '14]

$$x_{\mathrm{tot},c}(t) = \underbrace{x_{\mathrm{th},c}(t)}_{\text{Suspension Thermal Noise}} + \underbrace{x_{\mathrm{qu},c}(t)}_{\text{Quantum Noise}} + \underbrace{x_{\mathrm{DM},c}(t)}_{\text{DM Signal}}$$

- We neglect here some noise components

Toy Model: Noise

[Saulson '90; Gonzales, Saulson '94; Thorne '87]

- Thermal noise from fluctuation-dissipation theorem

[Callen, Welton '51; Callen, Greene '55]

$$S_{\rm th}(\omega) = \frac{4\,k_B\,T}{L^2\,\omega^2}\Re[Y(\omega)] = \frac{4\,k_B\,T}{L^2}\frac{\phi\,\omega_c^2/(m\,\omega)}{(\omega^2-\omega_c^2)^2+\omega_c^4\phi^2}$$

- Standard Quantum Limit

$$S_{\rm qu} = \frac{8\,\hbar}{m\,\omega^2\,L^2}$$

- The noise strain amplitude is

$$h_n = \sqrt{h_{\rm th}^2 + h_{\rm qu}^2} = \sqrt{S_{\rm th} + S_{\rm qu}}$$

Toy Model: DM Signal I

[Lee, Nugroho, MS '20; Tsuchida *et al.* '19]

- The DM signal is easier to model for a real eq.

$$m\,\ddot{x}_r + 2\,m\,\omega_r\,\xi\,\dot{x}_r + m\,\omega_r^2\left(1+\xi^2\right)x_r = \frac{q_R}{L}\,\delta(t)$$

- That has the solution

$$x_{\mathrm{DM}}(t) = \theta(t)\,\frac{q_R}{m\,\omega_r\,L}\exp\left(-\omega_r\,\xi\,t\right)\sin(\omega_r\,t)$$

- And we will need

$$|\tilde{x}_{\mathrm{DM}}(\omega)|^2 = \frac{q_R^2}{m^2\,L^2}\,\frac{1}{\left(\omega^2-\omega_r^2(1-\xi^2)\right)^2+4\,\omega_r^4\,\xi^2}$$

Toy Model: Coefficient Matching

[Lee, Nugroho, MS '20]

- KAGRA provides complex spring constants
- DM modeled with real coefficients
- Match oscillation frequency and damping

$$\omega_r = \omega_c \left(1 + \phi^2\right)^{1/4} \cos\left(\frac{1}{2} \arctan \phi\right) \approx \omega_c \left(1 + \frac{\phi^2}{8}\right)$$

$$\xi = \tan\left(\frac{1}{2} \arctan \phi\right) \approx \frac{\phi}{2} \left(1 - \frac{\phi^2}{4}\right)$$

Toy Model: DM Signal II

[Lee, Nugroho, MS '20; Tsuchida *et al.* '19]

- The DM signal after matching

$$|\tilde{x}_{\mathrm{DM}}(\omega)|^2 = \frac{q_R^2}{m^2\, L^2} \frac{1}{(\omega^2 - \omega_c^2)^2 + \omega_c^4 \phi^2}$$

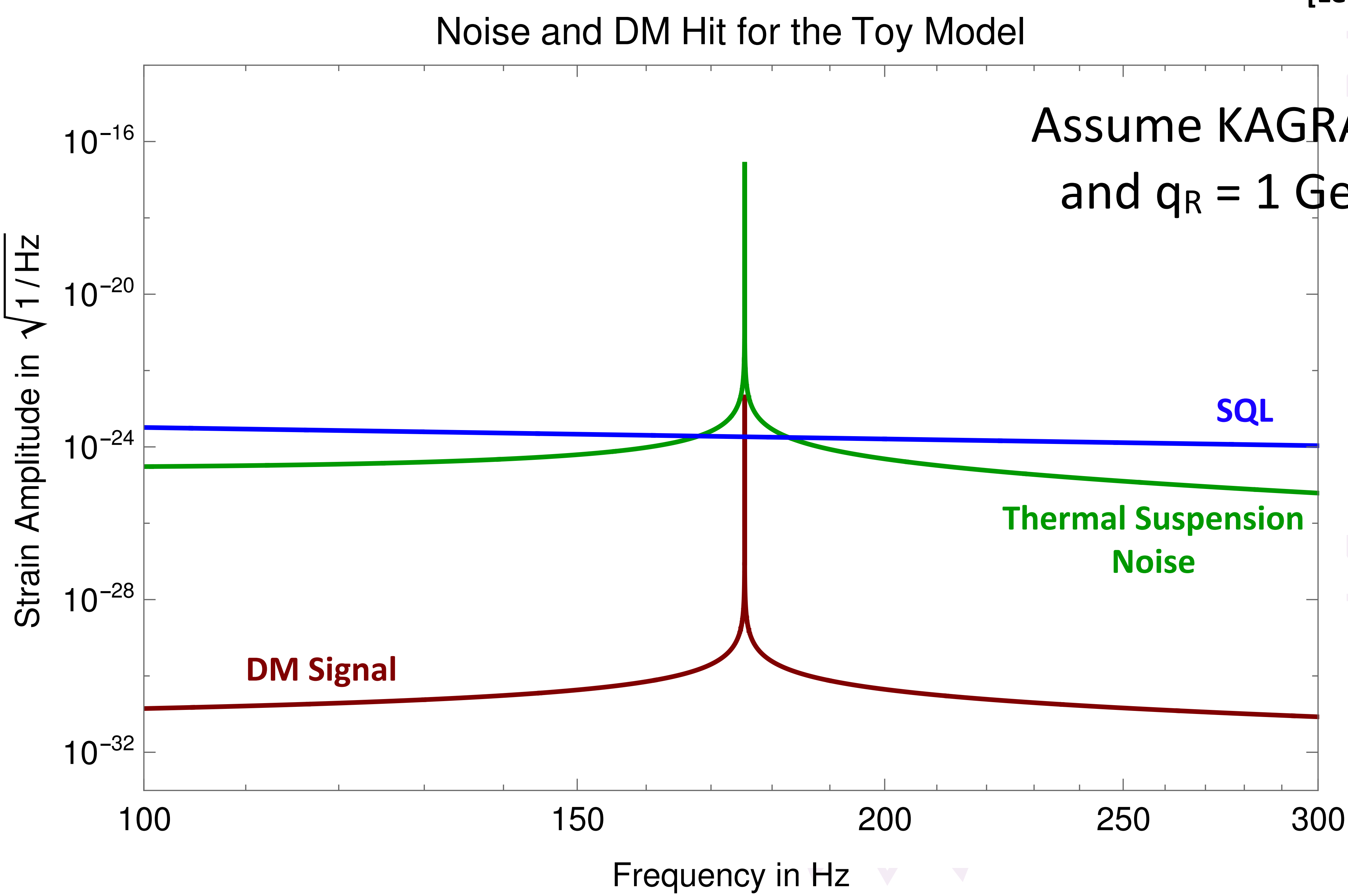
- The DM strain amplitude [Moore, Cole, Berry '14]

$$h_{\mathrm{DM}}(\omega) = \sqrt{\frac{2\,\omega}{\pi}}\, |\tilde{x}_{\mathrm{DM}}(\omega)|$$

Toy Model: Strain Amplitudes

[Lee, Nugroho, MS '20]

Assume KAGRA-like parameters and q_R = 1 GeV/c^2 x 220 km/s



Signal-to-Noise Ratio

[Lee, Nugroho, MS '20; Moore, Cole, Berry '14]

- The optimal SNR is given by

$$\varrho^2 = \int_{f_{\min}}^{f_{\max}} \mathrm{d}\, f \, \frac{4\,|\tilde{x}_{\mathrm{DM}}(2\,\pi\, f)|^2}{S_n(2\,\pi\, f)}$$

- Near the peak (FWHM) neglect quantum noise

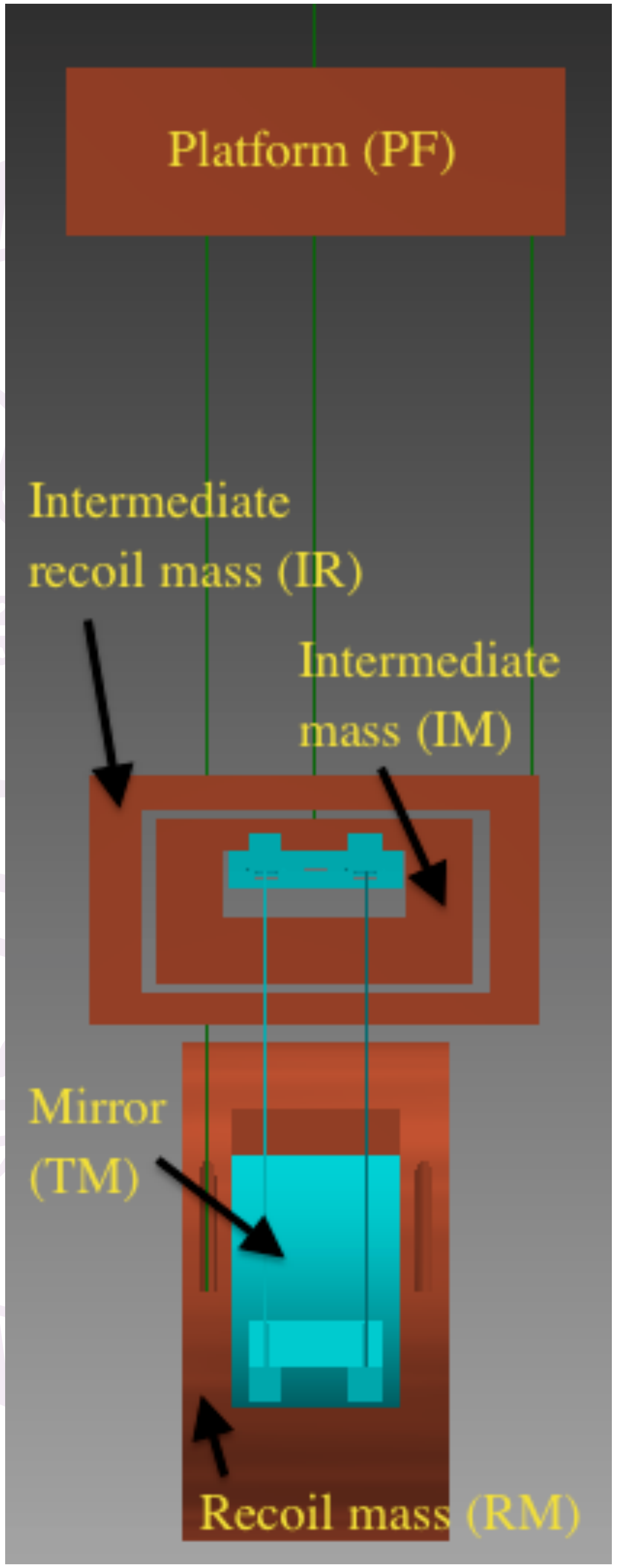
$$\varrho^2_{\mathrm{th}} = \frac{1}{2\pi}\frac{q_R^2}{m\,k_B\,T} = \frac{1}{2\pi}\frac{E_R}{E_{\mathrm{th}}} \underset{\uparrow}{=} 4.09 \times 10^{-24}$$

Numbers as in previous figure

- Need light, cold targets!

KAGRA

- KAGRA is a new gravitational wave detector in Japan
- Advantage: Cryogenic (T about 20 K)
- The mirror is a pendulum on springs on a pendulum (3 x coupled, damped harmonic oscillators)
- NTHU and other institutions in Taiwan are members



[Taken from the KAGRA PhD thesis of D. Chen, 2016]

Equations of Motion

[Lee, Nugroho, MS '20 based on KAGRA Document, JGW-T1707038v9]

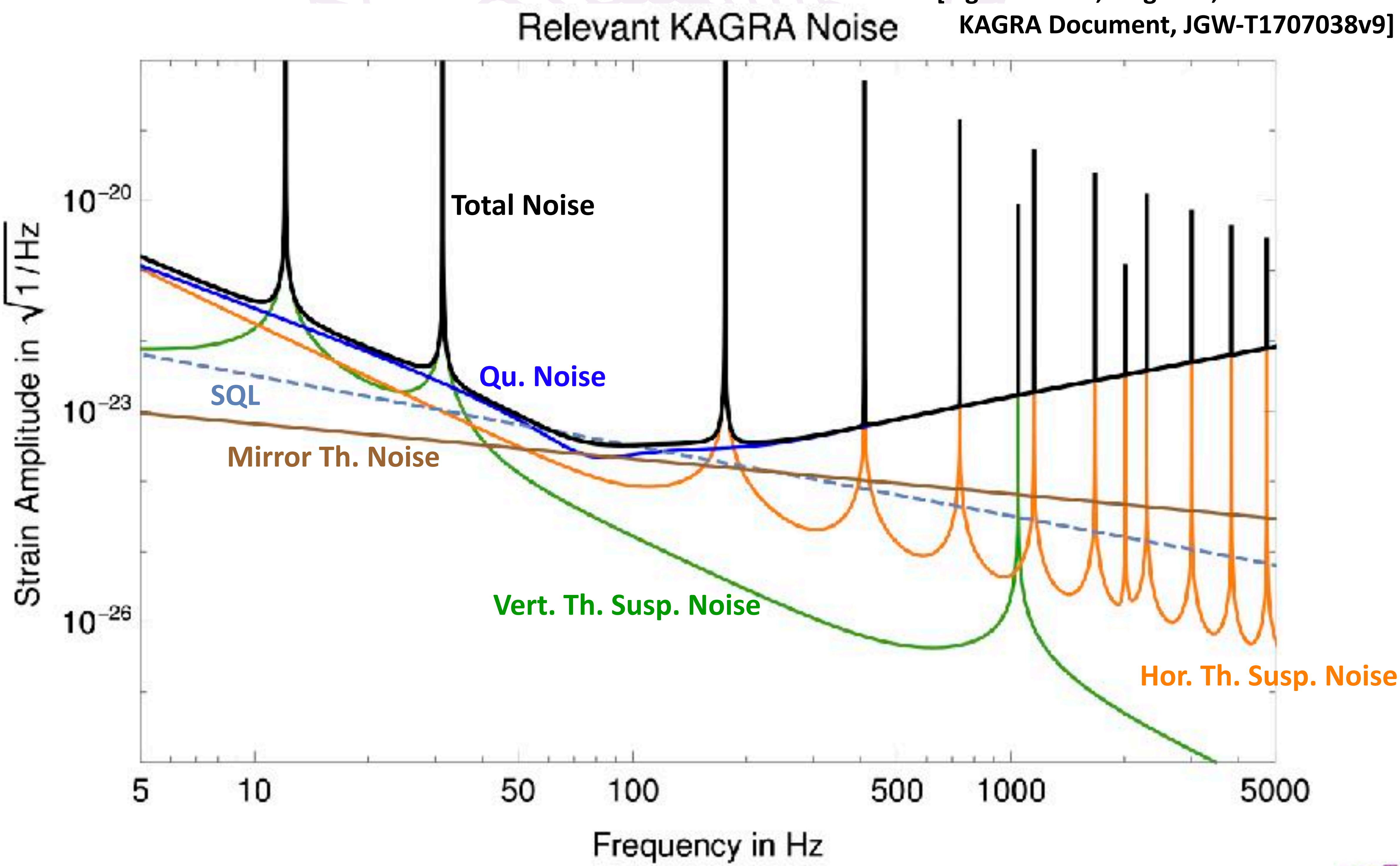
- The equations of motion take a 3x3 matrix form

$$\left(M \frac{\mathrm{d}^2}{\mathrm{d}\,t^2} + K_v \right) \vec{x}_v(t) = \frac{\vec{F}_{\mathrm{ext},v}(t)}{L}$$

- KAGRA can see vertical and horizontal modes
  - two sets of equations

KAGRA Noise

[Fig. from Lee, Nugroho, MS '20 based on KAGRA Document, JGW-T1707038v9]



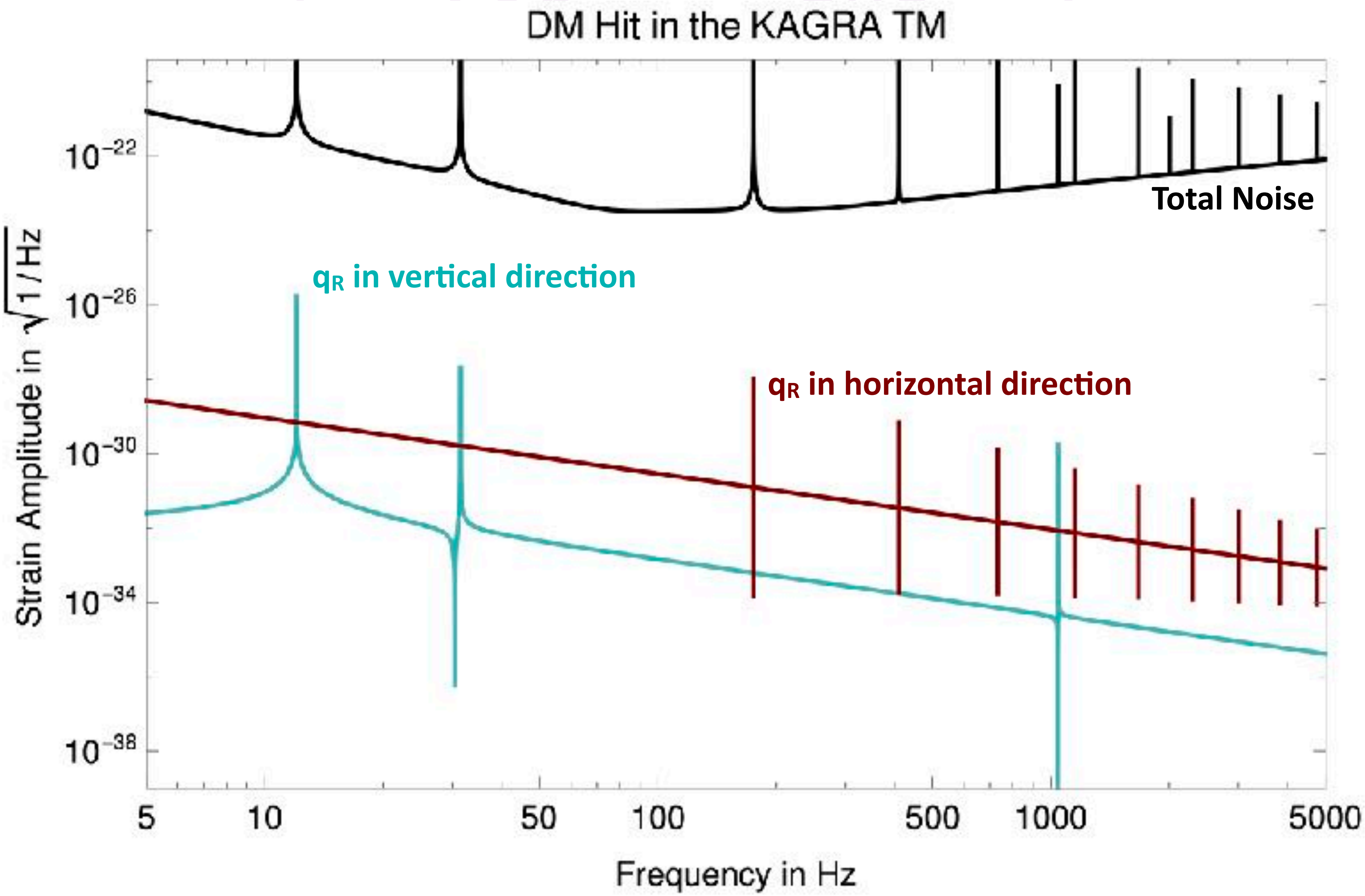
DM Signal

[Lee, Nugroho, MS '20]

- We only need the Fourier transform of the displacement
$$\vec{\tilde{x}}_v(t) = \left(-M\omega^2 + K_v\right)^{-1} \frac{\vec{\tilde{F}}_{\mathrm{ext},v}(t)}{L}$$
- We can then study the effect of the test mass for a hit in different components and directions

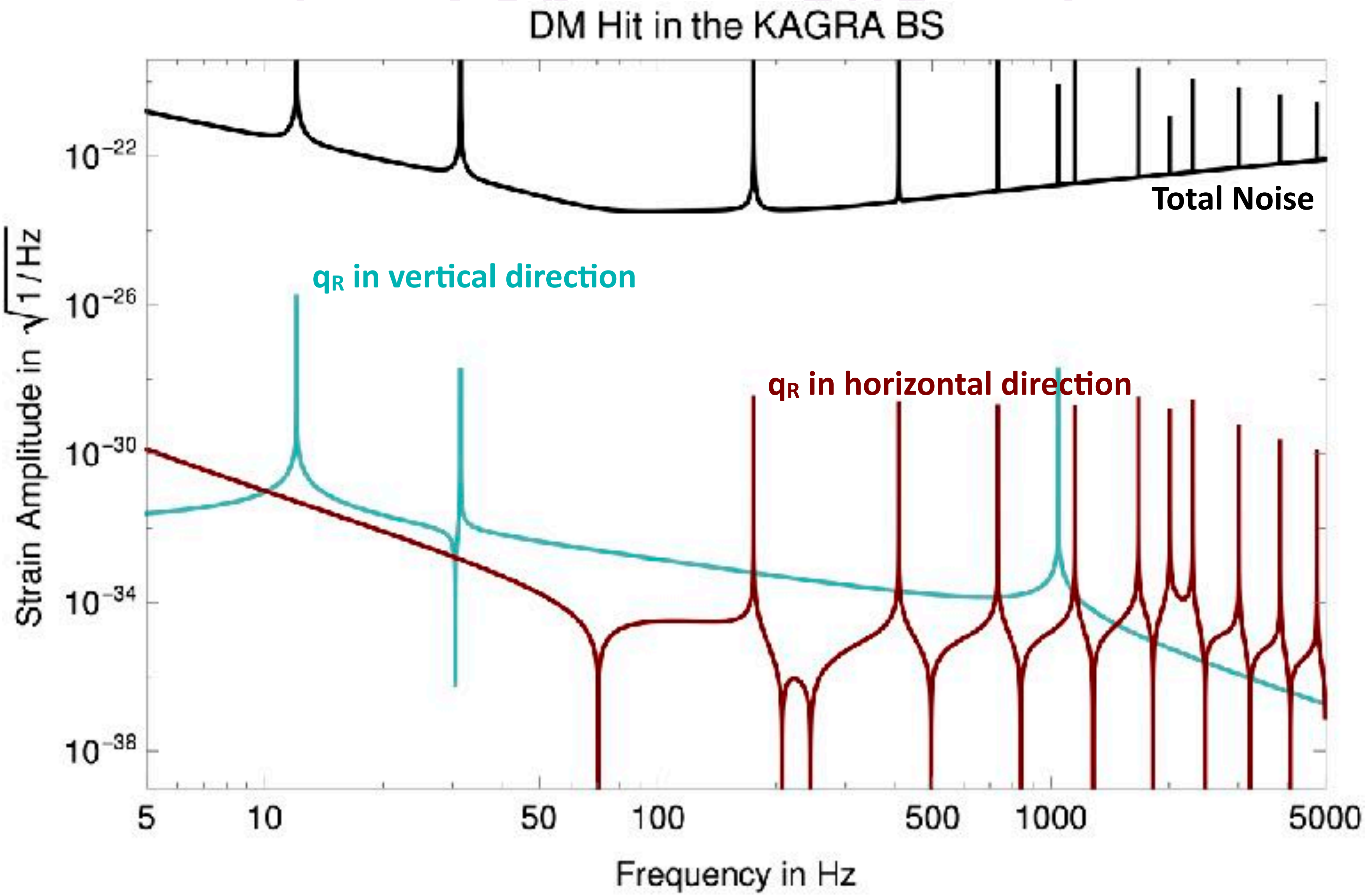
DM Signal at KAGRA

[Lee, Nugroho, MS '20]



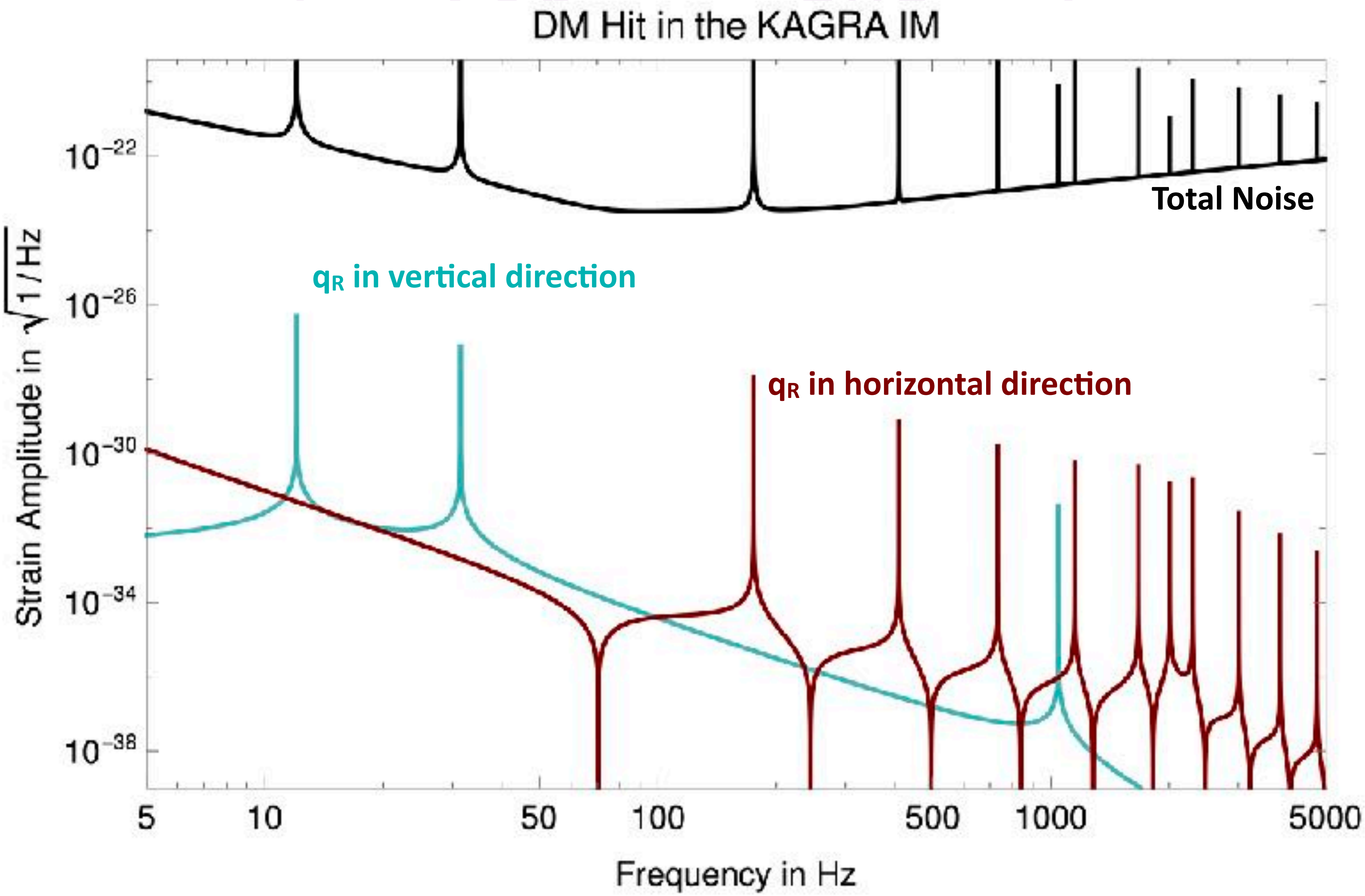
DM Signal at KAGRA

[Lee, Nugroho, MS '20]



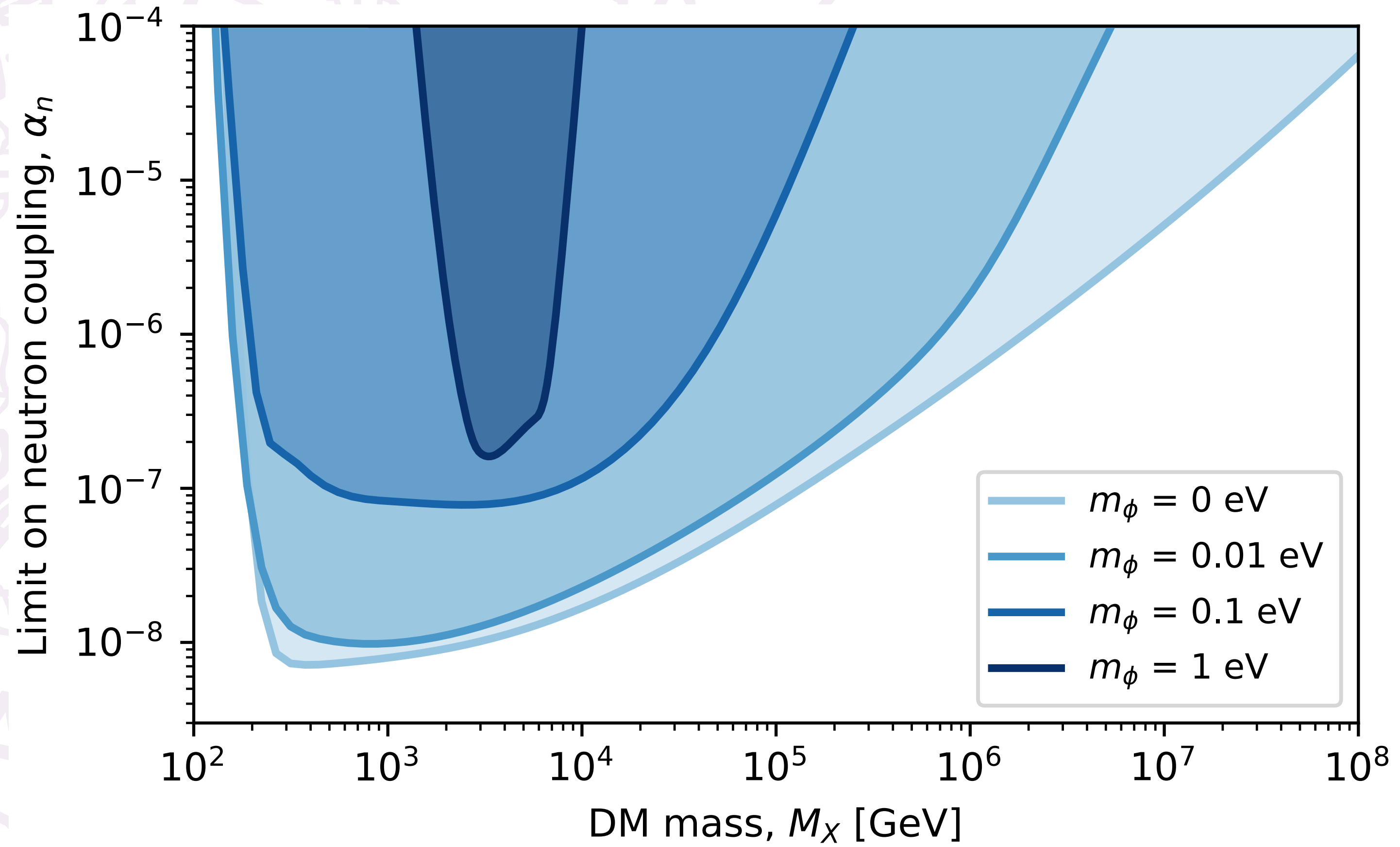
DM Signal at KAGRA

[Lee, Nugroho, MS '20]



Current Reality

- Optically levitated mass
- Target mass 1 ng
- Temperature 200 μK
- Several days exposure
- Experimental threshold 0.15 GeV



[Monteiro *et al.* '20]

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Summary and Conclusions

- Gravitational Wave Astronomy has just begun
- Impressive new technologies
- Can we use them to find cosmic relics?
  - Cosmic Neutrino Background? No (probably).
  - Dark Matter? Maybe.
- We need more research

Backup

Theory: Scattering

[Domcke, MS ’17; see also Duda *et al.* '01, ..., Opher '74]

- The basic formula

$$a_{G_F^2} = \Phi_\nu \, \frac{N_{AV}}{A \, m_{AV}} \, N_c \, \sigma_{\nu-A} \, \langle \Delta p \rangle$$

- Incoming flux: Φ_ν
- #nuclei in 1 g test material: $N_{AV}/(A m_A V)$
- Neutrino-nucleus cross-section: $\sigma_{\nu-A}$
- Coherence factor: N_c
- Average momentum transfer: $\langle \Delta p \rangle$

Theory: Scattering

[Domcke, MS ’17; see also Duda *et al.* '01, ..., Opher '74]

- Neutrinos can come in three kinematics
  - relativistic (R)
  - non-relativistic non-clustered (NR-NC)
  - non-relativistic clustered (NR-C)
- Two important numbers
  - The cross-section: $\sigma_{\nu-A} \approx 10^{-27}\ \mathrm{pb} = 10^{-63}\ \mathrm{cm}^2$
  - The coherence factor: $N_c = \frac{N_{AV}}{A\, m_{AV}}\, \rho\, \lambda_\nu^3 \sim 10^{20}$

Residual Gas

[Cheng, Primulando, MS '19]

- Gravitational wave detectors have ultra high vacua in their chambers
- We can estimate the hit rate from residual gas

$$
\begin{aligned}
R_{\mathrm{atm}} &= n\, A\, |v|\, f(v) \\
&\approx 8.3 \times 10^{9} \left(\frac{P}{10^{-10}\ \mathrm{mbar}} \sqrt{\frac{20\ \mathrm{K}}{T}} \frac{A}{\mathrm{mm}^2} \right) \frac{1}{\mathrm{s}}
\end{aligned}
$$

Residual Gas

[Cheng, Primulando, MS '19]

- What rate could we expect for DM?

$$R_{\mathrm{DM}} = (Z+N)^2\,\sigma_{\mathrm{DM}-N}\,\frac{M_T}{M_{\mathrm{mol}}}\,\frac{\rho_{\mathrm{DM}}}{M_{\mathrm{DM}}}\,\bar{v}_{\mathrm{DM}}$$
$$= 0.37\left(\frac{Z+N}{12}\,\frac{\sigma_{\mathrm{DM}-N}}{10^{-31}\,\mathrm{cm}^2}\,\frac{M_T}{10^{-3}\,\mathrm{g}}\,\frac{\rho_{\mathrm{DM}}}{0.3\,\mathrm{GeV/cm}^3}\,\frac{20\,\mathrm{MeV}}{M_{\mathrm{DM}}}\,\frac{\bar{v}_{\mathrm{DM}}}{341\,\mathrm{km/s}}\right)\frac{1}{\mathrm{s}}$$

- Can we cut on the background?
  - Yes! Cut on minimum recoil momentum

Residual Gas

[Cheng, Primulando, MS '19]

- No air flow: $\langle q_{\rm atm} \rangle$
- The width of the recoil momentum is

$$\sigma_{q_{\rm atm}} \approx 2.5 \times 10^{-24} \left(\frac{P}{10^{-10}\ \mathrm{mbar}} \frac{A}{\mathrm{mm}^2} \frac{\delta t}{0.1\ \mathrm{ns}} \sqrt{\frac{T}{20\ \mathrm{K}}} \right)^{1/2} \frac{\mathrm{kg\ m}}{\mathrm{s}}$$

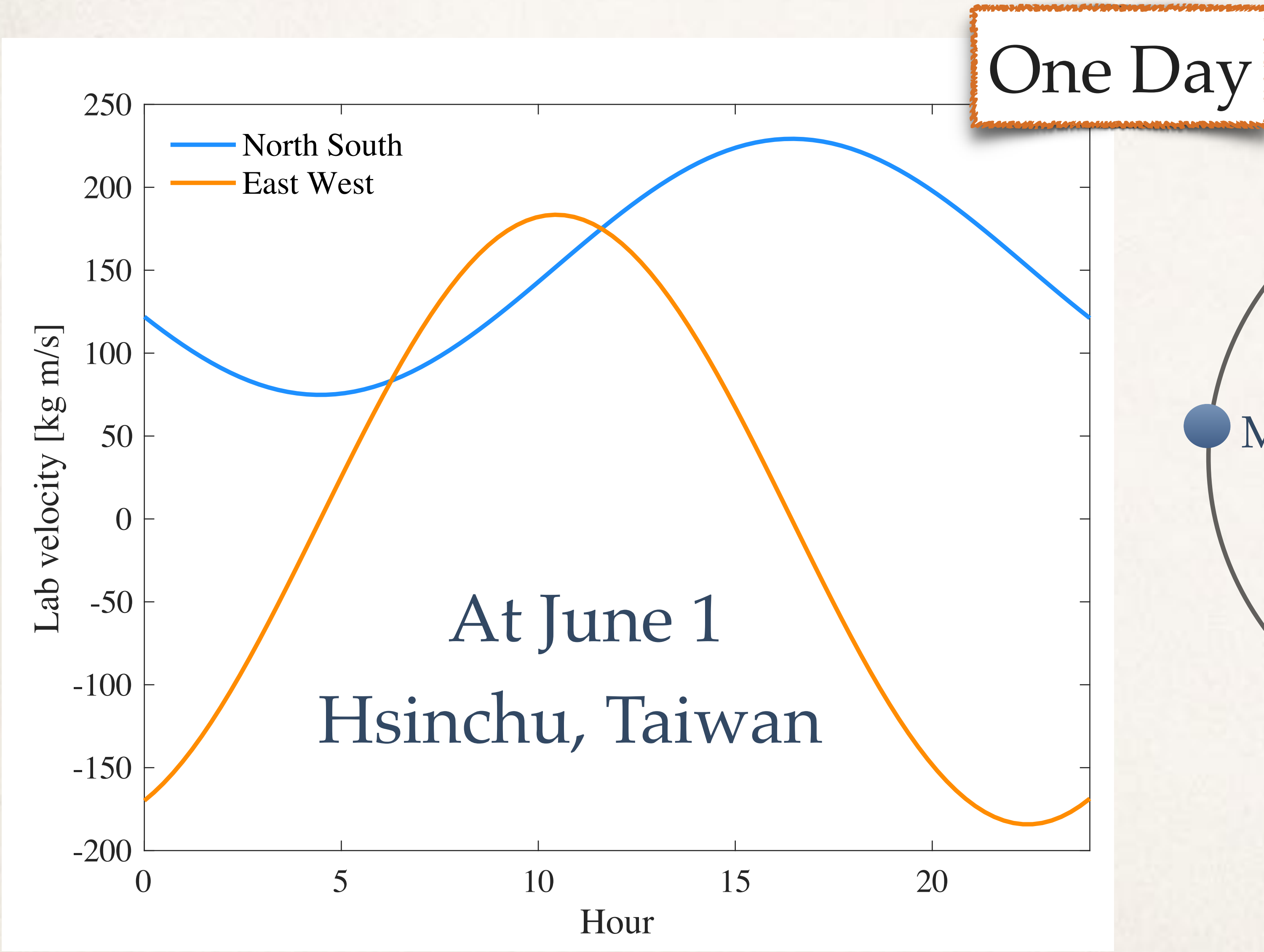
- Use LIGO resolution as naive estimate

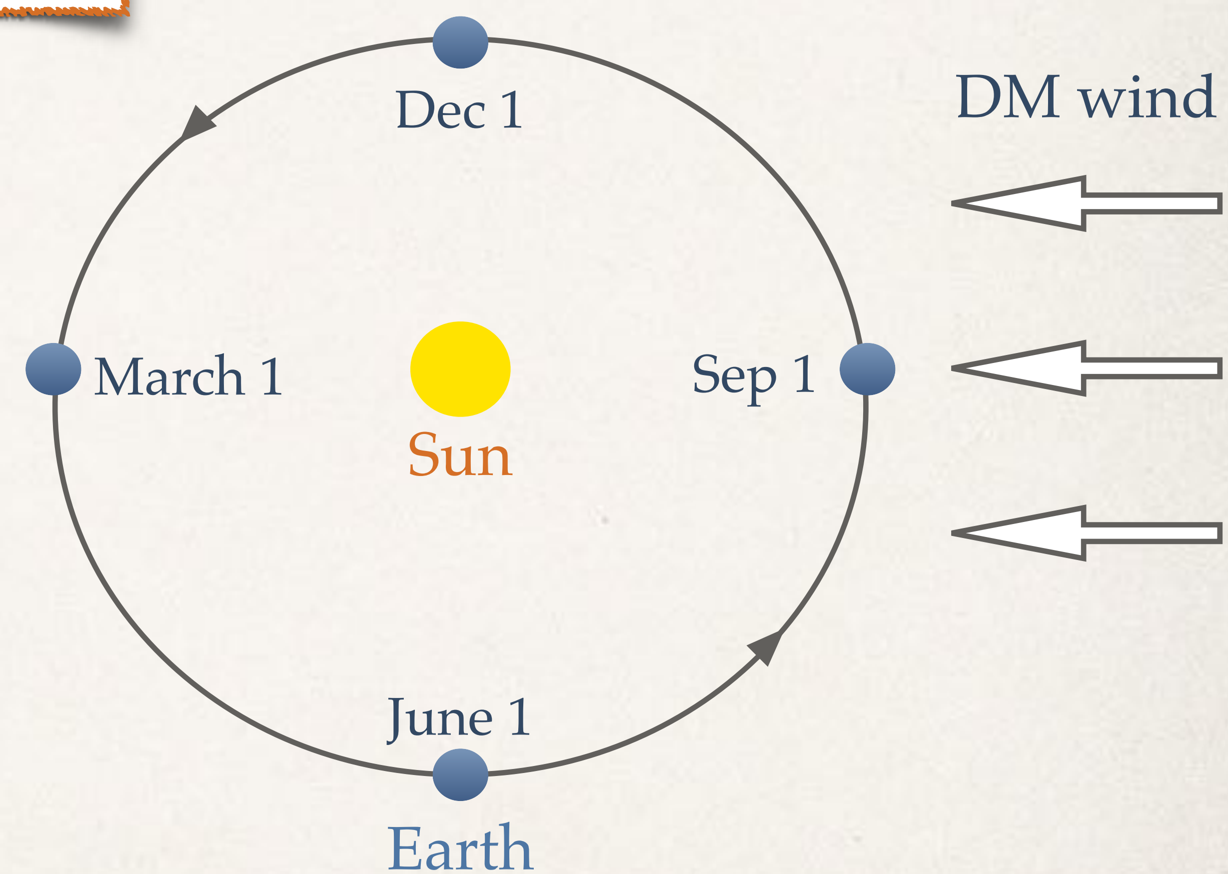
$$q_{\rm min} \equiv 2 \times 10^{-23}\ \mathrm{kg\ m/s} \approx 3.7 \times 10^{-5}\ \mathrm{GeV/c}$$

- Remaining gas hit rate $R_{\rm atm}^{\rm cut} \approx 5 \times 10^{-6}\ \mathrm{Hz}$

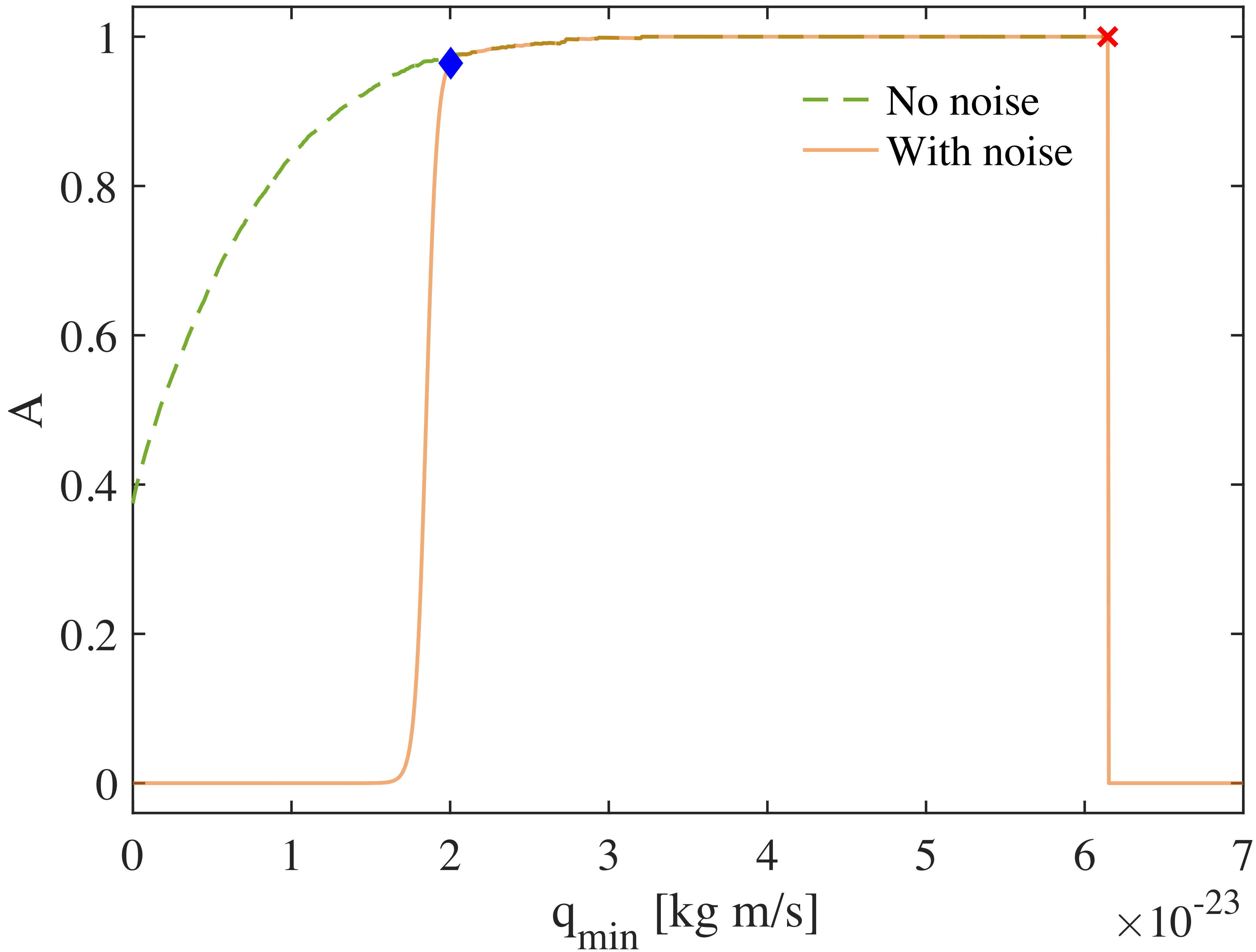
[slide taken from Ting Cheng now at MPIK]

Time Dependent Lab Velocity





Dependence of A on q_{min}



[Cheng, Primulando, MS '19]

Space-Based Experiments

- LISA target sensitivity (0.1 mHz < f < 1 Hz)

$$\sqrt{S_{\Delta g}} \leq 3\sqrt{2}\ \mathrm{fm\ s^{-2}}/\sqrt{\mathrm{Hz}} \times \sqrt{1 + (f/8\ \mathrm{mHz})^4}$$

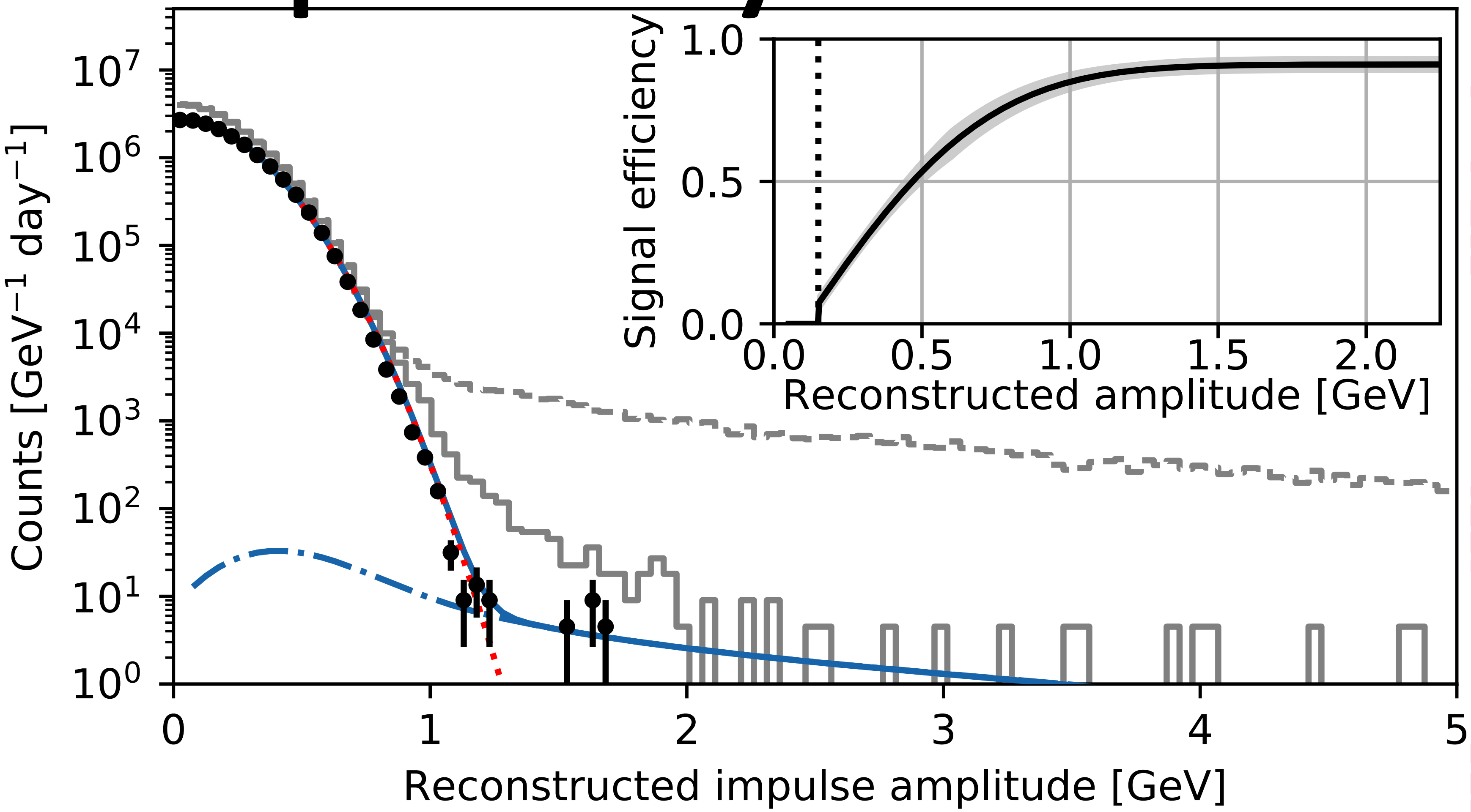
[LISA Pathfinder '16]

- Expected strain amplitude

$$\sqrt{S_{\Delta g,\mathrm{DM}}} \sim 4.1 \times 10^{-7} \sqrt{\frac{f}{\mathrm{Hz}}}\ \mathrm{fm\ s^{-2}}/\sqrt{\mathrm{Hz}}$$

[Lee, Nugroho, MS '20]

Optically Levitated Devices



[Monteiro *et al.* '20]

FIG. 2. Measured rate of reconstructed impulses after all cuts (black points), compared to the spectrum with only live-time selections applied (gray, solid) and with no cuts applied (gray, dashed). The Gaussian background (red, dotted), DM signal (blue, dot-dashed), and sum of background and signal (blue, solid) are also shown at the 95% CL upper limit, $\alpha_n = 8.5 \times 10^{-8}$, for $M_X = 5 \times 10^3$ GeV, $m_\phi = 0.1$ eV, and $f_X = 1$. (Inset) Overall signal efficiency versus amplitude (black) and estimated error (gray band) above the analysis threshold, $q_{\rm thr} = 0.15$ GeV (dotted).

Little (Incomplete) Overview

- $M_{DM} < 1$ MeV: quantum decoherence [Riedel '12; Riedel, Yavin '16]
- $M_{DM} < 10^{-4}$ eV: Sound of DM [Arvanitaki, Dimopoulos, van Tilburg '16; AURIGA '16]
- $M_{DM} < 10^{-11}$ eV: Variation of fundamental constants [Stadnik, Flambaum '14; '15; Grote, Stadnik '19]
- $10\ M_{\odot} < M_{DM} < 10^{5}\ M_{\odot}$: GW lensing [Jung, Shin '17]
- $M_{DM} < 10^{-11}$ eV: Dark Photons [Pierce, Riles, Zhao '18]
- $M_{DM} < 10^{-10}$ eV: Mirror oscillations [Morisaki, Suyama '18]

Ultralight DM

- Large occupation number ➙ acts like a classical field
- Motivated by string moduli fields
- Lagrangian

$$\mathcal{L}_\phi = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_\phi^2\phi^2$$

$$\mathcal{L}_{\phi-\mathrm{SM}} = \kappa\phi\left[\frac{d_e}{4e^2}F_{\mu\nu}F^{\mu\nu} - \frac{d_g\beta_3}{2g_3}G^A_{\mu\nu}G^{A\mu\nu} - \sum_{i=e,u,d}(d_{m_i}+\gamma_{m_i}d_g)m_i\bar{\psi}_i\psi_i\right]$$

[Morisaki, Suyama ’18]

Motion of Optical Instruments

[Morisaki, Suyama ’18]

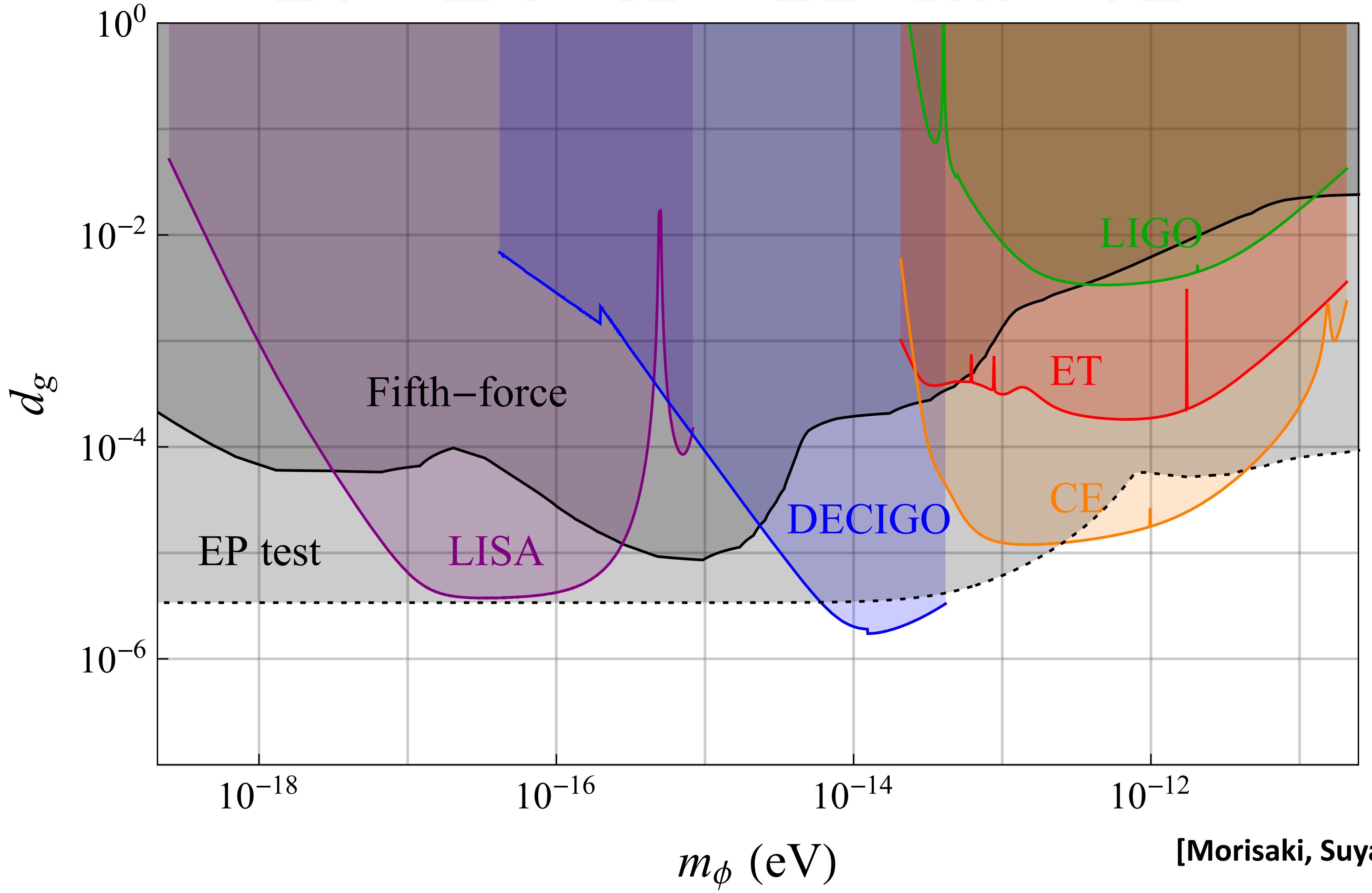
- The DM wave pushes optical instruments
- Assuming plane DM waves

$$x^i \simeq d_g^* \kappa \phi_{\vec{k}} \frac{k^i}{m_\phi^2} \sin(\omega_k t - \vec{k} \cdot \vec{x}_0 + \theta_{\vec{k}}) + \text{const.}$$

- where

$$\omega_k \sim m_\phi,\, \Delta\omega_k \sim m_\phi v_{\text{DM}}^2 \text{ and } |\vec{k}| \lesssim m_\phi v_{\text{DM}} \sim 10^{-3}\, m_\phi$$

Prospects in this Setup



[Morisaki, Suyama '18]