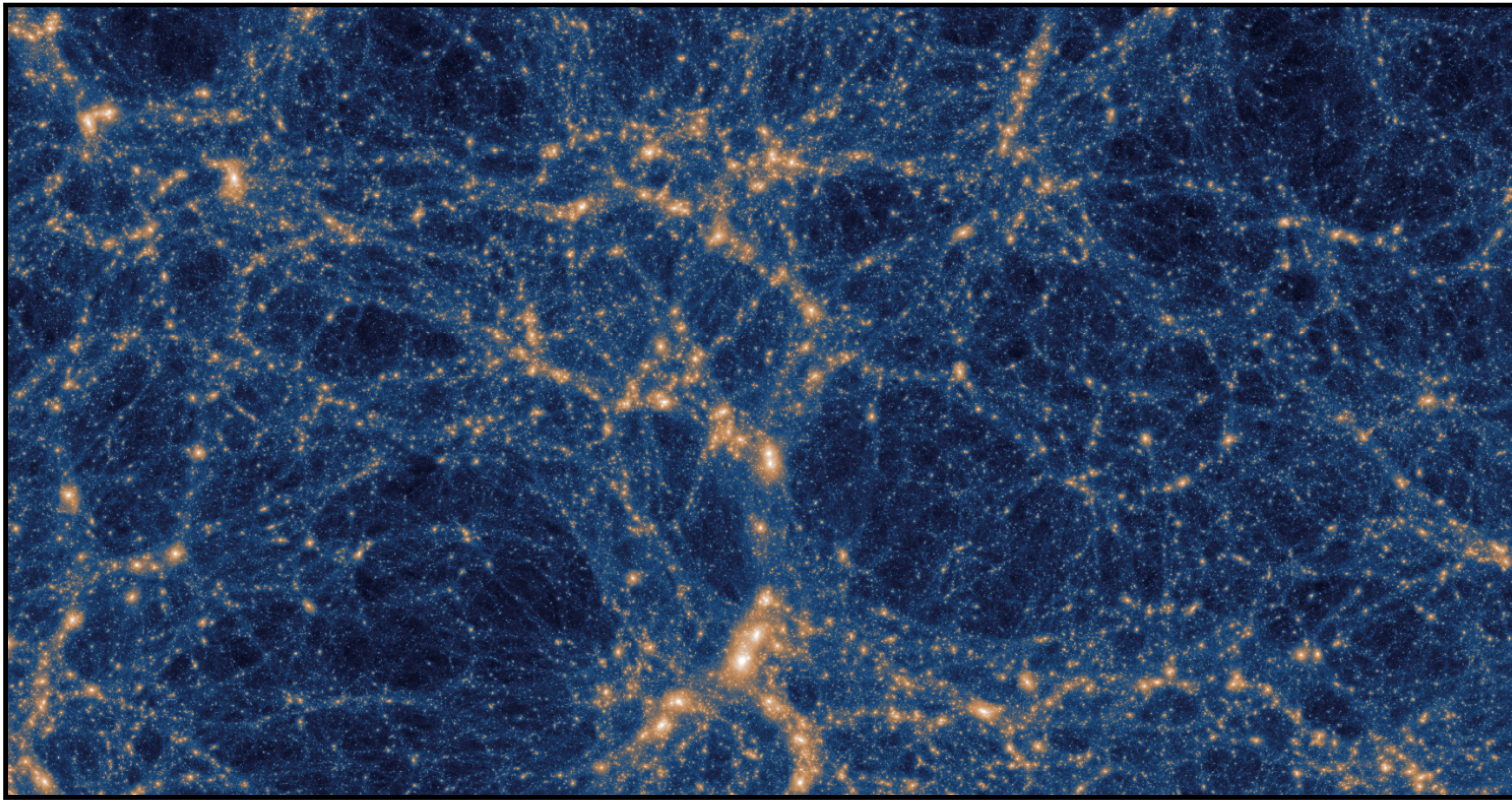

The Cosmological Bootstrap

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University of Amsterdam &
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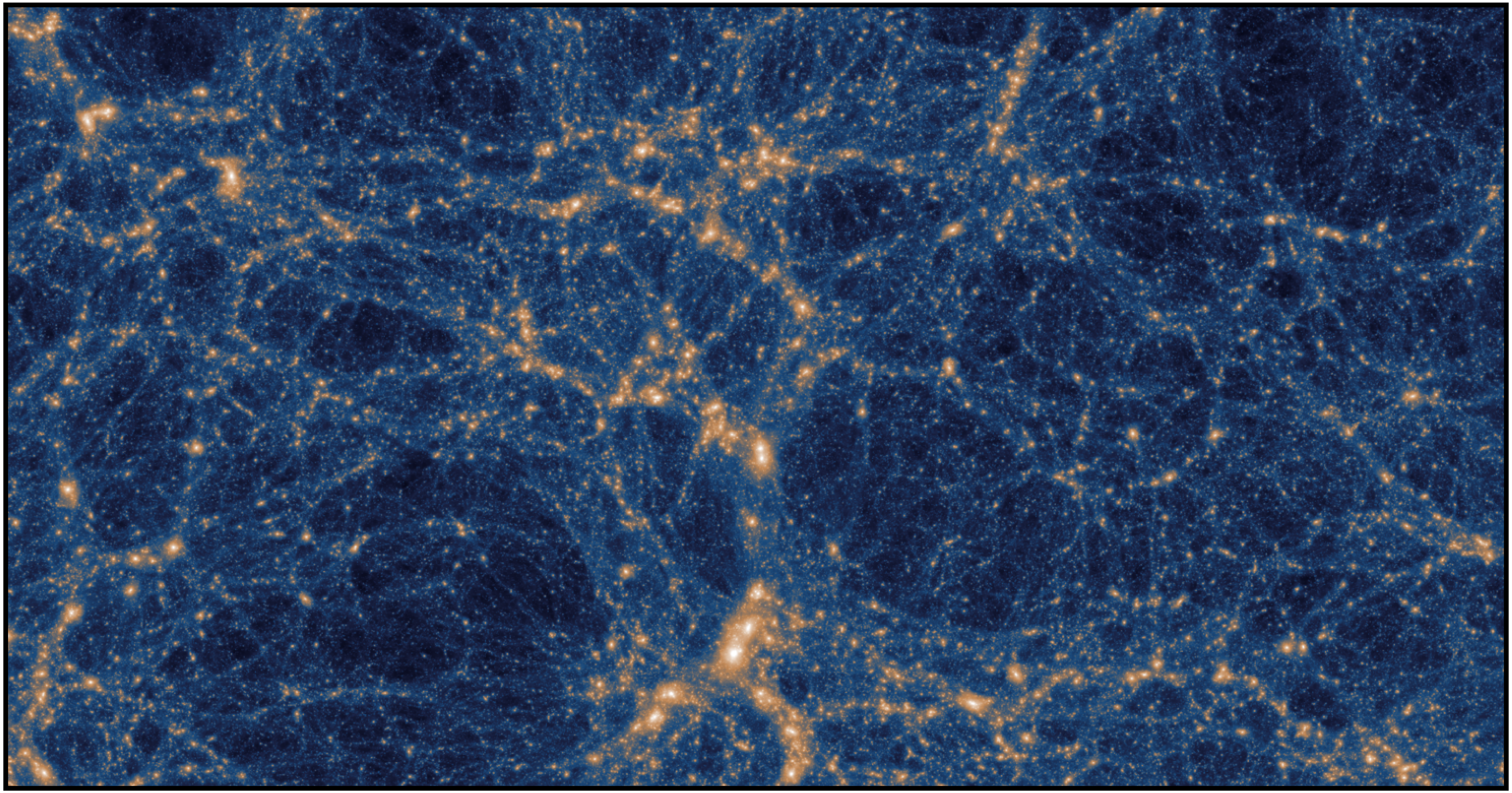
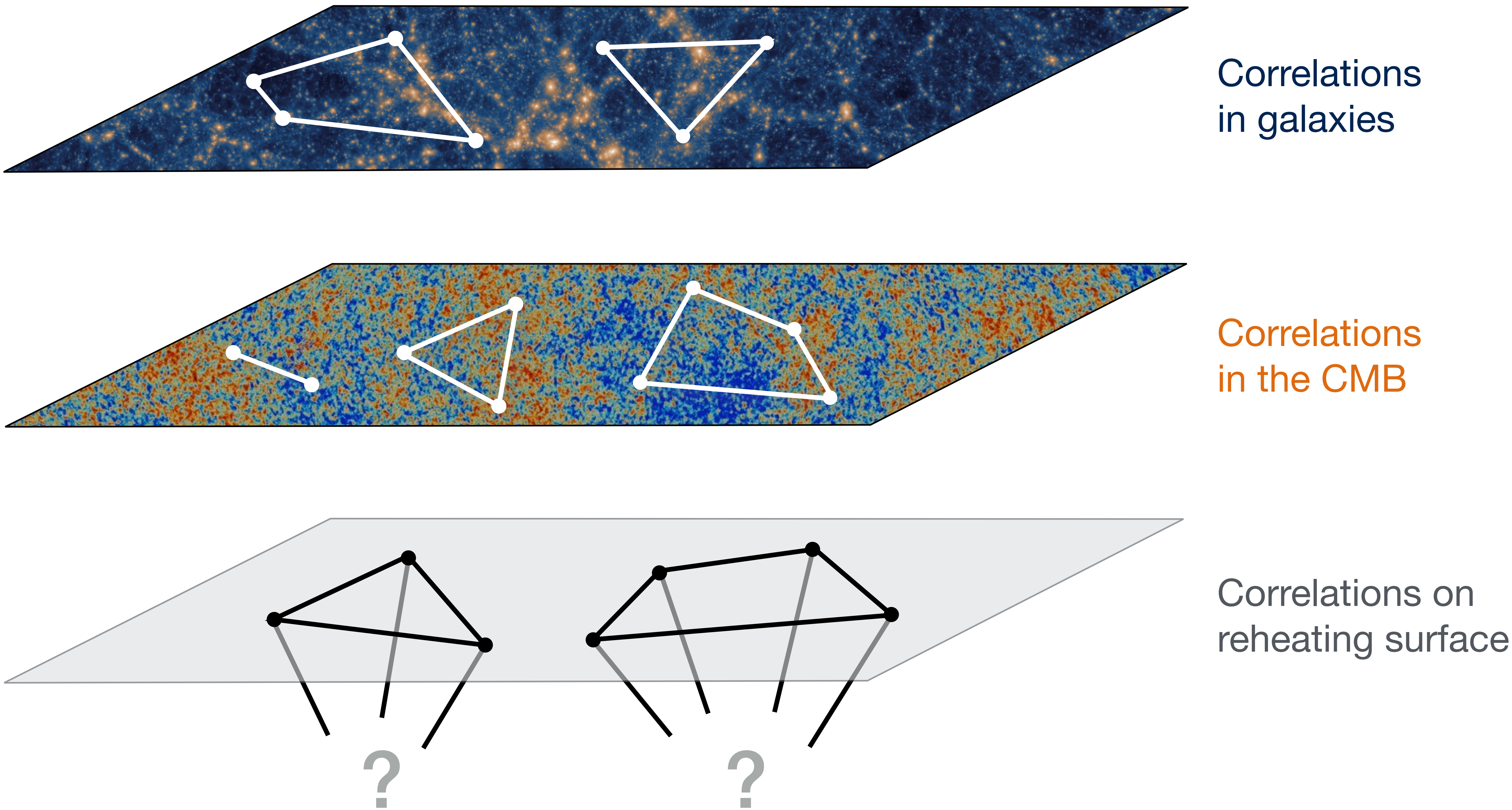
NCTS Annual Meeting
Dec 2020



Based on work with

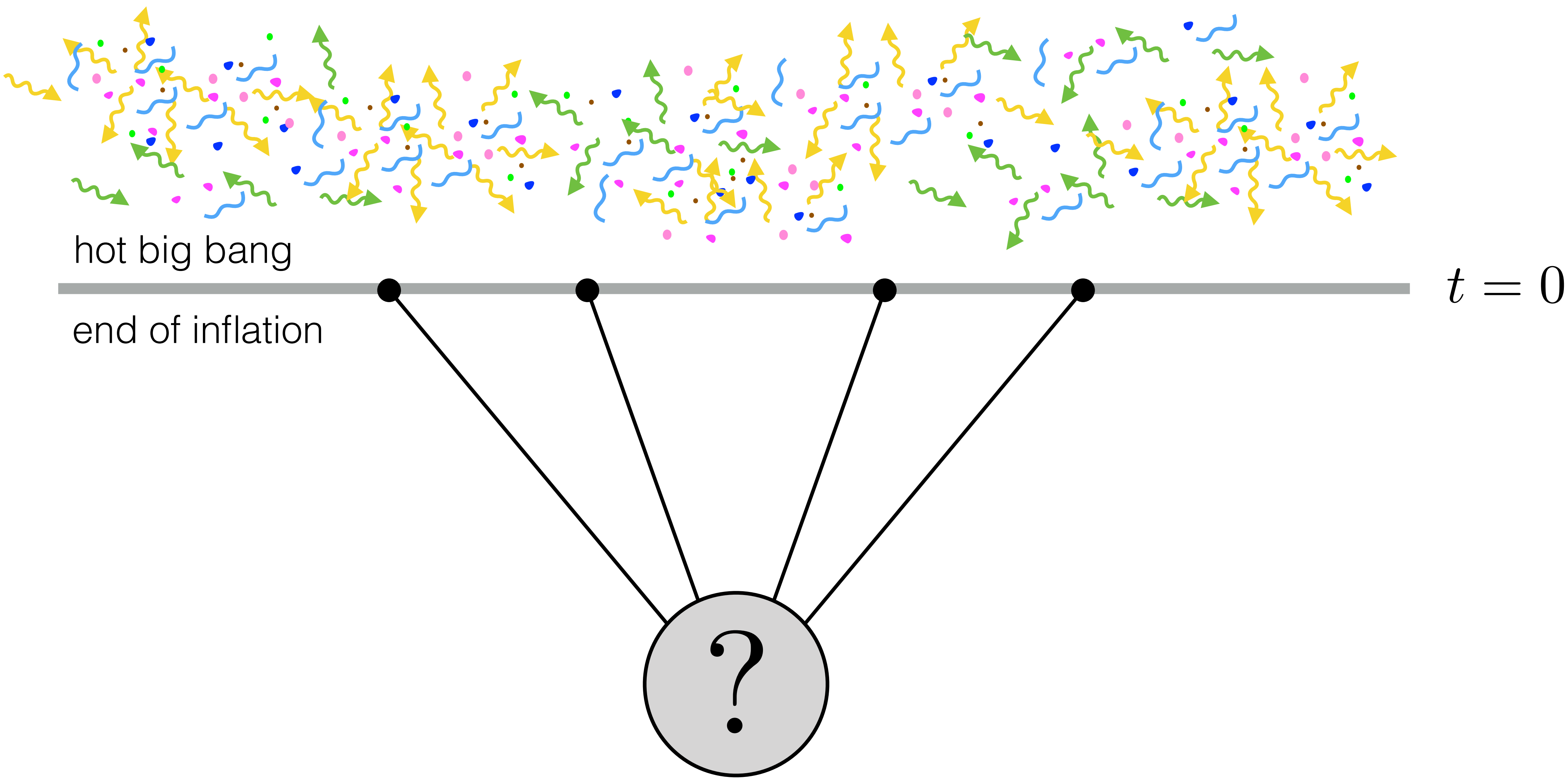
Nima Arkani-Hamed, Wei Ming Chen, Aaron Hillman, Hayden Lee, Manuel Loparco, Guilherme Pimentel, Carlos Duaso Pueyo and Austin Joyce

Cosmological structures are not distributed randomly, but are correlated over very large distances:



The correlations can be traced back to the beginning of the hot Big Bang.

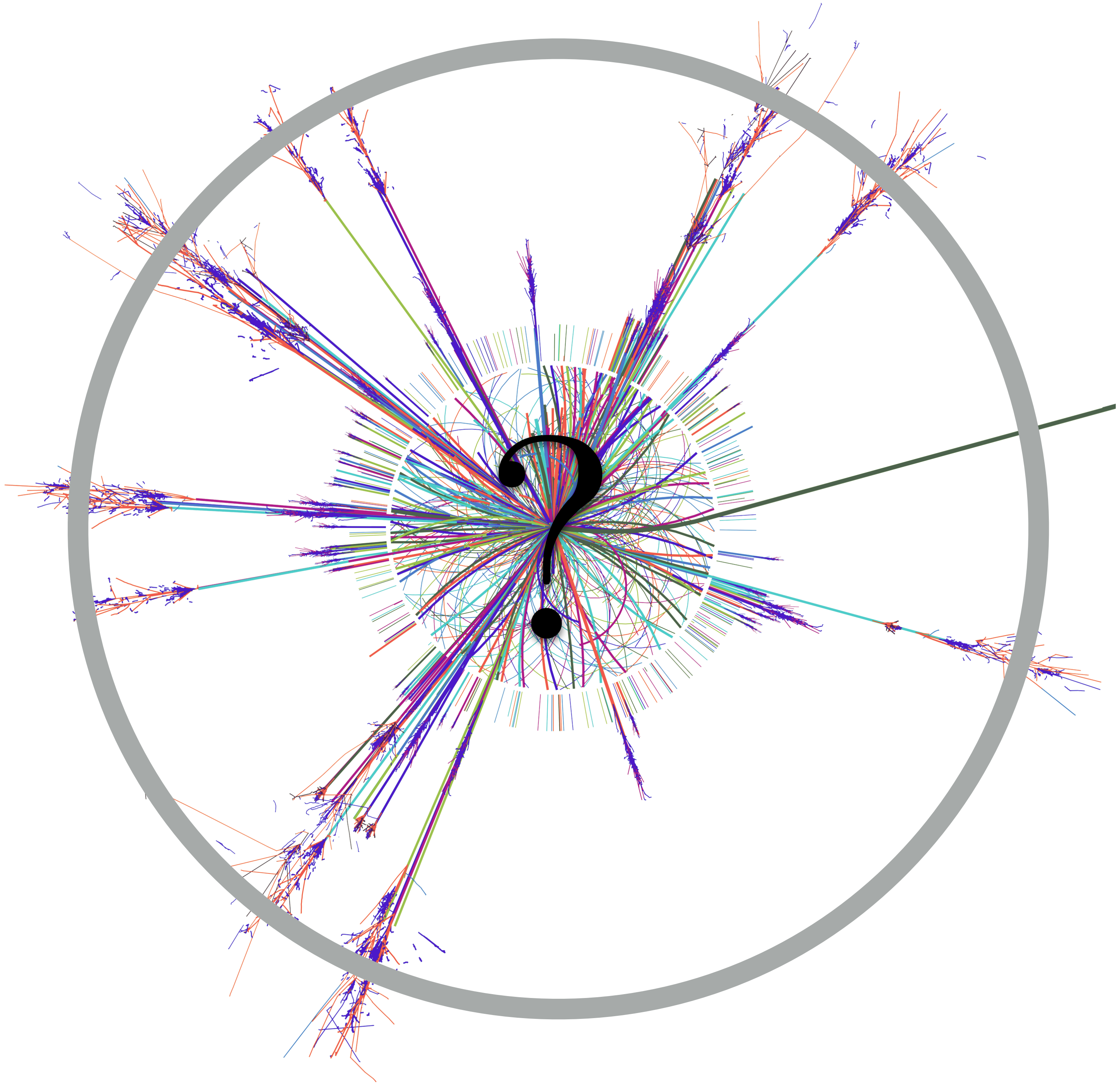
If inflation is correct, then the reheating surface is the future boundary of an approximate de Sitter spacetime:



Can we bootstrap these boundary correlators directly?

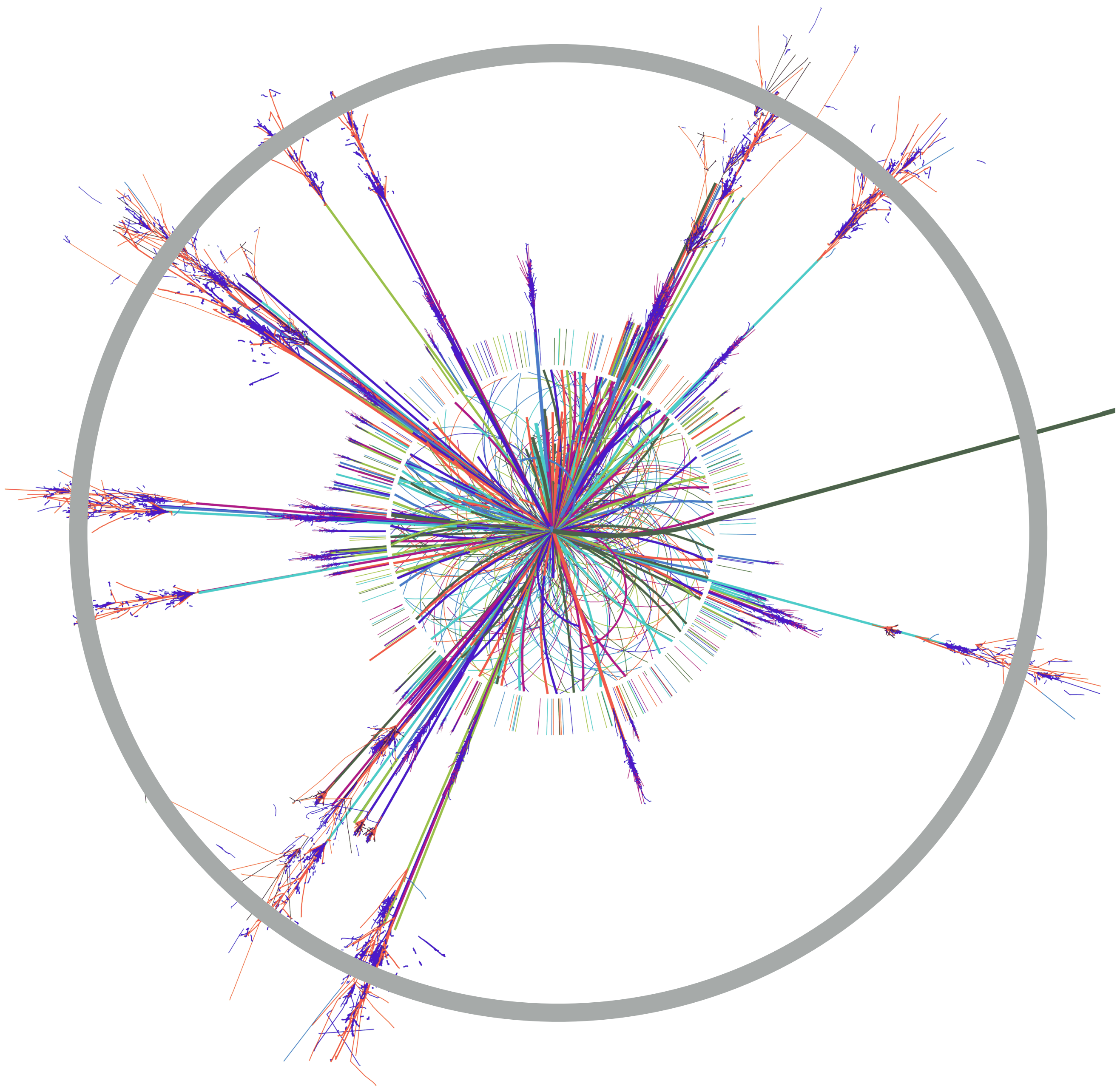
- Conceptual advantage: focus directly on observables.
- Practical advantage: simplify calculations.

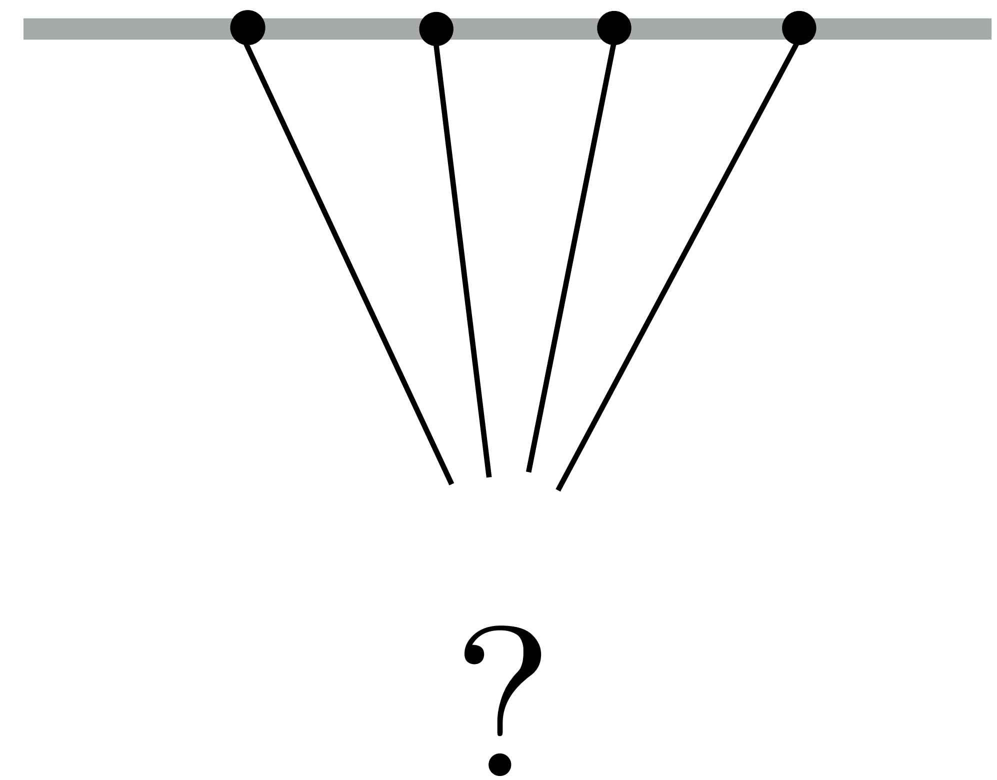
The bootstrap perspective has been very influential for **scattering amplitudes**:



In that case, the rules of **quantum mechanics** and **relativity** are very constraining.

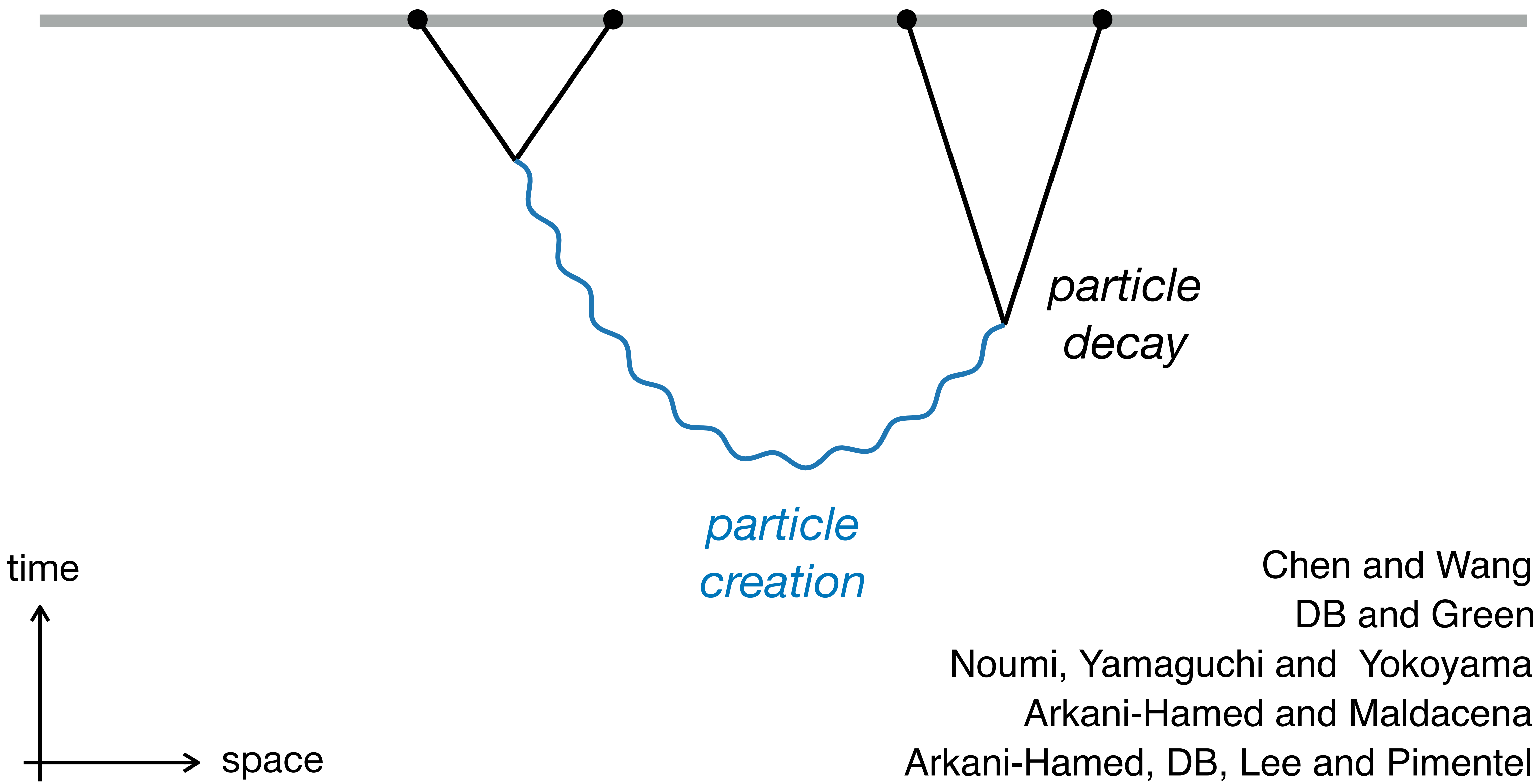
Does a similar **rigidity** exist for cosmological correlators?





Goal: Develop an understanding of cosmological correlators that parallels our understanding of flat-space scattering amplitudes.

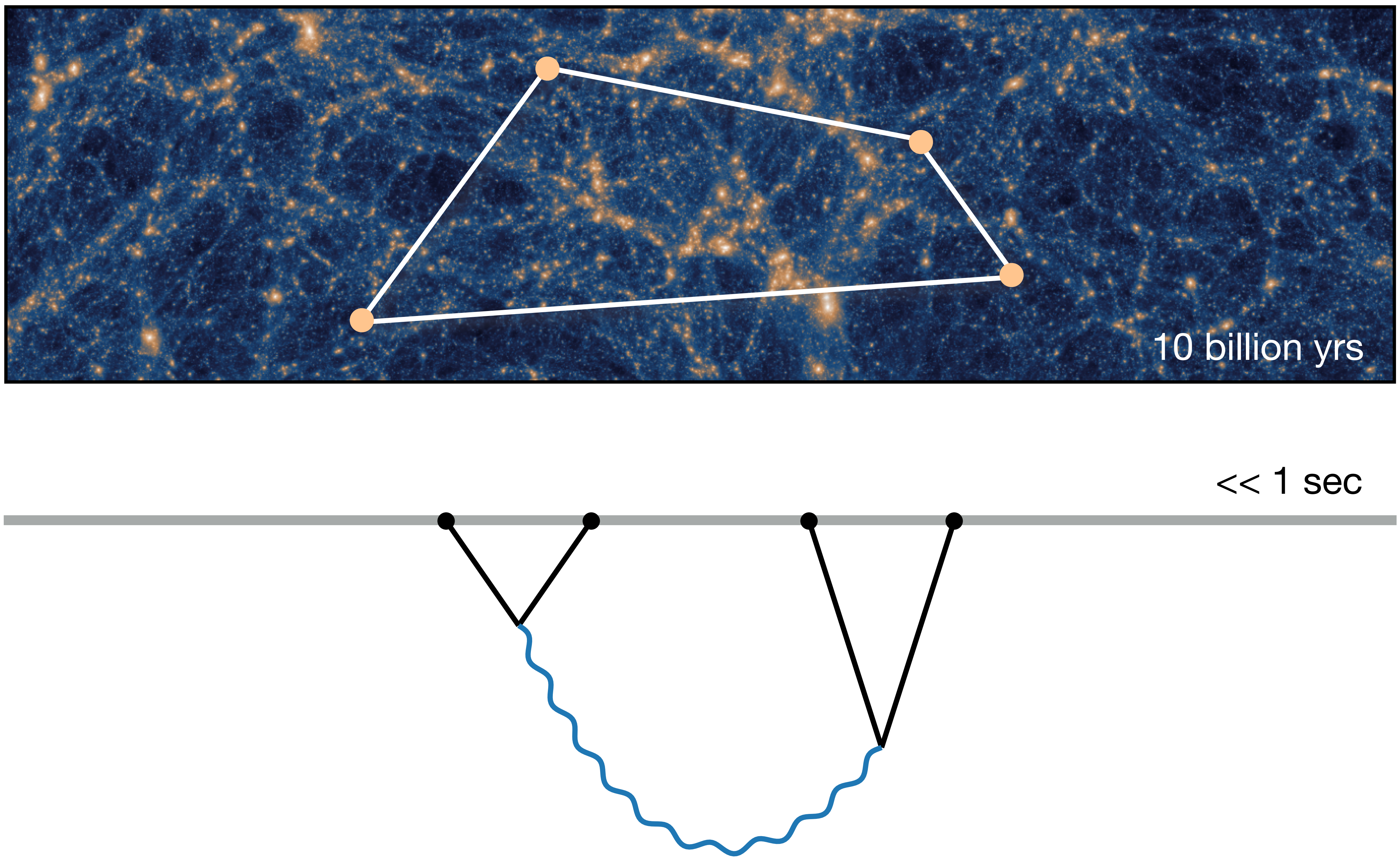
The connection to scattering amplitudes is also relevant because the early universe was like a giant **cosmological collider**:



Chen and Wang [2009]
DB and Green [2011]
Noumi, Yamaguchi and Yokoyama [2013]
Arkani-Hamed and Maldacena [2015]
Arkani-Hamed, DB, Lee and Pimentel [2018]

During inflation, the rapid expansion can produce very **massive particles** ($\sim 10^{14}$ GeV) whose decays lead to nontrivial correlations.

At late times, these correlations would leave an imprint in the distribution of galaxies.



Goal: Develop a systematic way to predict these signals.

Outline

I. Basics of the Bootstrap

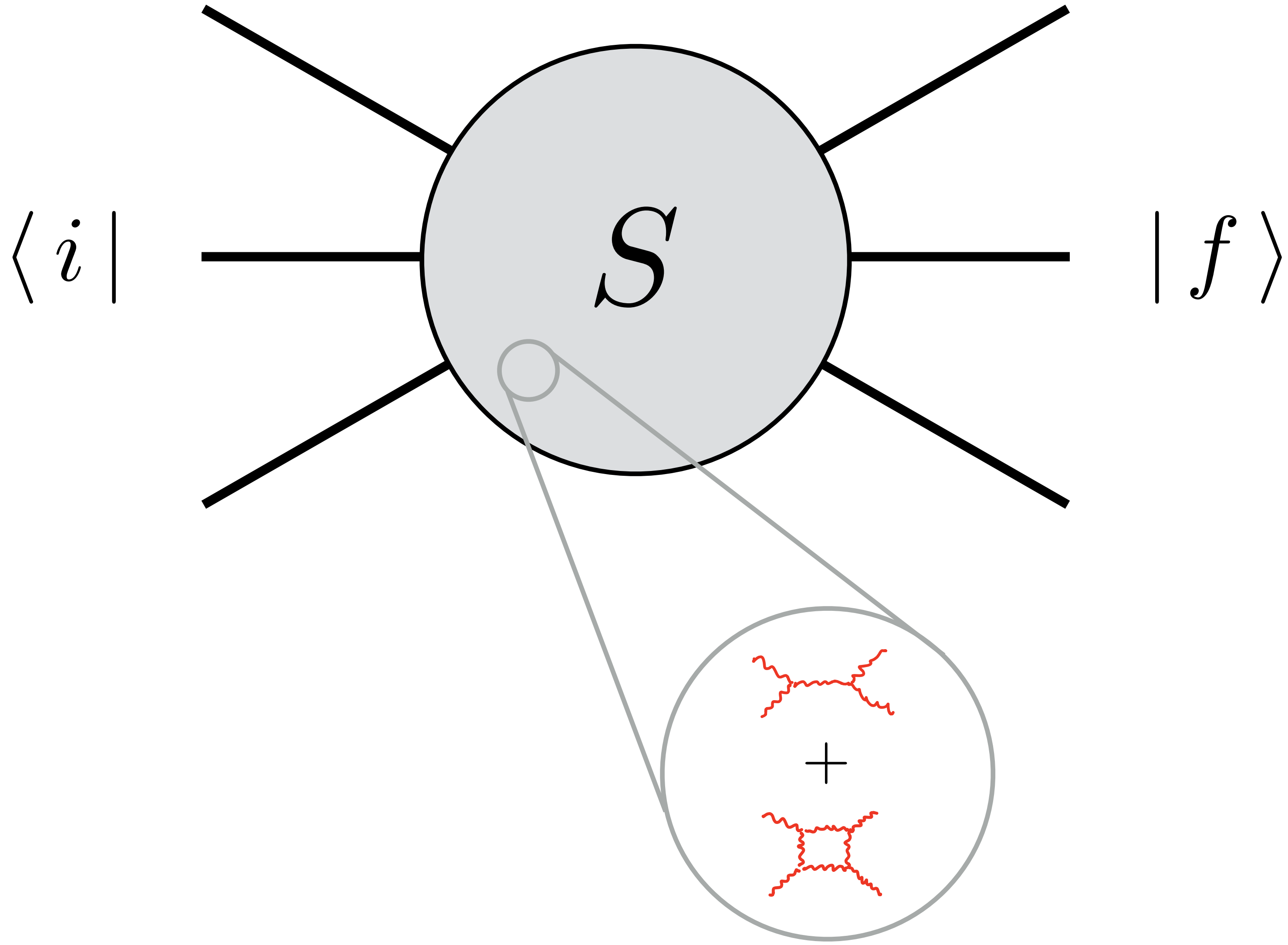
II. Spinning Correlators

III. Summary and Open Problems

I. Basics of the Bootstrap

S-matrix Bootstrap

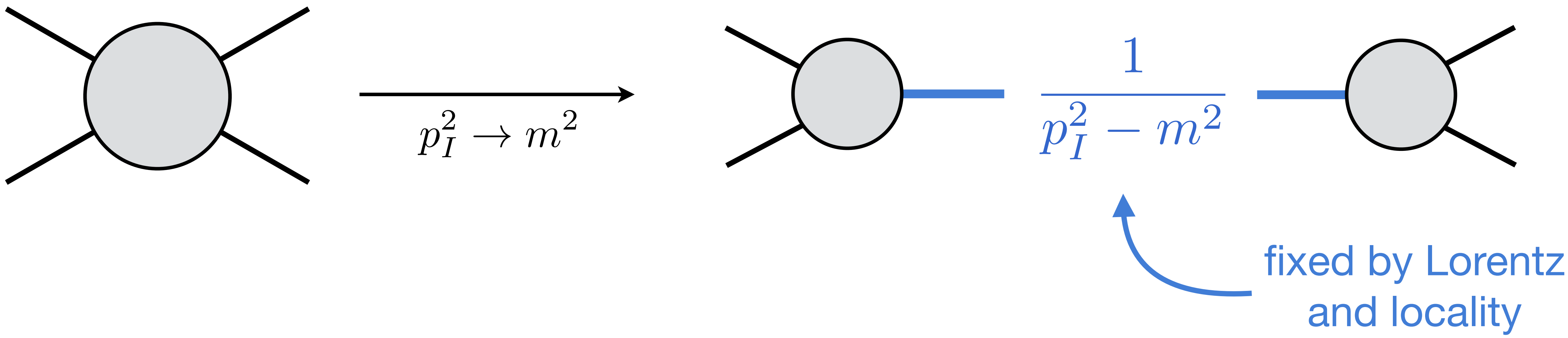
The fundamental observables in particle physics are scattering amplitudes:



Recently, there has be enormous progress in bypassing Feynman diagram expansions to write down on-shell amplitudes directly. [see Yu-tin's book]

S-matrix Bootstrap

Much of the physics of scattering amplitudes is controlled by **singularities**:



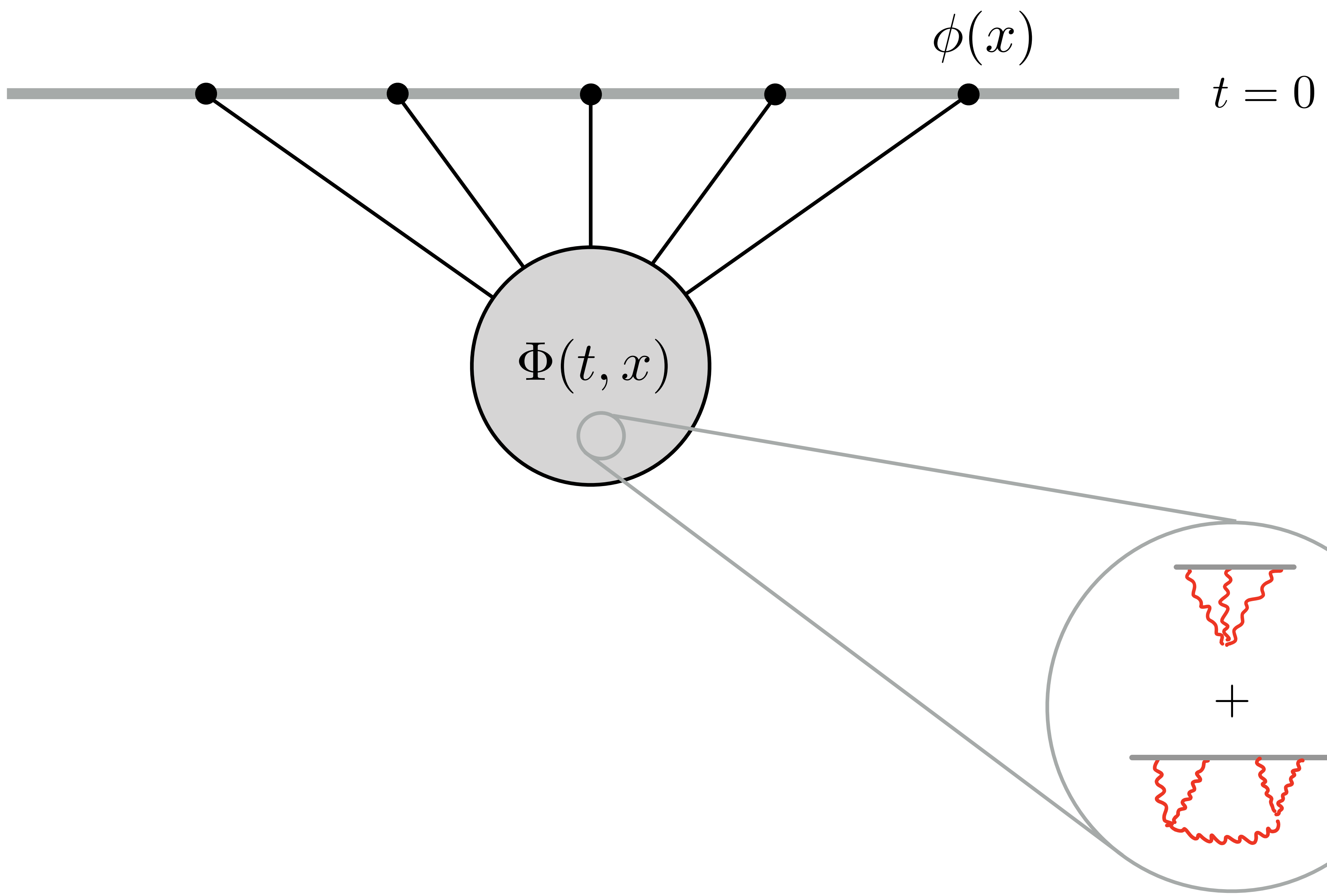
- Amplitudes factorize when intermediate particles go on-shell (← unitarity).
- Consistent factorization is very constraining for massless spinning particles:

spin 2 = **GR**	spin 1 = **YM**

Benincasa and Cachazo [2007]
McGady and Rodina [2010]

Cosmological Correlations

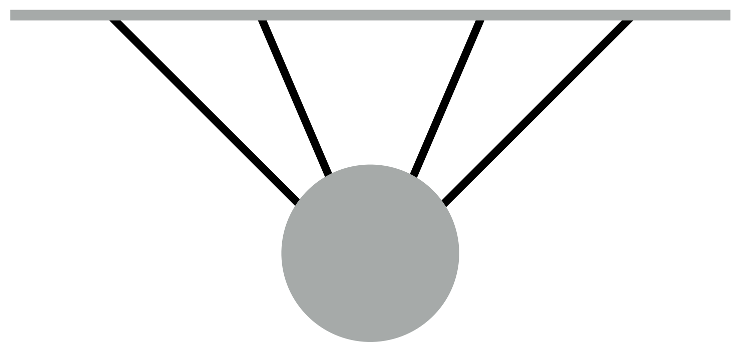
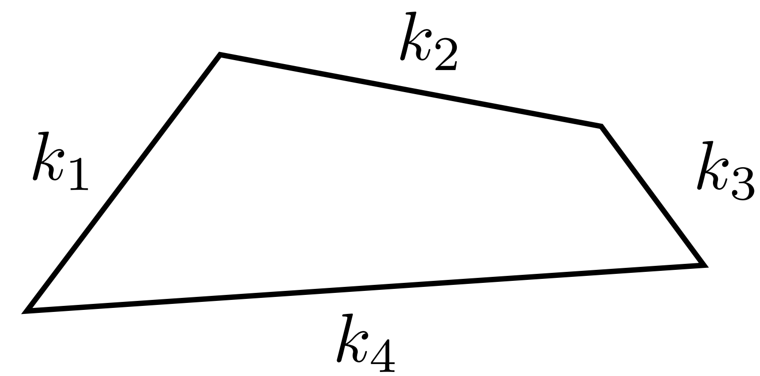
The fundamental observables in cosmology are correlation functions:



Much of the physics of these correlators is also controlled by **singularities**.

Total Energy Singularity

Every correlator has a singularity at vanishing total energy:

$F \equiv$  $=$ 

$$\sim \int_{-\infty}^{0} \mathrm{d}t \; e^{iEt} f(t) = \frac{A}{E^p} + \cdots$$

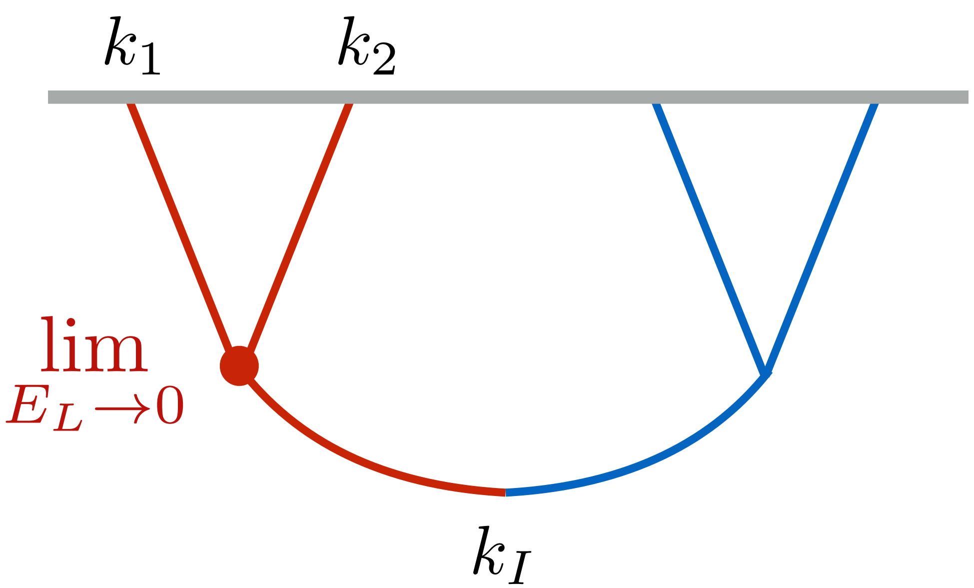
$$E \equiv k_1 + \cdots + k_N$$

- This singularity arises because energy is not conserved in cosmology.
- The residue of the singularity is the corresponding amplitude.

Raju [2012]
Maldacena and Pimentel [2011]

Partial Energy Singularities

Exchange diagrams lead to additional singularities:



$$= \frac{A_L \times F_R}{E_L^q} + \cdots$$

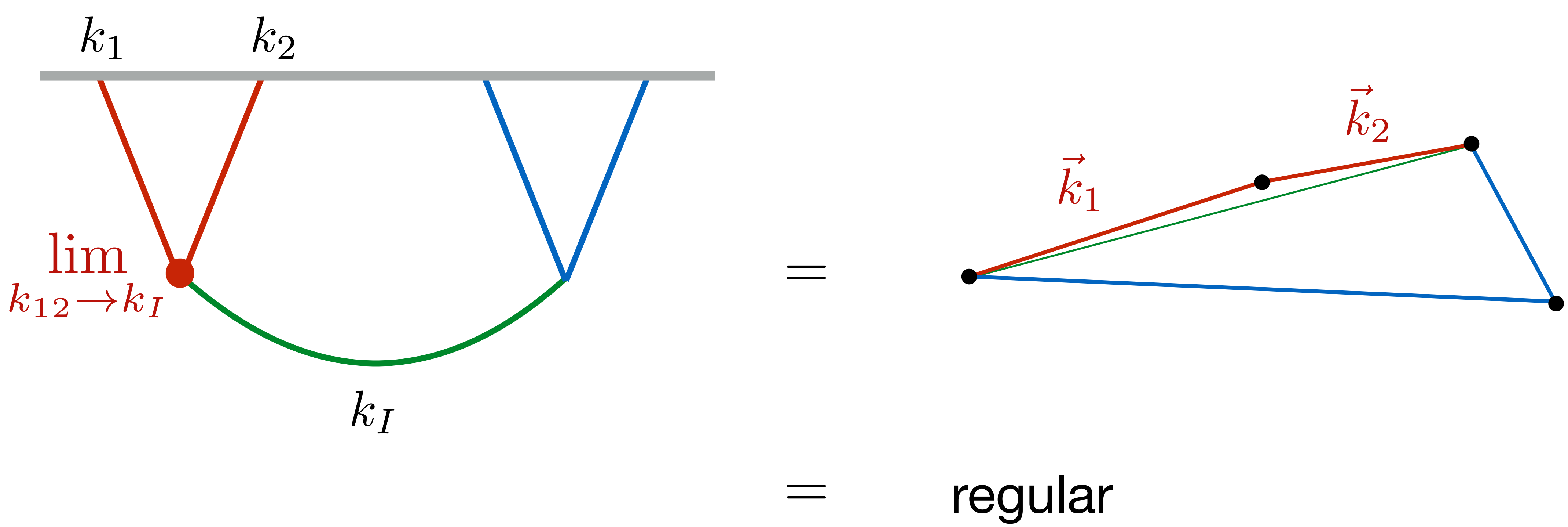
$$E_L = k_1 + k_2 + k_I$$

- This factorisation limit is an important constraint on physical correlators.
- It encodes the signature of **new particles**.

Arkani-Hamed, Benincasa and Postnikov [2017]
DB, Duaso Pueyo, Joyce, Lee and Pimentel [2020]

Folded Singularities

Vacuum initial conditions demand regularity in the folded limit:



- The absence of folded singularities is a nontrivial constraint.
- It encodes the **quantum origin** of the correlations.

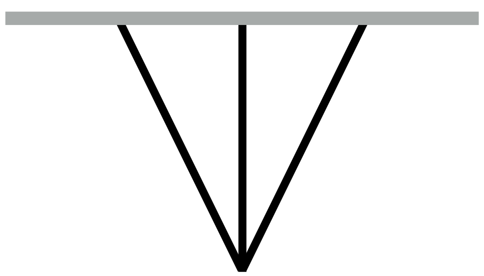
Flauger, Green and Porto [2013]
Arkani-Hamed and Maldacena [2015]
Green and Porto [2020]

Bootstrapping Correlators

Use these energy singularities as an **input** (not an output):

- Three-point correlators are fixed by the total energy singularity:

$$F_3 =$$



$$= \frac{A_3}{E^p}$$

- Symmetries are encoded in A_3 :

Lorentz invariance of A_3 $\longrightarrow$ de Sitter invariance of F_3

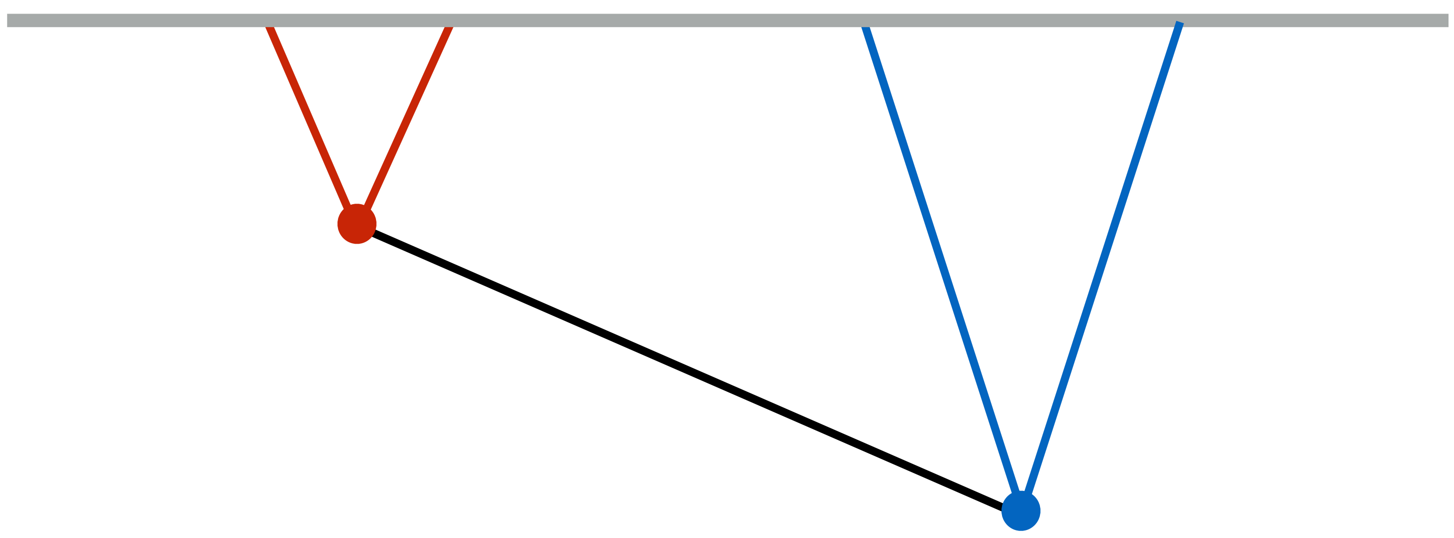
Arkani-Hamed, DB, Lee and Pimentel [2018]

Bootstrapping Correlators

Use these energy singularities as an **input**:

- In simple examples, higher-point correlators are fixed by their partial energy singularities (and the absence of folded singularities).

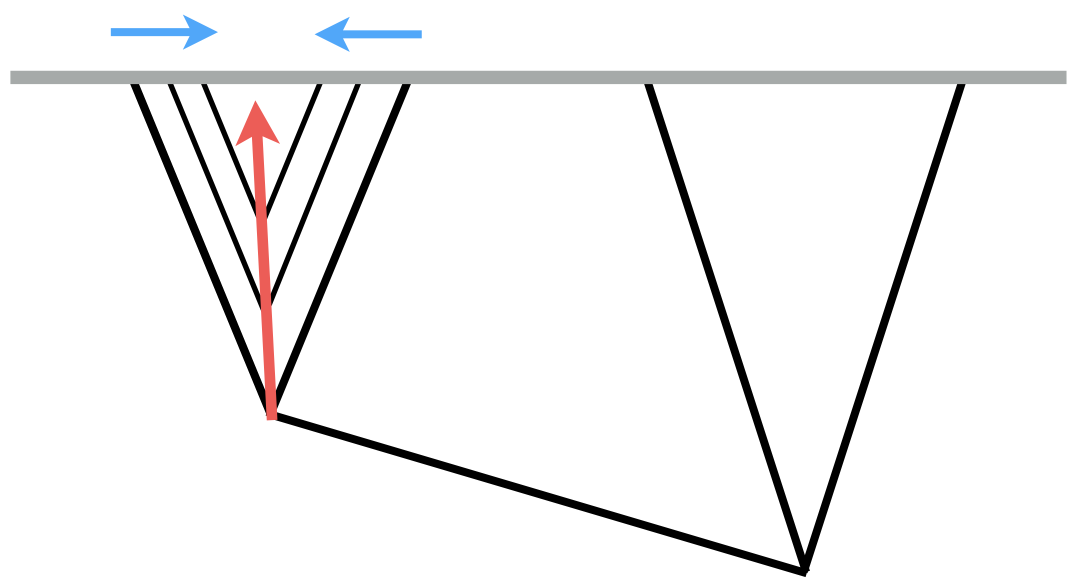
- In general, we need a way to connect the singular limits:

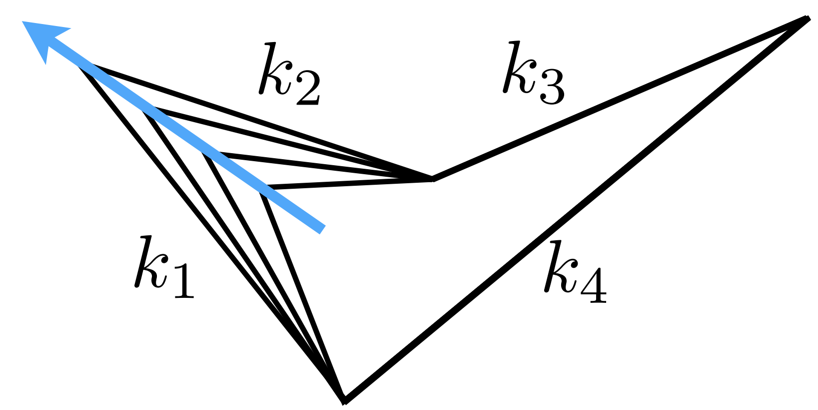


Arkani-Hamed, DB, Lee and Pimentel [2018]

Time Without Time

Time evolution in the bulk = momentum scaling on the boundary:





This implies a differential equation connecting the partial energy singularities:

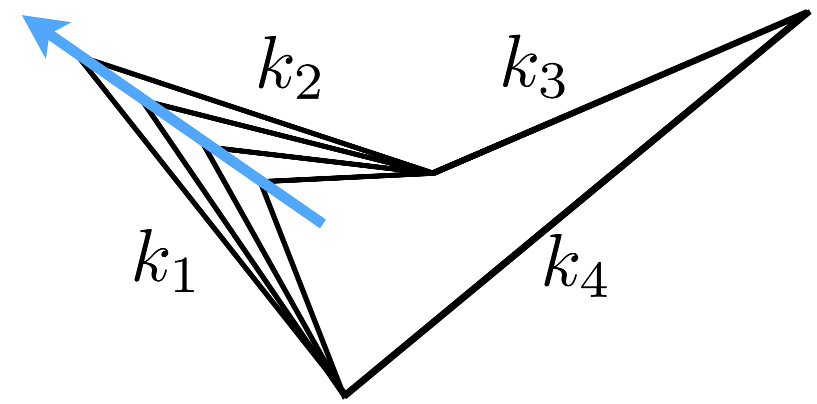
$$\left[(p^2-1)\partial_p^2 + 2p\,\partial_p - M^2\right] F_4 \;=\; C_4 \qquad p \equiv k_I/(k_1+k_2)$$

Arkani-Hamed, DB, Lee and Pimentel [2018]

Time Without Time

Time evolution in the bulk = momentum scaling on the boundary:

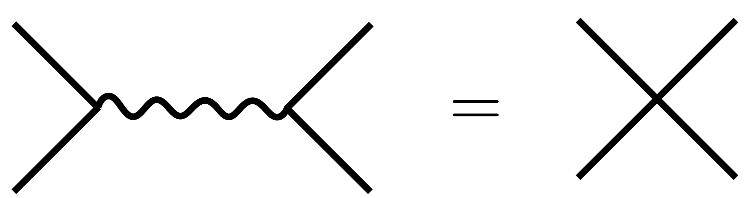
$$\left[(p^2-1)\partial_p^2 + 2p\,\partial_p - M^2\right] F_4 = C_4$$



exchange operator:
kills the partial energy singularity

contact interaction:
determined by the total energy singularity

cf. $(s - M^2)$

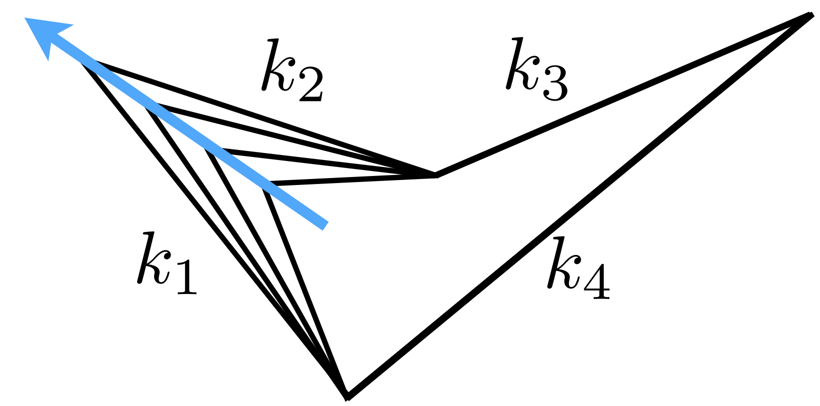


Arkani-Hamed, DB, Lee and Pimentel [2018]

Time Without Time

Time evolution in the bulk = momentum scaling on the boundary:

$$\left[(p^2-1)\partial_p^2 + 2p\,\partial_p - M^2\right] F_4 \;=\; C_4$$



exchange operator:
kills the partial energy singularity

contact interaction:
determined by the total energy singularity

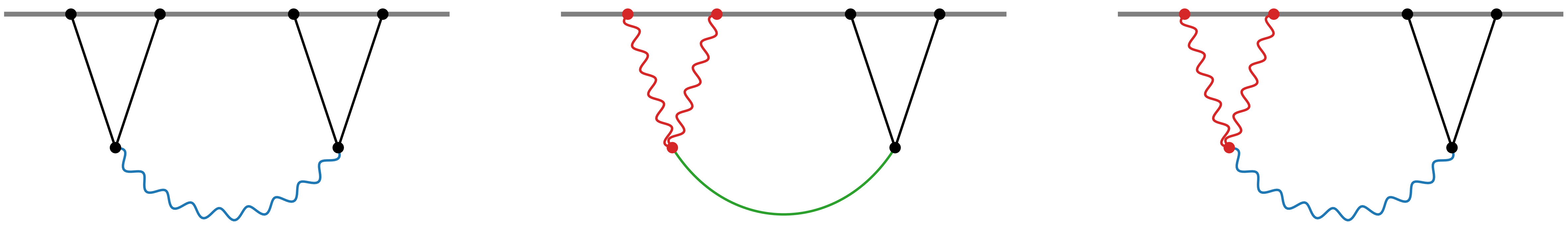
The solutions to the differential equation are specified by

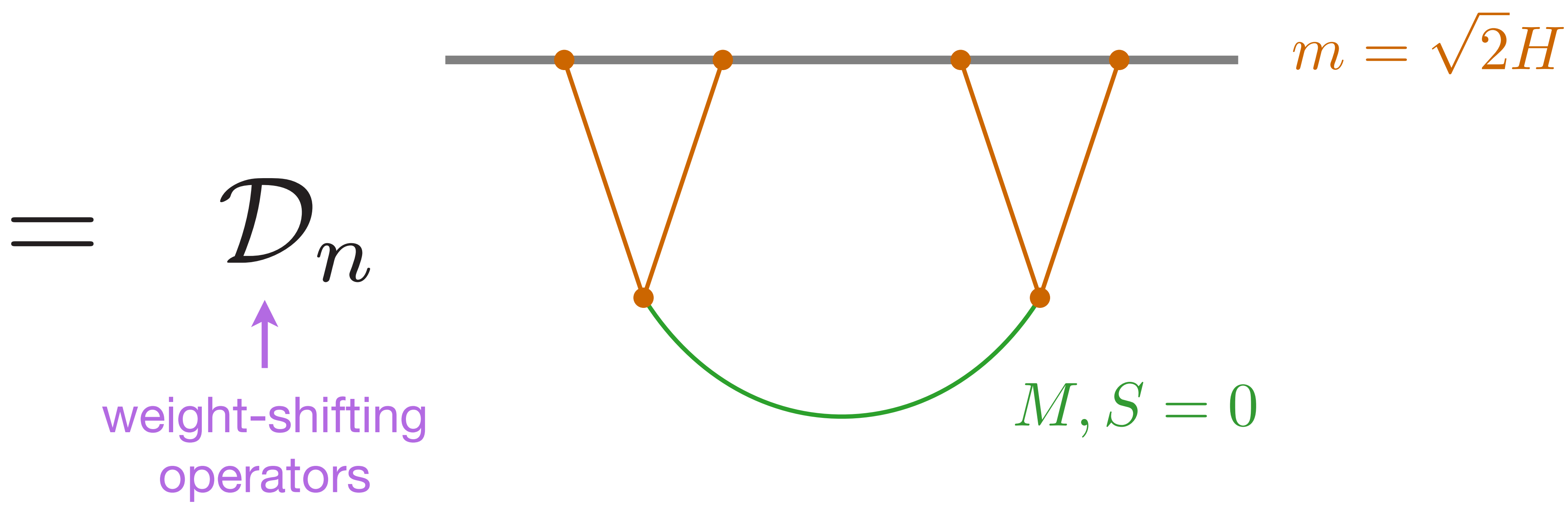
- Imposing the correct total energy singularity.
- Imposing the correct partial energy singularities.
- Requiring the absence of folded singularities.

Arkani-Hamed, DB, Lee and Pimentel [2018]

Exchange Solutions

There are **distinct solutions** for distinct microscopic processes during inflation (different masses and spin for internal and external particles):



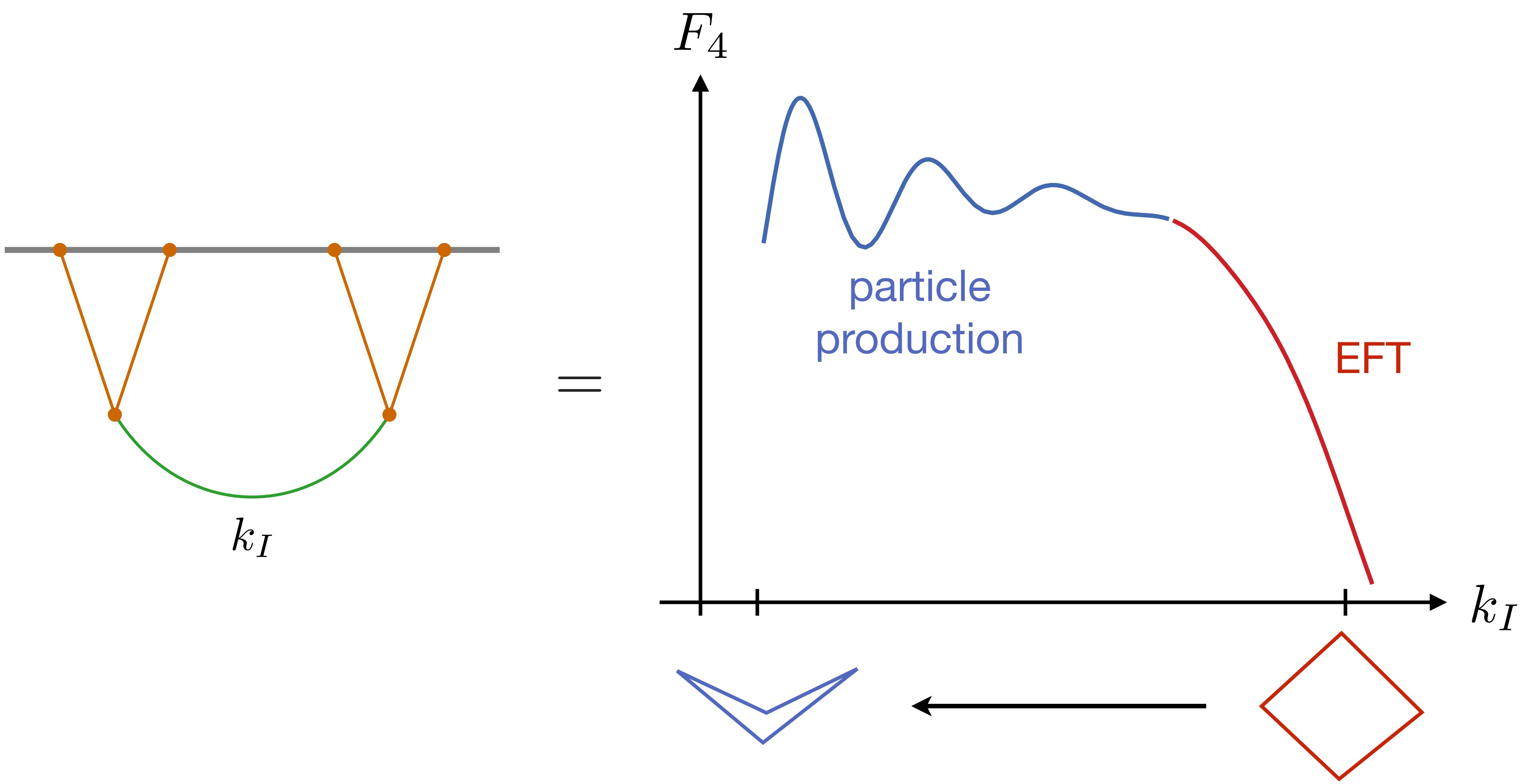


Remarkably, all solutions can be reduced to a **unique building block**.

Arkani-Hamed, DB, Lee and Pimentel [2018]

Seed Solution

Given the allowed singularities, the seed function has a unique solution:

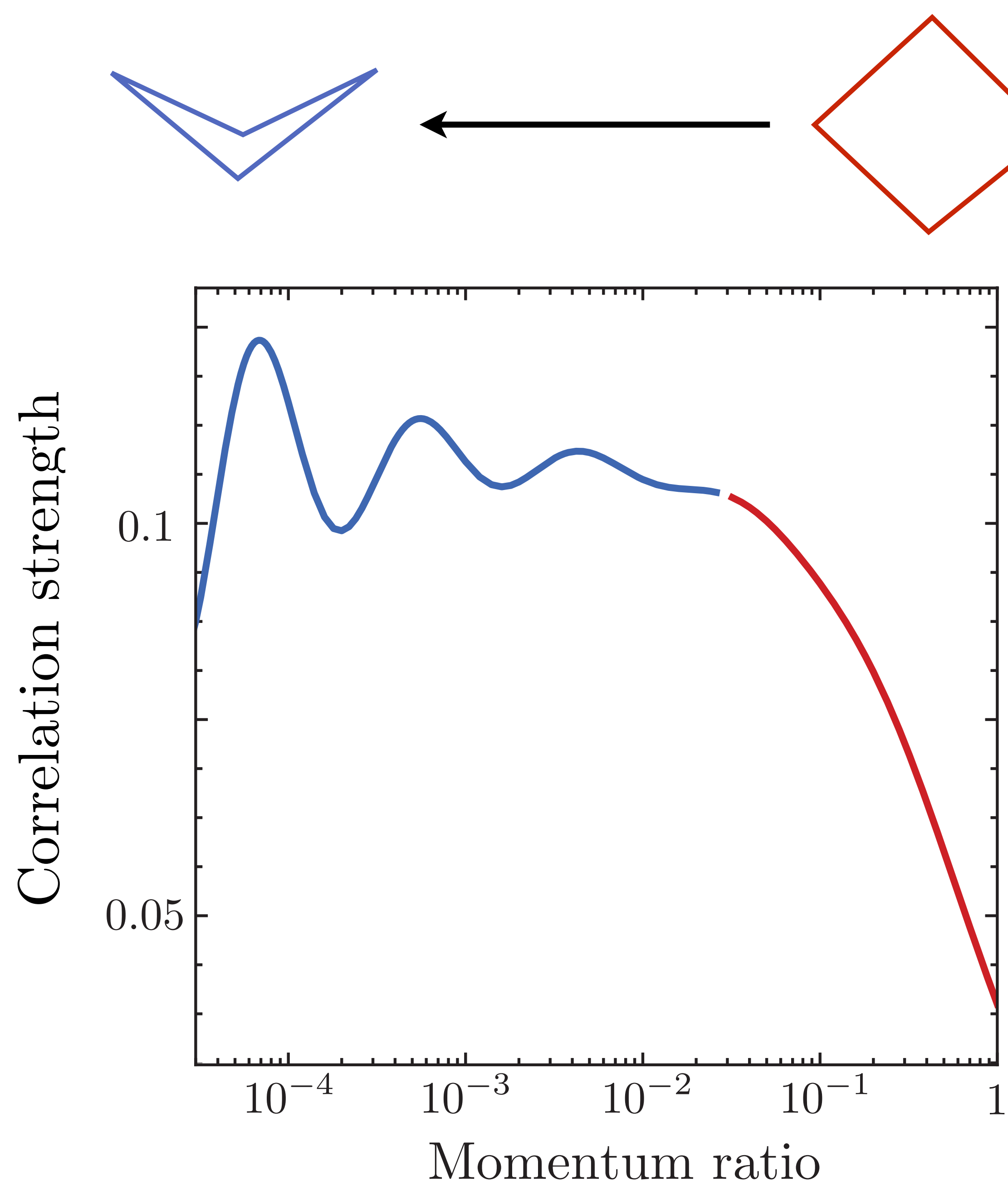


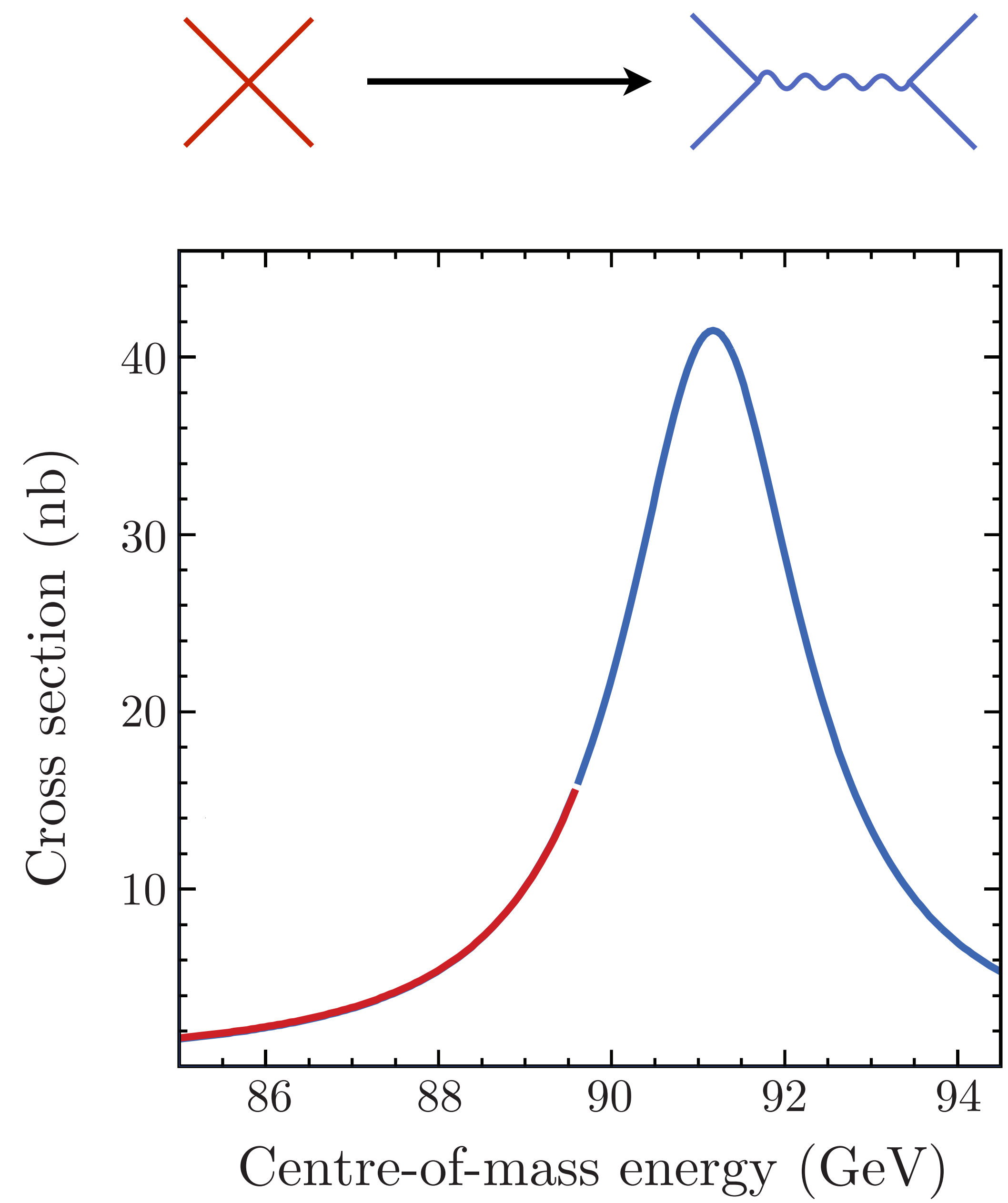
- The particle production piece is forced on us by the factorization limit.
- It dominates in the collapsed limit.

Arkani-Hamed, DB, Lee and Pimentel [2018]

Cosmological Collider Physics

The oscillatory feature is the analog of **resonances** in collider physics:



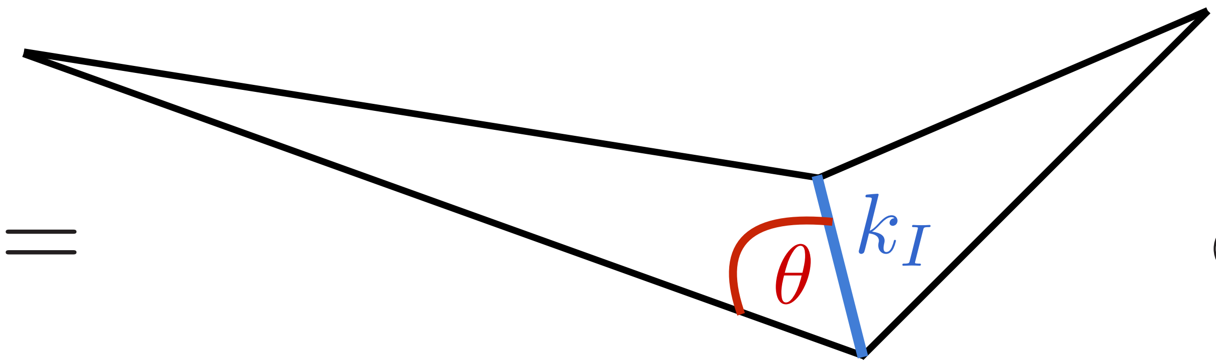


Arkani-Hamed and Maldacena [2015]

Particle Spectroscopy

The frequency of the oscillations depends on the **mass** of the particles:

$F_S =$



$\propto \sin[M \log k_I]\, P_S(\cos\theta)$

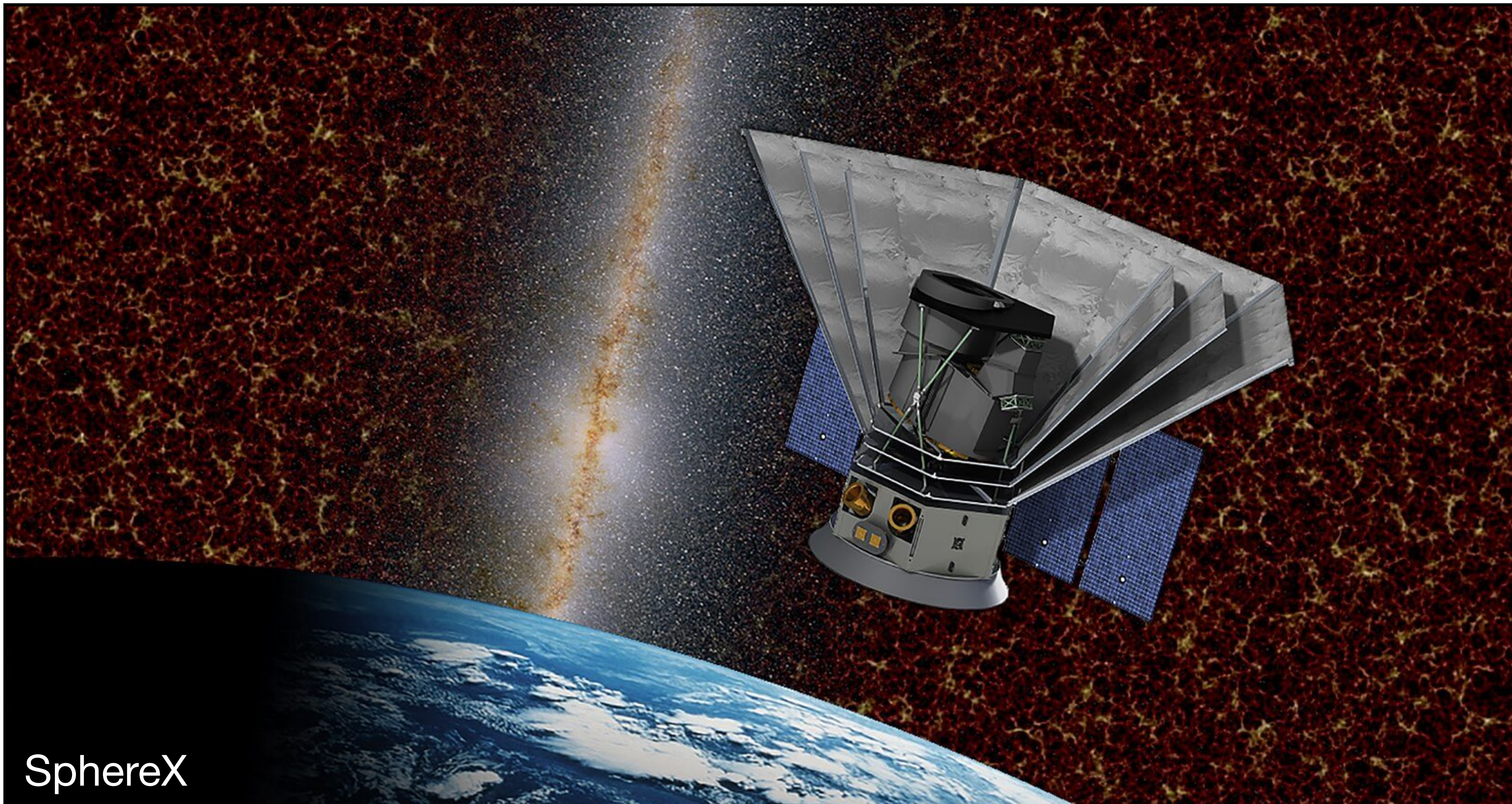
The angular dependence of the signal depends on the **spin** of the particles.

This is very similar to what we do in collider physics:

$$A = \frac{g^2}{s - M^2} P_S(\cos\theta)$$

Observational Prospects

The next generation of galaxy surveys will be sensitive to these signals from the early universe.



II. Spinning Correlators

So far, we have discussed scalar correlators.

Arkani-Hamed, DB, Lee and Pimentel [2018]
DB, Duaso Pueyo, Joyce, Lee and Pimentel [2019]

Now, we would like to extend the bootstrap to **spinning correlators**, especially to **massless** fields with spin.

DB, Duaso Pueyo, Joyce, Lee and Pimentel [2020]

Massless Particles in Inflation

- Fluctuations of all massless fields are amplified during inflation.
- Every inflationary model has two massless modes:

Inflaton	Graviton
ϕ	h_{ij}
Spin-0	Spin-2

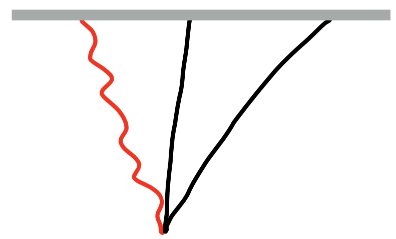
- Not much is known about tensor correlators beyond 3pt functions.
- Direct computations of spinning correlators are very complicated.
- Bootstrap methods are a necessity, not a luxury.

Spinning Correlators

The **three-point functions** of spinning fields can be constructed directly from the corresponding amplitudes.

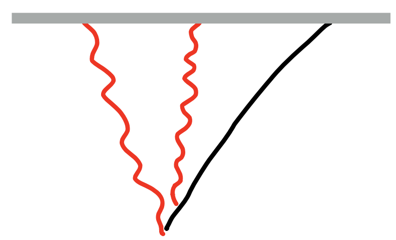
In flat space, we have

$$\langle J_S^- \phi\phi\rangle =$$



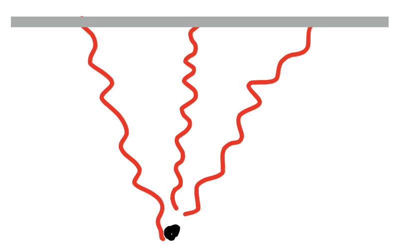
$$= \left(\frac{\langle 12\rangle\langle 13\rangle}{\langle 23\rangle}\right)^S \frac{1}{E}$$

$$\langle J_S^- J_S^- \phi\rangle =$$



$$= \langle 12\rangle^{2S}\frac{1}{E}$$

$$\langle J_S^+ J_S^- J_S^-\rangle =$$



$$= \left(\frac{\langle 23\rangle^3}{\langle 12\rangle\langle 13\rangle}\right)^S \frac{1}{E}$$

spin-S amplitude

total energy pole

DB, Duaso Pueyo, Joyce, Lee and Pimentel [2020]

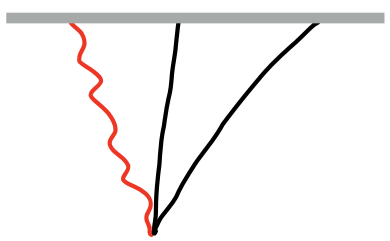
Spinning Correlators

Using the relation between the mode functions

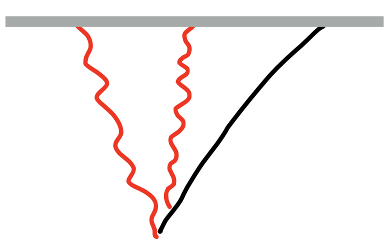
$$\phi^{\mathrm{dS}} = (1 - ikt)e^{ikt} = (1 - k\partial_k)\phi^{\mathrm{flat}}$$

these results can be uplifted to de Sitter space:

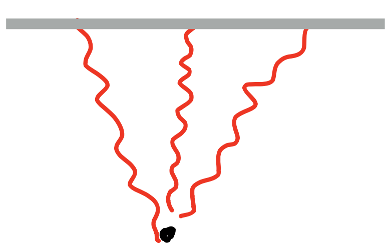
transmutation



$$= \left(\frac{\langle 12\rangle\langle 13\rangle}{\langle 23\rangle}\right)^S \prod_{j=1}^{S-1} \left[(2j-1) - k_1\partial_{k_1}\right] \frac{1}{E}$$



$$= \langle 12\rangle^{2S} \prod_{j=1}^{S-1} \left[(2j-1) - k_1\partial_{k_1}\right]\left[(2j-1) - k_2\partial_{k_2}\right] \frac{1}{E}$$

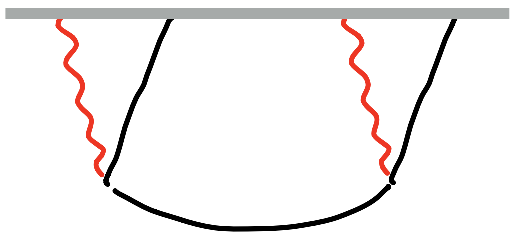


$$= \left(\frac{\langle 23\rangle^3}{\langle 12\rangle\langle 13\rangle}\right)^S \prod_{i=1}^{3}\prod_{j=1}^{S-1} \left[(2j-1) - k_i\partial_{k_i}\right] \left(\int_E^\infty \mathrm{d}E\right)^{2S-2} \frac{1}{E}$$

DB, Duaso Pueyo, Joyce, Lee and Pimentel [2020]

Spinning Correlators

Four-point functions are constrained by consistent factorization.
For example, the s-channel of the gravitational Compton correlator is



fixed by partial energy singularities

$$= (\vec{\xi}_1 \cdot \vec{k}_2)^2 (\vec{\xi}_3 \cdot \vec{k}_4)^2 \left[\frac{1}{E_L^2 E_R^2} \left(\frac{2 k_I k_1 k_3}{E^2} + \frac{2 k_1 k_3 + E_L k_3 + E_R k_1}{E} \right) \frac{1}{E_L E_R} \left(\frac{2 k_1 k_3}{E^3} + \frac{k_{13}}{E^2} + \frac{1}{E} \right) \right]$$

fixed by total energy singularity

unfixed

Fixing the subleading poles requires additional input.

DB, Duaso Pueyo, Joyce, Lee and Pimentel [2020]

Constraints from Unitarity

Unitarity in the bulk = factorization on the boundary:

$$F_4(k_i) + F_4^*(-k_i) = 2k_I^{2\Delta_I - 3}\tilde{F}_3^L \otimes \tilde{F}_3^R$$

cosmological optical theorem

Goodhew, Jazayeri and Pajer [2020]
Melzer and Sivaramakrishnan [2020]
Cespedes, Davis and Melville [2020]

shifted correlator

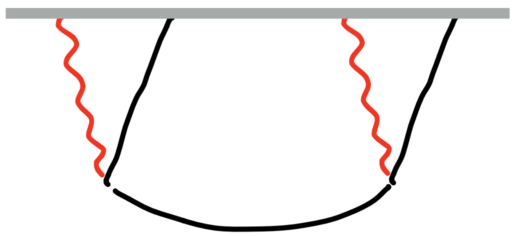
$$\tilde{F}_3^R \equiv \frac{1}{2k_I}\Big(F_3(k_{34} - k_I) - F_3(k_{34} + k_I)\Big)$$

- Unitarity requires consistent factorization away from the energy poles.
- This fixes the subleading singularities up to contact terms.
- Contact terms are fixed by the Ward-Takahashi identity.

DB, Chen, Joyce and others [2020]

From Flat Space to de Sitter

The correlator can also be obtained by uplifting the flat space result to de Sitter:



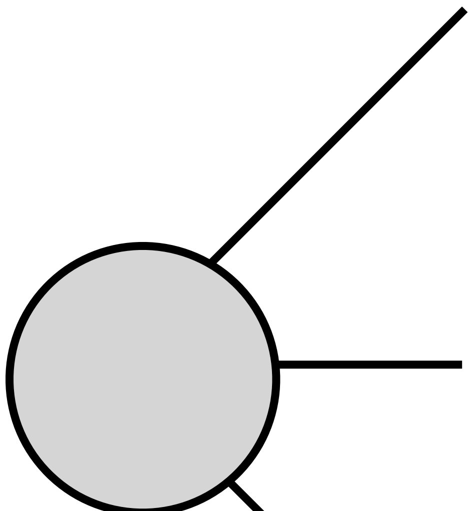
$$= (\vec{\xi}_1 \cdot \vec{k}_2)^2 (\vec{\xi}_3 \cdot \vec{k}_4)^2 \left[1 - k_1 \partial_{k_1}\right]\left[1 - k_3 \partial_{k_3}\right] \frac{1}{E E_L E_R}$$

$$= (\vec{\xi}_1 \cdot \vec{k}_2)^2 (\vec{\xi}_3 \cdot \vec{k}_4)^2 \left[\frac{1}{E_L^2 E_R^2}\left(\frac{2k_I k_1 k_3}{E^2} + \frac{2k_1 k_3 + E_L k_3 + E_R k_1}{E}\right) \frac{1}{E_L E_R}\left(\frac{2k_1 k_3}{E^3} + \frac{k_{13}}{E^2} + \frac{1}{E}\right)\right]$$

The physics is controlled by the leading-order poles. The subleading poles are linked to them by symmetry.

III. Summary and Open Problems

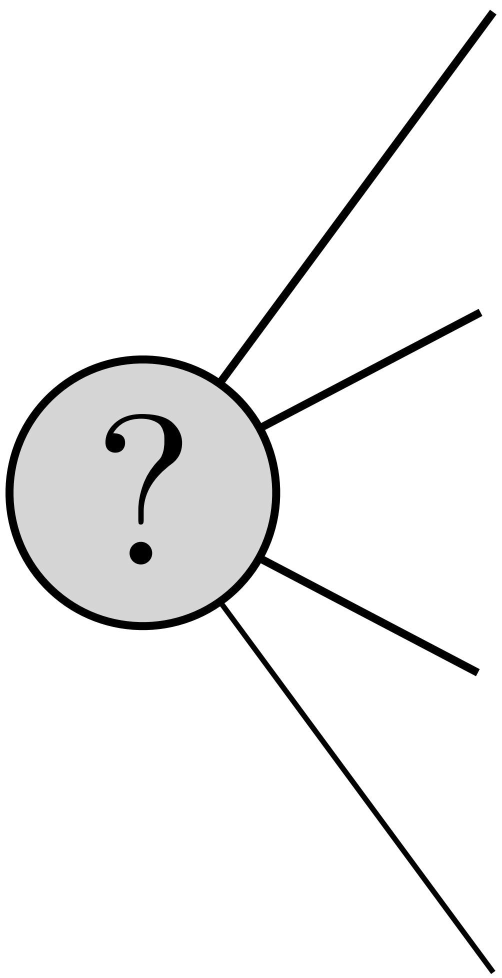
Summary



- **Singularities** control much of the structure of cosmological correlators.
- Simple building blocks are connected by **unitarity** and **local evolution**.
- **Simple seeds** can be transmuted into more complex correlators.

We now have a large amount of **theoretical data** to analyze.
Are there hidden structures to be discovered?

Open Problems



- The bootstrap is still performed channel by channel. Can we go **beyond Feynman diagrams**?
- Most explicit results are for dS-invariant correlators. Can we classify **boost-breaking correlators**?
- Can we go **beyond perturbation theory**? What are the nonperturbative bootstrap constraints?
- What are the extra constraints imposed by **ultraviolet completion**?

Thank you for your attention!