THE SILENCE OF BINARY KERR

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w. Rafael Aoude, Ming-Zhi Chung, Camila S. Machado, ManKuan Tam

Phys.Rev.Lett. 125 (2020) 18, 181602

(& earlier work w N. Arkani-Hamed, Tzu-Chen Huang, Jung-Wook Kim, and Sangmin Lee)

2020 NCTS Annual meeting (Dec 9th)

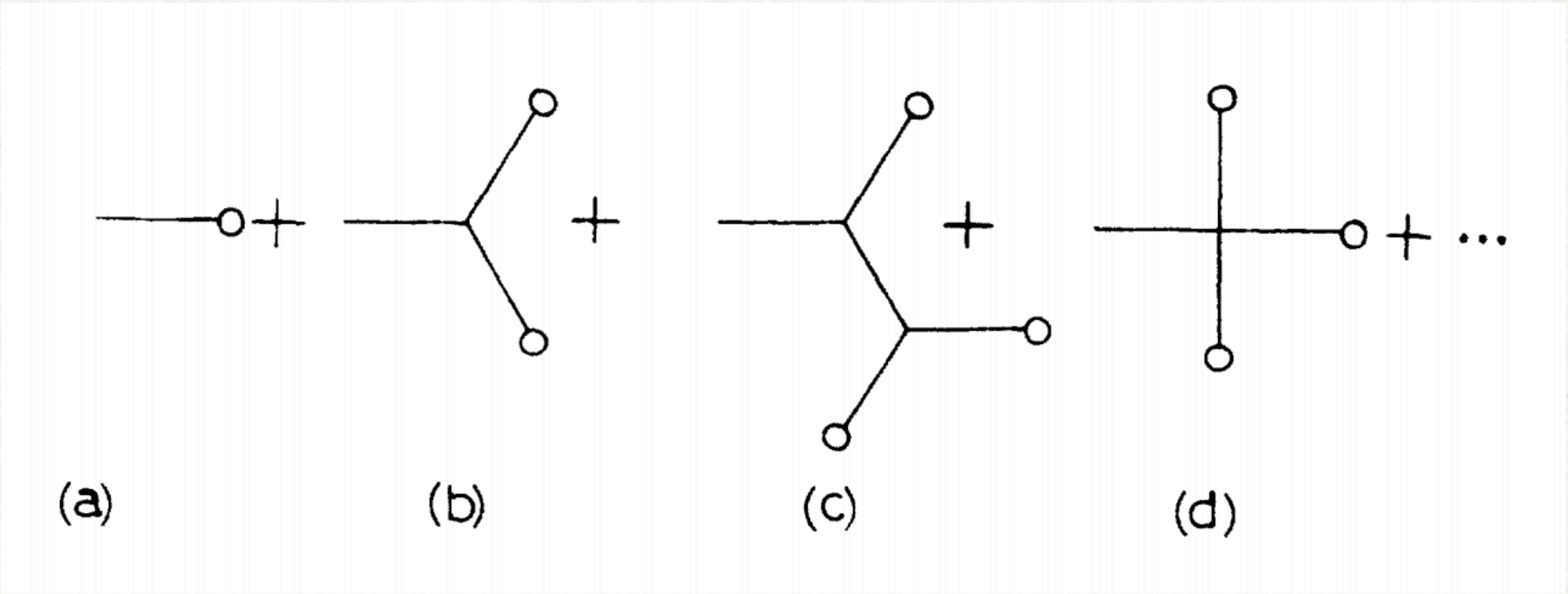
There has been a long history of development in the extraction of classical quantities from observables of quantum field theory

Vev of the gravitational field $\longrightarrow$ classical Schwarzschild solution

PHYSICAL REVIEW D VOLUME 7, NUMBER 8 15 APRIL 1973

Quantum Tree Graphs and the Schwarzschild Solution

M. J. Duff*
Physics Department, Imperial College, London SW7, England
(Received 7 July 1972)

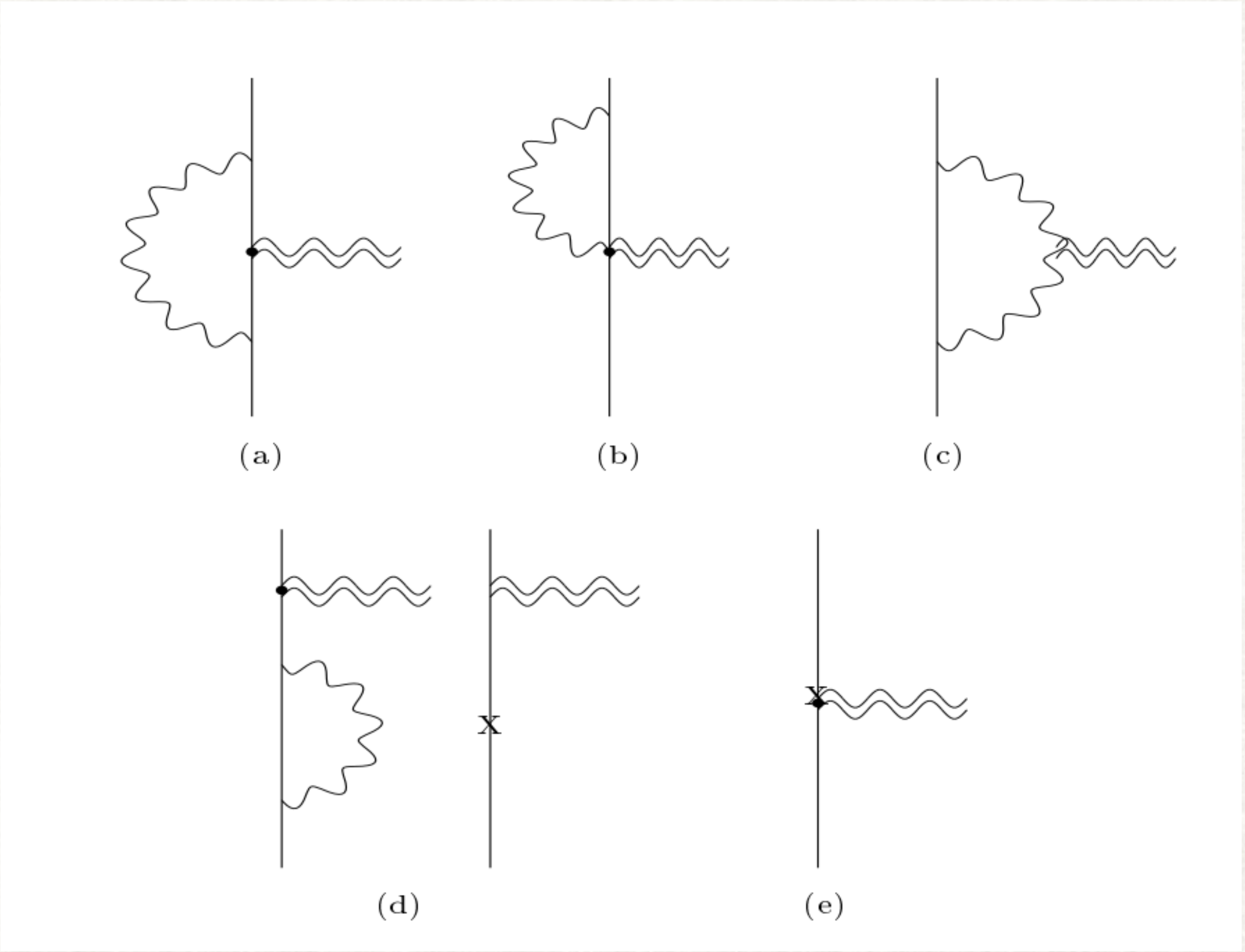


$$g^{ij} = \left(1 - \frac{2Gm}{r} + \frac{3G^2m^2}{r^2}\right)\eta^{ij} - \frac{G^2m^2}{r^2}\frac{x^i x^j}{r^2} + O(G^3)$$

The stress-tensor form factors $\longrightarrow$ Kerr Newman solution

J. F. DONOGHUE, B. R. HOLSTEIN, B. GARBRECHT AND T. KONSTANDIN
PHYS. LETT. B529 (2002) 132–142

N. E. J. BJERRUM-BOHR, J. F. DONOGHUE AND B. R. HOLSTEIN
PHYS. REV. D68 (2003) 084005



$$\begin{aligned} < p_2|T_{\mu\nu}|p_1 > &= \bar{u}(p_2)\Big[F_1(q^2)P_\mu P_\nu \frac{1}{m} \\ &- F_2(q^2)\big(\frac{i}{4m}\sigma_{\mu\lambda}q^\lambda P_\nu + \frac{i}{4m}\sigma_{\nu\lambda}q^\lambda P_\mu\big) \\ &+ F_3(q^2)(q_\mu q_\nu - g_{\mu\nu}q^2)\frac{1}{m}\Big] u(p_1) \end{aligned}$$

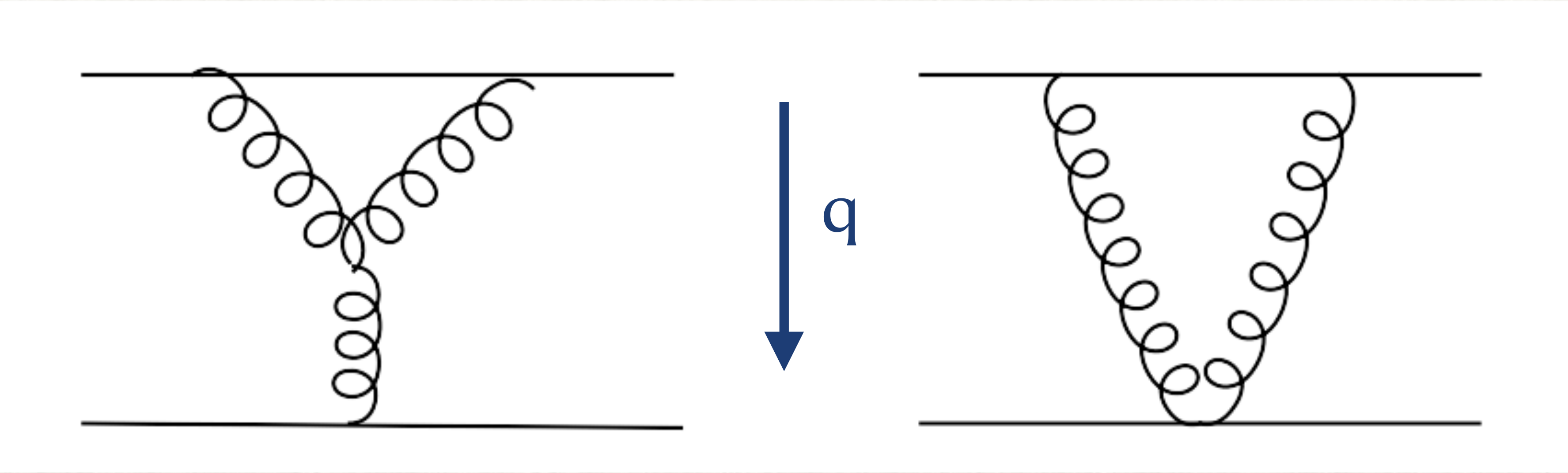
There has been a long history of development in the extraction of classical quantities from observables of quantum field theory

The on-shell scattering amplitude $\longrightarrow$ the conservative potential

D. NEILL , I. Z. ROTHSTEIN
NUCL. PHYS. B877 (2013) 177–189,

C. CHEUNG, I. Z. ROTHSTEIN AND M. P. SOLON
PHYS. REV. LETT. 121 (2018) 251101,

N. E. J. BJERRUM-BOHR, P. H. DAMGAARD, G. FESTUCCIA, L. PLANT´E AND P. VANHOVE
PHYS. REV. LETT. 121, 171601



$$iM = -iV$$

$$V_i^{cl}(\vec{r},\vec{p}) = \lim_{|\vec{r}\times\vec{p}|\to\infty} \int \frac{d^3\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} V_i(\vec{q},\vec{p})$$

$$\begin{aligned} V = &-\frac{G_N m_1 m_2}{r}\left(1+\frac{\vec{p}^2}{m_1 m_2}\left(1+\frac{3(m_1+m_2)^2}{2m_1 m_2}\right)\right) \\ &+\frac{G_N^2 m_1 m_2 (m_1+m_2)}{2r^2}\left(1+\frac{m_1 m_2}{(m_1+m_2)^2}\right) \\ &+\frac{G_N^2}{4r^2}\frac{\vec{p}^2}{m_1 m_2}\left(\frac{117 m_1^2 m_2^2 + 67(m_1 m_2^3 + m_1^3 m_2) + 10(m_1^4+m_2^4)}{m_1+m_2}\right) \end{aligned}$$

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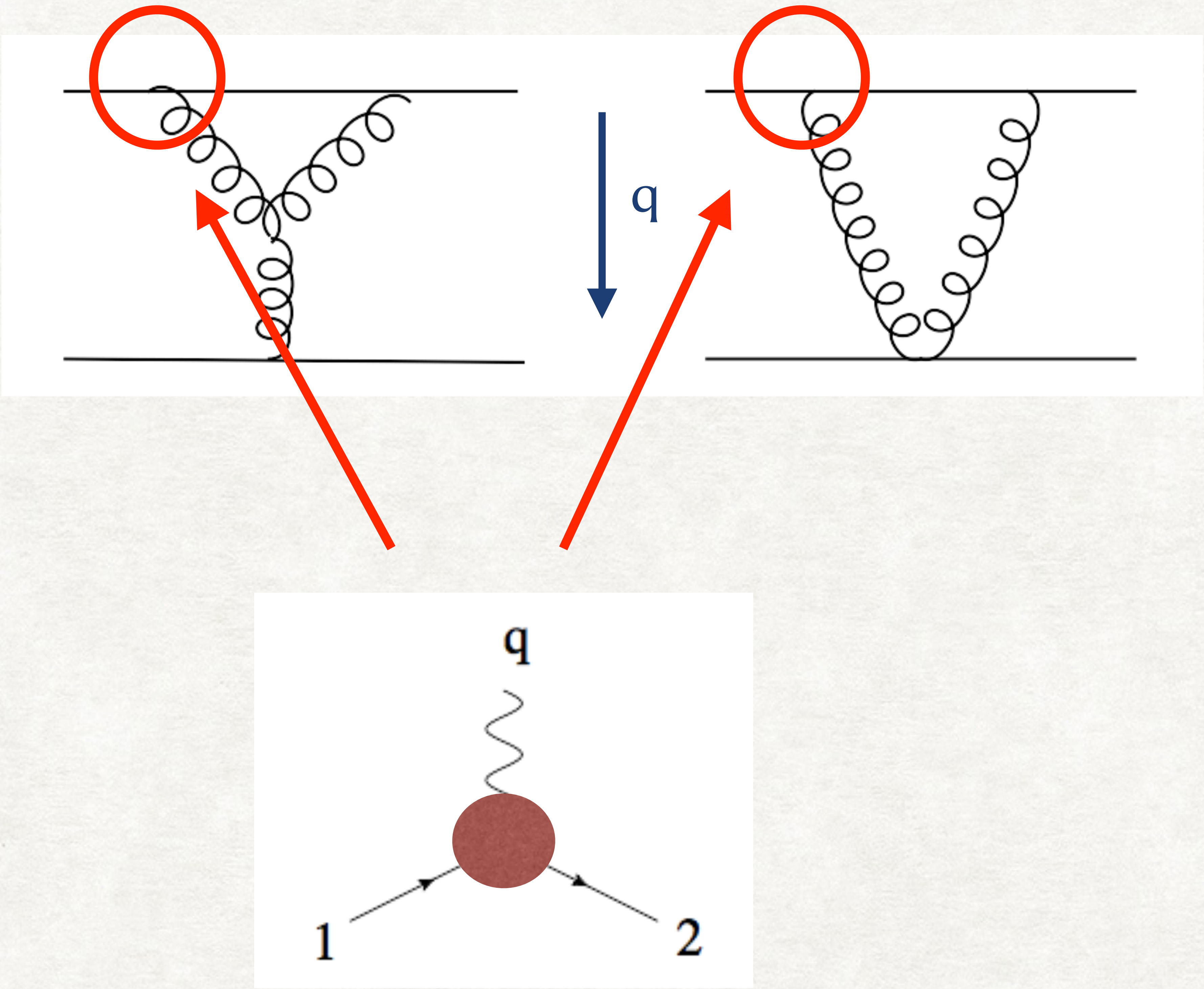

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$$V_i^{cl}(\vec{r},\vec{p}) = \lim_{|\vec{r}\times\vec{p}|\to\infty} \int \frac{d^3\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} V_i(\vec{q},\vec{p})$$

$$V = -\frac{G_N m_1 m_2}{r}\left(1+\frac{\vec{p}^2}{m_1 m_2}\left(1+\frac{3(m_1+m_2)^2}{2m_1 m_2}\right)\right) + \frac{G_N^2 m_1 m_2 (m_1+m_2)}{2r^2}\left(1+\frac{m_1 m_2}{(m_1+m_2)^2}\right) + \frac{G_N^2}{4r^2}\frac{\vec{p}^2}{m_1 m_2}\left(\frac{117 m_1^2 m_2^2 + 67(m_1 m_2^3 + m_1^3 m_2) + 10(m_1^4+m_2^4)}{m_1+m_2}\right)$$

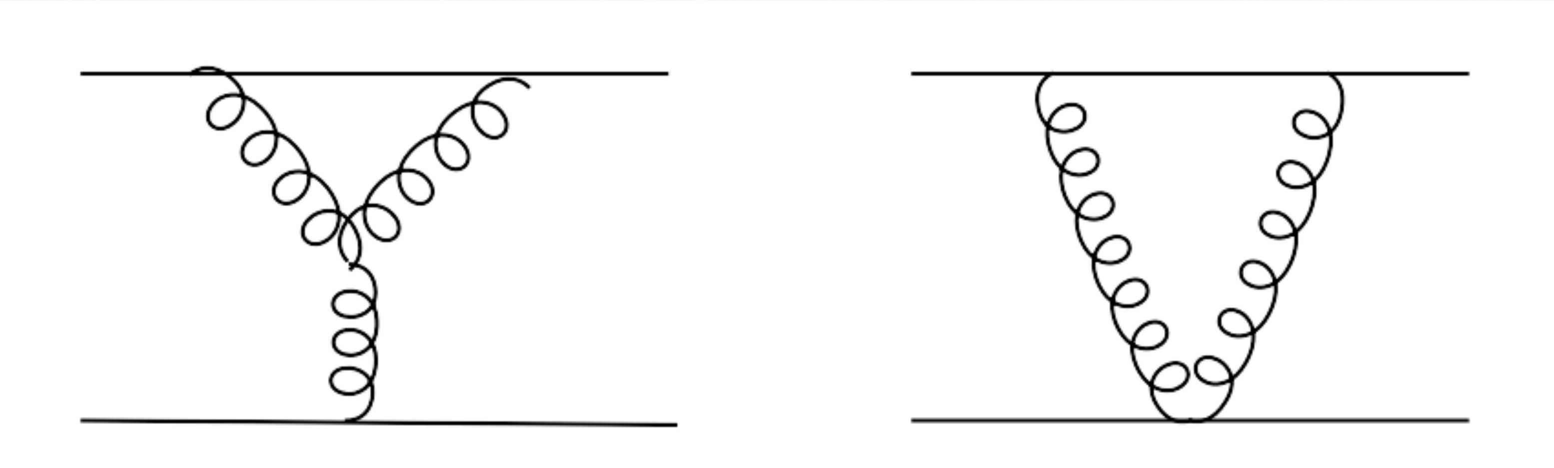
We are treating these classical entities as Quantum particles

BLACK HOLES <=> QUANTUM PARTICLES

No hair theorem: BH is characterized by (M, Q, |S|), with no reference to the origin of its makeup.

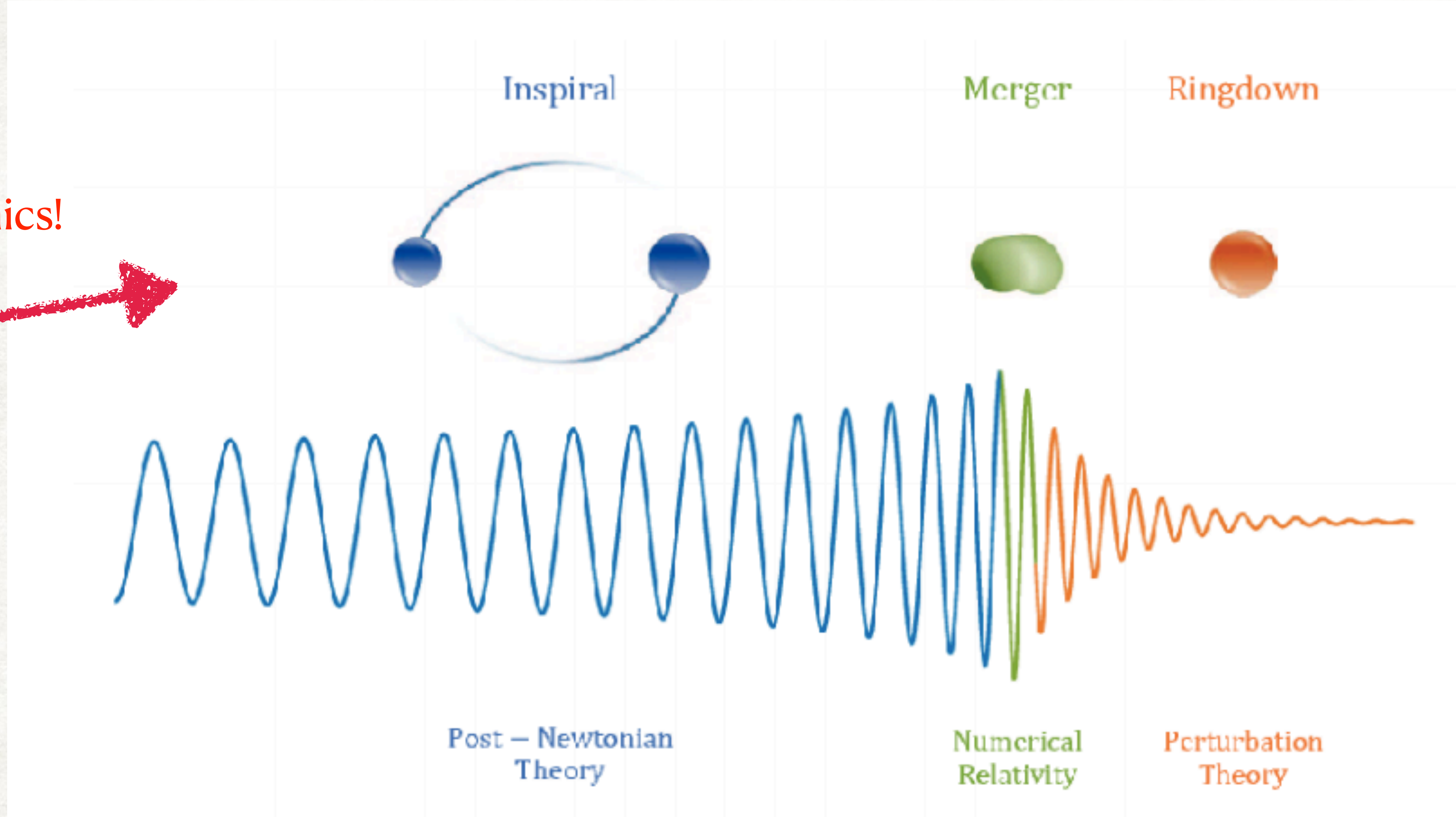
the same as elementary particles

Can the dynamics of black hole be captured by that of quantum particles ?



$$V = -\frac{G_N m_1 m_2}{r}\left(1+\frac{\vec{p}^2}{m_1 m_2}\left(1+\frac{3(m_1+m_2)^2}{2m_1 m_2}\right)\right) + \frac{G_N^2 m_1 m_2(m_1+m_2)}{2r^2}\left(1+\frac{m_1 m_2}{(m_1+m_2)^2}\right) + \frac{G_N^2}{4r^2}\frac{\vec{p}^2}{m_1 m_2}\left(\frac{117 m_1^2 m_2^2 + 67(m_1 m_2^3 + m_1^3 m_2) + 10(m_1^4 + m_2^4)}{m_1+m_2}\right)$$

Long distance dynamics!



Aren't all objects essentially point particles at long distances?

BLACK HOLES <=> QUANTUM PARTICLES

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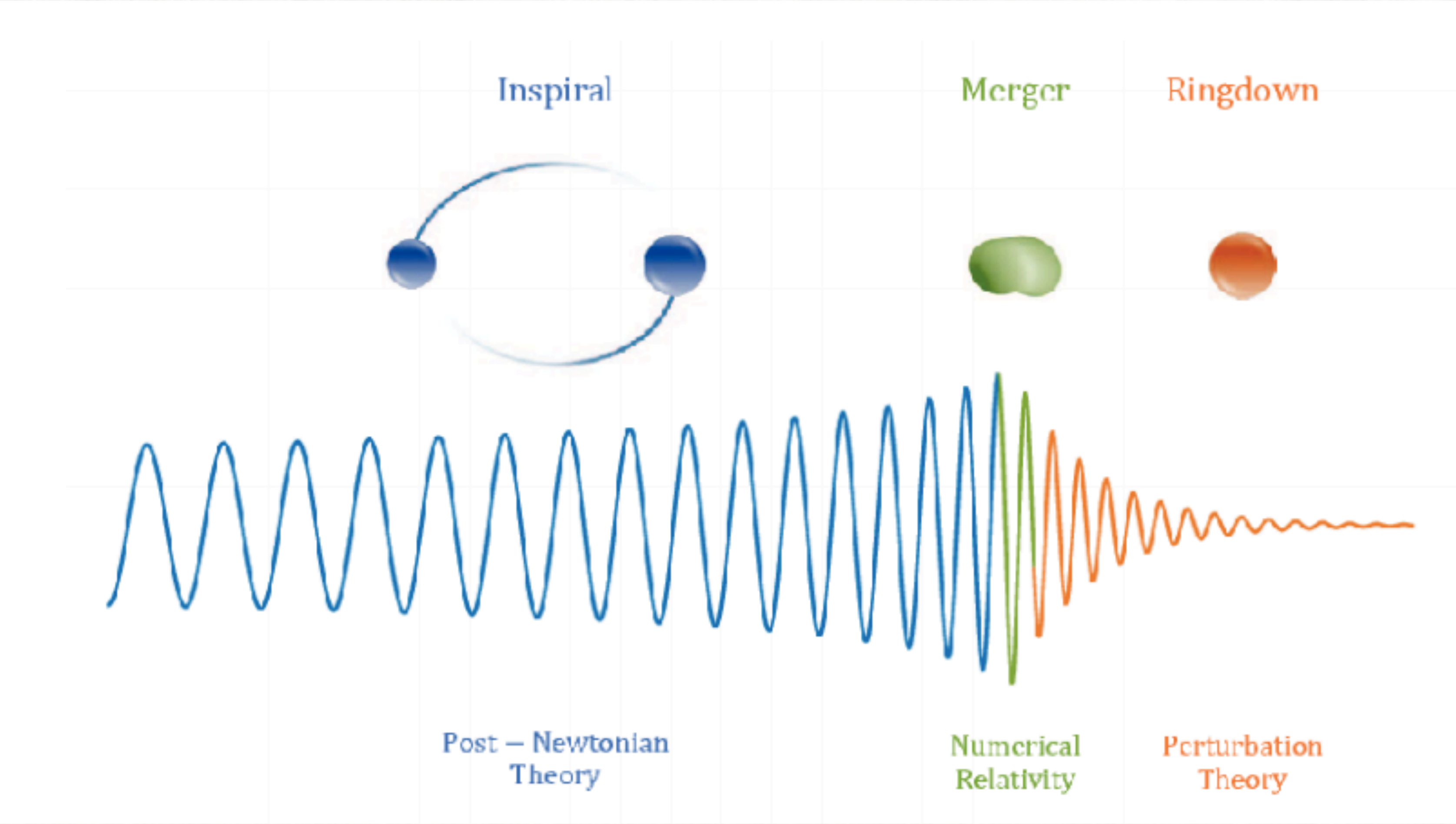


For point particles we introduce a world-line description,

the compact object are differentiated by the distinct ways it sources the back ground

GOLDBERGER AND ROTHSTEIN, PHYS. REV. D 73, 104029 (2006)

$$S = \int d\sigma \left\{ -m\sqrt{u^2} + c_E E^2 + c_B B^2 + \cdots \right\}$$

$$E_{\mu\nu} := R_{\mu\alpha\nu\beta} u^\alpha u^\beta$$
$$B_{\mu\nu} := \frac{1}{2} \epsilon_{\alpha\beta\gamma\mu} R^{\alpha\beta}{}_{\delta\nu} u^\gamma u^\delta$$

Tidal Love numbers: vanishing for BHs but not Neutron stars

BLACK HOLES <=> QUANTUM PARTICLES

This is more prominent when the object is spinning, spin degrees of freedom are included

Porto, PRD 73, 104031 (2006)

Porto and Rothstein PRD 78 (2008) 044013

Levi and Steinhoff JHEP 1509, 219 (2015)

$$S = \int d\sigma \left\{ -m\sqrt{u^2} - \frac{1}{2} S_{\mu\nu}\Omega^{\mu\nu} + L_{SI}\left[u^\mu, S_{\mu\nu}, g_{\mu\nu}(y^\mu)\right] \right\}$$

$$L_{SI} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{ES^{2n}}}{m^{2n-1}} D_{\mu_{2n}} \cdots D_{\mu_3} \frac{E_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}}$$
$$+ \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{BS^{2n+1}}}{m^{2n}} D_{\mu_{2n+1}} \cdots D_{\mu_3} \frac{B_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n}} S^{\mu_{2n+1}}$$

$$E_{\mu\nu} := R_{\mu\alpha\nu\beta} u^\alpha u^\beta$$
$$B_{\mu\nu} := \frac{1}{2}\epsilon_{\alpha\beta\gamma\mu} R^{\alpha\beta}{}_{\delta\nu} u^\gamma u^\delta$$

C#=1 for Kerr BHs !

Levi and Steinhoff JHEP 1509, 219 (2015)

Vines, Class. Quant. Grav. 35, no. 8, 084002 (2018)

We see that even far away, we have an infinite number of coefficients to characterize distinct objects!

BLACK HOLES <=> QUANTUM PARTICLES

- What characterizes the spin multi-poles of BHs from an on-shell view point ?

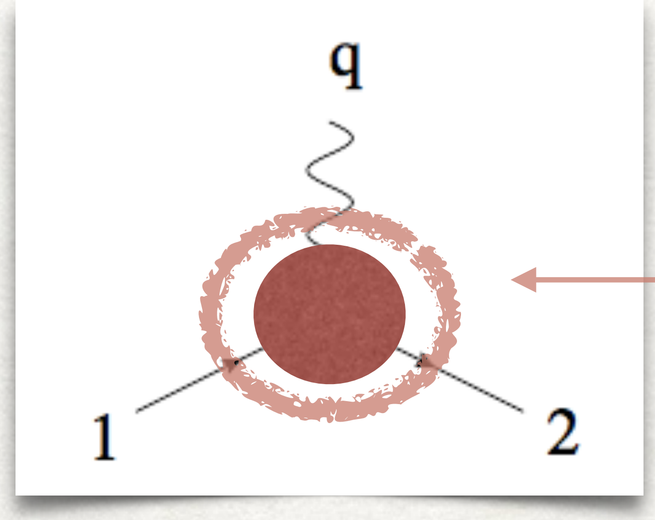
This appears challenging as there are no fundamental spinning particles beyond spin-2 while we need arbitrarily higher spin particles the capture the all order spin multiple.

- What are the underlying physical principles that selects these moments ?

The on-shell avatar of the no hair theorem ?

We consider the

Three-pt amp



The distinct three point amplitudes encode the distinct Wilson Coefficients

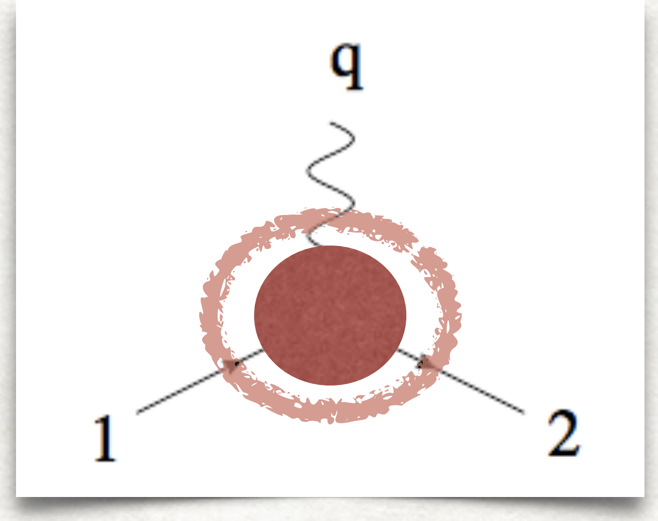
GENERAL MASSIVE AMPLITUDES IN 4D

N. Arkani-Hamed, Tzu-Chen Huang, Y-t H 1709.04891

For massive particles, the states transform as irrep under SU(2):

spin-s -> totally symmetric rank 2s tensor

$$O^{\{I_1 I_2 \cdots I_{2s}\}}$$



$$M^h_{\{I_1 I_2 \cdots I_{2s}\}\{J_1 J_2 \cdots J_{2s}\}}$$

For massless particles, the states transform as irrep under U(1):

spin-s -> it is just a phase rotation

$$O^h \to e^{ih\theta} O^h, \quad h = \pm s$$

$$p_\mu \to p_\mu \gamma^\mu = \begin{pmatrix} 0 & p_{a\dot{a}} \\ p^{\dot{a}a} & 0 \end{pmatrix} \to p_{a\dot{a}} = \begin{pmatrix} p_0 + p_3 & p_1 + ip_2 \\ p_1 - ip_2 & p_0 - p_3 \end{pmatrix}$$

$$det[p_{a\dot{a}}] = p^2$$

Massless : $p_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$

Massive : $p_{\alpha\dot{\alpha}} = \lambda^I_\alpha \tilde{\lambda}_{\dot{\alpha} I}$

GENERAL MASSIVE AMPLITUDES IN 4D

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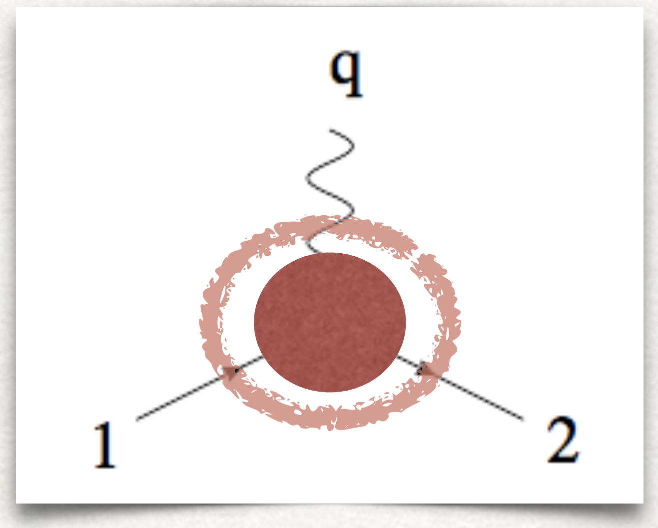
This implies that the amplitude must be proportional to

$$M^h_{\{I_1 I_2 \cdots I_{2s}\}\{J_1 J_2 \cdots J_{2s}\}} = (\lambda_{1I_1}^{\alpha_1}\lambda_{1I_2}^{\alpha_2}\cdots\lambda_{1I_{2s}}^{\alpha_{2s}})(\lambda_{2J_1}^{\beta_1}\lambda_{2J_2}^{\beta_2}\cdots\lambda_{2J_{2s}}^{\beta_{2s}})M^h_{\{\alpha_1\alpha_2\cdots\alpha_{2s}\}\{\beta_1\beta_2\cdots\beta_{2s}\}}$$



We will be interested in

$$M^h_{\{\alpha_1\alpha_2\cdots\alpha_{2s}\}\{\beta_1\beta_2\cdots\beta_{2s}\}} \longrightarrow \begin{array}{ccc} s=\frac{1}{2} & : & M^h_{\alpha\beta} \\ s=1 & : & M^h_{\{\alpha_1\alpha_2\}\{\beta_1\beta_2\}} \\ s=\frac{3}{2} & : & M^h_{\{\alpha_1\alpha_2\alpha_3\}\{\beta_1\beta_2\beta_3\}} \\ & \vdots & \end{array}$$

We need two vectors to span the SL(2,C) space:

$$(u_\alpha, v_\alpha) = (\lambda_{3,\alpha}, \epsilon_{\alpha\beta}\lambda_3^\beta)$$

We also need objects that can carry the U(1) little group weight of the massless leg.
We have λ_3 that transforms with negative U(1) weight. For positive:

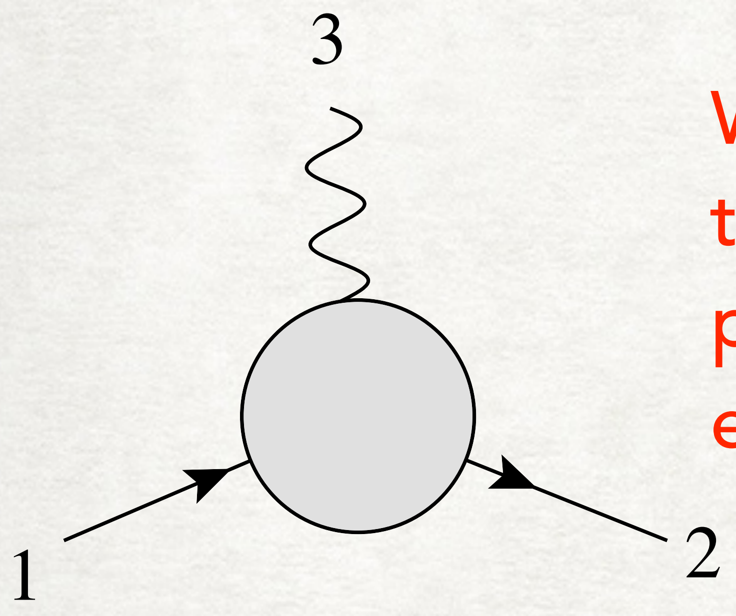
$$(p_1^2 = p_2^2 = m^2): \quad (p_1+p_3)^2 = p_2^2 \quad \rightarrow p_1\cdot p_3 = 0 \quad x\lambda_{3\alpha} = \frac{p_{1\alpha\dot{\alpha}}\tilde{\lambda}_3^{\dot{\alpha}}}{m}$$

Thus the kinematic building blocks are

$$(x,\ \lambda_{3\alpha},\ \epsilon_{\alpha\beta})$$

GENERAL MASSIVE AMPLITUDES IN 4D

Consider the three point amplitude with one massless and two equal mass



We have a parameterization of the three-point coupling that is purely kinematic in nature. For example for photon (h=1)

g-2 for electron and W

$$s=\frac{1}{2}: \qquad x\left(\epsilon_{\alpha\beta}+g_1\,x\frac{\lambda_\alpha\lambda_\beta}{m}\right)$$

$$s=1: \qquad x\left(\epsilon_{\alpha_1\beta_1}\epsilon_{\alpha_2\beta_2}+g_1\epsilon_{\alpha_2\beta_2}\,x\frac{\lambda_{\alpha_1}\lambda_{\beta_1}}{m}+g_2\,x^2\frac{\lambda_{\alpha_1}\lambda_{\beta_1}\lambda_{\alpha_2}\lambda_{\beta_2}}{m^2}\right)$$

$$\vdots$$

$$s=2: \qquad x\left(\epsilon^4+g_1\epsilon^3\,x\frac{\lambda^2}{m}+g_2\,\epsilon^2x^2\frac{\lambda^4}{m^2}+g_3\epsilon\frac{\lambda^6}{m^3}+g_4\frac{\lambda^8}{m^3}\right)$$

$$\vdots$$

These yields the "physical" parameterization of multipole moments. The presence of g-2 becomes trivial

$$\mathcal{M}_3^{\rm QED}=-ie\,\varepsilon_\mu^+(k_3)\bar{v}(p_2)\gamma^\mu u(p_1)=-i\sqrt{2}e\frac{[\mathbf{23}]\langle\zeta\mathbf{1}\rangle+\langle\mathbf{2}\zeta\rangle[3\mathbf{1}]}{\langle 3\zeta\rangle}$$

$$=i\sqrt{2}ex\frac{\langle 3\zeta\rangle\langle\mathbf{21}\rangle}{\langle 3\zeta\rangle}=i\sqrt{2}ex\langle\mathbf{21}\rangle$$

THE SIMPLEST MASSIVE-AMPLITUDE

Let's consider simplest possible amplitude is given by a pure x term

$$M_3(q^{+1}, \mathbf{1}^s, \mathbf{2}^s) = x\, m\epsilon^{2s}, \quad M_3(q^{+2}, \mathbf{1}^s, \mathbf{2}^s) = x^2\, m\epsilon^{2s}$$

Or after putting back the external polarization (spinors, vectors, tensors ..)

$$M_3(q^{+1}, \mathbf{1}^s, \mathbf{2}^s) = x\, m \left(\frac{\langle \mathbf{12}\rangle}{m}\right)^{2s}, \quad M_3(q^{+2}, \mathbf{1}^s, \mathbf{2}^s) = x^2\, m \left(\frac{\langle \mathbf{12}\rangle}{m}\right)^{2s}$$

For s=1/2, 1, 2 this gives QED, EW, and massive KK graviton minimal coupling.

see Neil Christensen et. a. Phys.Rev.D 98 (2018), 101 (2020) 6, 065019

But for s>2 there are no consistent fundamental higher-spin particles in flat space, what do these coupling describe ?

THE SIMPLEST MASSIVE-AMPLITUDE

To see what kind of interaction this describes, we compare this with amplitude from the 1-particle EFT

$$L_{SI} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{\mathrm{ES}^{2n}}}{m^{2n-1}} D_{\mu_{2n}} \cdots D_{\mu_3} \frac{E_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}}$$
$$+ \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{\mathrm{BS}^{2n+1}}}{m^{2n}} D_{\mu_{2n+1}} \cdots D_{\mu_3} \frac{B_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n}} S^{\mu_{2n+1}}$$

we find that

Guevara, Ochirov and Vines JHEP 1909, 056 (2019)
Chung, Huang, Kim and Lee, JHEP 1904, 156 (2019)

$$C_{\mathrm{S}^n} = 1 + \frac{n(n-1)}{4s} + \frac{n(n-1)(n^2-5n+10)}{32s^2} + \frac{n(n-1)(n^4-14n^3+71n^2-154n+144)}{384s^3} + \mathcal{O}(s^{-4})$$

The classical spin-limit corresponds to $\boxed{s \to \infty, \quad \hbar \to 0}$ with $\boxed{s\hbar}$ fix

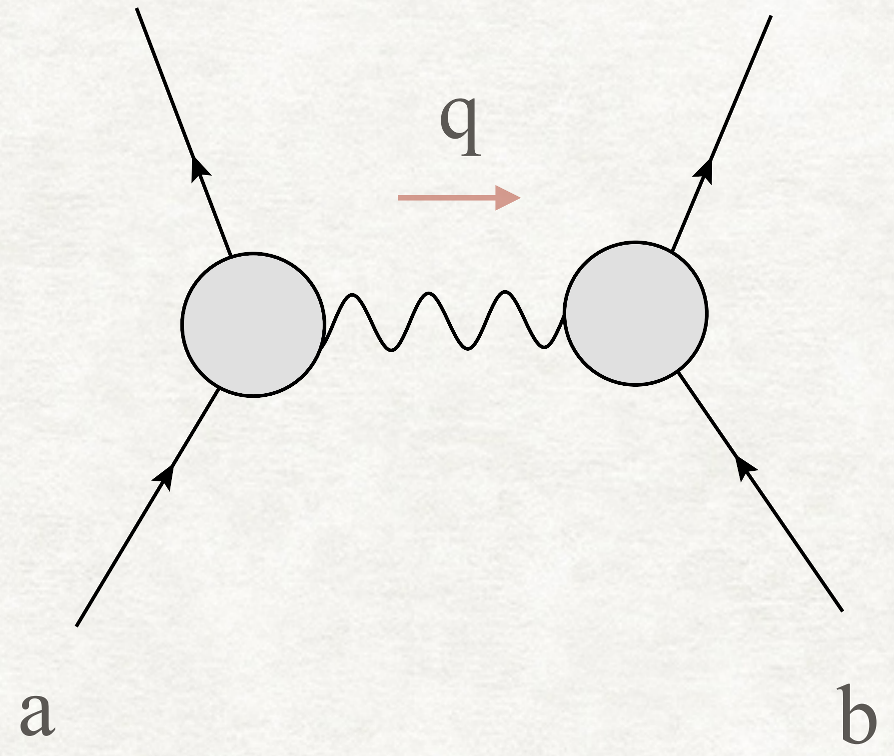
Minimal coupling for higher spins yields the leading order (in G) spin multipole for Kerr

THE SIMPLEST MASSIVE-AMPLITUDE

Minimal coupling in the large s limit is identical to Kerr BH:

$$M^{+2}_{s,min} = \frac{\kappa m x^2}{2}\frac{\langle \mathbf{21}\rangle^{2s}}{m^{2s}}$$

Let us check this on observables the classical gravitational potential: using minimal coupling at tree-level



$$V(p,q) = \left.\frac{M_4(s,t)}{4E_aE_b}\right|_{t\to 0} = \frac{\mathrm{Res}_t}{4E_aE_b\,q^2}$$

$$\mathrm{Res}_t = M^{+}_{3a}M^{-}_{3b} + M^{-}_{3a}M^{+}_{3b}$$

Exchange of positive / negative helicity graviton

Fourier transform to position space we find that

$$V^{\mathrm{BBN}}_{cl} = \left(-\cosh\left[\left(\frac{\vec{S}_a}{m_a}+\frac{\vec{S}_b}{m_b}\right)\times\vec{\nabla}\right] - 2\left(\frac{\vec{p}_a}{m_a}-\frac{\vec{p}_b}{m_b}\right)\cdot\sinh\left[\left(\frac{\vec{S}_a}{m_a}+\frac{\vec{S}_b}{m_b}\right)\times\vec{\nabla}\right]\right)\frac{Gm_am_b}{r}$$
$$+\frac{1}{2}\left(\left[\frac{\vec{p}_a}{m_a}\times\frac{\vec{S}_a}{m_a}-\frac{\vec{p}_b}{m_b}\times\frac{\vec{S}_b}{m_b}\right]\cdot\vec{\nabla}\right)\cosh\left[\left(\frac{\vec{S}_a}{m_a}+\frac{\vec{S}_b}{m_b}\right)\times\vec{\nabla}\right]\frac{Gm_am_b}{r}.$$

which matches to the classical GR result

Justin Vines 1709.06016

THE UNREASONABLE EFFECTIVENESS OF MINIMAL COUPLING

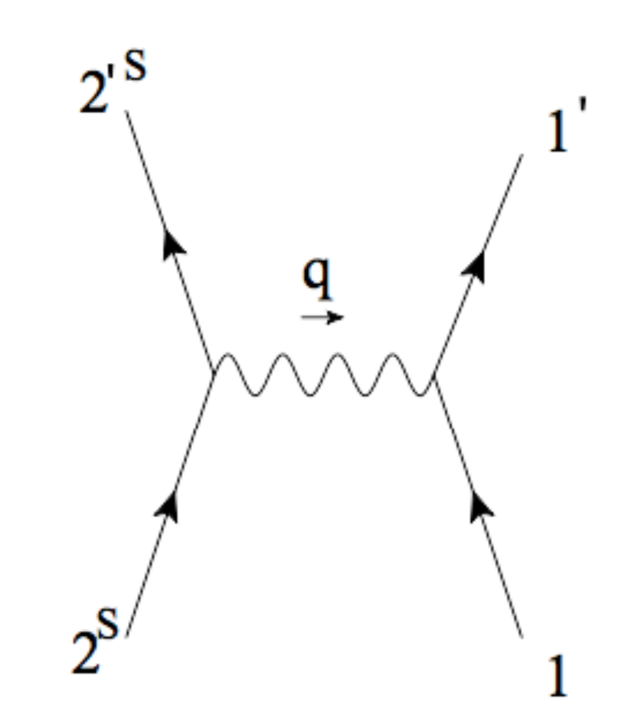
- For minimal coupling the spin-dependence exponentiates in the classical spin-limit

Arkani-Hamed, Huang, O'Connell, JHEP 01 (2020) 046

$$h=+1:\quad \frac{e_2}{\sqrt{2}}x\frac{\langle \mathbf{2}\mathbf{2}'\rangle^S}{m^{S-1}},\qquad h=-1:\quad \frac{e_2}{\sqrt{2}}\frac{1}{x}\frac{[\mathbf{2}\mathbf{2}']^S}{m^{S-1}} \quad\longrightarrow\quad \lim_{S\to\infty}\frac{e_2}{\sqrt{2}}m\,x\left(\mathbb{I}\pm\frac{1}{Sm}\bar{q}\cdot s\right)^S=\frac{e_2}{\sqrt{2}}mx\,e^{\pm\bar{q}\cdot a}$$

This induces an imaginary shift, relative to Schwarzschild, in any Fourier transform.
For example for the impulse

$$\Delta p_1^\mu=\frac{1}{4m_1m_2}\int \hat{d}^4q\,\hat{\delta}(q\cdot u_1)\hat{\delta}(q\cdot u_2)e^{-iq\cdot b}\,iq^\mu\,M_4\left(1,2\to 1',2'\right)|_{q^2\to 0}$$



This is simply the mysterious Janis Newman shift!

Newman, Janis
J. Math. Phys. 6, 915 (1965)

$$g_{\mu\nu}=g^0_{\mu\nu}+k_\mu k_\nu\phi(r),$$

$$\text{Schwarzschild}:\quad \phi_{\text{Sch}}(r)=\frac{r_0}{r},\quad k_\mu=(1,\hat{r})$$

$$\text{Kerr}:\quad \phi_{\text{Kerr}}(r)=\frac{r_0 r}{r^2+a^2\cos^2\theta},\quad k_\mu=(1,\hat{r})$$

$$\begin{aligned}\phi_{\text{Sch}}(r)|_{r\to r+ia\cos\theta}&=\frac{r_0}{2}\left(\frac{1}{r}+\frac{1}{\bar{r}}\right)\Big|_{r\to r+ia\cos\theta}\\&=\frac{r_0 r}{r^2+a^2\cos^2\theta}=\phi_{\text{Kerr}}(r)\,.\end{aligned}$$

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Arkani-Hamed, Huang, O'Connell, JHEP 01 (2020) 046

$$h=+1: \quad \frac{e_2}{\sqrt{2}}x\frac{\langle \mathbf{2 2'}\rangle^S}{m^{S-1}}, \quad h=-1: \quad \frac{e_2}{\sqrt{2}}\frac{1}{x}\frac{[\mathbf{2 2'}]^S}{m^{S-1}} \longrightarrow \lim_{S\to\infty}\frac{e_2}{\sqrt{2}}m\,x\left(\mathbb{I}\pm\frac{1}{Sm}\bar{q}\cdot s\right)^S = \frac{e_2}{\sqrt{2}}mxe^{\pm\bar{q}\cdot a}$$

This induces an imaginary shift, relative to Schwarzschild, in any Fourier transform.
This is simply the mysterious Janis Newman shift! Newman, Janis J. Math. Phys. 6, 915 (1965)

- We can augment minimal coupling with a pure phase rotation,

$$x \to xe^{i\theta}$$

This generates the Taub-NUT space time Huang, Kol, O'Connell, *Phys.Rev.D* 102 (2020) 4, 046005

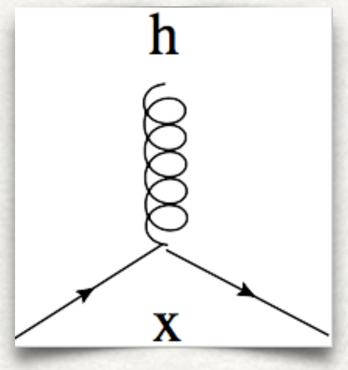
Indicates that the relation between Taub-NUT and Schwarzschild is an "Gravitational electricmagnetic duality rotation" Rotates super-translation charge with the dual supertranslation

MINIMAL UNIFICATION

We have seen that properties of BH solutions can now be cleanly cast into on-shell elements providing convenient basis to manifest the simplicity of BHs.

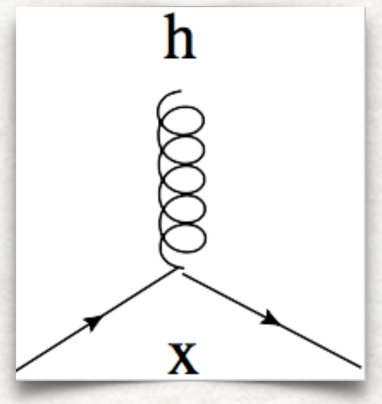
Schwarzschild

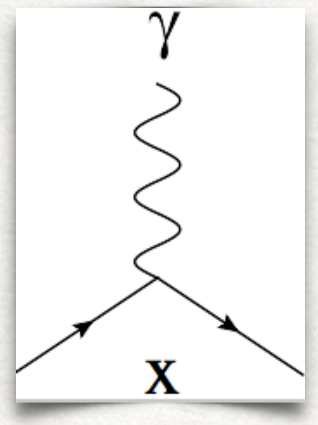
$$(0, x^2)$$



Reissner-Nordstorm

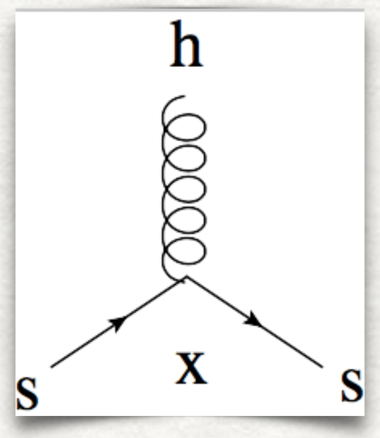
$$(x, x^2)$$





Kerr

$$\left(0, x^2 \frac{\langle \mathbf{12} \rangle^s}{m^s}\right)$$

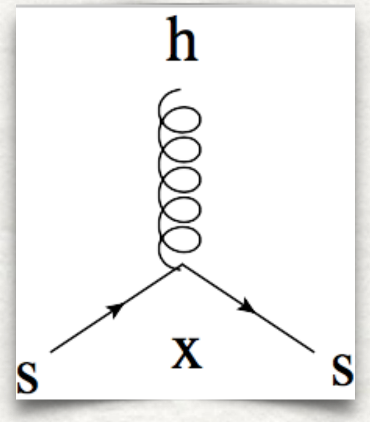


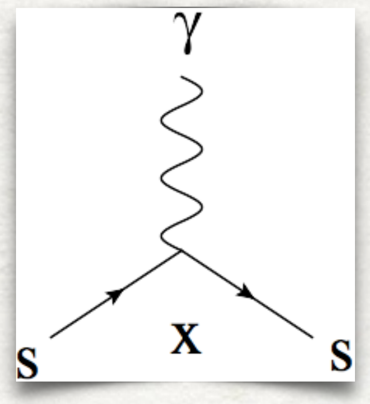
Guevara, Ochirov, Vines, JHEP 1909 (2019) 056

Arkani-Hamed, Huang, O'Connell, JHEP 01 (2020) 046

Kerr Newman

$$\left(x \frac{\langle \mathbf{12} \rangle^s}{m^s}, x^2 \frac{\langle \mathbf{12} \rangle^s}{m^s}\right)$$

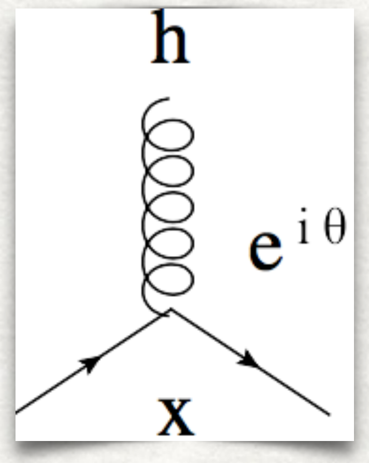




Moynihan, JHEP 01 (2020) 014
Chung, Huang, Kim, 1911.12775

Taub-NUT

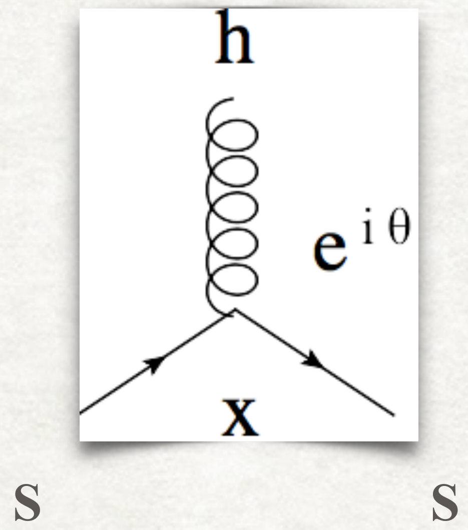
$$(0, x^2 e^{i\theta})$$



Huang, Kol, O'Connell,
Phys.Rev.D 102 (2020) 4, 046005

Kerr Taub-NUT

$$\left(0, x^2 \frac{\langle \mathbf{12} \rangle^s}{m^s} e^{i\theta}\right)$$



Emond, Huang, Kol,
Moynihan ,O'Connell, 2010.07861

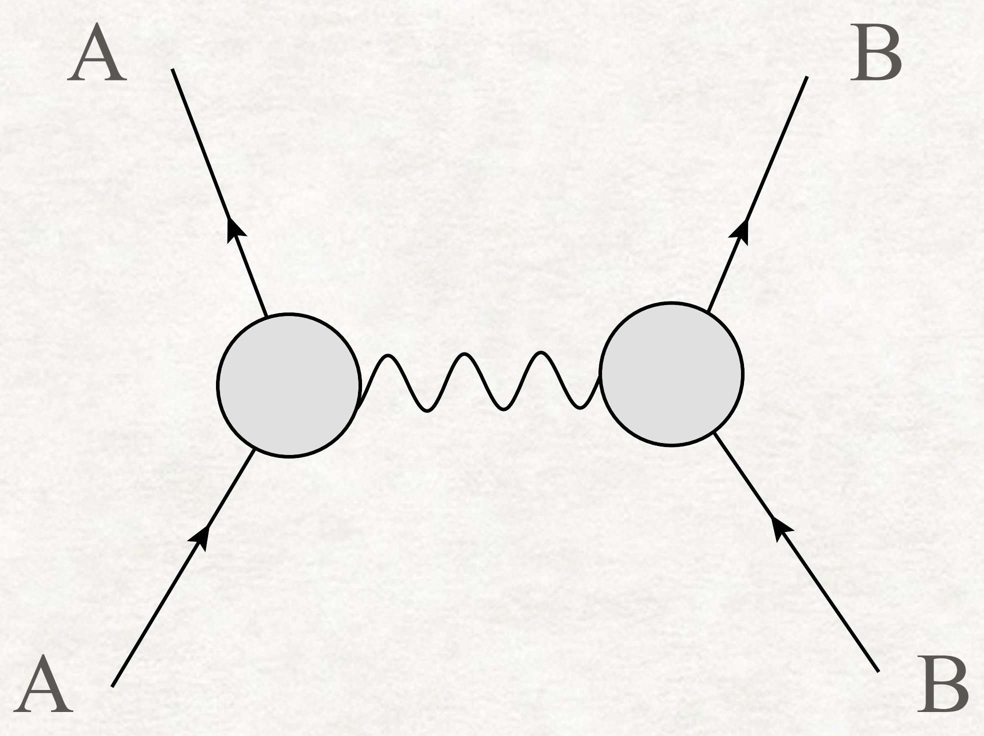
What is the underlying principle behind minimal coupling?

ENTANGLEMENT IN SPIN SPACE

S-matrix by nature is a unitary map between in and out states

$$|Out\rangle = \mathcal{S}|in\rangle$$

Consider the case of 2 -> 2 scattering



We can construct the reduced density matrix by simply tracing out the phase space of particle B's final state

$$\rho_{A,B}^{\mathrm{Out}} = |Out\rangle\langle Out| \quad \rightarrow \quad \rho_{A}^{\mathrm{Out}} = tr_B\left(\rho_{A,B}^{\mathrm{Out}}\right)$$

We can then compare the increase in entanglement entropy between in and out state

$$\Delta S \equiv -\mathrm{tr}\left[\rho^{\mathrm{out}} \log \rho^{\mathrm{out}}\right] + \mathrm{tr}\left[\rho^{\mathrm{in}} \log \rho^{\mathrm{in}}\right]$$

THE SPIN ENTANGLEMENT

Rafael Aoude, Ming-Zhi Chung, Y-T Huang
Camila S. Machado, and Man-Kuan Tam
Phys. Rev. Lett. 125, 181602

We consider the spin Hilbert space

$$\mathcal{H} = \mathcal{H}_{s_a} \otimes \mathcal{H}_{s_b}$$

Given an in-state, through the S-matrix we obtain an out-state in the same Hilbert space as

$$|\text{out}\rangle = (U_a \otimes U_b)\, \mathcal{S}\, |\text{in}\rangle$$

Once again we compute the reduced density matrix and corresponding entanglement entropy

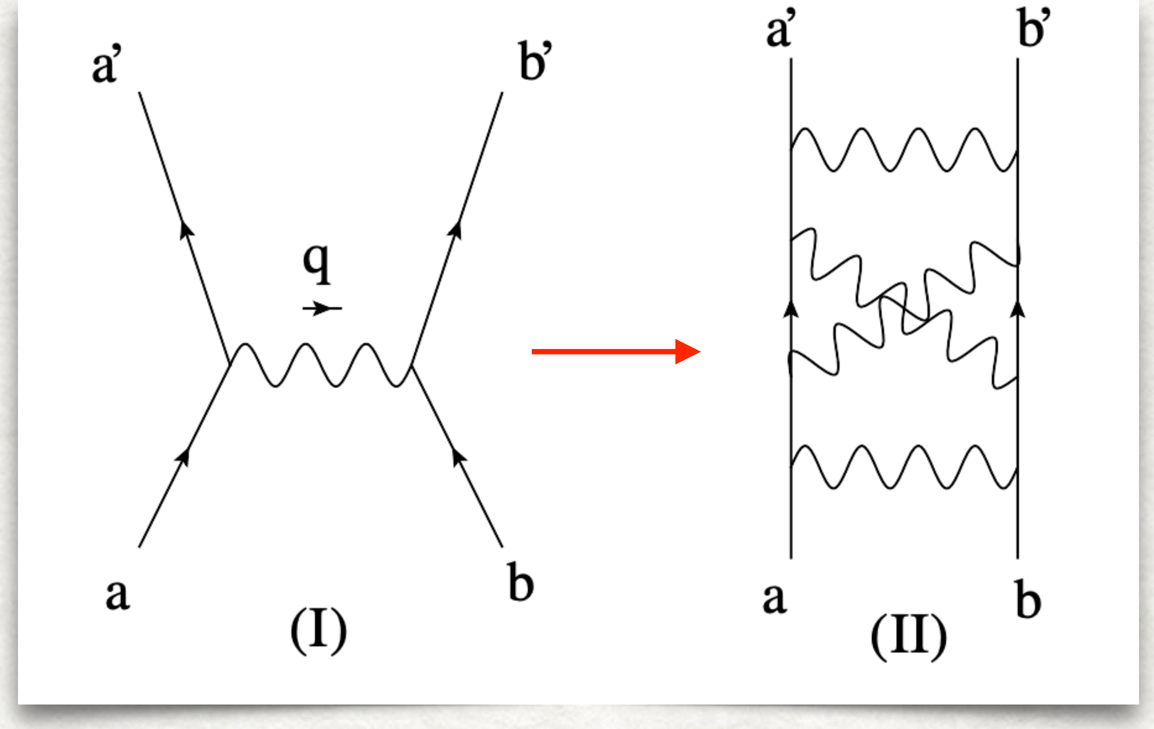
$$\rho^{\text{Out}}_{\text{A,B}} = |Out\rangle\langle Out| \quad \rightarrow \quad \rho^{\text{Out}}_{A} = tr_B\left(\rho^{\text{Out}}_{\text{A,B}}\right) \qquad \rightarrow \; S_{\text{VN}} = -tr_a[\rho_a^{Out} \log \rho_a^{Out}]$$

Ofcourse the entanglement entropy of the outstate depends on the instate, we can either consider the “difference” or the entanglement power

$$\mathcal{E}_a = 1 - \int \frac{d\Omega_a}{4\pi}\frac{d\Omega_b}{4\pi} \text{tr}_a \rho_a^2.$$

We will consider the 2-> 2 S-matrix in the Eikonal approximation

$$\chi(b) = \frac{1}{4|\vec{p}|E}\int \frac{d^2\vec{q}}{(2\pi)^2}\; e^{i\vec{q}\cdot\vec{b}}\overline{M}_{\text{tree}}(q^2). \rightarrow \quad \mathcal{S}_{\text{Eikonal}} = e^{i\chi(b)}$$



THE SPIN ENTANGLEMENT

We will consider the 2-> 2 S-matrix in the Eikonal approximation

$$\chi(b) = \frac{1}{4|\vec{p}|E} \int \frac{d^2\vec{q}}{(2\pi)^2} \; e^{i\vec{q}\cdot\vec{b}} \overline{M}_{\text{tree}}(q^2) \longrightarrow \mathcal{S}_{\text{Eikonal}} = e^{i\chi(b)}$$

We can now use the previous general formula for spinning body. For fixed s_a, s_b

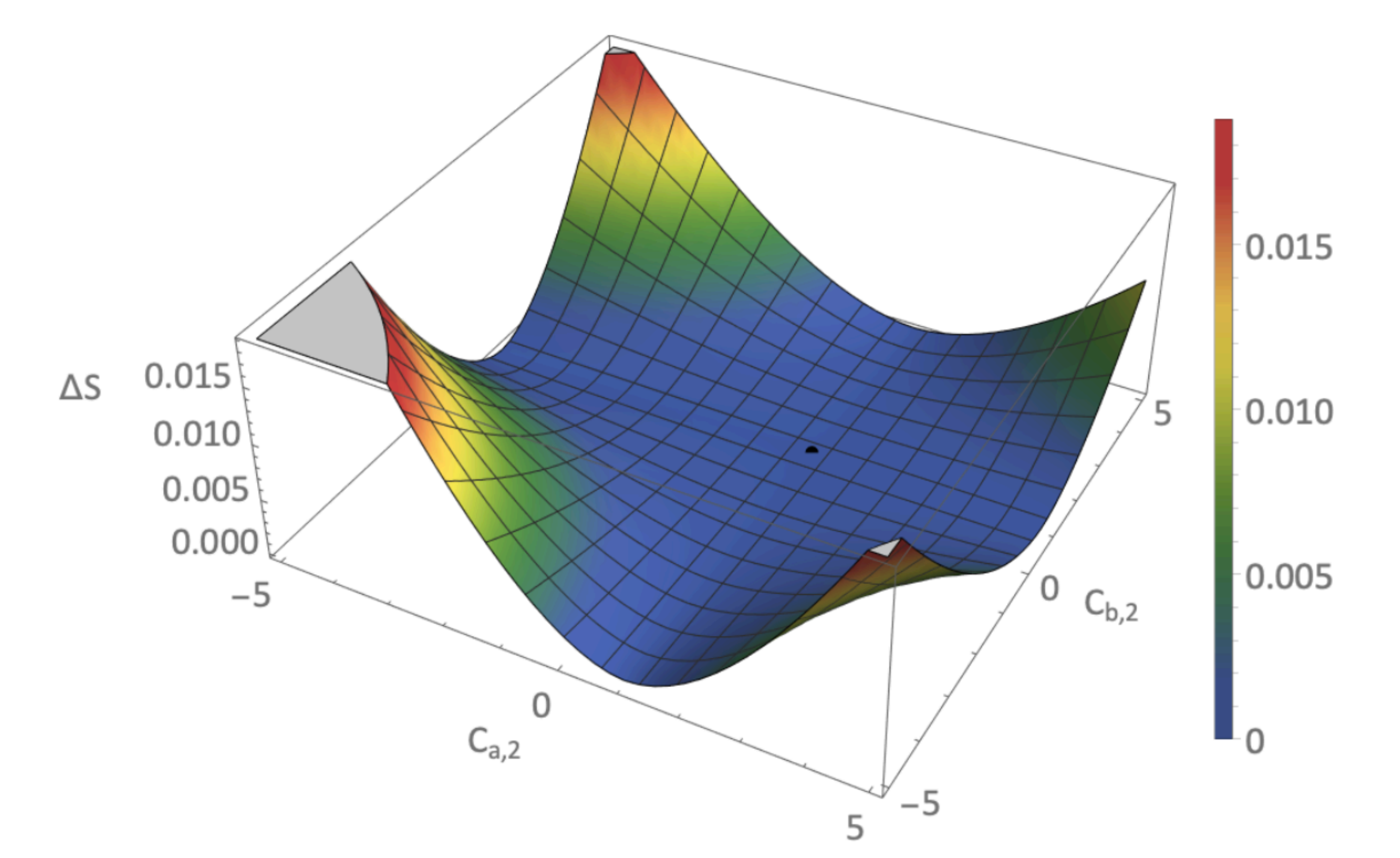
$$\overline{M}_{\text{tree}}(q^2) = -\frac{16\pi G m_a^2 m_b^2}{q^2} \tag{11}$$
$$\times \left\{ \sum_{m=0}^{\lfloor s_a \rfloor} \sum_{n=0}^{\lfloor s_b \rfloor} A_{2m,2n} \left(\mathbb{T}_a^{2m} \otimes \mathbb{T}_b^{2n}\right) \right.$$
$$+ \frac{m_a^2 m_b}{E} \sum_{m=0}^{\lceil s_a \rceil -1} \sum_{n=0}^{\lfloor s_b \rfloor} A_{2m+1,2n} \left(\text{Sym}\left[\mathbb{E}_a \mathbb{T}_a^{2m}\right] \otimes \mathbb{T}_b^{2n}\right)$$
$$+ \frac{m_a m_b^2}{E} \sum_{m=0}^{\lfloor s_a \rfloor} \sum_{n=0}^{\lceil s_b \rceil -1} A_{2m,2n+1} \left(\mathbb{T}_a^{2m} \otimes \text{Sym}\left[\mathbb{E}_b \mathbb{T}_b^{2n}\right]\right)$$
$$\left. + \sum_{m=0}^{\lceil s_a \rceil -1} \sum_{n=0}^{\lceil s_b \rceil -1} A_{2m+1,2n+1} \left(\mathbb{T}_a^{2m+1} \otimes \mathbb{T}_b^{2n+1}\right) \right\},$$

$$A_{0,0} = c_{2\Theta}\,, \quad A_{1,0} = \frac{i(2E r_a c_\Theta - m_b c_{2\Theta})}{m_a^2 m_b r_a}\,, \tag{13}$$
$$A_{1,1} = \frac{c_{2\Theta} s_\Theta^2}{E^2 r_a r_b} + \frac{c_{2\Theta}}{m_a m_b} - \frac{2 c_\Theta s_\Theta^2}{E m_a r_a} - \frac{2 c_\Theta s_\Theta^2}{E m_b r_b}\,,$$
$$A_{2,0} = \frac{C_{a,2} c_{2\Theta}}{2 m_a^2} + \frac{m_b^2 c_{2\Theta} s_\Theta^2}{2E^2 m_a^2 r_a^2} - \frac{2 m_b c_\Theta s_\Theta^2}{E m_a^2 r_a}\,,$$
$$A_{2,1} = i\left(\frac{E C_{a,2} c_\Theta}{m_a^3 m_b^2} - \frac{C_{a,2} c_{2\Theta}}{2 m_a^2 m_b^2 r_b} + \frac{c_{2\Theta}}{4E^2 m_a^2 r_a^2 r_b} - \frac{c_{4\Theta}}{8E^2 m_a^2 r_a^2 r_b} - \frac{c_\Theta}{2E m_a^2 r_a m_b r_b} + \frac{c_{3\Theta}}{2E m_a^2 r_a m_b r_b} - \frac{c_{2\Theta}}{m_a^3 r_a m_b} - \frac{1}{8E^2 m_a^2 r_a^2 r_b} - \frac{c_\Theta}{4E m_a^3 r_a^2} + \frac{c_{3\Theta}}{4E m_a^3 r_a^2}\right),$$
$$A_{2,2} = \frac{C_{a,2} C_{b,2} c_{2\Theta}}{4 m_a^2 m_b^2} - \frac{C_{a,2} c_\Theta s_\Theta^2}{E m_a m_b^2 r_b} - \frac{C_{b,2} c_\Theta s_\Theta^2}{E m_a^2 r_a m_b} + \frac{c_{2\Theta} s_\Theta^4}{4E^4 r_a^2 r_b^2} - \frac{c_\Theta s_\Theta^4}{E^3 m_a r_a^2 r_b} - \frac{c_\Theta s_\Theta^4}{E^3 r_a m_b r_b^2} + \frac{c_{2\Theta} s_\Theta^2}{E^2 m_a r_a m_b r_b} + \frac{C_{b,2} c_{2\Theta} s_\Theta^2}{4E^2 m_a^2 r_a^2} + \frac{C_{a,2} c_{2\Theta} s_\Theta^2}{4E^2 m_b^2 r_b^2}\,,$$

THE SPIN ENTANGLEMENT

To see the dependence on the Wilson coefficients, lets start with spin-1

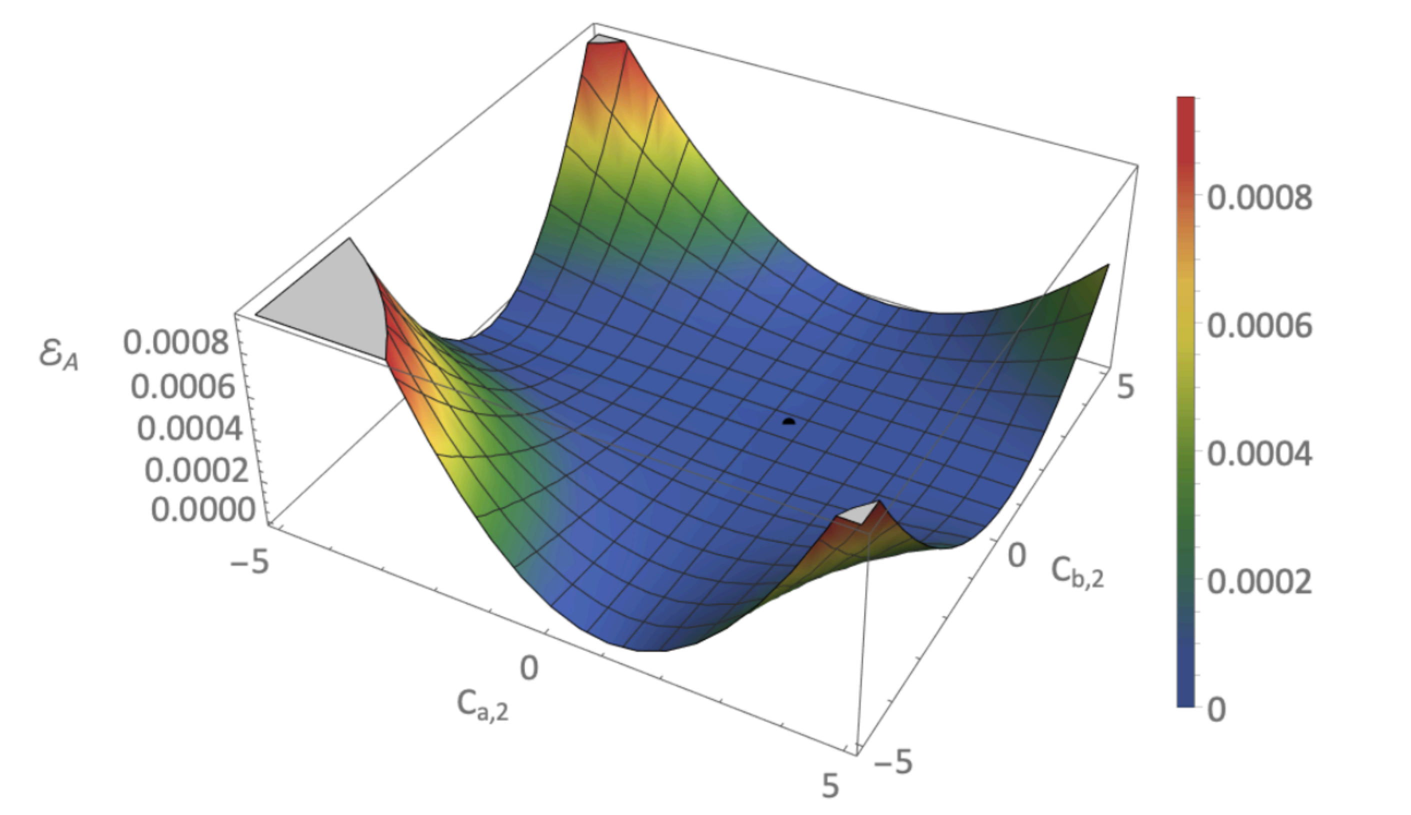
$$
\begin{aligned}
A_{0,0} &= c_{2\Theta}\,, \quad A_{1,0} = \frac{i(2Er_a c_\Theta - m_b c_{2\Theta})}{m_a^2 m_b r_a}\,, \qquad\qquad (13)\\
A_{1,1} &= \frac{c_{2\Theta} s_\Theta^2}{E^2 r_a r_b} + \frac{c_{2\Theta}}{m_a m_b} - \frac{2c_\Theta s_\Theta^2}{E m_a r_a} - \frac{2c_\Theta s_\Theta^2}{E m_b r_b}\,,\\
A_{2,0} &= \frac{C_{a,2} c_{2\Theta}}{2m_a^2} + \frac{m_b^2 c_{2\Theta} s_\Theta^2}{2E^2 m_a^2 r_a^2} - \frac{2m_b c_\Theta s_\Theta^2}{E m_a^2 r_a}\,,\\
A_{2,1} &= i\left(\frac{E C_{a,2} c_\Theta}{m_a^3 m_b^2} - \frac{C_{a,2} c_{2\Theta}}{2m_a^2 m_b^2 r_b} + \frac{c_{2\Theta}}{4E^2 m_a^2 r_a^2 r_b}\right.\\
&\quad - \frac{c_{4\Theta}}{8E^2 m_a^2 r_a^2 r_b} - \frac{c_\Theta}{2E m_a^2 r_a m_b r_b} + \frac{c_{3\Theta}}{2E m_a^2 r_a m_b r_b}\\
&\quad \left. - \frac{c_{2\Theta}}{m_a^3 r_a m_b} - \frac{1}{8E^2 m_a^2 r_a^2 r_b} - \frac{c_\Theta}{4E m_a^3 r_a^2} + \frac{c_{3\Theta}}{4E m_a^3 r_a^2}\right),\\
A_{2,2} &= \frac{C_{a,2} C_{b,2} c_{2\Theta}}{4m_a^2 m_b^2} - \frac{C_{a,2} c_\Theta s_\Theta^2}{E m_a m_b^2 r_b} - \frac{C_{b,2} c_\Theta s_\Theta^2}{E m_a^2 r_a m_b}\\
&\quad + \frac{c_{2\Theta} s_\Theta^4}{4E^4 r_a^2 r_b^2} - \frac{c_\Theta s_\Theta^4}{E^3 m_a r_a^2 r_b} - \frac{c_\Theta s_\Theta^4}{E^3 r_a m_b r_b^2}\\
&\quad + \frac{c_{2\Theta} s_\Theta^2}{E^2 m_a r_a m_b r_b} + \frac{C_{b,2} c_{2\Theta} s_\Theta^2}{4E^2 m_a^2 r_a^2} + \frac{C_{a,2} c_{2\Theta} s_\Theta^2}{4E^2 m_b^2 r_b^2}\,,
\end{aligned}
$$



(I) Relative Von Neumann entropy.

$(C_{a,2}, C_{b,2}) = (1, 1)$

$\Delta S \approx 1.54 \times 10^{-9}$



(II) Entanglement power.

$\mathcal{E}_a \approx 1.10 \times 10^{-10}$

We see that at the minimal coupling value both Relative Entanglement and Entanglement reaches minimum and near zero!

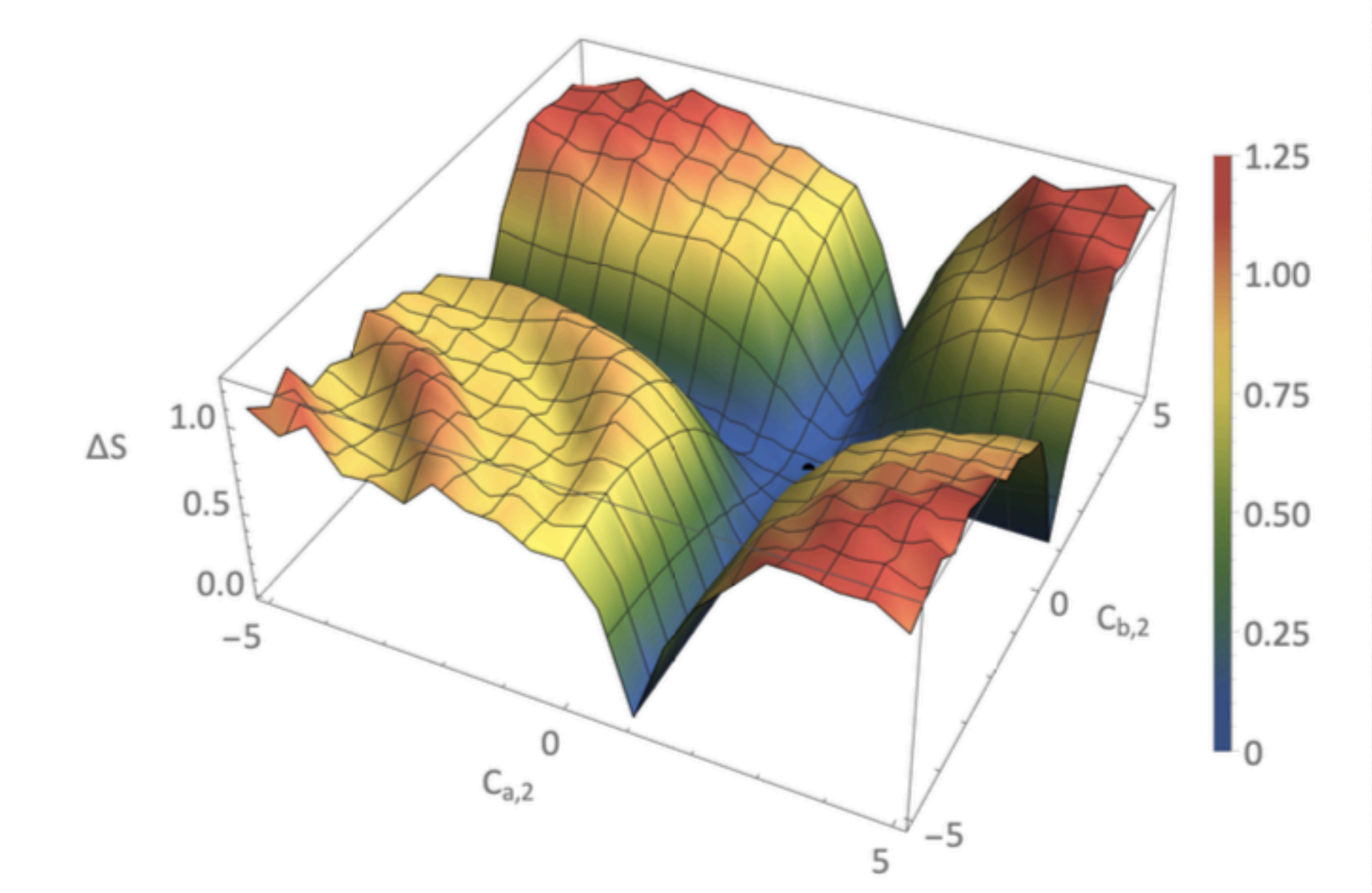
$|\vec{p}_a| = |\vec{p}_b| = |\vec{p}|,\ m_a = m_b = m,\ \vec{b} = (b, 0, 0)$

$Gm^2 = 10^{-4},\ |\vec{p}|b = 1000,\ |\vec{p}|/m = 100$

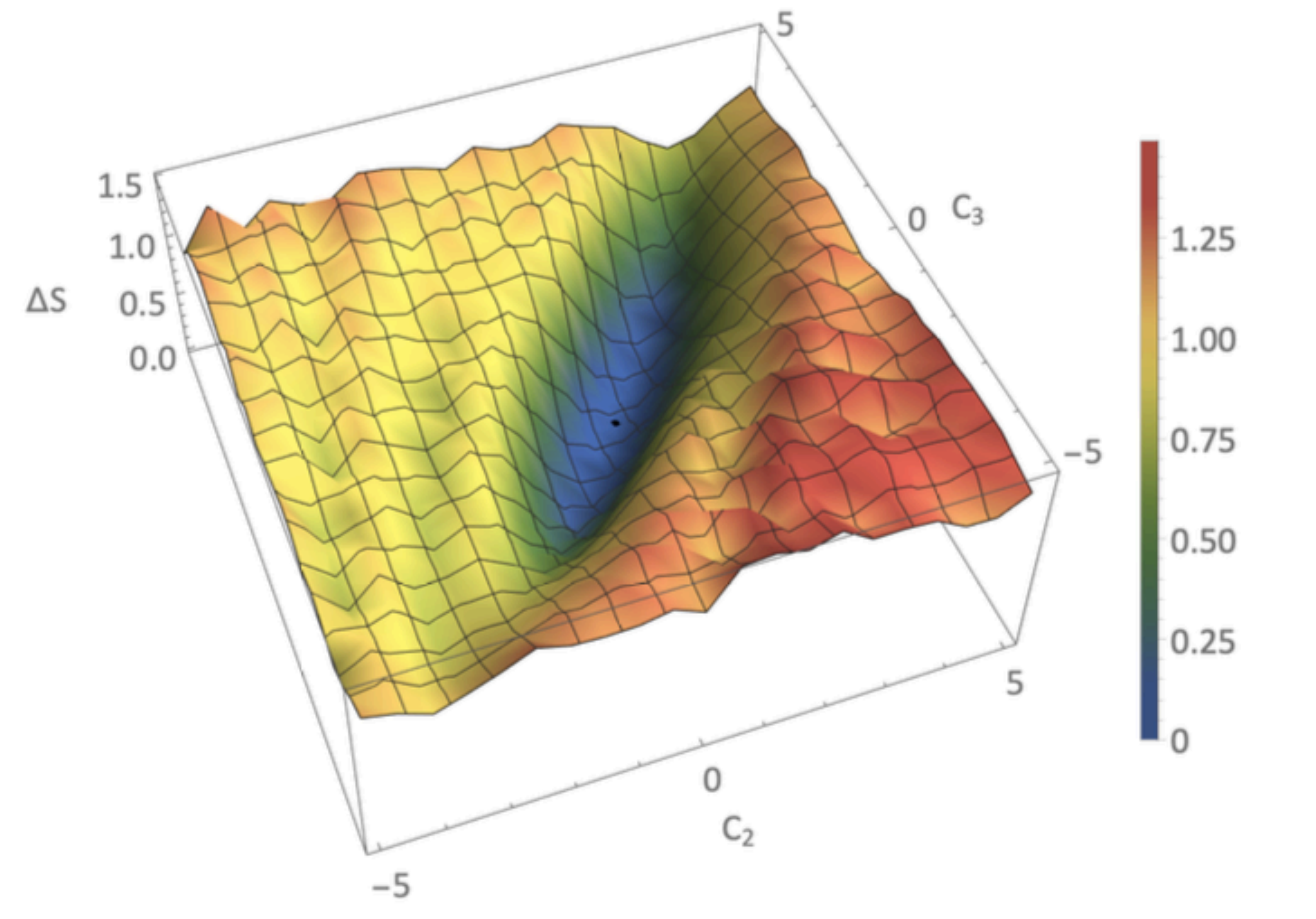
THE SPIN ENTANGLEMENT

For spin-3 we have 5 Wilson coefficient for each particle

$(C_{a,2}, C_{b,2})$



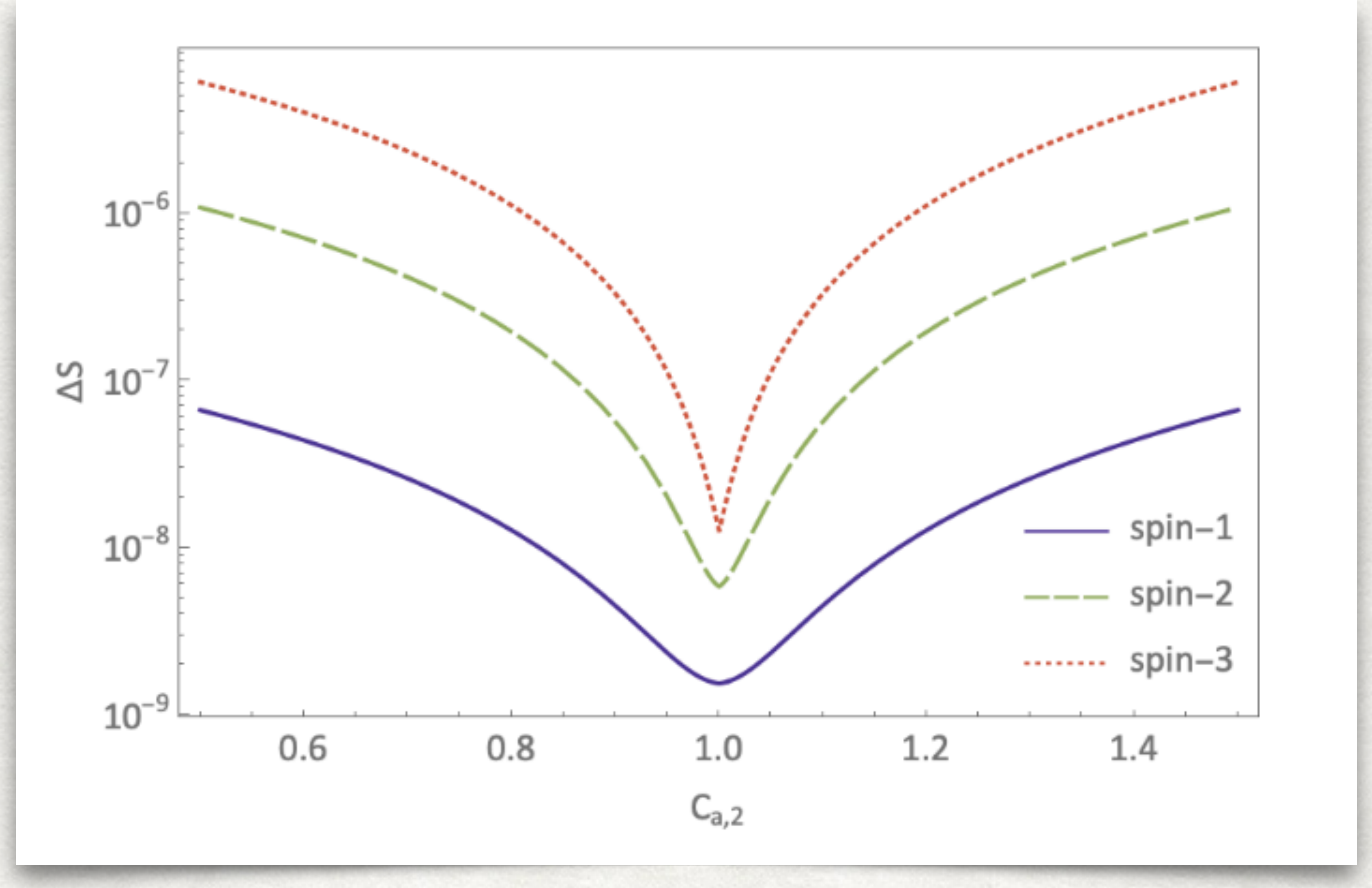
$C_{a,2} = C_{b,2} = C_2,\ C_{a,3} = C_{b,3} = C_3$



$\vec{b} = (b, 0, 0),\ Gm^2 = 10^{-4},\ |\vec{p}|b = 1000,\ |\vec{p}|/m = 100$

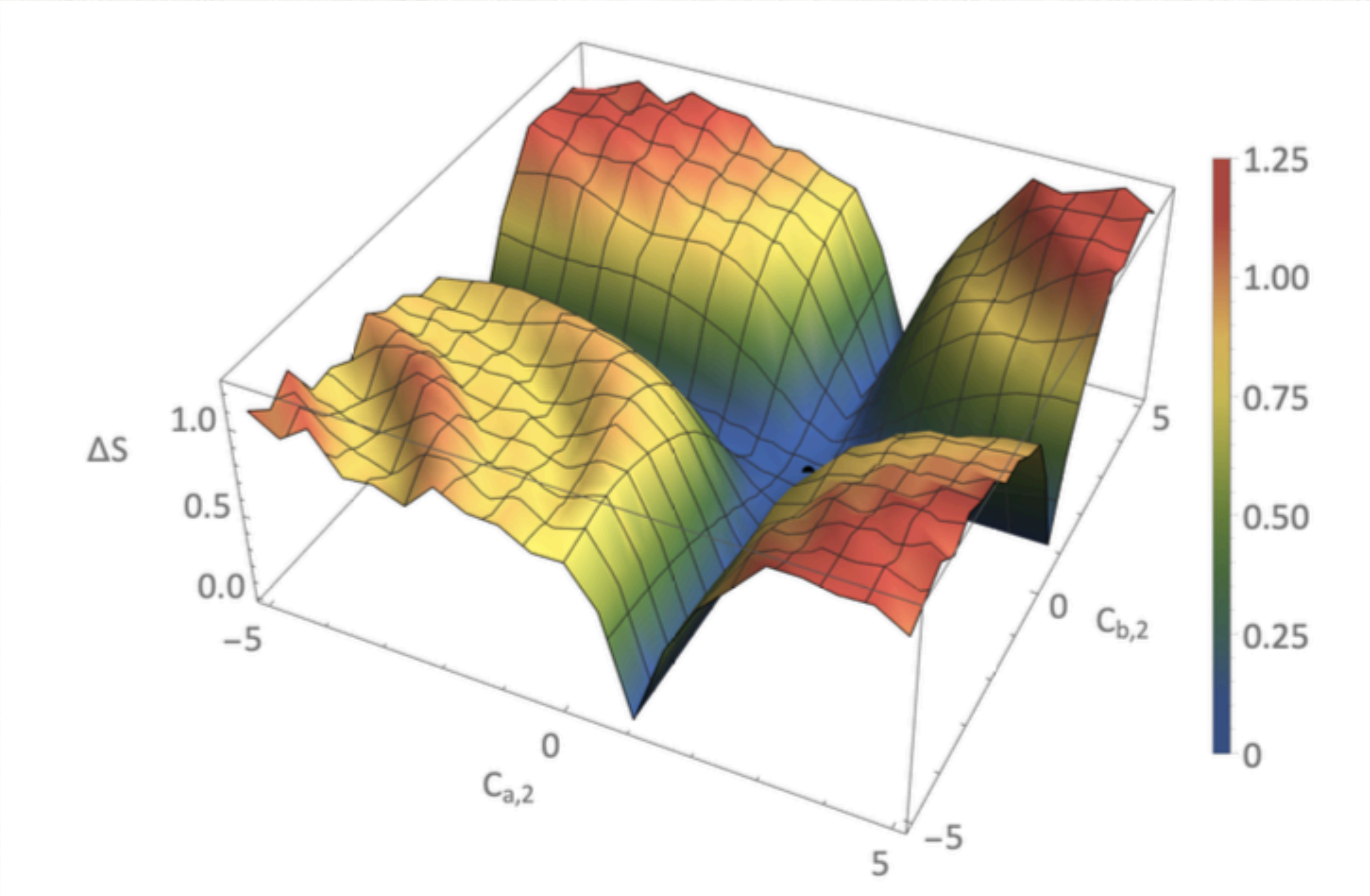
$\Delta S \approx 1.26 \times 10^{-8}$

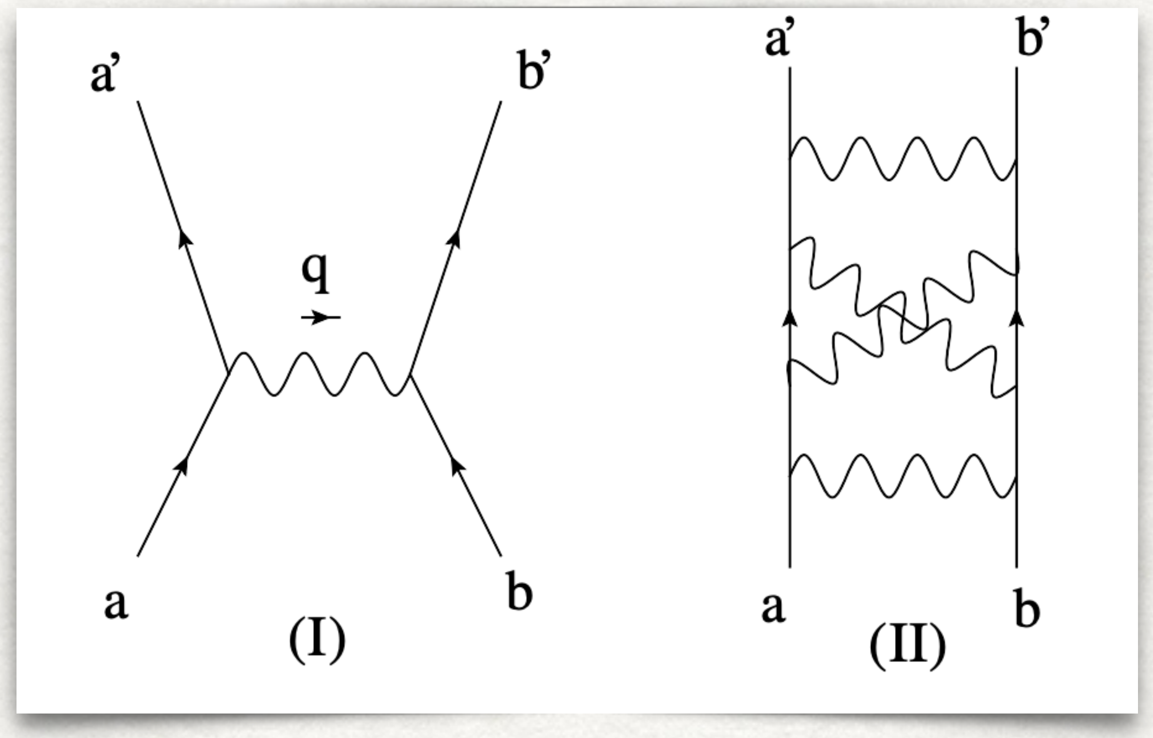
We see that in all case $(C_{a,2}, C_{b,2})$ yields the dominant contribution to entanglement production



THE SPIN ENTANGLEMENT

It is remarkable that the spin-spin couplings for black holes, essentially vanish in the Eikonal limit !





$$\chi_1 = \frac{\xi E}{|\boldsymbol{p}|}\left[-a_1^{(0)} \ln \boldsymbol{b}^2 - \frac{2a_1^{(1,i)}}{\boldsymbol{b}^2}(\boldsymbol{p}\times \boldsymbol{S}_i)\cdot \boldsymbol{b} + a_1^{(2,1)}\left(\frac{2}{\boldsymbol{b}^2}\boldsymbol{S}_{1\perp}\cdot \boldsymbol{S}_{2\perp} - 4\frac{\boldsymbol{S}_{1\perp}\cdot \boldsymbol{b}\,\boldsymbol{S}_{2\perp}\cdot \boldsymbol{b}}{\boldsymbol{b}^4}\right)\right];$$

Zvi Bern, Andres Luna , Radu Roiban, Chia-Hsien Shen, Mao Zeng 2005.03071

THE SPIN ENTANGLEMENT

What is special about C=1 becomes transparent when we consider the spin-mixing terms in the Eikonal phase, in the relativistic limit

For example for spin-1, with p/m>>1

$$\chi = \begin{pmatrix} \log b^2 & 0 & \frac{\alpha(C_{b2}-1)}{b^2} & \cdots \\ 0 & \log b^2 & 0 & \cdots \\ \frac{\alpha(C_{b2}-1)}{b^2} & 0 & \log b^2 & \cdots \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

We see that in the off-diagonal spin-flip component vanishes for C=1 (BH value)!

Similar conclusion for higher spin analysis.

At 1 PM, the BH moments are such that in the Eikonal limit, one achieves vanishing spin rotations.

THE SPIN ENTANGLEMENT

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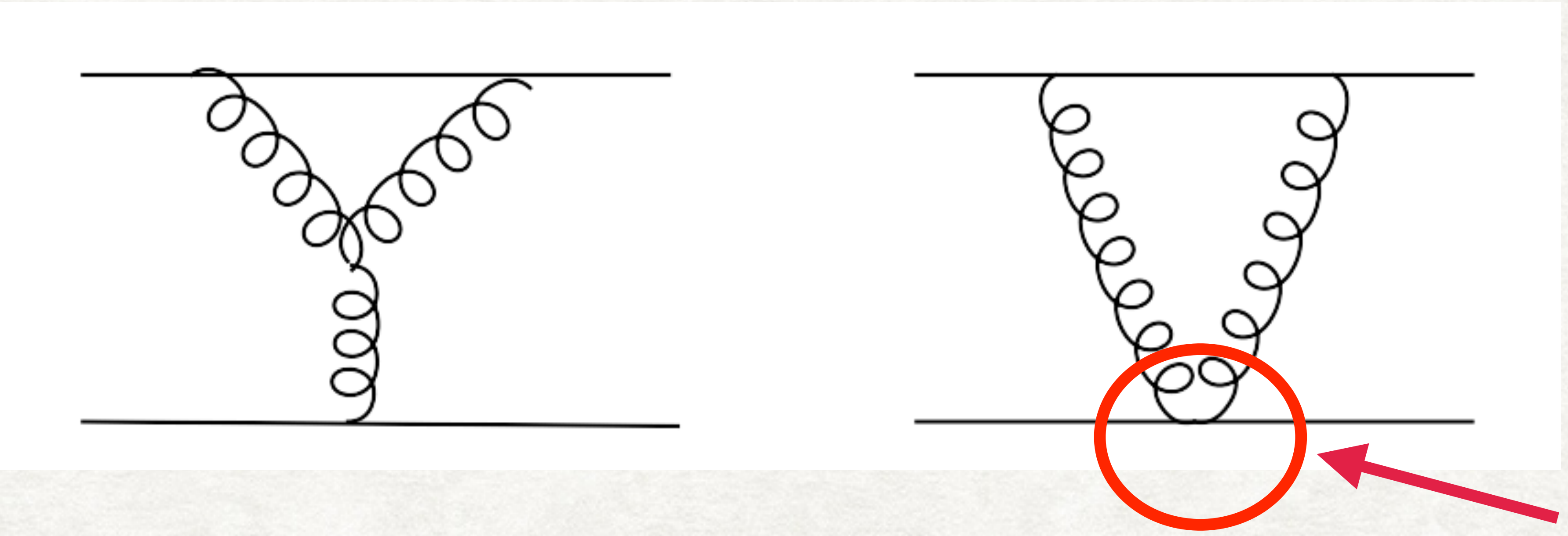
$$\chi = \begin{pmatrix} \log b^2 & 0 & \frac{\alpha(C_{b2}-1)}{b^2} & \cdots \\ 0 & \log b^2 & 0 & \cdots \\ \frac{\alpha(C_{b2}-1)}{b^2} & 0 & \log b^2 & \cdots \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

At 1 PM, the BH moments are such that in the Eikonal limit, one achieves vanishing spin rotations.

2PM ?



Guidance for gravitational Compton amplitude ?

CONCLUSION AND OUTLOOK

- We have seen that in terms of on-shell basis, properties of BH solutions are cleanly captured

  → Minimal coupling generates BH multipole expansion

- The simplicity in the on-shell basis reflect hidden relations for the classical solutions:

  double copy, complex shifts, duality transformations

- The physical principle behind spin-minimal coupling appears to be near zero spin-entanglement
- What is the story beyond leading G ?
- Quantum corrections ? Anomalous multipole moments ?
- Charged BH, SUSY black holes, BPS