Regular black hole models

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Based on:

"Information loss problem and a 'black hole` model with a closed apparent horizon", V.F., JHEP 1405 (2014) 049;

"Notes on non-singular models of black holes", V. F., Phys.Rev. D94 (2016) no.10, 104056;

"Quantum radiation from a sandwich black hole", V. F. and A. Zelnikov, Phys.Rev. D95 (2017) no.4, 044042;

"Quantum radiation from an evaporating non-singular black hole", V. F. and A. Zelnikov, Phys.Rev. D95 (2017) no.12, 124028.

According to GR: Singularity exists inside a black hole.

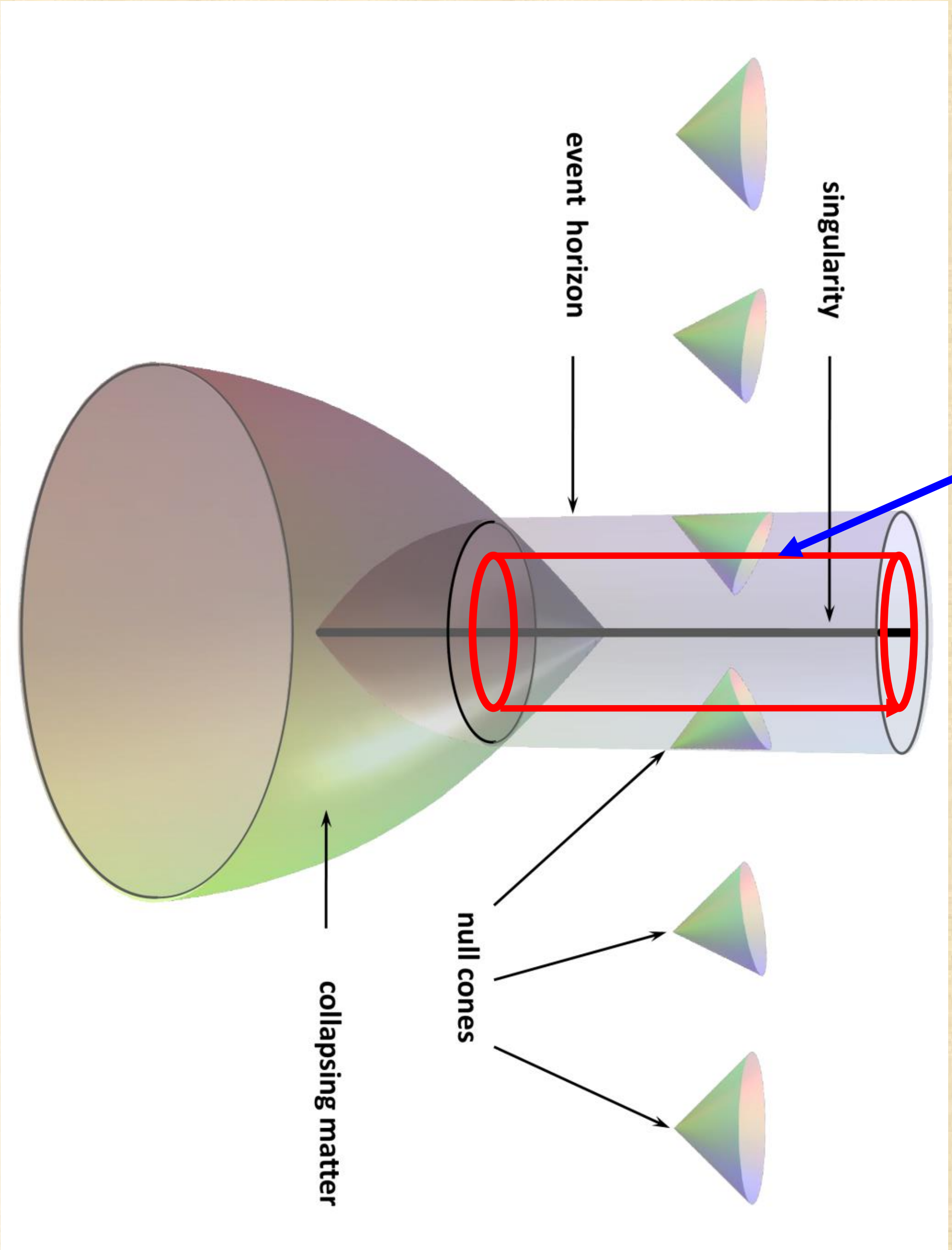
Theorems on singularities: Penrose and Hawking.

There exists a curvature singularity inside
a stationary BH in the Einstein gravity $\Rightarrow$
This theory is UV incomplete.

Expectations: When curvature becomes high (e.g. reaches the Planckian value) the classical GR should be modified. Singularities of GR would be resolved.

Consider a black hole with the gravitational radius r_g. What is spatial volume inside it?

Is it $\sim r_g^3$? WRONG!!!

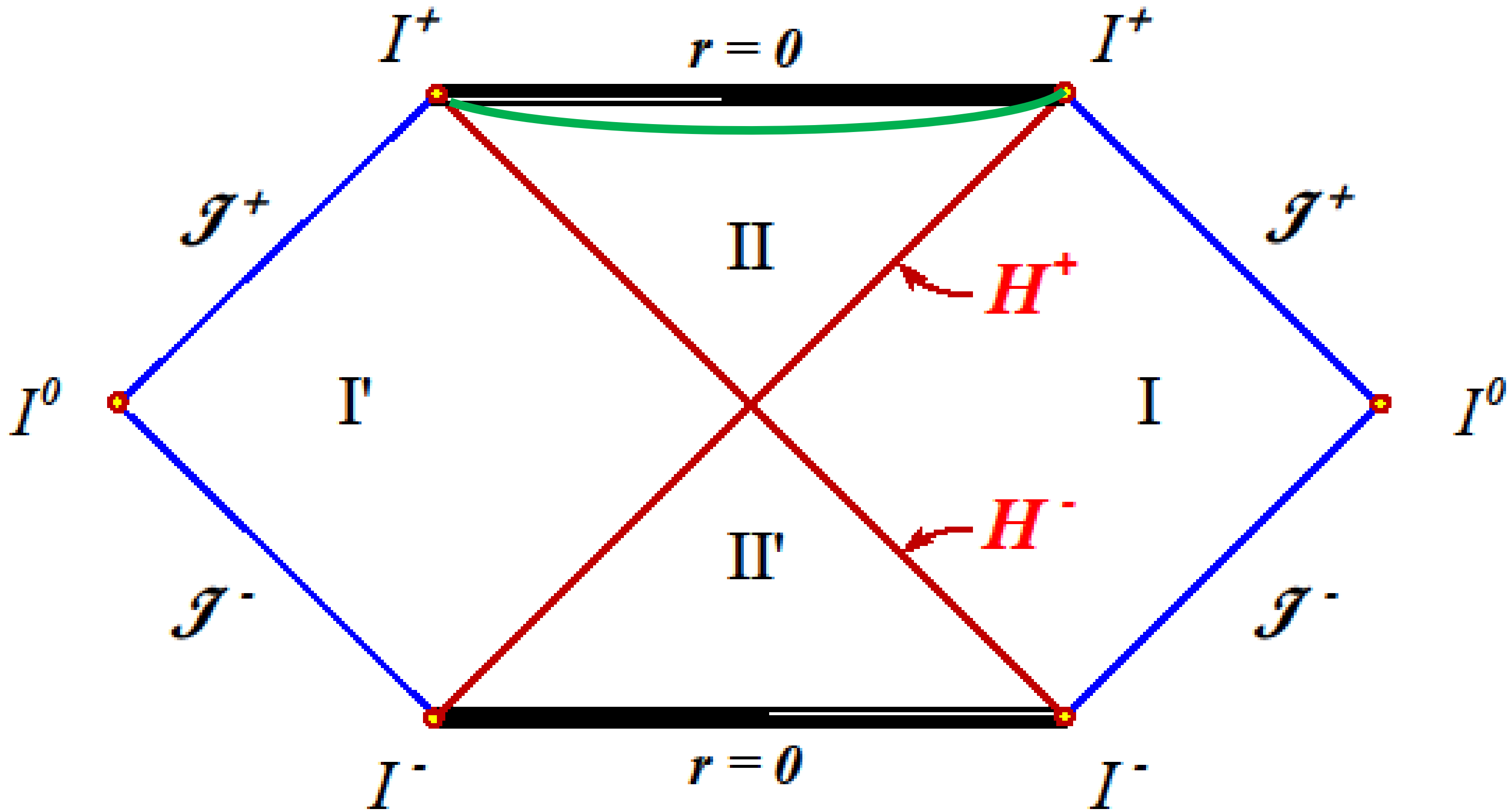


r plays the role of time inside BH

Slice $r = \text{const}$ has topology $S^2 \times R$

Spatial volume $\sim r_g^2 \times | g_{tt} |^{1/2} \, (t/c) >>> r_g^3$

Contracting anisotropic universe



I^+
$r = 0$
I^+
$\mathcal{J}^+$
II
H^+
$\mathcal{J}^+$
I^0
I'
I
I^0
H^-
II'
$\mathcal{J}^-$
$\mathcal{J}^-$
I^-
$r = 0$
I^-

$$\mathbf{R}^2 = R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} = \frac{48M^2}{r^6},$$

$$\mathbf{R}^2 \sim \ell^{-4} \Rightarrow r_* = \ell \left(\frac{R_g}{\ell} \right)^{1/3}$$

$$M \sim 4 \times 10^6 M_\odot, \quad R_g \sim 10^{12}\, cm,$$

$$\ell \sim 10^{-33}\,\mathrm{cm}, \quad r_* \sim 10^{15}\, \ell$$

$$ds^2 = -\frac{dr^2}{\frac{2M}{r}-1} + (\frac{2M}{r}-1)dt^2 + r^2 d\omega^2,$$

$$\tau = \int dr \sqrt{\frac{r}{2M-r}} \approx \frac{2}{3\sqrt{2M}} r^{3/2},$$

$$ds^2 \sim -d\tau^2 + a\,\tau^{-2/3} dt^2 + b\,\tau^{4/3} d\omega^2$$

$$T_{evap} \sim t_{Pl}(M / m_{Pl})^3, \quad M = 10^{15} g,$$

$$T_{evap} \sim 10^{17} \sec, L_{evap} \sim 10^{28} cm,$$

$$r_* = l_{Pl}(M / m_{Pl})^{1/3},$$

$$L \sim l_{Pl}(M / m_{Pl})^{11/3} \sim L_{evap}(M / m_{Pl})^{2/3}$$

$$\sim 10^{13} L_{evap}$$

Phenomenological description

(i) There exists a critical energy scale parameter μ .

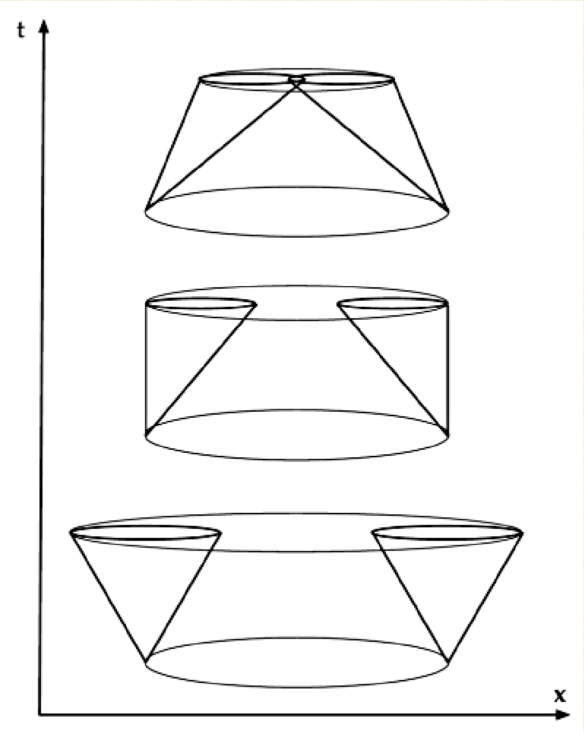
The corresponding fundamental length is $\ell = \dfrac{\hbar}{\mu c}$;

(ii) $\mu \ll \mu_{Pl}$ (similar to the inflaton mass parameter in cosmology);

(iii) Limiting curvature condition: $|\Re| \le \dfrac{C}{\ell^2}$. C is a universal constant, defined by the theory and independent of the parameters of the solution.

[Markov, JETP Lett. 36, 265 (1982)]

`Quasi-local definition' of BH: Apparent horizon



A compact smooth surface *B* is called a trapped surface if both, in- and out-going null surfaces, orthogonal to *B*, are non-expanding .

A trapped region is a region inside *B*.

A boundary of all trapped regions is called an apparent horizon.

Null energy condition: $T_{\mu\nu} l^{\mu} l^{\mu} \geq 0$

Trapped surface + NEC

=Event horizon existence

In a ST obeying the null-energy condition the apparent horizon lies inside (or coincides with) the true event horizon.

In classical physics in order to prove the existence of a BH (in an exact mathema-tical sense) it is not necessary to wait infinite time, but it is sufficient to check the existence of the trapped surface `now’.

In quantum physics the energy conditions could be violated. An example is an evaporating black hole (negative energy flux through the horizon reduces its mass).

It is possible, in principle, that the apparent horizon exists, but there is no event horizon.

We shall focus on this option.

General form of SS metric in advanced time coordinates

$$ds^2 = -\alpha^2 f\, dv^2 + 2\alpha\, dv\, dr + r^2 d\Omega^2 ,$$

$f = (\nabla r)^2 = g^{\mu\nu} r_{,\mu} r_{,\nu}$. $\quad f(v,r)|_{r\to\infty} = 1.$

Apparent horizon: $f = 0$. Red-shift function: α(v,r). In a static ST: $\xi_{\mathrm{t}}^2 = -\alpha^2 f$,

$\Re^2 \sim 4\left(\frac{[(\nabla r)^2 - 1]}{r^2}\right)^2 \Rightarrow$ If ST is regular at $r = 0$, the apparent horizon cannot cross this line.

ST is regular at $r=0$, if curvature invariants are finite there:

$$f=1+\tfrac{1}{2}f_2(v)r^2+\ldots\ ,$$

$$\alpha=\alpha_0(v)[1+\tfrac{1}{2}\alpha_2(v)r^2+\ldots].$$

We use normalization: $\alpha(v,r)|_{r\to\infty}=1$, then the rate of the proper time at the center, τ, and the rate of the Killing time at infinity, v, are connected as: $d\tau=\alpha_0(v)dv$.

(i) An apparent horizon in a regular metric cannot cross $r = 0$.

(ii) It has two branches: outer- and inner-horizons.

(iii) Non-singular BH model with a closed apparent horizon

[V.F. and G.Vilkovisky, Phys. Lett., 106B, 307 (1981)]

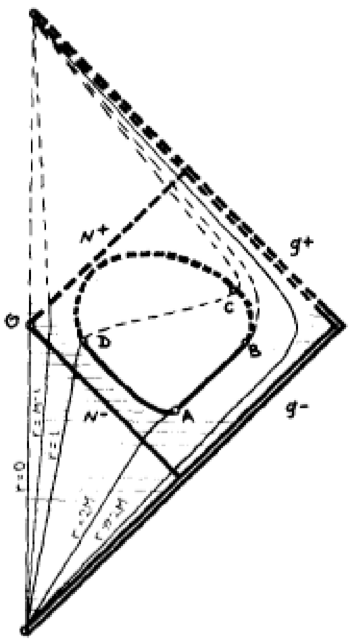


Fig. 1. Penrose diagram for the collapse of the null shell ($M \gg 1$). Solid (dashed) lines are used for the known (hypothetical) details of the picture. The shaded region is the region of validity of the obtained asymptotic solution. The line $N^- \cup N^+$ is the world line of the null shell. The closed and dashed bold line $ABCD$ is the apparent horizon. The light lines are the level lines r = const.

Static SS non-singular black-hole metrics

Non-singular $\Leftrightarrow$ regular at the center and obeying the limiting curvature property.

Remark: All stationary BH solutions in General Relativity can be written in the form, where the metric coefficients are rational functions of the coordinates.

We assume that $f = \dfrac{P_n(r)}{Q_n(r)}$, where $P_n(r)$ and $Q_n(r)$

are polynomials of the same order n, so that

$$f = \frac{r^n + \ldots}{r^n + \ldots} \to 1 \text{ at } r \to \infty.$$

Metrics with $n \leq 2$ cannot be consistent metrics of a non-singular black hole. [V.F. PR D94,104056 (2016)].

Example (n=3): Hayward metric [2007]:

$$f = 1 - \frac{2Mr^2}{r^3 + 2M\ell^2}, \quad (\alpha = 1).$$

Non-singular evaporating black hole: $M = M(v)$.

Non-singular model of a black hole

$$dS^2 = -\alpha^2 f dv^2 + 2\alpha dv dr + r^2 d\omega^2 ,$$

$$f = 1 - \frac{2Mr^2}{r^3 + 2M\ell^2} ,$$

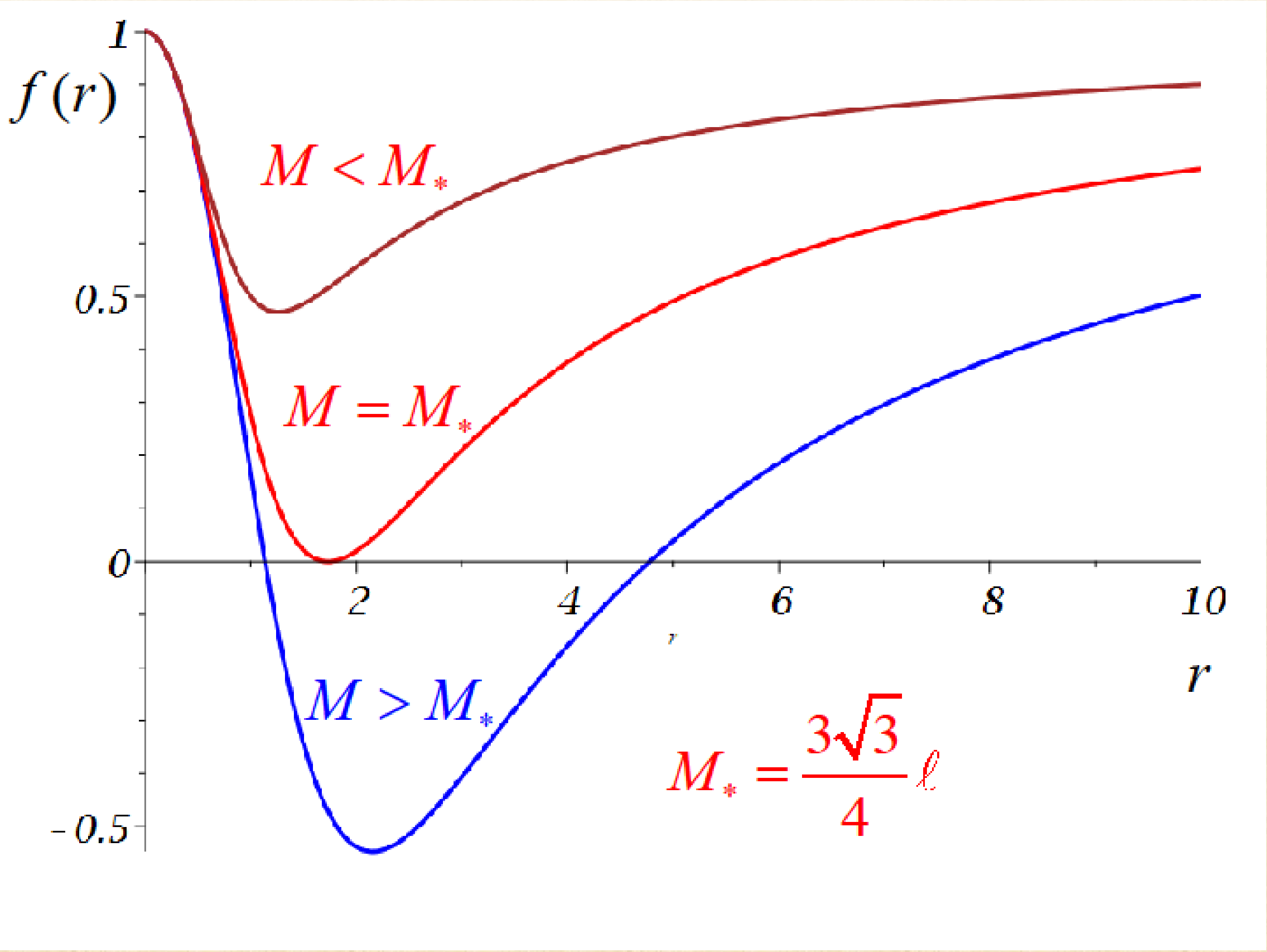
Standard: α=1;

Modified: $\alpha = \dfrac{r^n + \ell^n}{r^n + \ell^n + (2M)^k \ell^{n-k}}.$

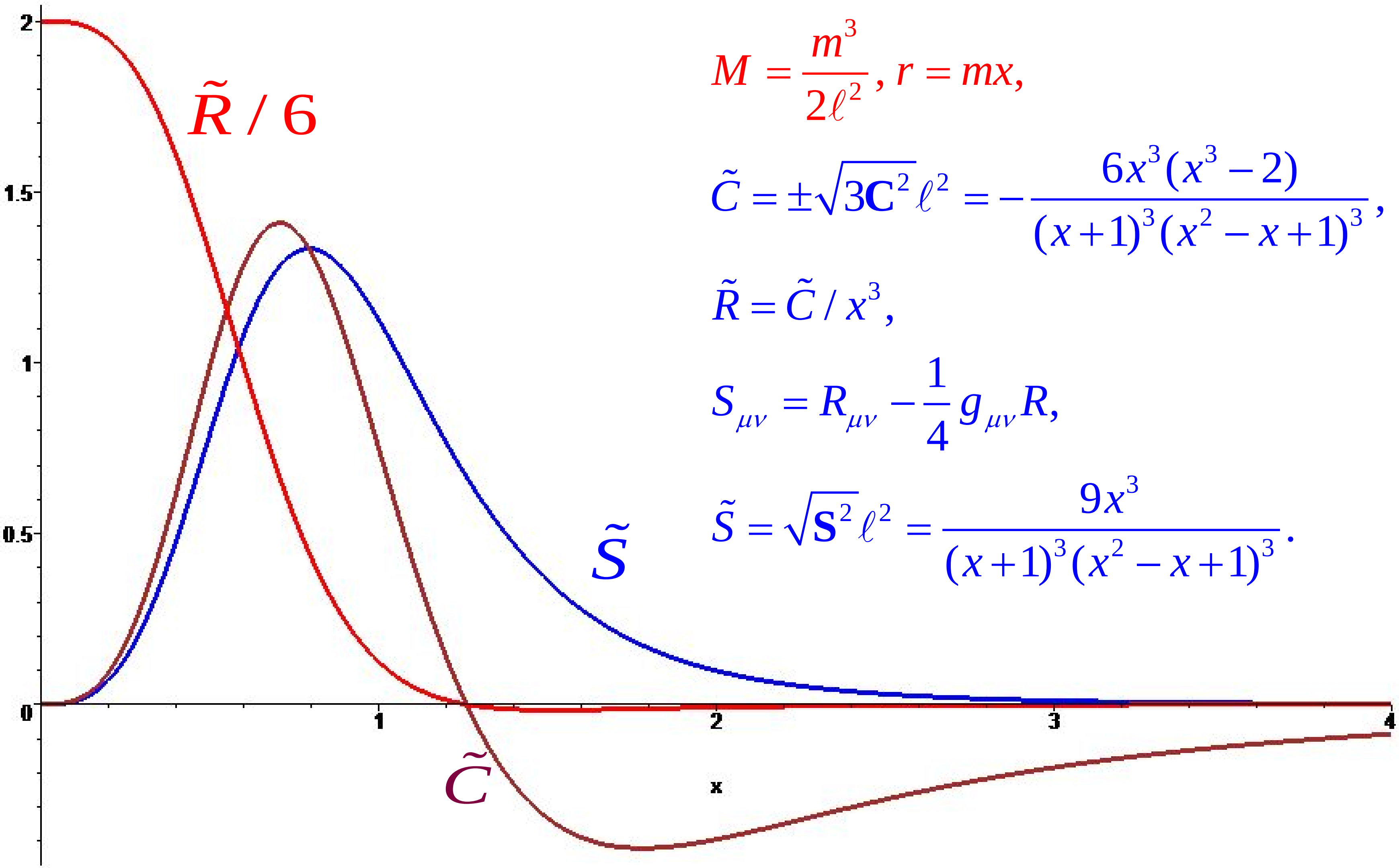
For α=1 $\Rightarrow m = (2\ell^2 M)^{1/3}, \quad r = mx,$

$$f = 1 - \frac{m^2}{\ell^2} \frac{x^2}{1+x^3}$$

$$m_* = (27/4)^{1/6}\,\ell \Leftrightarrow M_* = \frac{3\sqrt{3}}{4}\,\ell$$



Mass gap is a characteristic feature of non-singular black holes.



$$M = \frac{m^3}{2\ell^2},\ r = mx,$$

$$\tilde{C} = \pm\sqrt{3\mathbf{C}^2}\,\ell^2 = -\frac{6x^3(x^3-2)}{(x+1)^3(x^2-x+1)^3},$$

$$\tilde{R} = \tilde{C}/x^3,$$

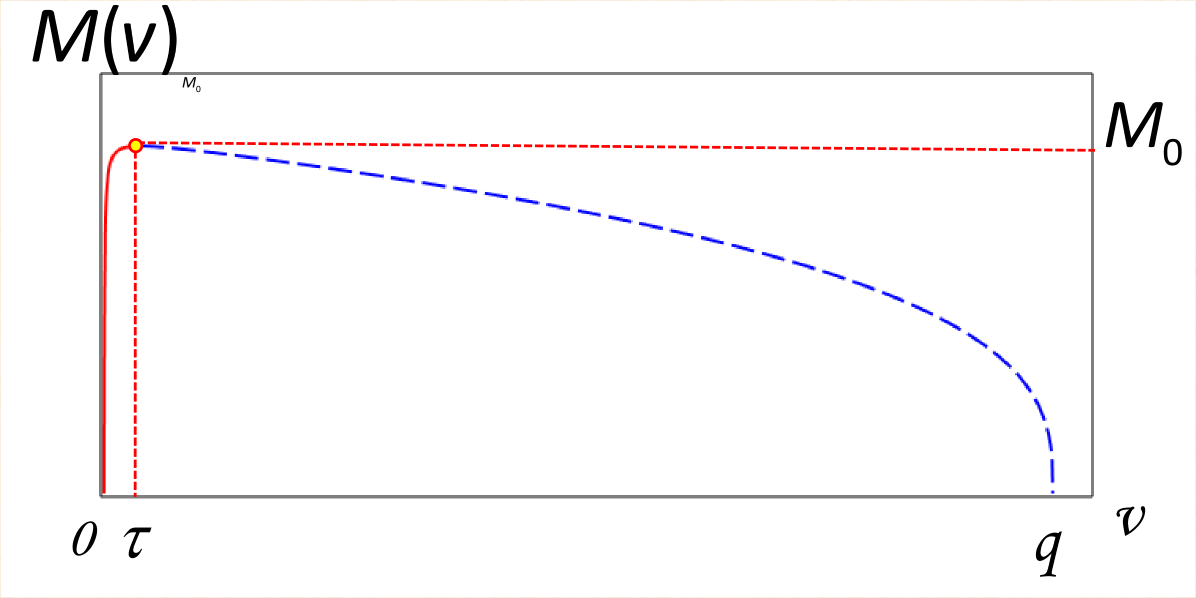
$$S_{\mu\nu} = R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R,$$

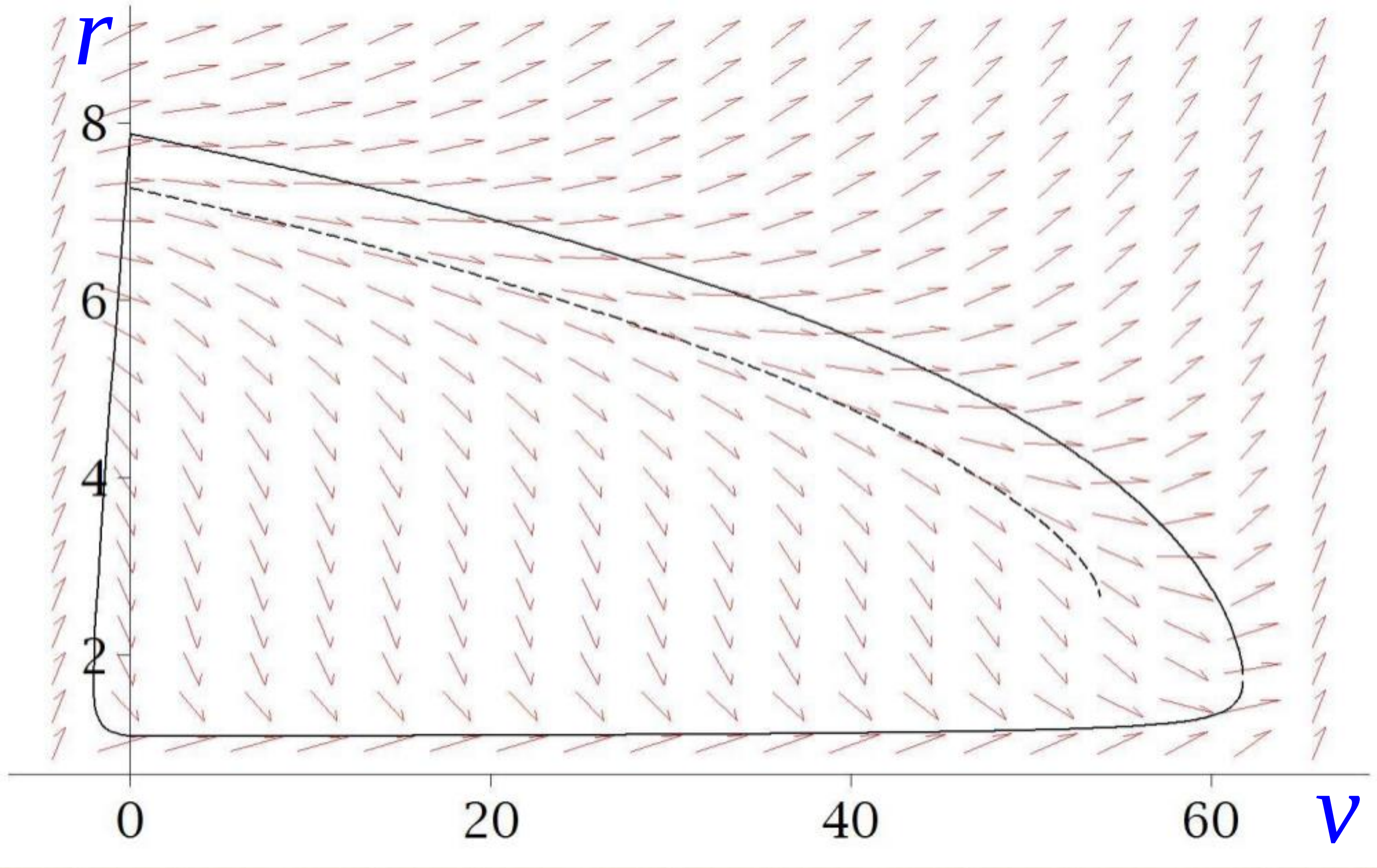
$$\tilde{S} = \sqrt{\mathbf{S}^2}\,\ell^2 = \frac{9x^3}{(x+1)^3(x^2-x+1)^3}.$$

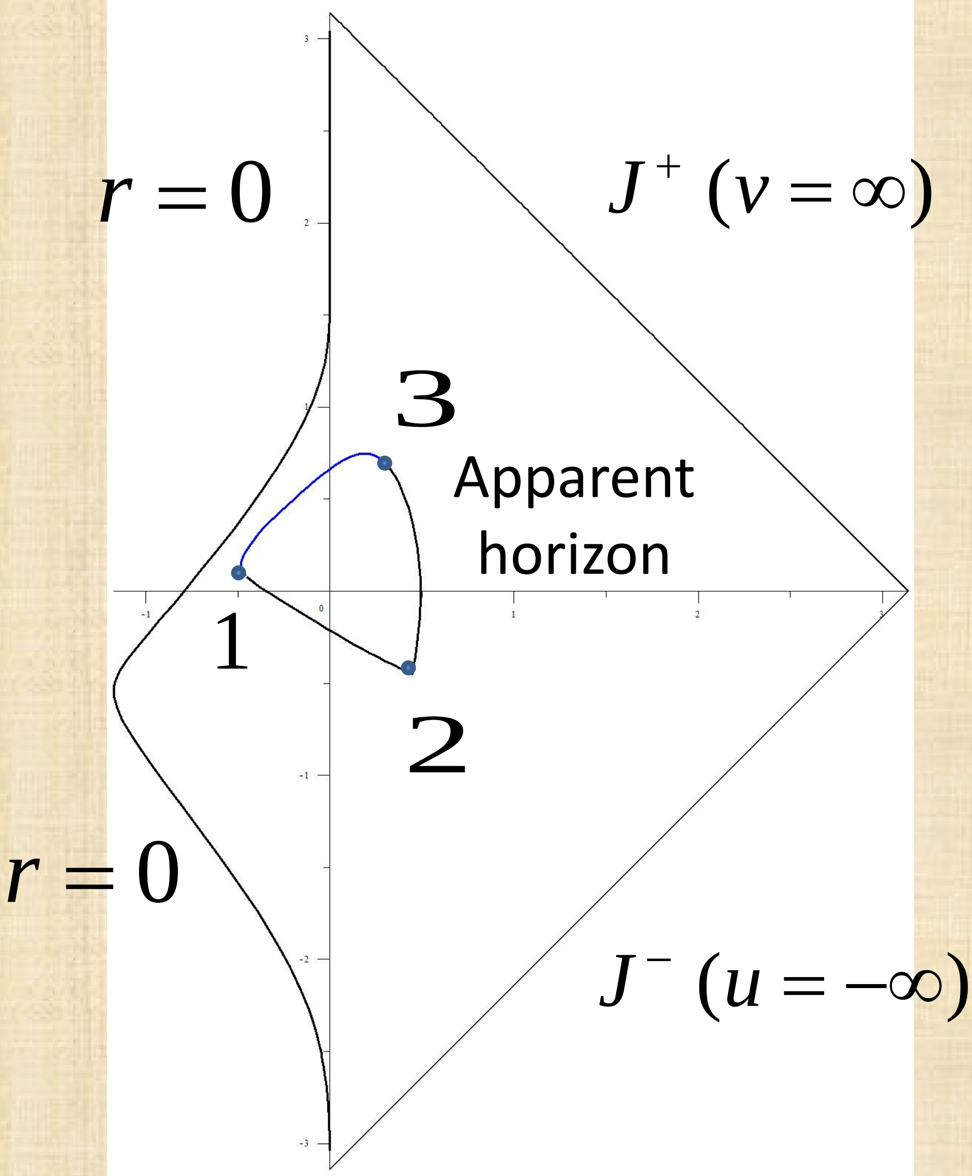
Non-singular model of an evaporating black hole

$$dS^2 = -f dv^2 + 2dvdr + r^2 d\omega^2 ,$$

$$f = 1 - \frac{2M(v)r^2}{r^3 + 2M(v)\ell^2} \quad (\alpha = 1)$$




r
8
6
4
2
0
20
40
60
v


$r=0$
$J^+\ (v=\infty)$
3
Apparent
horizon
1
2
$r=0$
$J^-\ (u=-\infty)$

Information Loss Puzzle

$$\begin{pmatrix}\text{Pure}\\ \text{state}\end{pmatrix} \overset{\text{Collapse}}{\Rightarrow} \begin{pmatrix}\text{Black}\\ \text{hole}\end{pmatrix} \overset{\text{Evaporation}}{\Rightarrow} \begin{pmatrix}\text{Thermal}\\ \text{radiation}\end{pmatrix}$$

A model of a black hole with a closed apparent horizon is one of the options that was discussed in the connection with the information loss paradox.

Aharonov, Casher, Nussinov [1987]; Carlitz, Willey [1987]; Preskill [1992]

"The final stage of the evaporation process must take a very long time," $T_{ev} \geq M^4$.

Self-consistency problem [Bolashenko and V.F. (1986)]

Quantum effects

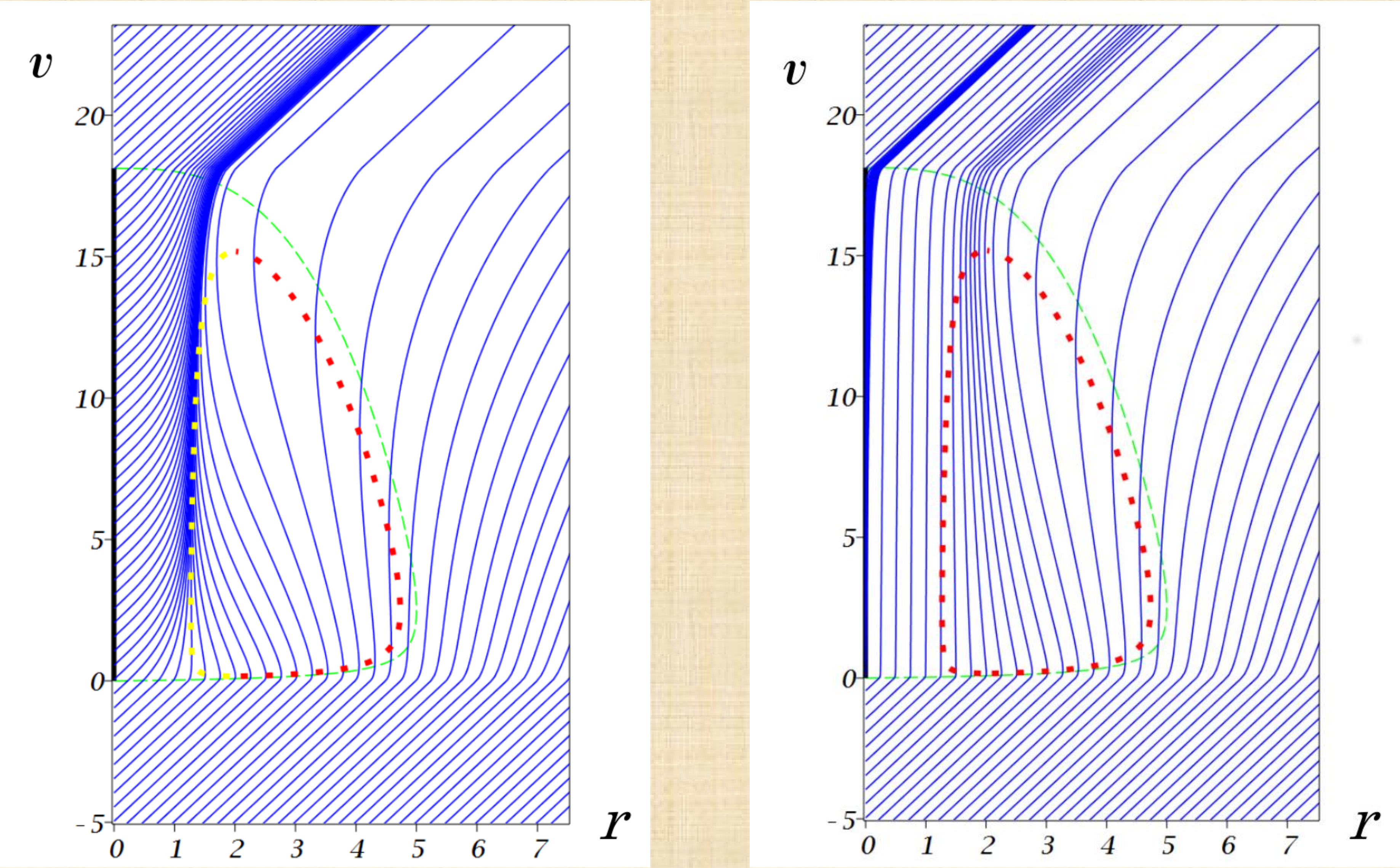
- a quantum massless scalar field, in the background of a non-singular black hole.
- 2D approximation.
- Expectation value of the stress-energy tensor can be easily obtained from the known conformal anomaly. [Christensen and Fulling, PD, D15, 2088 (1977)].
- It can also be derived from Polyakov effective action. [V.F. and Vilkovisky, in Quantum Gravity, p.267 (1984)]

Polyakov action:

$$W[g] = -\frac{b}{2}\int d^2x \sqrt{g}\, R \frac{1}{\Box} R,$$

$$\left\langle T^{\mu\nu} \right\rangle = -\frac{1}{2\sqrt{g}} \frac{\delta W[g]}{\delta g_{\mu\nu}},$$

$$\left\langle T^{\mu\nu} \right\rangle g_{\mu\nu} = 2bR.$$



$\mu_0 = 5, \quad \tau = 2.5, \quad \alpha = 1$ (left) and $\alpha = \dfrac{1+r^5}{1+r^5+\mu_0^3}$ (right)

Sequence of events (as seen by an external observer):

(i) black hole formation;

(ii) Hawking radiation;

(iii) radiation from the black hole interior;

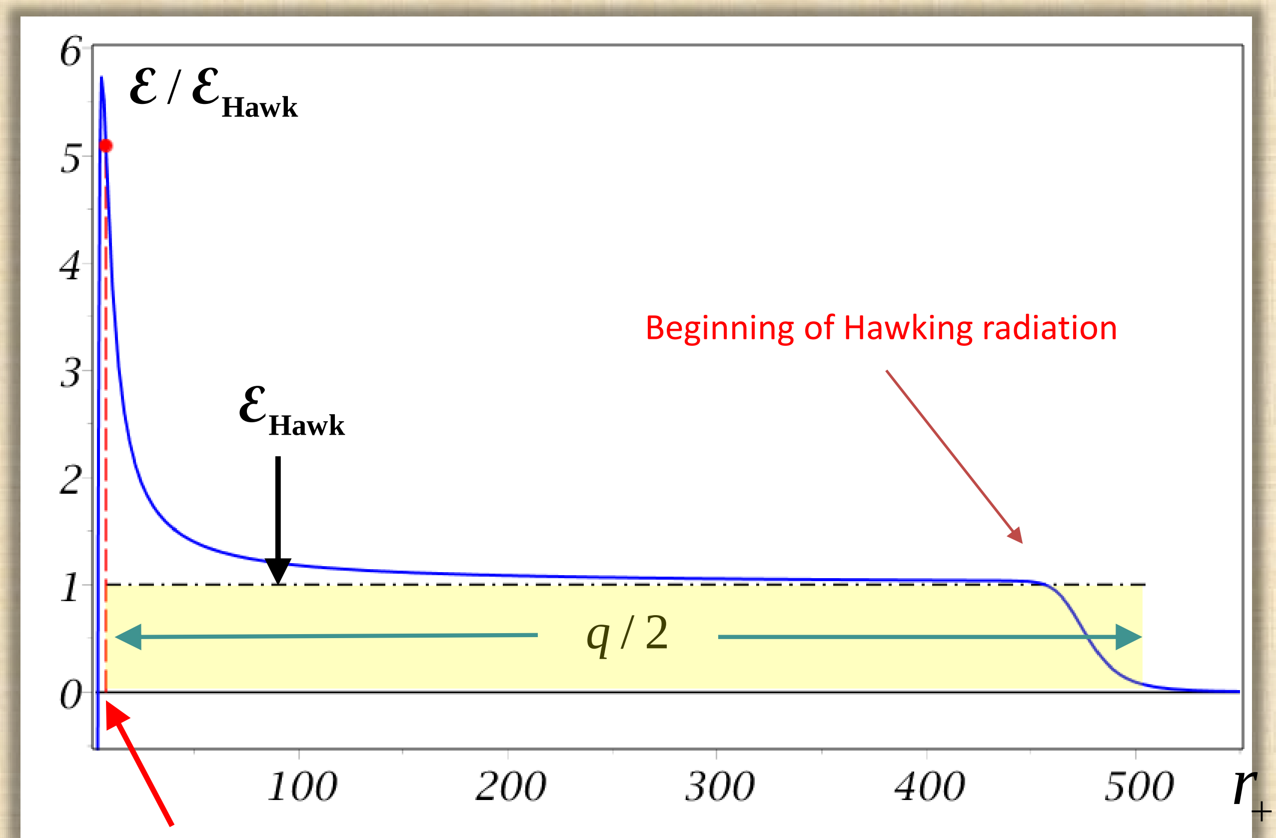
(iv) ourburst of radiation from the inner domain (near inner horizon);

(v) Mass inflation; $\kappa \sim -\alpha_0$;

(vi) Total emitted energy is always positive, its density can be negative during short time.

Hawking radiation

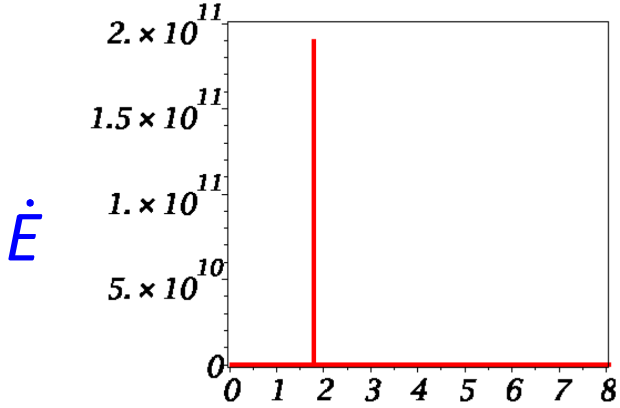
The Hawking result for the quantum energy flux from a black hole is correctly reproduced, when the mass parameter p and the lifetime of the blackhole q are large. The shape of the curve is almost the same for both standard and modified models. Duration of the almost constant tail of quantum radiation is approximately equal to q (lifetime of the black hole).



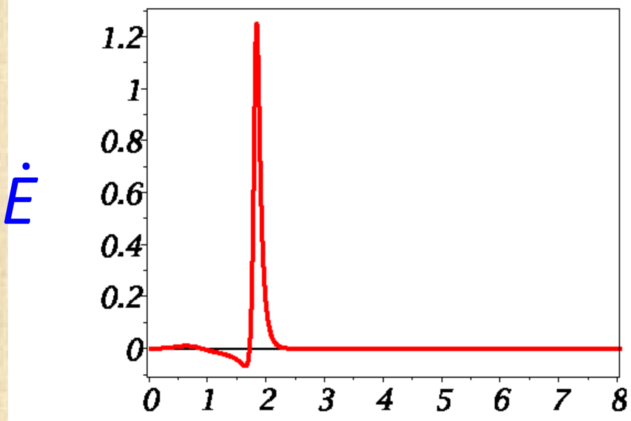
Outer horizon $(r_1 = 8)$

$q = 1000$

$$\mathcal{E}_{Hawk} = \frac{\kappa_1{}^2}{48\pi} = \frac{1}{192\,\pi\,p^2}$$



$M = 3, \;\; \alpha = 1$



$M = 3, \;\; \alpha_0 \sim M^{-3}$

Main results:

(i) Properly reproduced Hawking radiation from the oute horizon (for $M \gg \ell$);

(ii) For α=1 -- huge outburst of the quantum radiation from the inner horizon: $\Delta E \sim \exp(\Delta v/\ell)$. This radiation comes from the inner horizon during time interval $\Delta u_{+} \sim \exp(-\Delta v/\ell)$;

(iii) Mass inflation mechanism (Israel, Poisson [1990]);

(iv) For a special choice of red-shift factor ($\alpha_0 \ll 1$) outburst of the energy can be reduced to the power law;

(v) Self-consistency problem remains [Bolashenko and V.F. (1986)].

Lessons

- Mass gap for mini black-hole formation;
- Quantum effects inside a singular black holes are important;
- Hayward model is over simplified;
- Back reaction is very important, especially near the inner horizon;

- Non-locality can help, by smearing the "sharp" inner horizon structure;
- One can expect formation of a geon-type small size and mass object at the final stage of the evaporation with a very small red-shift factor α_0 at its center, slowly evaporating.
- Possible life-time $\sim M^4$ instead of M^3.