

Cosmological Relaxation: Theory and Experiment

A potential solution to the cosmological
constant problem

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Work with Peter Graham and Surjeet Rajendran

arxiv: 1902.06793 and 1709.01999

Cosmological Constant

$$\int d^4x \sqrt{-g} (M_{pl}^2 R + \mathcal{L}(\phi_{sm}, \partial\phi_{sm}) + \Lambda_0)$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{M_{pl}^2} (T_{\mu\nu} + g_{\mu\nu} \Lambda)$$

$$T_{\mu\nu} = \text{diag}(\rho, p, p, p)$$

CC is time-independent
(normal matter redshifts).

$$H^2 = \frac{1}{3M_{pl}^2} (\rho + \Lambda)$$

CC-dominated spacetime grows
exponentially fast.

$$\dot{H} = -\frac{1}{2M_{pl}^2} (\rho + p)$$

$$a = a_0 e^{Ht}$$

CC Contributions

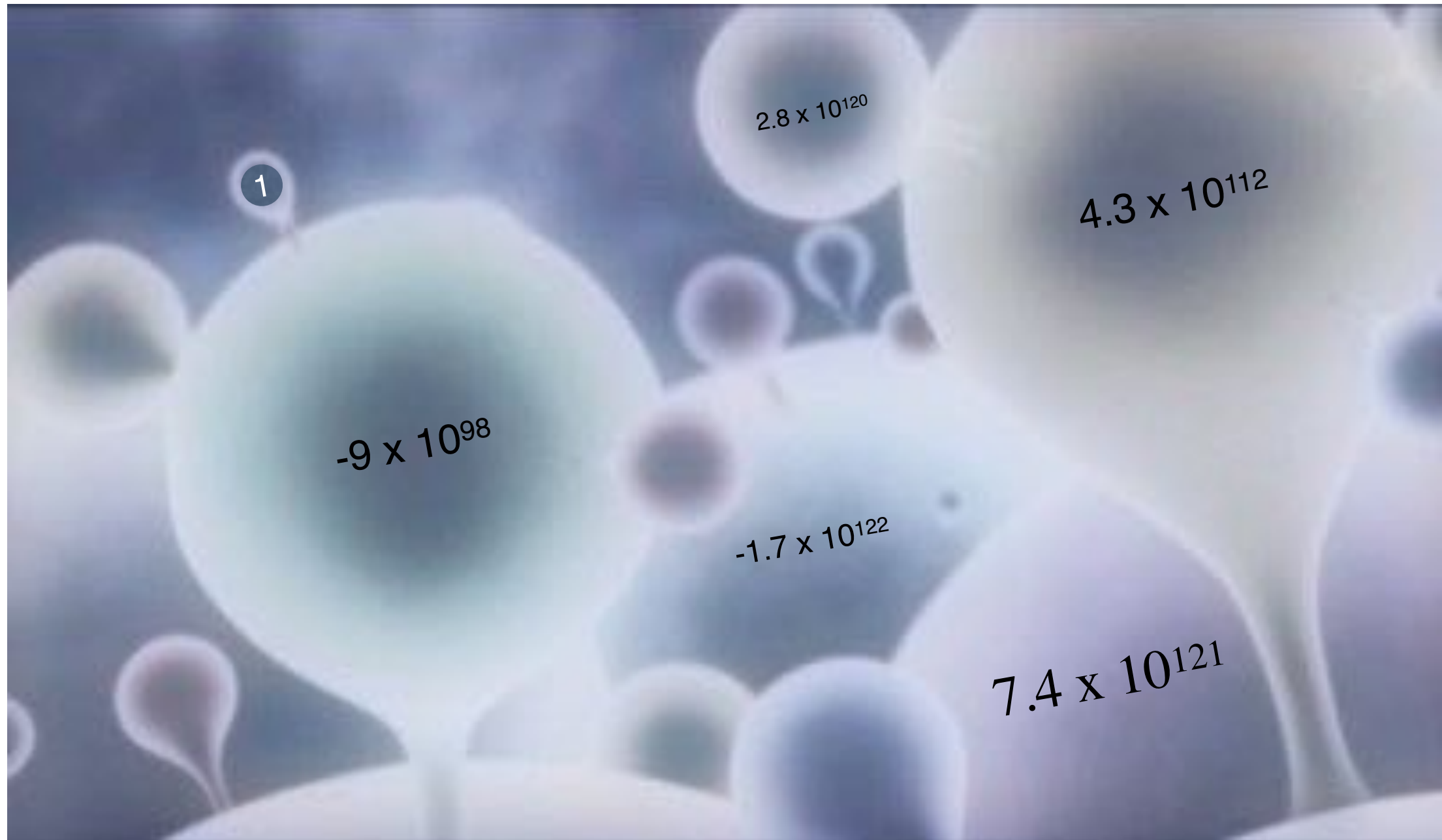
The delicateness of the **cosmological constant**.

$$\delta\Lambda \sim \begin{array}{c} \text{[Diagram 1: Circle]} + \text{[Diagram 2: Circle with wavy line]} + \text{[Diagram 3: Circle with wavy line and internal loop]} + \dots \\ + \Lambda_0 + \text{[Diagram 4: Wavy line]} + \dots \end{array}$$

Divergent and
finite
(*e.g.*, $m_e^4 \ln m_e$)

$$\text{Naive : } M_{pl}^4 \sim 10^{123} \rho_{D.E.}$$

Explanation: It's Anthropic

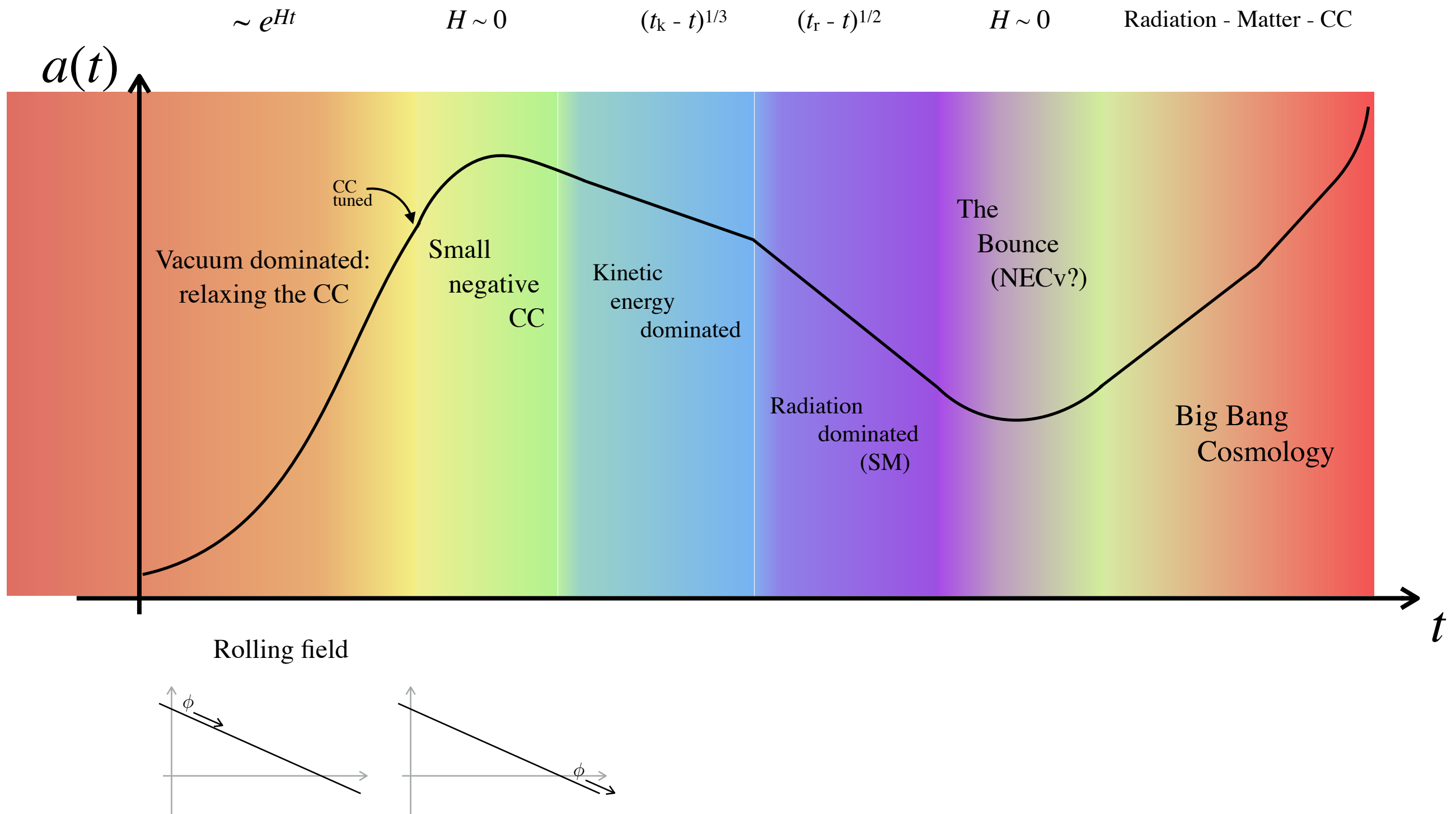


Structure only forms when CC is tiny... (assumes given $\delta q/q$)

A 'historical' solution

Our Model: Summary

Evolution of the scale factor

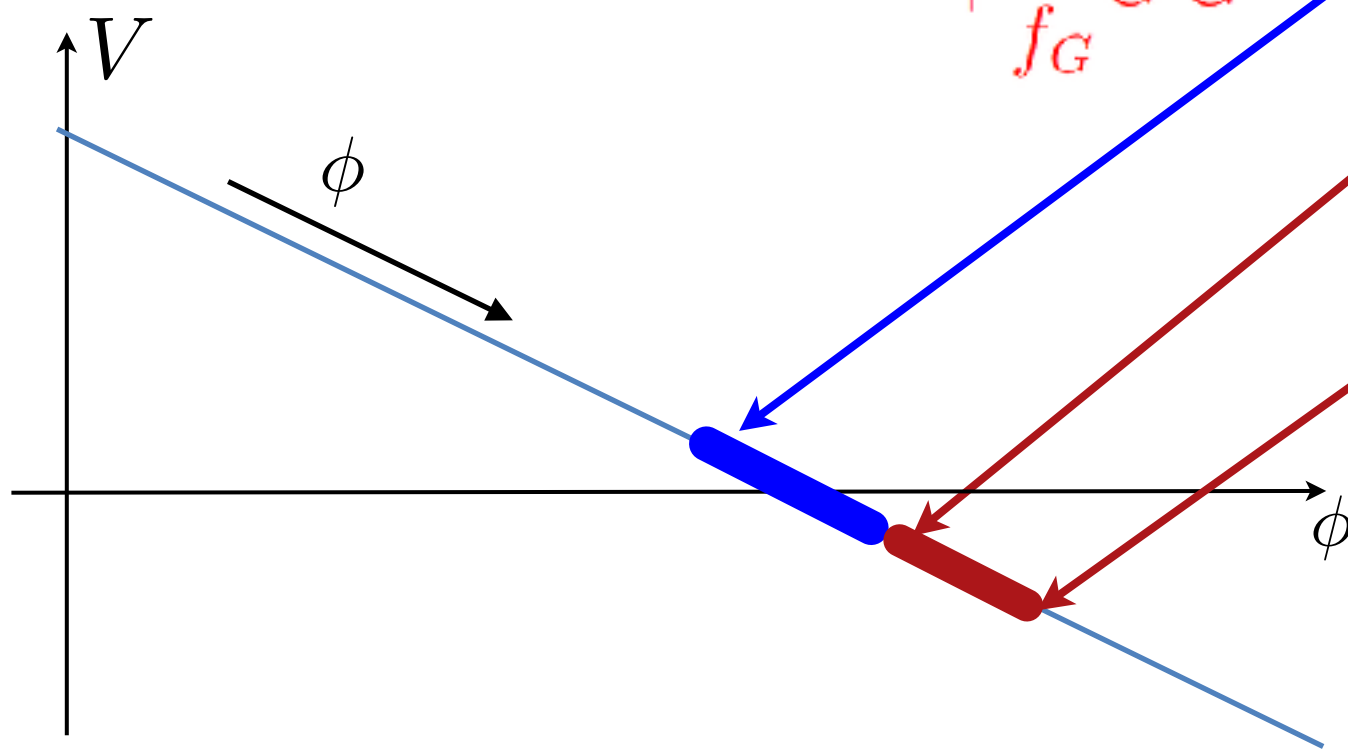


Our Model: Summary

Evolution of the rolling scalar field

Simplest Model:

$$\mathcal{L} \supset g^3 \phi + \text{arbitrary CC} + \frac{\phi}{f} F' \tilde{F}' + \frac{\phi}{f_G} G' \tilde{G}'$$



- ϕ slow rolls (extremely long time), CC drops
- Critical point: at $\phi \sim M_{\text{pl}}$, ϕ fast rolls through zero, universe starts to crunch
- Kinetic energy blue-shifts as universe crunches
- Reheating: kinetic energy converted to radiation, ϕ is stopped
- Bounce occurs (our mechanism independent of particular bounce model), regular post-inflation cosmology afterwards
- High Hubble scale freezes ϕ until today, CC fixed at small value (set by g)

CC Solution: Initial Expansion

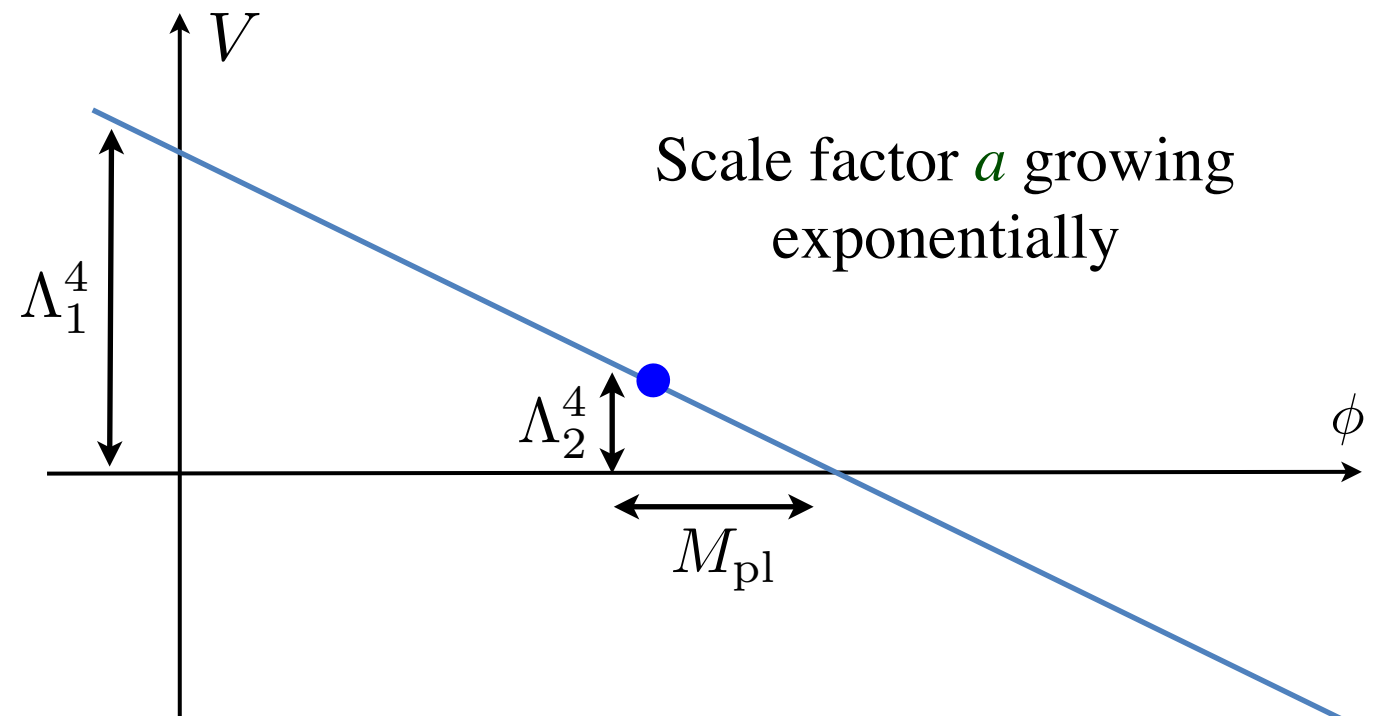
Simplest Model: $\mathcal{L} \supset g^3 \phi + \text{arbitrary CC} + \frac{\phi}{f} F' \tilde{F}' + \frac{\phi}{f_G} G' \tilde{G}'$

Avoid eternal inflation at top:

$$H^3 \lesssim V' \rightarrow g \gtrsim \frac{\Lambda_1^2}{M_{\text{pl}}}$$

Fast roll begins: $g^3 M_{\text{pl}} \sim \Lambda_2^4$

Together $\rightarrow \Lambda_1^3 \lesssim \Lambda_2^2 M_{\text{pl}}$



So to get today's CC $\Lambda_2 \sim \text{meV}$

Can solve CC problem up to $\Lambda_1 \lesssim 10 \text{ MeV}$

Dynamical relaxation first tried by Abbott (1985) and Banks (1984).
Suffered from eternal inflation and an empty universe.



CC Solution: Roll to Negative CC

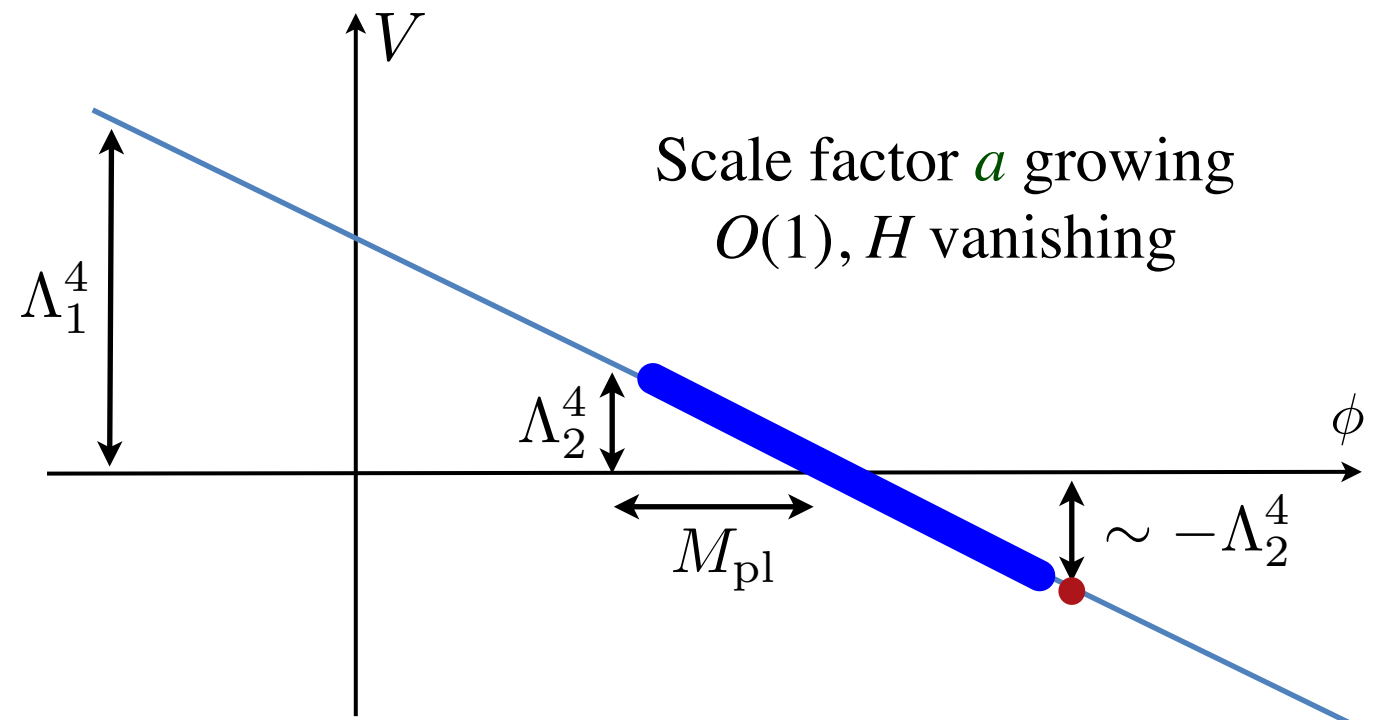
Simplest Model: $\mathcal{L} \supset g^3 \phi + \text{arbitrary CC} + \frac{\phi}{f} F' \tilde{F}' + \frac{\phi}{f_G} G' \tilde{G}'$

Avoid eternal inflation at top:

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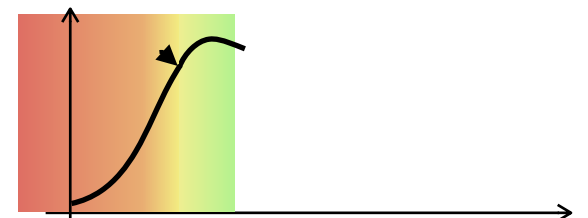


$$H^2 = \frac{1}{3M_{\text{pl}}} \left(\frac{1}{2} \dot{\phi}^2 - g^3 \phi \right)$$

$$\dot{H} = -\frac{1}{2M_{\text{pl}}} \dot{\phi}^2$$

Hubble decreasing monotonically

Vanishes in a finite roll of $\phi \sim M_{\text{pl}}$



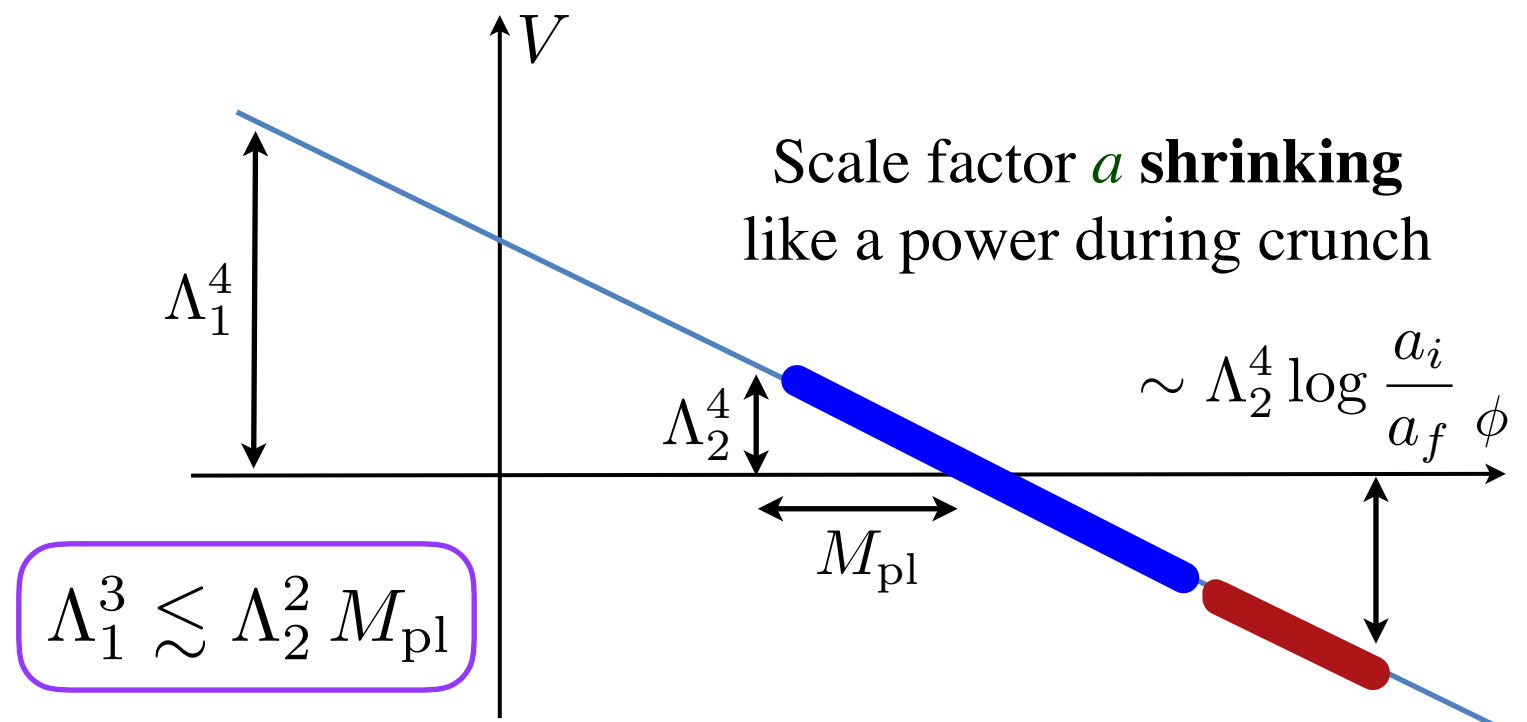
CC Solution: Crunch - K.E. Dom.

Simplest Model: $\mathcal{L} \supset g^3 \phi + \text{arbitrary CC} + \frac{\phi}{f} F' \tilde{F}' + \frac{\phi}{f_G} G' \tilde{G}'$

Avoid eternal inflation at top:

$$H^3 \lesssim V' \rightarrow g \gtrsim \frac{\Lambda_1^2}{M_{\text{pl}}}$$

Fast roll begins: $g^3 M_{\text{pl}} \sim \Lambda_2^4$

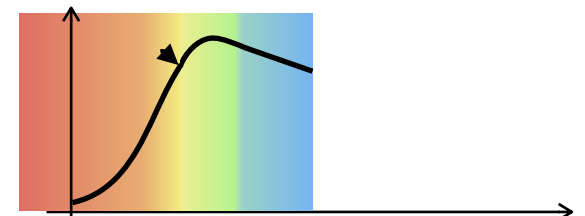


Hubble anti-friction accelerates ϕ rapidly: $\ddot{\phi} + 3H \dot{\phi} - g^3 \phi = 0$

During kinetic energy dominance: $\dot{\phi} \propto \frac{1}{a^3}$

$$\Delta\phi \sim \int dt \dot{\phi} \sim \int da \frac{\dot{\phi}}{\dot{a}} \sim \int \frac{da}{a} \frac{\dot{\phi}}{H} \sim \sqrt{3} M_{\text{pl}} \log \frac{a_i}{a_f}$$

Can crunch to extremely small scales while maintaining a small CC!



CC Solution: Stopping & Reheating

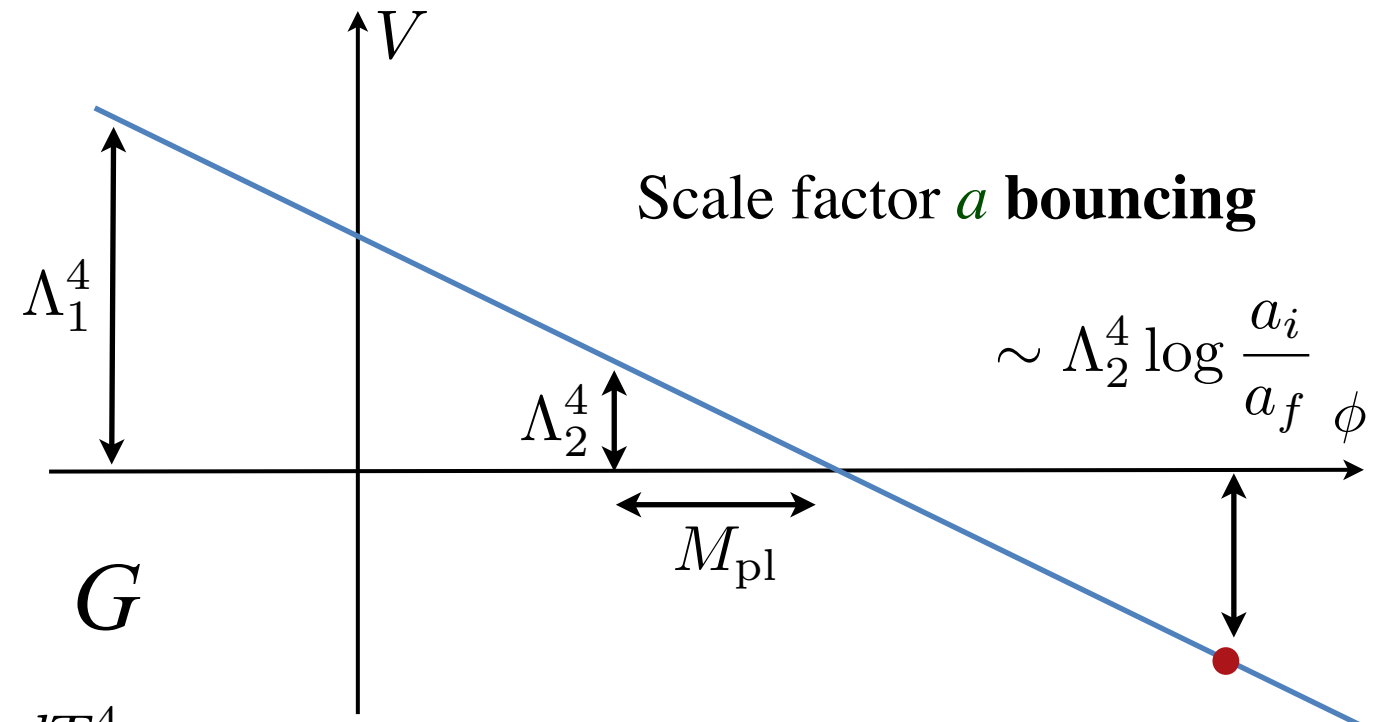
Simplest Model: $\mathcal{L} \supset g^3 \phi + \text{arbitrary CC} + \frac{\phi}{f} F' \tilde{F}' + \frac{\phi}{f_G} G' \tilde{G}' + \text{coupling btwn groups}$

Thermal bath causes extra friction term
with coupling to pure Yang-Mills
(e.g. Laine & Vuorinen 2017)

$$\Gamma_{\text{th}} \sim \frac{\alpha^3 T^3}{f^2}$$

$$\ddot{\phi} + (3H + \Gamma_{\text{th}}) \dot{\phi} - g^3 \phi = 0$$

(Take $\Gamma_{\text{th}} \gg H$)



This friction heats thermal bath further $\frac{dT^4}{dt} \sim \Gamma_{\text{th}} \dot{\phi}^2 \rightarrow \text{a runaway}$

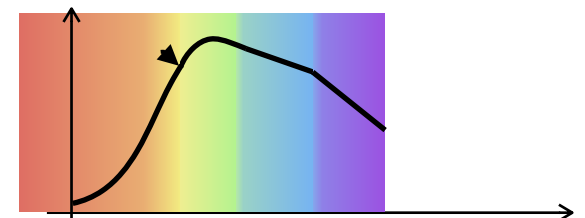
Almost all kinetic energy dumped in a time $\sim \Gamma_{\text{th}}^{-1}$ faster than Hubble time $\rightarrow f_G^2 \lesssim \alpha^3 T_{\text{reheat}} M_{\text{pl}}$

What starts the runaway? For small H , e.o.m. is $\ddot{A}'_{\pm} + \left(m_{A'}^2 + k^2 \mp \frac{\dot{\phi}}{f} k \right) A'_{\pm} = 0$ Anber & Sorbo (2009)

Once $\frac{\dot{\phi}}{f} \gtrsim m_{A'}$, then A'_+ modes become unstable

\rightarrow Coupling between groups causes reheating

all ϕ 's kinetic energy rapidly heats that sector once $\dot{\phi}$ large \rightarrow
motion of ϕ stops, CC is fixed



Reheating Details

Simplest Model: $\mathcal{L} \supset g^3 \phi + \text{arbitrary CC} + \frac{\phi}{f} F' \tilde{F}' + \bar{\psi}(\not{D} + m)\psi + \frac{m}{2} A' A'$

Last step — reheat the standard model!

Can add mixing with photon (hypercharge):

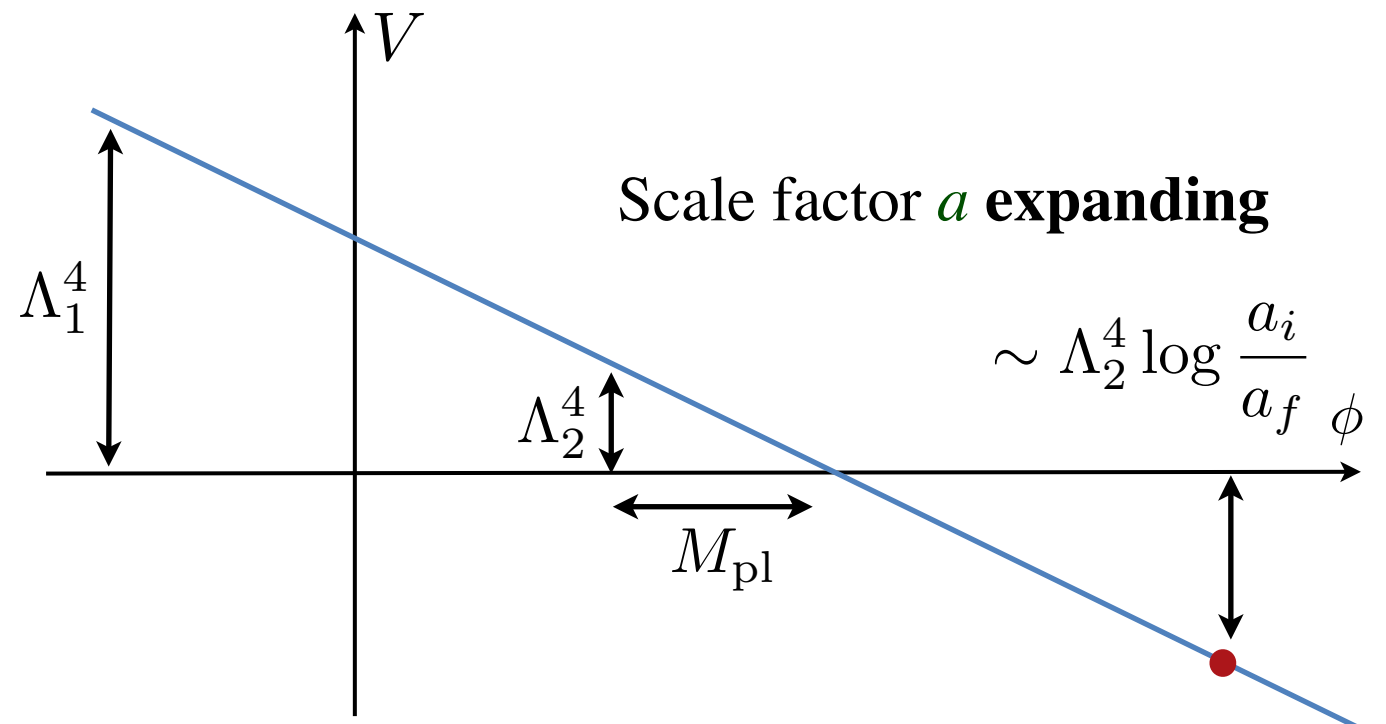
$$\mathcal{L} \supset \epsilon F'_{\mu\nu} F^{\mu\nu}$$

Will cause decays of new sector into SM with rate:

$$\Gamma_{decay} \sim \alpha \epsilon^2 m_{A'}$$

This sets the Hubble time of decay, and temperature:

$$T_d \sim \alpha^{1/2} \epsilon \sqrt{m_{A'} M_p}$$

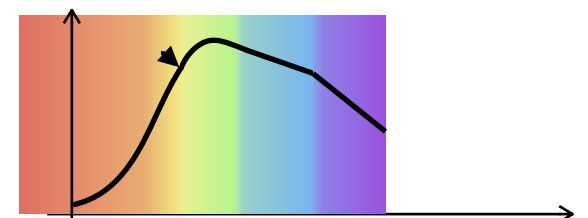


Also produces a direct coupling to photons:

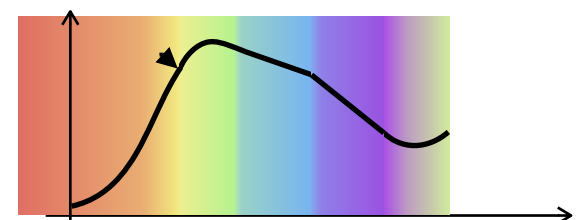
$$\epsilon^2 (\phi/f) F_{\mu\nu} \tilde{F}^{\mu\nu}$$

If $f > \epsilon^2 M_p$, then dynamics don't change.

However, possible to produce a similar story with photons directly!



The Bounce



Bouncing in Flat FRW

Example: a *flat, homogenous, isotropic* universe

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho \qquad \frac{\ddot{a}}{a} = -\frac{4\pi}{3}G(\rho + 3p)$$

$$\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 = -\frac{12\pi}{3}G(\rho + p)$$

At minimum, $\dot{a} = 0, \ddot{a} > 0 \implies \rho + p < 0$

Perfect fluid, $p = w\rho \implies \rho > 0$ and $w < -1$

or $\implies \rho < 0$ and $w > 1$

Matter must violate the Null Energy Condition

In general, $n^\mu n^\nu T_{\mu\nu} < 0$ for any null n^μ

Null Energy Condition

In general, $n^\mu n^\nu T_{\mu\nu} < 0$ for any null n^μ

Potential Issues:

Negative energy states - vacuum decay, other instabilities.

However, an effective low-energy theory may have NECv without instabilities and one may hope to avoid them in a UV completion.

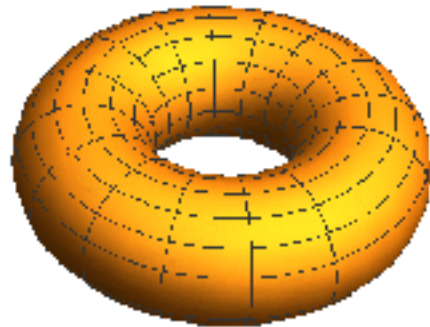
Similar issue when fluid has $w < 0$ — naively, $c_s^2 < 0$, instabilities. However, short-distance physics (*e.g.*, domain-wall networks) have additional d.o.f. and stability.

Null Energy Condition

There are known (real & effective) violations without microscopic instabilities in **compact spaces**:

Casimir in compact dimensions:

$$Mink^4 \times T^2$$



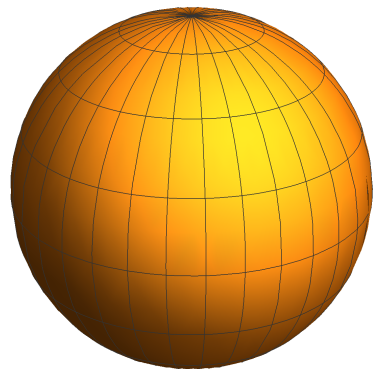
Massless fields:

$$T_{\mu\nu} = \rho \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 \end{pmatrix}$$

Positive spatial curvature:

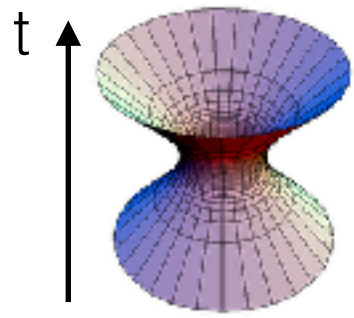
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G \left(\rho - \frac{\kappa}{a^2}\right)$$

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi}{3}G (\rho + 3p)$$

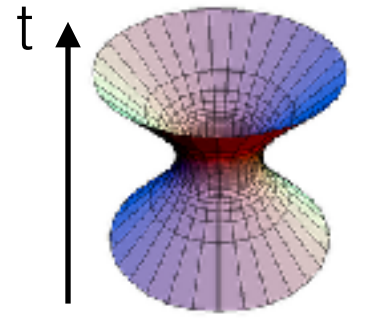


$$\left(\frac{\ddot{a}}{a}\right) - \left(\frac{\dot{a}}{a}\right)^2 = -4\pi G \left(\rho + p - \frac{2}{3} \frac{\kappa}{a^2}\right)$$

Effective NEC_v as seen from
Friedmann Eq.s



Bouncing Cosmology



Need congruence of converging geodesics to diverge.

Raychaudhuri's Equation: e.g., $\hat{\theta} = \hat{g}_{\mu\nu} \nabla^\mu U^\nu$ Congruence: U^μ

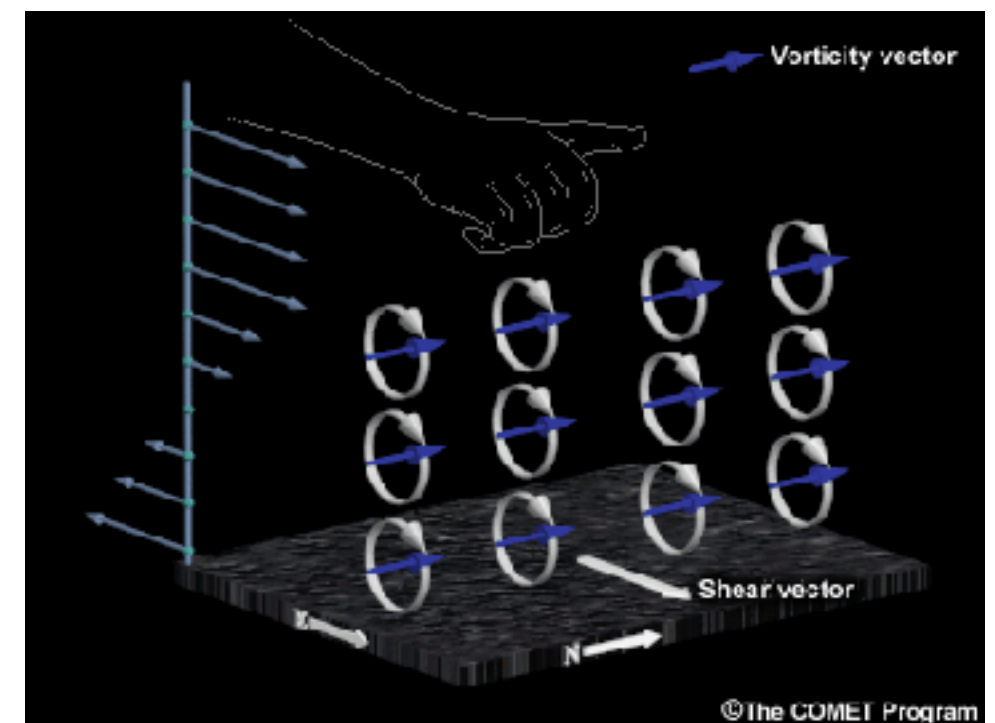
$$\frac{d\hat{\theta}}{d\lambda} = -\frac{1}{2}\hat{\theta}^2 - 2\hat{\sigma}^2 + 2\hat{\omega}^2 - 8\pi G T_{\mu\nu} U^\mu U^\nu$$

From converging to diverging geodesics requires: $\frac{d\hat{\theta}}{d\lambda} > 0$

Need $T_{\mu\nu} U^\mu U^\nu < 0$ or $\hat{\omega} \neq 0$

↓
Null Energy
Violation

↓
Vorticity

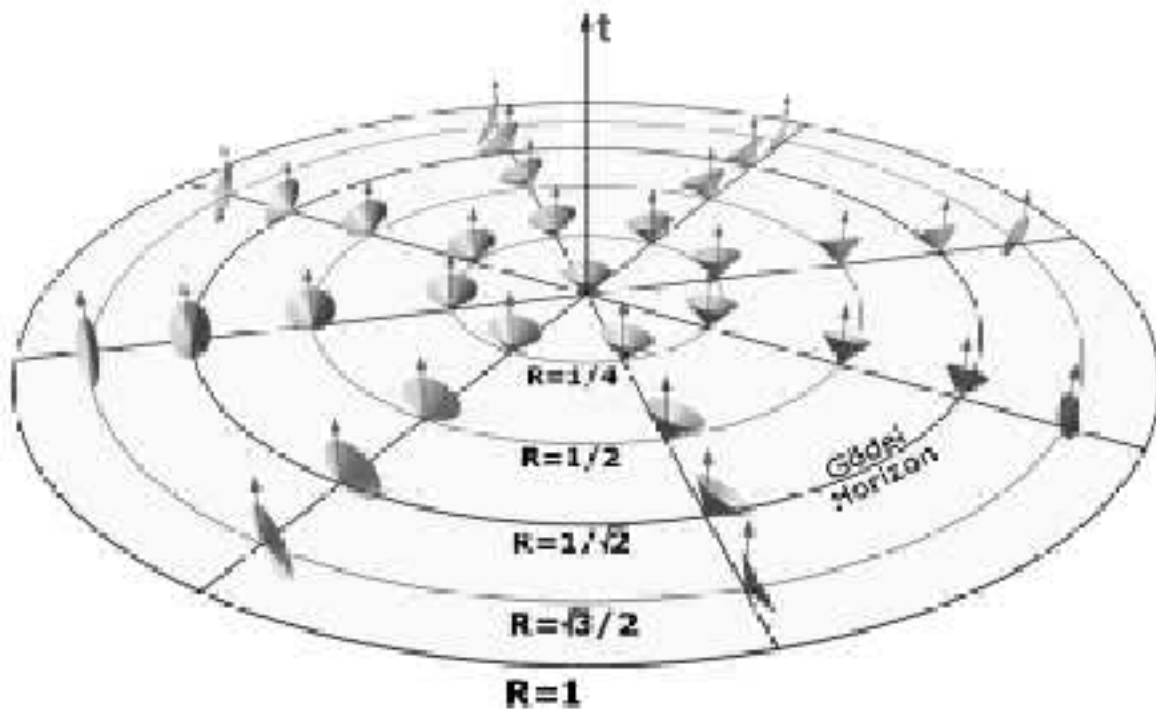


Godel Metric

$$ds^2 = \frac{2}{\omega^2} \left(-dt^2 + dr^2 + dy^2 - (\sinh^4 r - \sinh^2 r) d\phi^2 - 2\sqrt{2} \sinh^2 r d\phi dt \right)$$

Cosmological Constant +
Spinning Dust

Stationary Universe:
Gravity balanced by rotation



Closed time-like curves for $r > 1$ (not our universe)

Can we put vorticity in compact dimensions?

Metric Ansatz

Spacetime : $\sim R^4 \times T^3$

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 + L^2(d\theta^2 + d\phi_1^2 + d\phi_2^2) - 2\epsilon L(\sin\theta dt d\phi_1 + \cos\theta dt d\phi_2)$$

↓
FRW

↓
Standard
Compact

↓
Vorticity
Geodesics along
 R^4 forced to move into
extra-dimensions

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Plug in a bouncing $a(t)$ and use Einstein's Equations to get
energy-momentum tensor

The Energy-Momentum Tensor

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 + L^2(d\theta^2 + d\phi_1^2 + d\phi_2^2) - 2\epsilon L(\sin\theta dt d\phi_1 + \cos\theta dt d\phi_2)$$

to $\mathcal{O}(\epsilon^2)$:

$$T_{tt} = -M_7^5 \left(3\epsilon^2 \frac{\ddot{a}}{a} + 3(\epsilon^2 - 1) \frac{\dot{a}^2}{a^2} - \frac{3\epsilon^2}{4L^2} \right)$$

$$T_{ii} = -M_7^5 \left(-2(\epsilon^2 - 1)\ddot{a}a - (\epsilon^2 - 1)\dot{a}^2 + \frac{\epsilon^2 a^2}{4L^2} \right)$$

$i = x, y, z$

Consider 4D geodesic during bounce. NEC?

$$\text{for } L^2 \frac{\ddot{a}}{a}, \epsilon^2 \ll 1 \Rightarrow T_{tt} + \frac{T_{xx}}{a^2} \simeq M_7^5 \left(\frac{\epsilon^2}{2L^2} - 2\frac{\ddot{a}}{a} \right)$$

Vorticity combats gravity!

The Energy-Momentum Tensor

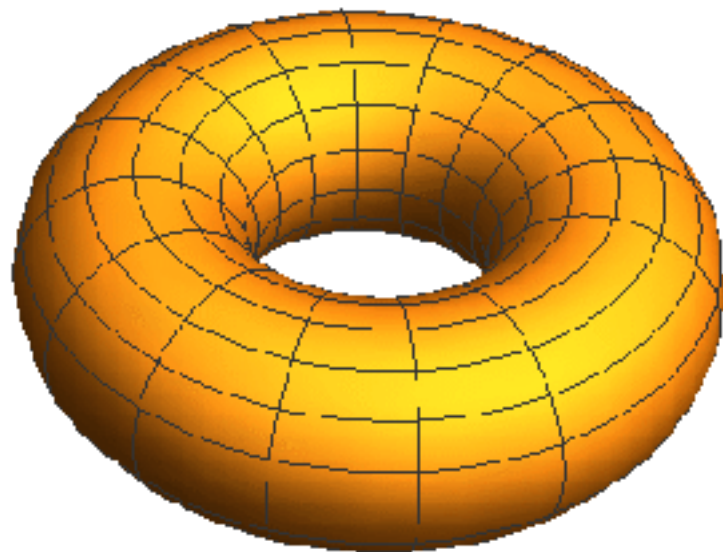
$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 + L^2(d\theta^2 + d\phi_1^2 + d\phi_2^2) - 2\epsilon L(\sin\theta dt d\phi_1 + \cos\theta dt d\phi_2)$$

However, must have NECv in compact directions.

$$e.g., \text{ for } n^\mu = (1, 0, 0, 0, 0, n_6, 0) \Rightarrow n^\mu n^\nu T_{\mu\nu} = \frac{\epsilon}{L^2} \sin\theta - 3\frac{\ddot{a}}{a}$$

with n_6 fixed by $n^\mu n^\nu g_{\mu\nu} = 0$

But we can use Casimir!



$$T_{\mu\nu} - T_{\mu\nu}^C = T_{\mu\nu}^D$$

Can show T^D can preserve the **Dominant Energy Condition**

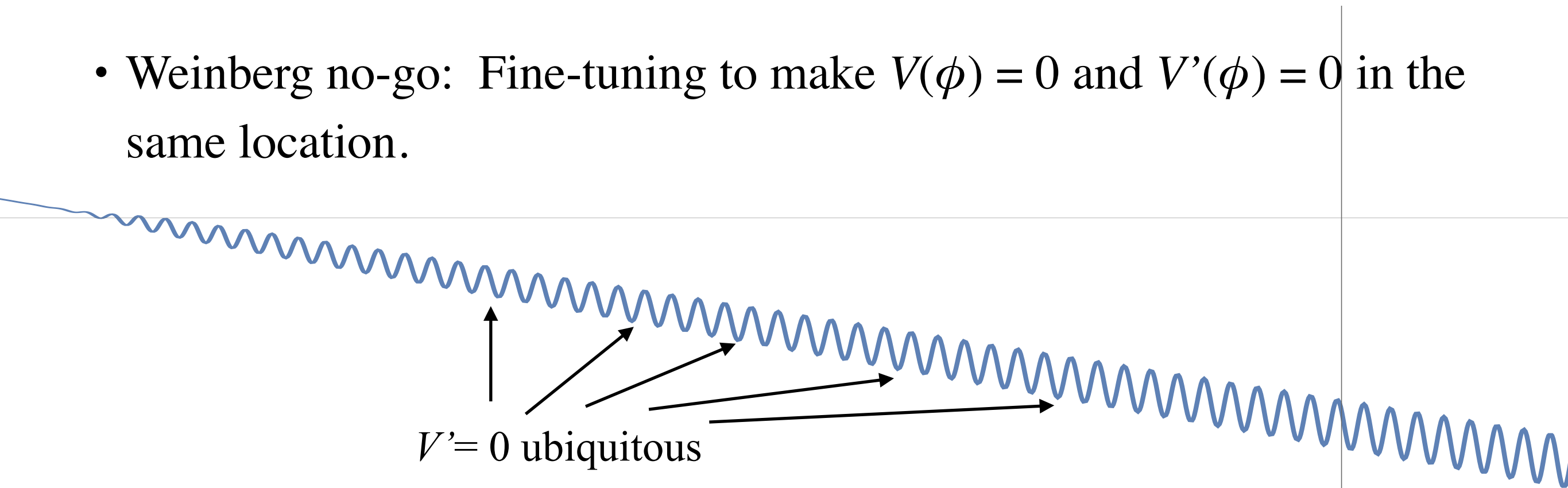
Don't know: Microphysics of T^D
Coming soon!

Why Relaxing/Bouncing for the Cosmological Constant?

So far, our model (with a bounce) naturally produces a negative cosmological constant $-(\text{meV})^4$ and big-bang cosmology from a larger, $(10 \text{ MeV})^4$

Why Relaxing/Bouncing for the Cosmological Constant?

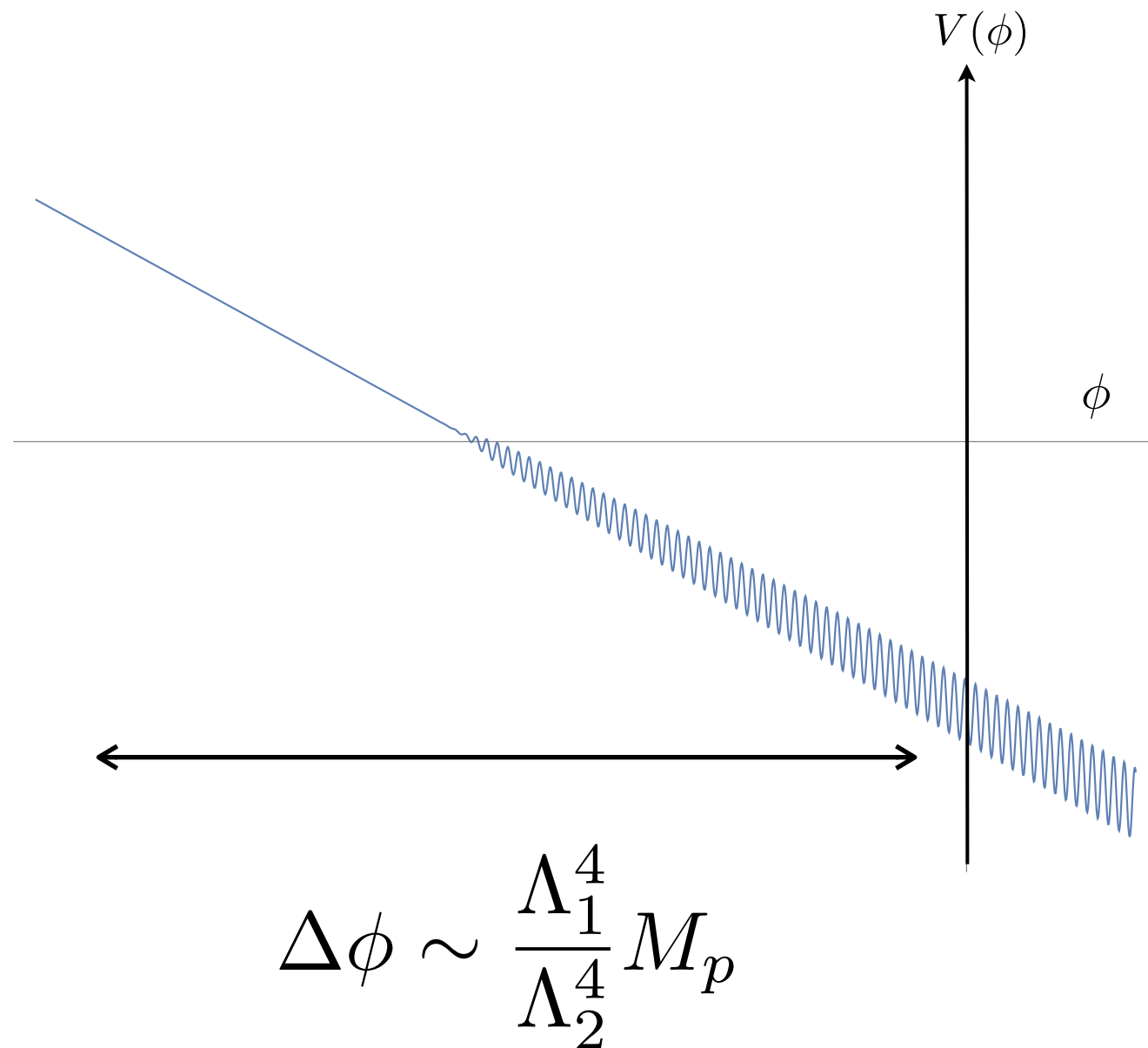
- Weinberg no-go: Fine-tuning to make $V(\phi) = 0$ and $V'(\phi) = 0$ in the same location.



- Phase transitions: If the CC is tuned in the early universe, phase transitions (QCD, EW, etc) will de-tune it.

Relaxing can tune the CC in the *Standard Model vacuum*. The bounce reheats above phase transitions and cosmic evolution brings us back to the tuned CC.

Concerns: Field Traversal



Examples in string theory suggest that traversing more than M_p brings down large numbers of states.

Looks naively bad w.r.t. the Weak Gravity Conjecture.

Effective field theories below the Planck scale, however, can reproduce such effects without large traversals in the UV

Clockwork: Choi, Im (2015),
DEK, Rattazzi (2015)

Other complaints: tiny global charges, large excursions can't be tested in our universe, ...

Aside: A Clockwork Axion

$$V(\phi) = \sum_{j=0}^N \left(-m^2 \phi_j^\dagger \phi_j + \frac{\lambda}{4} |\phi_j^\dagger \phi_j|^2 \right) + \sum_{j=0}^{N-1} \left(\epsilon \phi_j^\dagger \phi_{j+1}^3 + h.c. \right)$$

$$\phi_j \rightarrow U_j \equiv f e^{i\pi_j / (\sqrt{2}f)}$$

$$\frac{1}{2} \sum_{j=0}^N \partial_\mu \pi_j \partial^\mu \pi_j + \epsilon f^4 \sum_{j=0}^{N-1} e^{i(3\pi_{j+1} - \pi_j) / (\sqrt{2}f)} + h.c. + \dots$$

Aside: A Clockwork Axion

$$M_{ij}^2 = \epsilon f^2 \begin{pmatrix} 1 & -3 & 0 & 0 & & & \\ -3 & 10 & -3 & 0 & & & \\ 0 & -3 & 10 & -3 & & & \\ 0 & 0 & -3 & 10 & & & \\ & & & & \ddots & & \\ & & & & & \ddots & \\ & & & & & & 10 & -3 \\ & & & & & & -3 & 9 \end{pmatrix}$$

eigenvalues:

$$m_\theta^2 = \epsilon f^2 (10 - 6 \cos \theta), \quad 0 \leq \theta < 2\pi$$

plus zero mode

$$\vec{a}_{(0)}^T = \mathcal{N} \left(1 \quad \frac{1}{3} \quad \frac{1}{9} \quad \dots \quad \frac{1}{3^N} \right)$$

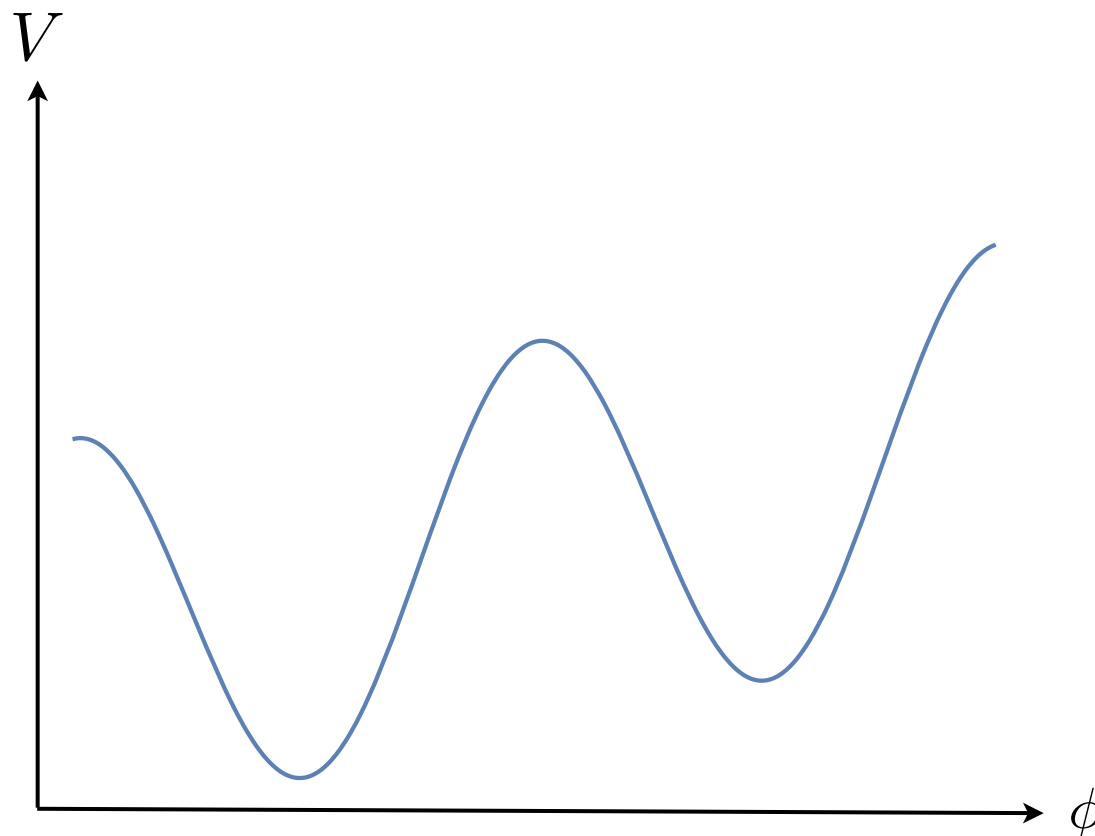
An effective large ‘ f ’ from suppressed w.f. overlap: $\sim \frac{a}{32\pi^2 (3^N f)} H^{\mu\nu} \tilde{H}_{\mu\nu}$

The Add-ons:

Making a fully phenomenologically viable model

A Positive CC Model

Add another field (e.g. an axion) with two minima split by $\sim \text{meV}$
this scale is what actually sets today's CC

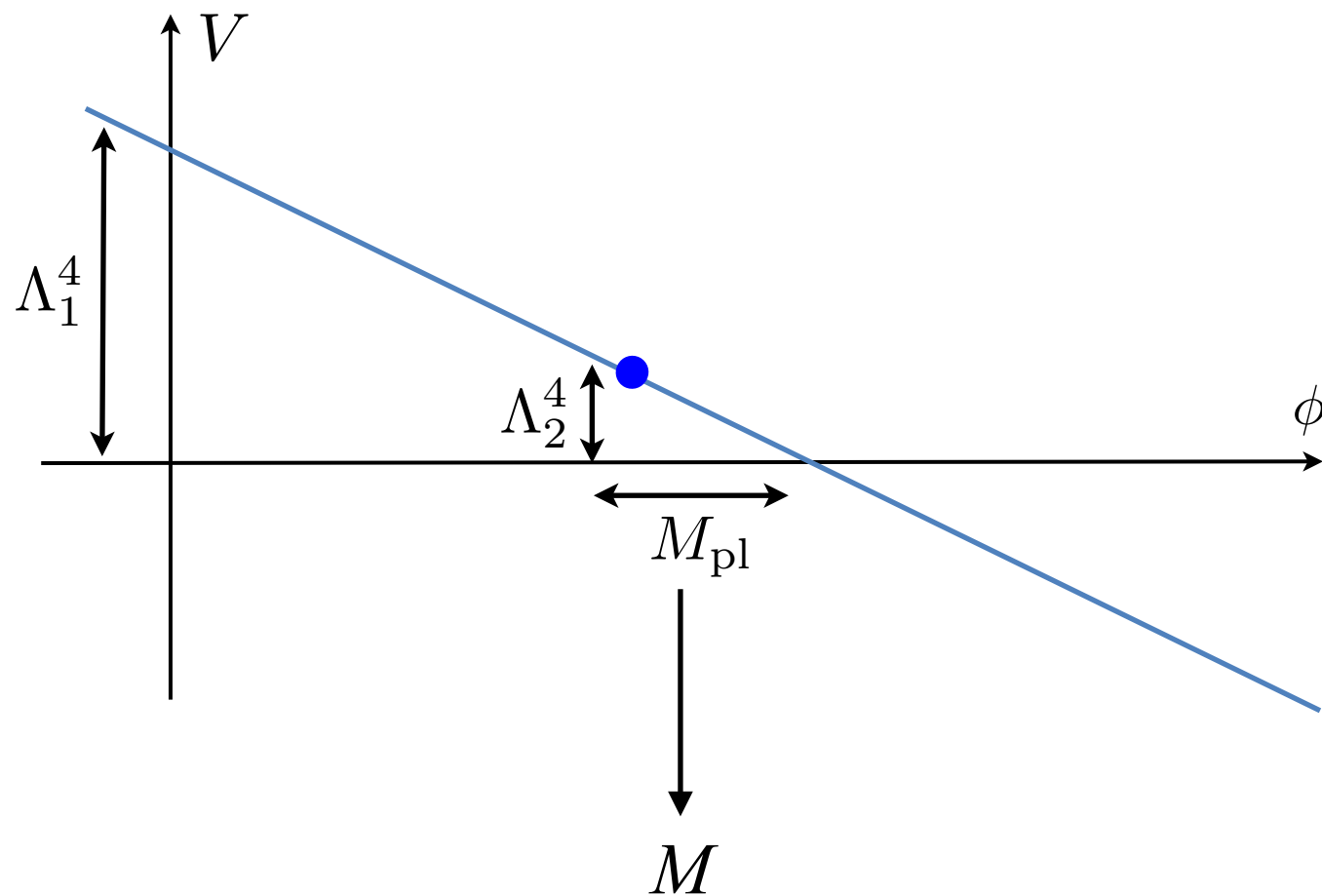


As universe heats during contraction, temperature gets arbitrarily high
resets field which can then naturally settle later in higher minima

Still preliminary: Ugly scalar field theories work, possibly confining YM

Higher CC

Must raise untuned initial CC above \sim weak scale to be consistent with LHC, etc.



$$g^3 M \sim \Lambda_2^4$$

$$\Lambda_1^3 < \Lambda_2^2 M_{\text{pl}} \sqrt{\frac{M_{\text{pl}}}{M}}$$

$$M \sim \Lambda_1 \sim 10 \text{ GeV}$$

$$H^3 \lesssim V' \rightarrow g \gtrsim \frac{\Lambda_1^2}{M_{\text{pl}}}$$

Fast roll begins: $g^3 M_{\text{pl}} \sim \Lambda_2^4$

$$\Lambda_1^3 \lesssim \Lambda_2^2 M_{\text{pl}}$$

Add $\frac{\phi}{f} F' \tilde{F}'$

For small H , $\ddot{A}'_{\pm} + \left(m_{A'}^2 + k^2 \mp \frac{\dot{\phi}}{f} k \right) A'_{\pm} = 0$

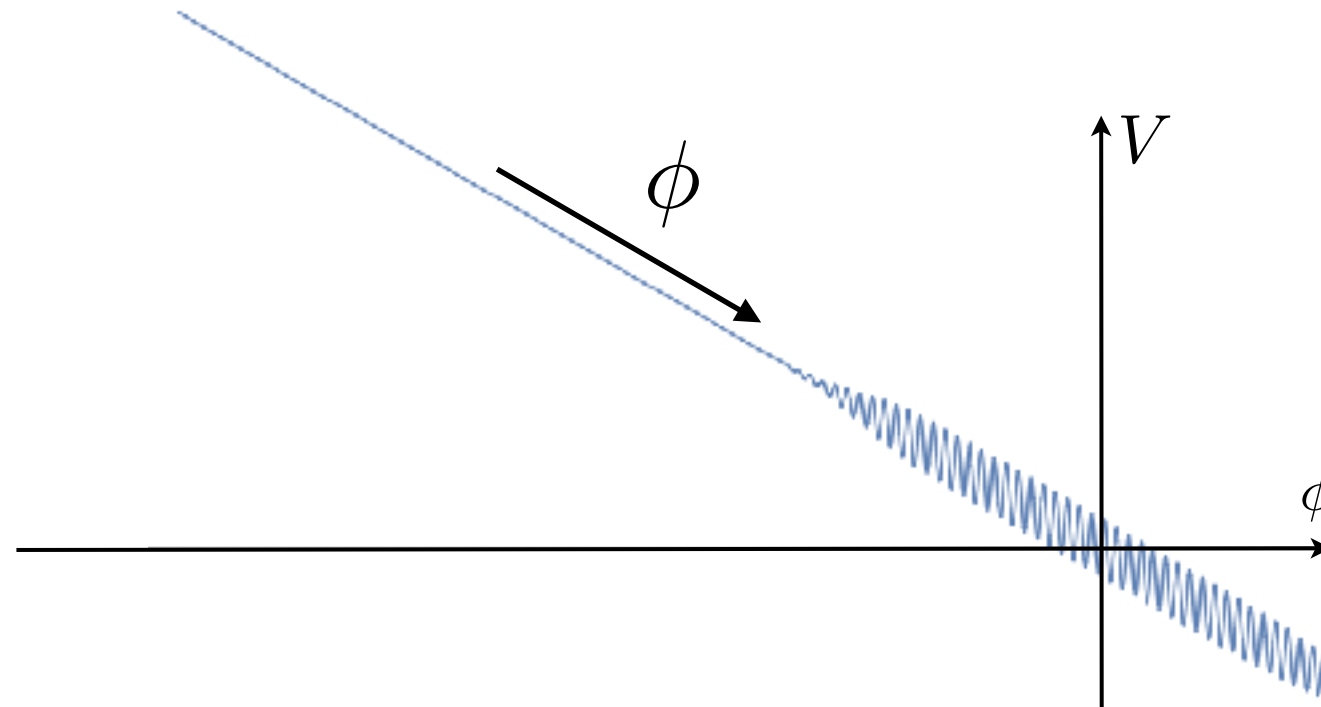
Once $\frac{\dot{\phi}}{f} \gtrsim m_{A'}$, then A'_+ modes become unstable

Anber & Sorbo (2009)

Higher Stages

Must raise untuned initial CC above \sim weak scale to be consistent with LHC, etc.

A second axion with a steeper slope



Now need to stop in an actual barrier instead of falling below zero CC

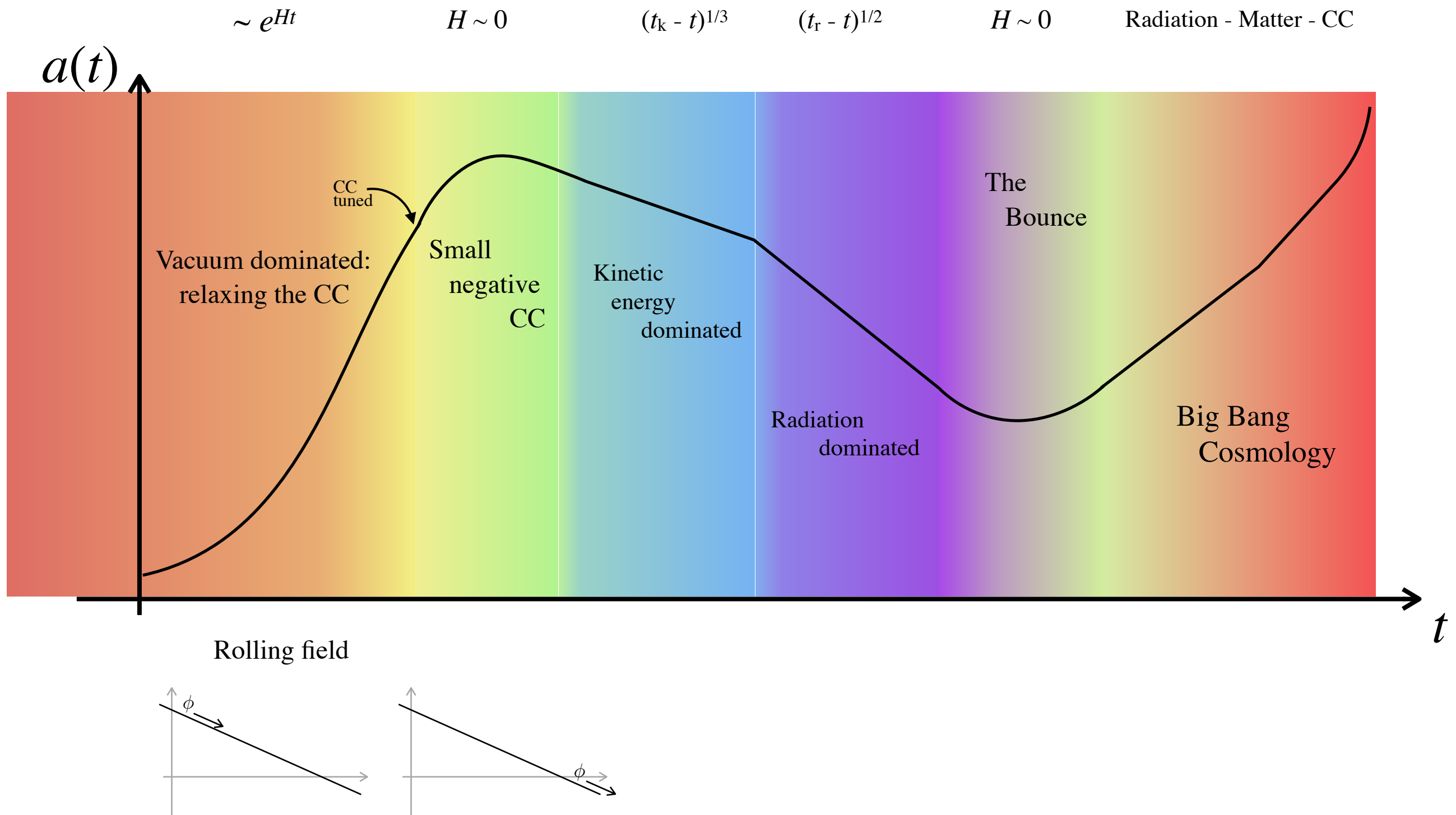
Naturally will get this stage to go first and tune CC down to at least 10 MeV

Appears to raise the CC to at least 1 TeV with just one more axion

Many stages may allow us to raise all the way to nearly M_{pl}

Cosmology

Timeline of the Universe

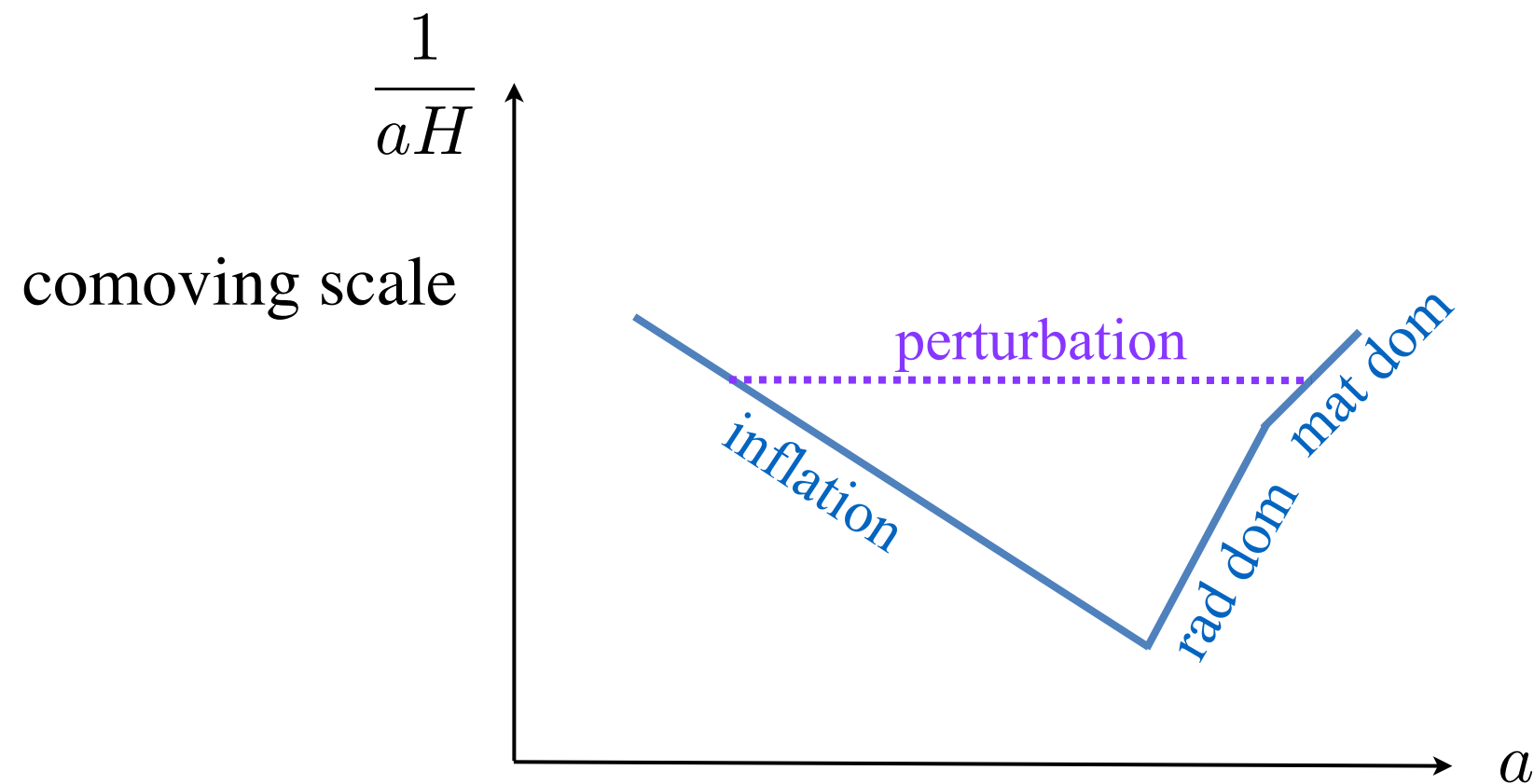


Cosmology

- CC relaxation happens during a period of slow-roll inflation. Can this be our inflation?
- This makes universe very flat, homogeneous, etc.
- But had to inflate down to $\rho \sim \text{meV}^4$ — far too low to start BBN etc.
- So need contraction and a bounce to raise energy density and “start” the universe
- Does this mess up homogeneity, flatness, etc?

Can we get perturbation spectrum right?

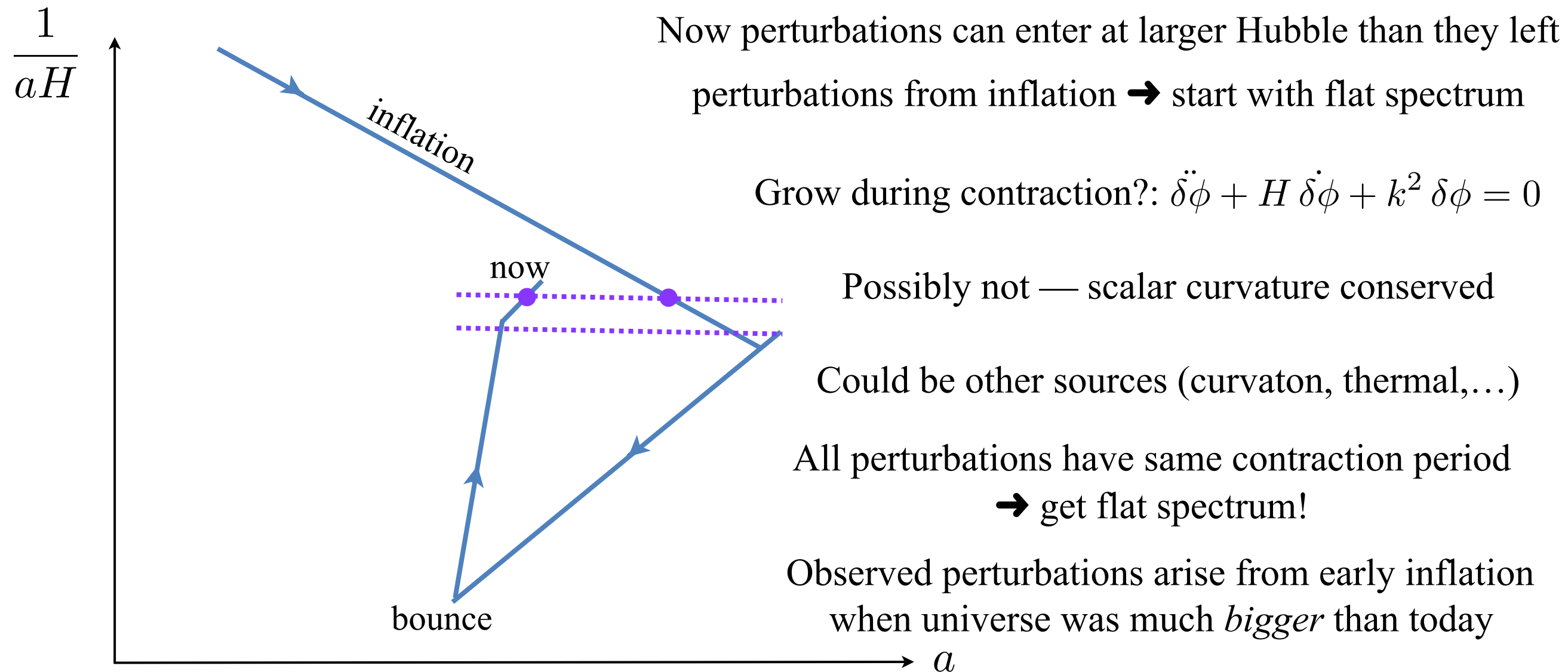
Standard Cosmology



Normally Hubble of inflation is much larger than Hubble today

Our Cosmology

Just assume expand with steeper slope than contract, then flat spectrum is automatic



So our model is: during inflation the CC drops, then gets stuck at reheating, giving today's small CC

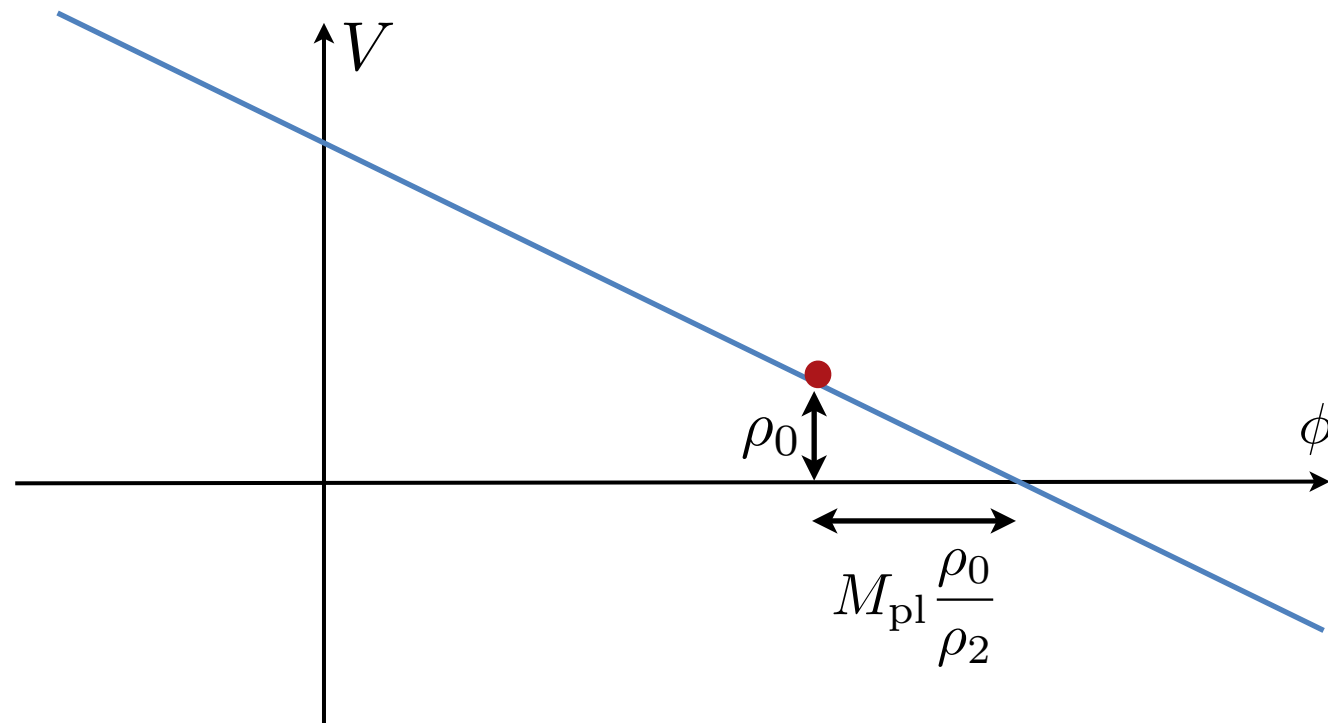
still preliminary...

Signatures

Signatures: Dark Energy E.O.S.

Rolling field should be rolling today

The equation of state, w , will depend on how much was added to make CC positive



ρ_0 = measured CC today

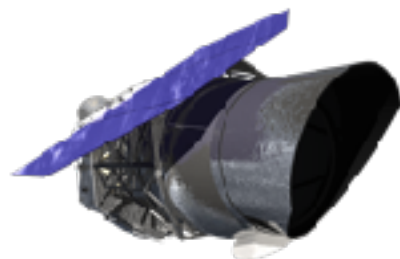
ρ_2 = what we scanned down to

$$\delta w \sim \frac{\dot{\phi}^2}{\rho_0} \sim \left(\frac{\rho_2}{\rho_0} \right)^2$$

Currently constrained at the
5% level

Many upcoming experiments will measure w better, e.g. WFIRST, Euclid, gravitational waves...

Could make a discovery, hard to rule out parameter space



Signatures: Coupling to Photons

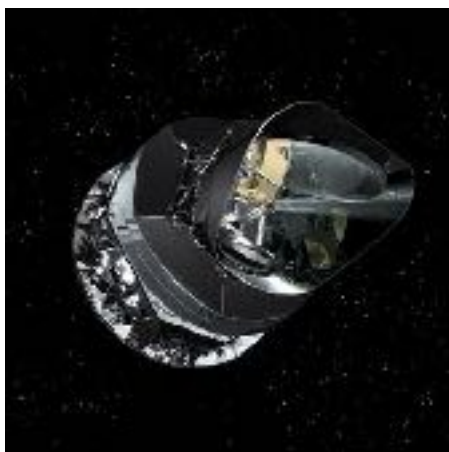
Rolling field should be rolling today

Our axion has to have non-gravitational couplings, could couple to photons $\frac{\phi}{f_\gamma} F \tilde{F}$

$$\ddot{A}_\pm + \left(k^2 \mp \frac{\dot{\phi}}{f} k \right) A_\pm = 0$$

Rolling axion \rightarrow polarization rotation of light $\Delta\theta \sim \frac{\Delta\phi}{f_\gamma} \rightarrow$ Cosmic birefringence in CMB

Current limit from Planck (and BICEP/Keck, etc): $\Delta\theta < 0.5^\circ \simeq .009 \text{ Rad}$



$$\delta w \left(\frac{M_{pl}}{f} \right)^2 < 10^{-4}$$

Future CMB polarization measurements (*e.g.*, CMB-S4, etc): $\delta w \left(\frac{M_{pl}}{f} \right)^2 < 2 \times 10^{-9} !$

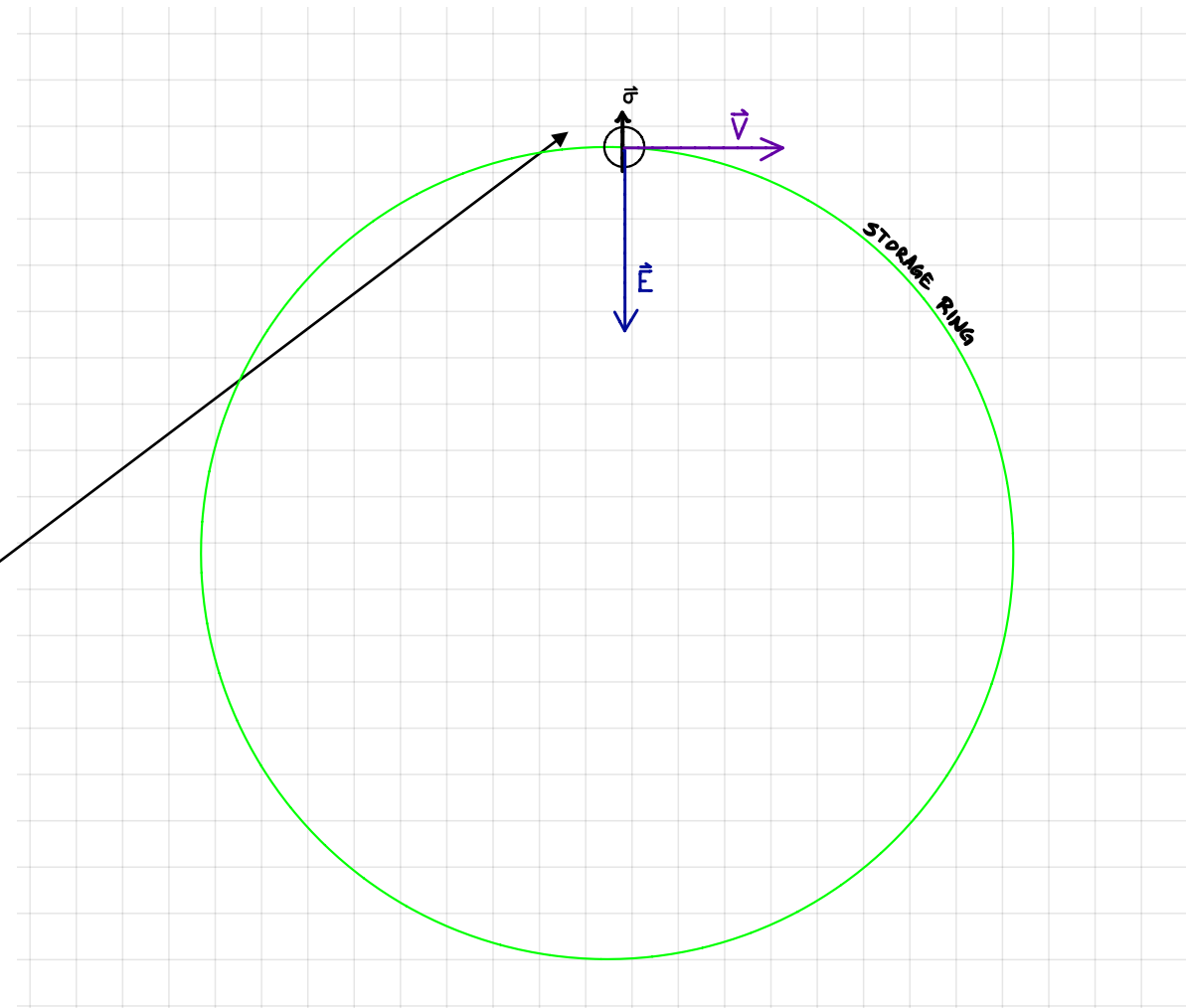
Signatures: Coupling to Fermions

ϕ has non-gravitational couplings — could couple to SM fermions $\frac{\partial_\mu \phi}{f} \bar{\psi} \gamma^\mu \gamma^5 \psi$

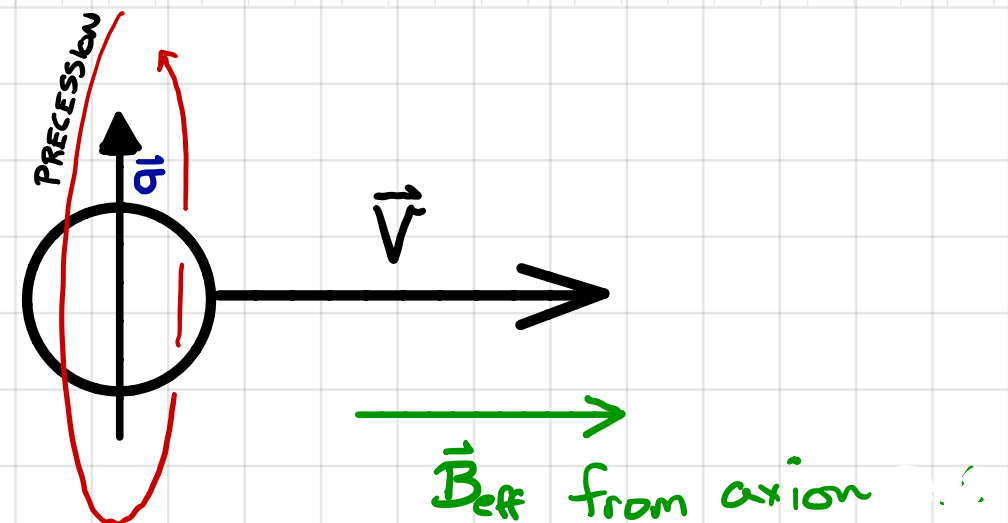
Spatial gradient of axion field \rightarrow spin precession in ‘axion wind’ $H = \frac{\vec{\nabla} \phi}{f} \cdot \vec{\sigma}_\psi$

Cosmological axion field **dominantly homogeneous**, but can use a **highly boosted** experiment!

Spin fixed to be radial at magic momentum without signal.



$\vec{\nabla} \phi$ acts as an effective magnetic field acting on the spin causing precession out of the plane



Signatures: Coupling to Fermions

Proton storage ring EDM experiment:

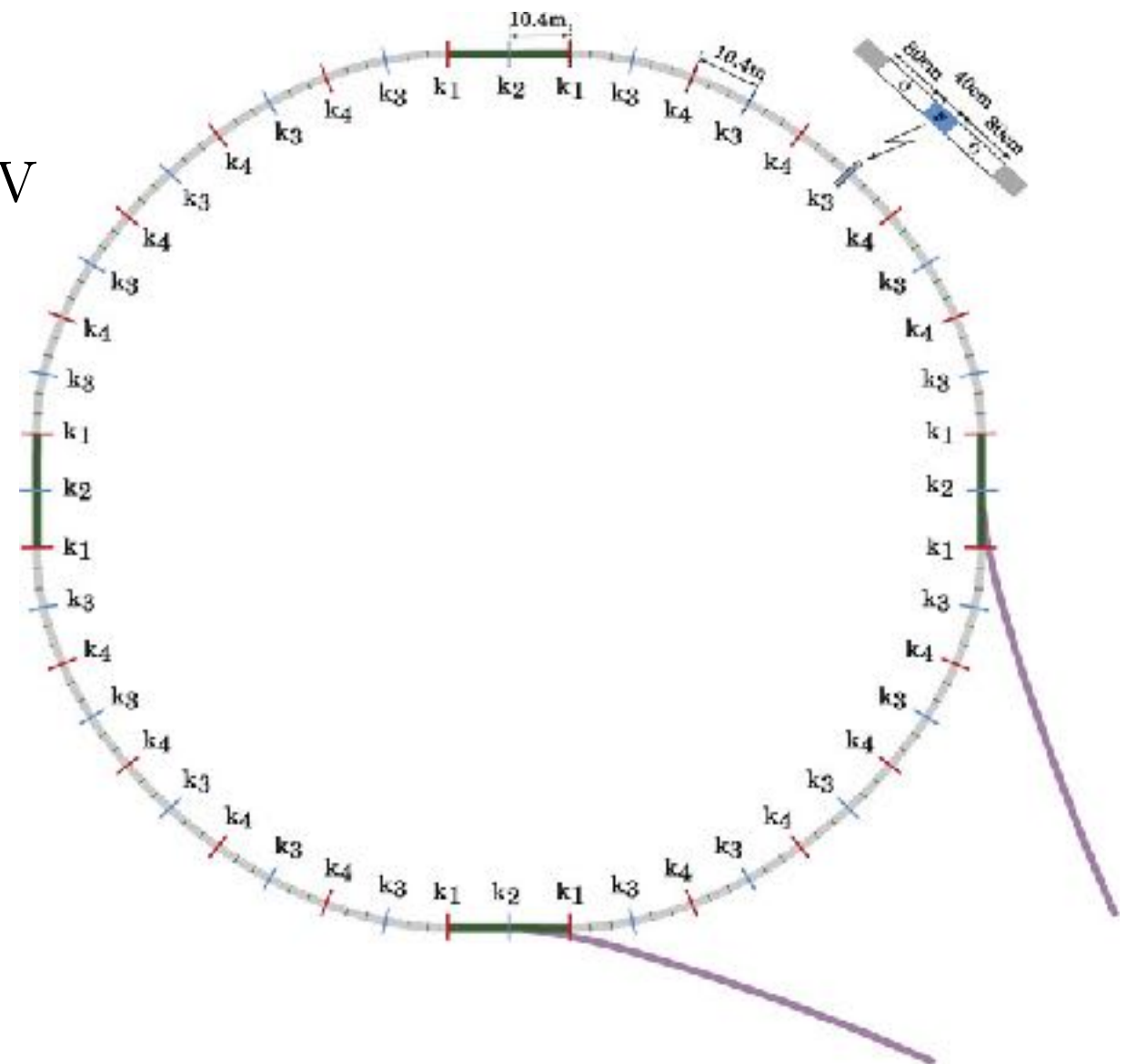
V. Anastassopoulos, et al (2016)

Stores protons for ~ 1000 s

Measure spin precession to 10^{-6} rad

Allows it to push past astro bounds $f \sim 10^9$ GeV

Appears to be the best ‘dark energy in the lab’ experiment



in progress with Yannis Semertzidis, S Haciomeroglu, Z Omarov...

Summary

Demonstrated a calculable solution to CC problem (assuming a bounce).

Only known solution with no sickness all the way up to M_{pl}

- Dynamical relaxation like a 1D landscape (avoids Weinberg's “theorem” by large field range).
- However has deterministic dynamics which drives universe to low CC, instead of anthropics

Future

- we have a calculable bounce but still have to get it in our model
- check inflation...
- better models/other ideas?
- observables — look for it!

Thank you!